Seminar
COMPASS Collaboration Meeting

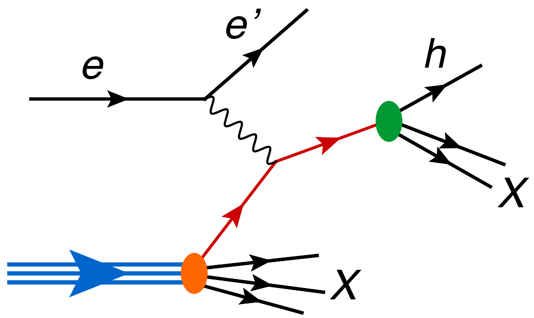
CERN, Geneva, Switzerland: August 28, 2015.

QUARK HADRONIZATION IN NJL-JET MODEL

Hrayr Matevosyan

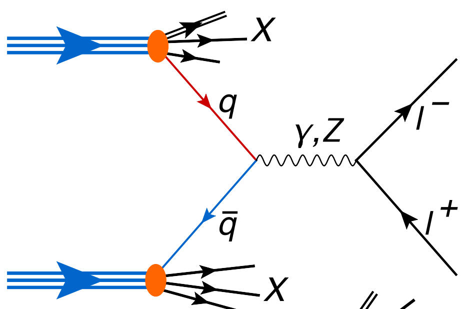
Collaborators:
A.W. Thomas, A. Kotzinian & W. Bentz

FACTORIZATION AND *UNIVERSALITY IN DIS*



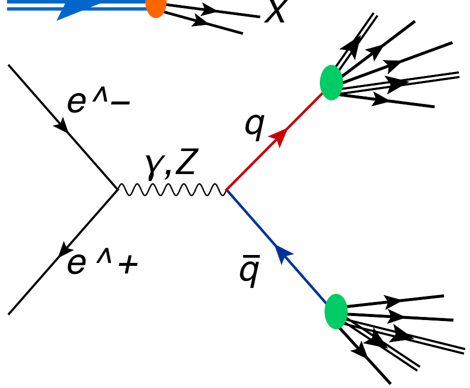
- SEMI INCLUSIVE DIS (SIDIS)

$$\sigma^{eP\to ehX} = \sum_q f_q^P \otimes \sigma^{eq\to eq} \otimes D_q^h$$



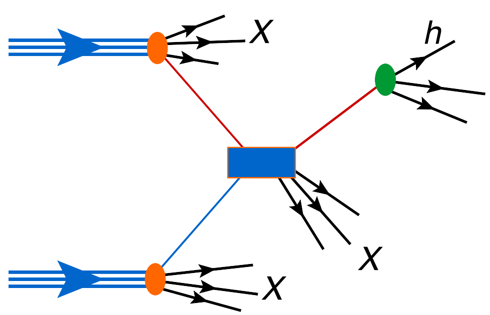
- DRELL-YAN (DY)

$$\sigma^{PP\to l^+l^-X} = \sum_{q,q'} f_q^P \otimes f_{\bar{q}}^P \otimes \sigma^{q\bar{q}\to l^+l^-}$$



- e^+e^-

$$\sigma^{e^+e^-\to hX} = \sum_q \sigma^{e^+e^-\to q\bar{q}} \otimes (D_q^h + D_{\bar{q}}^h)$$

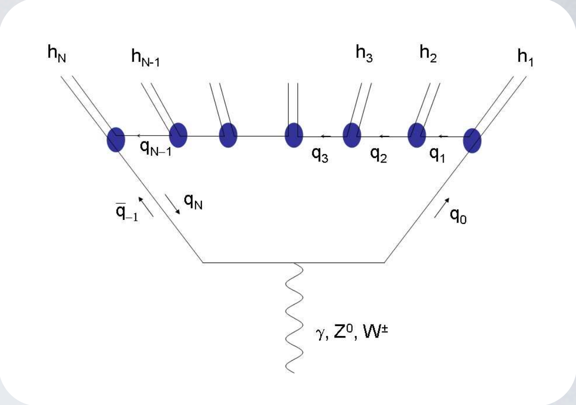


- Hadron Production

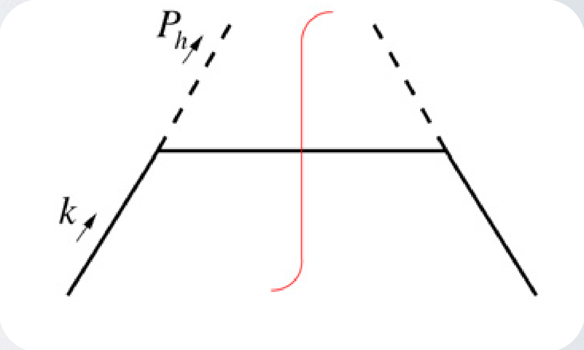
$$\sigma^{PP\to hX} = \sum_{q,q'} f_q^P \otimes f_{q'}^P \otimes \sigma^{qq'\to qq'} \otimes D_q^h$$

(SOME of the) MODELS FOR FRAGMENTATION

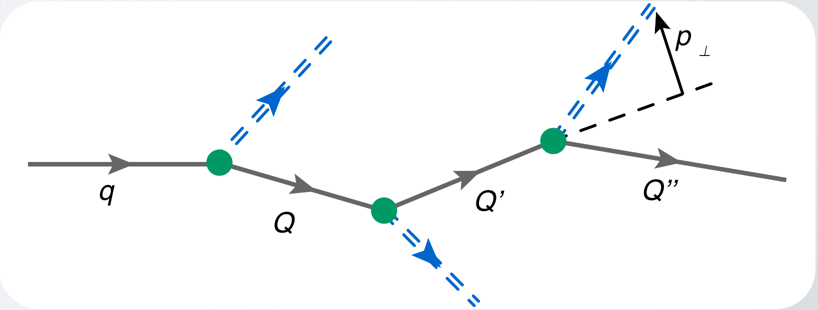
- *Lund String Model*
  - Very Successful implementation in *JETSET, PYTHIA*.
  - Highly Tunable - Limited Predictive Power.
  - No Spin Effects - Formal developments by X. Artru et al but no quantitative results!



- *Spectator Model*
  - Quark model calculations with empirical form factors.
  - No unfavored fragmentations.
  - Need to tune parameters for small z dependence.



- *NJL-jet Model*
  - *Multi-hadron* emission framework with effective quark model input.
  - *Monte-Carlo framework* allows flexibility in including the transverse momentum, spin effects, two-hadron correlations, etc.



MOTIVATION

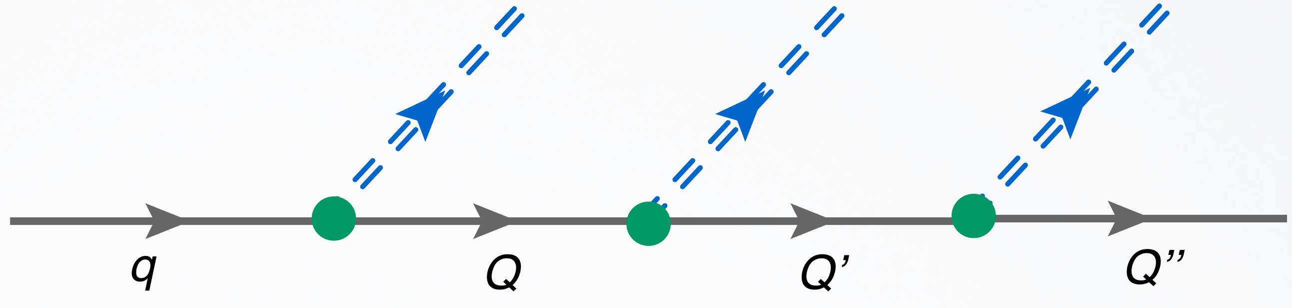
- Build a model for multi-hadron emission process using *microscopic* quark models as input.
- A *robust* and *expandable* Monte Carlo framework for describing both *Favored and Unfavored* fragmentation functions.
- NO model parameters fitted to fragmentation data!
- Momentum and quark flavor conservation is imposed.
- Extensions to TMD, Polarized Quark Fragmentation, Dihadron Fragmentations.

THE QUARK JET MODEL

Field, Feynman, Nucl.Phys.B136:1,1978.

Assumptions:

- **Number Density interpretation**
- **No re-absorption**
- **∞ hadron emissions**



$$D_q^h(z) = \hat{d}_q^h(z) + \int_z^1 \hat{d}_q^Q(y)dy \cdot D_Q^h(\frac{z}{y})\frac{1}{y}$$

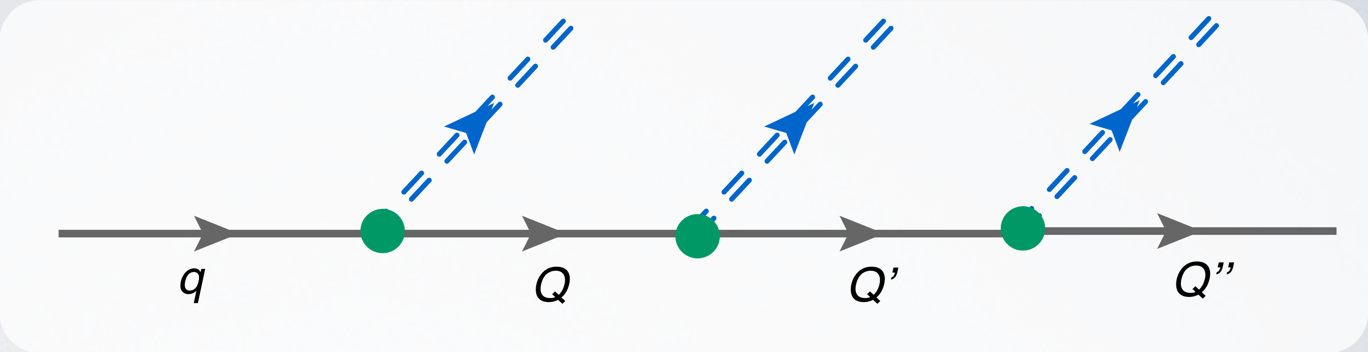
$$\hat{d}_q^h(z) = \hat{d}_q^{Q'}(1-z)|_{h=\bar{Q}'q}$$

THE QUARK JET MODEL

Field, Feynman, Nucl.Phys.B136:1,1978.

Assumptions:

- Number Density interpretation
- No re-absorption
- ∞ hadron emissions



Probability of finding hadron h with mom. frac. [z, z+dz] in a jet of quark q

The probability scales with mom. fraction

$$D_q^h(z)dz = \hat{d}_q^h(z)dz + \int_z^1 \hat{d}_q^Q(y)dy \cdot D_Q^h(\frac{z}{y})\frac{dz}{y}$$

Prob. of emitting at step 1

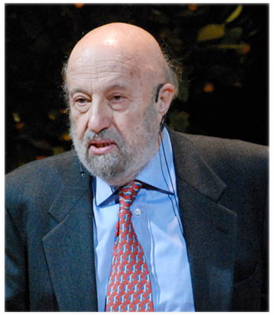
Prob. of mom. [y, y+dy] is transferred to jet at step 1.

NAMBU--JONA-LASINIO MODEL

Yoichiro Nambu and Giovanni Jona-Lasinio:

"Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I"

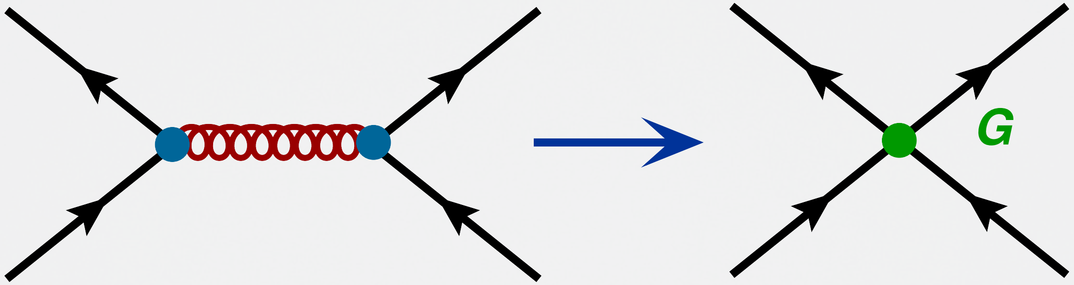
Phys.Rev. 122, 345 (1961)



Effective Quark model of QCD

- Effective Quark Lagrangian

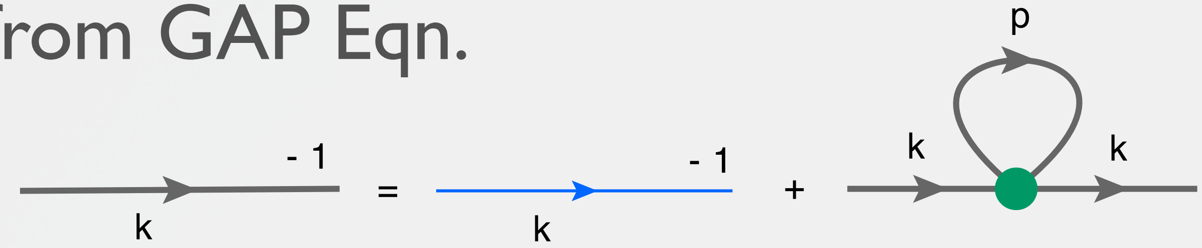
$$\mathcal{L}_{NJL} = \overline{\psi}_q(i\partial\!\!\!/ - m_q)\psi_q + G(\overline{\psi}_q\Gamma\psi_q)^2$$

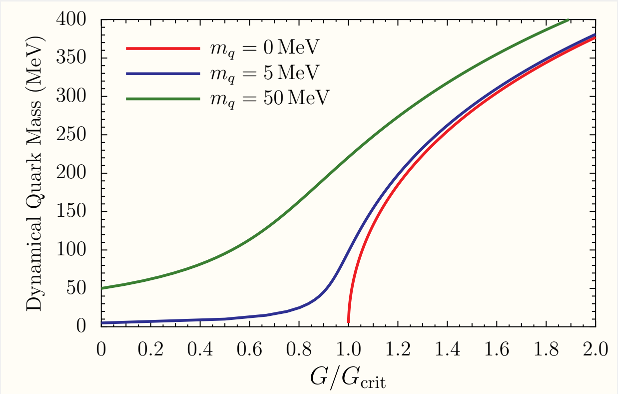


- Low energy chiral effective theory of QCD.
- Covariant, has the same flavor symmetries as QCD.

NAMBU--JONA-LASINIO MODEL

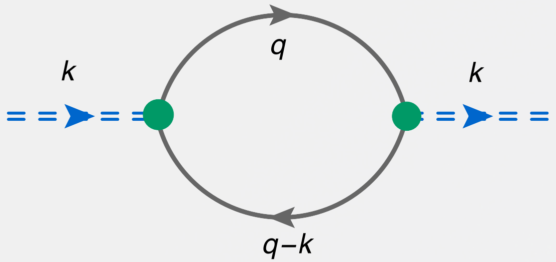
•Dynamically Generated Quark Mass from GAP Eqn.

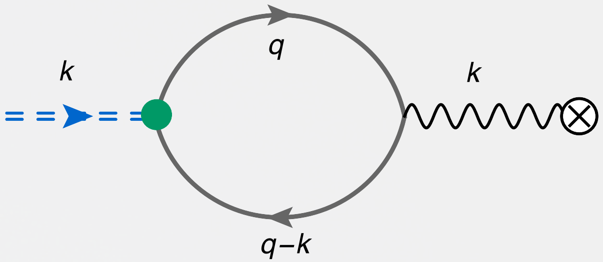




•Pion mass and quark-pion coupling from t-matrix pole.

•Pion decay constant





Fixing Model Parameters

•Use Lepage-Brodsky Invariant Mass cut-off regularization scheme.

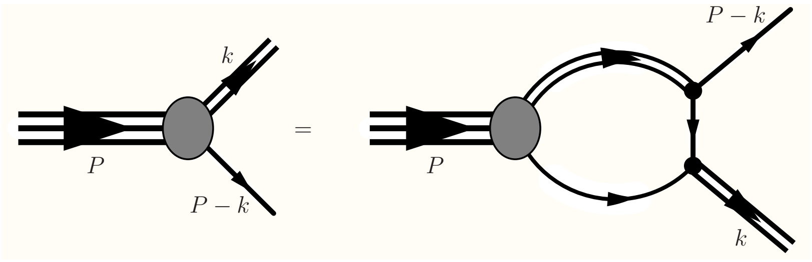
$$M_{12} \le \Lambda_{12} = \sqrt{\Lambda_3^2 + M_1^2} + \sqrt{\Lambda_3^2 + M_2^2}$$

• Choose a $M_{u(d)}$ and use physical f_π, m_π, m_K, to fix model parameters Λ_3, G, M_s and calculate g_{hqQ} .

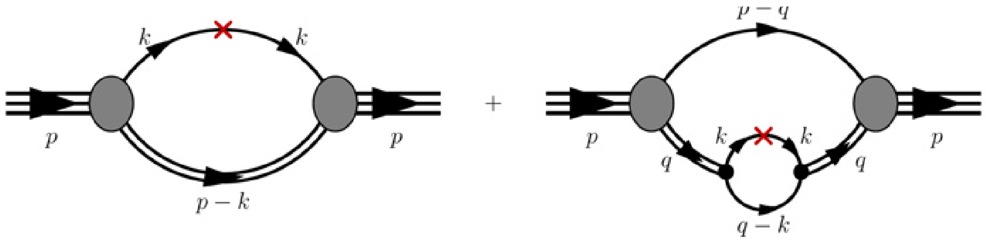
NJL: INTEGRATED NUCLEON PDFS

I. C. Cloet, W. Bentz, and A. W. Thomas, PLB 621, 246 (2005).

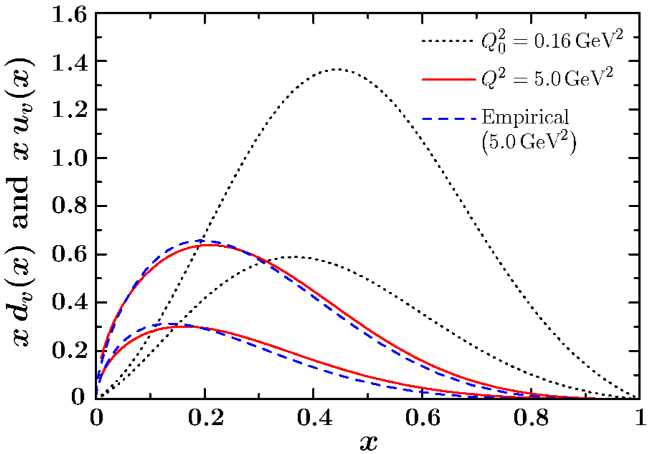
- **Quark-diquark description of Nucleon using relativistic Faddeev approach**

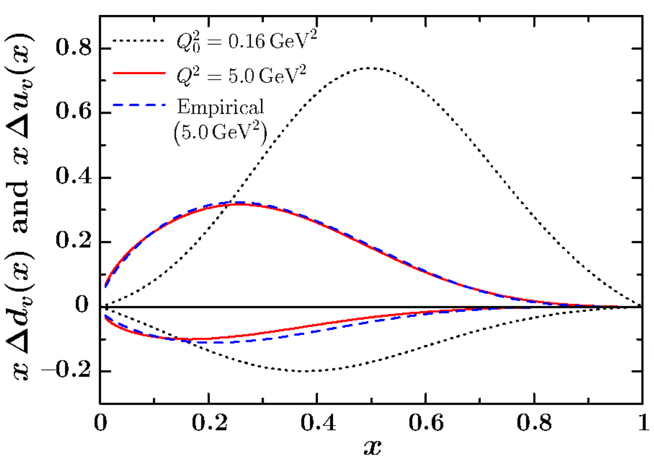


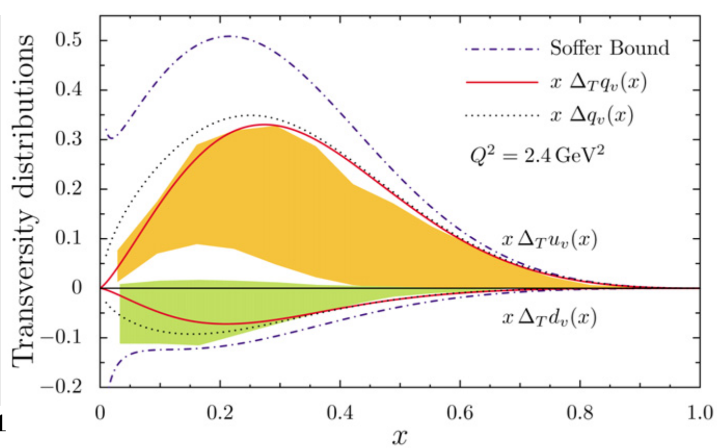
- **PDFs from Feynman diagrams**



A good description of both unpolarized and polarized PDFs.



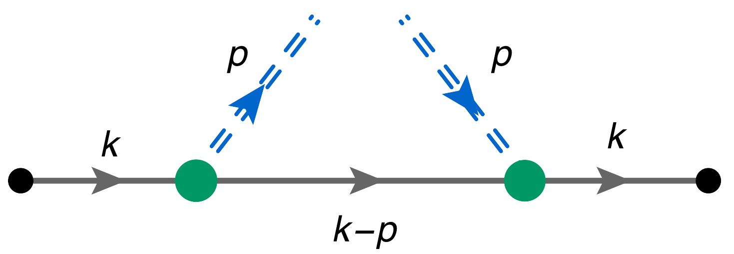


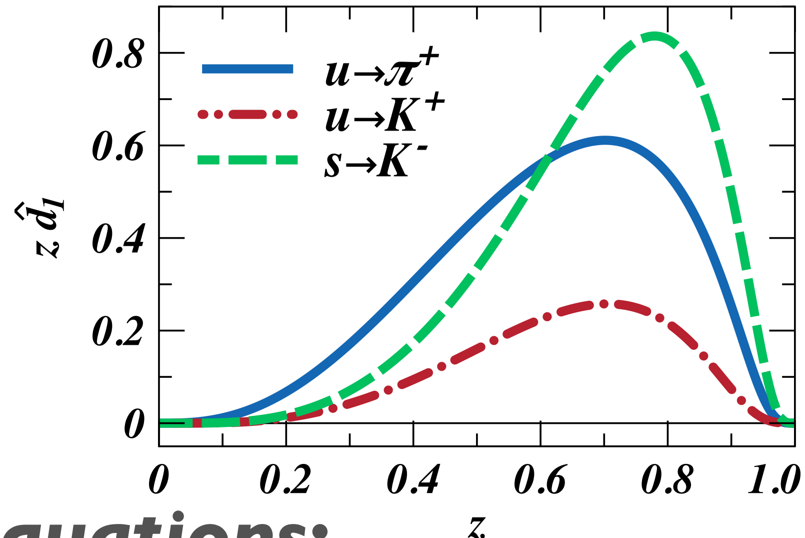


SOLUTIONS OF THE INTEGRAL EQUATIONS

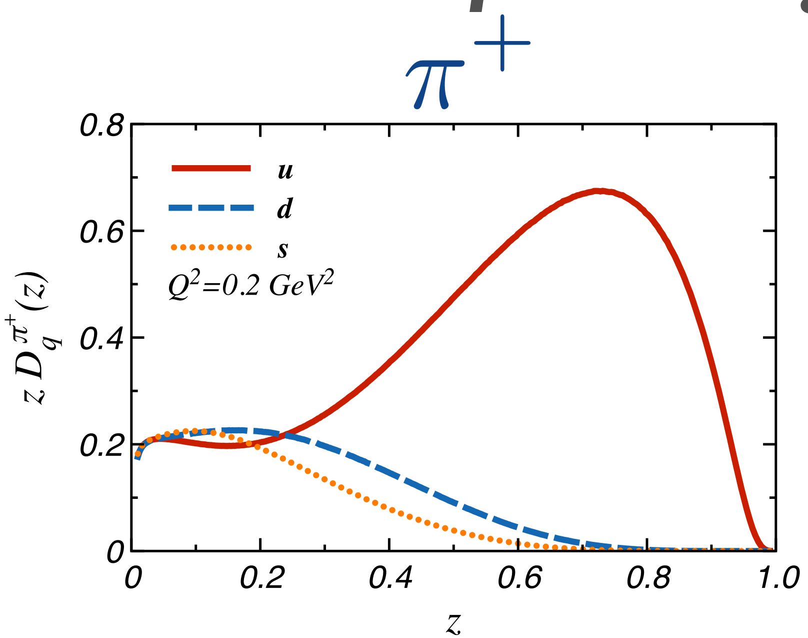
H.M., Thomas, Bentz, PRD. 83:074003, 2011

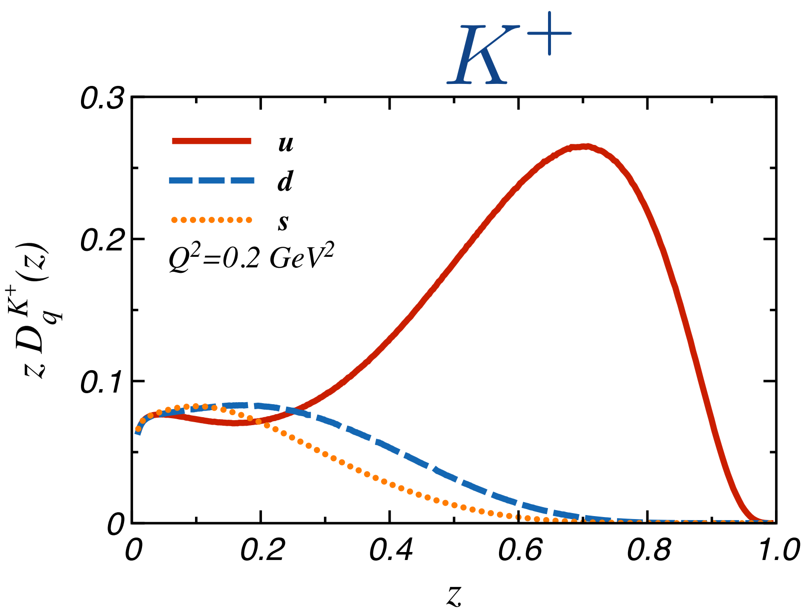
✦ *Input elementary probabilities from NJL:*





✦ *Solutions of the integral equations:*

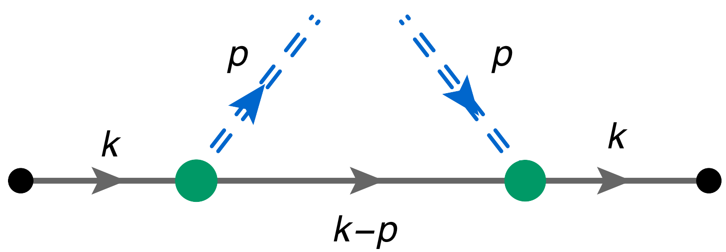


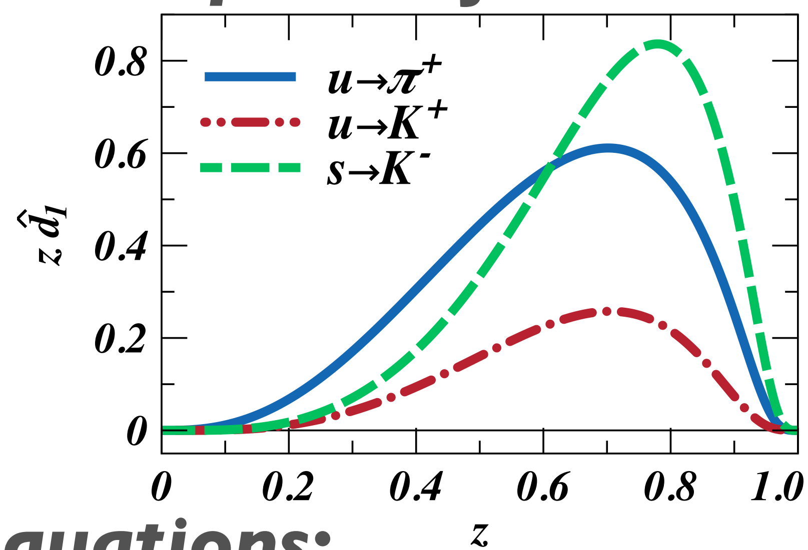


SOLUTIONS OF THE INTEGRAL EQUATIONS

H.M., Thomas, Bentz, PRD. 83:074003, 2011

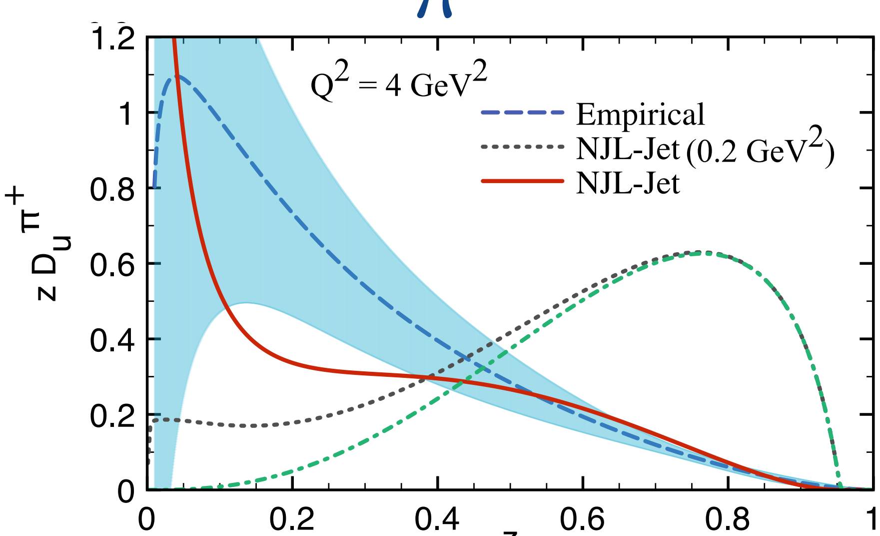
✦ *Input elementary probabilities from NJL:*



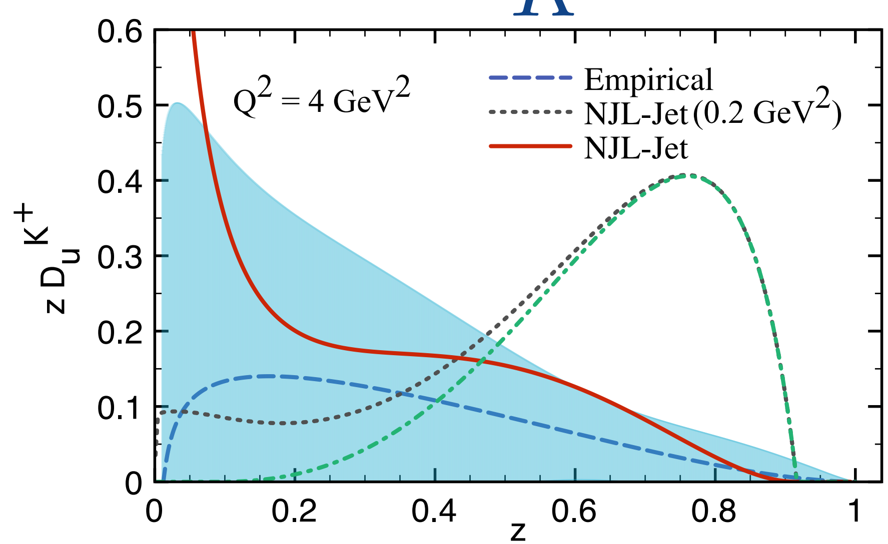


✦ *Solutions of the integral equations:*

π^+



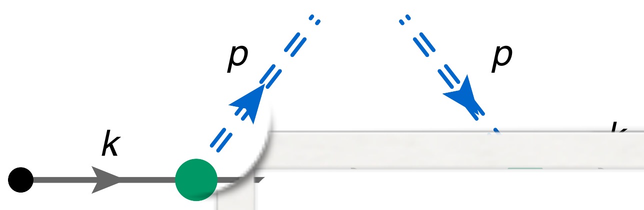
K^+

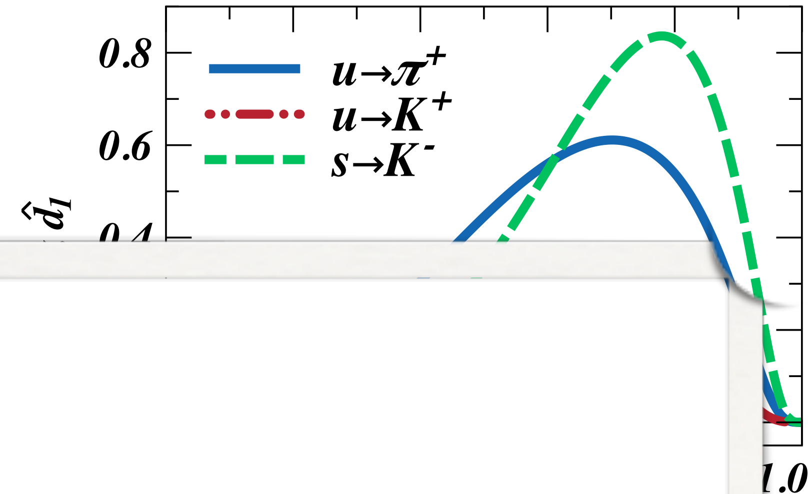


SOLUTIONS OF THE INTEGRAL EQUATIONS

H.M., Thomas, Bentz, PRD. 83:074003, 2011

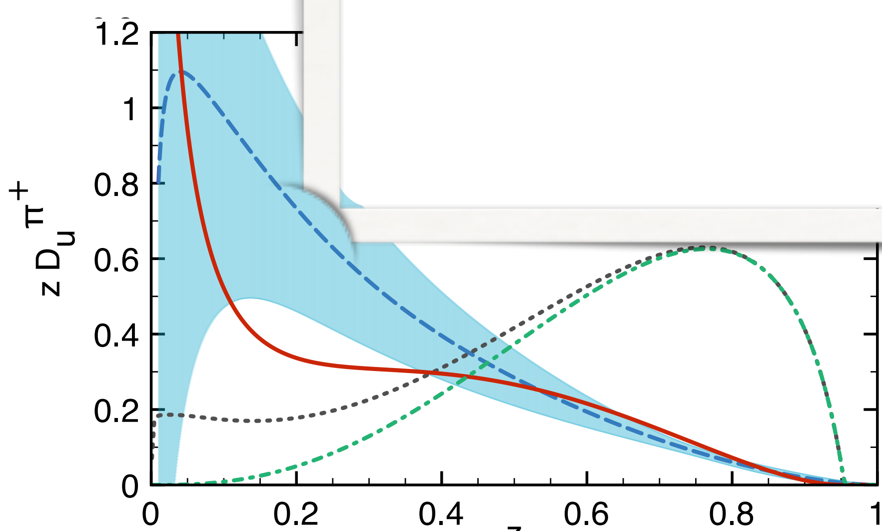
✦ ***Input elementary probabilities from NJL:***

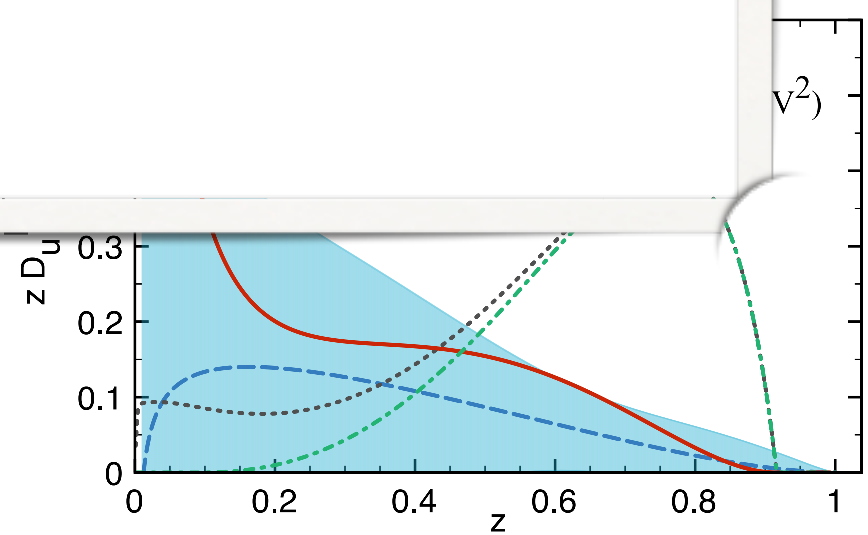




✦ ***Solutio***

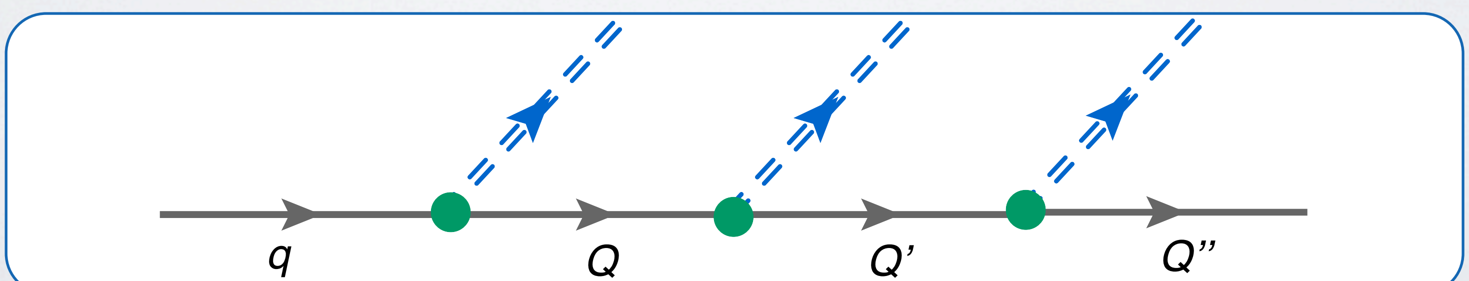
What happens for finite number of emitted hadrons?





MONTE-CARLO (MC) APPROACH

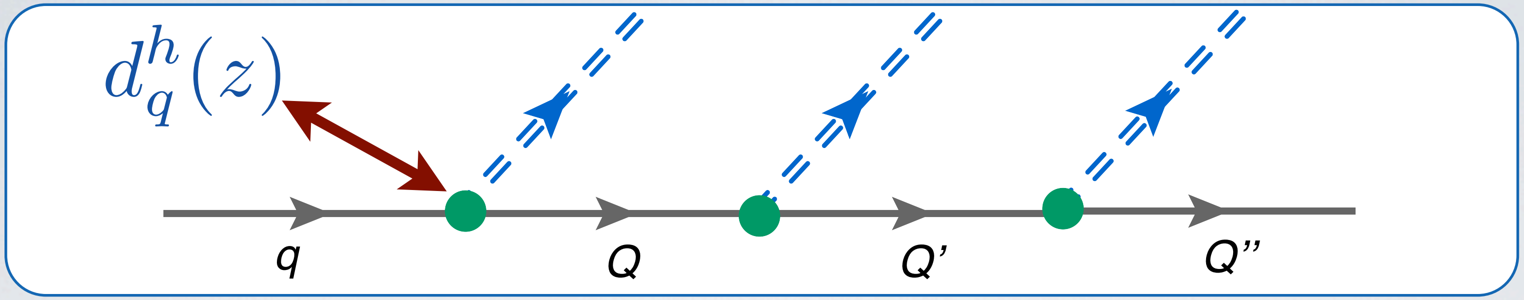
✦ Using the ***probabilistic*** interpretation of fragmentation funcs. to include the effect of ***multiple*** hadron emissions.



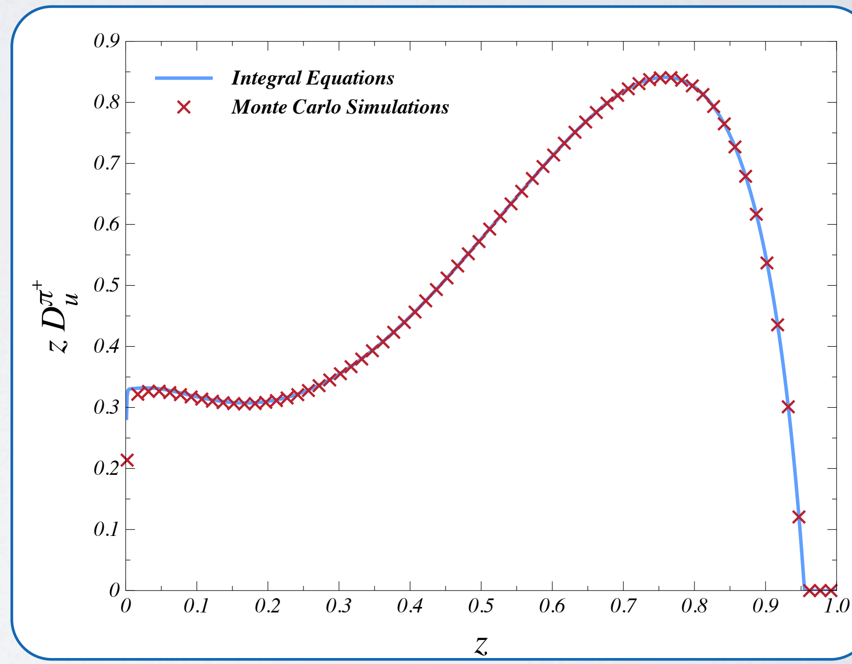
INTEGRATED FRAGMENTATIONS FROM MC

H.M., Thomas, Bentz, PRD. 83:07400; PRD.83:114010, 2011.

- Input: One hadron emission probability



- Sample the emitted hadron type and z according to input splitting.



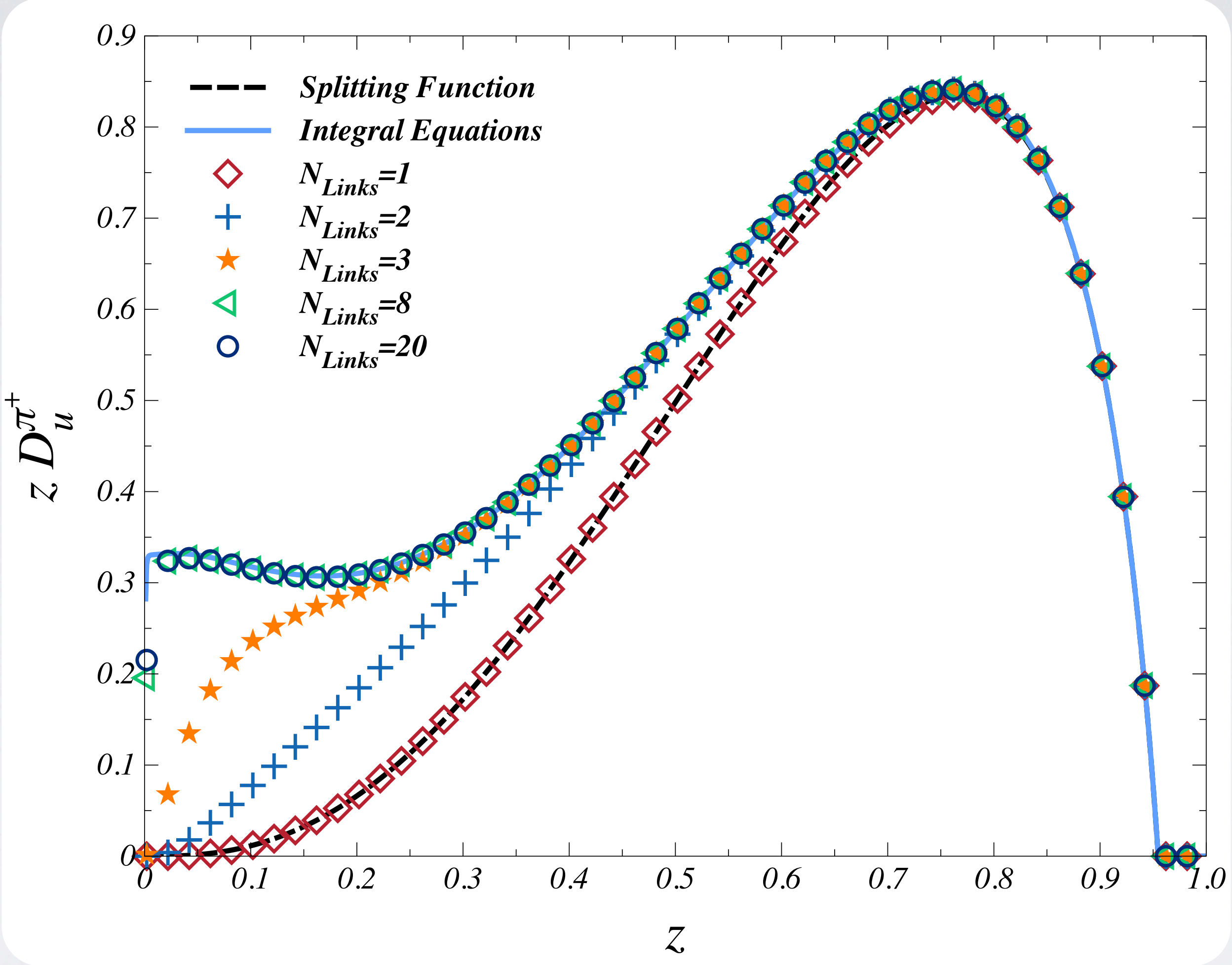
- ***CONSERVE***: Momentum and Quark Flavor in each step.

- Repeat for decay chains with the same initial quark.

$$D_q^h(z)\Delta z = \langle N_q^h(z, z+\Delta z)\rangle \equiv \frac{\sum_{N_{Sims}} N_q^h(z, z+\Delta z)}{N_{Sims}}$$

DEPENDENCE ON NUMBER OF EMITTED HADRONS

- Restrict the number of emitted hadrons, N_{Links} in MC.



- We reproduce the splitting function and the full solution perfectly.
- The low z region is saturated with just a few emissions.

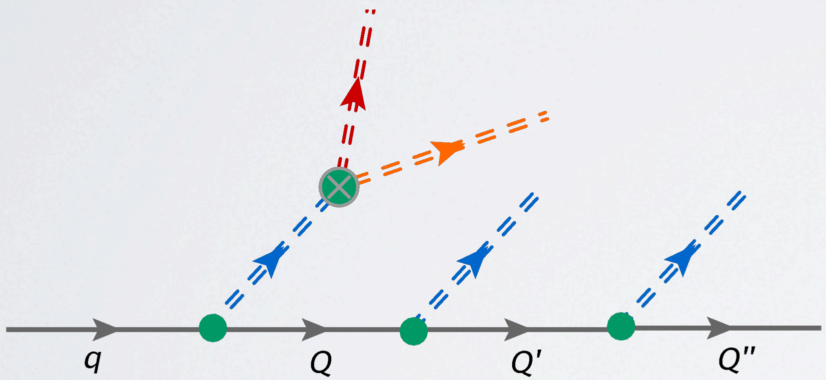
MORE CHANNELS

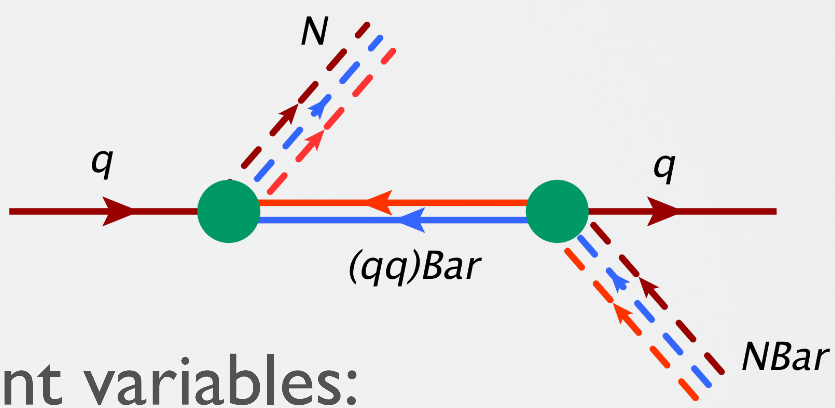
H.M., Thomas, Bentz, PRD. 83:074003, 2011

- Calculate quark splittings to vector mesons, Nucleon Anti-Nucleon: $d_q^h(z)$

$$h = \rho^0, \rho^\pm, K^{*0}, \overline{K}^{*0}, K^{*\pm}, \phi, N, \bar{N}$$

- Add the decay of the resonances:

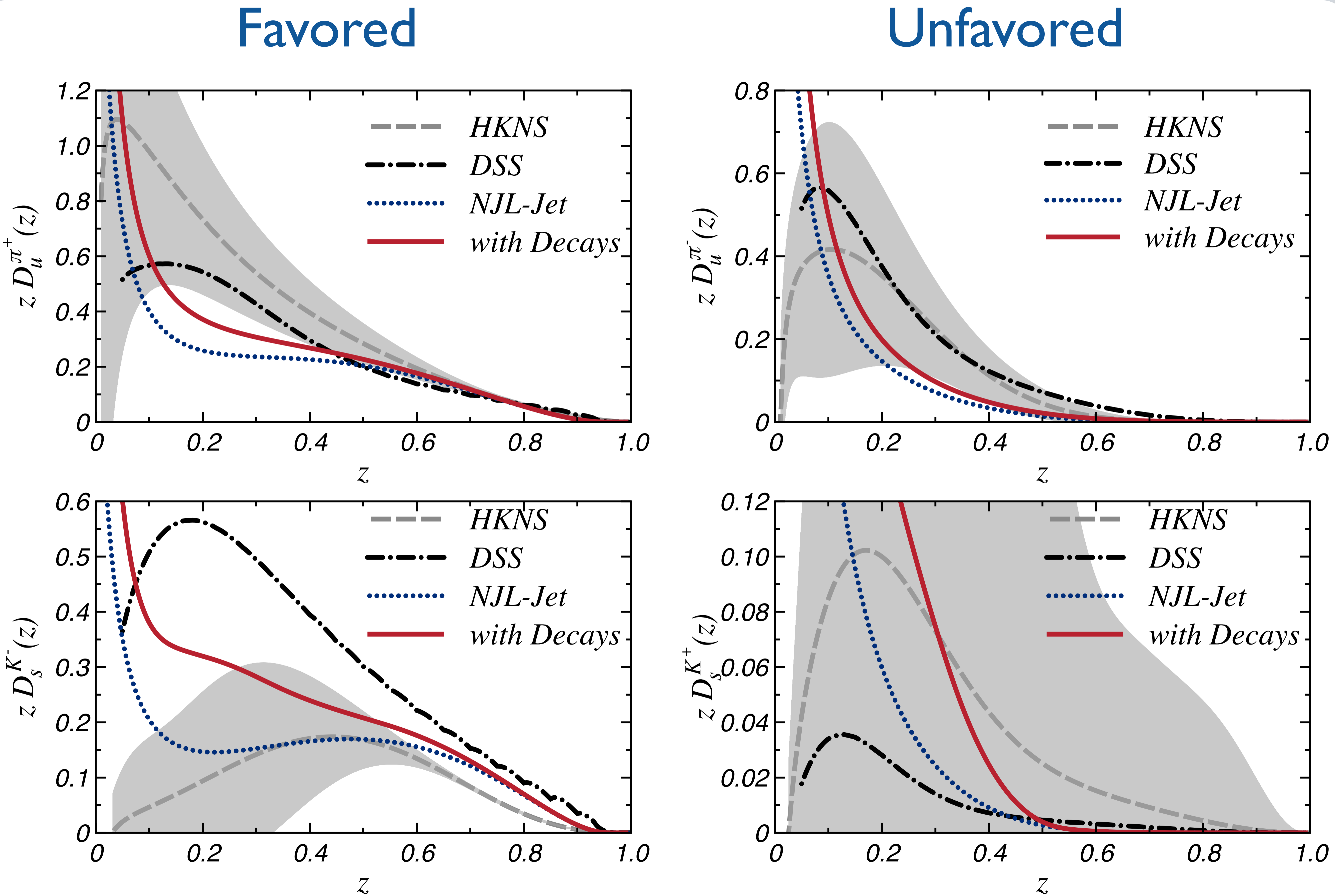




- Decay cross-section in light-front variables:

$$dP^{h\to h_1,h_2}(z_1) = \begin{cases} \frac{C_h^{h_1 h_2}}{8\pi}\, dz_1 & \text{if } z_1 z_2\, m_h^2 - z_2 m_{h1}^2 - z_1 m_{h2}^2 \geq 0;\; z_1 + z_2 = 1, \\ 0 & \text{otherwise.} \end{cases}$$

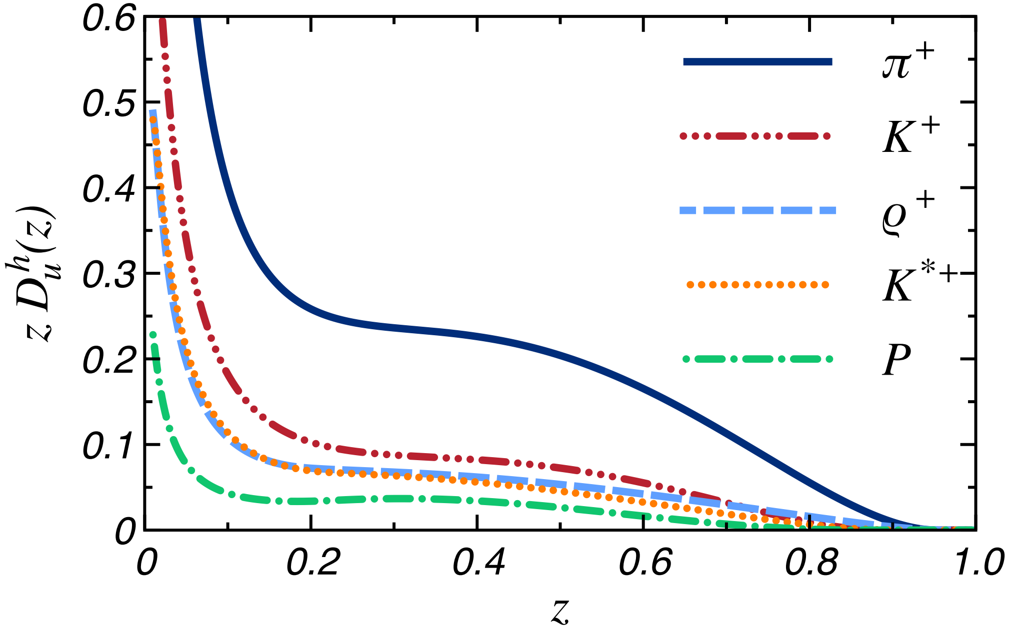
Results with VM decays: $Q^2 = 4\ \mathrm{GeV}^2$



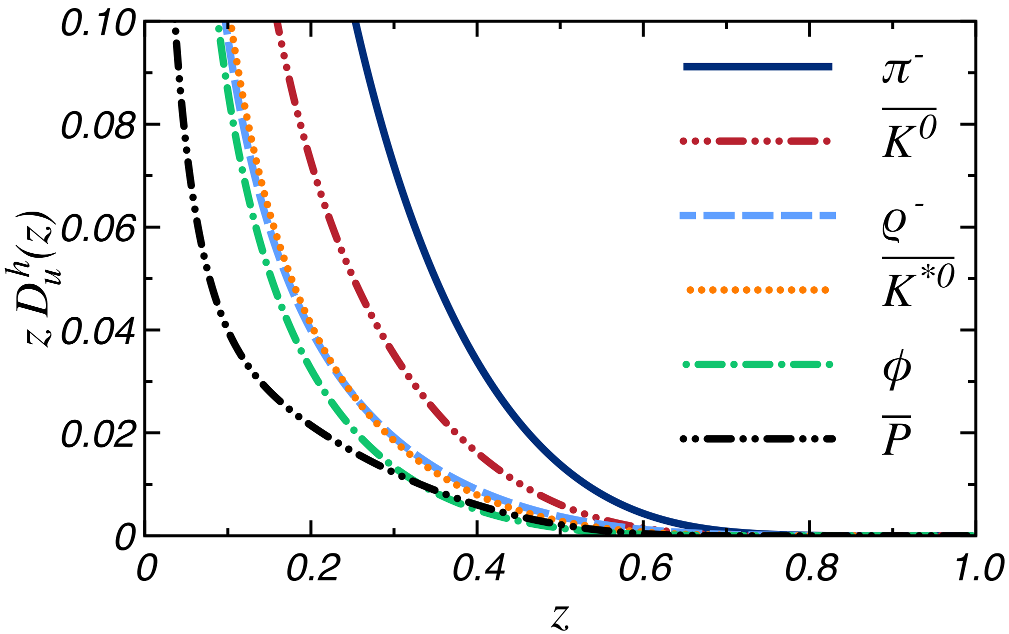
Results: Fragmentations to All Hadrons

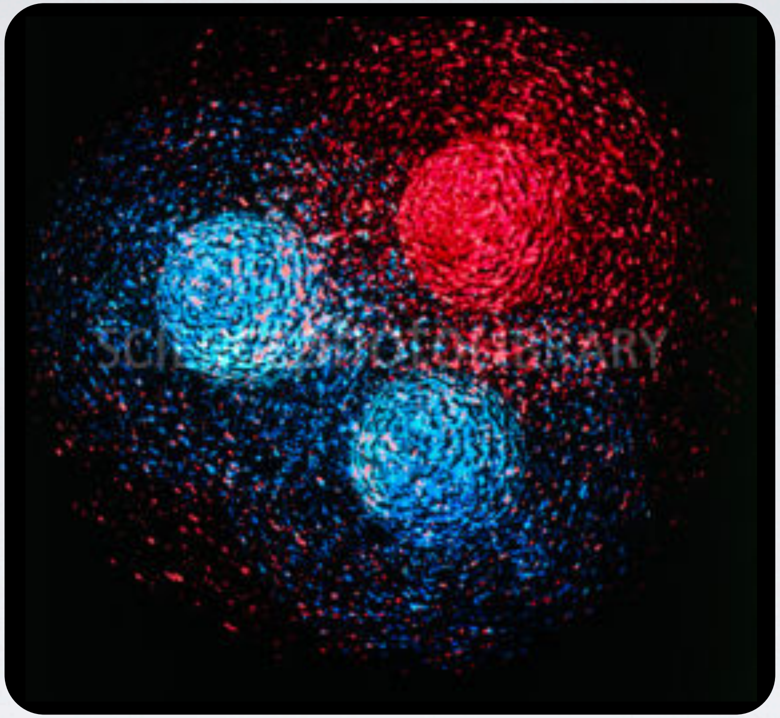
$Q^2 = 4\ \mathrm{GeV}^2$

Favored



Unfavored

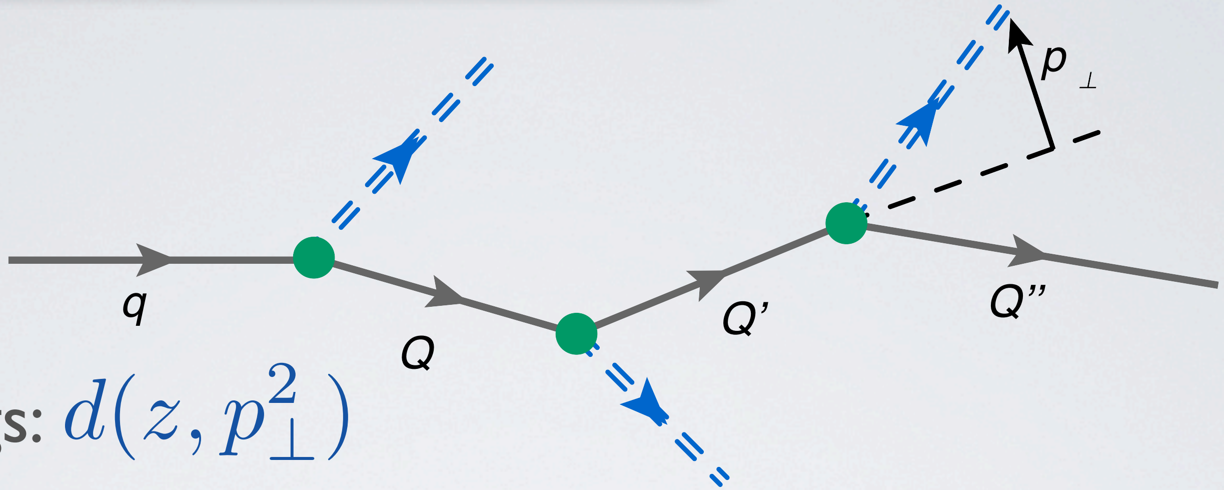




TRANSVERSE MOMENTUM DEPENDENCE

INCLUDING THE TRANSVERSE MOMENTUM

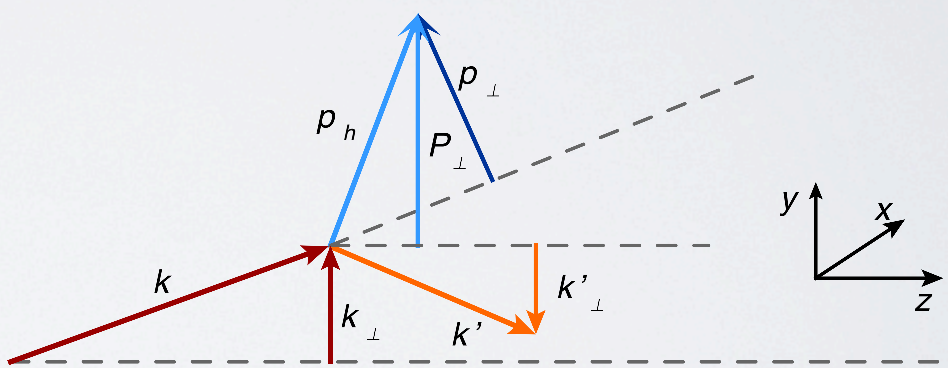
H.M.,Bentz, Cloet, Thomas, PRD.85:014021, 2012



- TMD splittings: $d(z, p_\perp^2)$
- Conserve transverse momenta at each link.

$$\mathbf{P}_\perp = \mathbf{p}_\perp + z\mathbf{k}_\perp$$

$$\mathbf{k}_\perp = \mathbf{P}_\perp + \mathbf{k}'_\perp$$



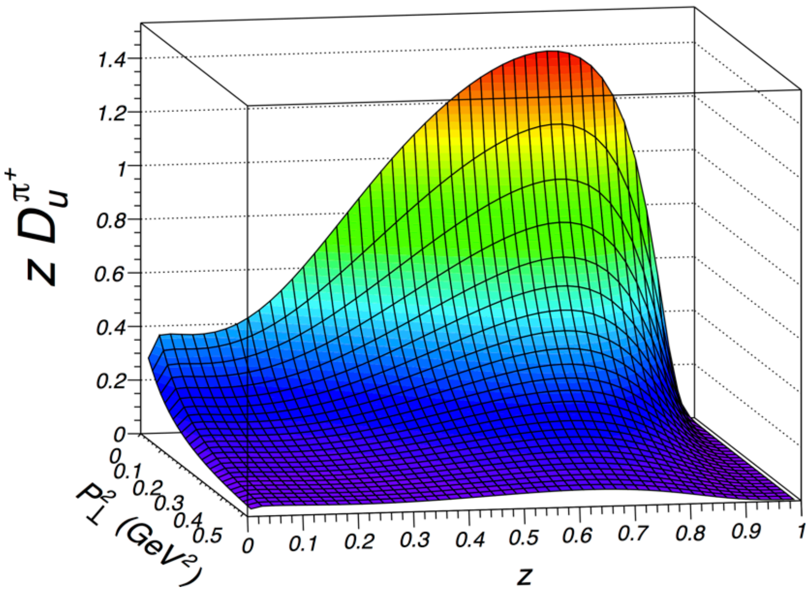
- Calculate the Number Density

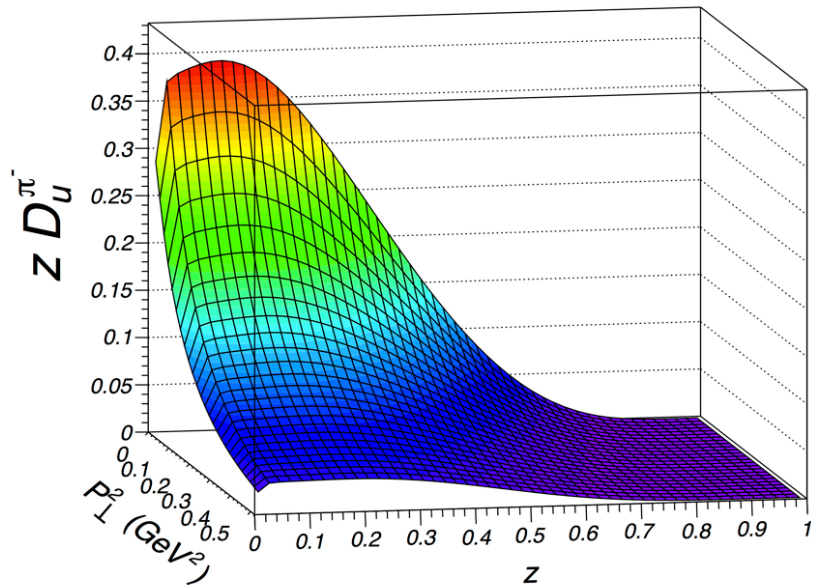
$$D_q^h(z, P_\perp^2)\Delta z\ \pi\Delta P_\perp^2 = \frac{\sum_{N_{Sims}} N_q^h(z, z+\Delta z, P_\perp^2, P_\perp^2 + \Delta P_\perp^2)}{N_{Sims}}.$$

TMD FRAGMENTATION FUNCTIONS

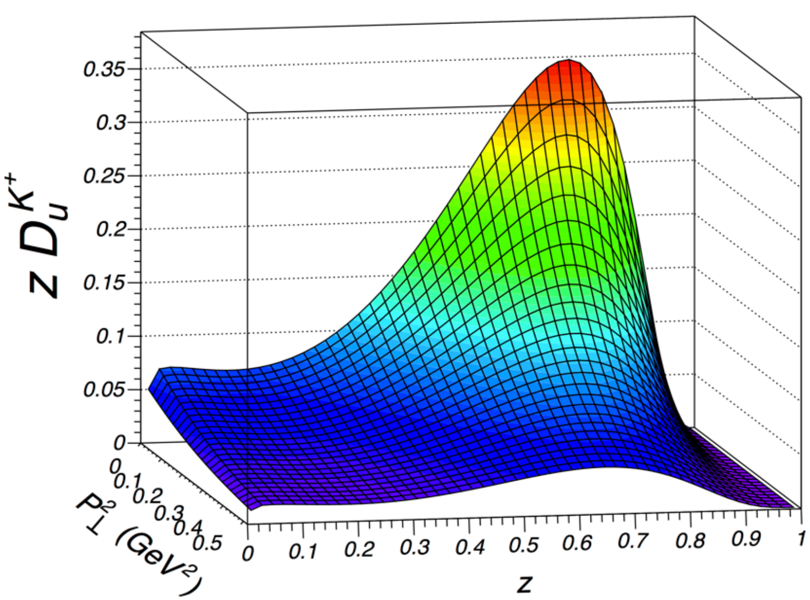
FAVORED

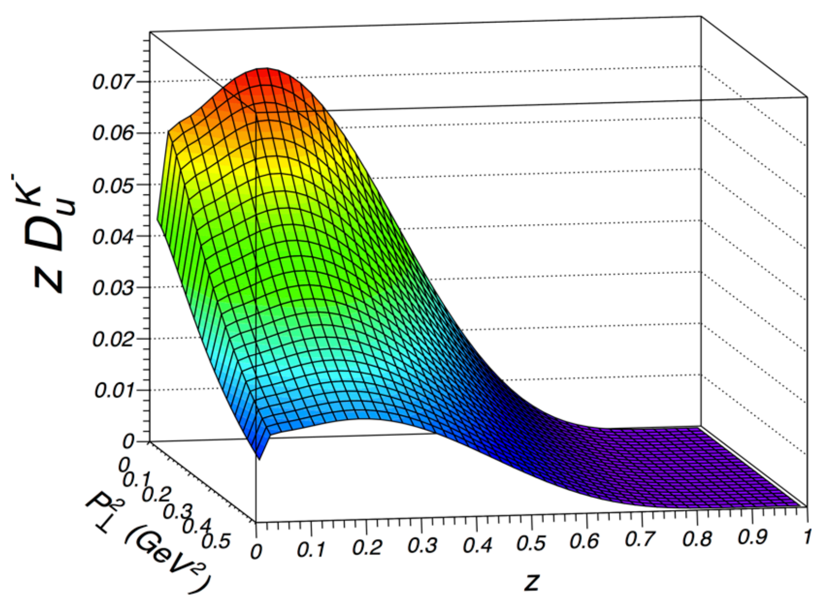
• UNFAVORED





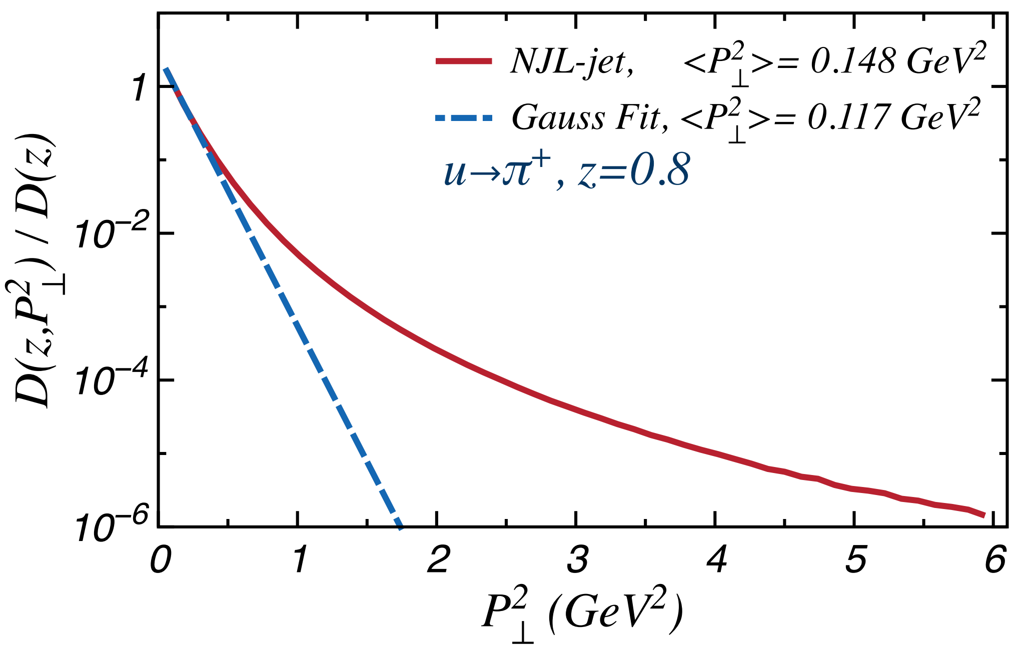
π

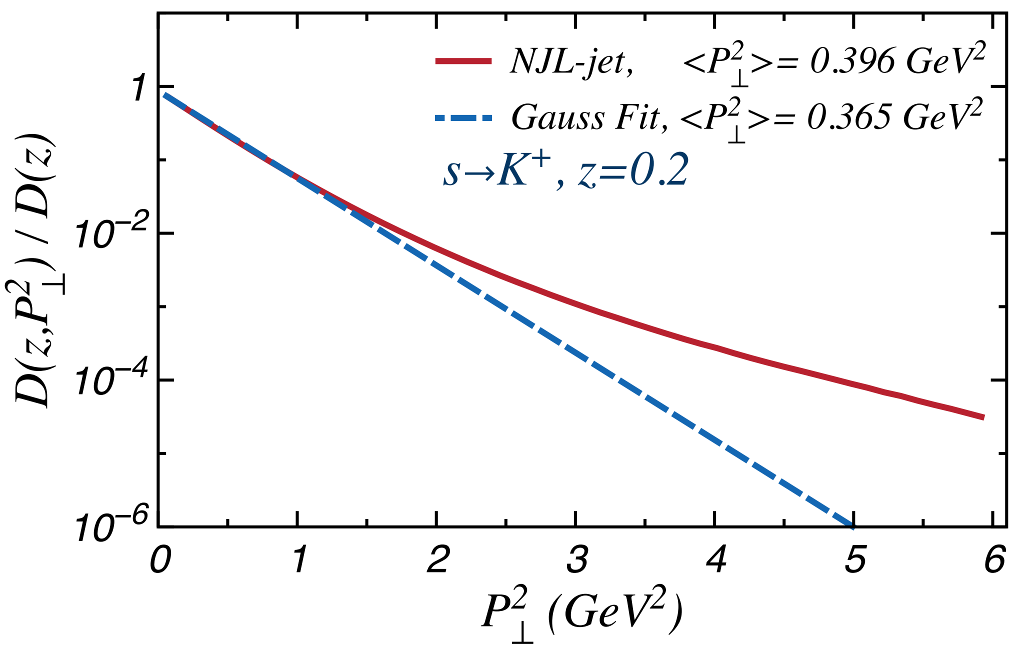




K

COMPARISON WITH GAUSSIAN ANSATZ





- Average TM: $\langle P_\perp^2 \rangle \equiv \frac{\int d^2\mathbf{P}_\perp \; P_\perp^2 D(z, P_\perp^2)}{\int d^2\mathbf{P}_\perp \; D(z, P_\perp^2)}$
- Gaussian ansatz assumes: $D(z, P_\perp^2) = D(z) \frac{e^{-P_\perp^2/\langle P_\perp^2 \rangle}}{\pi \langle P_\perp^2 \rangle}$

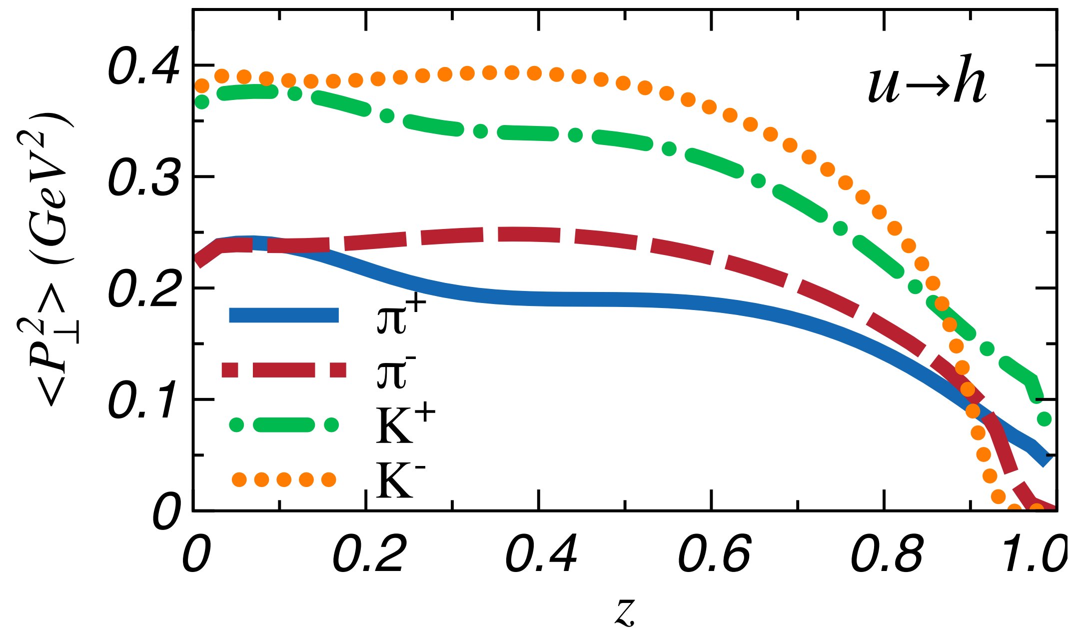
AVERAGE TRANSVERSE MOMENTA VS Z

FRAGMENTATION

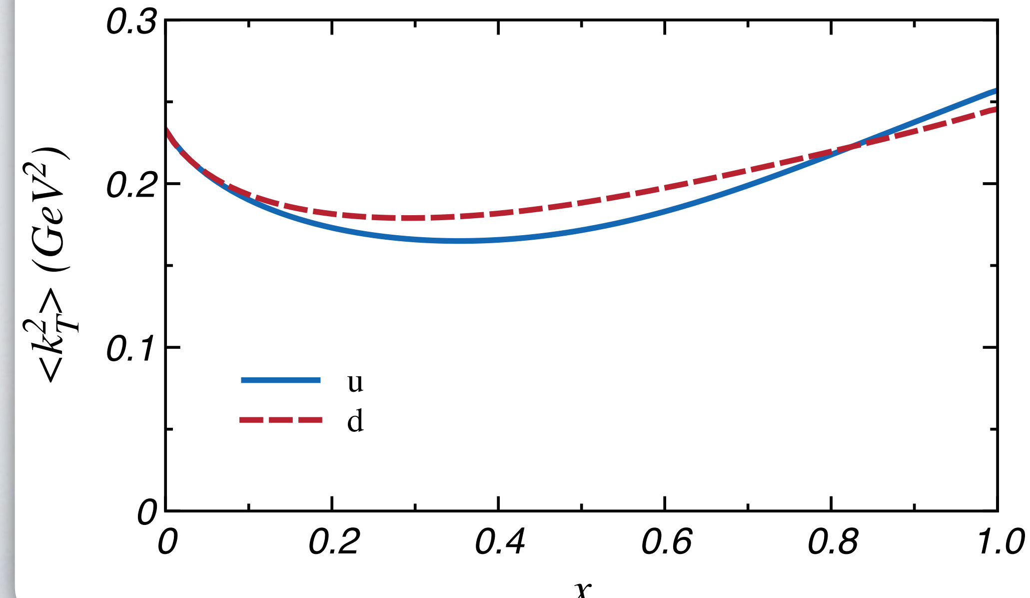
$$\langle P_\perp^2 \rangle_{unf} > \langle P_\perp^2 \rangle_f$$

✦ Indications from HERMES data:

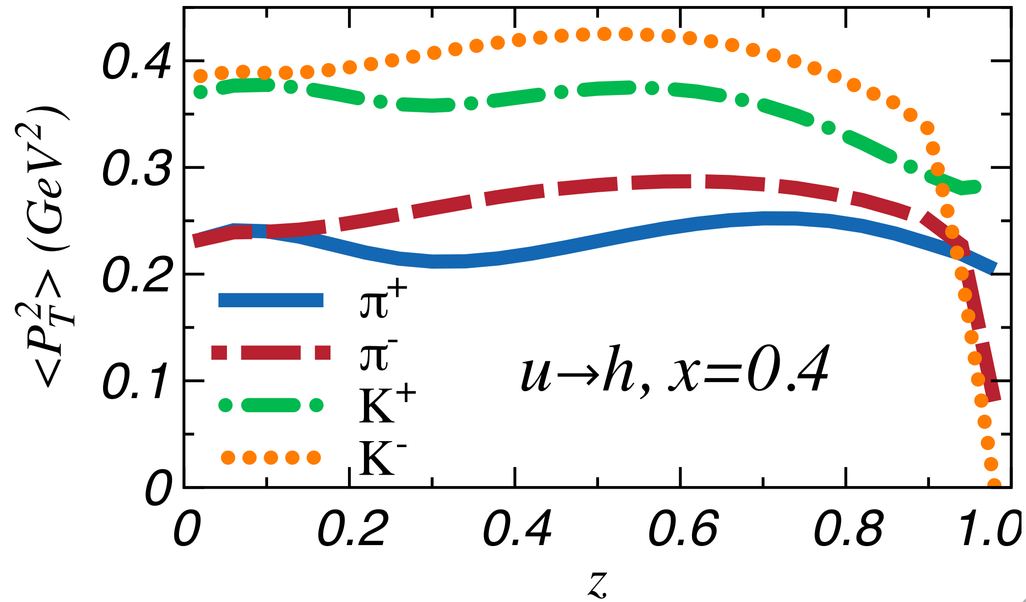
A. Signori, et al: JHEP 1311, 194 (2013)

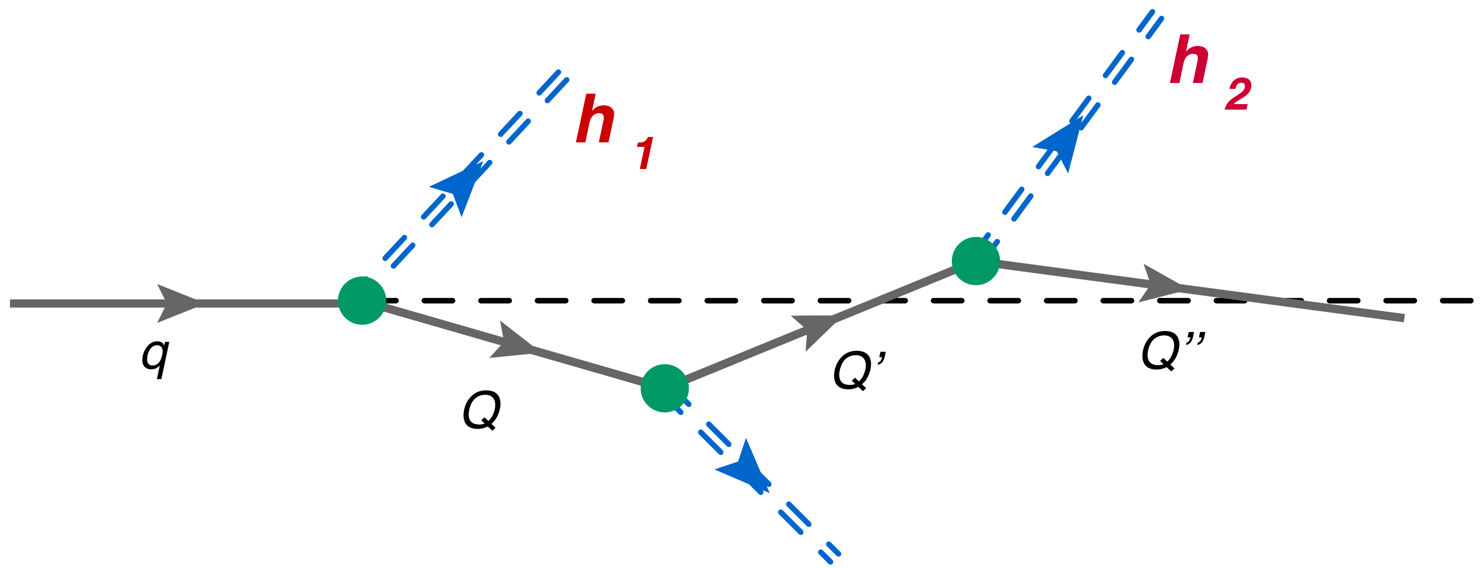


PDF



SIDIS



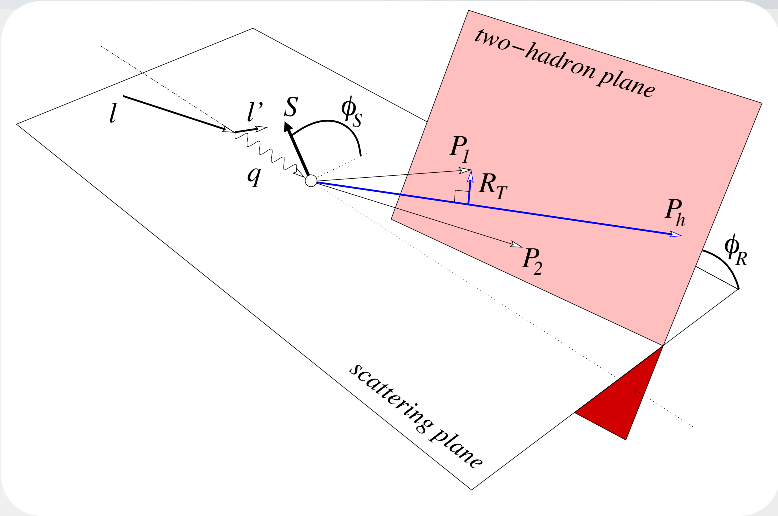


TWO HADRON CORRELATIONS: DIHADRON FRAGMENTATION FUNCTIONS

ACCESS TO TRANSVERSITY PDF FROM DFF

M. Radici, et al: PRD 65, 074031 (2002).

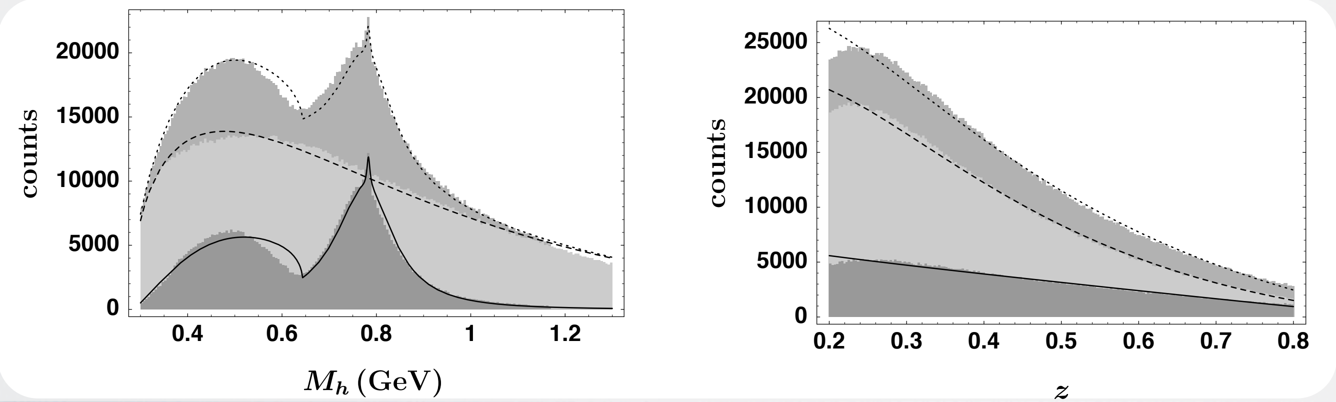
- In two hadron production from polarized target the cross section factorizes *collinearly* - no TMD!
- Allows clean access to *transversity*.
- *Unpolarized* and *Interference* Dihadron FFs are needed!



$$\frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \propto \sin(\phi_R + \phi_S) \frac{\sum_q e_q^2 \; h_1^q(x)/x \; H_1^{\sphericalangle q}(z, M_h^2)}{\sum_q e_q^2 \; f_1^q(x)/x \; D_1^q(z, M_h^2)}$$

- Empirical Model for D_1^q have been fitted to PYTHIA simulations.

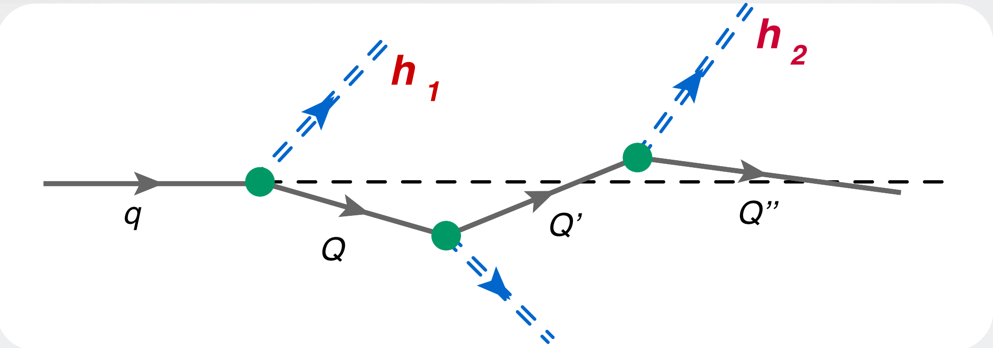
A. Bacchetta and M. Radici, PRD 74, 114007 (2006).



Experiments: BELLE, HERMES, COMPASS.

UNPOLARIZED DIHADRON FRAGMENTATIONS

H.M. Thomas, Bentz, PRD.88:094022, 2013.



- The probability density for observing two hadrons:

$$P_1 = (z_1 k^-, P_1^+, \boldsymbol{P}_{1,\perp}),\ \ P_1^2 = M_{h1}^2$$

$$P_2 = (z_2 k^-, P_2^+, \boldsymbol{P}_{2,\perp}),\ \ P_2^2 = M_{h2}^2$$

- The corresponding number density:

$$D_q^{h_1 h_2}(z, M_h^2)\ \Delta z\ \Delta M_h^2 = \langle N_q^{h_1 h_2}(z, z+\Delta z; M_h^2, M_h^2 + \Delta M_h^2) \rangle$$

$$z = z_1 + z_2 \qquad M_h^2 = (P_1 + P_2)^2$$

- Kinematic Constraint.

$$z_1 z_2 M_h^2 - (z_1 + z_2)(z_2 M_{h1}^2 + z_1 M_{h2}^2) \geq 0$$

- In MC simulations record all the pairs in every decay chain.

THE EFFECT OF VECTOR MESONS (VM)

- A naive assumption: VMs should have modest contribution due to relatively small production probability $P(\pi^+)/P(\rho^+) \approx 1.7$
- ***But***: Combinatorial factors enhance VM contribution significantly!
- Let's consider only two hadron emission

Direct: $u \to d + \pi^+ \to u + \pi^- + \pi^+$

VM: $u \to d + \pi^+ \to u + \rho^- + \pi^+$, $\rho^- \to \pi^- \pi^0$

...

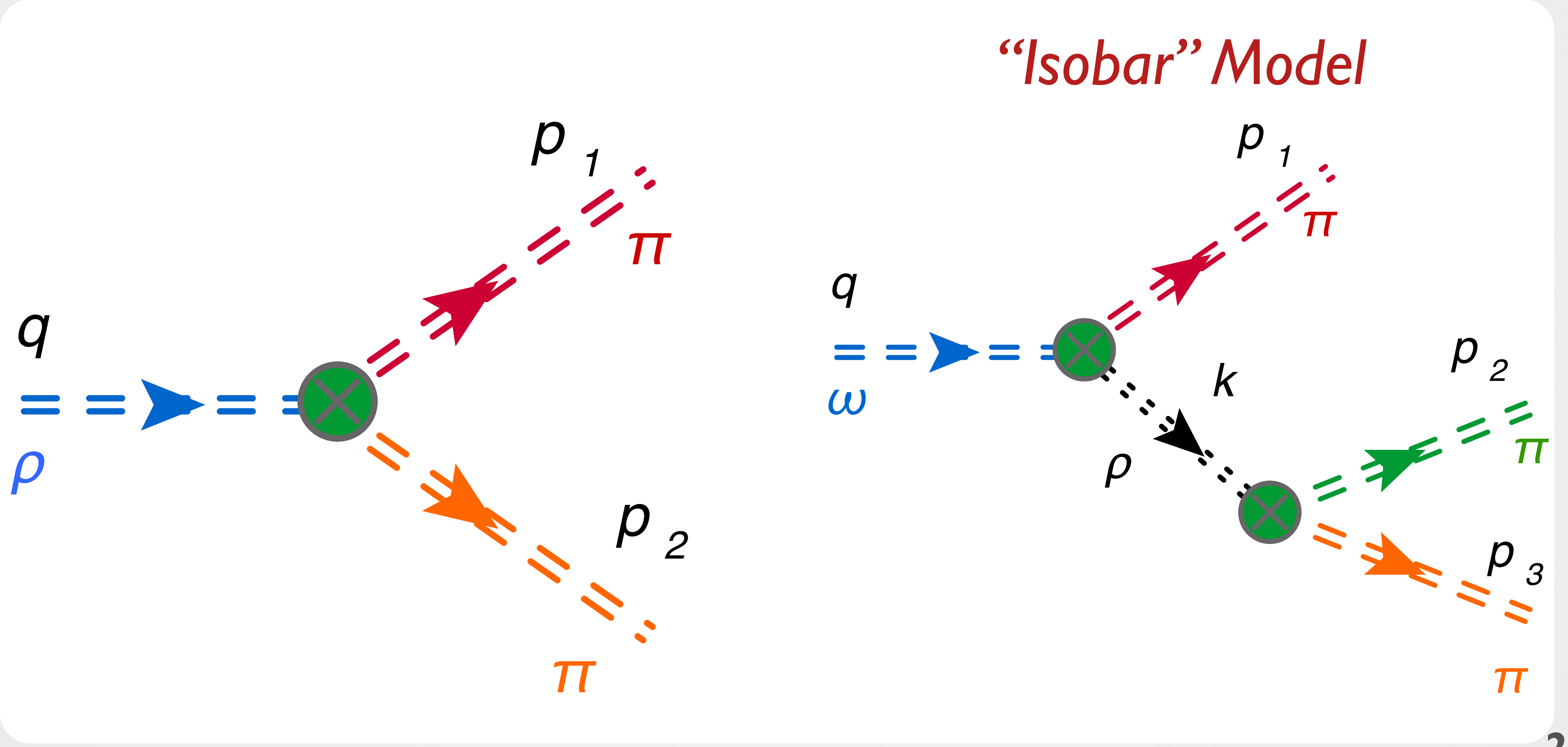
$u \to u + \rho^0 \to u + \rho^0 + \rho^0$, $\rho^0 \to \pi^+ \pi^-$, $\rho^0 \to \pi^+ \pi^-$

$$P_{Dir}(\pi^+\pi^-)/P_{VM}(\pi^+\pi^-) \approx \frac{1}{4}$$

2- AND 3-BODY DECAYS

The M_h^2 spectrum of pseudoscalars is strongly affected by VM decays.

- We include only the 2-body decays ρ, K^*.
- Both 2- and 3-body decays of ω, ϕ .



2- AND 3-BODY DECAYS

The M_h^2 spectrum of pseudoscalars is strongly affected by VM decays.

- We include only the 2-body decays ρ, K^*.
- Both 2- and 3-body decays of ω, ϕ .

Achasov et al. (SND), PRD 68, 052006, (2003).

- 2-body decay amplitude: $$M(p_1, p_2) = \frac{g_V^{h_1 h_2} \epsilon^\mu (p_{2\mu} - p_{1\mu})}{D_V(q^2)}$$
- Resonance propagator:

$$D_V(s) = m_V^2 - s - i\sqrt{s}\Gamma_V(s)$$

$$\Gamma_V(s) = \frac{m_V^2}{s}\Gamma_V \left(\frac{q(s)}{q(m_V^2)}\right)^3$$

Relative Momentum of daughters in their CM frame.

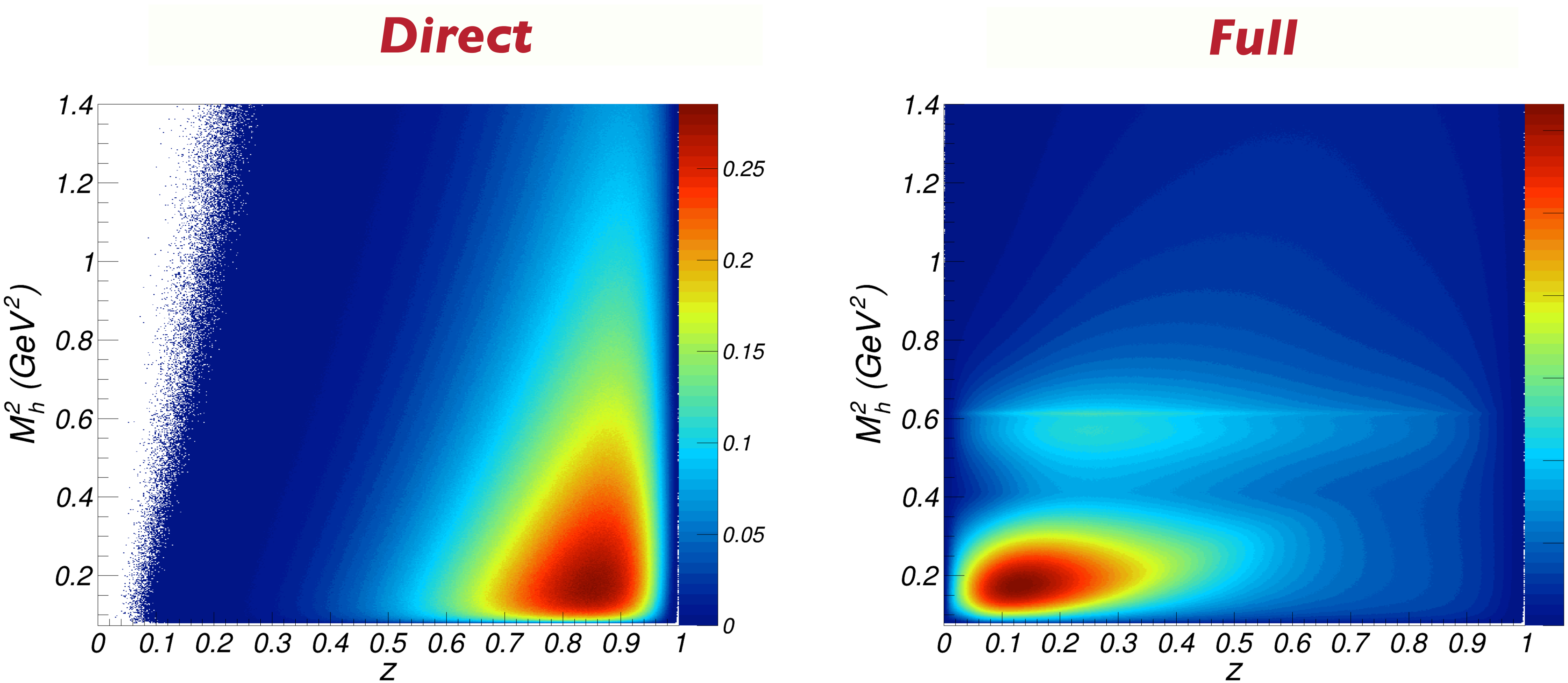
- 3-body decay amplitude (ignore small width):

$$M(p_1, p_2, p_3) = \varepsilon_{\mu\alpha\beta\gamma}\epsilon^\mu p_1^\alpha p_2^\beta p_3^\gamma \sum_{i=0,\pm} \frac{g_{V\rho_i\pi}\ g_{\rho_i\pi\pi}}{D_{\rho_i}(v_i^2)}$$

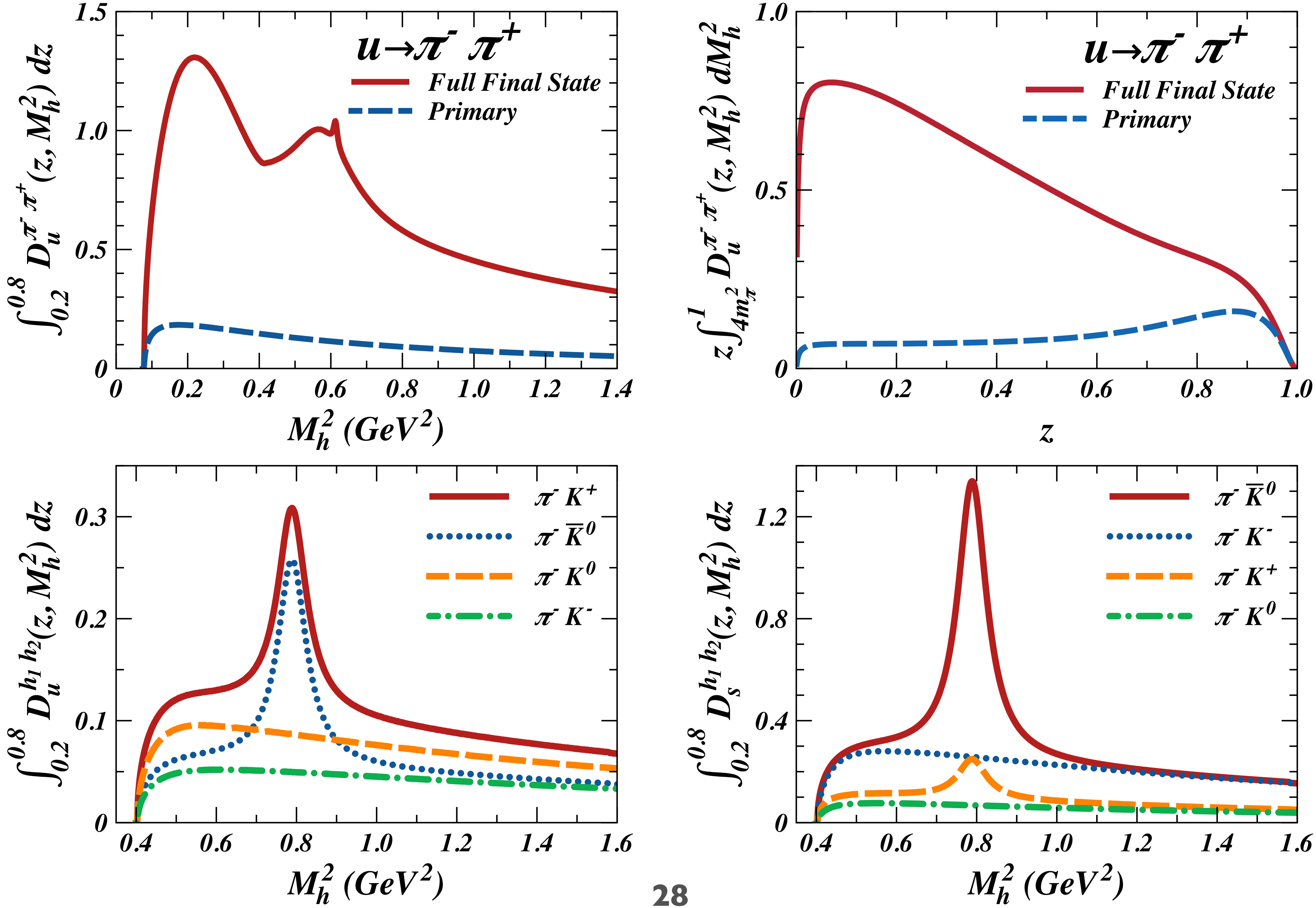
- Simulate 2- and 3-body phase space in LC.

RESULTS FOR PION DFF $u \to \pi^- \pi^+$

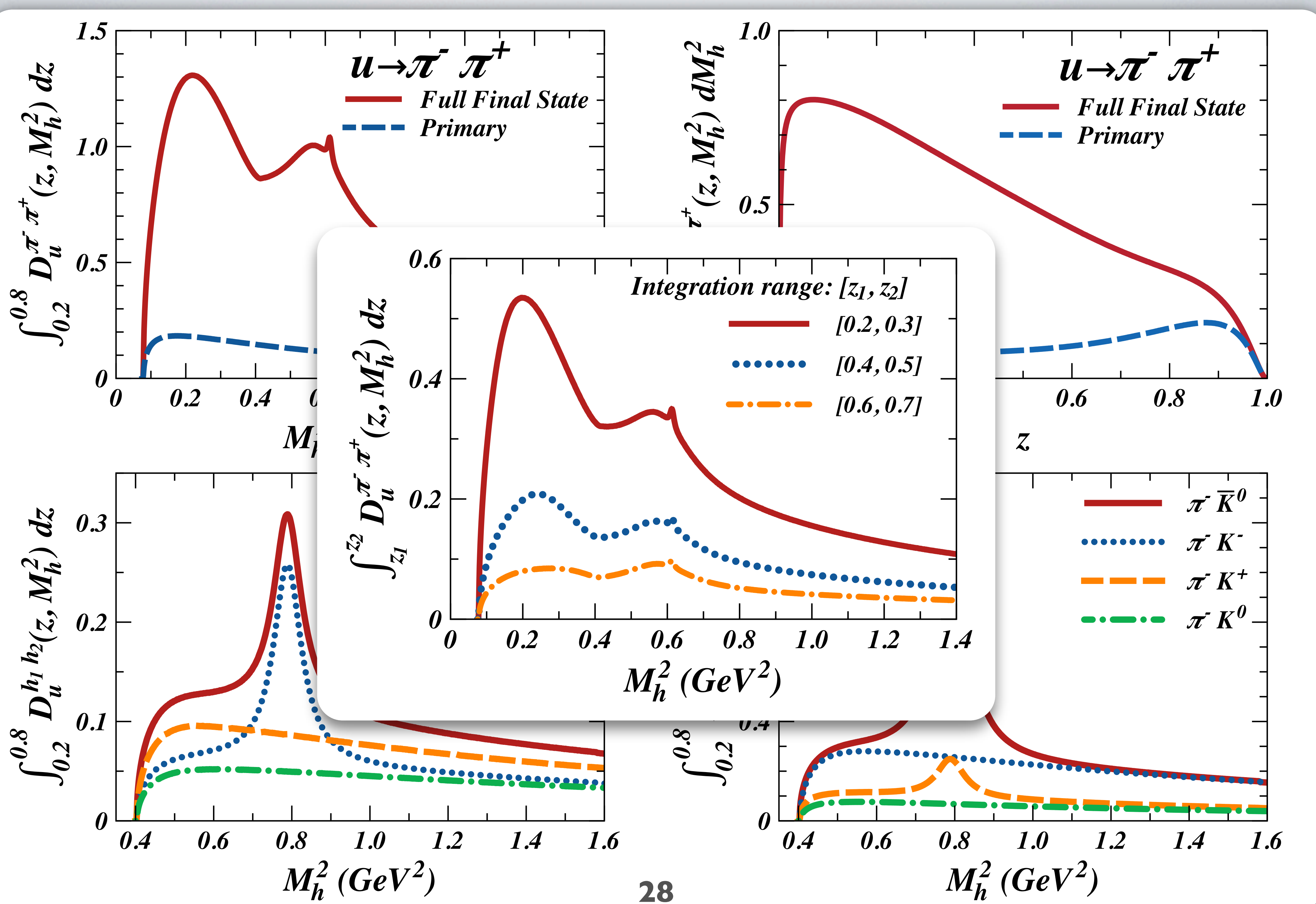
$N_{Links} = 2$



RESULTS FOR DFFS $N_{Links} = 8$



RESULTS FOR DFFS $N_{Links} = 8$

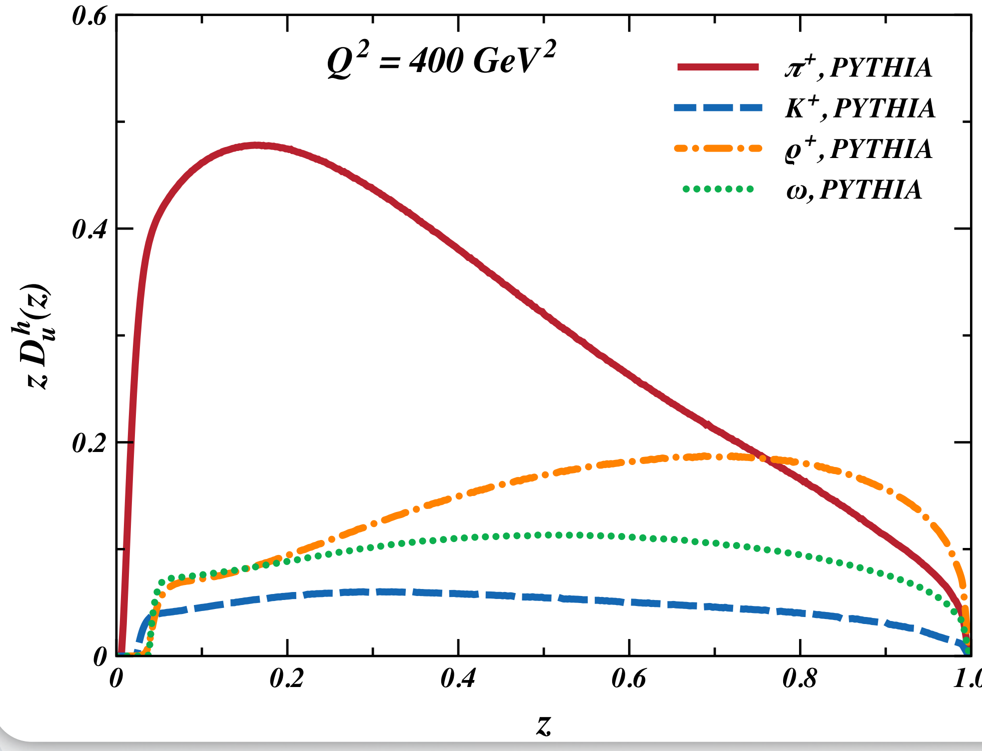


PYTHIA SIMULATIONS

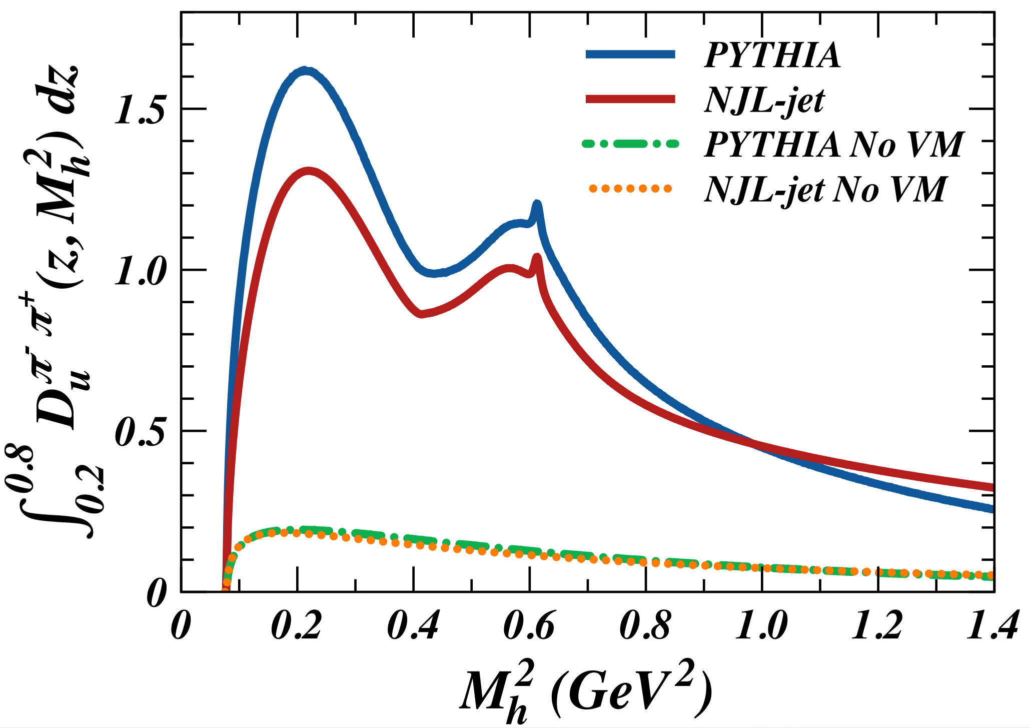
- Setup hard process with back to back $q\ \bar{q}$ along **z** axis.
- ***Only Hadronize.*** Allow the same resonance decays as NJL-jet.
- Assign hadrons with positive p_z to q fragmentation.

$$E_q = 10\ \text{GeV}$$

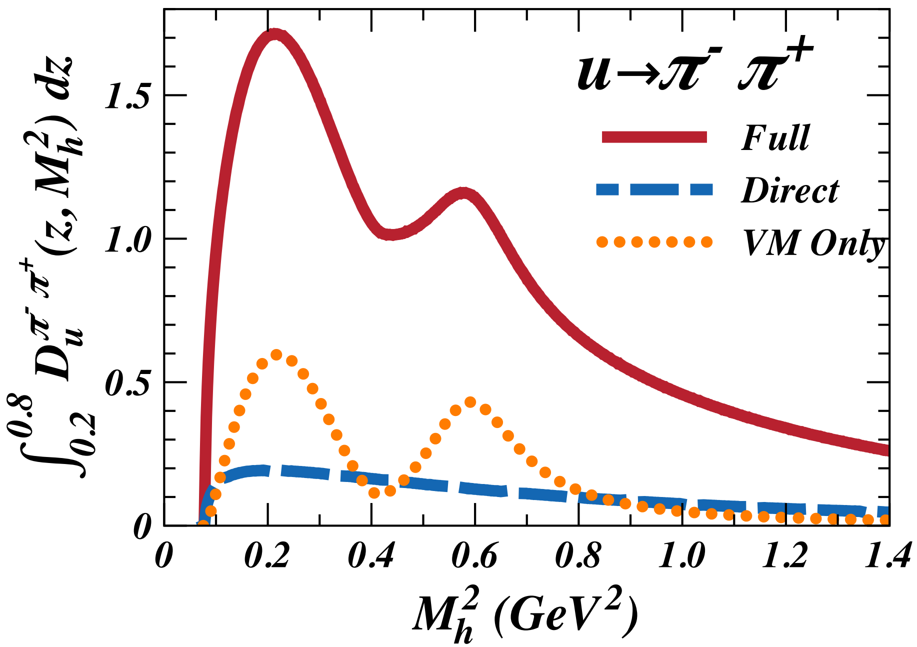
Single Hadron

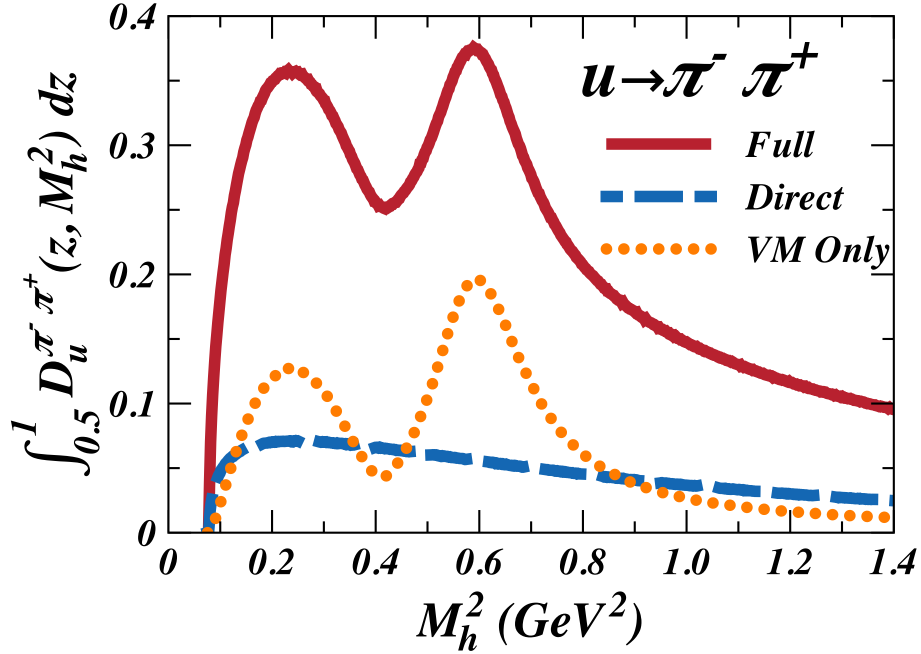


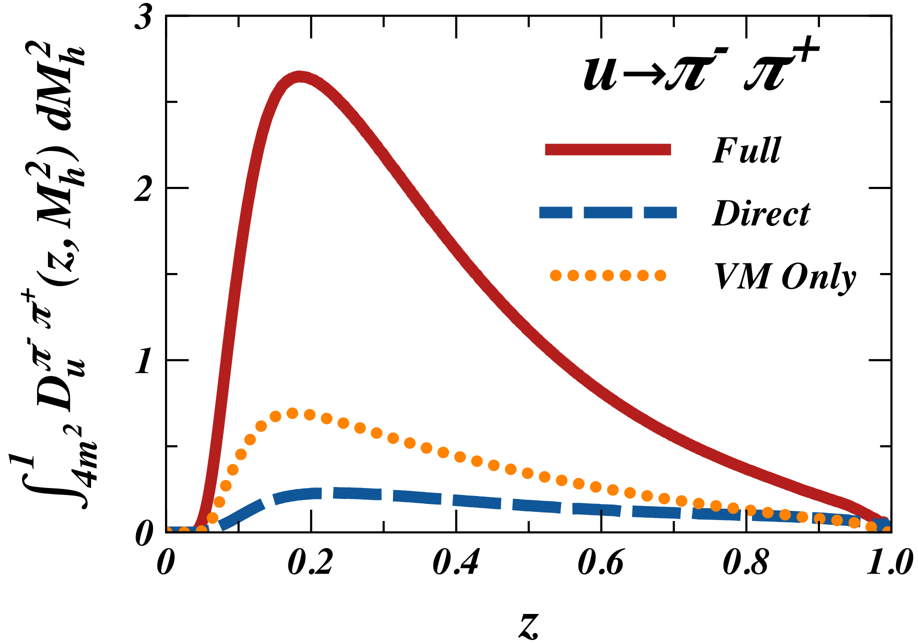
Dihadron



PYTHIA RESULTS FOR $u \to \pi^- \pi^+$







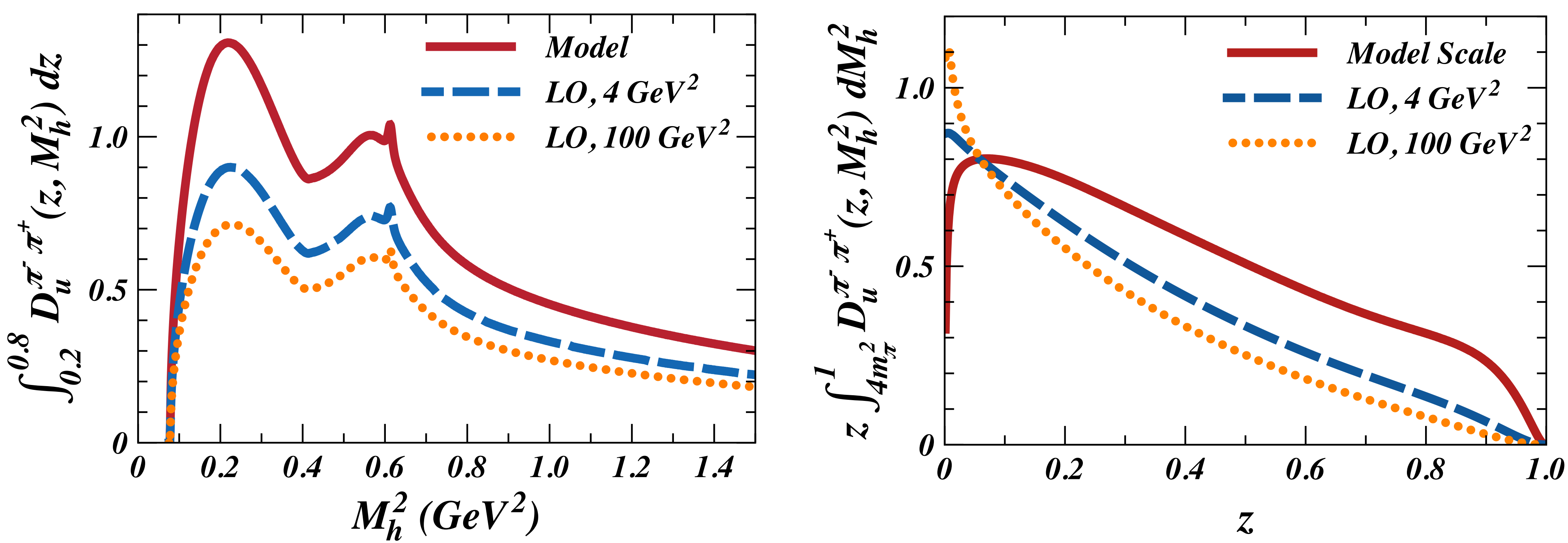
EVOLUTION OF DFF

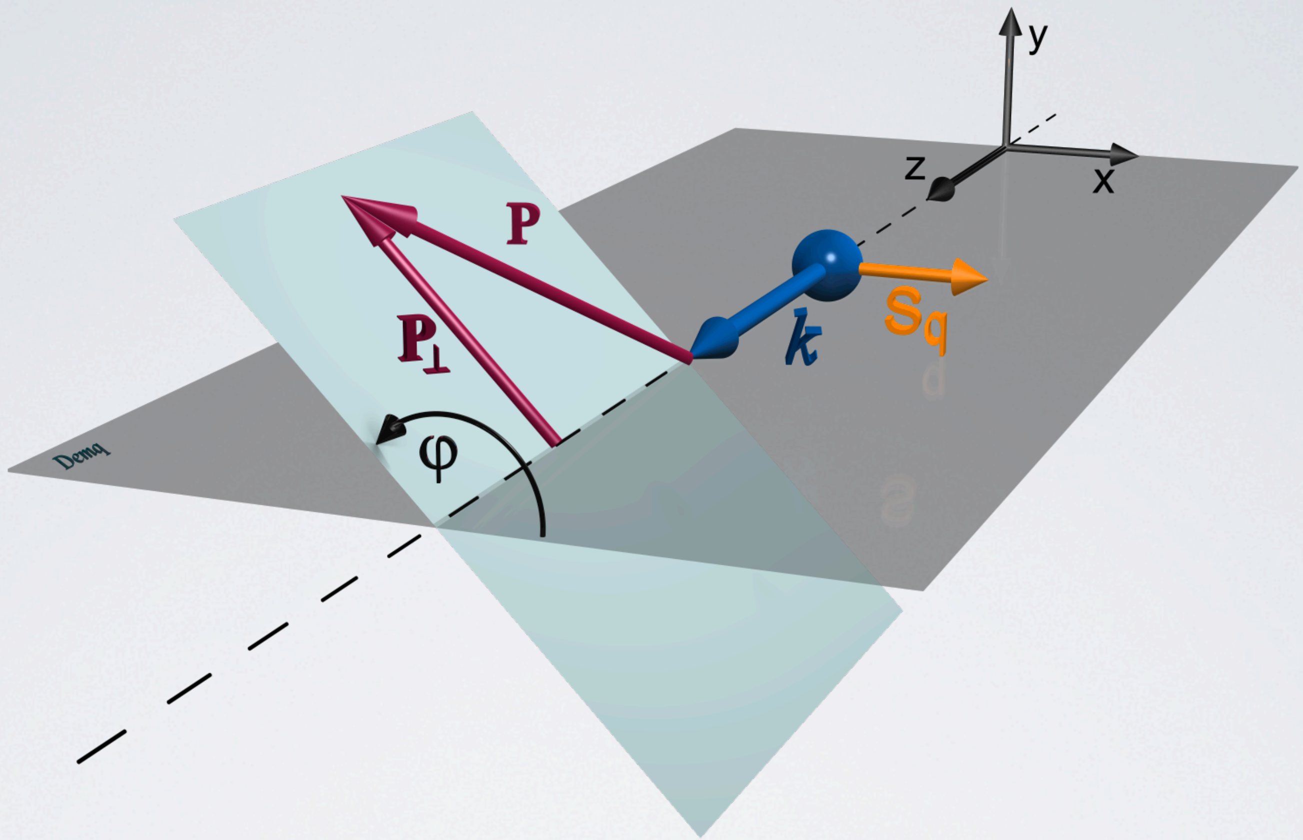
Bacchetta et. al., Phys.Rev. D79, 034029 (2009).

At leading order:

$$\frac{d}{d\log Q^2} D_{1,q}(z, M_h^2, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_z^1 \frac{du}{u} D_{1,q'}\left(\frac{z}{u}, M_h^2, Q^2\right) P_{q'q}(u)$$

$$u \to \pi^- \pi^+$$





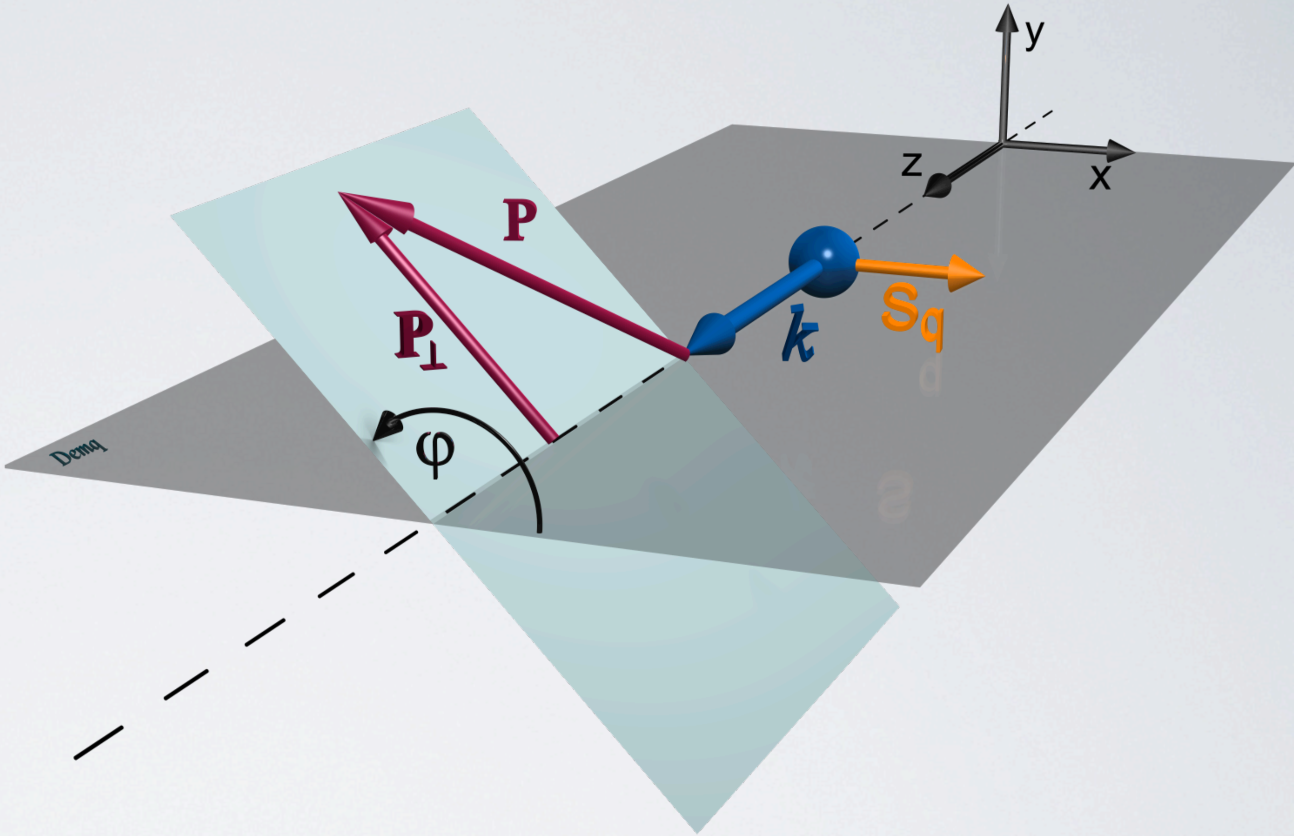
TRANSVERSELY POLARIZED QUARK FRAGMENTATION:

COLLINS EFFECT AND TWO-HADRON CORRELATIONS

COLLINS FRAGMENTATION FUNCTION

- **Collins Effect:**

  Azimuthal Modulation of Transversely Polarized Quark' Fragmentation Function.



Unpolarized

$$D_{h/q^\uparrow}(z, P_\perp^2, \varphi) = D_1^{h/q}(z, P_\perp^2) - H_1^{\perp h/q}(z, P_\perp^2)\frac{P_\perp S_q}{z m_h}\sin(\varphi)$$

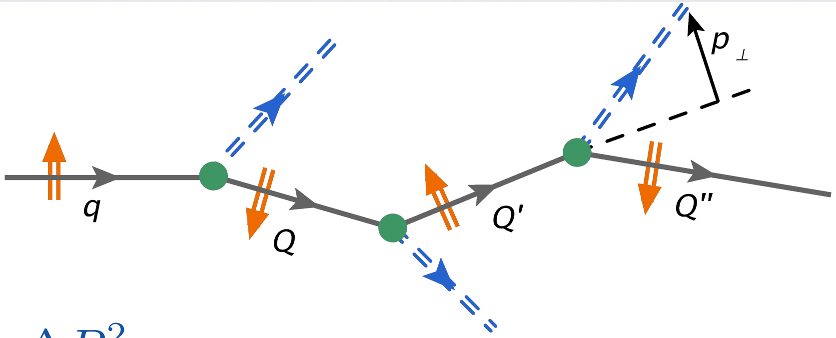
Collins

- **Chiral-ODD: Needs to be coupled with another chiral-odd quantity to be observed.**

COLLINS FRAGMENTATION FUNCTION FROM NJL-JET

H.M.,Bentz, Thomas, PRD.86:034025, 2012.

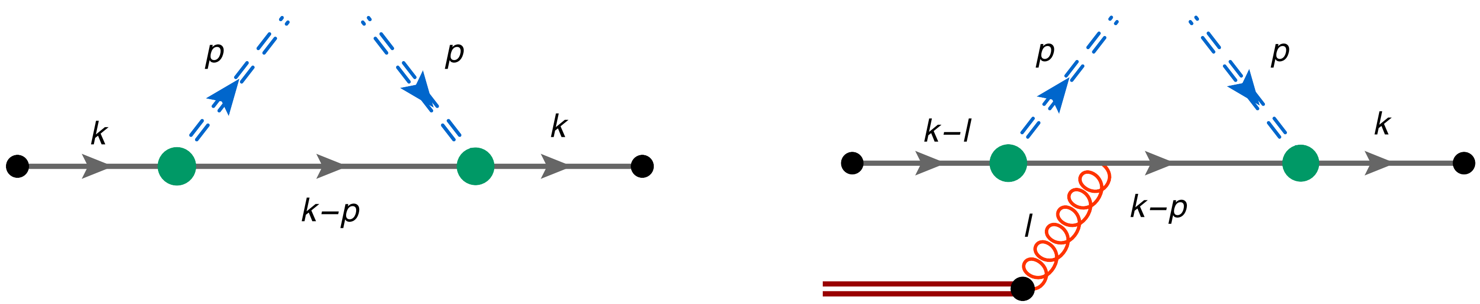
- **Extend the NJL-jet Model to Include the Quark's Spins.**



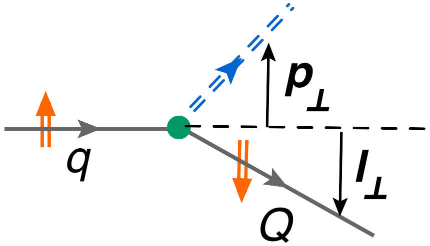
$$D_{h/q^\uparrow}(z, P_\perp^2, \varphi)\, \Delta z\, \frac{\Delta P_\perp^2}{2}\, \Delta\varphi = \left\langle N_{q^\uparrow}^h(z, z+\Delta z; P_\perp^2, P_\perp^2 + \Delta P^2; \varphi, \varphi + \Delta\varphi) \right\rangle$$

- **Model Calculated Elementary Collins Function as Input**

A. Bacchetta et. al., PLB659, 234 (2008).



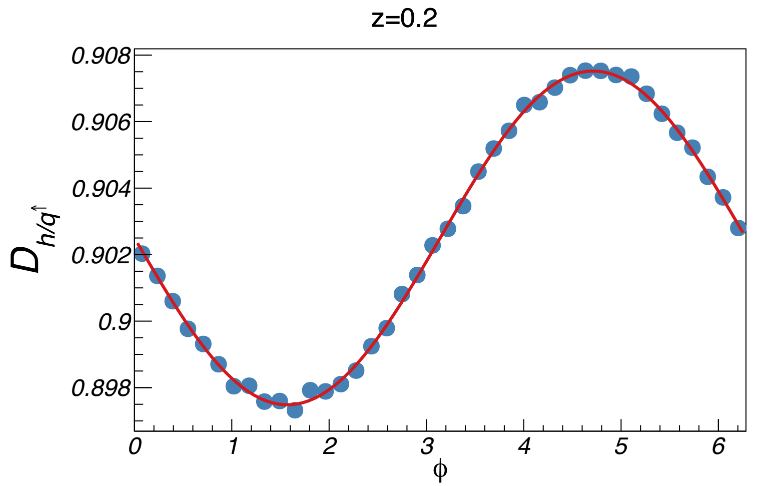
- ***Spin flip probability:*** $\mathcal{P}_{SF}$

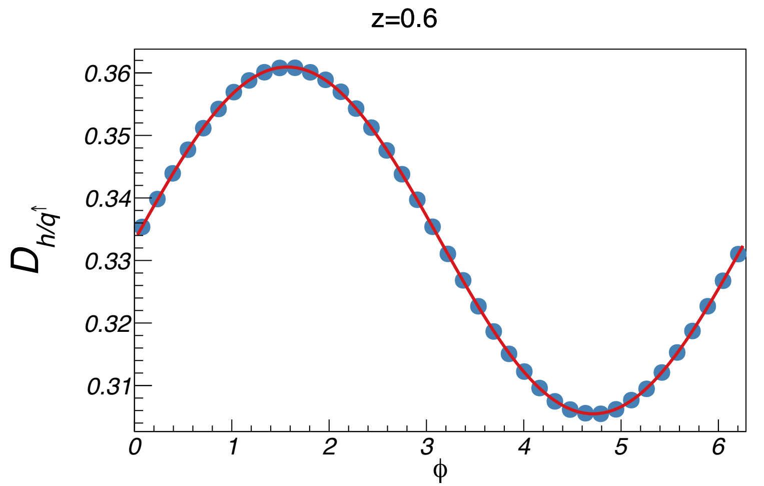


INTEGRATED POLARIZED FRAGMENTATIONS

- Integrate Polarized Fragmentations over $P_\perp^2$

$$D_{h/q^\uparrow}(z,\varphi) \equiv \int_0^\infty dP_\perp^2 \; D_{h/q^\uparrow}(z,P_\perp^2,\varphi) = \frac{1}{2\pi}\left[D_1^{h/q}(z) \; - 2H_{1(h/q)}^{\perp(1/2)}(z) S_q \sin(\varphi)\right]$$

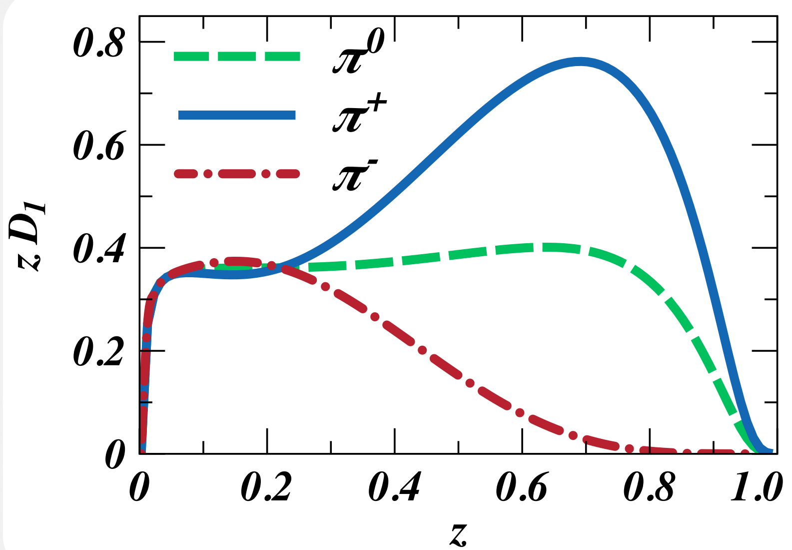




$$D_1^{h/q}(z) \equiv \pi \int_0^\infty dP_\perp^2 \; D_1^{h/q}(z,P_\perp^2)$$

$$H_{1(h/q)}^{\perp(1/2)}(z) \equiv \pi \int_0^\infty dP_\perp^2 \frac{P_\perp}{2zm_h} H_1^{\perp h/q}(z,P_\perp^2)$$

- Fit with form: $F(c_0,c_1) = c_0 - c_1 \sin(\varphi)$



COLLINS EFFECT - MK2

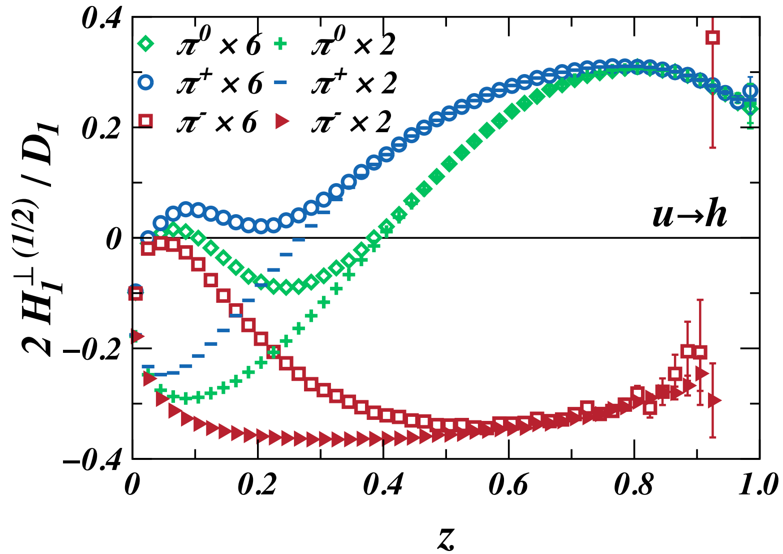
MK2 Model Assumptions: **H.M., Kotzinian, Thomas, PLB731 208-216 (2014).**

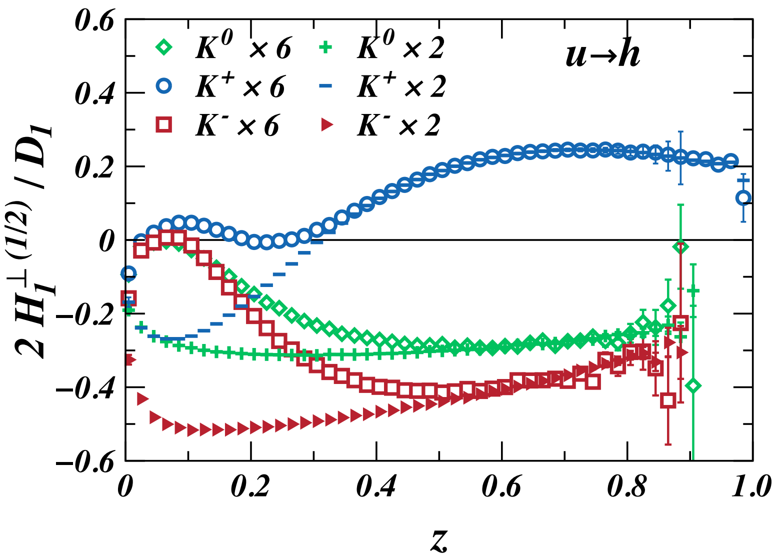
1. Allow for Collins Effect only in a SINGLE emission vertex - N_L^{-1} scaling of the resulting Collins function.
2. Use constant values for $\mathcal{P}_{SF}$.

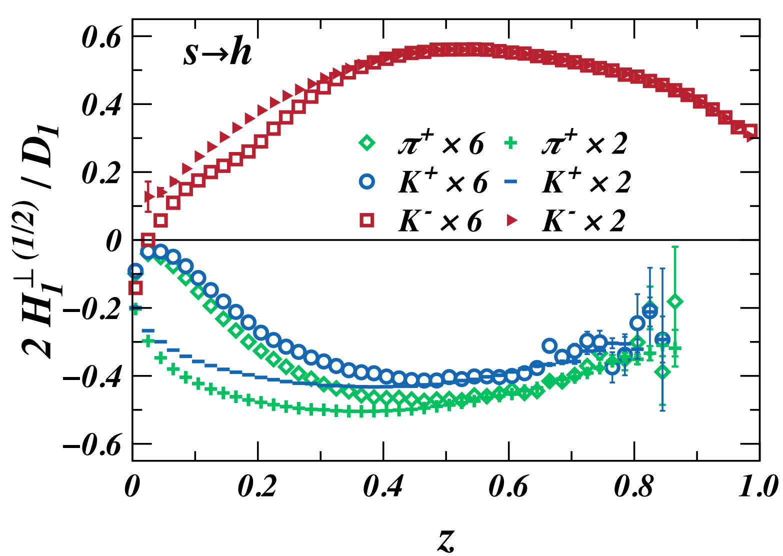
$$\mathcal{P}_{SF} = 1$$

✦ The results for N_L=2 and N_L=6, scaled up by a factor N_L.

$$F(c_0, c_1) = c_0 - c_1 \sin(\varphi)$$







COLLINS EFFECT - MK2

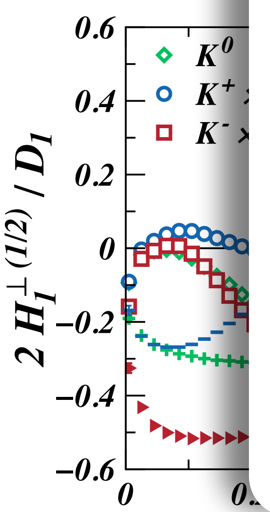
MK2 Model Assumptions:

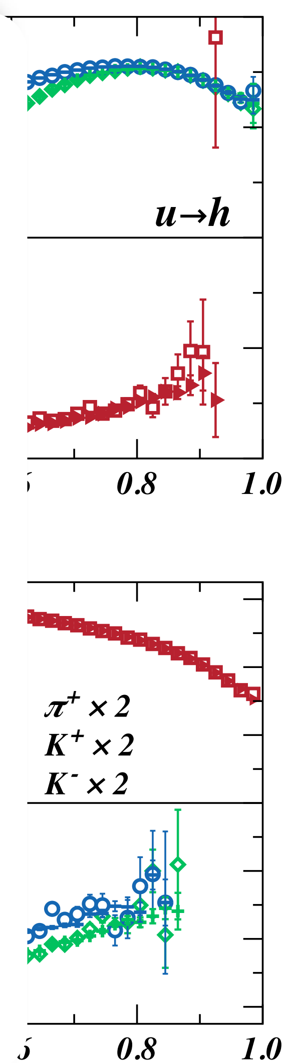
H.M., Kotzinian, Thomas, PLB731 208-216 (2014).

1. Allow for Collins Effect only in a SINGLE emission vertex - N_L^{-1} scaling of the resulting Collins function.
2. Use constant values for $\mathcal{P}_{SF}$.

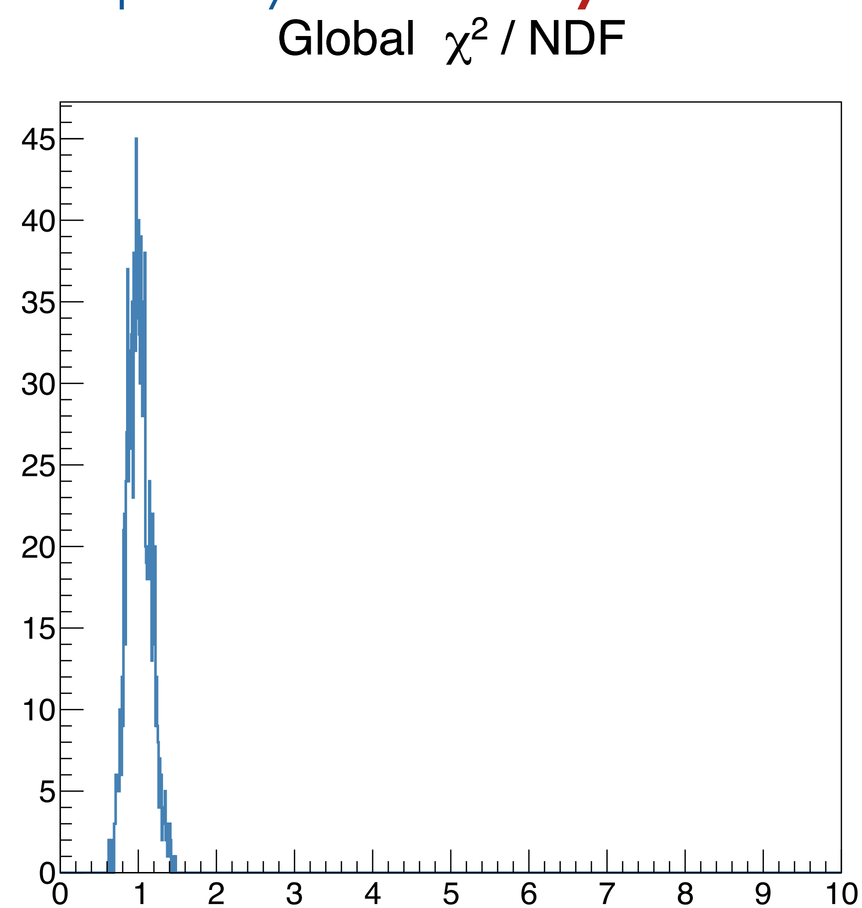
✦ The resu
scaled u

$F(c_0,$





Fit quality for **every** z slice.

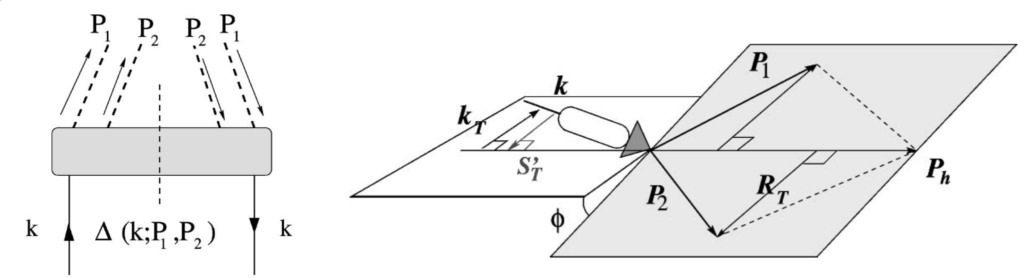


z

z

TWO-HADRON FRAGMENTATION

A. Bianconi, et al: PRD 62, 034008 (2000). M. Radici, et al: PRD 65, 074031 (2002).



- Kinematic Variables:

$$P_1 = \left[\xi P_h^- , \frac{M_1^2 + \vec{R}_T^2}{2\,\xi P_h^-}, \vec{R}_T \right], \qquad k = \left[\frac{P_h^-}{z}, z\frac{k^2 + \vec{k}_T^2}{2 P_h^-}, \vec{k}_T \right] \qquad z \equiv z_h = z_1 + z_2$$

$$P_2 = \left[(1-\xi) P_h^- , \frac{M_2^2 + \vec{R}_T^2}{2(1-\xi) P_h^-}, -\vec{R}_T \right] \qquad \mathbf{R} = \frac{\mathbf{P}_1 - \mathbf{P}_2}{2} \qquad \xi = \frac{z_1}{z_1 + z_2}$$

- The relevant terms of the quark correlator at leading order for a **Transversely Polarized Quark:**

Unpolarized

$$\Delta^{[\gamma^-]} = D_1(z_h, \xi, k_T^2, R_T^2, \mathbf{k}_T \cdot \mathbf{R}_T)$$

Interference

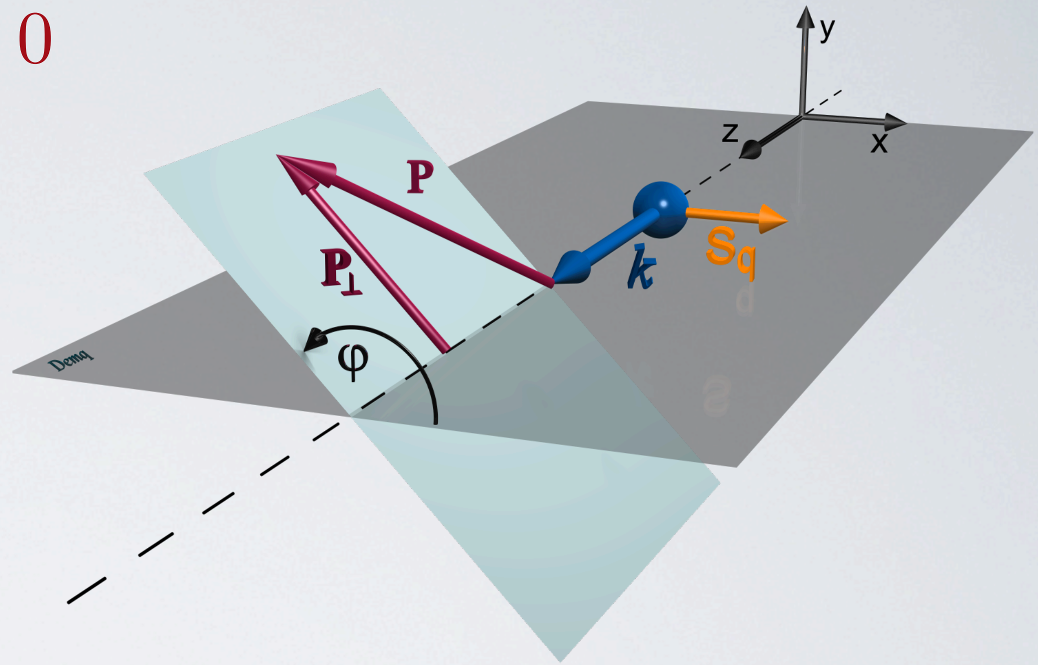
$$\Delta^{[i\sigma^{i-}\gamma_5]} = \frac{\epsilon_T^{ij} R_{Tj}}{M_1 + M_2} H_1^{\sphericalangle}(z_h, \xi, k_T^2, R_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) + \frac{\epsilon_T^{ij} k_{Tj}}{M_1 + M_2} H_1^{\perp}(z_h, \xi, k_T^2, R_T^2, \mathbf{k}_T \cdot \mathbf{R}_T)$$

- **IFFS are Chiral-ODD: Need to be coupled with another chiral-odd quantity to be observed (e.g. transversity).**

TWO-HADRON FRAGMENTATION

✦ **Transformation to frame $\mathbf{k}_T = 0$**

$$k = (k^-, k^+, \mathbf{0})$$
$$\mathbf{k}_T = -\mathbf{P}_T / z_h$$



$$\mathbf{P}_T = \mathbf{P}^{\perp}_{h_1} + \mathbf{P}^{\perp}_{h_2}$$
$$\mathbf{R} = (\mathbf{P}^{\perp}_{h_1} - \mathbf{P}^{\perp}_{h_2})/2$$

✦ **Integrate over one or other momentum:**

$$D^{h_1 h_2}_{q^\uparrow}(\varphi_R) = D^{h_1 h_2}_{1,q} + \sin(\varphi_R - \varphi_S)\mathcal{F}[H_1^{\sphericalangle}, H_1^{\perp}]$$
$$D^{h_1 h_2}_{q^\uparrow}(\varphi_T) = D^{h_1 h_2}_{1,q} + \sin(\varphi_T - \varphi_S)\mathcal{F}'[H_1^{\sphericalangle}, H_1^{\perp}]$$

✦ **The IFF surviving after $\mathbf{k}_T$ integration is redefined as**

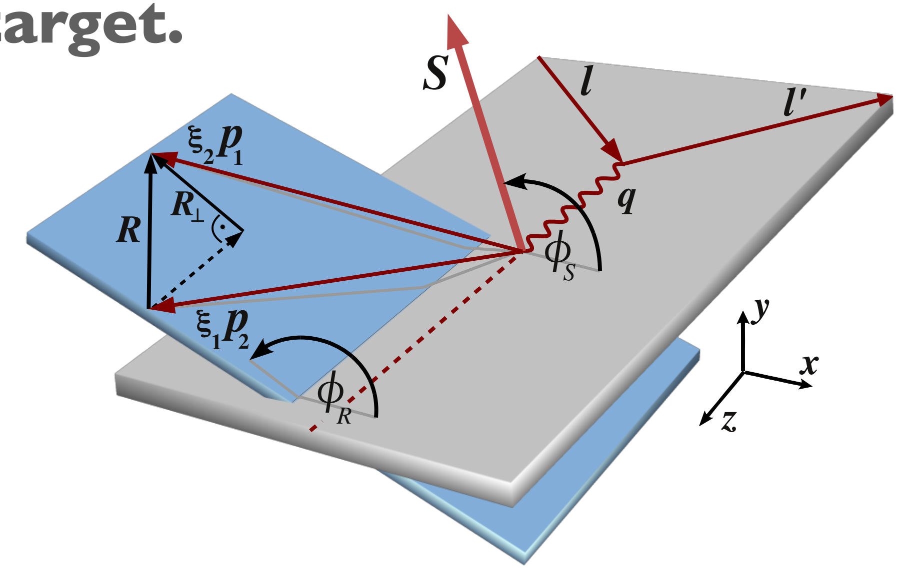
A. Bacchetta, M. Radici: PRD 69, 074026 (2004).

$$H_1^{\sphericalangle}(z_h, \xi, M_h^2) \equiv \int d^2\mathbf{k}_T \left[H_1^{\sphericalangle' e}(z_h, \xi, M_h^2, k_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) + \frac{k_T^2}{2M_h^2} H_1^{\perp o}(z_h, \xi, k_T^2, R_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) \right]$$

RECENT COMPASS RESULTS

COMPASS, PLB736, 124-131 (2014).

- **SIDIS with transversely polarized target.**



- **Collins single spin asymmetry:**

$$A_{Coll} = \frac{\sum_q e_q^2 \, \Delta_T q \otimes H_1^{\perp h/q}}{\sum_q e_q^2 \; q \otimes D_1^{h/q}}$$

- **Two hadron single spin asymmetry:**

$$A_{UT}^{\sin\phi_{RS}} = \frac{|\boldsymbol{p}_1 - \boldsymbol{p}_2|}{2M_{h^+h^-}} \frac{\sum_q e_q^2 \cdot h_1^q(x) \cdot H_{1,q}^{\sphericalangle}(z,\, M_{h^+h^-}^2,\, \cos\theta)}{\sum_q e_q^2 \cdot f_1^q(x) \cdot D_{1,q}(z,\, M_{h^+h^-}^2,\, \cos\theta)}$$

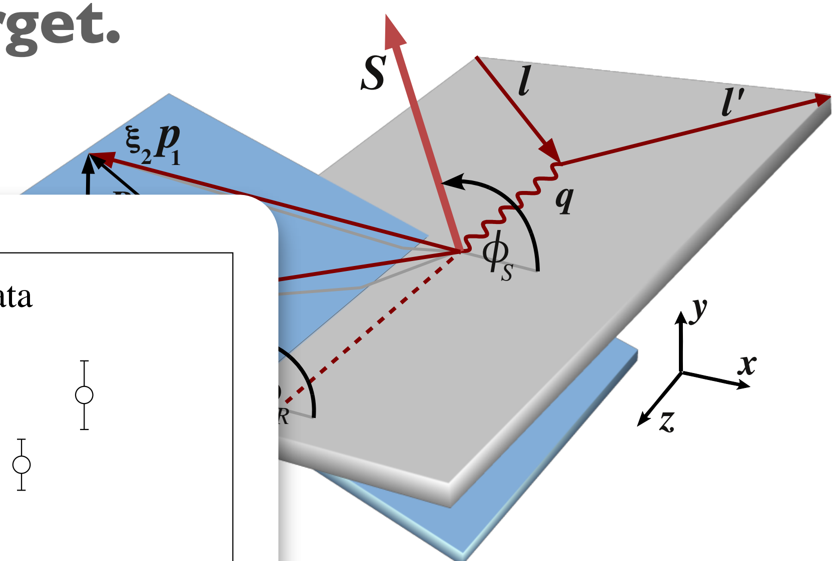
- **Note the choice of the vector**

$$\boldsymbol{R}_{Artru} = \frac{z_2 \boldsymbol{P}_1 - z_1 \boldsymbol{P}_2}{z_1 + z_2}$$

RECENT COMPASS RESULTS

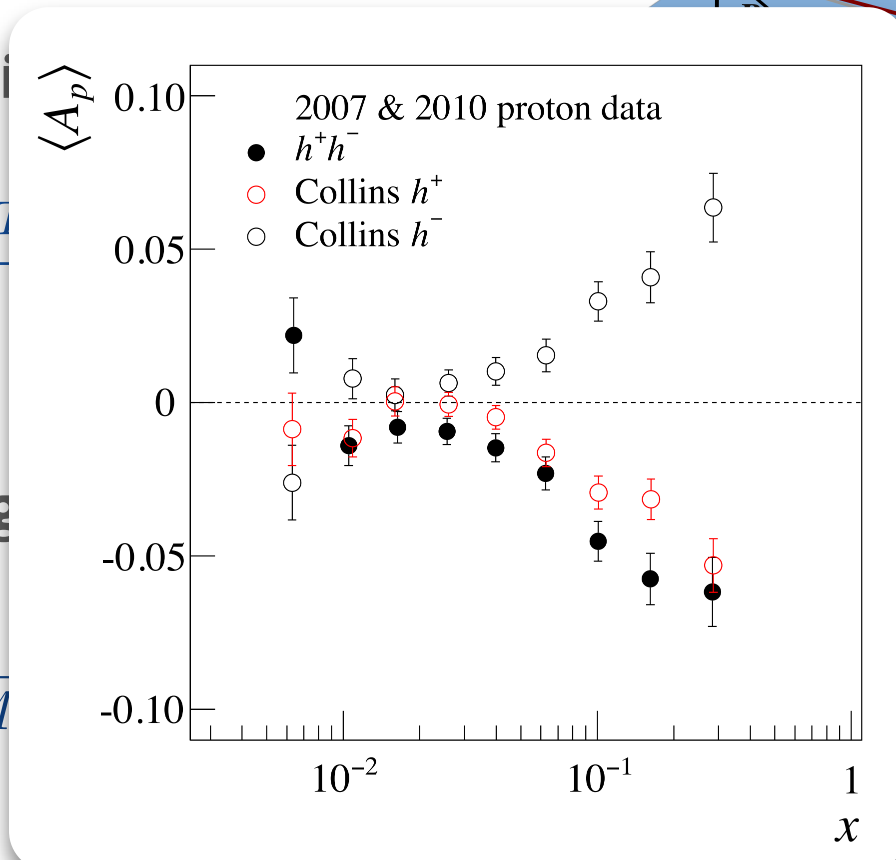
COMPASS, PLB736, 124-131 (2014).

✦ **SIDIS with transversely polarized target.**



✦ **Collins single spi**

$$A_{Coll} = \frac{\sum_q e_q^2 \, \Delta_T}{\sum_q e_q^2}$$



✦ **Two hadron sing**

$$A_{UT}^{\sin \phi_{RS}} = \frac{|\boldsymbol{p}_1}{2M} \quad \frac{\cos\theta)}{\cos\theta)}$$

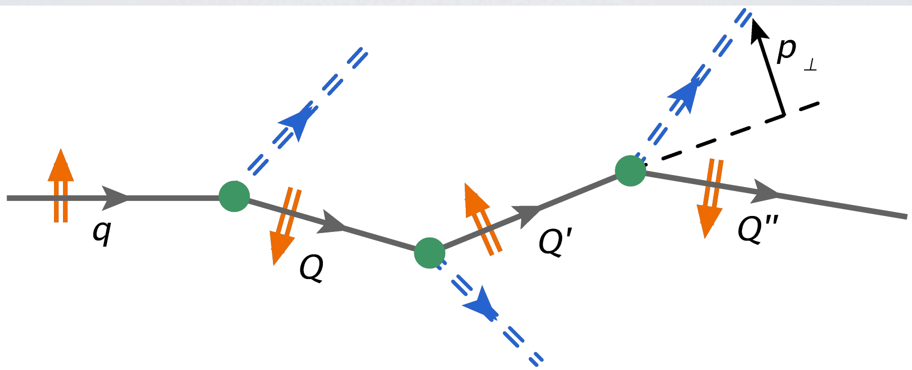
✦ **Note the choice of the vector**

$$\boldsymbol{R}_{Artru} = \frac{z_2 \boldsymbol{P}_1 - z_1 \boldsymbol{P}_2}{z_1 + z_2}$$

POLARIZED QUARK DIFF IN QUARK-JET.

H.M., Kotzinian, Thomas, PLB731 208-216 (2014).

- **Use the NJL-jet Model including Collins effect (Mk 2) to study DiFFs.**



$$D_{q^\uparrow}^{h_1 h_2}(z, M_h^2, \varphi_R)\ \Delta z\ \Delta M_h^2\ \Delta\varphi_R = \left\langle N_{q^\uparrow}^{h_1 h_2}(z, z+\Delta z; M_h^2, M_h^2+\Delta M_h^2; \varphi_R, \varphi_R+\Delta\varphi_R)\right\rangle.$$

- ***Choose a constant Spin flip probability:*** $\mathcal{P}_{SF}$

- ***Simple model to start with:***

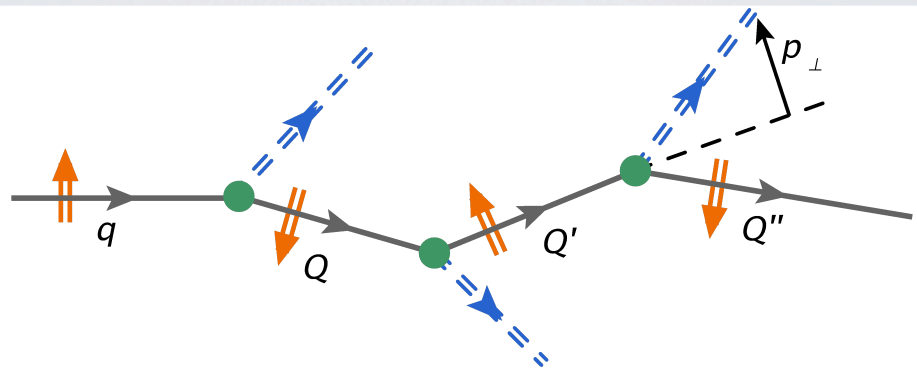
  Only pions and extreme ansatz for the Collins term in elementary function.

$$d_{h/q^\uparrow}(z, \mathbf{p}_\perp) = d_1^{h/q}(z, p_\perp^2)(1 - 0.9\sin\varphi)$$

POLARIZED QUARK DIFF IN QUARK-JET.

H.M., Kotzinian, Thomas, PLB731 208-216 (2014).

- **Use the NJL-jet Model including Collins effect (Mk 2) to study DiFFs.**



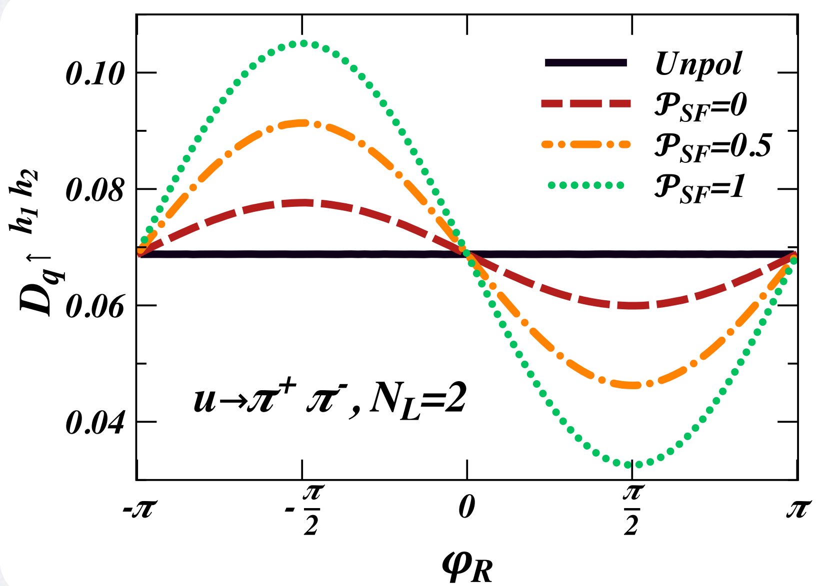
$$D_{q^\uparrow}^{h_1 h_2}(z, M_h^2, \varphi_R)\ \Delta z\ \Delta M_h^2\ \Delta\varphi_R = \left\langle N_{q^\uparrow}^{h_1 h_2}(z, z+\Delta z; M_h^2, M_h^2+\Delta M_h^2; \varphi_R, \varphi_R+\Delta\varphi_R)\right\rangle .$$

- ***Choose a constant Spin flip probability:*** $\mathcal{P}_{SF}$

- ***Simple model to start with:***

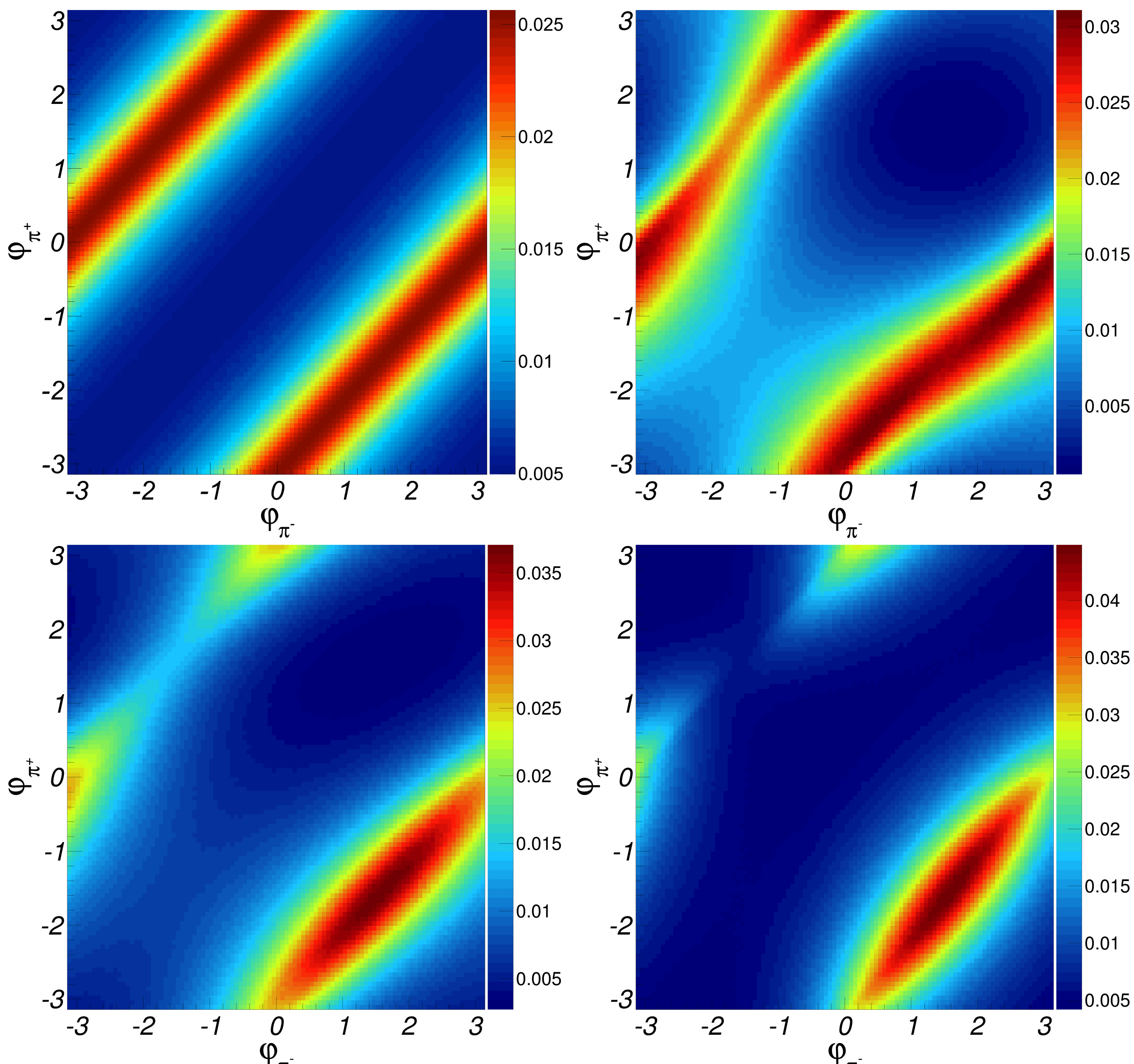
  Only pions and extreme ansatz for the Collins term in elementary function.

$$d_{h/q^\uparrow}(z, \mathbf{p}_\perp) = d_1^{h/q}(z, p_\perp^2)(1 - 0.9\sin\varphi)$$



ANGULAR CORRELATIONS: $u \to \pi^+\pi^-$

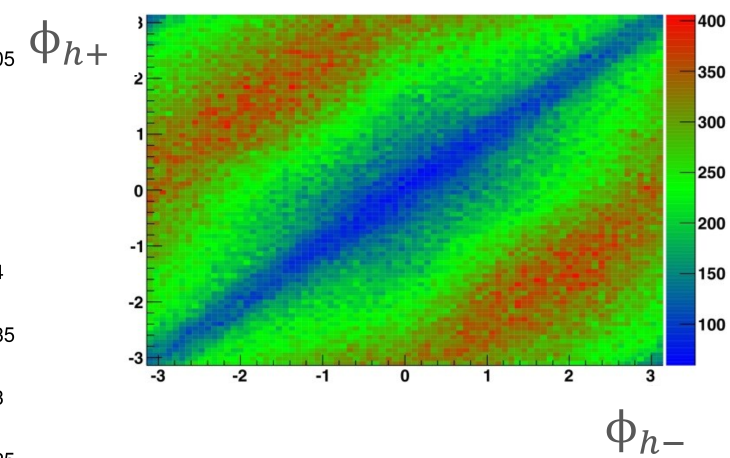
Unpolarized | $\mathcal{P}_{SF} = 0$



$\mathcal{P}_{SF} = 0.5$ | $\mathcal{P}_{SF} = 1$

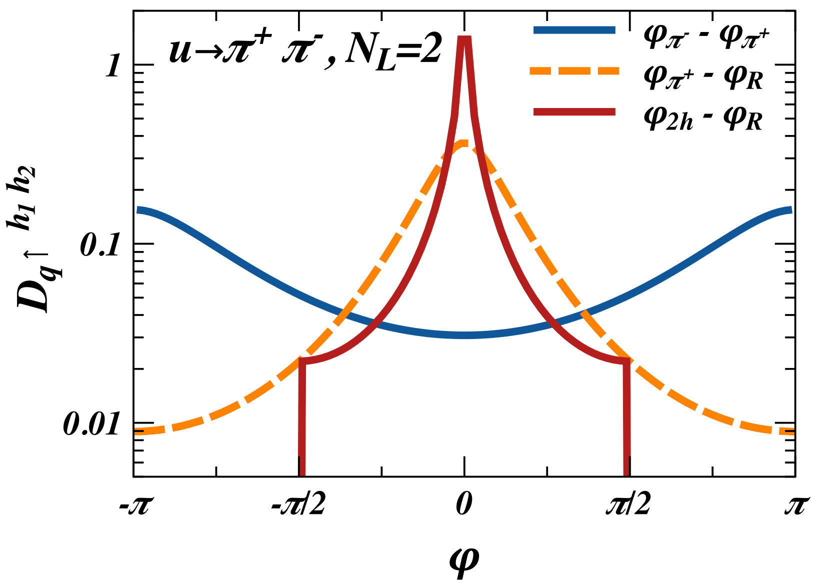
COMPASS Preliminary:

F. Bradamante - COMO 2013.



ANGULAR CORRELATIONS: $u \to \pi^+\pi^-$

Quark-Jet



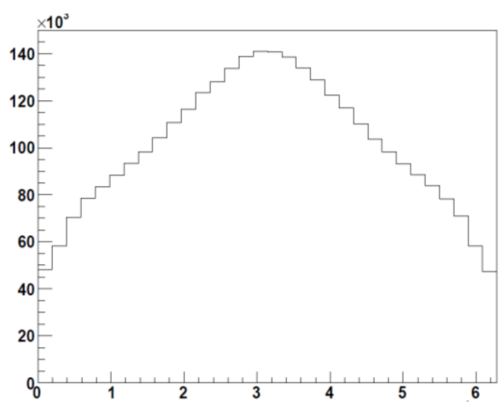
- We define:

$$\mathbf{P}_{2h} = \frac{\widehat{\mathbf{P}}_1 - \widehat{\mathbf{P}}_2}{2}$$

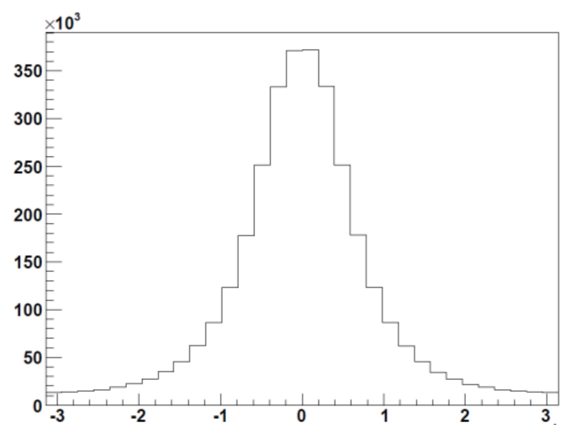
- No Spin Dependence Included!

COMPASS Results

F. Bradamante - COMO 2013.

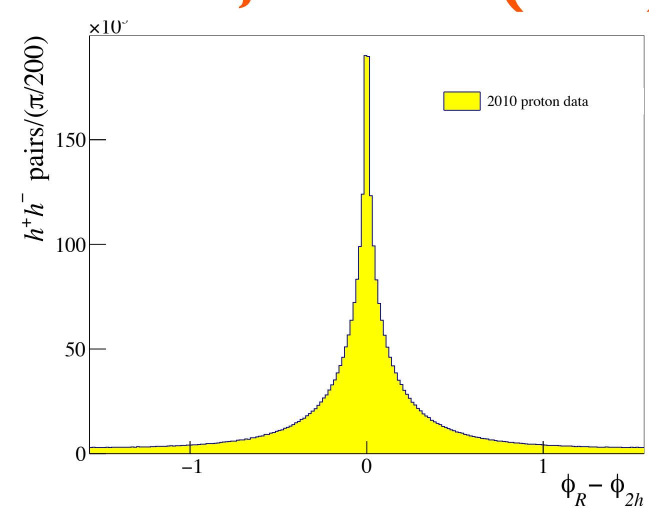


$\phi_{h+} - \phi_{h-}$

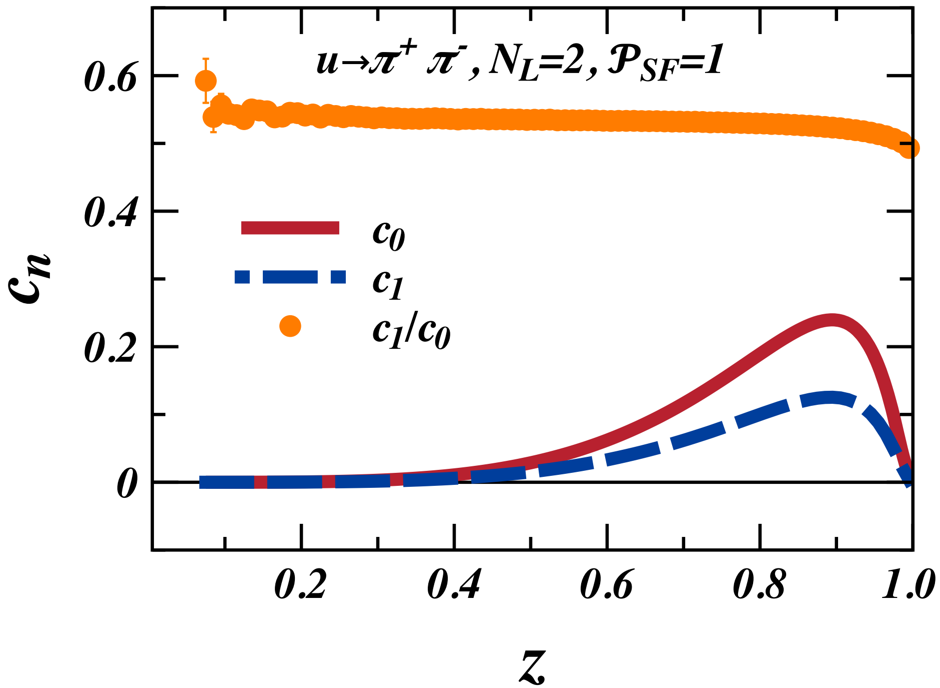


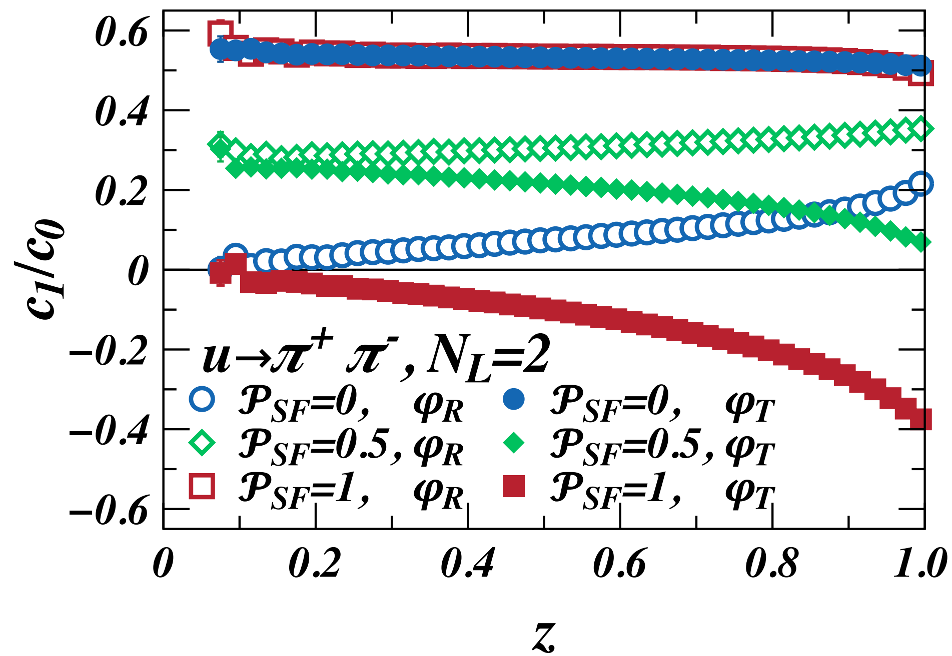
$\phi_{h+} - \phi_R$

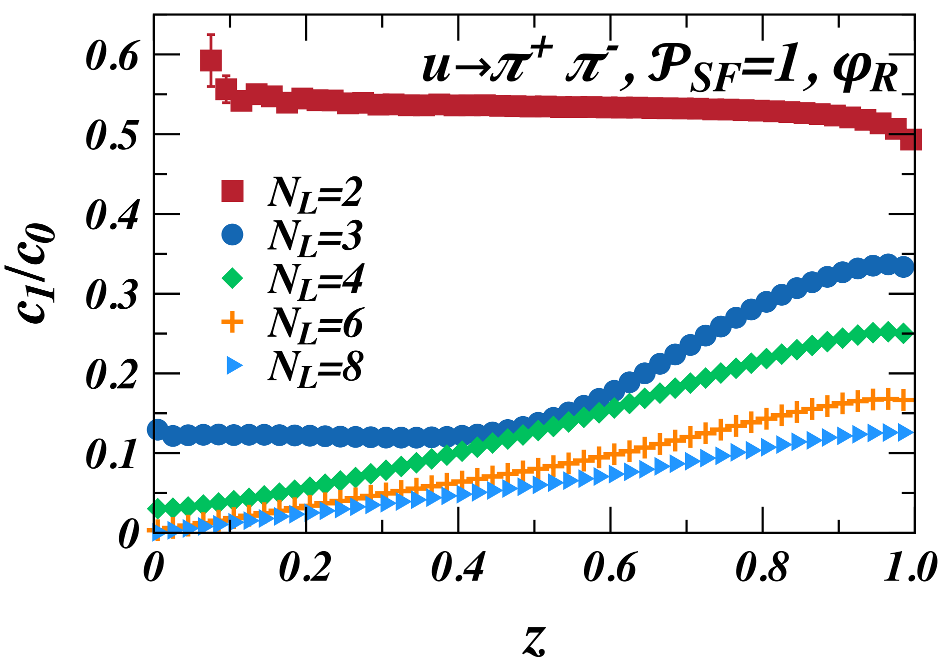
PLB736, 124-131 (2014).

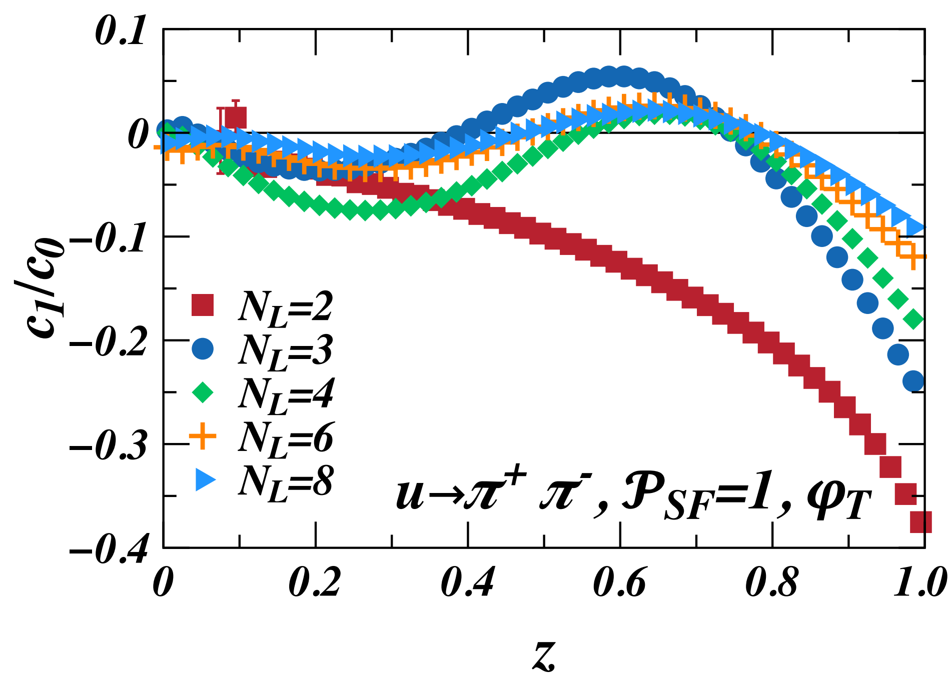


ANALYZING POWERS



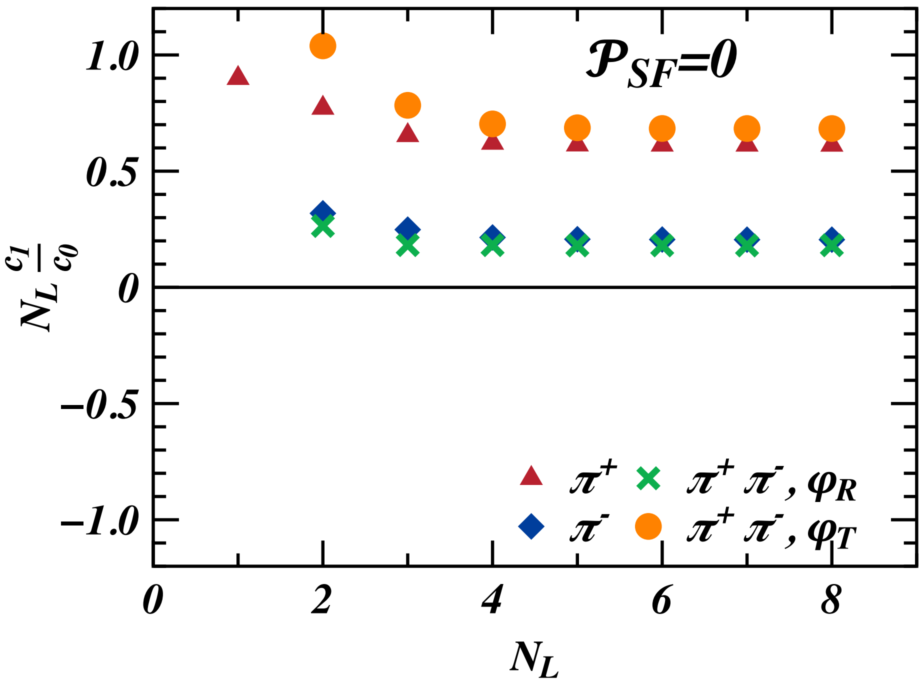


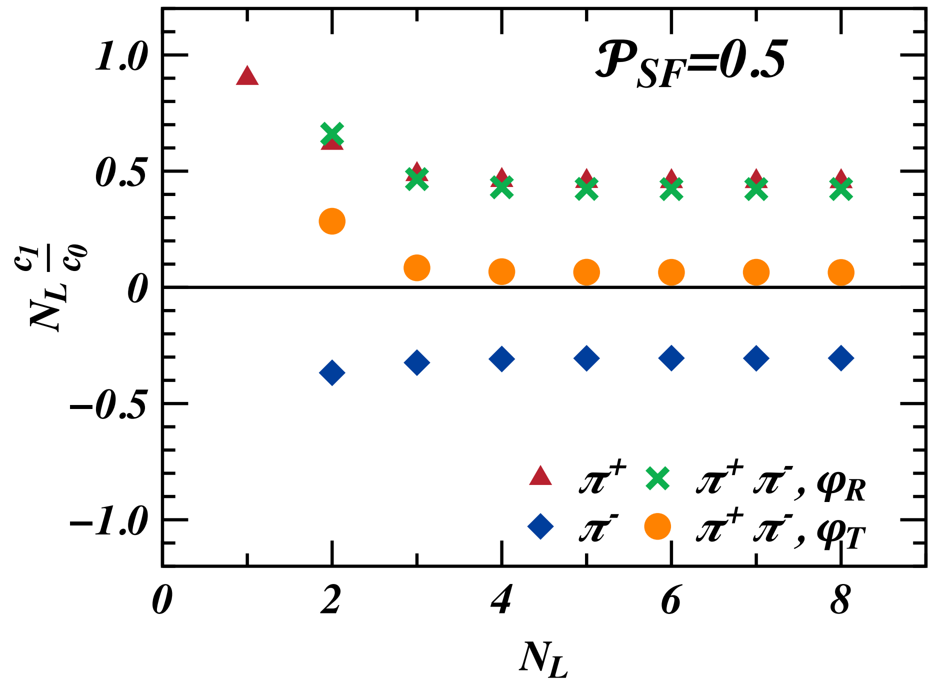


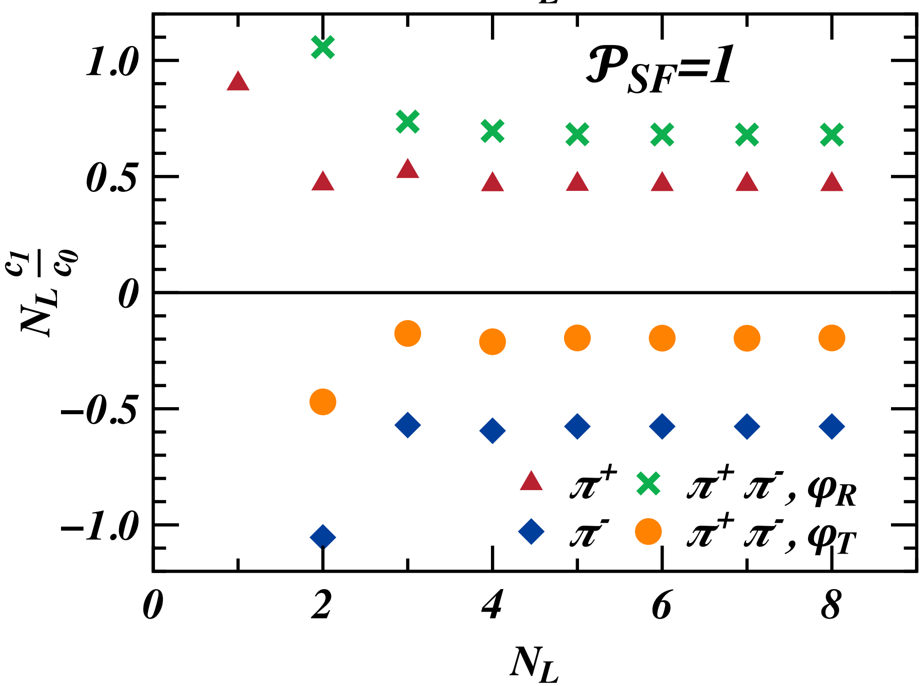


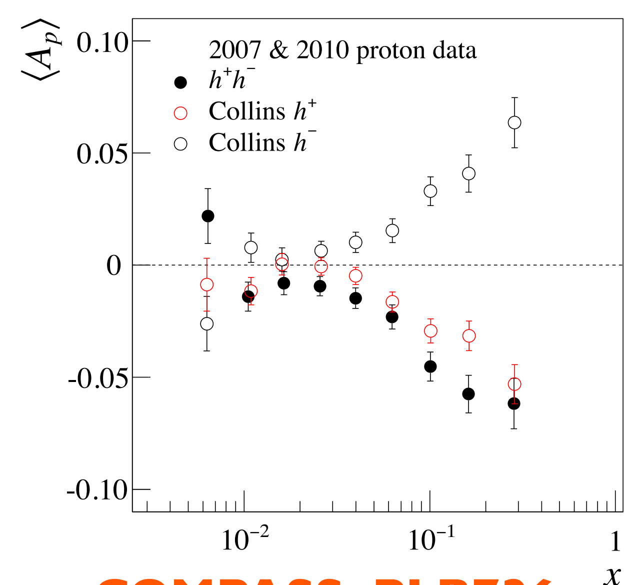
INTEGRATED ANALYZING POWERS

$z_{1,2} > 0.2, z > 0.2$









COMPASS: PLB736, 124-131 (2014).

Single Hadron

$$N_h \propto \sigma_{UU}(1+\sin(\phi_C)A_C G)$$

$$\phi_C = \phi_h - \phi_{S'}$$
$$= \phi_h + \phi_S - \pi$$

$$A_C = \frac{\sum_q e_q^2\, \Delta_T q \otimes H_1^{\perp h/q}}{\sum_q e_q^2\, q \otimes D_1^{h/q}}$$

DiHadron

$$N_{h^+h^-} \propto \sigma_{UU}(1+\sin(\phi_{RS})A_{UT}^{\sin\phi_{RS}} F)$$

$$\phi_{RS} = \phi_R - \phi_{S'}$$
$$= \phi_R + \phi_S - \pi$$

$$A_{UT}^{\sin\phi_{RS}} \propto \frac{\sum_q e_q^2 \cdot h_1^q \cdot H_{1,q}^{\sphericalangle}}{\sum_q e_q^2 \cdot f_1^q \cdot D_{1,q}}$$

NJL-Jet Fits

$$D_{q^\uparrow} = c_0 - \sin(\phi) c_1$$

124-131 (2014).

INTEGRATED ANALYZING POWERS

Single Hadron

$$N_h \propto \sigma_{UU}(1 + \sin(\phi_C) A_C G)$$

$$\phi_C = \phi_h - \phi_{S'}$$
$$= \phi_h + \phi_S - \pi$$

$$A_C = \frac{\sum_q e_q^2 \, \Delta_T q \otimes H_1^{\perp h/q}}{\sum_q e_q^2 \, q \otimes D_1^{h/q}}$$

DiHadron

$$N_{h^+h^-} \propto \sigma_{UU}(1 + \sin(\phi_{RS}) A_{UT}^{\sin\phi_{RS}} F)$$

$$\phi_{RS} = \phi_R - \phi_{S'}$$
$$= \phi_R + \phi_S - \pi$$

$$A_{UT}^{\sin\phi_{RS}} \propto \frac{\sum_q e_q^2 \cdot h_1^q \cdot H_{1,q}^{\sphericalangle}}{\sum_q e_q^2 \cdot f_1^q \cdot D_{1,q}}$$

NJL-Jet Fits

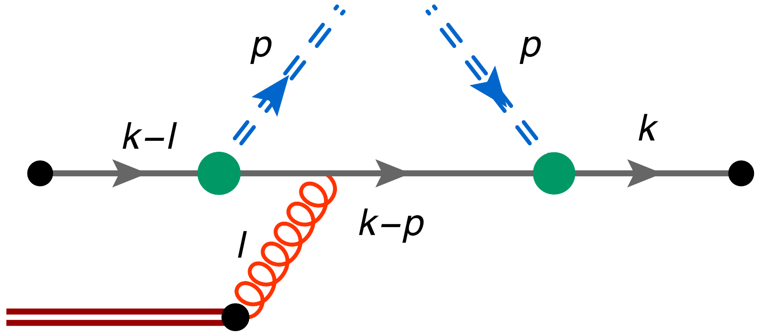
$$D_{q\uparrow} = c_0 - \sin(\phi) c_1$$

124-131 (2014).

IMPROVED MODEL FOR COLLINS EFFECT

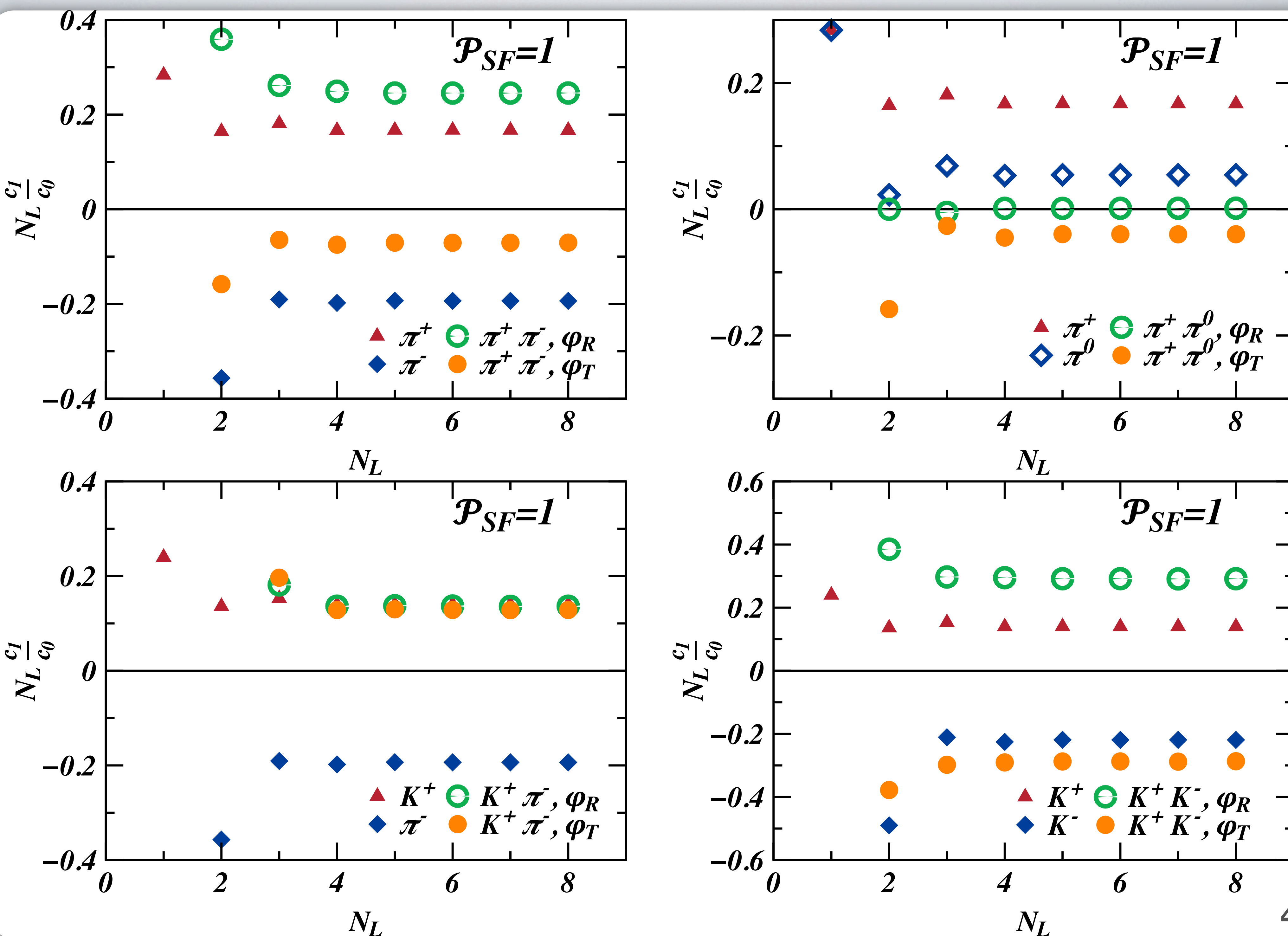
✦**Use the *spectator model* for Collins function.**

$$H_1^{\perp h/q}(z, P_\perp^2)\frac{P_\perp S_q}{z m_h}\sin(\varphi)$$



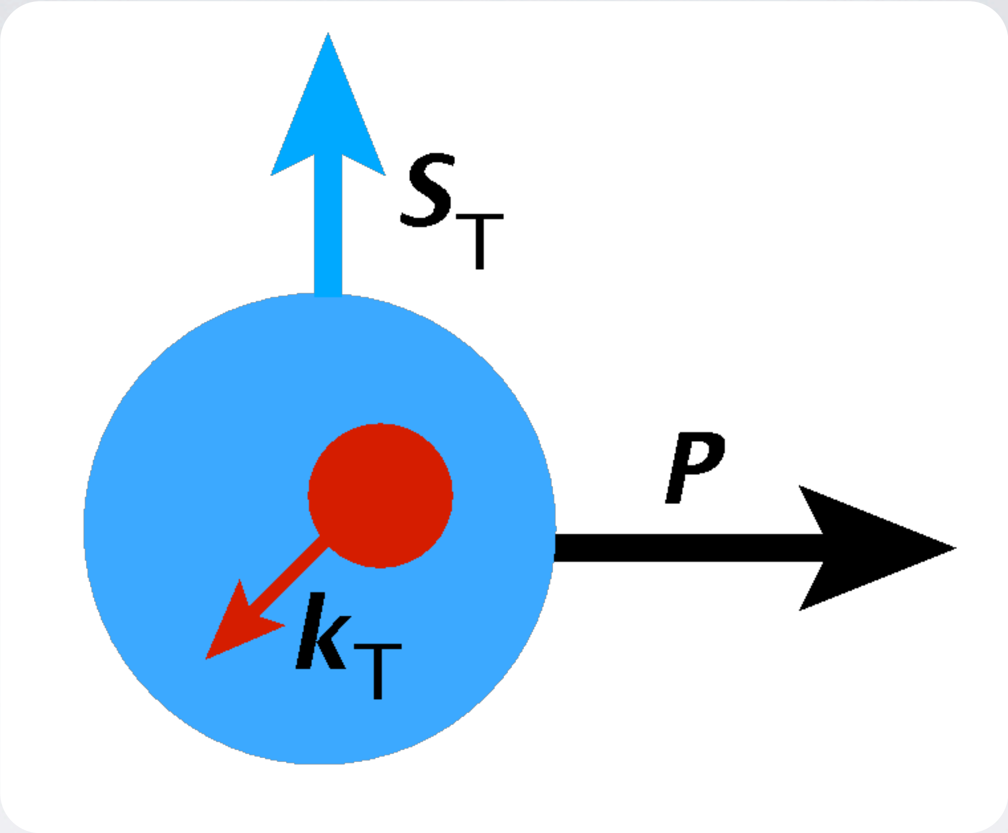
✦**Include both *pion and kaon* channels.**

IMPROVED MODEL RESULTS



CONCLUSIONS: NJL-jet

- ***Multi-hadron*** emissions are ***essential*** to complete description of both Favored and Unfavored fragmentation functions!
- The ***NJL-Jet*** model provides a ***robust*** and ***extendable*** framework for microscopic description of various fragmentation phenomena using MC simulations: TMD, Collins, DiHadron.
- ***NJL-Jet*** MC helps us to test and understand important aspects of various processes using a specific underlying quark model:
  - **z** dependence of $\langle P_\perp^2 \rangle$.
  - Effect of VM decays on Dihadron FFs.
  - The role of the Collins mechanism in IFFs.
- Further developments of the model are underway:
  - Including ***vector mesons*** in polarized fragmentations.
  - Exploring the ***target fragmentation***.



CHALLENGES IN EXTRACTING SIVERS PDF FROM SIDIS

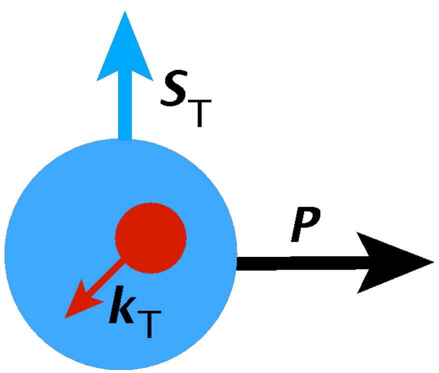
SIVERS PDF

N/q	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	$h_1\ h_{1T}^\perp$

D. Sivers, Phys.Rev. D41 (1990).

- Correlation of k_T and S_T
- Proposed by Dennis Sivers in 1990 to explain the single spin asymmetry in $pp^\uparrow \to \pi + X$

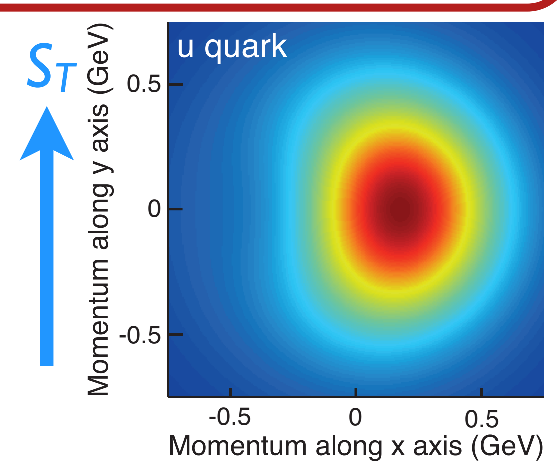
$S_T k_T \sin(\varphi_k - \varphi_S)$



$$f_\uparrow^q(x, \vec{k}_T) = f_1^q(x, k_T) + \frac{[\vec{S} \times \vec{k}_T]_3}{M} f_{1T}^{\perp q}(x, k_T)$$

- Naively *T-odd*, gauge-link should be included in the definition.
- Accessible in Polarized SIDIS, Drell-Yan.

$$f_{1T}^{\perp SIDIS} = -f_{1T}^{\perp DY}$$



EIC White Paper, arXiv:1212.1701

mPYTHIA 6.4

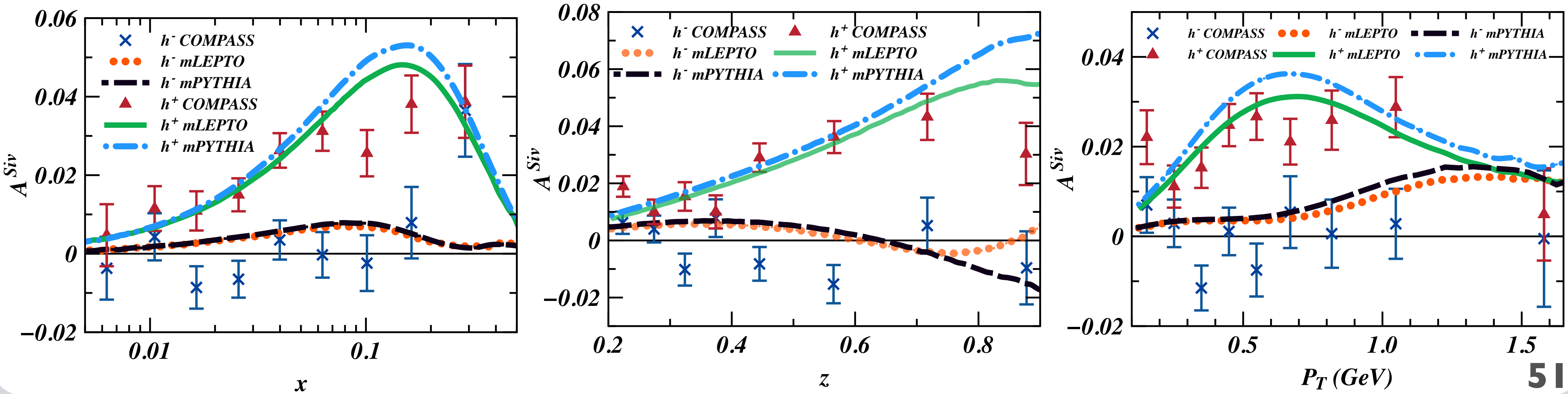
Sivers Effect in PYTHIA and Simulations for SIDIS(/DY/PP)

EVENT GENERATORS + SIVERS EFFECT

Kotzinian, H.M., Thomas: PRL.113, 062003 ; PRD.90, 074006 ; 1407.6572 (2014);

- Use ***PYTHIA 6.4*** (and ***LEPTO*** earlier) (*F77-yuk*).
- Incorporate dynamical hadronization mechanism: one, two,... hadron FFs.
- Sivers effect modulates quark TM's azimuthal angle: *relatively easy* to include in MC generators.
- Use Sivers PDF extraction from *Torino group*.
- Event generators allow to study *exp. kinematics effects*.

❖ ***Does it work?***



LO APPROXIMATION FOR SSA

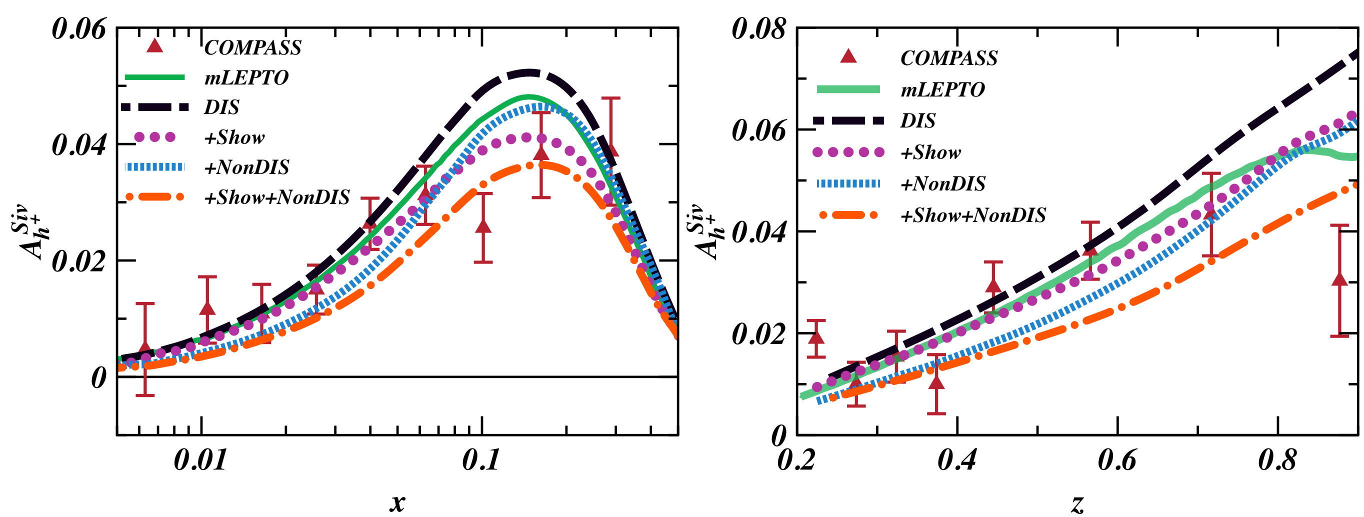
- Fits for *Sivers PDF* from HERMES and COMPASS data utilize *LO DIS-only* expressions for *SSAs*.

M. Anselmino et. al: PRD 86, 014028 (2012).

$$A_{UT}^{\sin(\phi_h-\phi_S)} = \frac{\sum_q \int d\phi_S d\phi_h d^2\boldsymbol{k}_\perp \Delta^N \hat{f}_{q/p^\uparrow}(x,k_\perp,Q)\sin(\varphi-\phi_S)\frac{d\hat{\sigma}^{\ell q\to \ell q}}{dQ^2}\hat{D}_q^h(z,p_\perp,Q)\sin(\phi_h-\phi_S)}{\sum_q \int d\phi_S d\phi_h d^2\boldsymbol{k}_\perp \hat{f}_{q/p}(x,k_\perp,Q)\frac{d\hat{\sigma}^{\ell q\to \ell q}}{dQ^2}\hat{D}_q^h(z,p_\perp,Q)}.$$

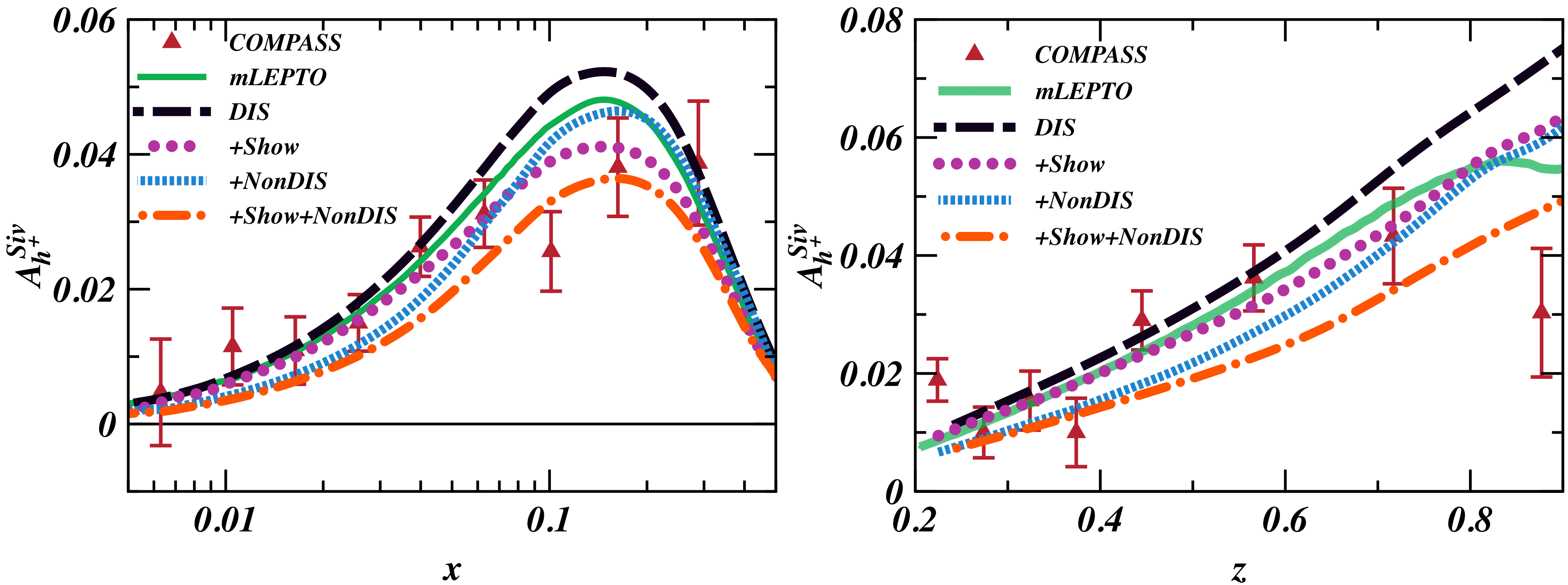
- **Is this justified at COMPASS energies?**
- ***Test using mPYTHIA: turn on non-DIS effects (VMD, GVMD, "direct") and parton showering (QCD+QED).***

H.M et al., arXiv:1502.02669 (2015).



LO APPROXIMATION FOR SSA

H.M et al., arXiv:1502.02669 (2015).

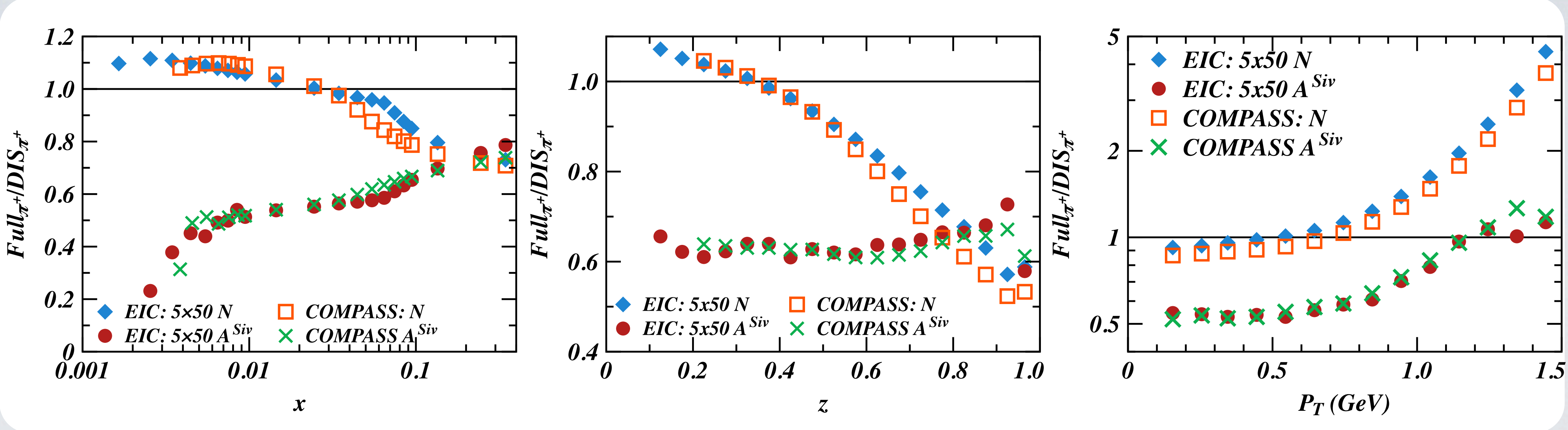


- **Significant** effects, but still agrees with data!
- Current Sivers PDF extractions *may* be underestimated.
- Note: no *model-independent* way to exclude non-DIS effects.

Can We Still Use These Parametrizations?

H.M et al., arXiv:1502.02669 (2015).

- ***How reliable are our SSA predictions for other experiments?***
- ***Construct Ratios of Full (non-DIS + showers) to LO DIS results for multiplicities and Sivers SSAs at COMPASS and EIC.***



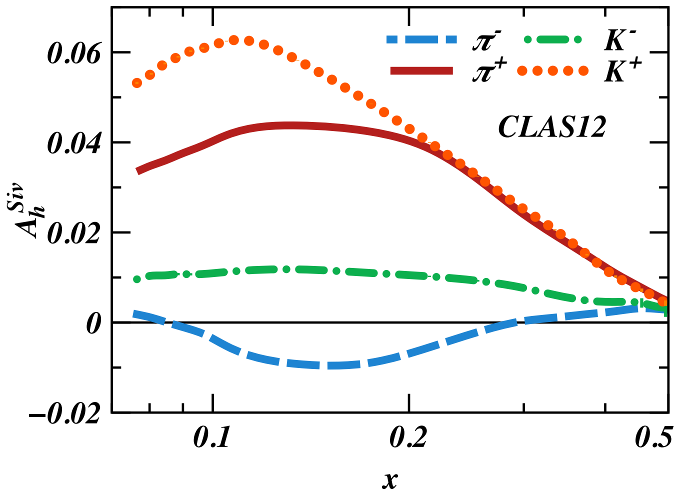
- The Ratios are *very close* between *COMPASS* and *EIC*:
- We can *reliably estimate* SSAs if we use only *LO DIS* terms with the *current parametrization* of Sivers PDFs.

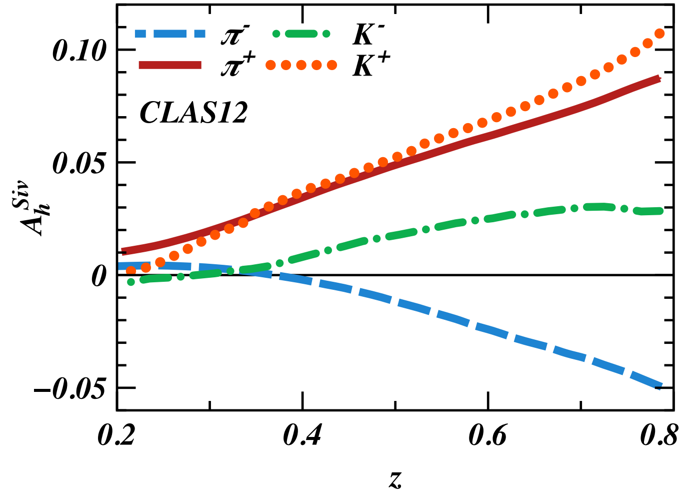
Sivers SSAs

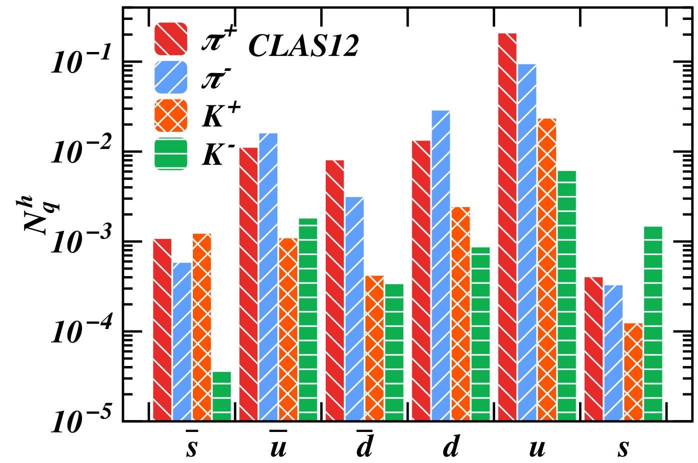
H.M et al., arXiv:1502.02669 (2015).

❖ ***Exploring large x at CLAS 12 and small x at EIC .***

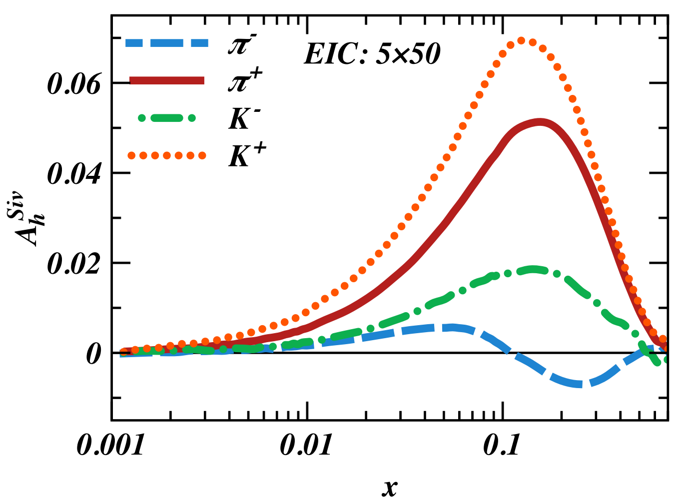
✦ **CLAS 12.**

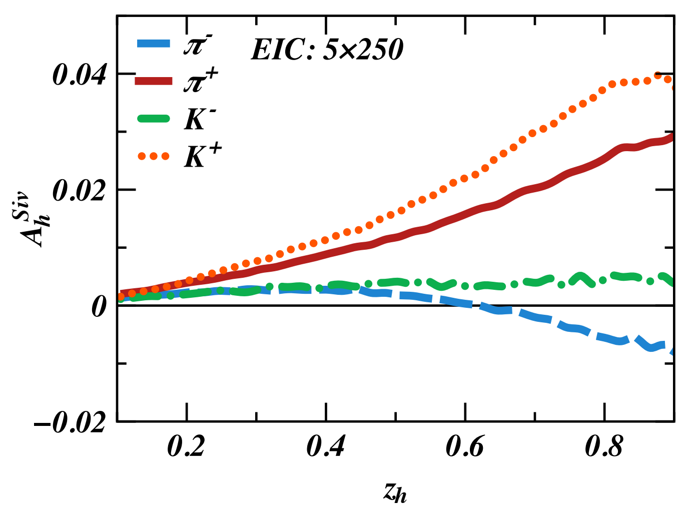


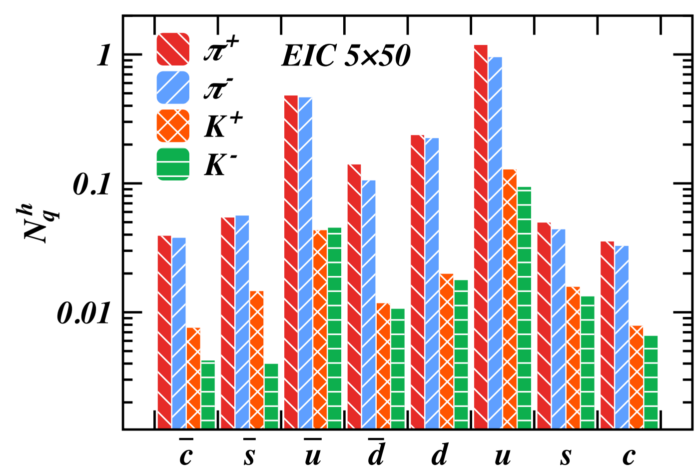




✦ **EIC 5x50 GeV (e p).**







✦ **Significant (measurable?) SSAs!**

CONCLUSIONS II: SIVERS

- LEPTO and PYTHIA ***MC*** event generators have been modified to predict Sivers SSAs for both one and two hadrons:
  - mLEPTO for COMPASS: *dihadron SSA ≅ single hadron SSA*.
  - CLAS12, SoLID and EIC predictions: Measurable SSAs.
  - *Non-DIS processes and showers* should be considered in the extractions of the Sivers PDF .


Thanks!

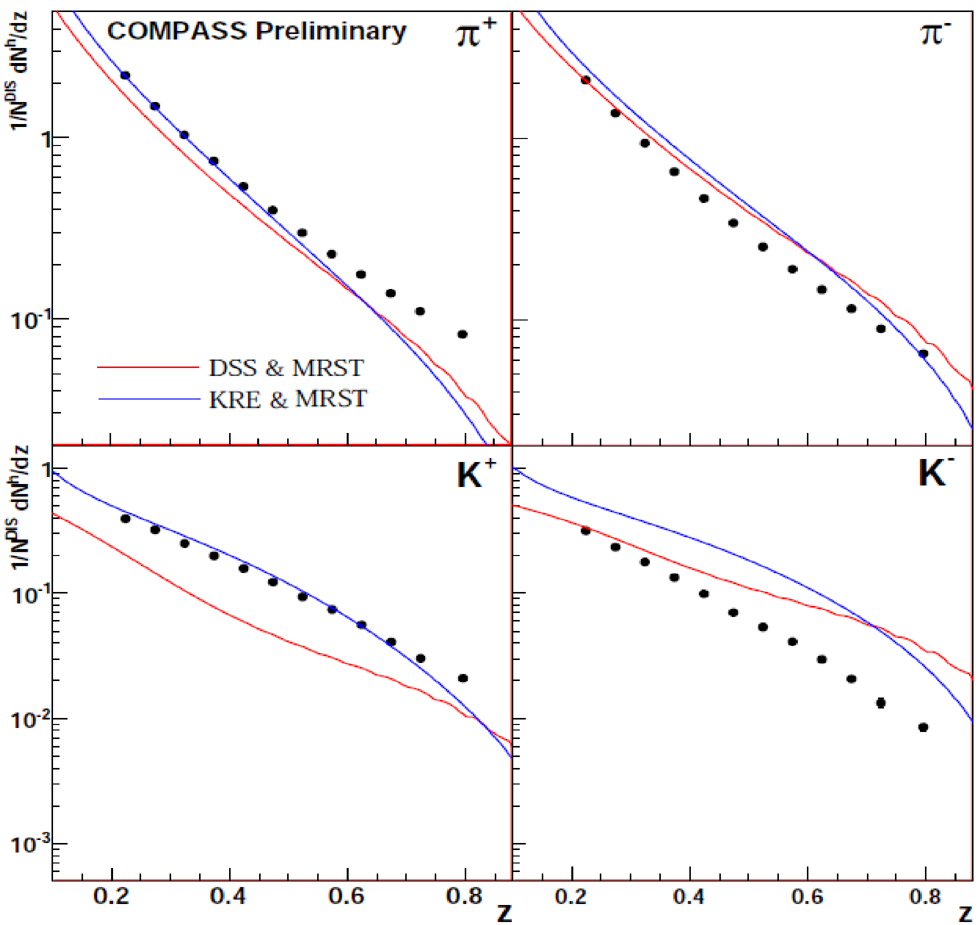
BACKUP SLIDES

Unfavored FFs NOT well known!

Hadron Multiplicities

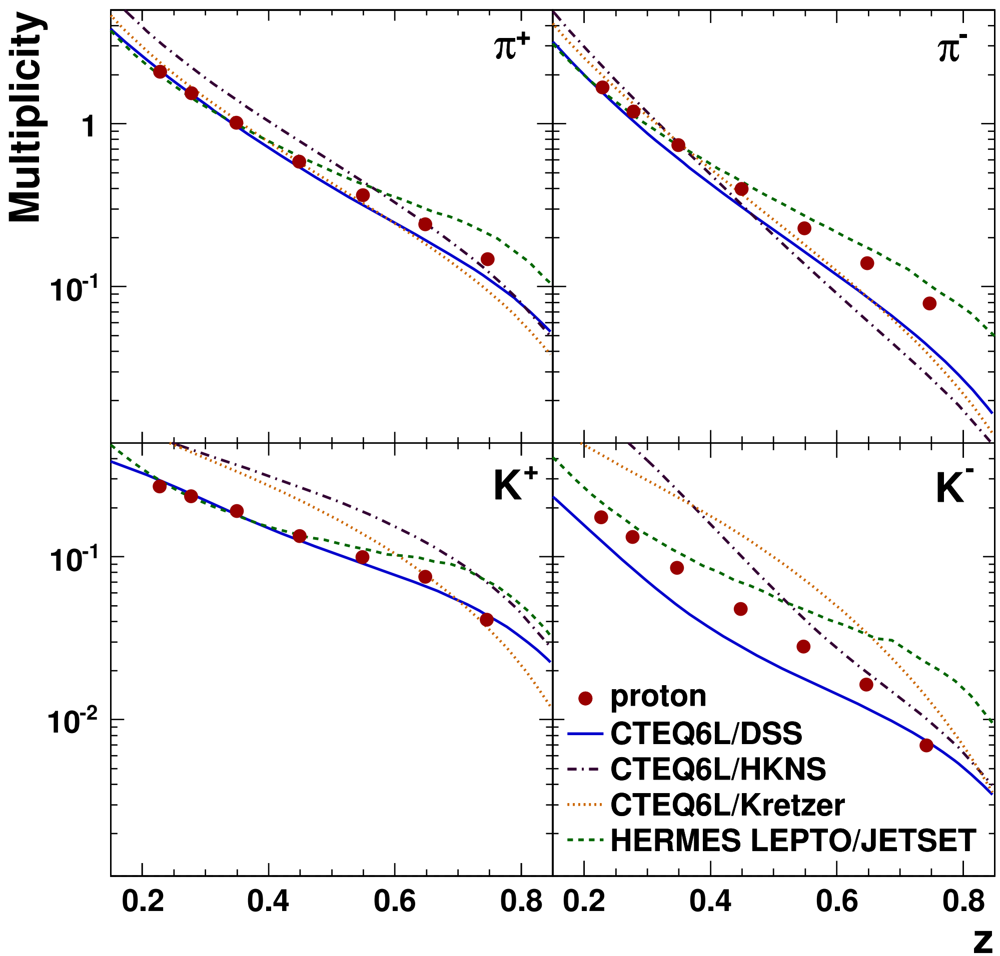
- Preliminary from COMPASS

Talk by C.Franco at CIPANP 2012.



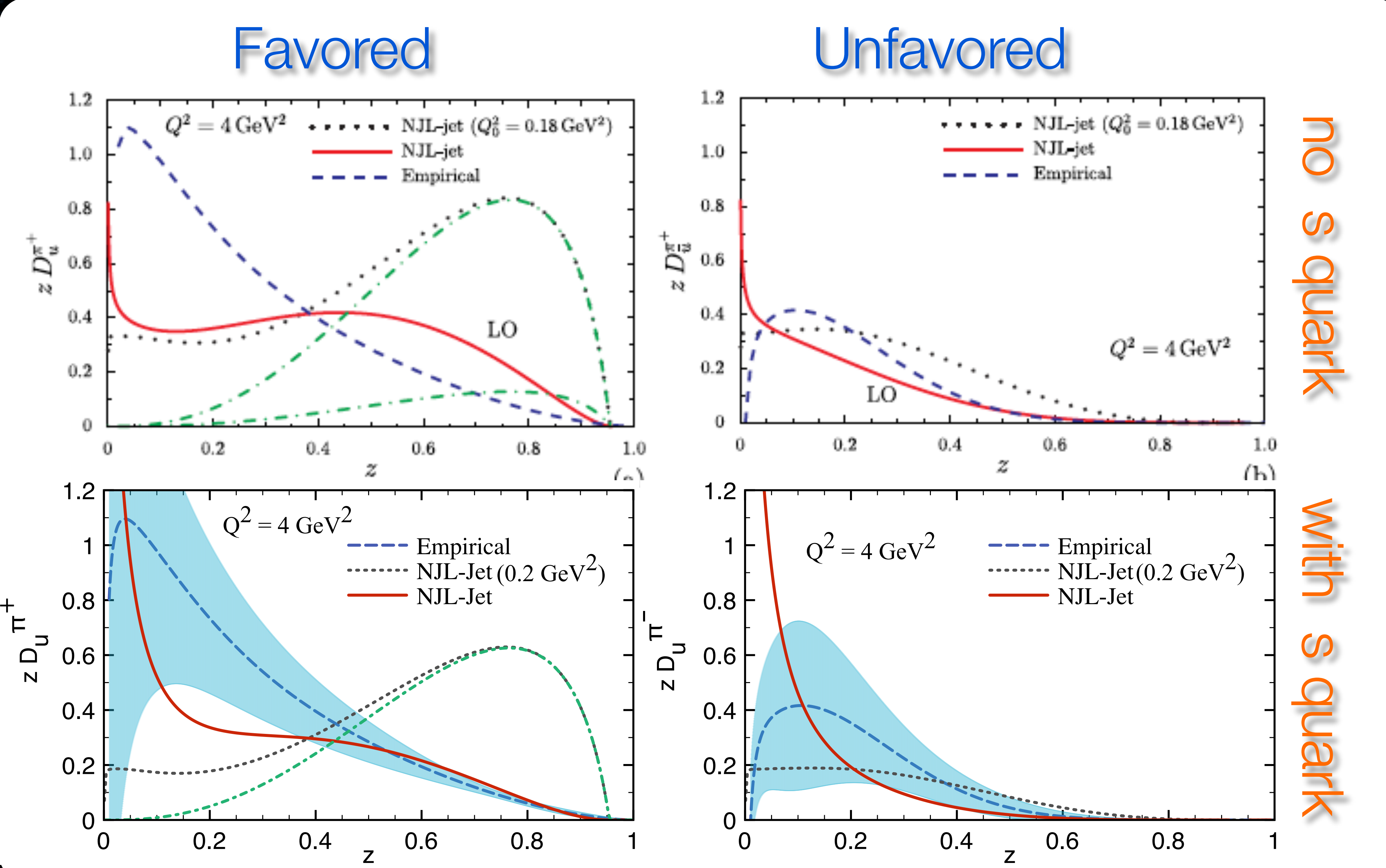
- Also results from HERMES

Phys. Rev. D 87, 074029 (2013)

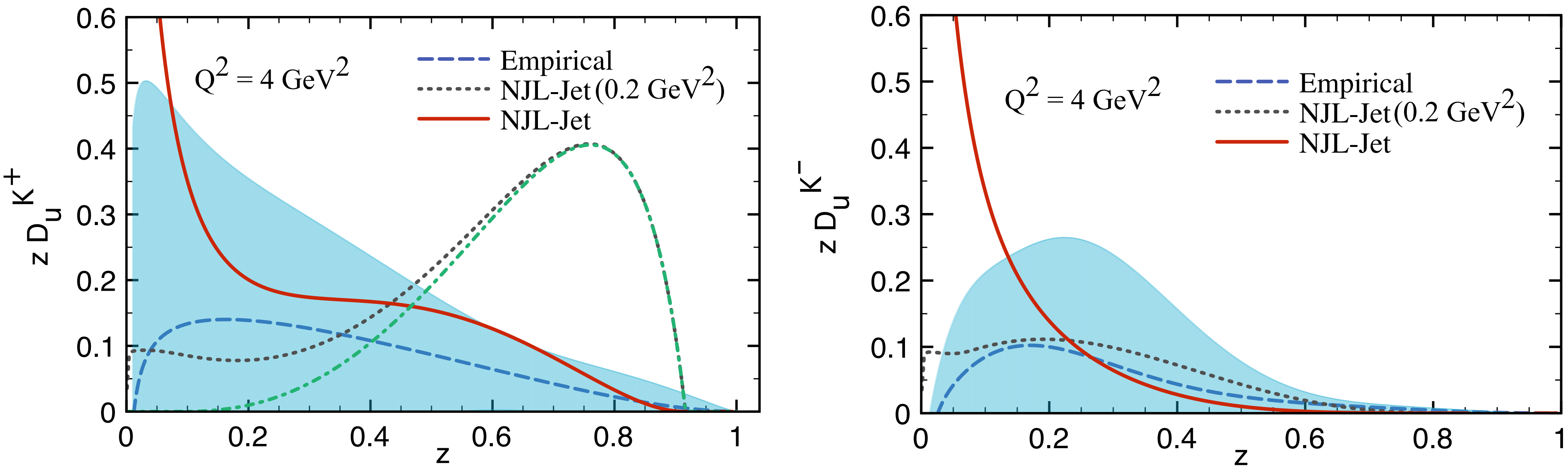


Strangeness Effect in Pion

Ito et al. Phys.Rev.D80:074008,2009



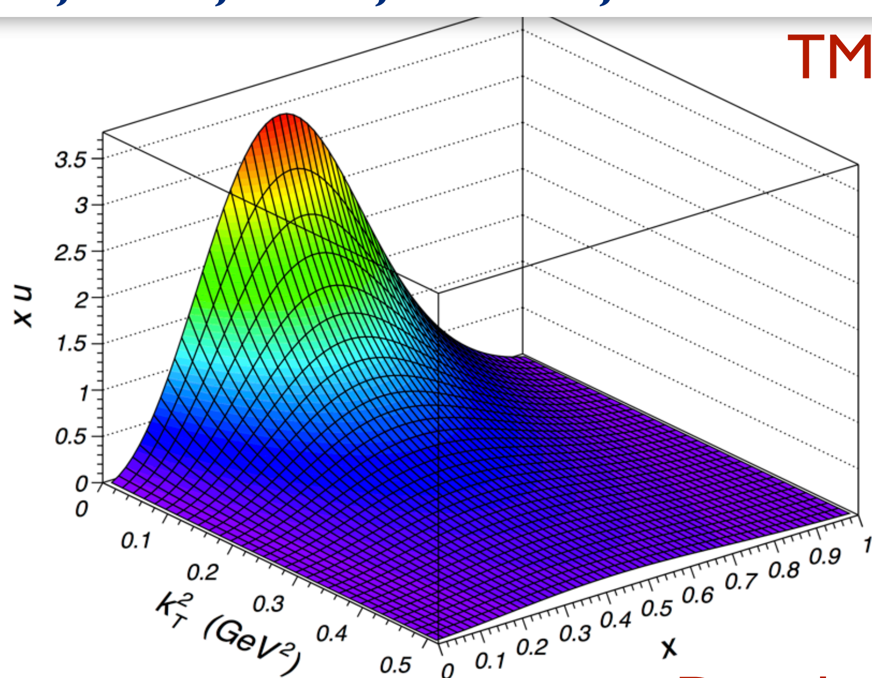
Results for Kaon

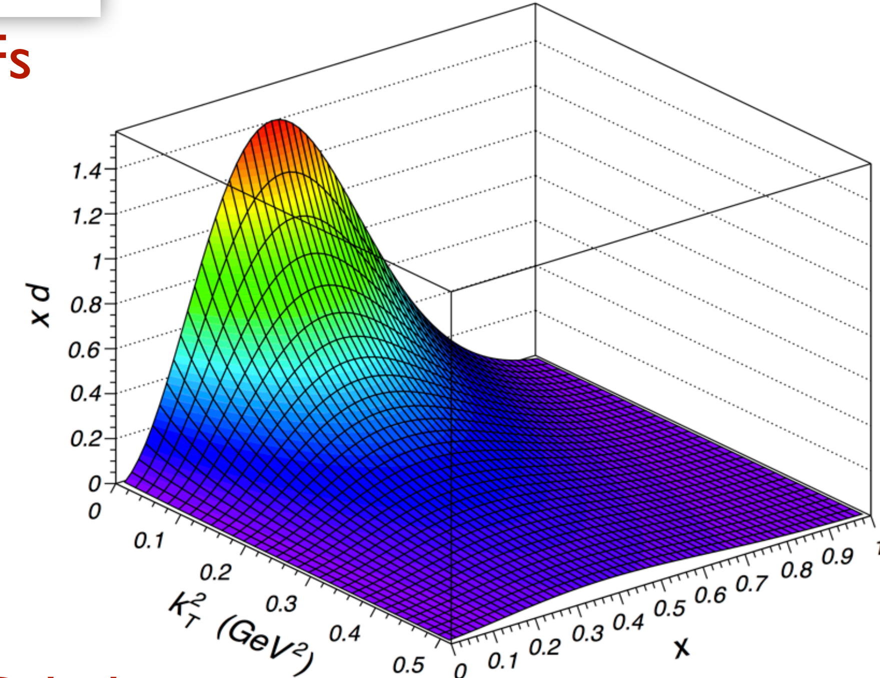


NJL: NUCLEON PDFS - TMD RESULTS

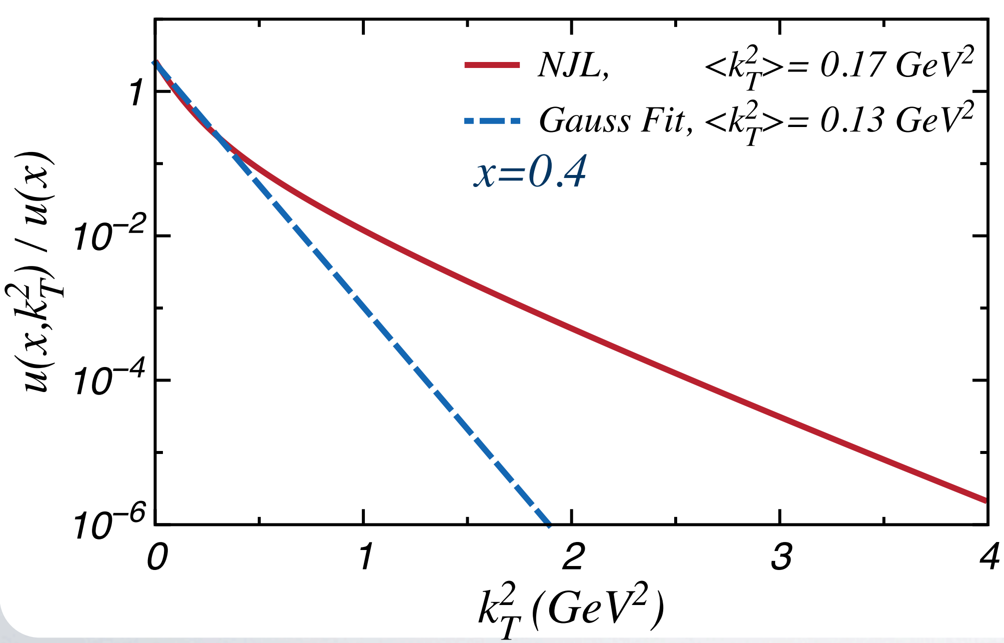
H.M.,Bentz, Cloet, Thomas, PRD.85:014021, 2012

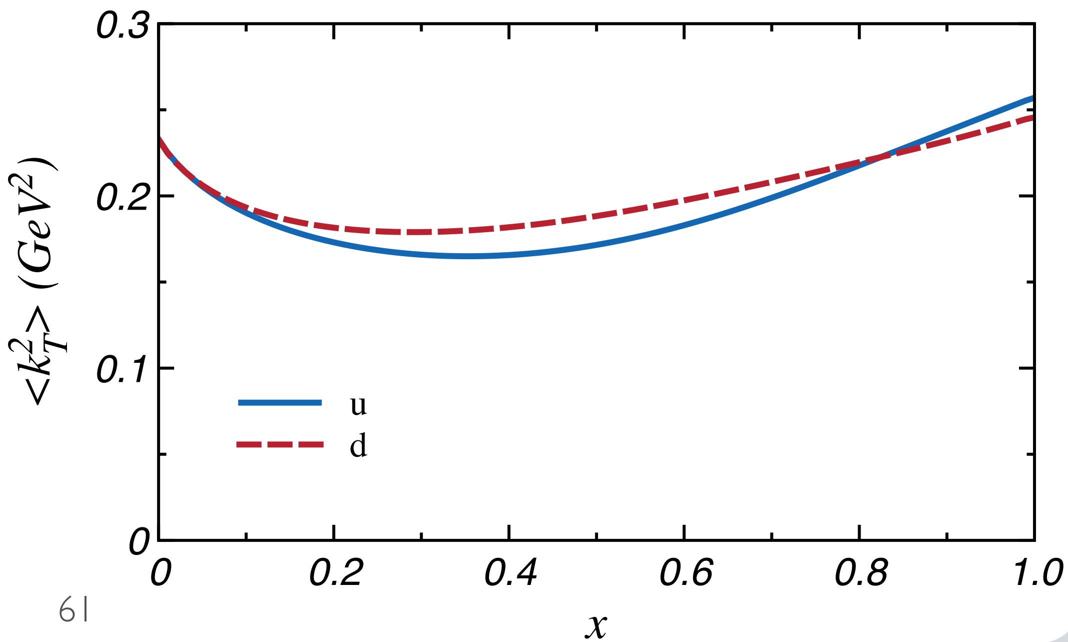
TMD PDFs





Details of TMD behavior





THE TREATMENT OF VM DECAYS: COMPARISON TO PYTHIA.

- 2-body decay amplitude: non-relativistic Breit-Wigner:

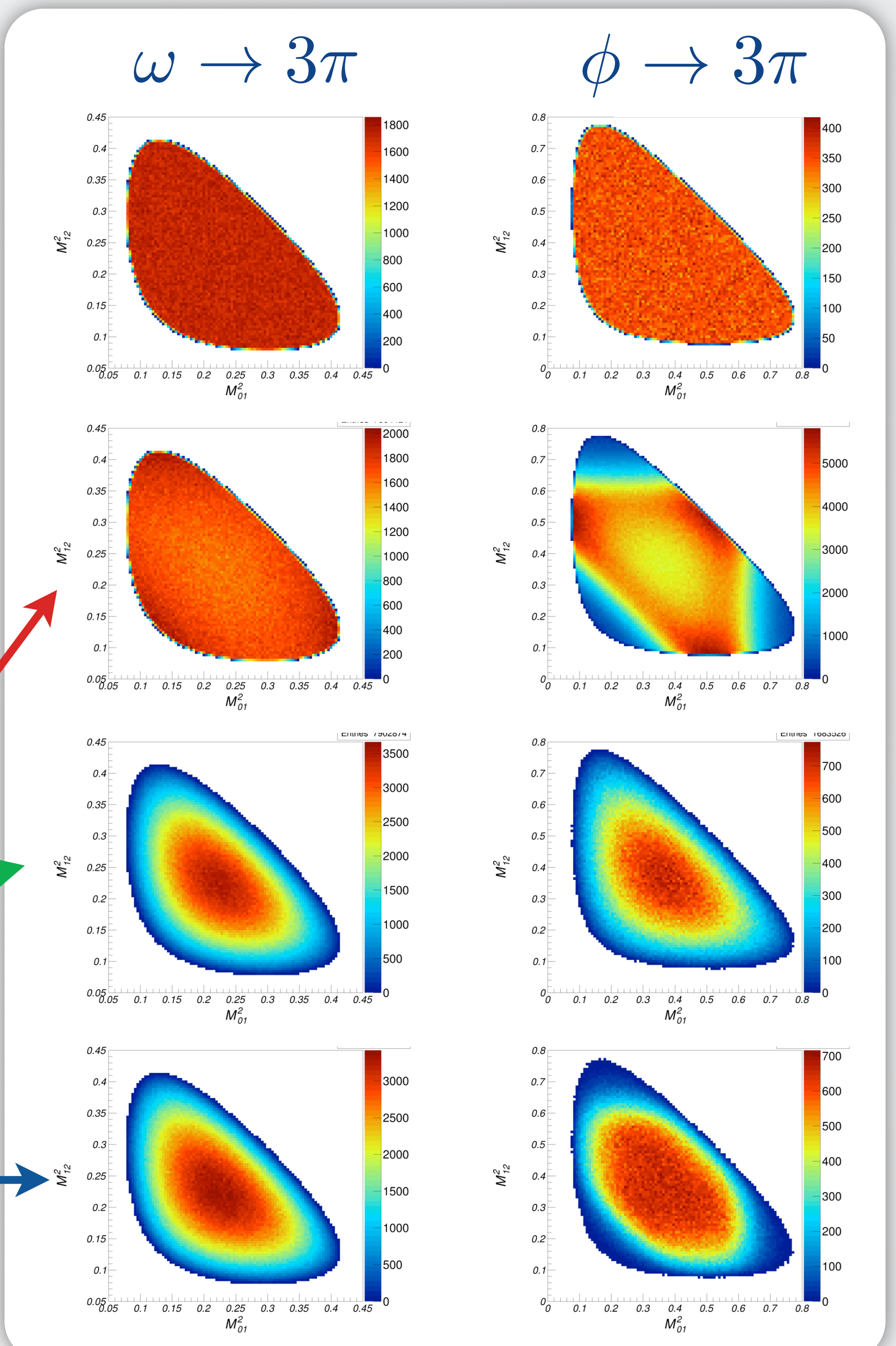
$$\mathcal{P}(m)dm \propto \frac{1}{(m-m_0)^2+\Gamma^2/4}dm$$

- Constant decay width of VM.

$$\Gamma_V(s) = \frac{m_V^2}{s}\Gamma_V\left(\frac{q(s)}{q(m_V^2)}\right)^3$$

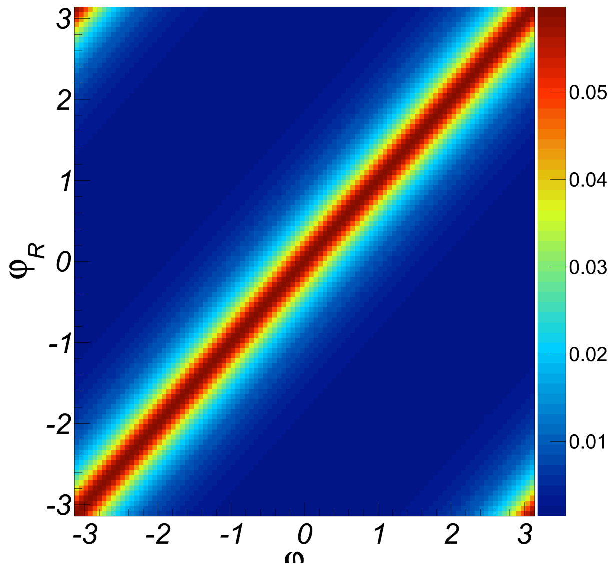
- 3-body decay amplitude:
  - Point-like coupling (PYTHIA).
  - "Isobar" model (HERWIG, NJL-jet).

$$M = \varepsilon_{\mu\alpha\beta\gamma}\epsilon^\mu p_1^\alpha p_2^\beta p_3^\gamma \sum_{i=0,\pm} \frac{g_{V\rho_i\pi}\, g_{\rho_i\pi\pi}}{D_{\rho_i}(v_i^2)}$$

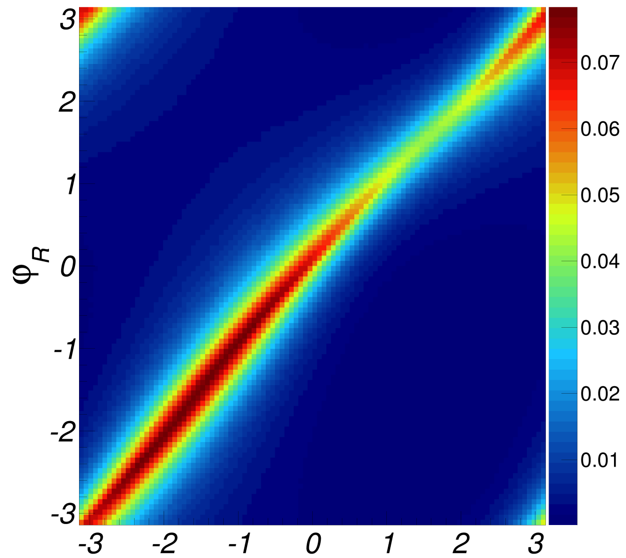


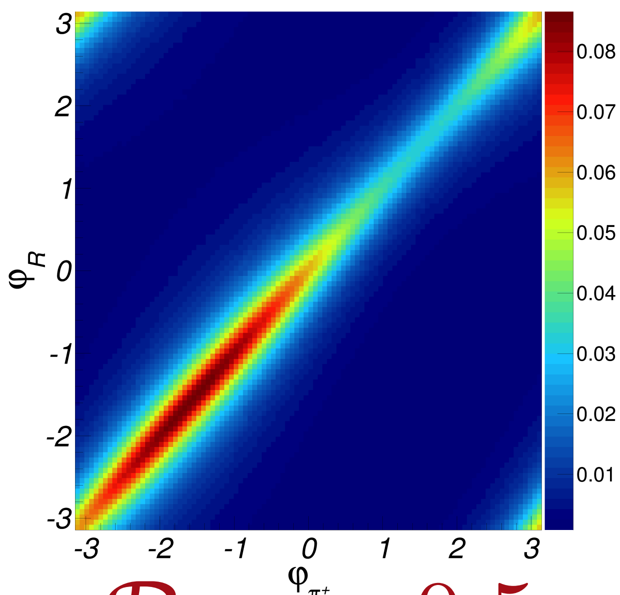
ANGULAR CORRELATIONS: $u \to \pi^+\pi^-$

Unpolarized

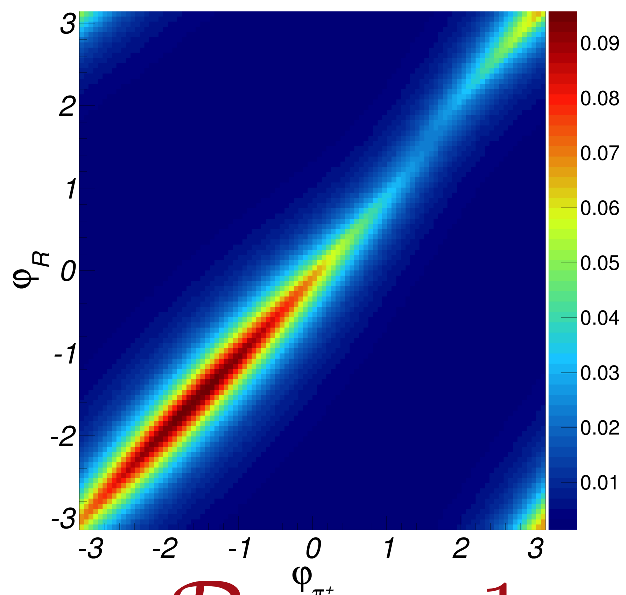


$\mathcal{P}_{SF} = 0$





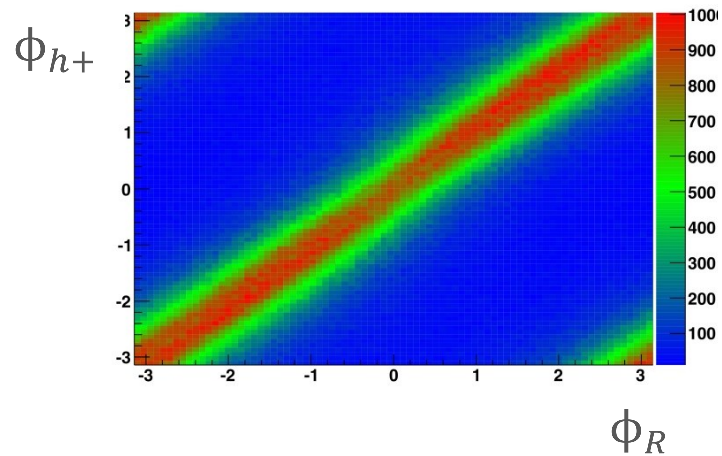
$\mathcal{P}_{SF} = 0.5$



$\mathcal{P}_{SF} = 1$

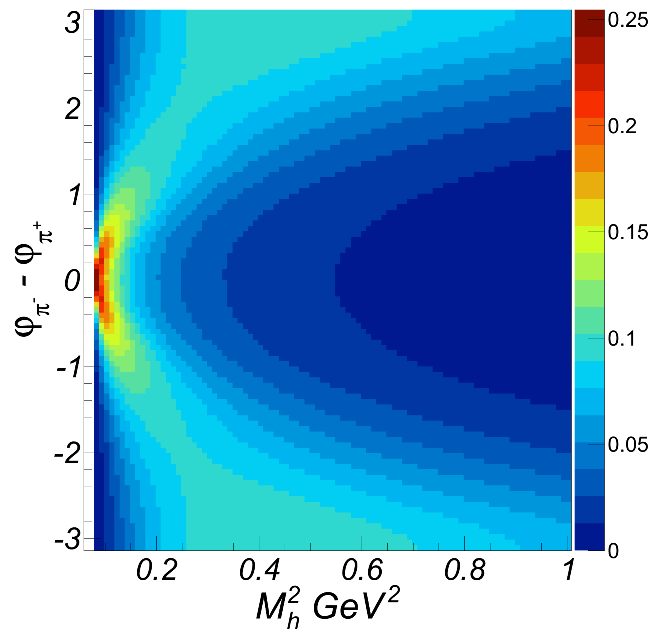
COMPASS Preliminary:

F. Bradamante - COMO 2013.



ANGULAR CORRELATIONS: $u \to \pi^+\pi^-$

$$M_h^2 = \frac{(z_1+z_2)(z_2 M_{h1}^2 + z_1 M_{h2}^2) + (z_2 \mathbf{P}_{1\perp} - z_1 \mathbf{P}_{2\perp})^2}{z_1 z_2}$$



TMDS FROM SIDIS $e\,N \rightarrow e\,h\,X$

- Cross-section factorizes: $P_T^2 \ll Q^2$

Fragmentation

$$\frac{d\sigma^{lN\to l'hX}}{dx\,dQ^2\,dz\,d^2P_T} = \sum_q f_1^q(x, k_T^2, Q^2) \otimes d\sigma^{lq\to lq} \otimes D_q^h(z, p_\perp^2, Q^2)$$

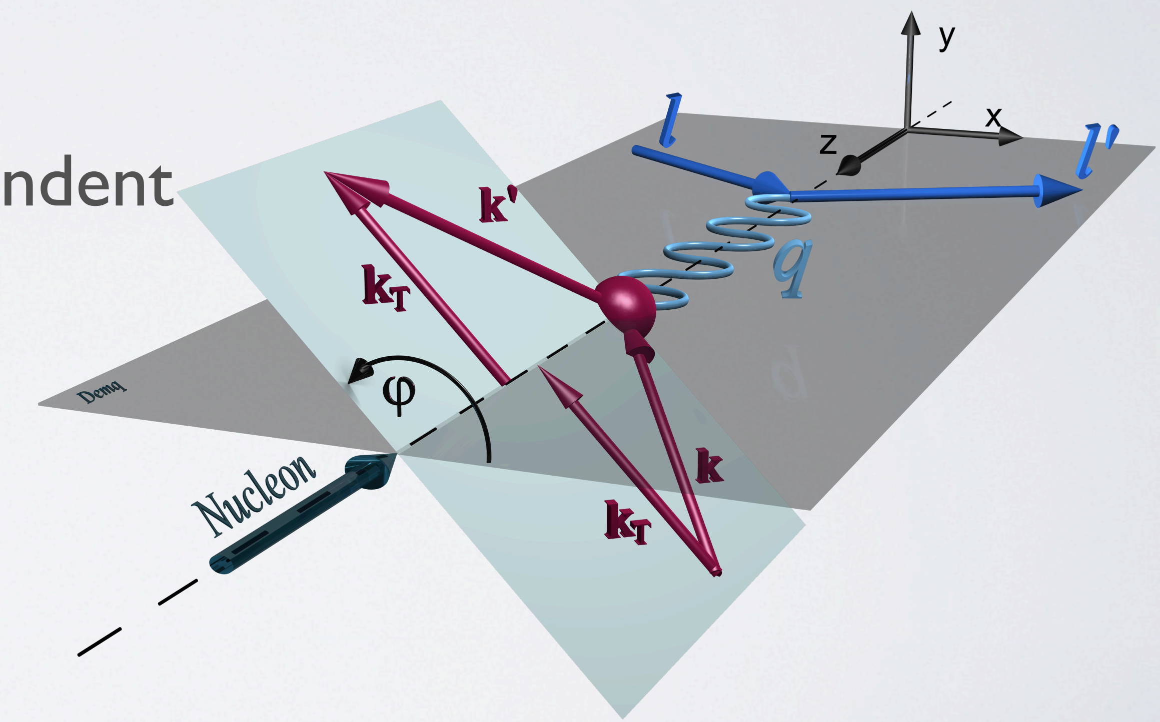
Distribution

$$\mathbf{P_T} = \mathbf{P}_\perp + z\mathbf{k_T}$$

- Transverse Momentum Dependent (TMD) PDFs and FFs:

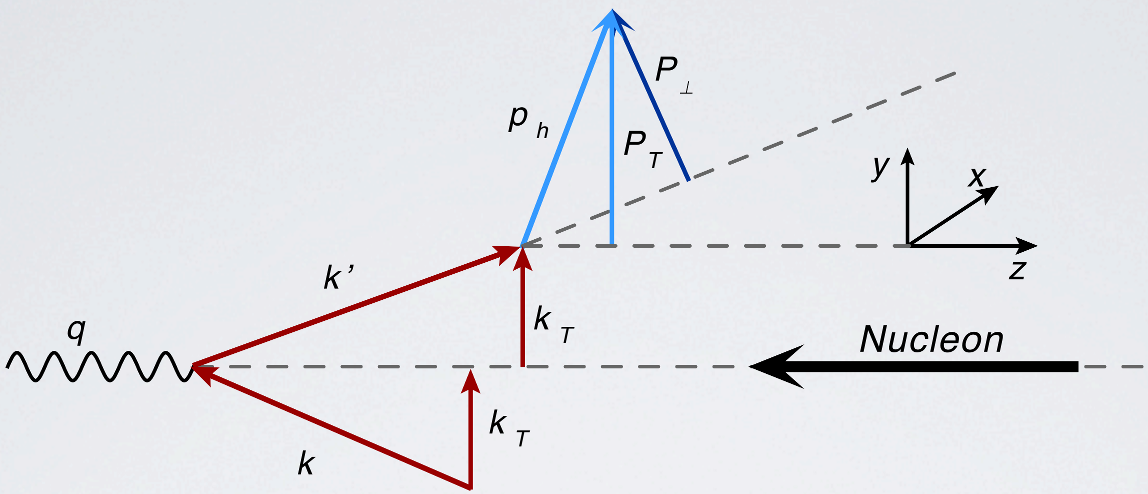
$$\int d^2\mathbf{k}_\perp f(x, k_\perp^2) = f(x)$$

$$\int d^2\mathbf{P}_\perp D(z, P_\perp^2) = D(z)$$

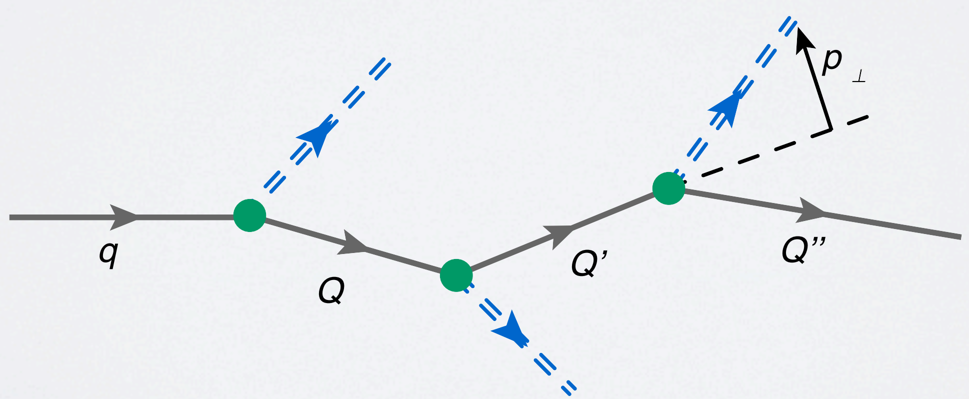


- Access to nucleon's transverse structure.
- NJL provides microscopic description of TMD PDFs and FFs!

THE TRANSVERSE MOMENTA OF HADRONS IN SIDIS



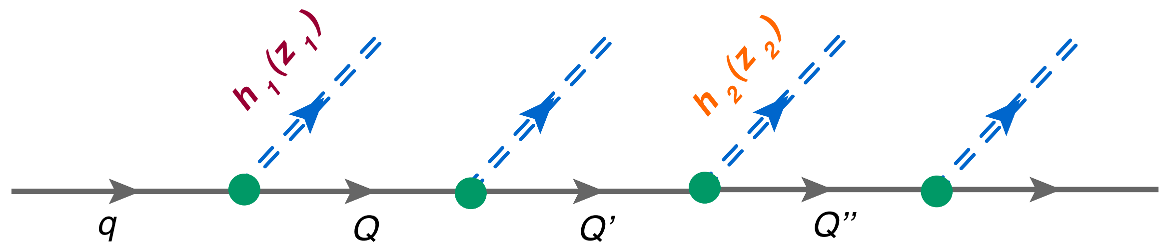
- Use TMD quark distribution functions from the NJL model .
- Use NJL-Jet hadronization model.



- Evaluate the cross-section using MC simulation.

UNPOLARIZED DIHADRON FRAGMENTATIONS

Casey, HM, Thomas: PRD 85, 114049 (2012). Casey, Cloet, HM, Thomas: PRD 86, 114018 (2012).



- The probability density for observing two hadrons:

$$D_q^{h_1 h_2}(z_1, z_2)\ \Delta z_1\ \Delta z_2 = \langle N_q^{h_1 h_2}(z_1, z_1 + \Delta z_1; z_2, z_2 + \Delta z_2) \rangle$$

- First Explorations within NJL-jet model in Integral Eq. formalism.

$u \to \pi^+ \pi^-$

Model Scale

Evolved to $Q^2 = 4\ \mathrm{GeV}^2$