Seminar at COMPASS Collaboration Meeting
CERN: July 27, 2017.

"DIHADRON CORRELATIONS IN POLARIZED QUARK HADRONIZATION."

Hrayr Matevosyan

THE UNIVERSITY *of* ADELAIDE

Collaborators: A. Kotzinian and A.W. Thomas.

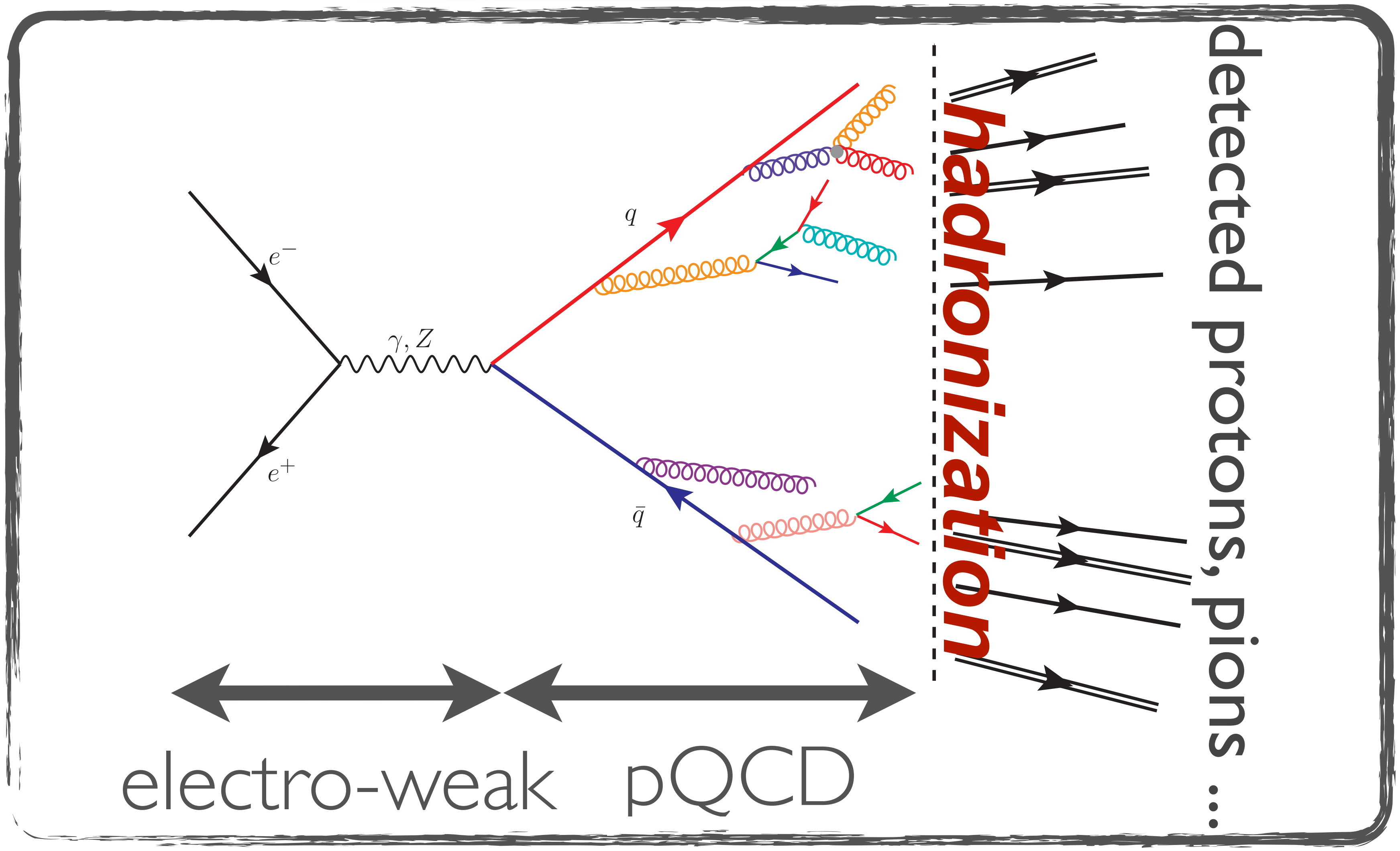
Outlook

- *Introduction and Motivation.*
- *Quark-jet framework for hadronization.*
- *Dihadron FFs: longitudinally and transversely polarised quark.*
- *Conclusions.*

Hadronization: $e^- e^+ \to hX$

- **The conjecture of *Confinement*:**

✦ ***NO free quarks or gluons have been directly observed: only HADRONS.***



✦ **Hadronization**: describes the process where colored quarks and gluons form colourless hadrons (in deep inelastic scattering).

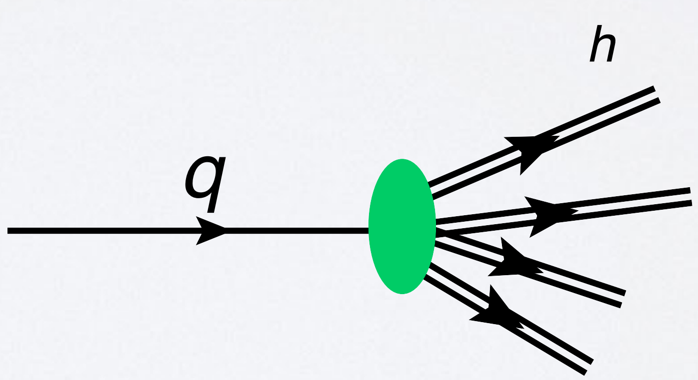
Fragmentation Functions

- The non-perturbative, universal functions encoding parton hadronization are the: ***Fragmentation Functions (FF)***.

$$\boxed{\frac{1}{\sigma}\frac{d}{dz}\sigma(e^-e^+ \to hX) = \sum_i \mathcal{C}_i(z,Q^2) \otimes D_i^h(z,Q^2)}$$

- Unpolarized FF is the number density for parton *i* to produce hadron *h* with LC momentum fraction **z**.

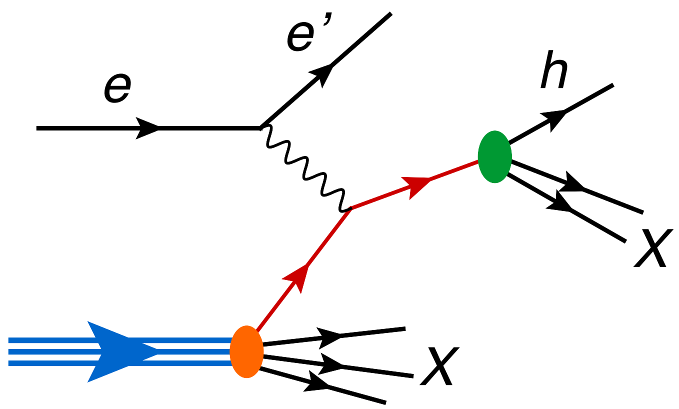
$$\boxed{D_i^h(z,Q^2)}$$



- **z** is the light-cone mom. fraction of the parton carried by the hadron

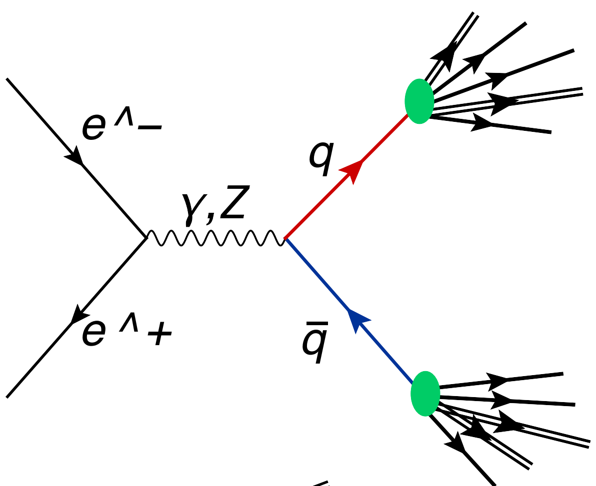
$$z = \frac{p^-}{k^-} \approx z_h = \frac{2E_h}{Q} \qquad\qquad a^\pm = \frac{1}{\sqrt{2}}(a^0 \pm a^3)$$

FACTORIZATION AND UNIVERSALITY



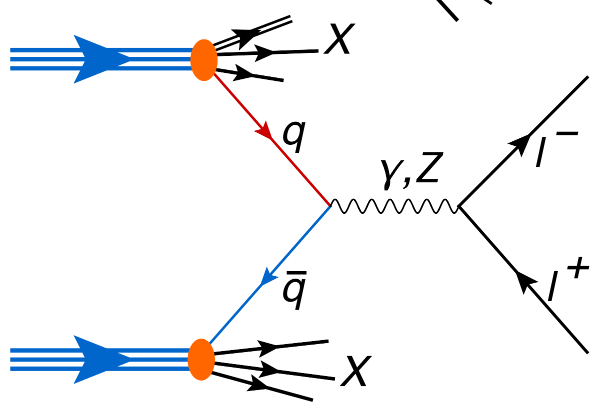
- SEMI INCLUSIVE DIS (SIDIS)

$$\sigma^{eP\to ehX} = \sum_q f_q^P \otimes \sigma^{eq\to eq} \otimes D_q^h$$



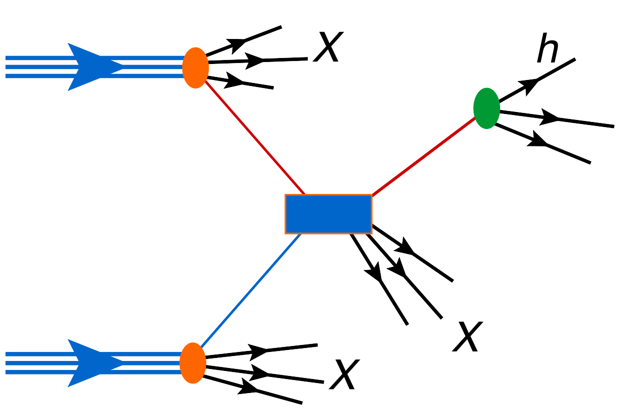
- e^+e^-

$$\sigma^{e^+e^-\to hX} = \sum_q \sigma^{e^+e^-\to q\bar{q}} \otimes (D_q^h + D_{\bar{q}}^h)$$



- DRELL-YAN (DY)

$$\sigma^{PP\to l^+l^-X} = \sum_{q,q'} f_q^P \otimes f_{\bar{q}}^P \otimes \sigma^{q\bar{q}\to l^+l^-}$$



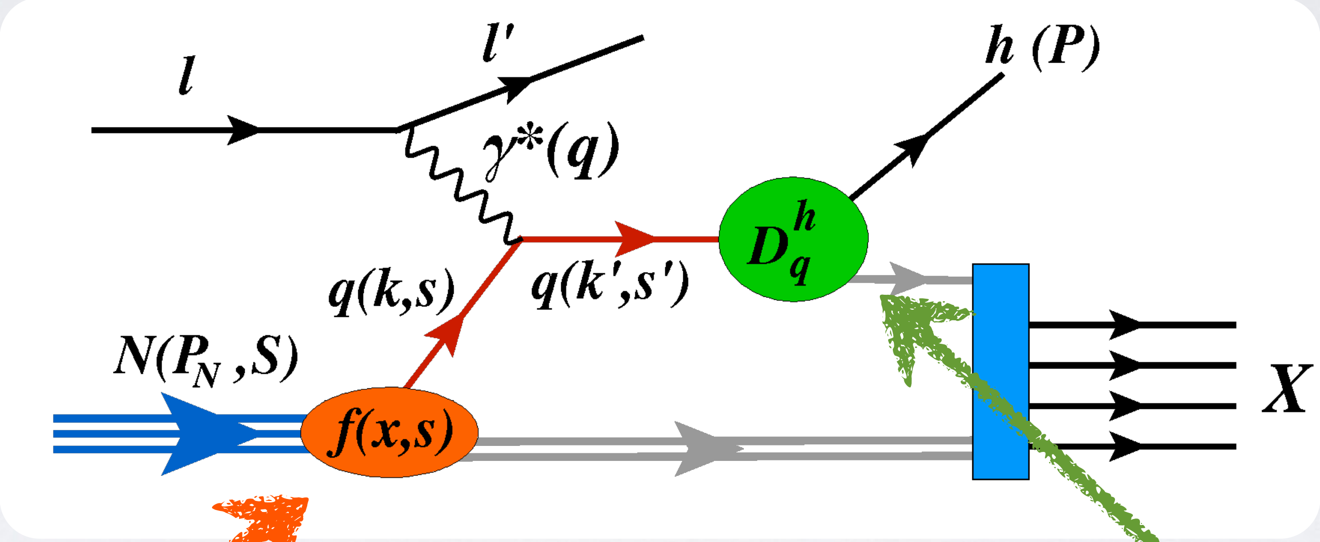
- Hadron Production

$$\sigma^{PP\to hX} = \sum_{q,q'} f_q^P \otimes f_{q'}^P \otimes \sigma^{qq'\to qq'} \otimes D_q^h$$

MEASURING PDFS WITH TRANSVERSE MOMENTUM DEPENDENCE

- ***Measurement of the <u>transverse momentum</u> of the produced hadron in SIDIS provides access to <u>TMD PDFs/FFs</u>.***

  - *SIDIS Process with TM of hadron measured.*



- *TMD PDFs*

N/q	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	$h_1\ h_{1T}^\perp$

- *TMD FFs*

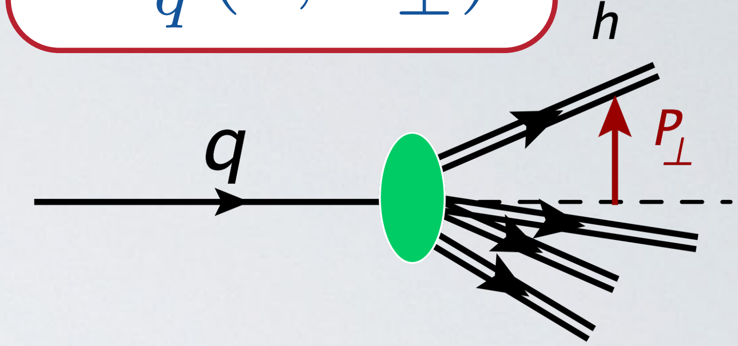
q/h	U
U	D_1
L	
T	$H_1^\perp$

*unpol/spinless h!

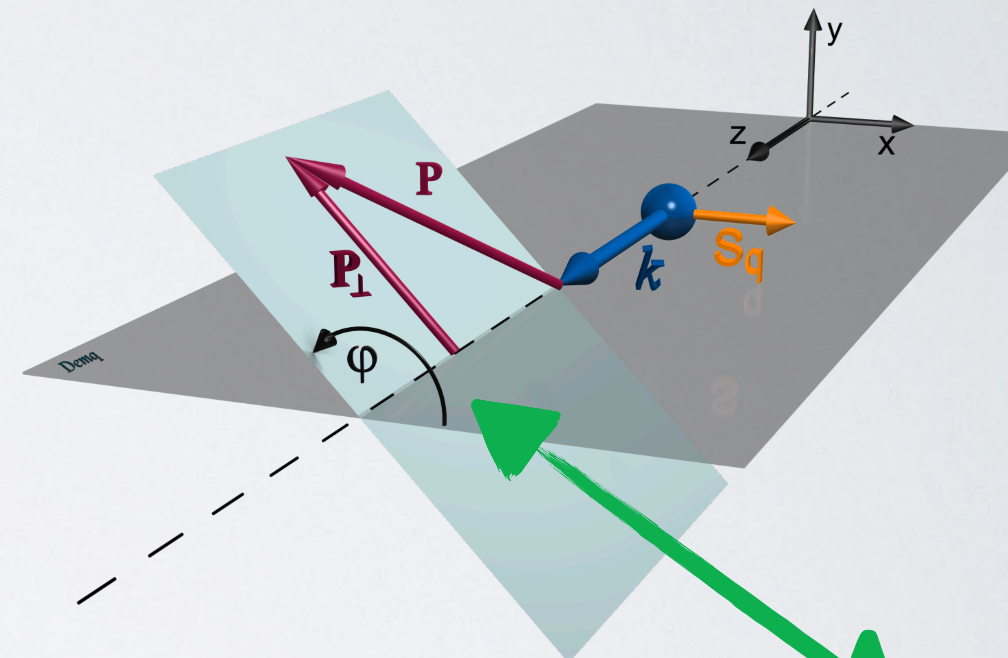
TMD FFs and Collins Fragmentation Function

- **Unpolarized TMD FF:** number density for quark ***q*** to produce unpolarized hadron ***h*** carrying LC fraction **z** and TM $P_\perp$.

$$D_q^h(z, P_\perp^2)$$



- **Collins Effect:** Azimuthal Modulation of Transversely Polarized Quark' FF. Fragmenting quark's transverse spin couples with produced hadron's TM!



$$D_{h/q^\uparrow}(z, P_\perp^2, \varphi) = D_1^{h/q}(z, P_\perp^2) - H_1^{\perp h/q}(z, P_\perp^2)\frac{P_\perp S_q}{z m_h}\sin(\varphi)$$

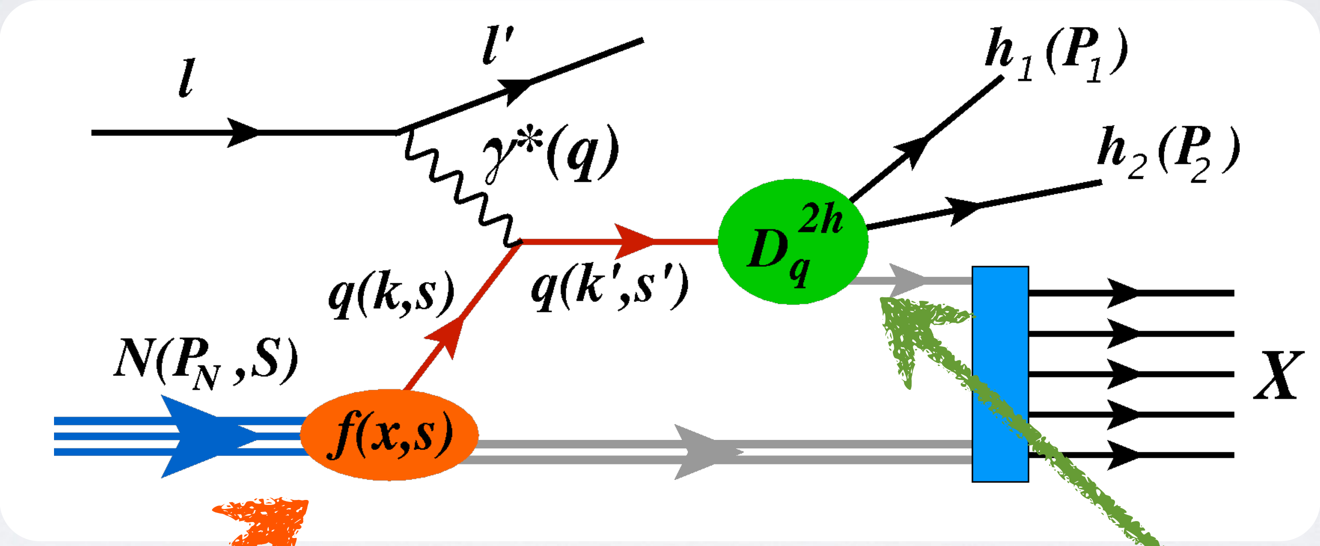
Unpolarized

Collins

- *Collin FF* is **Chiral-ODD:** Should to be coupled with another chiral-odd PDF/FF in observables.

TMD PDFs with Two-Hadron FFs

- ***Measuring <u>two-hadron</u> semi-inclusive DIS: an additional method for accessing TMD PDFs.***
  - *SIDIS Process with TM of hadrons measured.*



- *TMD PDFs*

N/q	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	$h_1 \; h_{1T}^\perp$

- *TMD DiFFs*

q/h₁h₂	U
U	D_1
L	$G_1^\perp$
T	$H_1^\perp \quad H_1^\sphericalangle$

*unpol/spinless h!

TWO HADRON CORRELATIONS:
DIHADRON FRAGMENTATION FUNCTIONS

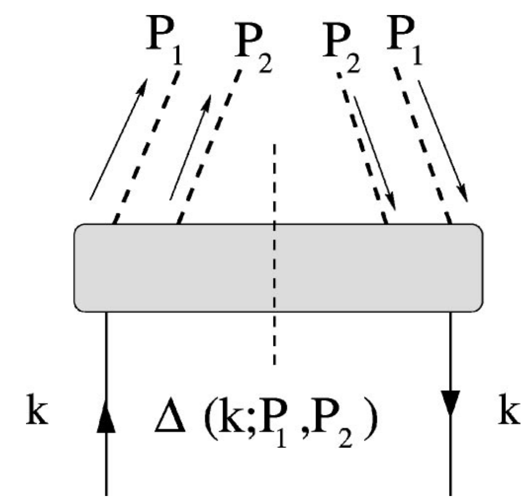
Two-Hadron Kinematics

A. Bianconi et al: PRD 62, 034008 (2000).

✦ **Total and Relative TM of hadron pair.**

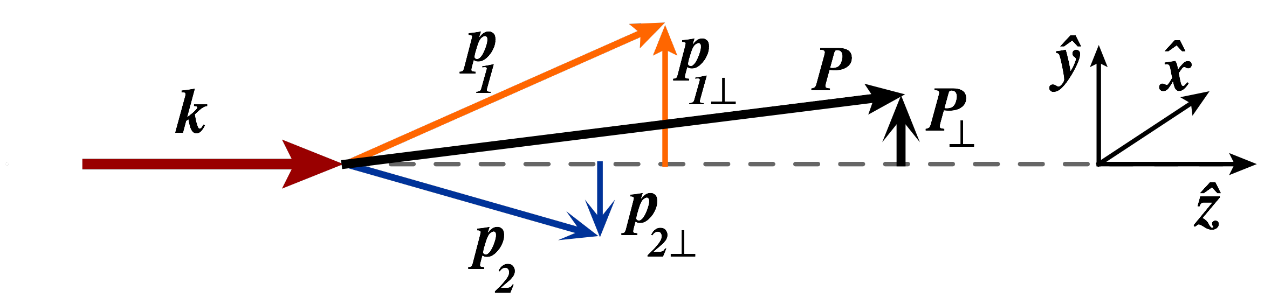
$$P = P_1 + P_2 \qquad z = z_1 + z_2$$

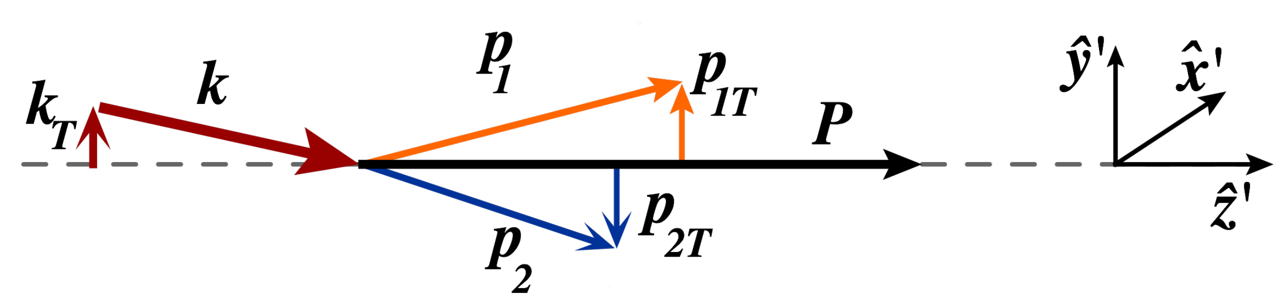
$$R = \frac{1}{2}(P_1 - P_2) \quad \xi = \frac{z_1}{z} = 1 - \frac{z_2}{z}$$



✦ Two Coordinate systems:

- $\perp$: modelling hadronization
- T: field-theoretical definition of DiFFs





✦ Lorentz Boost:

$$\boldsymbol{P}_{1T} = \boldsymbol{P}_{1\perp} + z_1 \boldsymbol{k}_T$$

$$\boldsymbol{P}_{2T} = \boldsymbol{P}_{2\perp} + z_2 \boldsymbol{k}_T$$

$$\boldsymbol{k}_T = -\frac{\boldsymbol{P}_\perp}{z}$$

✤ Relative TM in two systems

$$\boldsymbol{R}_\perp = \frac{1}{2}(\boldsymbol{P}_{1\perp} - \boldsymbol{P}_{2\perp})$$

$$\boldsymbol{R}_T = \frac{z_2 \boldsymbol{P}_{1\perp} - z_1 \boldsymbol{P}_{2\perp}}{z}$$

Field-Theoretical Definitions

- ***The quark-quark correlator.***

$$\Delta_{ij}(k;P_1,P_2)=\sum_X\int d^4\zeta e^{ik\cdot\zeta}\langle 0|\psi_i(\zeta)|P_1P_2,X\rangle\langle P_1P_2,X|\bar{\psi}_j(0)|0\rangle$$

$$\Delta^{\Gamma}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)=\frac{1}{4z}\int dk^+\mathrm{Tr}[\Gamma\Delta(k,P_1,P_2)]|_{k^-=P_h^-/z}$$

- ***The definitions of DiFFs from the correlator.***

Quark Polarization

$$\Delta^{[\gamma^-]}=D_1(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

Unpolarised

$$\Delta^{[\gamma^-\gamma_5]}=\frac{\epsilon_T^{ij}R_{Ti}k_{Tj}}{M_1M_2}G_1^{\perp}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

Longitudinal

$$\Delta^{[i\sigma^{i-}\gamma_5]}=\frac{\epsilon_T^{ij}R_{Tj}}{M_1+M_2}H_1^{\sphericalangle}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)+\frac{\epsilon_T^{ij}k_{Tj}}{M_1+M_2}H_1^{\perp}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

Transverse

Number Densities

- ***The full number density:***

$$F(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T;\boldsymbol{s}) = D_1(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

$$+\ s_L\,\frac{(\boldsymbol{R}_T\times\boldsymbol{k}_T)\cdot\hat{\boldsymbol{z}}}{M_1M_2}G_1^{\perp}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

$$+\ \frac{(\boldsymbol{s}_T\times\boldsymbol{R}_T)\cdot\hat{\boldsymbol{z}}}{M_1+M_2}H_1^{\sphericalangle}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

$$+\ \frac{(\boldsymbol{s}_T\times\boldsymbol{k}_T)\cdot\hat{\boldsymbol{z}}}{M_1+M_2}H_1^{\perp}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

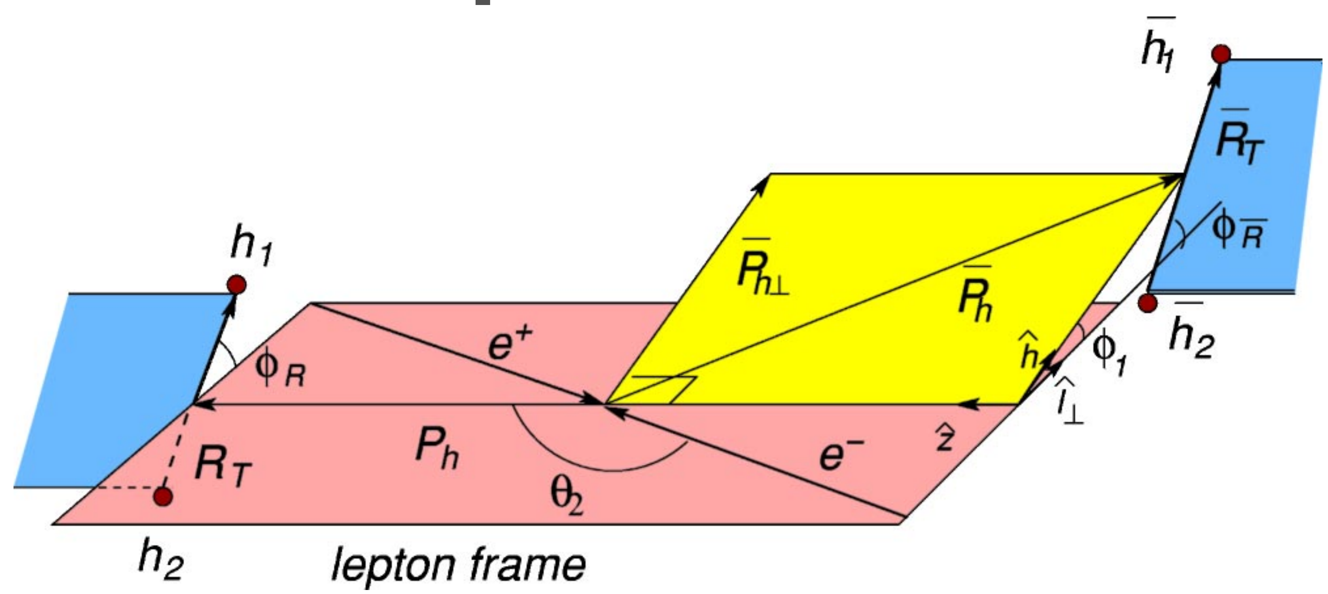
- ***The differential number of hadron pairs:***

$$dN_q^{h_1h_2} = F_q^{h_1h_2}(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T;\boldsymbol{s})\;dz\;d\xi\;d^2\boldsymbol{k}_T\;d^2\boldsymbol{R}_T$$

Back-to-back *two* hadron pairs in e^+e^-

D. Boer et al: PRD 67, 094003 (2003).

- *Can access both helicity and transverse pol. dependent DiFFs:*

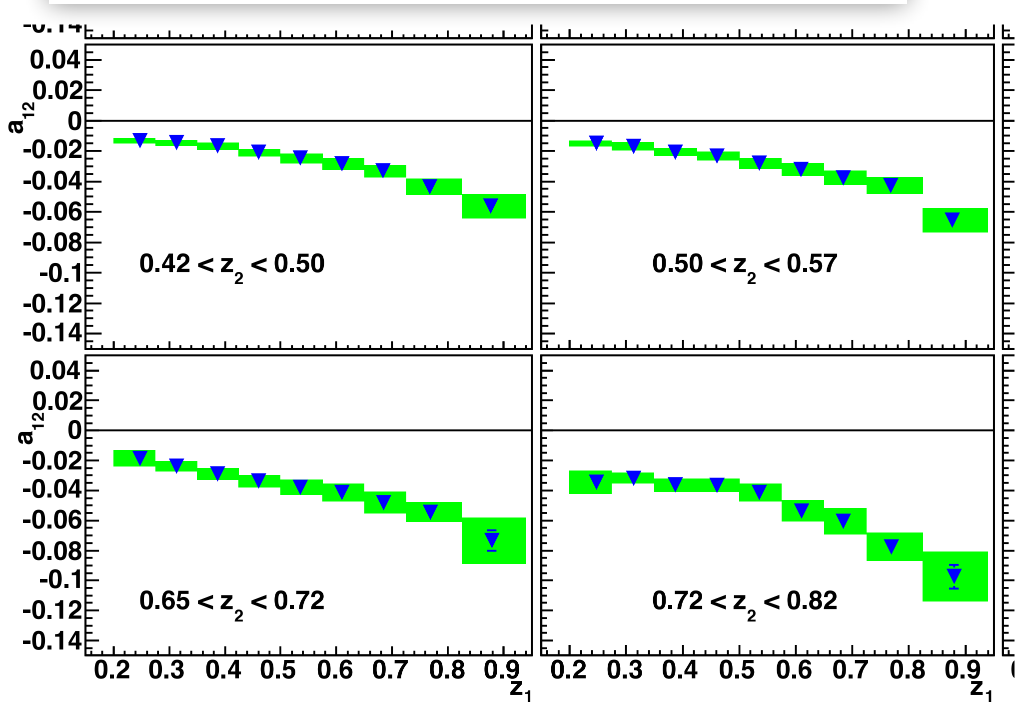


$$A^{\cos(\varphi_R+\varphi_{\bar{R}})} \sim \frac{H_1^{\sphericalangle}(z,M_h^2)\bar{H}_1^{\sphericalangle}(\bar{z},M_{\bar{h}}^2)}{D_1(z,M_h^2)\ \bar{D}_1(\bar{z},M_{\bar{h}}^2)}$$

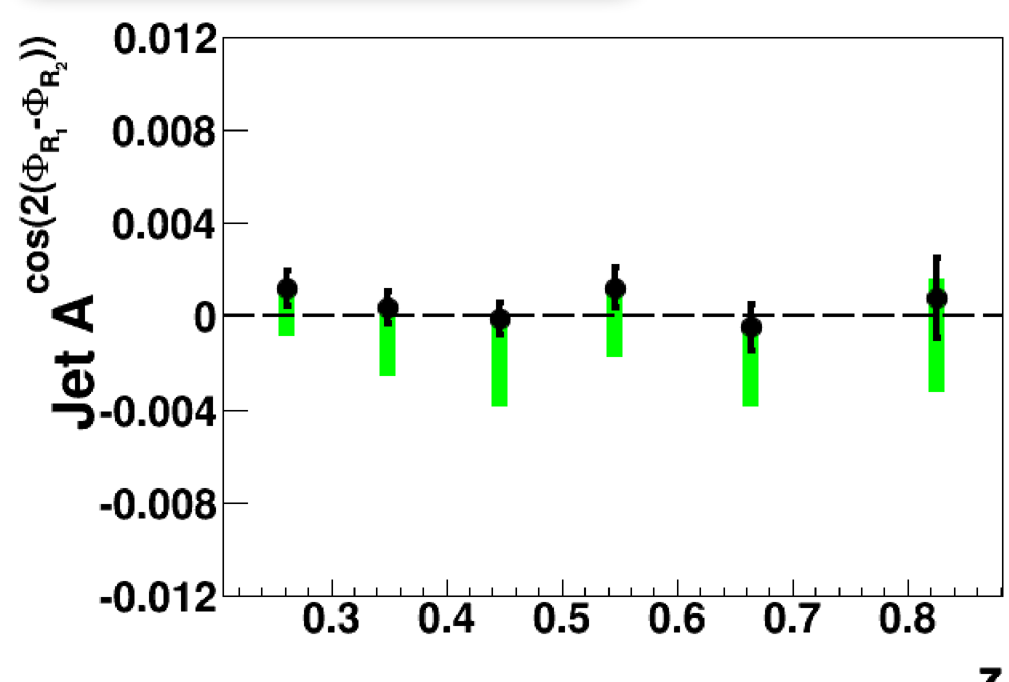
$$A^{\cos(2(\varphi_R-\varphi_{\bar{R}}))} \sim \frac{G_1^{\perp}(z,M_h^2)\bar{G}_1^{\perp}(\bar{z},M_{\bar{h}}^2)}{D_1(z,M_h^2)\ \bar{D}_1(\bar{z},M_{\bar{h}}^2)}$$

✦ ***BELLE results.***

Phys.Rev.Lett. 107 (2011) 072004



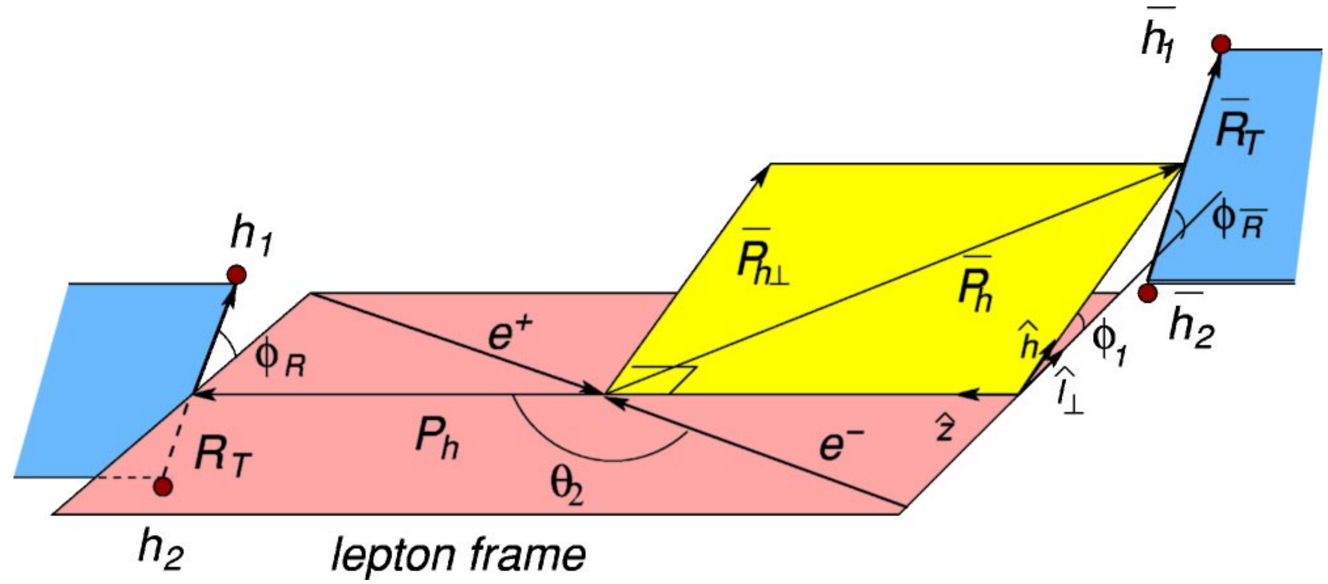
PoS DIS2015 (2015) 216



Back-to-back *two* hadron pairs in e$^+$e$^-$

D. Boer et al: PRD 67, 094003 (2003).

- ***Can access both helicity and transverse pol. dependent DiFFs:***

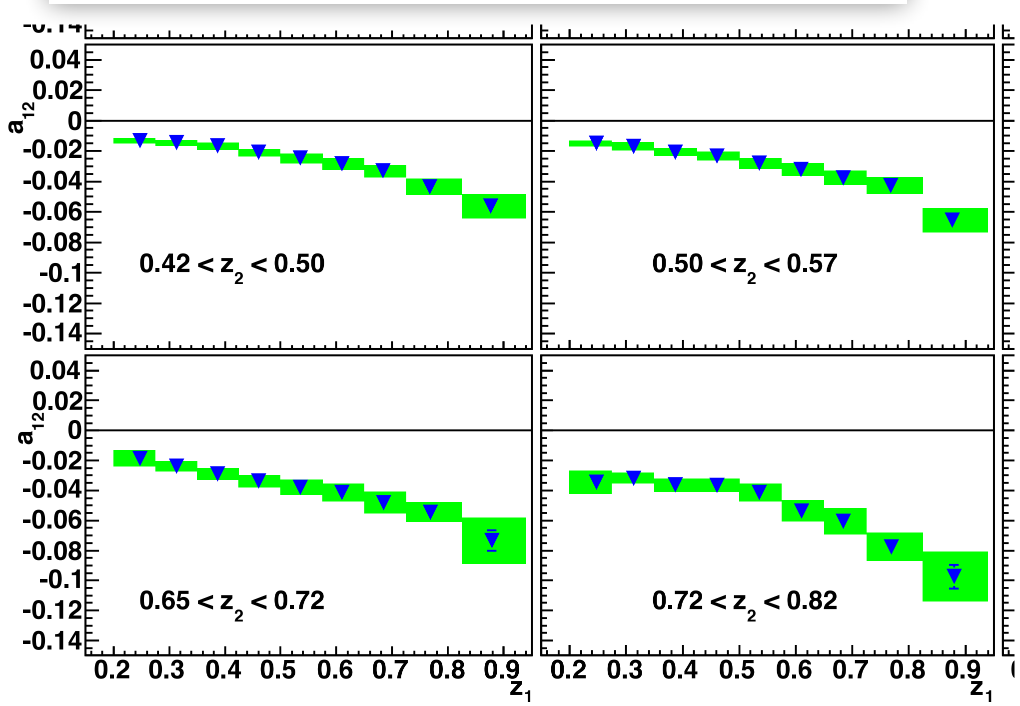


$$A^{\cos(\varphi_R+\varphi_{\bar{R}})} \sim \frac{H_1^{\sphericalangle}(z,M_h^2)\bar{H}_1^{\sphericalangle}(\bar{z},M_{\bar{h}}^2)}{D_1(z,M_h^2)\ \bar{D}_1(\bar{z},M_{\bar{h}}^2)}$$

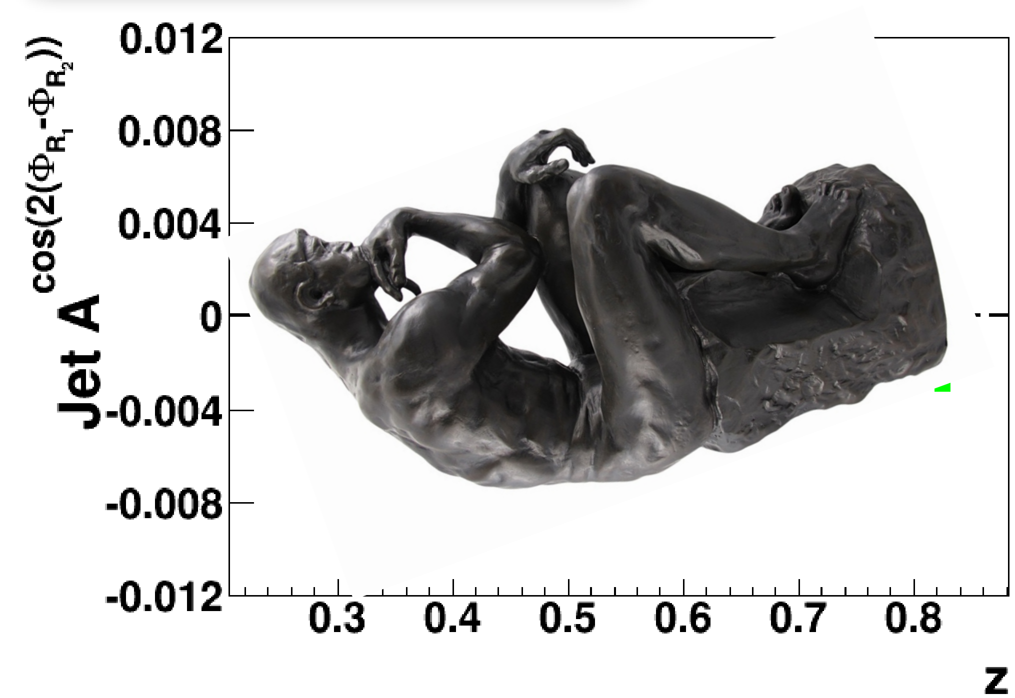
$$A^{\cos(2(\varphi_R-\varphi_{\bar{R}}))} \sim \frac{G_1^{\perp}(z,M_h^2)\bar{G}_1^{\perp}(\bar{z},M_{\bar{h}}^2)}{D_1(z,M_h^2)\ \bar{D}_1(\bar{z},M_{\bar{h}}^2)}$$

✦ ***BELLE results.***

Phys.Rev.Lett. 107 (2011) 072004



PoS DIS2015 (2015) 216



Moments of DiFFs in e$^+$e$^-$

- ***Entering the integrated cross-section expressions.***

$$D_1(z, M_h^2) = \int d\xi \int d\varphi_R \int d^2\boldsymbol{k}_T \; D_1(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)$$

$$G_1^{\perp}(z, M_h^2) = \int d\xi \int d\varphi_R \int d^2\boldsymbol{k}_T \; (\boldsymbol{k}_T \cdot \boldsymbol{R}_T) \; G_1^{\perp}(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)$$

$$H_{1,e^+e^-}^{\sphericalangle}(z, M_h^2) = \int d\xi \int d\varphi_R \int d^2\boldsymbol{k}_T \; |\boldsymbol{R}_T| H_1^{\sphericalangle}(z_h, \xi, k_T^2, R_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)$$

$$H_{1,e^+e^-}^{\perp}(z, M_h^2) = \int d\xi \int d\varphi_R \int d^2\boldsymbol{k}_T \; |\boldsymbol{k}_T| H_1^{\perp}(z_h, \xi, k_T^2, R_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)$$

Moments of DiFFs in e^+e^-

- ***Entering the integrated cross-section expressions.***

$$D_1(z, M_h^2) = \int d\xi \int d\varphi_R \int d^2\boldsymbol{k}_T\ D_1(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)$$

$\cos(\varphi_R - \varphi_k)$ **moment**

$$G_1^\perp(z, M_h^2) = \int d\xi \int d\varphi_R \int d^2\boldsymbol{k}_T\ (\boldsymbol{k}_T \cdot \boldsymbol{R}_T)\, G_1^\perp(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)$$

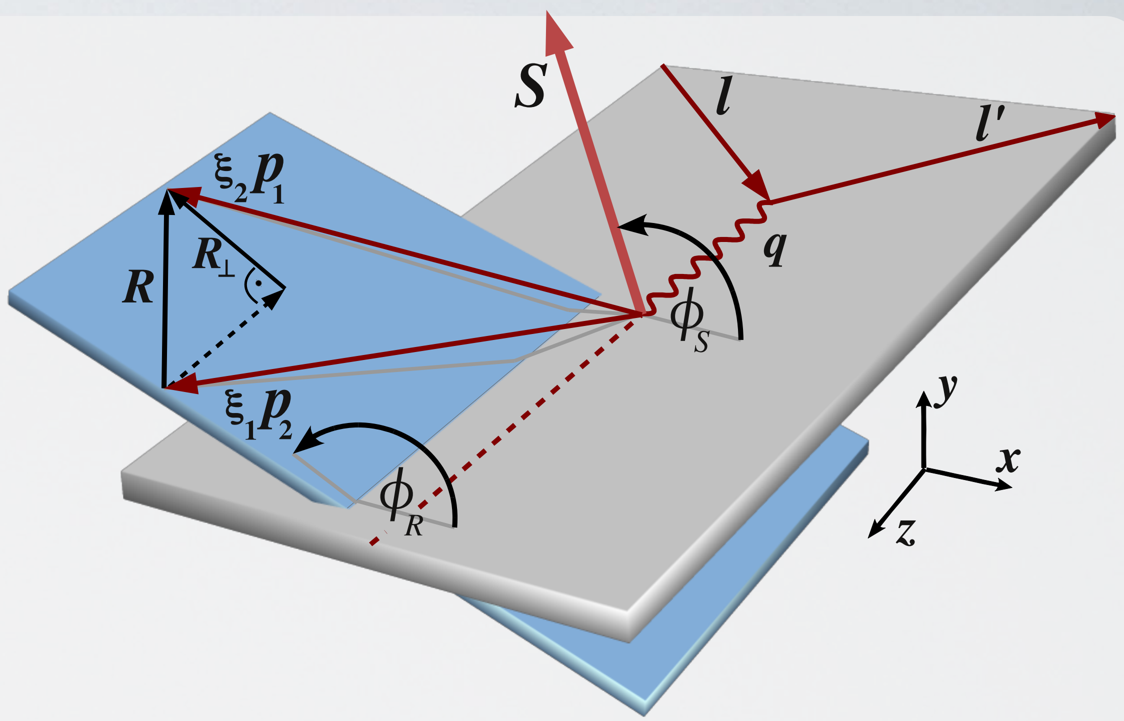
$$H_{1,e^+e^-}^{\sphericalangle}(z, M_h^2) = \int d\xi \int d\varphi_R \int d^2\boldsymbol{k}_T\ |\boldsymbol{R}_T| H_1^{\sphericalangle}(z_h, \xi, k_T^2, R_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)$$

$$H_{1,e^+e^-}^{\perp}(z, M_h^2) = \int d\xi \int d\varphi_R \int d^2\boldsymbol{k}_T\ |\boldsymbol{k}_T| H_1^{\perp}(z_h, \xi, k_T^2, R_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)$$

Access Transversity PDF using DiFFs in *SIDIS*

M. Radici, et al: PRD 65, 074031 (2002).

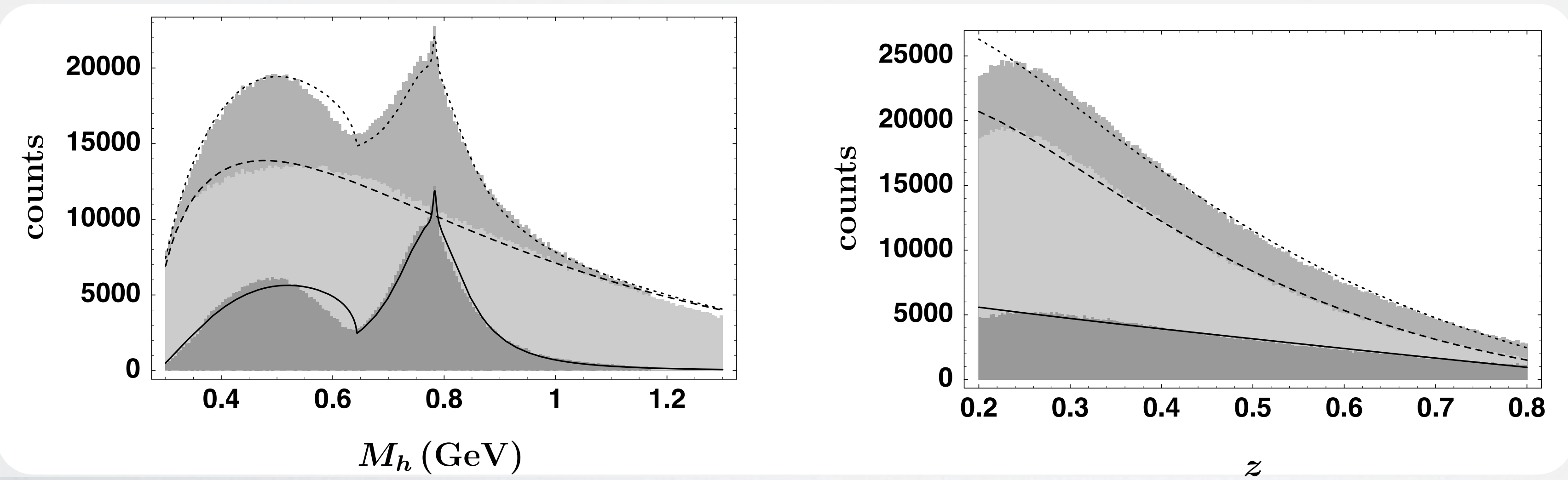
- In two hadron production from polarized target the cross section factorizes *collinearly* - no TMD!
- Allows clean access to *transversity*.
- *Unpolarized* and *Interference* Dihadron FFs are needed!



$$\frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \propto \sin(\phi_R + \phi_S) \frac{\sum_q e_q^2 \; h_1^q(x)/x \;\; H_1^{\sphericalangle q}(z, M_h^2)}{\sum_q e_q^2 \; f_1^q(x)/x \;\; D_1^q(z, M_h^2)}$$

- Empirical Model for D_1^q has been fitted to PYTHIA simulations.

A. Bacchetta and M. Radici, PRD 74, 114007 (2006).



Experiments:
BELLE,
HERMES,
COMPASS.

Moments of DiFFs in SIDIS

A. Bacchetta, M. Radici: PRD 69, 074026 (2004).

- *Here transversely polarised DiFFs are **admixture of cos Fourier moments** of both unintegrated DiFFs:*

$$H_{1,SIDIS}^{\sphericalangle}(z, M_H^2) = \left[H_1^{\sphericalangle[0]} + H_1^{\perp[1]} \right]$$

$$H_{1,SIDIS}^{\perp}(z, M_H^2) = \left[H_1^{\perp[0]} + H_1^{\sphericalangle[1]} \right]$$

- *Generated by* $\cos(\varphi_{RK})$ *dependences of unintegrated DiFFs:*

$\varphi_{RK} \equiv \varphi_R - \varphi_k$

$$H_1^{\sphericalangle}(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)$$
$$H_1^{\perp}(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)$$

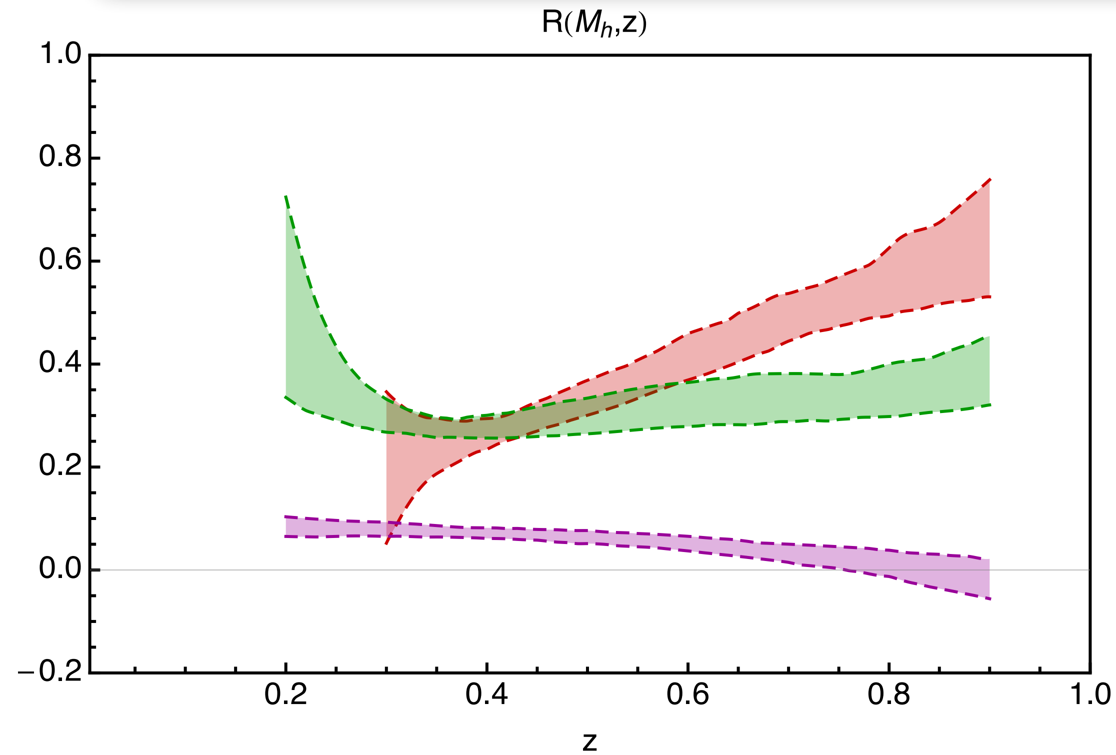
- ***Might be differences with those measured in e^+e^- !***

Current Challenges

1) Phenomenological Extractions of DiFFs.

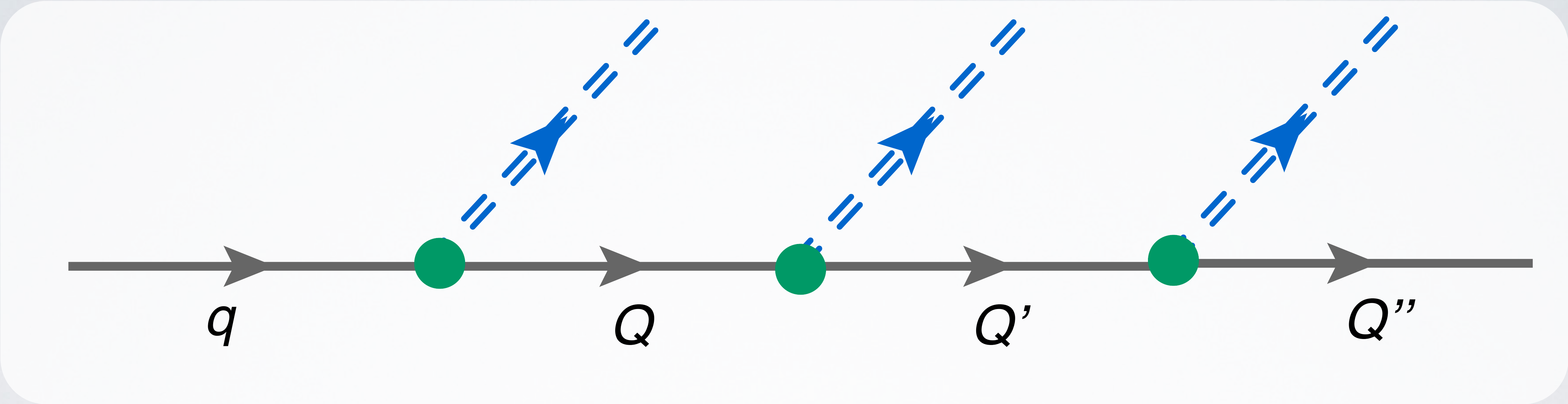
- Unpolarised DiFFs from *PYTHIA*
- Still Large Uncertainties.
- Simplistic Approximations.
- Limited kinematic region.

Radici et al: JHEP JHEP 1505 (2015) 123.



2) Full Event Generators:

- ***No*** Mainstream MC generator ***includes spin*** in Full Hadronization *yet*: *PYTHIA, HERWIG, SHERPA…*
- MC generators are needed to support mapping of the 3D structure of nucleon at *JLab12, BELLE II, EIC.*



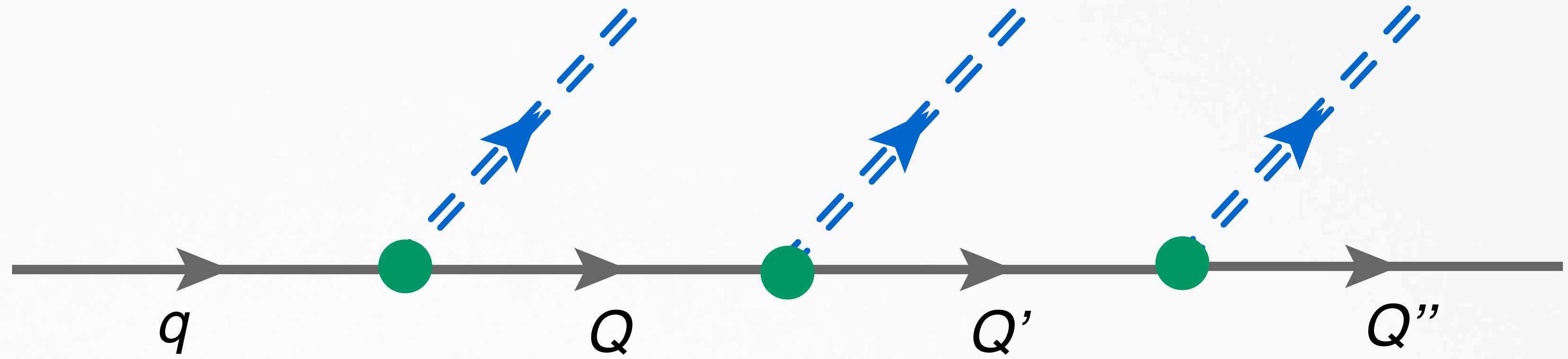
The Quark-jet Framework

THE QUARK JET MODEL

Field, Feynman, Nucl.Phys.B136:1,1978.

Assumptions:

- Number Density interpretation
- No re-absorption
- ∞ hadron emissions



$$D_q^h(z) = \hat{d}_q^h(z) + \int_z^1 \hat{d}_q^Q(y)dy \cdot D_Q^h(\frac{z}{y})\frac{1}{y}$$

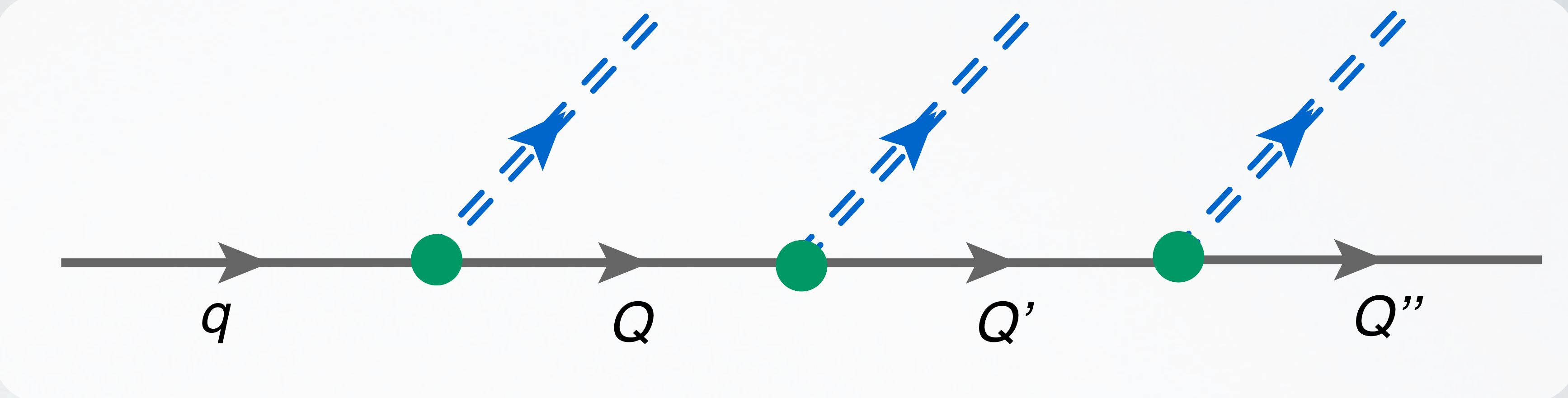
$$\hat{d}_q^h(z) = \hat{d}_q^{Q'}(1-z)|_{h=\bar{Q}'q}$$

THE QUARK JET MODEL

Field, Feynman, Nucl.Phys.B136:1,1978.

Assumptions:

- Number Density interpretation
- No re-absorption
- ∞ hadron emissions



Probability of finding hadron h with mom. frac. [z, z+dz] in a jet of quark q

The probability scales with mom. fraction

$$D_q^h(z)dz = \hat{d}_q^h(z)dz + \int_z^1 \hat{d}_q^Q(y)dy \cdot D_Q^h\left(\frac{z}{y}\right)\frac{dz}{y}$$

Prob. of emitting at step 1

Prob. of mom. [y, y+dy] is transferred to jet at step 1.

ELEMENTARY SPLITTINGS

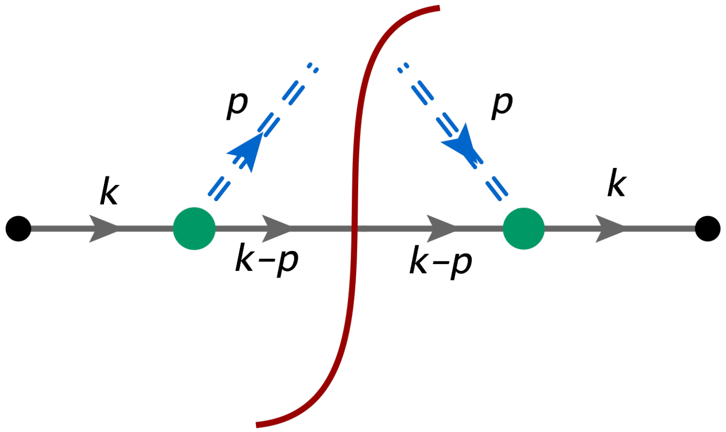
H.M., Thomas, Bentz, PRD. 83:07400; PRD.83:114010, 2011.

- **Quark-quark correlator:**

$$\Delta_{ij}(z,p_\perp) = \frac{1}{2N_c\, z}\sum_X \int \frac{d\xi^+ d^2\boldsymbol{\xi}_\perp}{(2\pi)^3}\, e^{ip\cdot\xi} \times \langle 0|\mathcal{U}_{(\infty,\xi)}\,\psi_i(\xi)|h,X\rangle_{\text{out out}}\langle h,X|\bar\psi_j(0)\,\mathcal{U}_{(0,\infty)}|0\rangle\Big|_{\xi^-=0}$$

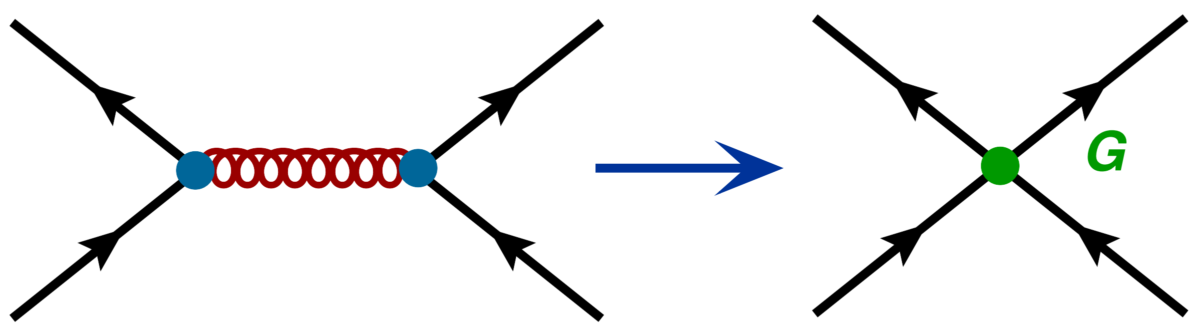
- **One-quark truncation of the wavefunction:** $q \to Qh$

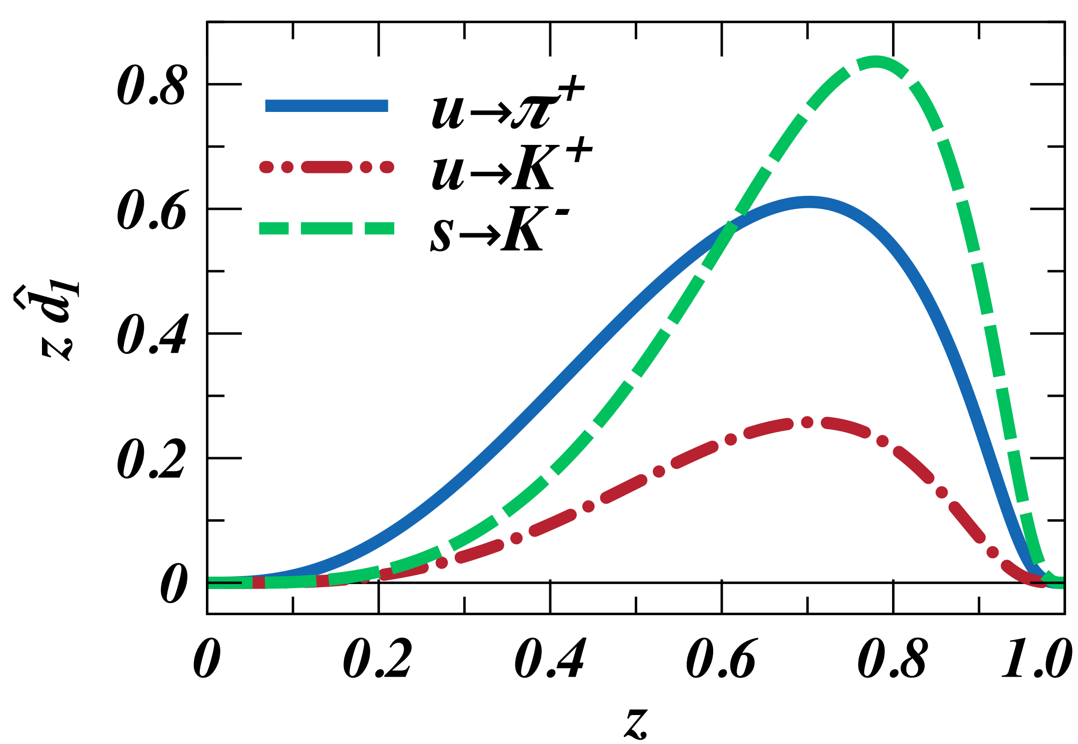
$$d_q^h(z,p_\perp^2) = \frac{1}{2}\mathrm{Tr}[\Delta_0(z,p_\perp^2)\gamma^+]$$



- **Use Nambu--Jona-Lasinio (NJL) Effective quark model:**

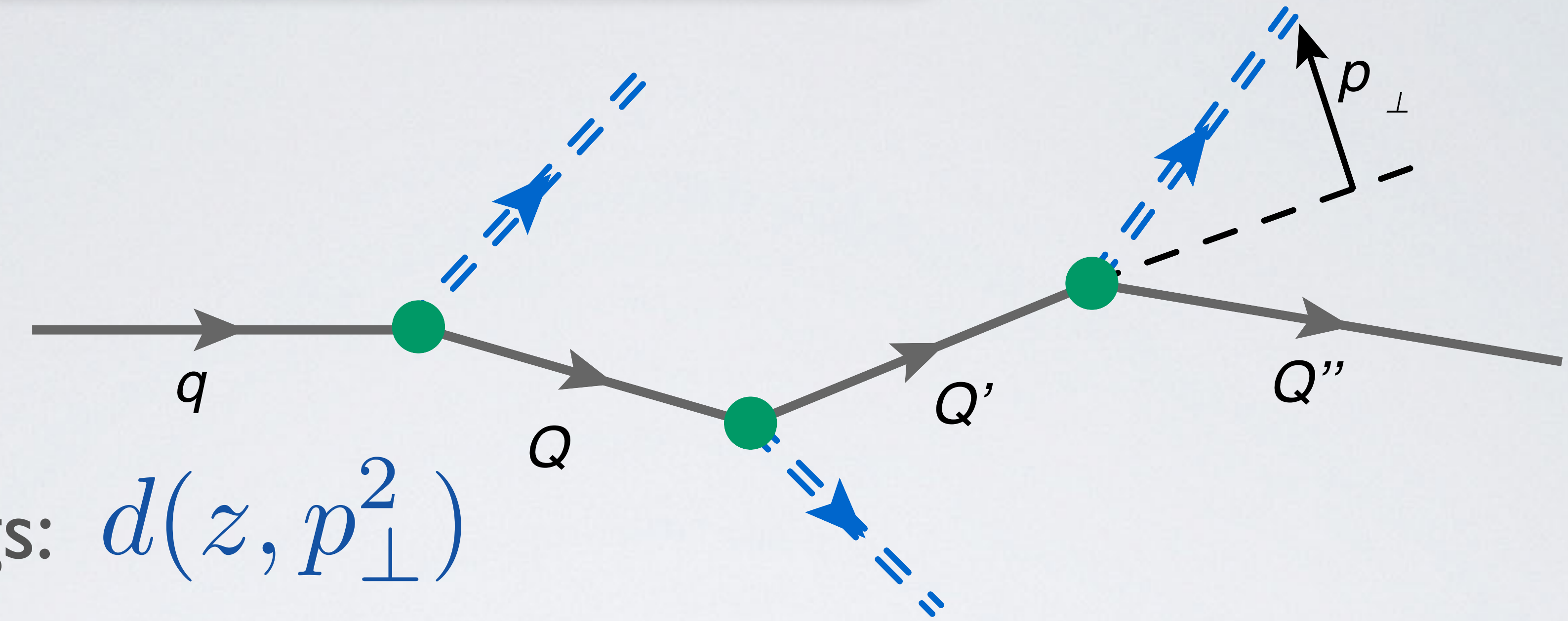
$$\mathcal{L}_{NJL} = \overline{\psi}_q(i\partial\!\!\!/ - m_q)\psi_q + G(\overline{\psi}_q\Gamma\psi_q)^2$$





INCLUDING THE *TRANSVERSE MOMENTUM*

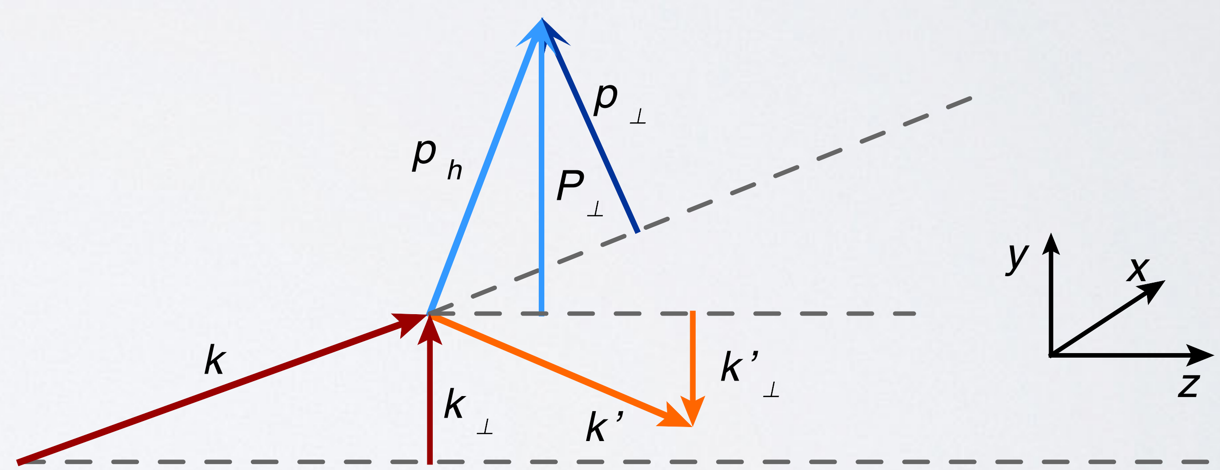
H.M.,Bentz, Cloet, Thomas, PRD.85:014021, 2012



- TMD splittings: $d(z, p_\perp^2)$
- Conserve transverse momenta at each link.

$$\boxed{\mathbf{P}_\perp = \mathbf{p}_\perp + z\mathbf{k}_\perp}$$

$$\boxed{\mathbf{k}_\perp = \mathbf{P}_\perp + \mathbf{k}'_\perp}$$

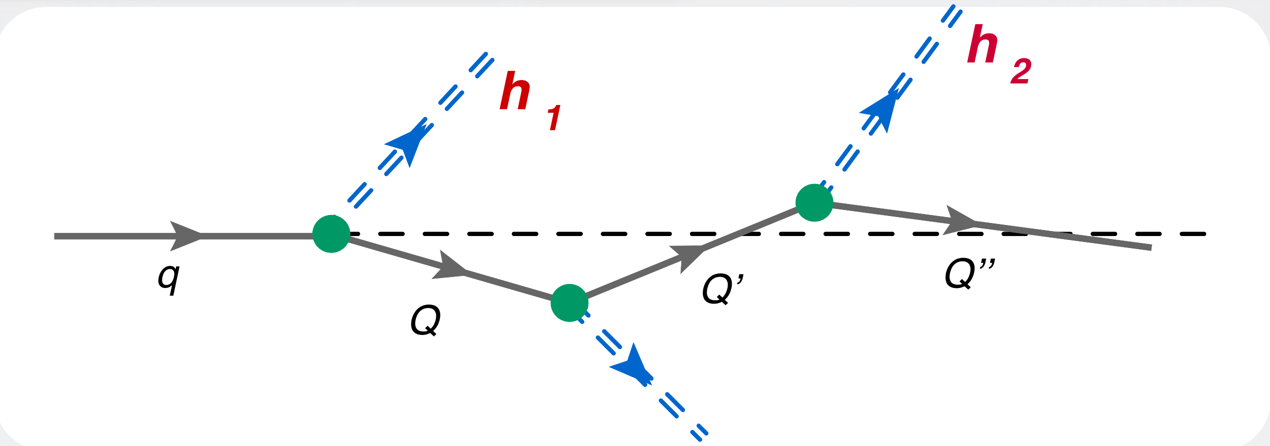


- Calculate the Number Density

$$D_q^h(z, P_\perp^2)\Delta z\ \pi\Delta P_\perp^2 = \frac{\sum_{N_{Sims}} N_q^h(z, z+\Delta z, P_\perp^2, P_\perp^2 + \Delta P_\perp^2)}{N_{Sims}}.$$

UNPOLARIZED DIHADRON FRAGMENTATIONS

H.M. Thomas, Bentz, PRD.88:094022, 2013.



- The probability density for observing two hadrons:

$$P_1 = (z_1 k^-, P_1^+, \boldsymbol{P}_{1,\perp}),\ P_1^2 = M_{h1}^2$$

$$P_2 = (z_2 k^-, P_2^+, \boldsymbol{P}_{2,\perp}),\ P_2^2 = M_{h2}^2$$

- The corresponding number density:

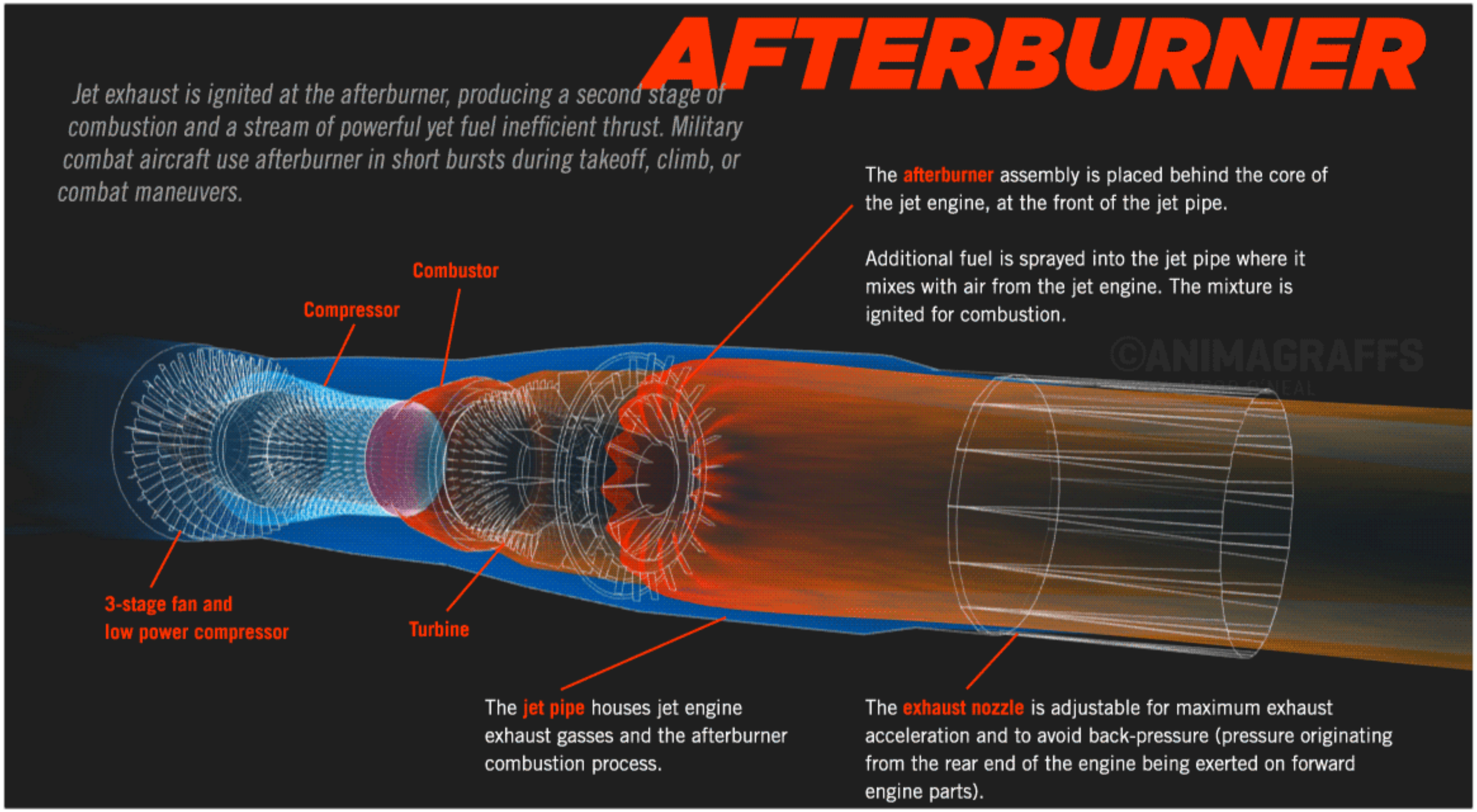
$$\boxed{D_q^{h_1 h_2}(z, M_h^2)\ \Delta z\ \Delta M_h^2 = \langle N_q^{h_1 h_2}(z, z+\Delta z; M_h^2, M_h^2 + \Delta M_h^2)\rangle}$$

$$z = z_1 + z_2 \quad M_h^2 = (P_1 + P_2)^2$$

- Kinematic Constraint.

$$\boxed{z_1 z_2 M_h^2 - (z_1 + z_2)(z_2 M_{h1}^2 + z_1 M_{h2}^2) \geq 0}$$

- In MC simulations record all the pairs in every decay chain.

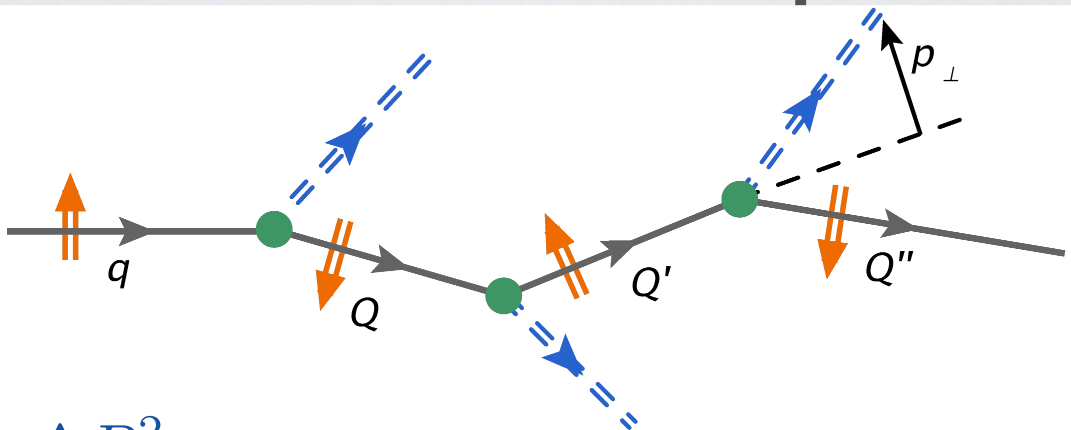


POLARISATION IN QUARK-JET FRAMEWORK

POLARIZATION IN QUARK-JET FRAMEWORK

H.M.,Bentz, Thomas, PRD.86:034025, (2012). H.M., Kotzinian, Thomas, PLB731 208-216 (2014).

- Extend Quark-jet Model to include Spin.

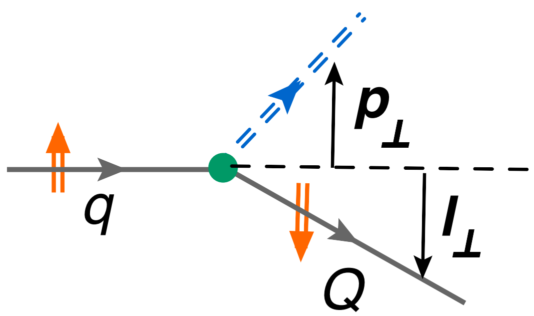


$$D_{h/q^\uparrow}(z, P_\perp^2, \varphi)\,\Delta z\,\frac{\Delta P_\perp^2}{2}\,\Delta\varphi = \left\langle N_{q^\uparrow}^h(z, z+\Delta z; P_\perp^2, P_\perp^2+\Delta P^2; \varphi, \varphi+\Delta\varphi)\right\rangle$$

- Input Elementary Collins Function: *Model* or *Parametrization*

- *Calc. Spin of the remnant quark:* $\mathbf{S}'$

Previously: constant values for spin flip probability: $\mathcal{P}_{SF}$



✦ Use fit form to extract unpol. and Collins FFs from $D_{h/q^\uparrow}$.

$$F(c_0, c_1) \equiv c_0 - c_1 \sin(\varphi_C)$$

$$D_{h/q^\uparrow}(z, p_\perp^2, \varphi) = D^{h/q}(z, p_\perp^2) - H^{\perp h/q}(z, p_\perp^2)\frac{p_\perp s_T}{z m_h}\sin(\varphi_C)$$

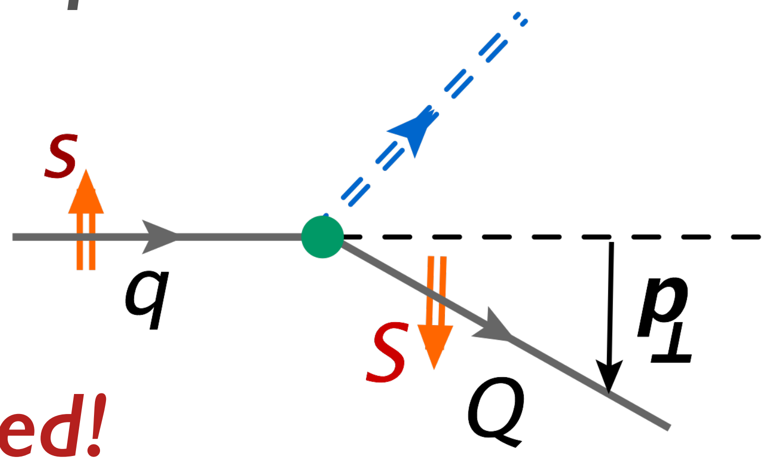
SPIN TRANSFER

✦NJL-jet MKIII:

Bentz et al, Phys.Rev. D94 034004 (2016).

- *The probability for the process* $q \to Q$, *initial spin* $\mathbf{s}$ *to* $\mathbf{S}$

$$F^{q\to Q}(z, \mathbf{p}_\perp; \mathbf{s}, \mathbf{S}) = \alpha_{\mathbf{s}} + \boldsymbol{\beta}_{\mathbf{s}} \cdot \mathbf{S}$$

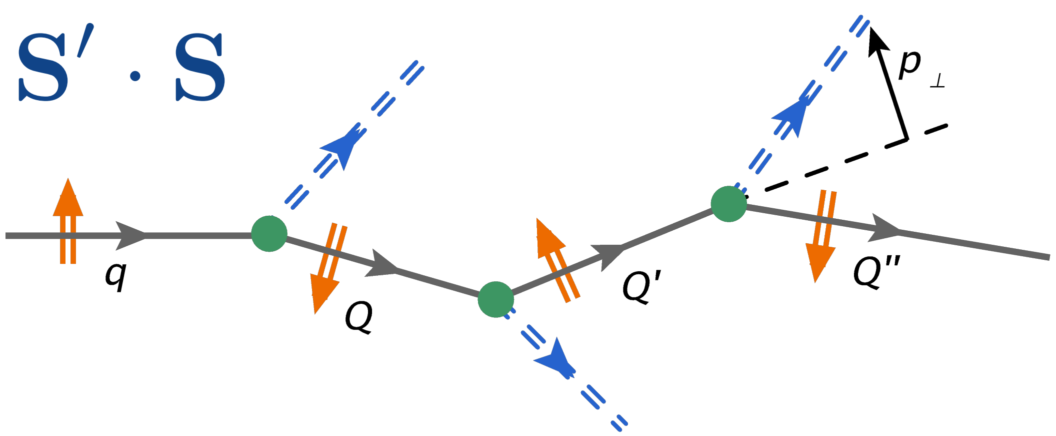


- *Intermediate quarks in quark-jet are unobserved!*

Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii: **QUANTUM ELECTRODYNAMICS (1982).**

$$F^{q\to Q}(z, \mathbf{p}_\perp; \mathbf{s}, \mathbf{S}) \sim \mathrm{Tr}[\rho^{\mathbf{S}'}\rho^{\mathbf{S}}] \sim 1 + \mathbf{S}' \cdot \mathbf{S}$$

$$\mathbf{S}' = \frac{\boldsymbol{\beta}_{\mathbf{s}}}{\alpha_{\mathbf{s}}}$$



- *Remnant quark's* $\mathbf{S}'$ *uniquely determined by* $z, \mathbf{p}_\perp$ *and* $\mathbf{s}$ *!*

- *Process probability is the same as transition to unpolarized state.*

$$F^{q\to Q}(z, \mathbf{p}_\perp; \mathbf{s}, \mathbf{0}) = \alpha_s$$

REMNANT QUARK'S POLARISATION

✦ **We can express the spin of the remnant quark** $\mathbf{S}' = \dfrac{\beta_{\mathbf{s}}}{\alpha_{\mathbf{s}}}$ **in terms of *quark-to-quark TMD FFs*.**

$$\alpha_q \equiv D(z, \boldsymbol{p}_\perp^2) + (\boldsymbol{p}_\perp \times \boldsymbol{s}_T) \cdot \hat{\boldsymbol{z}} \frac{1}{z\mathcal{M}} H^\perp(z, \boldsymbol{p}_\perp^2)$$

$$\beta_{q\parallel} \equiv s_L \, G_L(z, \boldsymbol{p}_\perp^2) - (\boldsymbol{p}_\perp \cdot \boldsymbol{s}_T) \frac{1}{z\mathcal{M}} H_L^\perp(z, \boldsymbol{p}_\perp^2)$$

$$\boldsymbol{\beta}_{q\perp} \equiv \boldsymbol{p}'_\perp \frac{1}{z\,\mathcal{M}} D_T^\perp(z, \boldsymbol{p}_\perp^2) - \boldsymbol{p}_\perp \frac{1}{z\mathcal{M}} s_L G_T(z, \boldsymbol{p}_\perp^2) + \boldsymbol{s}_T \, H_T(z, \boldsymbol{p}_\perp^2) + \boldsymbol{p}_\perp (\boldsymbol{p}_\perp \cdot \boldsymbol{s}_T) \frac{1}{z^2\mathcal{M}^2} H_T^\perp(z, \boldsymbol{p}_\perp^2)$$

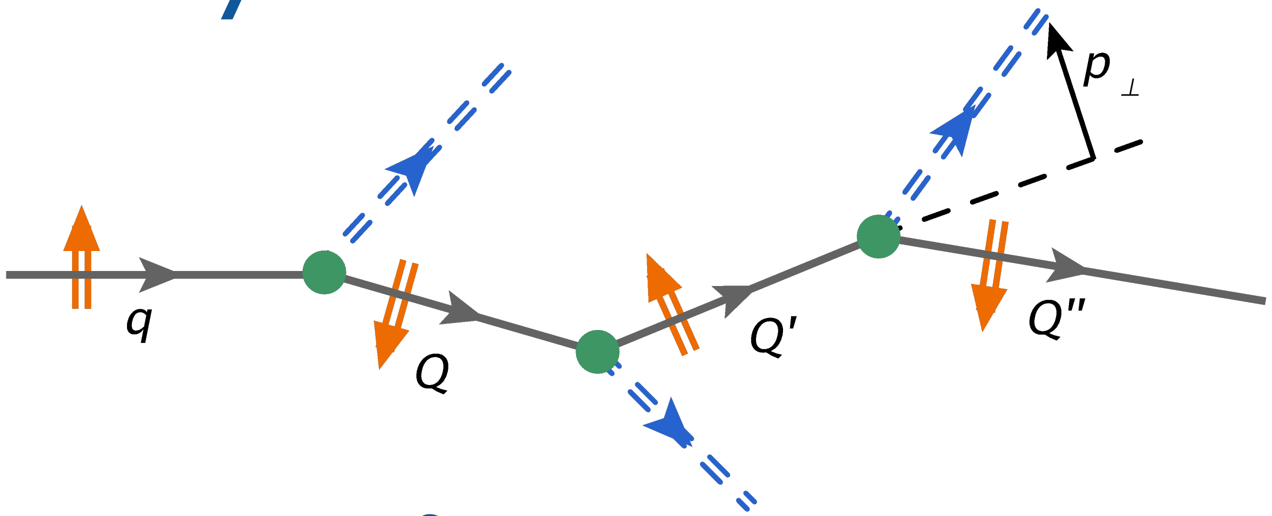
$$F^{q \to Q}(z, \boldsymbol{p}_\perp; \boldsymbol{s}, \boldsymbol{S})$$

Q/q	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	$H_{1T} H_{1T}^\perp$

MC SIMULATION OF FULL HADRONIZATION

HM et al, Phys. Rev. D95 04021, (2017)

- ✦ We can consider many hadron emissions.



- ✦ We can sample the $h, z, p_{\perp}^{2}, \varphi_{h}$ using

$$\boxed{f^{q\to h}(z, p_{\perp}^{2}, \varphi_{h}; \mathbf{S}_{T})}$$

- ✦ Determine the momenta in the initial frame and calculate

$$\Delta N = \langle N_{q}^{h_1 h_2}(z, z+\Delta z, \varphi, \varphi+\Delta\varphi, ...)\rangle$$

- ✦ *Calculate the remnant quark's spin:* $\mathrm{S}' = \frac{\boldsymbol{\beta}_{\mathbf{s}}}{\alpha_{\mathbf{s}}}$
- ✦ We only need the "elementary" splittings.

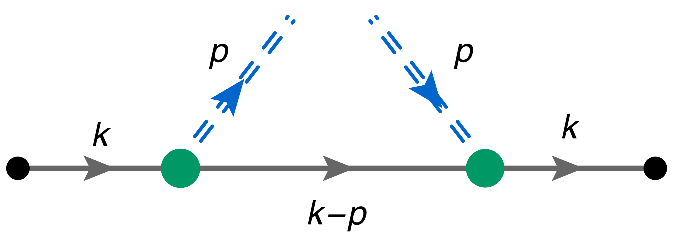
$$\boxed{f^{q\to h}} \qquad \boxed{f^{q\to Q}}$$

Model Calculations of $q \to Q$ Splittings

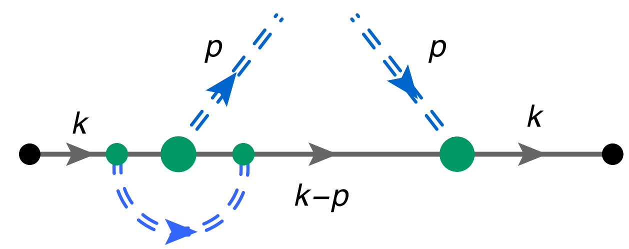
E.G. - Meissner et al, PLB 690, 296 (2010).

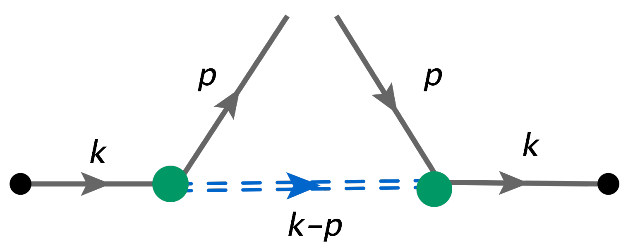
✦ We can use the same "spectator" type calculations as for pion.

T-even **T-odd**

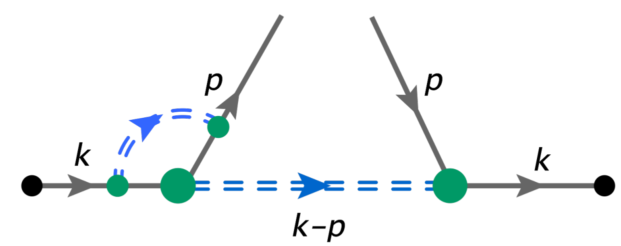


$$q \to h$$





$$q \to Q$$



✦ Positivity Constraints on TMD FFs:

Bacchetta et al, P.R.L. 85, 712 (2000).

$$(H_L^{\perp[1]})^2 + (D_T^{\perp[1]})^2 \leq \frac{p_\perp^2}{4z^2M^2}(D+G_L)(D-G_L) \leq \frac{p_\perp^2}{4z^2M^2}D^2$$

$$(G_T^{[1]})^2 + (H^{\perp[1]})^2 \leq \frac{p_\perp^2}{4z^2M^2}(D+G_L)(D-G_L) \leq \frac{p_\perp^2}{4z^2M^2}D^2$$

✦ T-odd parts from previous models *violate positivity*!

$$(\hat{G}_T^{[1]})^2 = (\hat{H}_L^{\perp[1]})^2 = \frac{p_\perp^2}{4z^2M^2}(\hat{D}+\hat{G}_L)(\hat{D}-\hat{G}_L) \leq \frac{p_\perp^2}{4z^2M^2}\hat{D}^2$$

$$\boxed{\hat{H}^\perp(z,p_\perp^2) = 0, \quad \hat{D}_T^\perp(z,p_\perp^2) = 0.}$$

Model Calculations of $q \to Q$ Splittings

✦ **Simple Model that is positive-definite:**

$$\hat{d}(z,p_\perp^2) = 1.1\, \hat{d}_{tree}(z,p_\perp^2),$$

✦ **Use Collins-ansatz for T-odd**

J. C. Collins, NPB 396, 161 (1993)

$$\frac{p_\perp}{zM} \frac{\hat{h}^{\perp(q\to h)}(z,p_\perp^2)}{\hat{d}^{(q\to h)}(z,p_\perp^2)} = 0.4 \frac{2\, p_\perp M_Q}{p_\perp^2 + M_Q^2}$$

$$d_T^\perp = -h^\perp$$

✦ **Ensures the inequalities**

$$(H_L^{\perp[1]})^2 + (D_T^{\perp[1]})^2 \leq \frac{p_\perp^2}{4z^2M^2}(D+G_L)(D-G_L) \leq \frac{p_\perp^2}{4z^2M^2}D^2$$

$$(G_T^{[1]})^2 + (H^{\perp[1]})^2 \leq \frac{p_\perp^2}{4z^2M^2}(D+G_L)(D-G_L) \leq \frac{p_\perp^2}{4z^2M^2}D^2$$

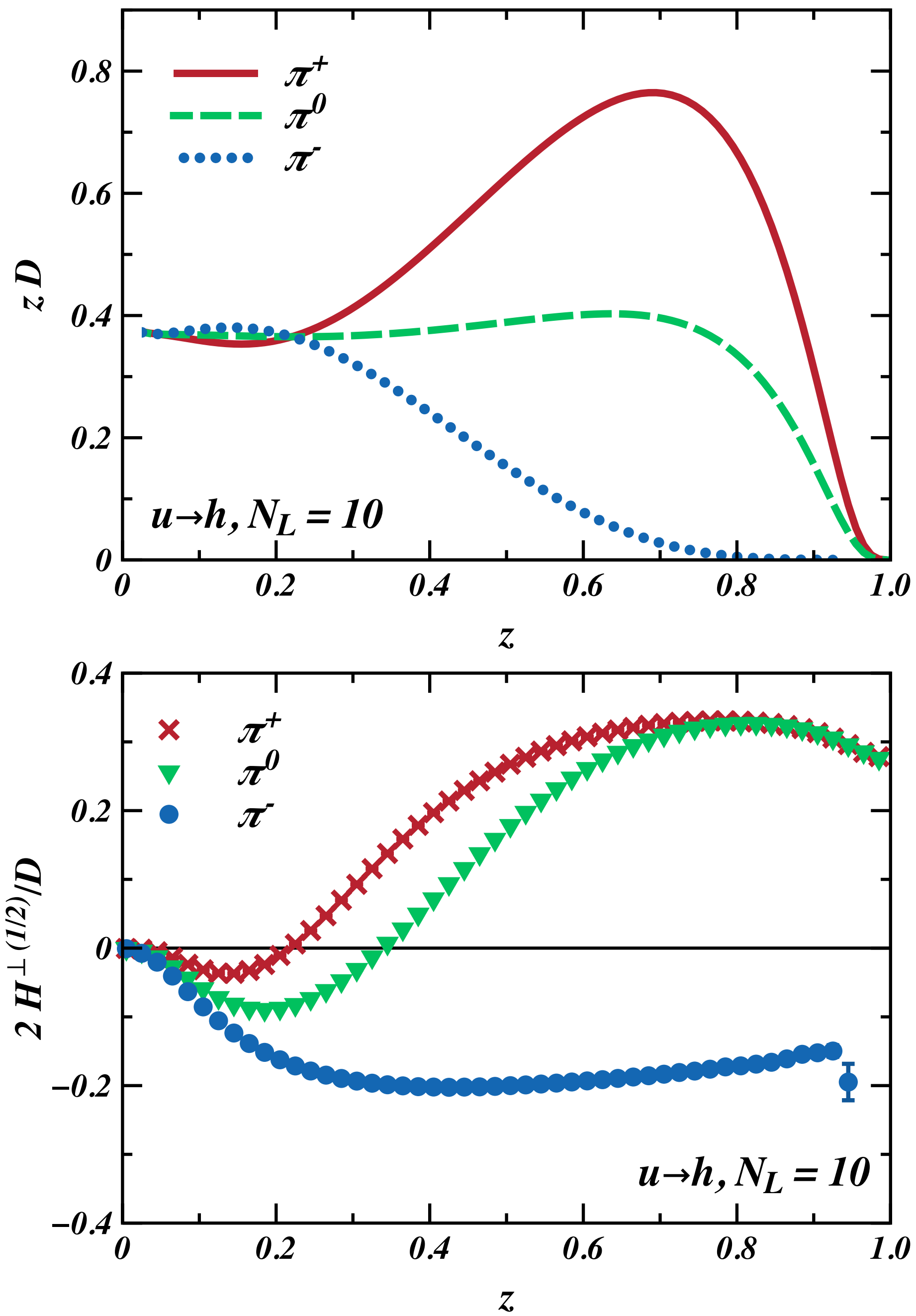
✶ **Also: *Evolution - mimicking ansatz***

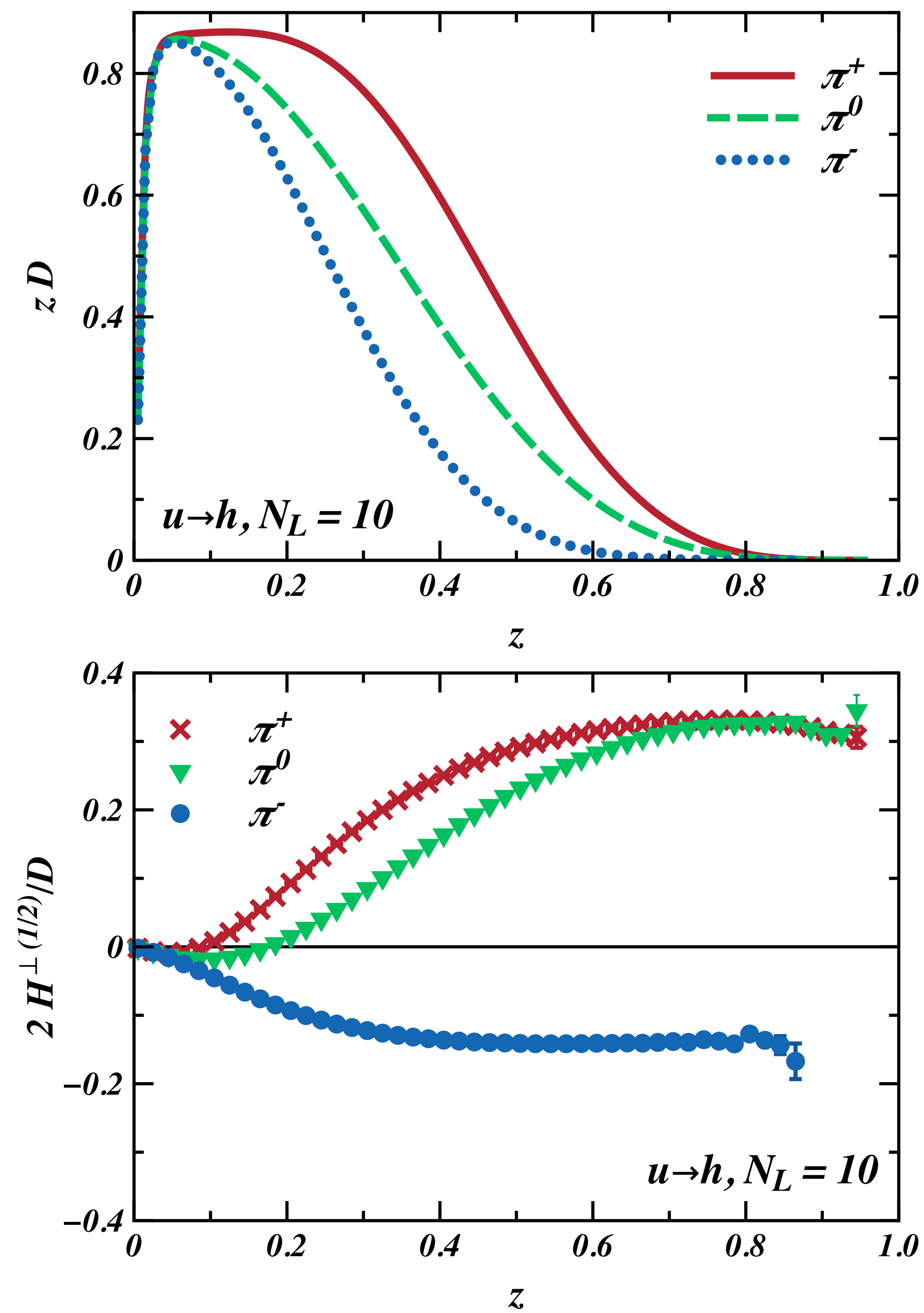
$$\boxed{\hat{d}'(z,p_\perp^2) = (1-z)^4 \hat{d}(z,p_\perp^2)}$$

Results for Collins Effect

HM et al, Phys. Rev. D95 04021, (2017)

- *NJL* Model
- Evolution-mimicking *Ansatz.*

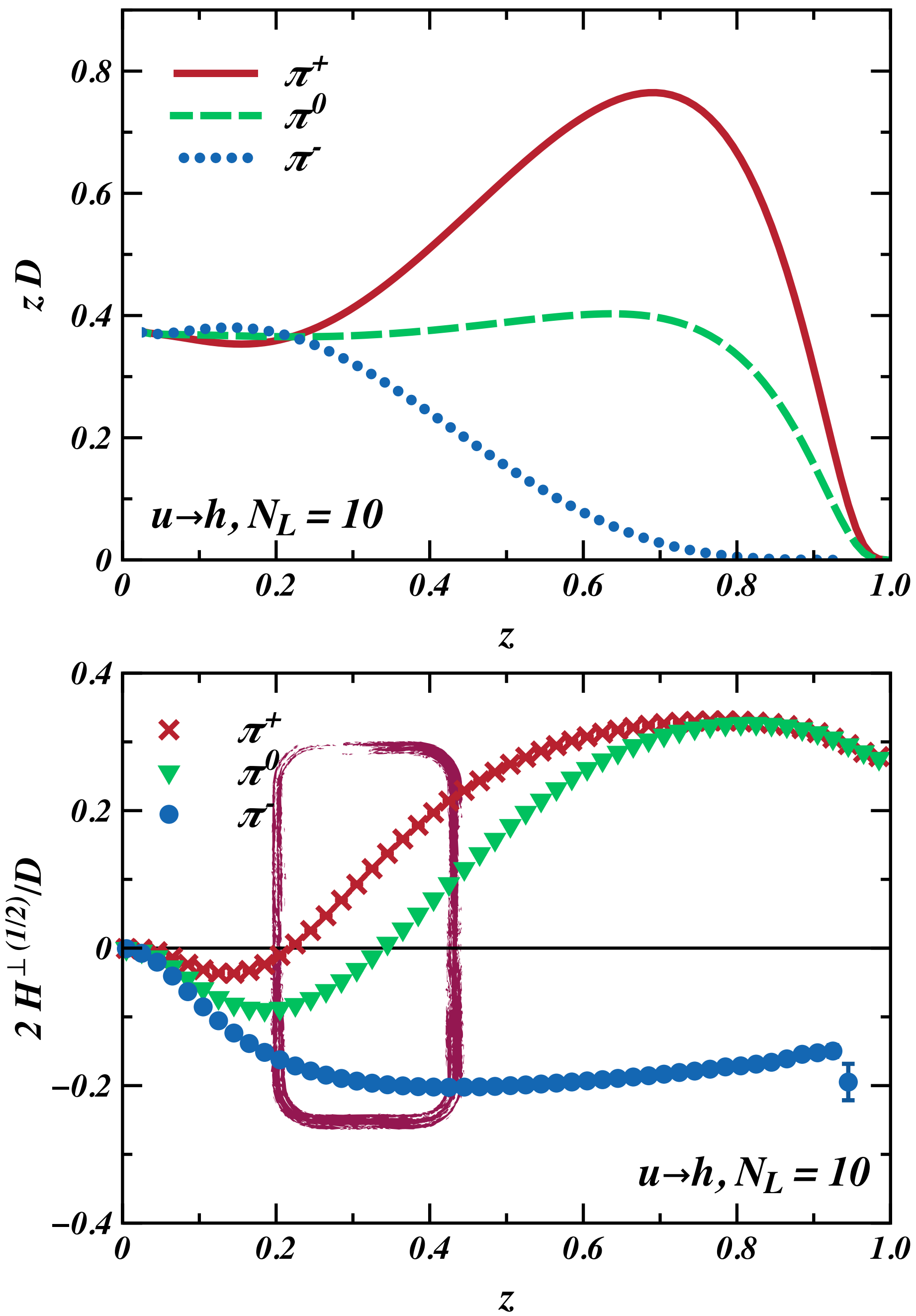




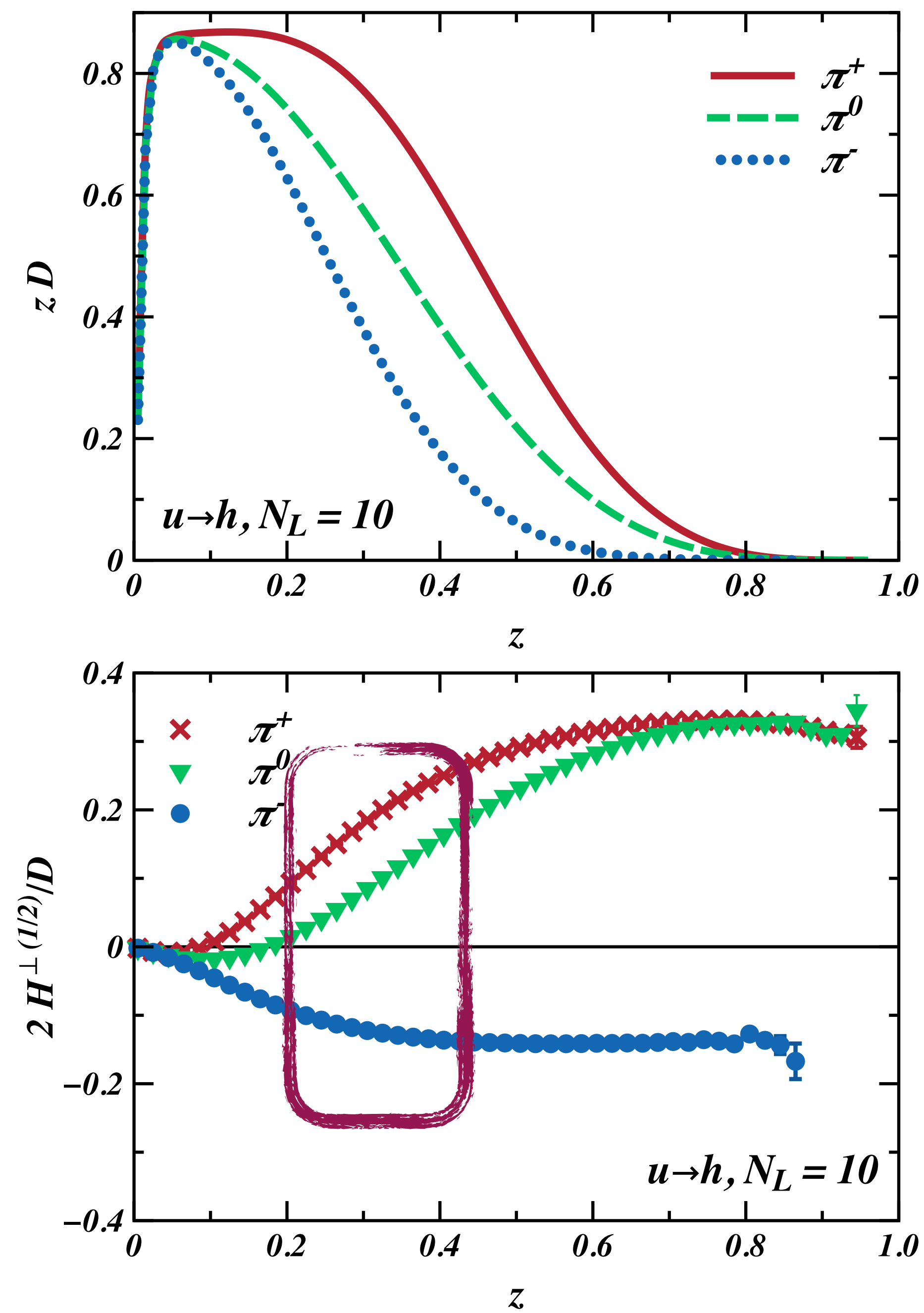
Results for Collins Effect

HM et al, Phys. Rev. D95 04021, (2017)

- ***NJL* Model**



- **Evolution-mimicking *Ansatz.***



✦ ***Opposite sign and similar size*** in mid-z range for charged pions. (Similar to empirical extractions).

✦ ***Dependence on model inputs:*** can be tuned to data.

Longitudinal Polarisation
in DiHadron FFs

DIFFS FROM THE NUMBER DENSITY

HM et al, arXiv:1707.04999, (2017)

- Can only calculate number density form MC simulations.
- Extract DiFFs from specific angular modulations.

$$F(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T;s_L) = D_1(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\cos(\varphi_{RK})) - s_L \frac{R_T k_T \sin(\varphi_{RK})}{M_1 M_2} G_1^{\perp}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\cos(\varphi_{RK}))$$

- Unpolarized DiFF: *straight forward integration* of number density.

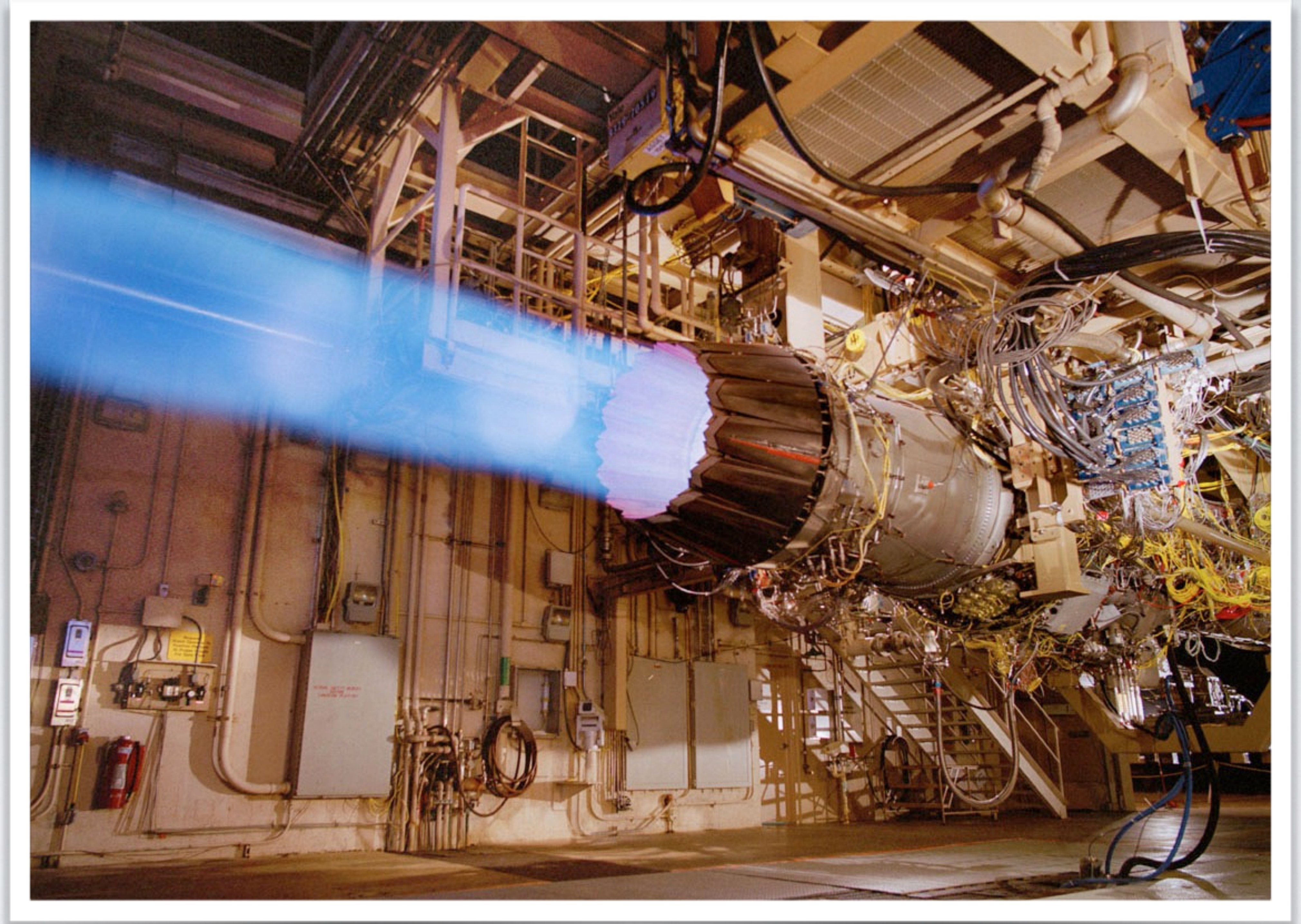
$$D_1(z,M_h^2) = \int d\xi \int d\varphi_R \int d^2\boldsymbol{k}_T \; D_1(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

$$\boxed{D_1(z,M_h^2) = \int d\xi \int d\varphi_R \int d^2\boldsymbol{k}_T \; F(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T;s_L)}$$

- Need $\cot(\varphi_{RK})$ to extract *helicity dependent DiFF!*

$$G_1^{\perp}(z,M_h^2) = \int d\xi \int d\varphi_R \int d^2\boldsymbol{k}_T(\boldsymbol{k}_T\cdot\boldsymbol{R}_T)\; G_1^{\perp}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

$$\boxed{G^{\perp}(z,M_h^2) = -\frac{M_1 M_2}{s_L} \int d\xi \int d\varphi_R \int d^2\boldsymbol{k}_T \; \cot(\varphi_{RK}) F(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T;s_L)}$$

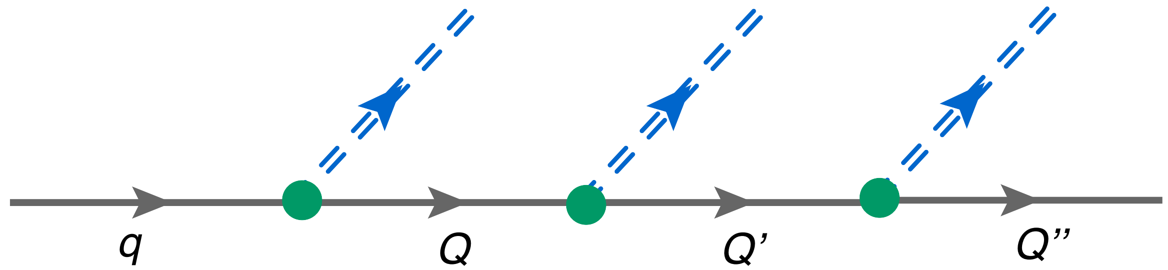


VALIDATION TESTS

The Total Number of Pion Pairs

✦ **Validate MC by analytically calculating the total number of pion pairs produced for given N_L.**

$$(\overbrace{\pi^+,\pi^-,\pi^+,\ldots,}^{N_L-n_0}\overbrace{\pi^0,\pi^0,\pi^0}^{n_0}).$$



$$\boxed{\mathcal{N}^{(\pi^+\pi^-)}(N_L) = \sum_{n_0=0}^{n_0=N_L} C_{N_L}^{n_0}\left(\frac{2}{3}\right)^{N_L-n_0}\left(\frac{1}{3}\right)^{n_0} U\left(\frac{N_L-n_0}{2}\right)\, D\left(\frac{N_L-n_0}{2}\right).}$$

✦ **Extraction from DiFFs.**

$$\mathcal{N}_{MC}^{(\pi^+\pi^-)}(N_L) = \int_0^1 dz\; D_{1,[N_L]}^{u\to\pi^+\pi^-}(z)$$

N_L	$\mathcal{N}^{(\pi^+\pi^-)}$	$\mathcal{N}_N^{(\pi^+\pi^-)}$	$\mathcal{N}_{MC}^{(\pi^+\pi^-)}$	$\mathcal{N}_{MC,z_{min}}^{(\pi^+\pi^-)}$
2	$\frac{4}{9}$	0.44444	0.4444	0.350175
3	$\frac{28}{27}$	1.03704	1.03694	0.683999
4	$\frac{152}{81}$	1.87654	1.87641	0.959588
5	$\frac{712}{243}$	2.93004	2.92992	1.11531
6	$\frac{3068}{729}$	4.2085	4.20882	1.18162
7	$\frac{12484}{2187}$	5.70828	5.70867	1.20282
8	$\frac{48752}{6561}$	7.43057	7.43047	1.20809

✓ **MC simulations and Integral Expressions agree very well!**

✓ **z cuts allow fast convergence with N_L.**

LONGITUDINAL POLARISATION

✦ **DiFF for longitudinally polarized quark:** $s_L \; (\boldsymbol{k}_T \times \boldsymbol{R}_T) \cdot \hat{z}$

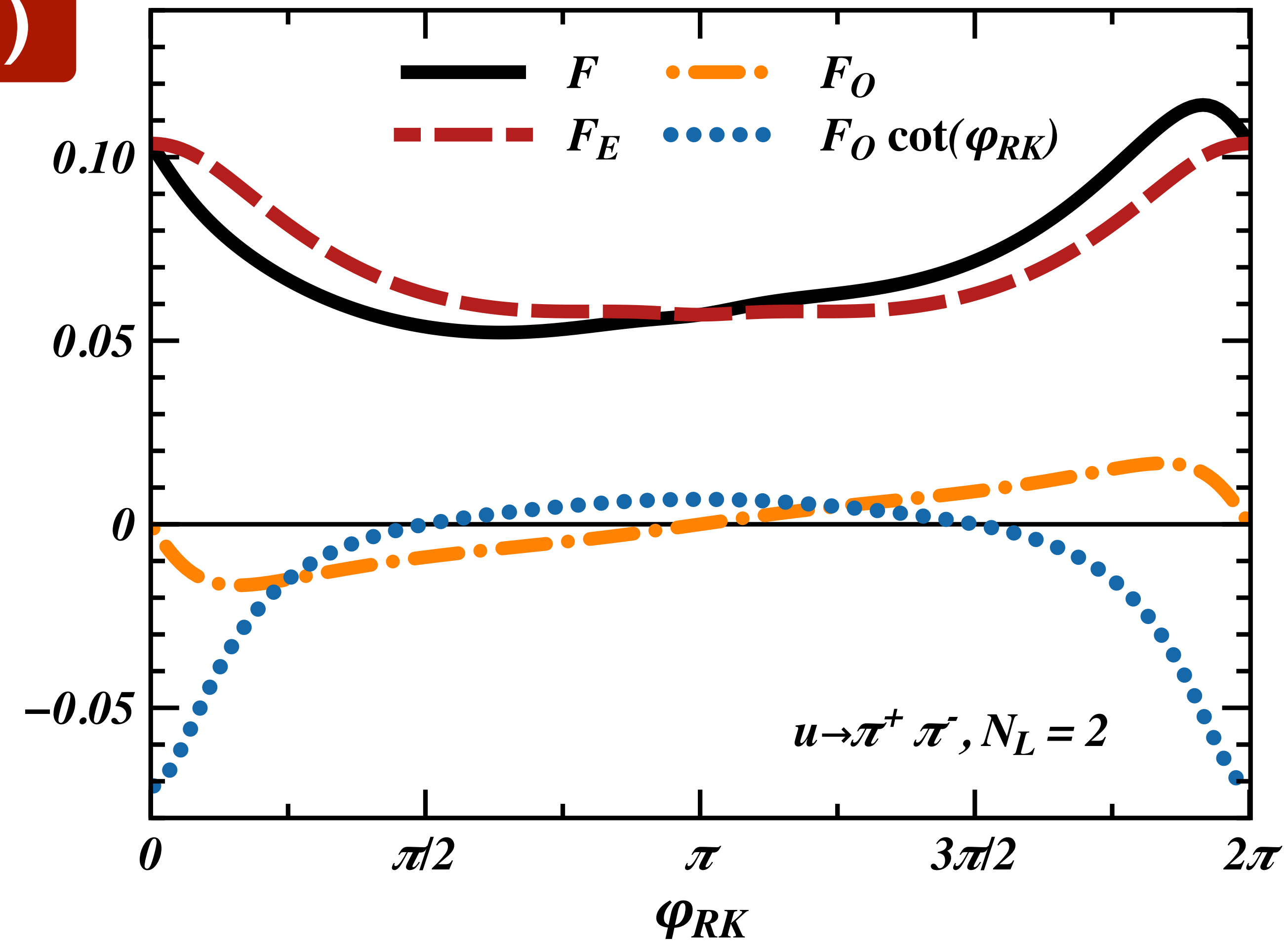
$$\tilde{G}_1^{\perp}(z) = -\frac{1}{s_L} \int d\xi \int d^2\boldsymbol{R}_T \int d^2\boldsymbol{k}_T \;\; \cot(\varphi_{RK}) F(z, \xi, \boldsymbol{k}_T, \boldsymbol{R}_T; s_L).$$

✦ **The extraction method works: the angular dependence for N_L=2.**

(given *large enough* statistics!)

$$F_E(\varphi_{RK}) = \frac{F(\varphi_{RK}) + F(2\pi - \varphi_{RK})}{2}$$

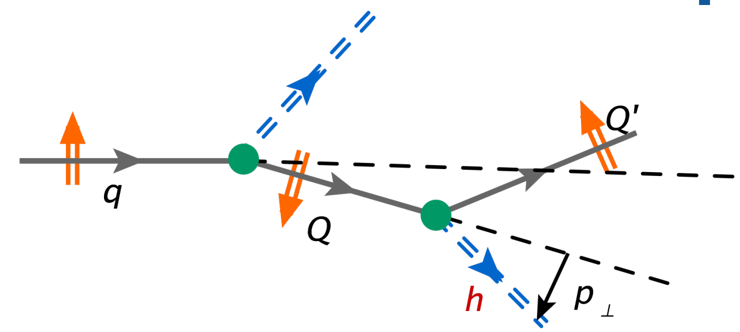
$$F_O(\varphi_{RK}) = \frac{F(\varphi_{RK}) - F(2\pi - \varphi_{RK})}{2}$$



VALIDATION: 2 PRODUCED HADRONS

✦ **Validate MC simulations by comparing to explicit Integral Expressions (IE). Only pions produced in the first two steps!**

$$F^{(2)}_{q\to h_1 h_2} = \sum_{q_1} \hat{f}^{q\to q_1+h_1} \otimes \hat{f}^{q_1\to h_2}.$$

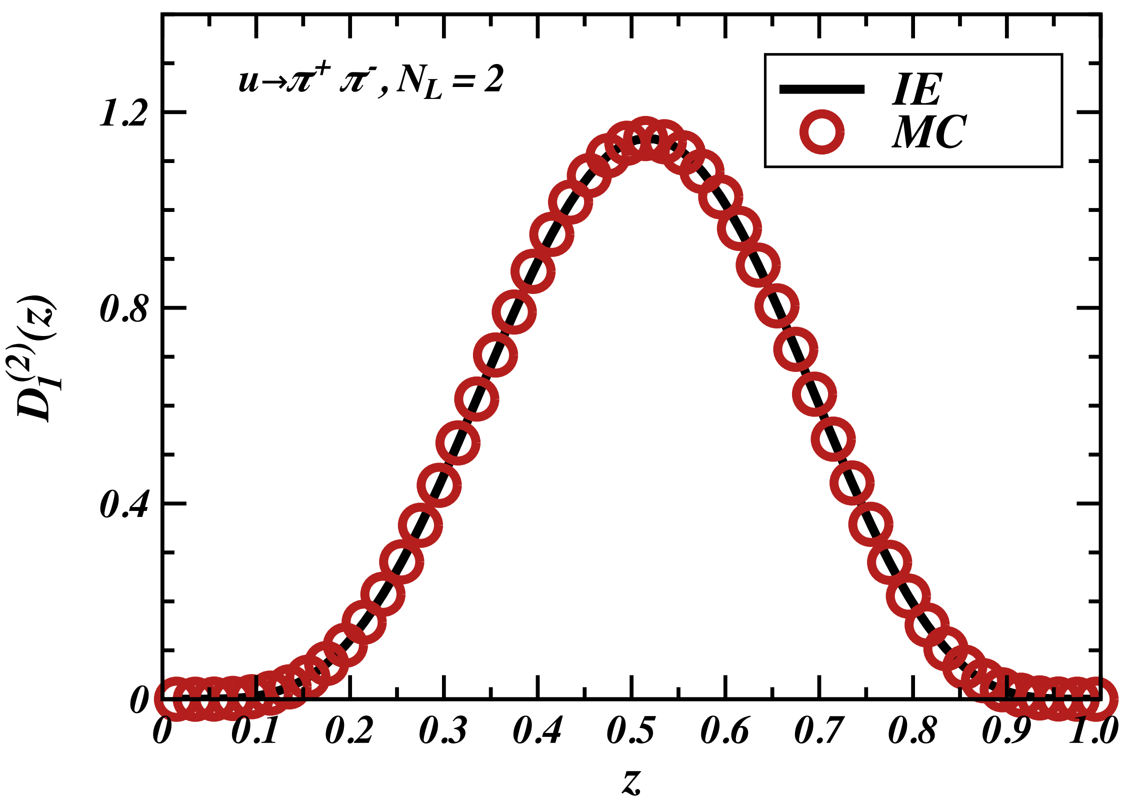


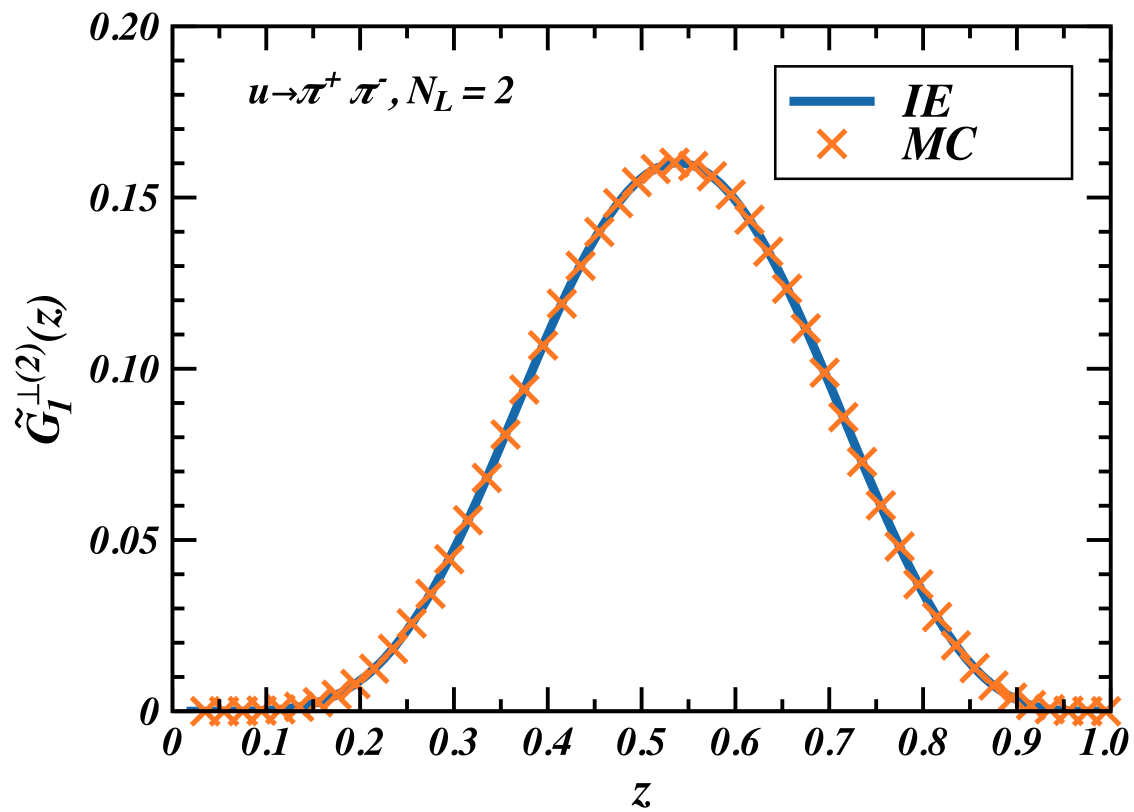
$$D_1^{(2)}(z) = \hat{D}^{q\to q_1} \otimes \hat{D}^{q_1\to h}$$

$$G_1^{\perp(2)}(z) = \hat{G}_T^{q\to q_1} \otimes \hat{H}^{\perp(q_1\to h)}$$

Spin rotation

Collins effect at 2-nd emission



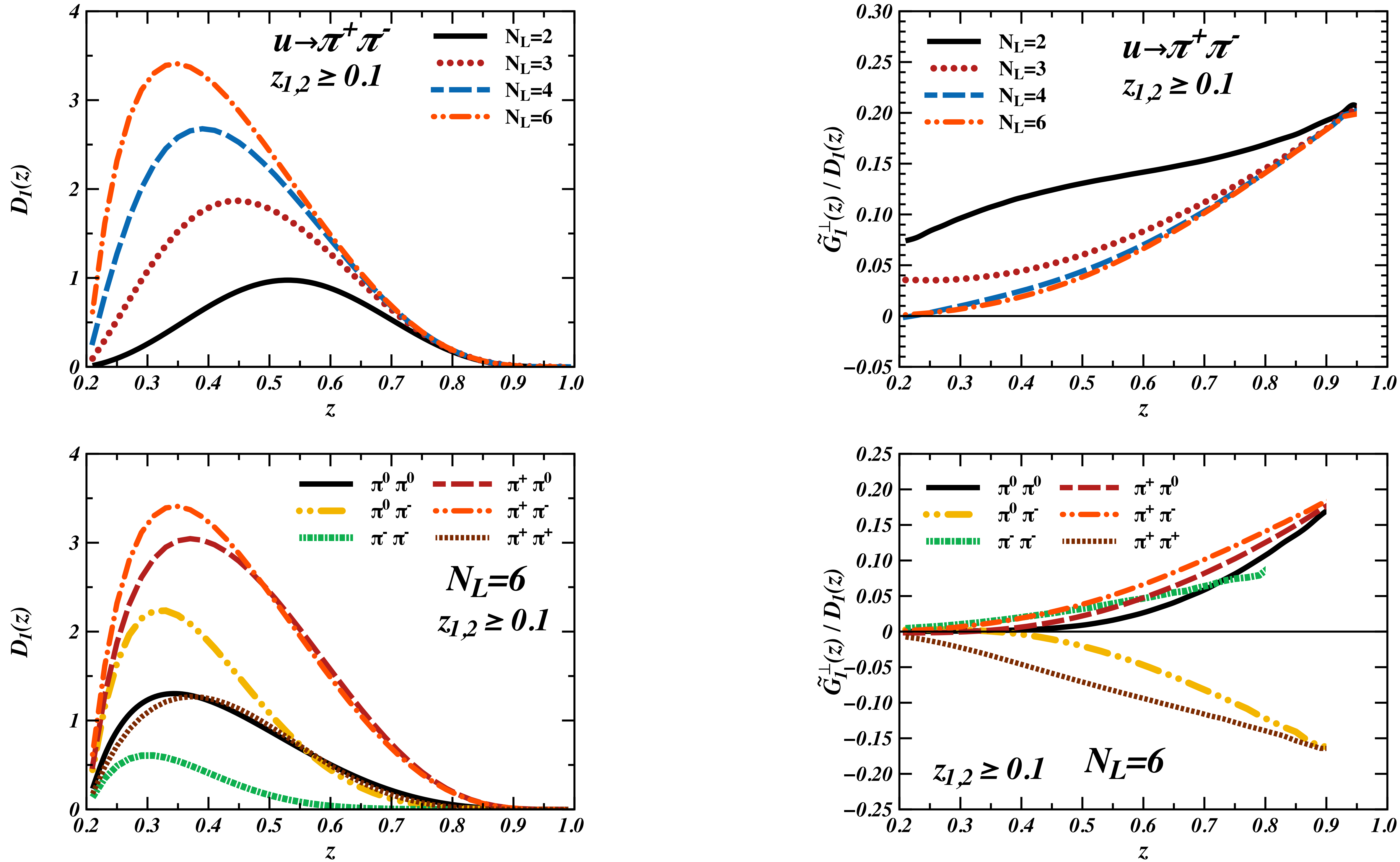


✓ **Collins effect generates helicity dep. two-hadron correlation!**

RESULTS

Results for helicity dependant DiFFs

✦ Results for helicity DiFFs, ***N_L*** dependence, various pairs. Cuts: $z_{1,2} \geq 0.1$

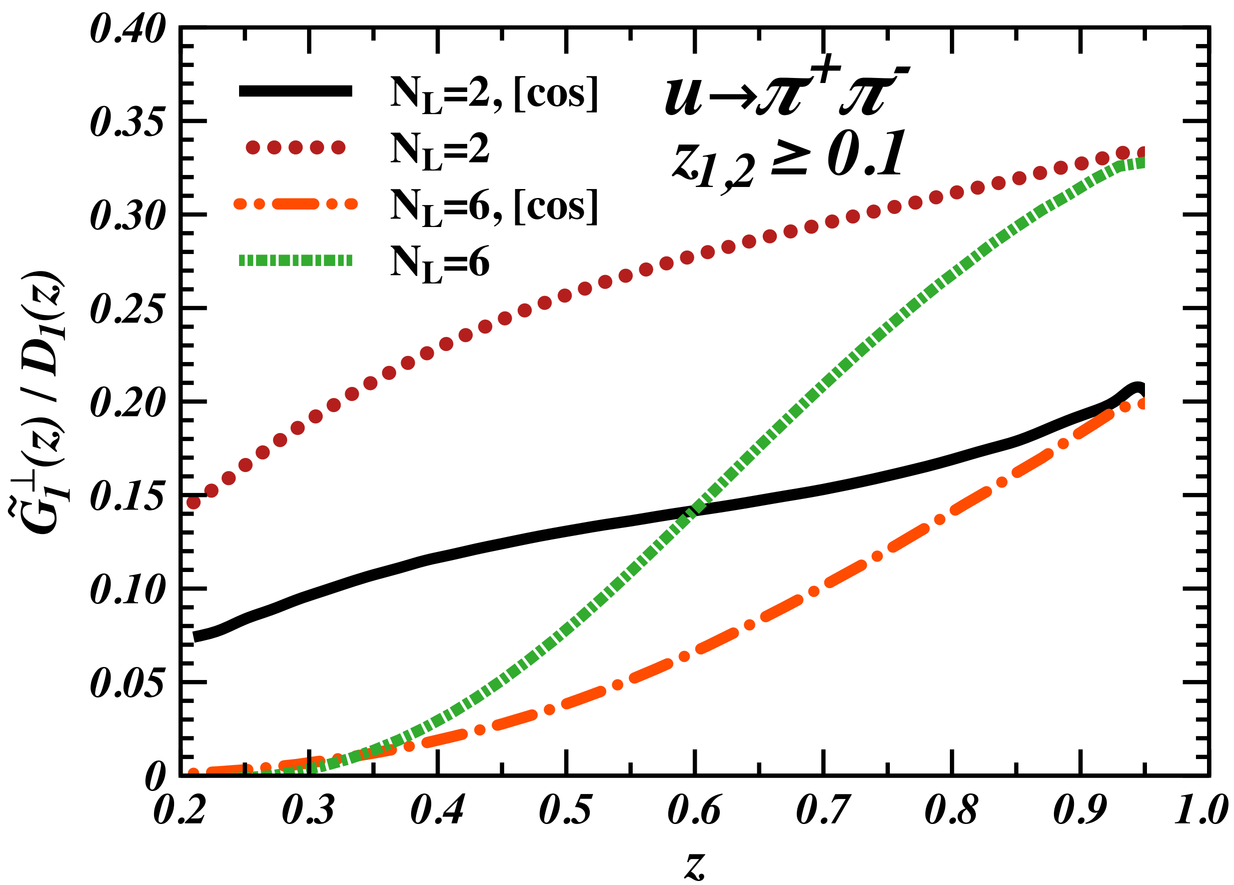


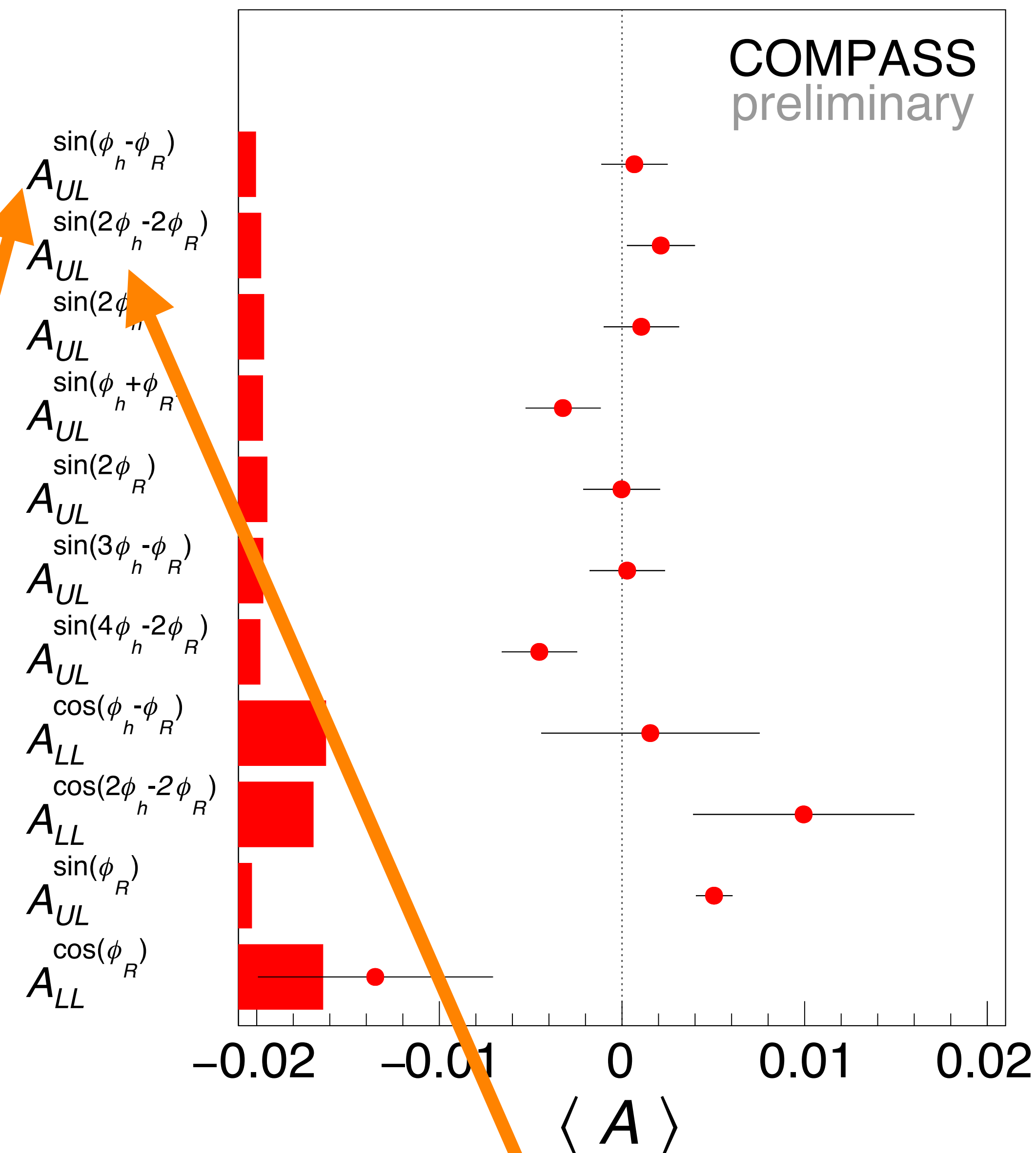
✦ Non-zero signal for various channels, ***sign change for*** $\pi^+\pi^+$ ***pairs*** !

✦ $z_{1,2} \geq 0.1$ ***cut enhances the analysing power at high-z for larger N_L*** !

Helicity DiFFs in SIDIS

- First two moments from quark-jet
- *SIDIS extraction in COMPASS*





$$G^\perp(\cos(\varphi_k - \varphi_R)) = G^{\perp[0]} + \cos(\varphi_k - \varphi_R) G^{\perp[1]} \ldots$$

✦ First moment is suppressed with respect to the second one.

Transverse Polarisation in DiHadron FFs

TRANSVERSELY POL. DIFFS FROM NUMBER DENSITY

- **Slightly more complicated procedure:**

$$F(\varphi_R, \varphi_k; \boldsymbol{s}_T) = D_1(\cos(\varphi_{RK})) + a_R \sin(\varphi_R - \varphi_s) H_1^{\sphericalangle}(\cos(\varphi_{RK})) + a_K \sin(\varphi_k - \varphi_s) H_1^{\perp}(\cos(\varphi_{RK}))$$

- **e+e- DiFFs:**

$$H_1^{\sphericalangle, e^+e^-}(z) = -\frac{2}{s_T}\left\langle \frac{\cos(\varphi_k - \varphi_s)}{\sin(\varphi_{RK})} F \right\rangle$$

$$H_1^{\perp, e^+e^-}(z) = \frac{2}{s_T}\left\langle \frac{\cos(\varphi_R - \varphi_s)}{\sin(\varphi_{RK})} F \right\rangle$$

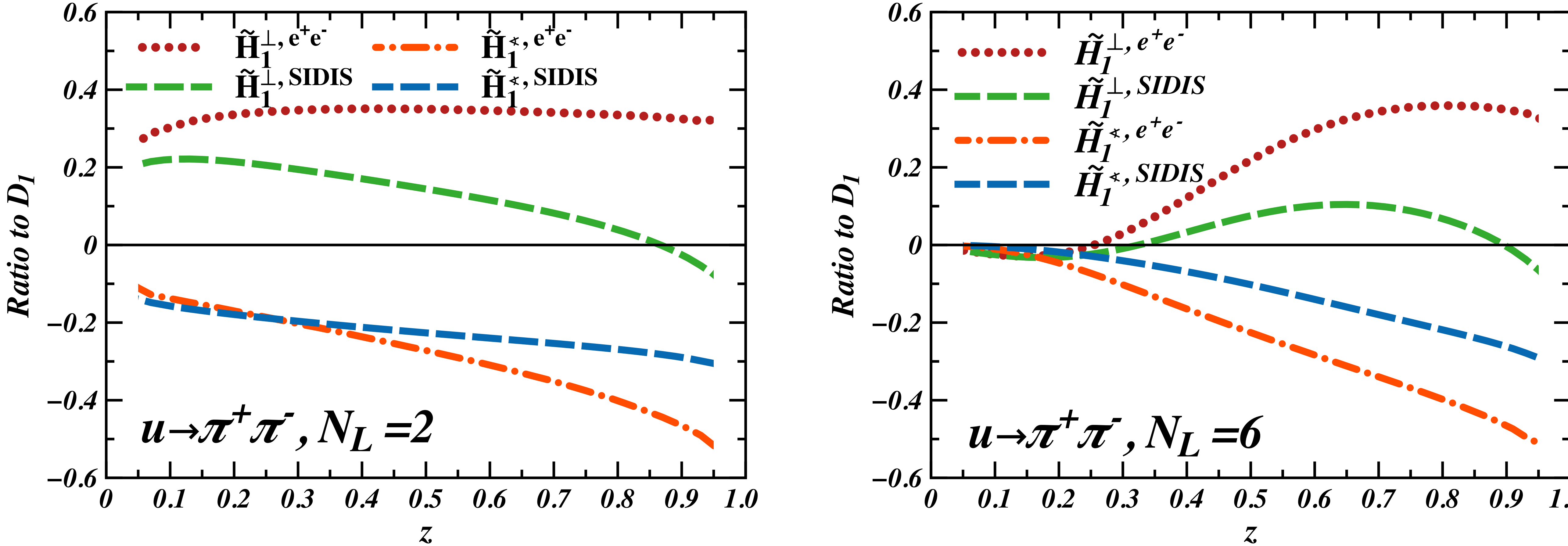
- **SIDIS DiFFs:**

$$H_1^{\sphericalangle, SIDIS}(z) = \frac{2}{s_T}\left\langle \sin(\varphi_R - \varphi_s) F \right\rangle$$

$$H_1^{\perp, SIDIS}(z) = \frac{2}{s_T}\left\langle \sin(\varphi_k - \varphi_s) F \right\rangle$$

Analysing Power for Longitudinal Spin

✦ Comparing the analysing powers for all polarized DiFFs.



✦ Alternate signs for the two DiFFs.

✦ Significant differences between SIDIS and e+e- results!

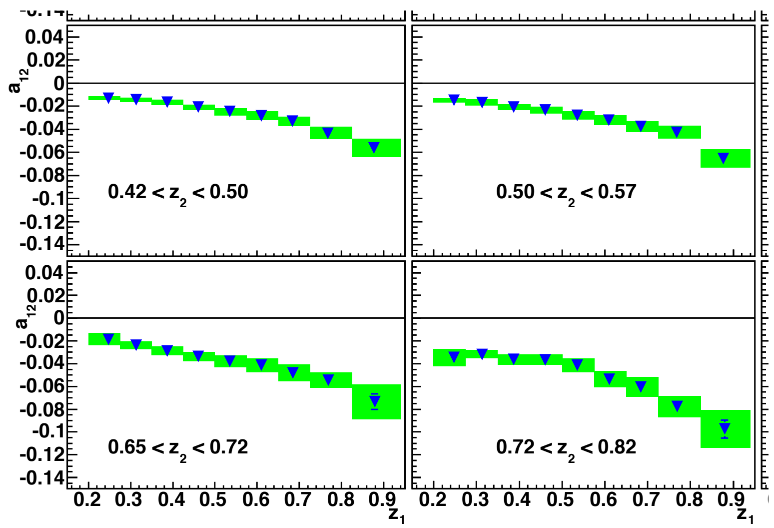
✦ *The e+e- cross section derivations under review!*

Analysing powers for DiFFs in e⁺e⁻

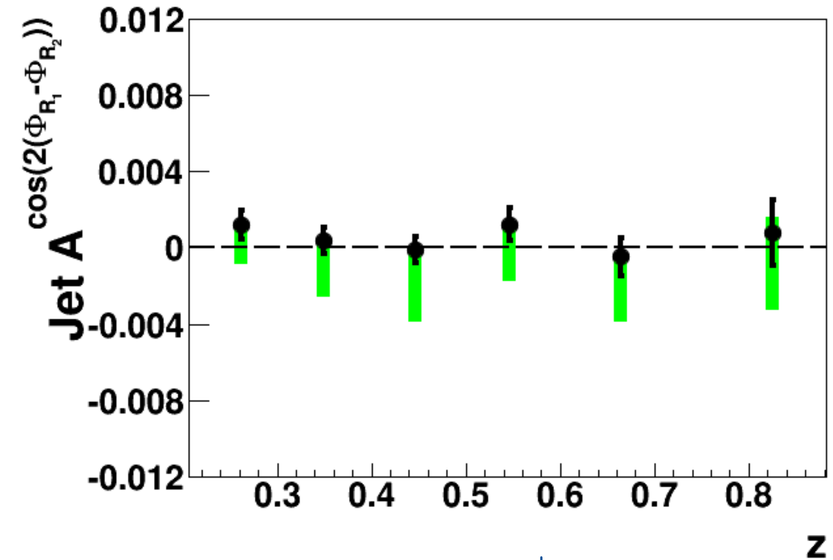
Phys.Rev.Lett. 107 (2011) 072004 | PoS DIS2015 (2015) 216

- ***BELLE results:***

$\sim H_q^{\sphericalangle}(z_1, m_1^2) H_{\bar{q}}^{\sphericalangle}(z_2, m_2^2)$

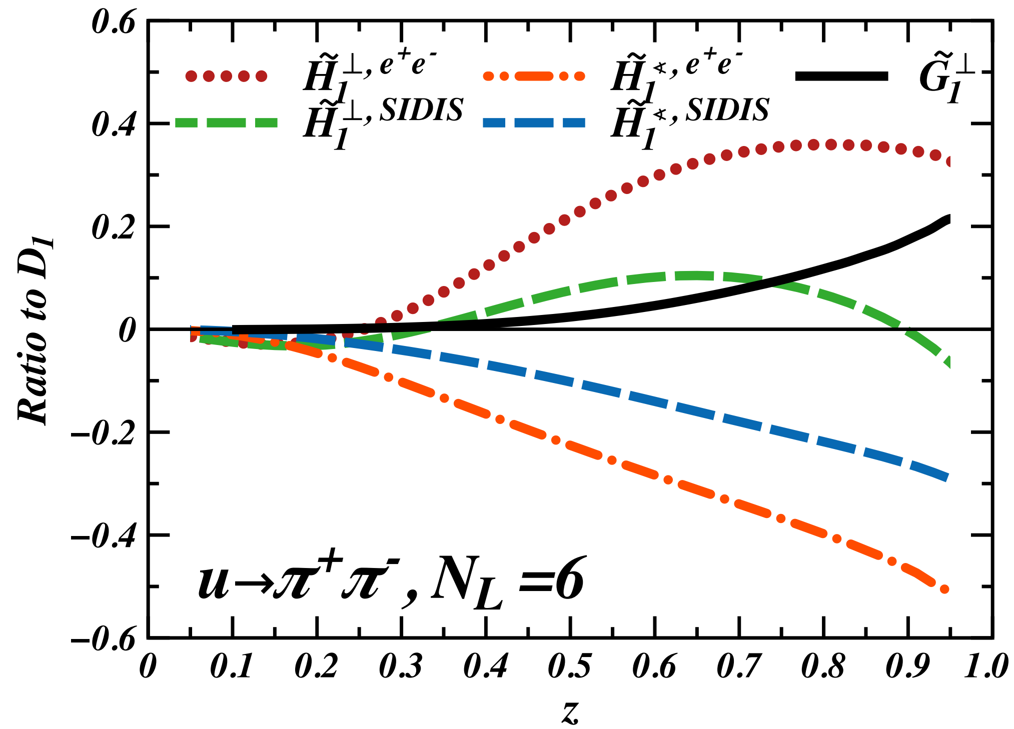


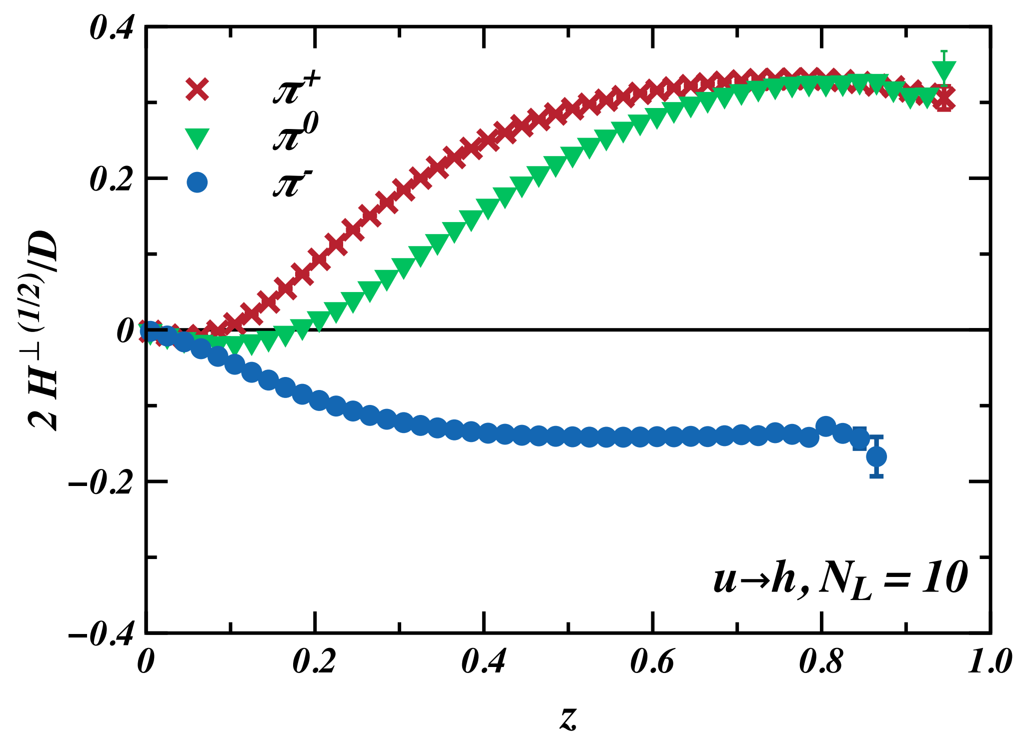
$\sim G_q^{\perp}(z_1, m_1^2) G_{\bar{q}}^{\perp}(z_2, m_2^2)$



  - ***Significant*** signal for $H_1^{\sphericalangle}$
  - ***No signal for*** $G_1^{\perp}$

- **The analysing powers of DiFFs from quark-jet framework.**
  - *Possible explanation:* $G_1^{\perp}$ ***naturally smaller than*** $H_1^{\sphericalangle}$ **.**





The Effect of Vector Mesons (VM) on (unpol) DiFFs

INCLUSION OF VECTOR MESONS AND (STRONG) DECAYS

- A naive assumption: VMs should have modest contribution due to relatively small production probability $P(\pi^+)/P(\rho^+) \approx 1.7$
- ***But***: Combinatorial factors enhance VM contribution significantly!
- Let's consider only two hadron emission

Direct: $u \to d + \pi^+ \to u + \pi^- + \pi^+$

VM: $u \to d + \pi^+ \to u + \underbrace{\rho^-}_{\to \pi^-\pi^0} + \pi^+$

...

$u \to u + \rho^0 \to u + \underbrace{\rho^0}_{\to \pi^+\pi^-} + \rho^0 \to \pi^+\pi^-$

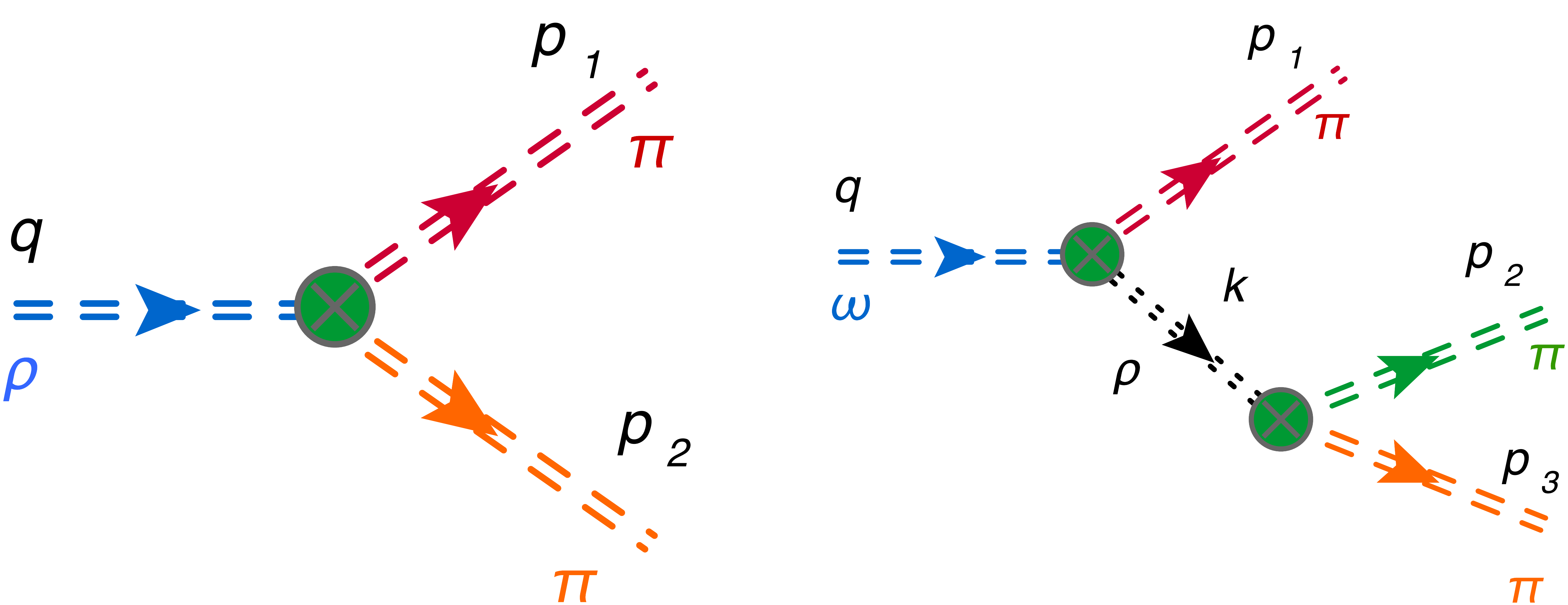
$$\boxed{P_{Dir}(\pi^+\pi^-)/P_{VM}(\pi^+\pi^-) \approx \frac{1}{4}}$$

2- AND 3-BODY DECAYS

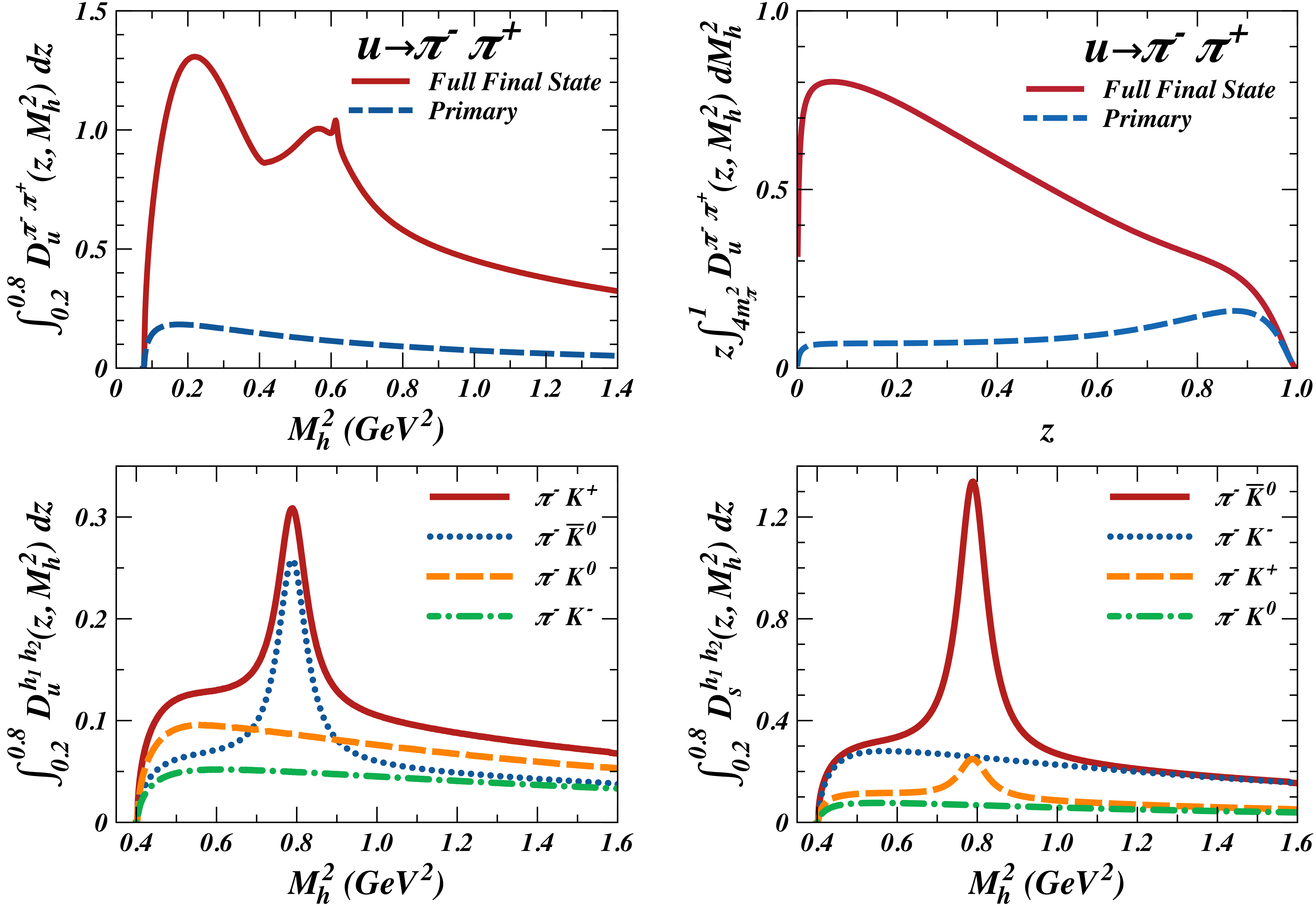
The M_h^2 spectrum of pseudoscalars is strongly affected by VM decays.

- We include only the 2-body decays ρ, K^*.
- Both 2- and 3-body decays of ω, ϕ .

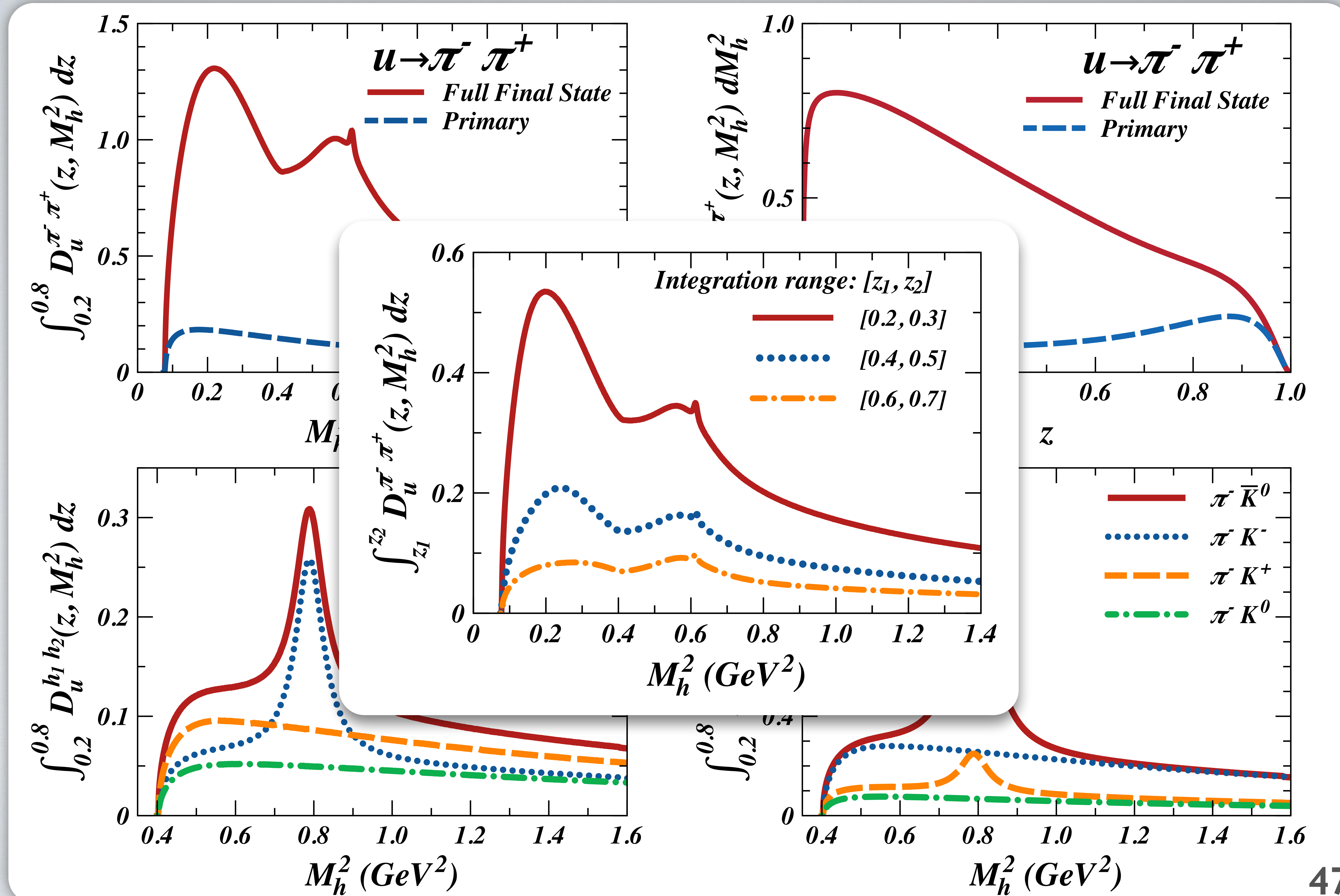
Achasov et al. (SND), PRD 68, 052006, (2003).



Effect of VMs on Unpol. DiFFs

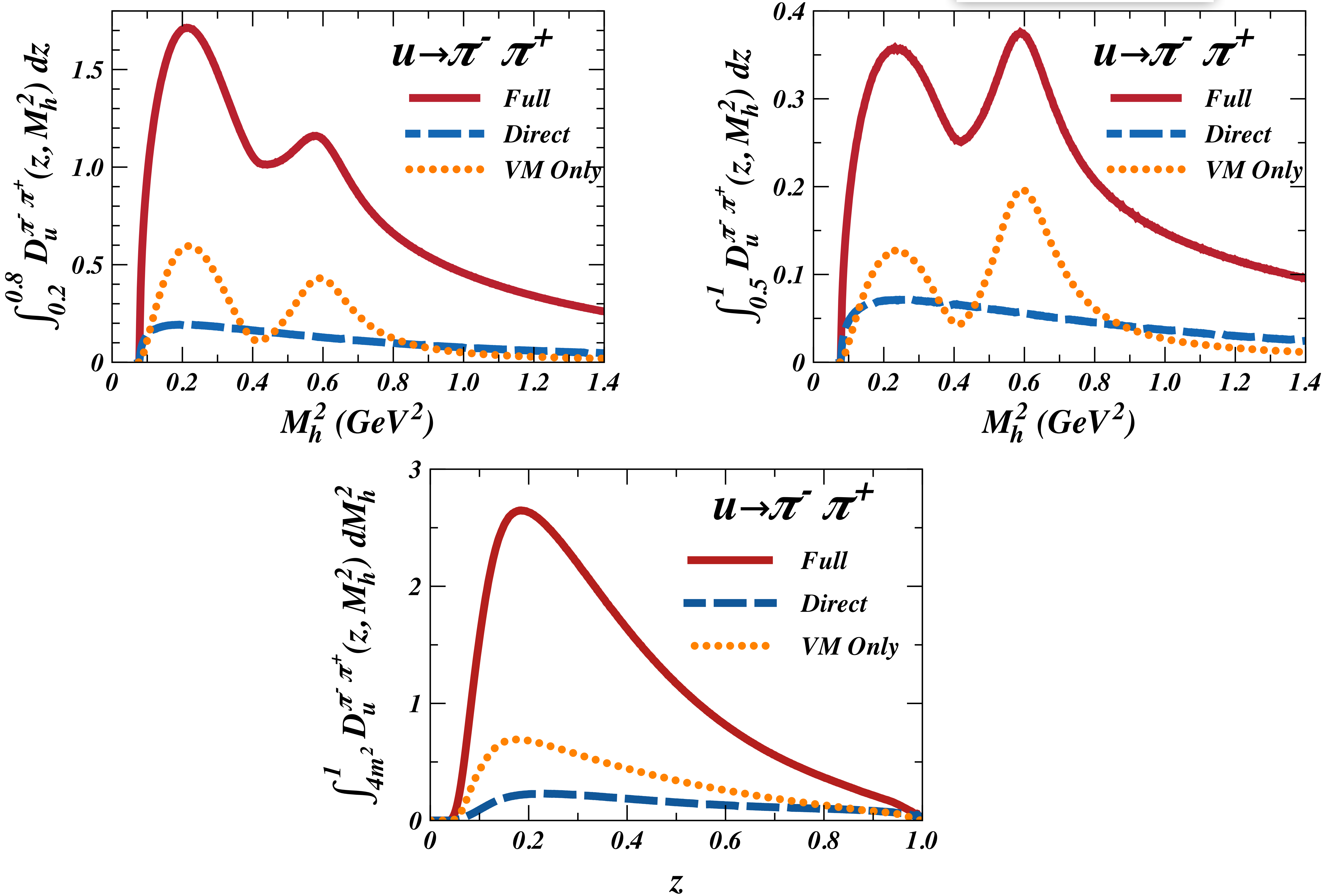


Effect of VMs on Unpol. DiFFs



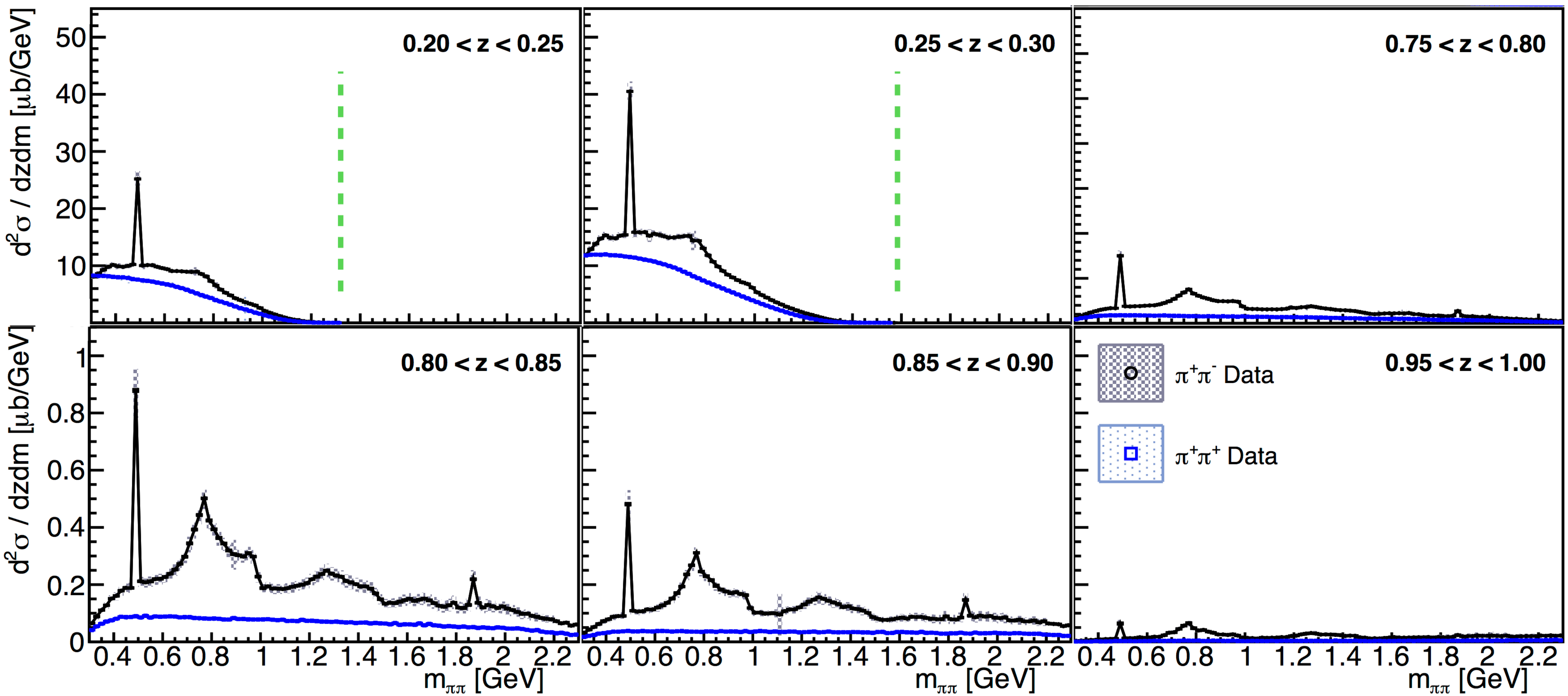
PYTHIA RESULTS FOR $u \rightarrow \pi^- \pi^+$

arXiv:1706.08348



Recent BELLE Results

- Invariant mass dependence of unroll DiFFs: arXiv:1706.08348



- **Note:** $D(z, M_h)dM_h = 2M_h\ D(z, M_h^2)dM_h$
- **Large z favours large M_h !**
- ***Non-resonant channels have no M_h structure, but are amplified!***

CONCLUSIONS

- ❖ Hadronization Models are needed to calculate polarised TMD FFs and DiFFs, and study various correlations between them.

- ❖ Polarised hadronisation in *MC generators: support for future experiments* to map the 3D structure of nucleon (*COMPASS, JLab12, BELLE II, EIC*).

- ❖ The *quark-jet* framework describes hadronization of a quark with arbitrary polarization via spin density matrix formalism.

- ❖ *All 3* DiHadron spin correlations from single-hadron effects in quark-jet!

- ❖ *Naturally small* signal for helicity-dependent DiFFs.

- ❖ *Sizeable* differences for IFF in SIDIS and **e^+e^-** (pending a review the cross section derivations for **e^+e^-**).

Thanks!

BACKUP SLIDES

Number Densities

- ***The full number density***

$$
\begin{aligned}
F(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T;\boldsymbol{s}) = \quad & D_1(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\cos(\varphi_{RK})) \\
& - s_L \frac{R_T k_T \sin(\varphi_{RK})}{M_1 M_2} G_1^{\perp}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\cos(\varphi_{RK})) \\
& + \frac{s_T R_T \sin(\varphi_R - \varphi_s)}{M_1 + M_2} H_1^{\sphericalangle}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\cos(\varphi_{RK})) \\
& + \frac{s_T k_T \sin(\varphi_k - \varphi_s)}{M_1 + M_2} H_1^{\perp}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\cos(\varphi_{RK}))
\end{aligned}
$$

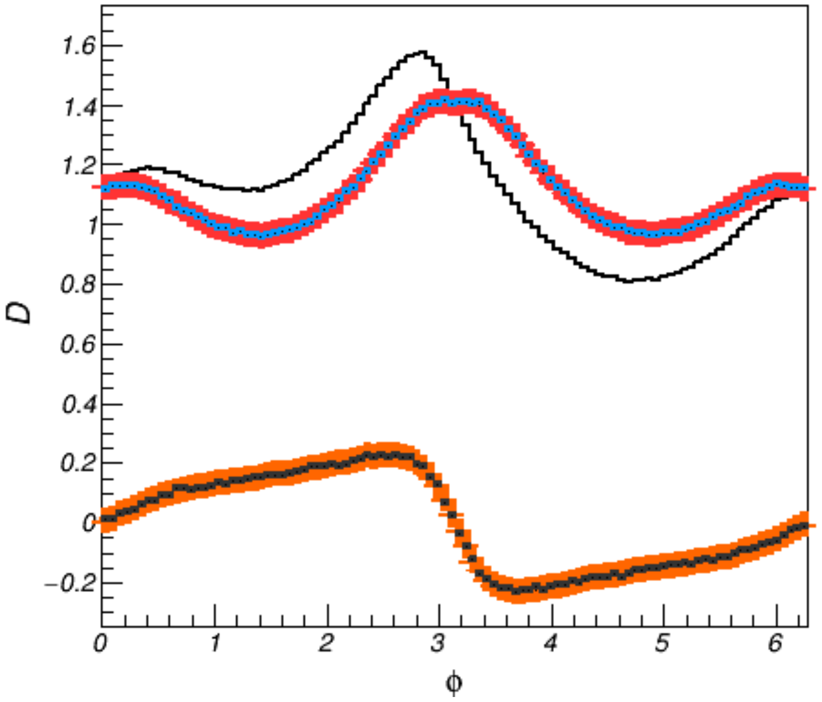
- ***The differential element***

$$
dN(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T;\boldsymbol{s}) = F(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T;\boldsymbol{s})\,dz\,d\xi\,d^2\boldsymbol{R}_T\,d^2\boldsymbol{k}_T
$$

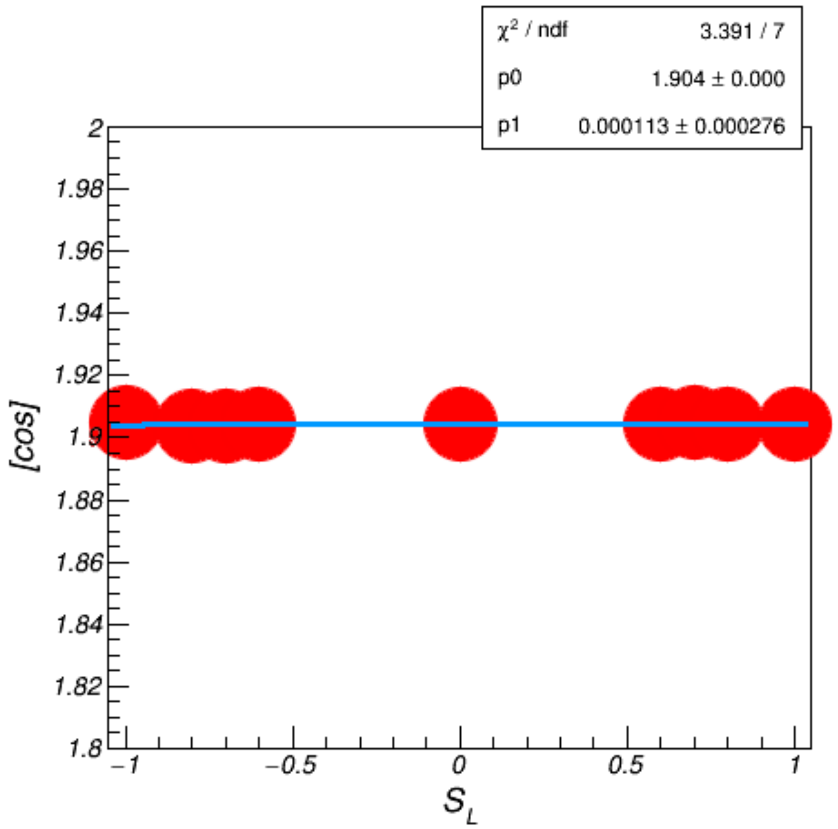
Longitudinal Spin

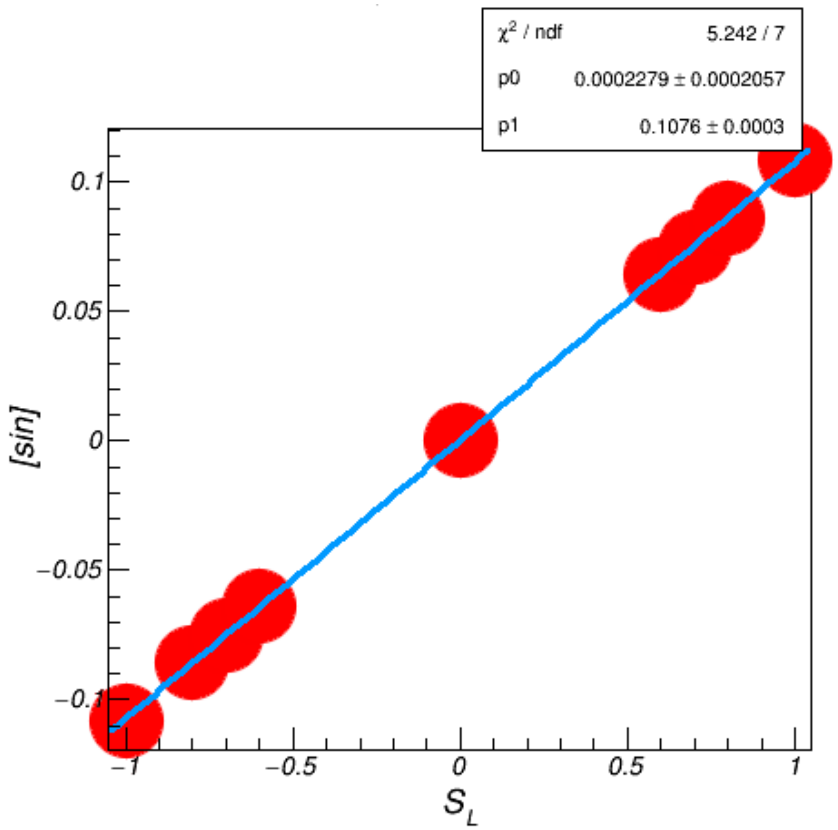
✦ **FF for longitudinally polarized quark:** $(\mathbf{R} \times \mathbf{T}) \cdot \mathbf{s}_L$

$$D_{q^{\rightarrow}}^{h_1 h_2}(\varphi_{R-T}) = D_q^{h_1 h_2}[cos(\varphi_{R-T})] + s_L \sin(\varphi_{R-T}) \mathcal{G}[cos(\varphi_{R-T})]$$
$$\varphi_{R-T} \equiv \varphi_R - \varphi_T$$



✦ **Proof of linear dependence on s_L: 9 values of $(s_L, \mathbf{s}_T)$ for $N_L = 6$.**

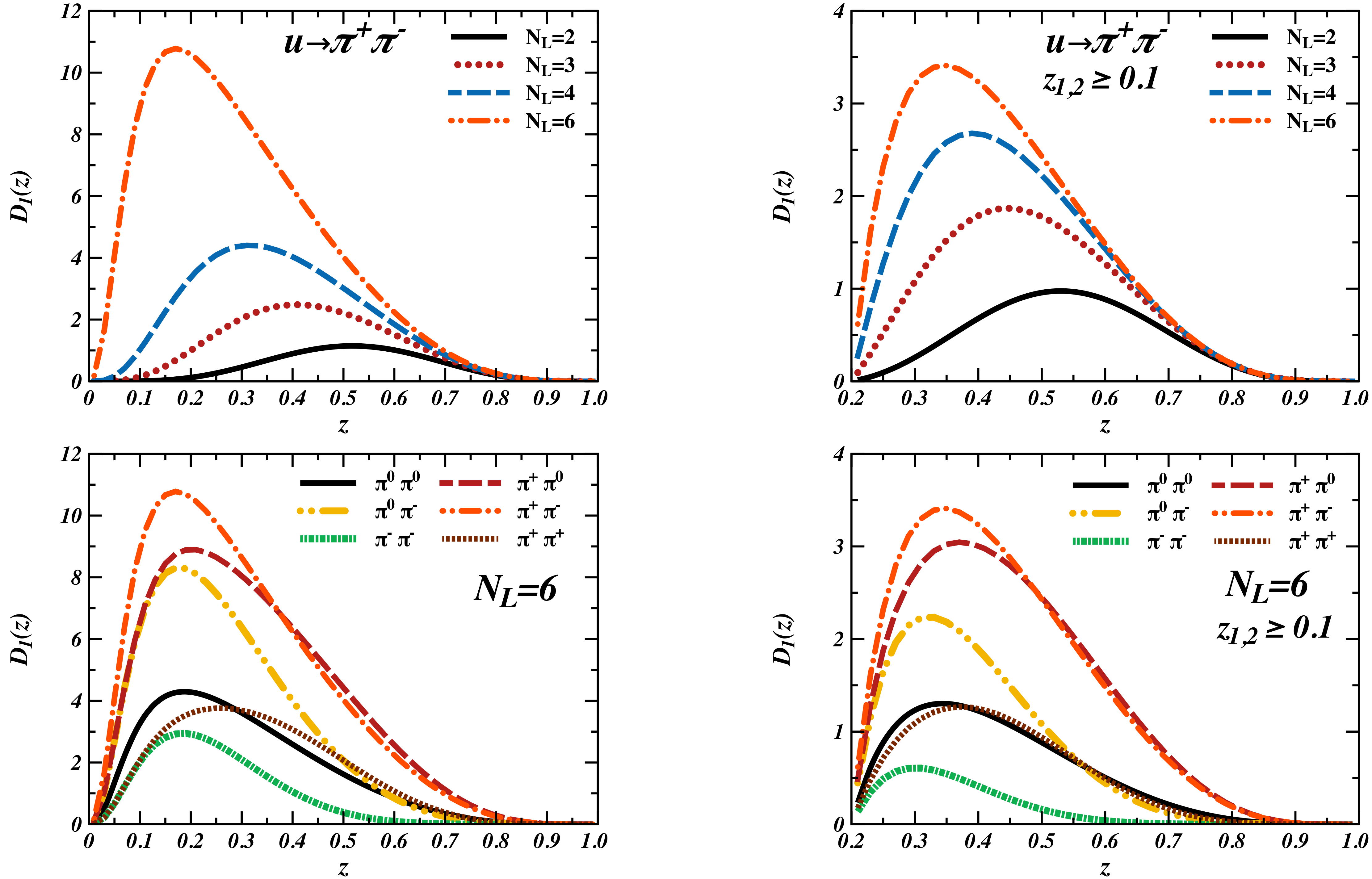




Results for unpolarized DiFF

✦ **Results for unpolarized DiFFs, *N_L* dependence, various pairs:**

- ***No Cuts***
- **z Cuts: $z_{1,2} \geq 0.1$**



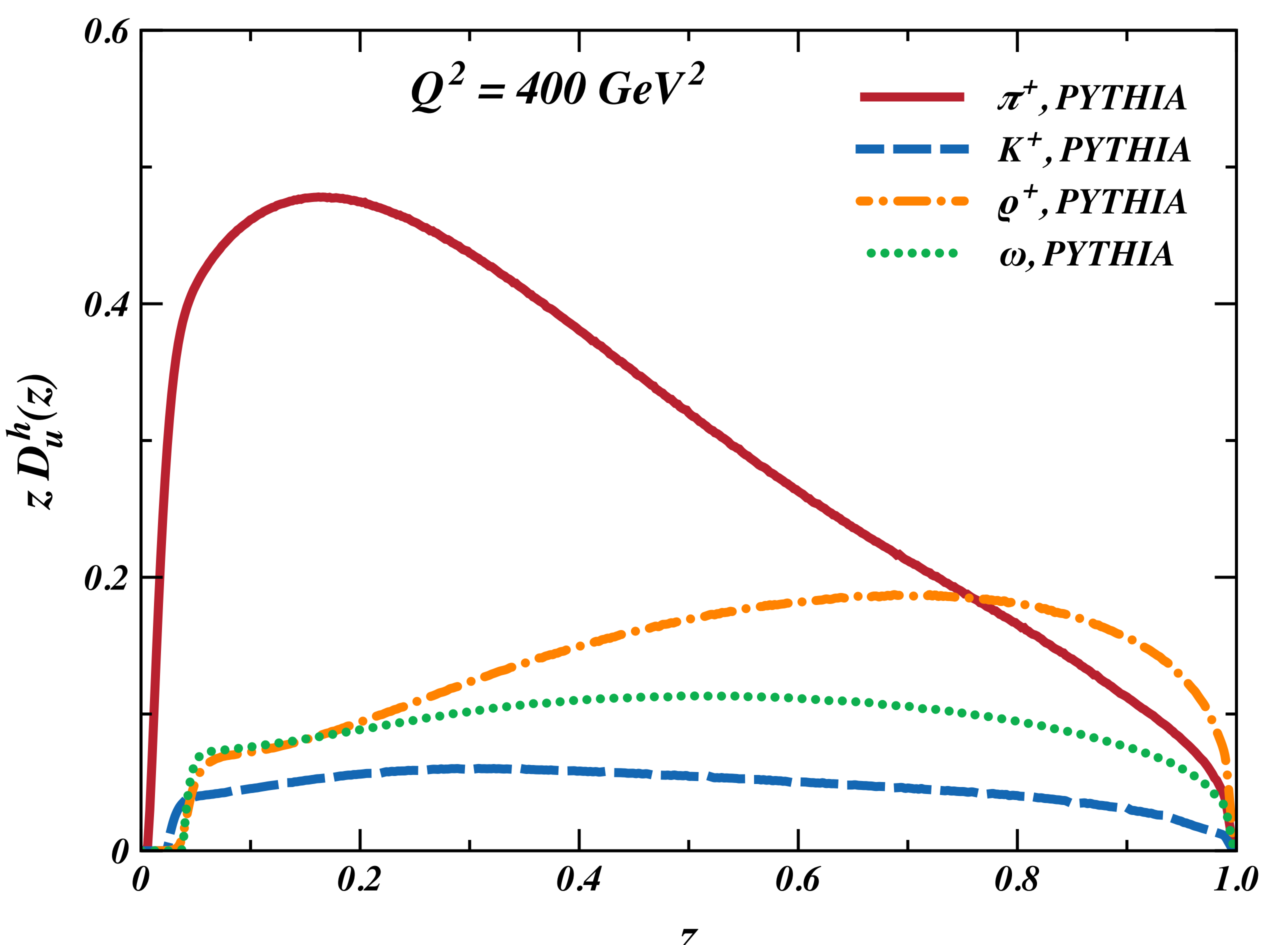
✦ $z_{1,2} \geq 0.1$ ***cut brings in convergence with N_L* !**

PYTHIA SIMULATIONS

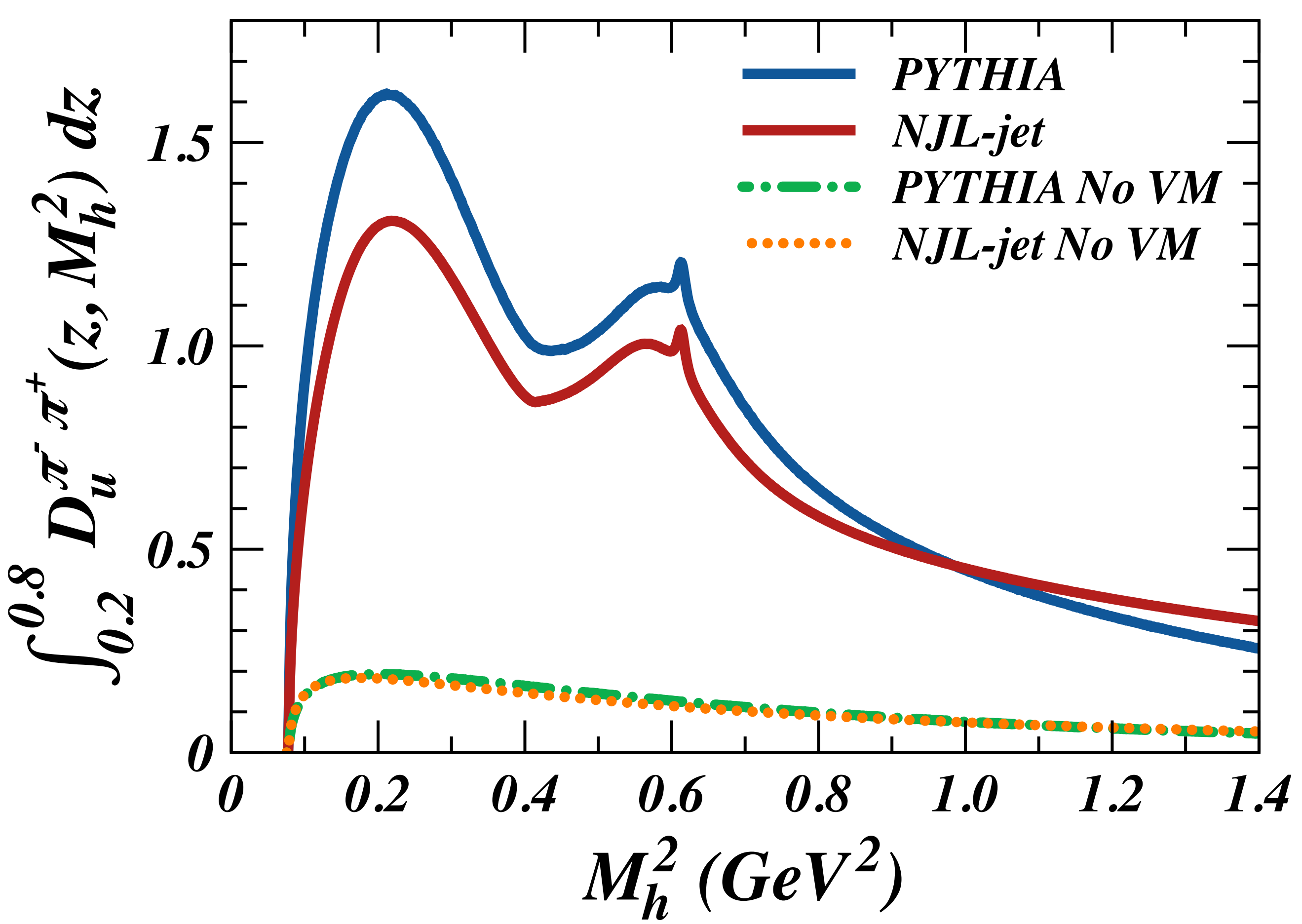
- Setup hard process with back to back $q\ \bar{q}$ along **z** axis.
- ***Only Hadronize.*** **Allow the same resonance decays as NJL-jet.**
- Assign hadrons with positive p_z to q fragmentation.

$$E_q = 10\ \mathrm{GeV}$$

Single Hadron



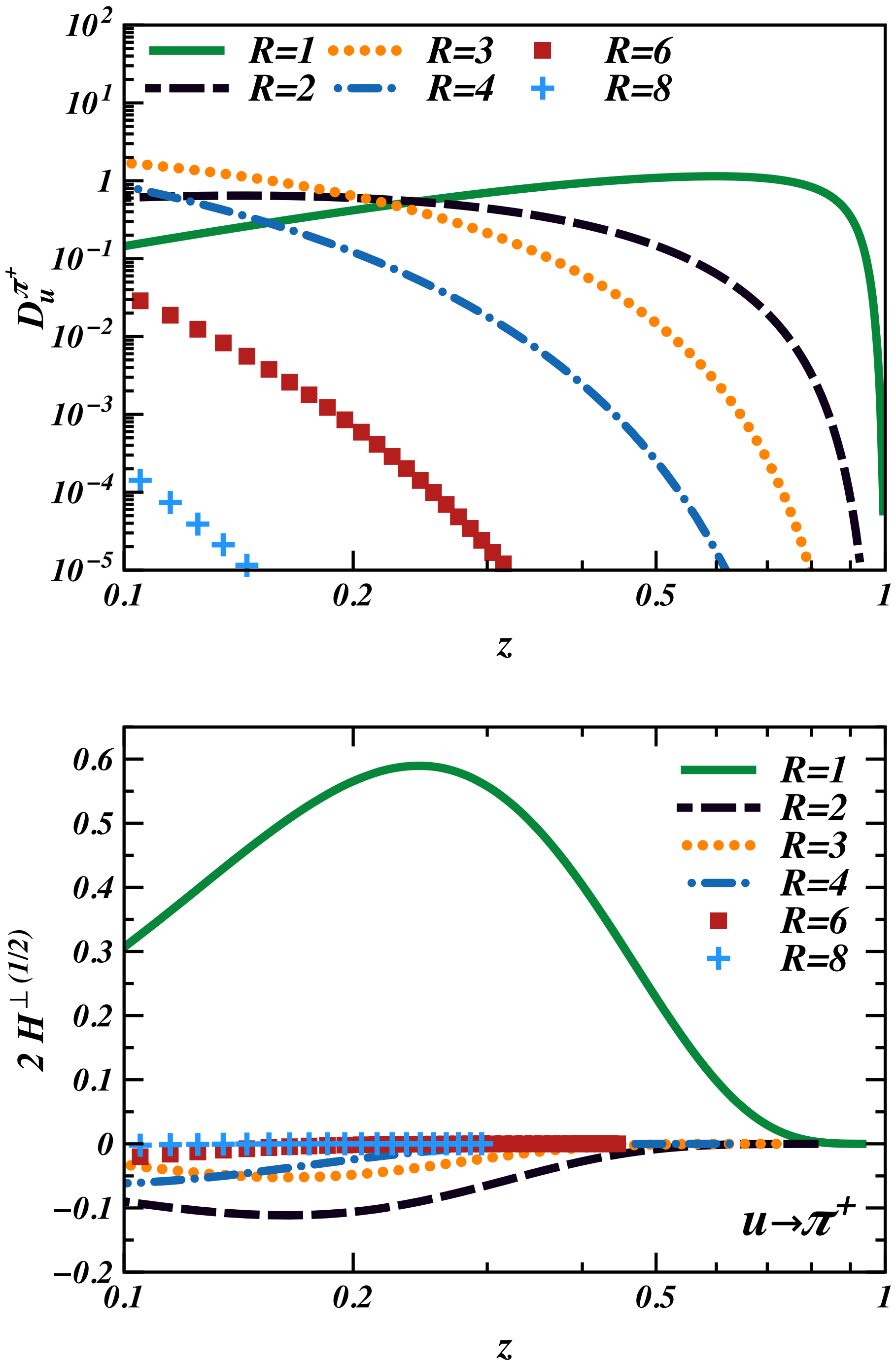
Dihadron



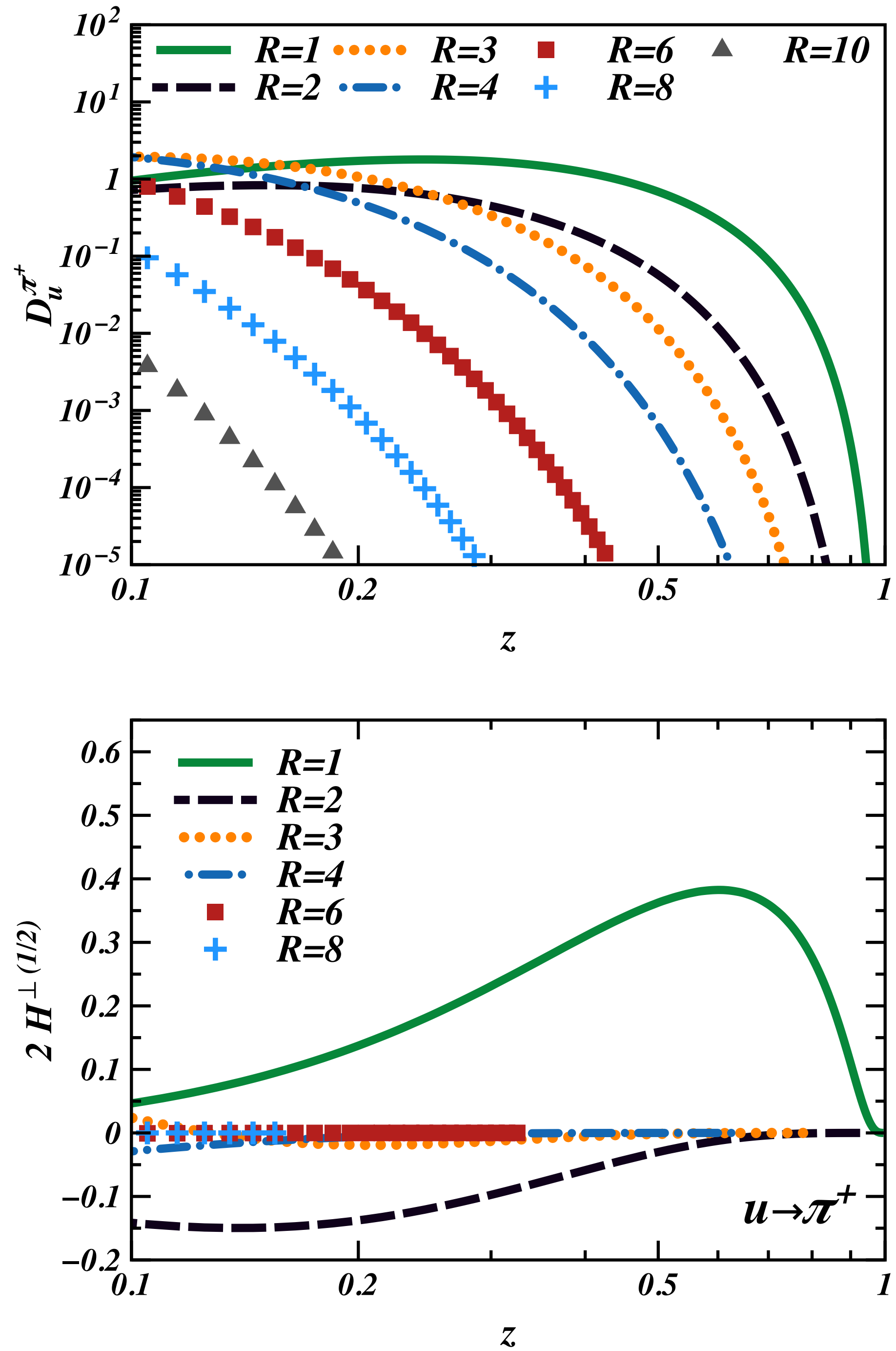
Saturations of FFs with h Rank

✦ FFs vs Rank of produced hadron.

▸ ***NJL* Model**



▸ **Evolution-mimicking *Ansatz*.**



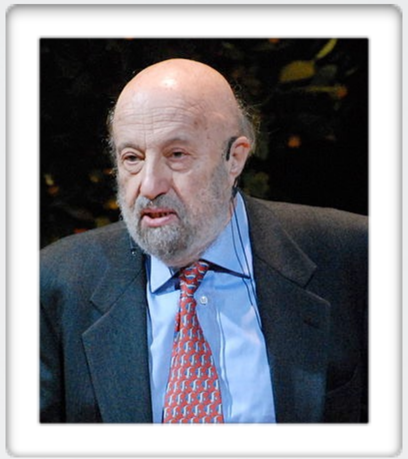
✓ Hadrons of *Rank > 4* are negligible for FFs at *z > 0.1*

NAMBU--JONA-LASINIO MODEL

Yoichiro Nambu and Giovanni Jona-Lasinio:

"Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I"

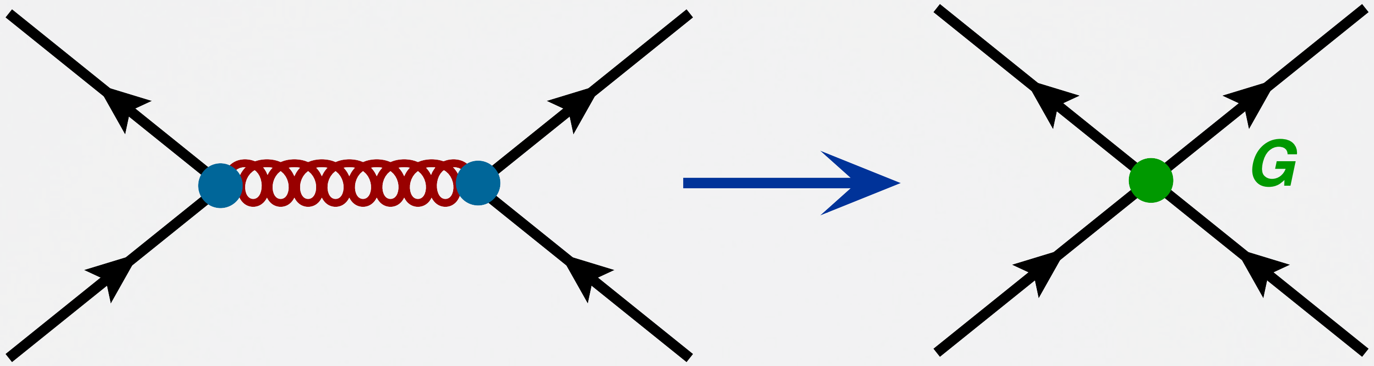
Phys.Rev. 122, 345 (1961)



Effective Quark model of QCD

- Effective Quark Lagrangian

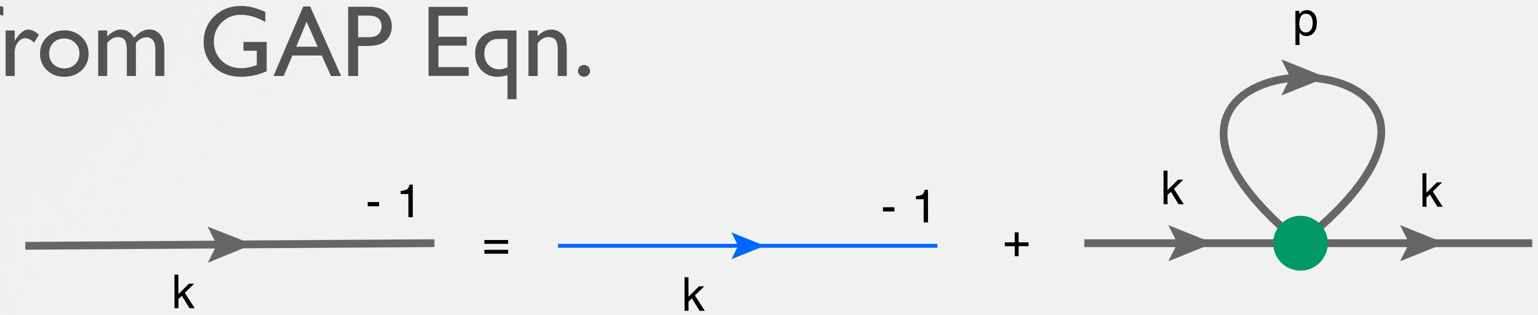
$$\mathcal{L}_{NJL} = \overline{\psi}_q(i\partial\!\!\!/ - m_q)\psi_q + G(\overline{\psi}_q\Gamma\psi_q)^2$$

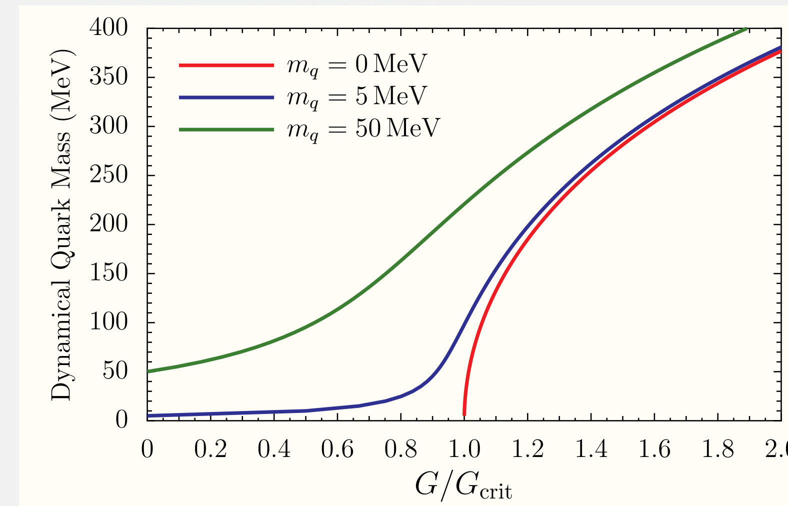


- Low energy chiral effective theory of QCD.
- Covariant, has the same flavor symmetries as QCD.

NAMBU--JONA-LASINIO MODEL

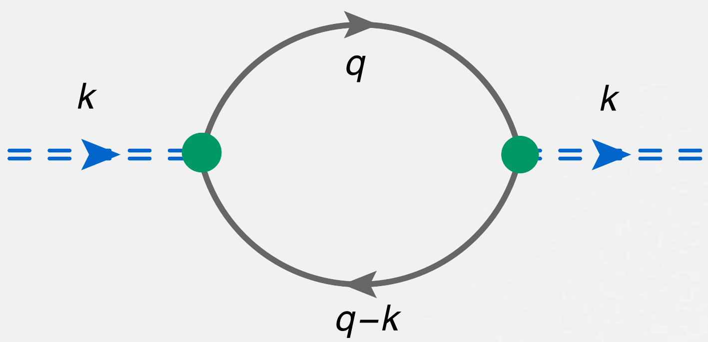
•Dynamically Generated Quark Mass from GAP Eqn.

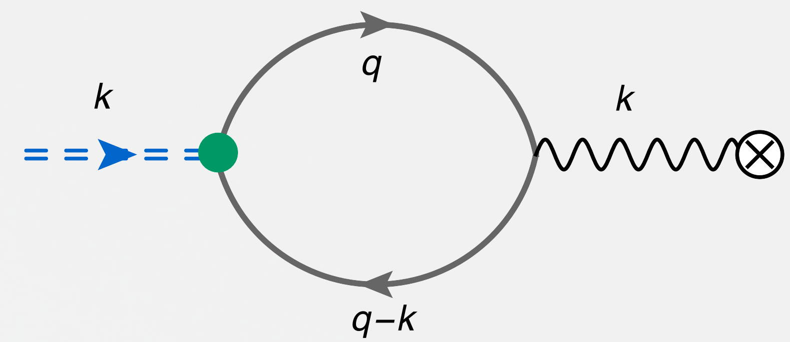




•Pion mass and quark-pion coupling from t-matrix pole.

•Pion decay constant





Fixing Model Parameters

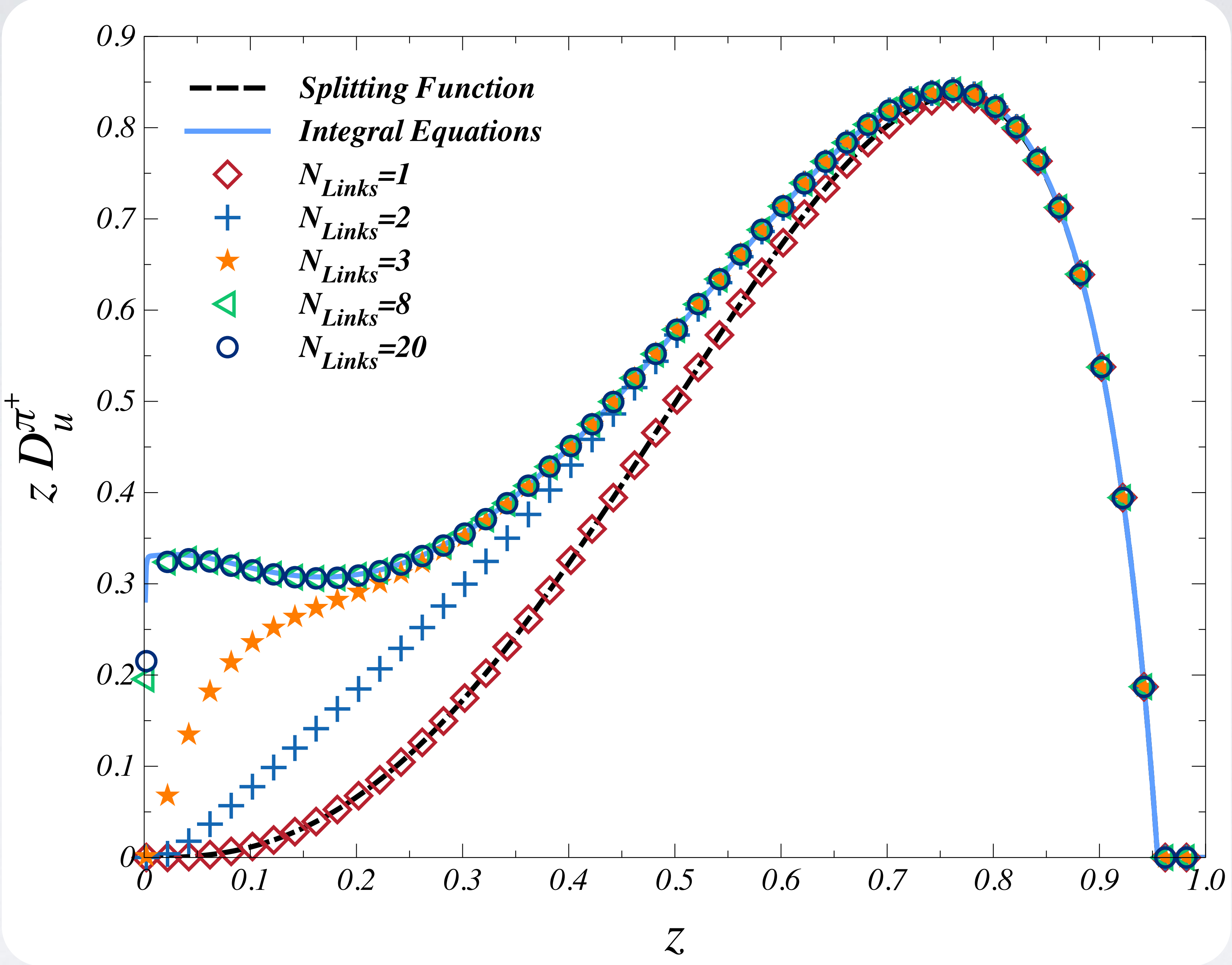
•Use Lepage-Brodsky Invariant Mass cut-off regularisation scheme.

$$M_{12} \leq \Lambda_{12} = \sqrt{\Lambda_3^2 + M_1^2} + \sqrt{\Lambda_3^2 + M_2^2}$$

• Choose a $M_{u(d)}$ and use physical f_π, m_π, m_K to fix model parameters $\Lambda_3,\ G, M_s$ and calculate g_{hqQ}.

DEPENDENCE ON NUMBER OF EMITTED HADRONS

- Restrict the number of emitted hadrons, N_{Links} in MC.

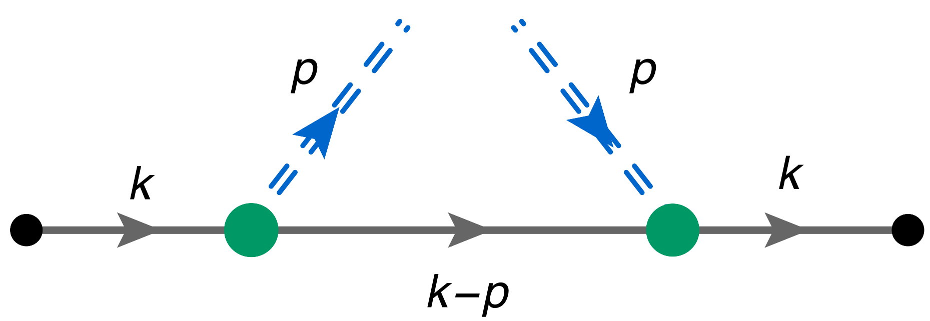


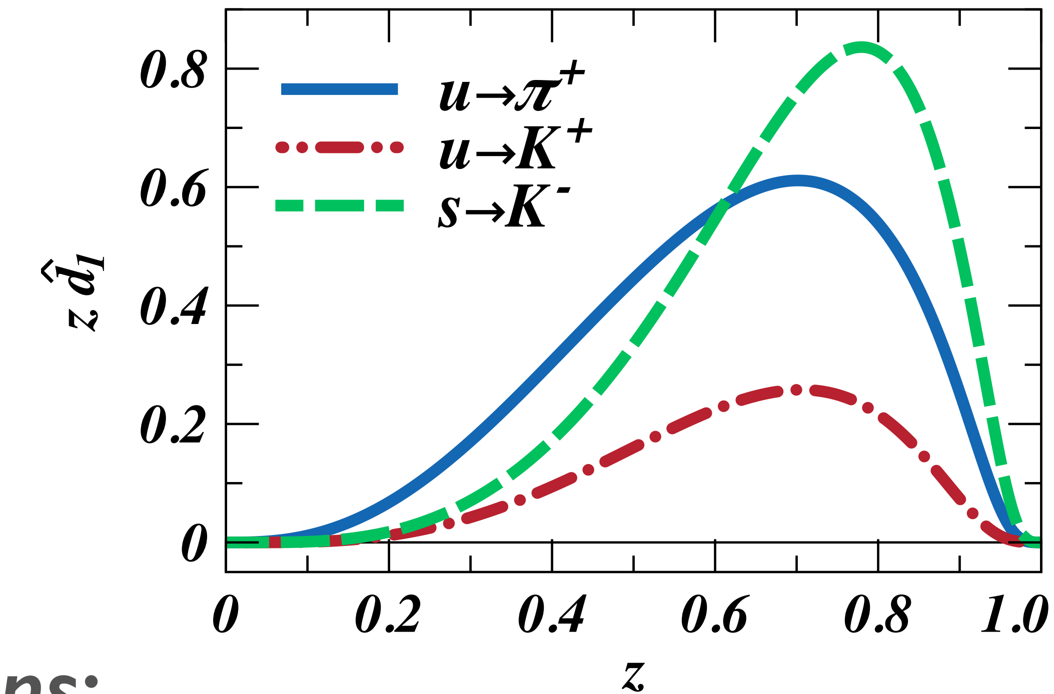
- We reproduce the splitting function and the full solution perfectly.
- The low z region is saturated with just a few emissions.

SOLUTIONS OF THE INTEGRAL EQUATIONS

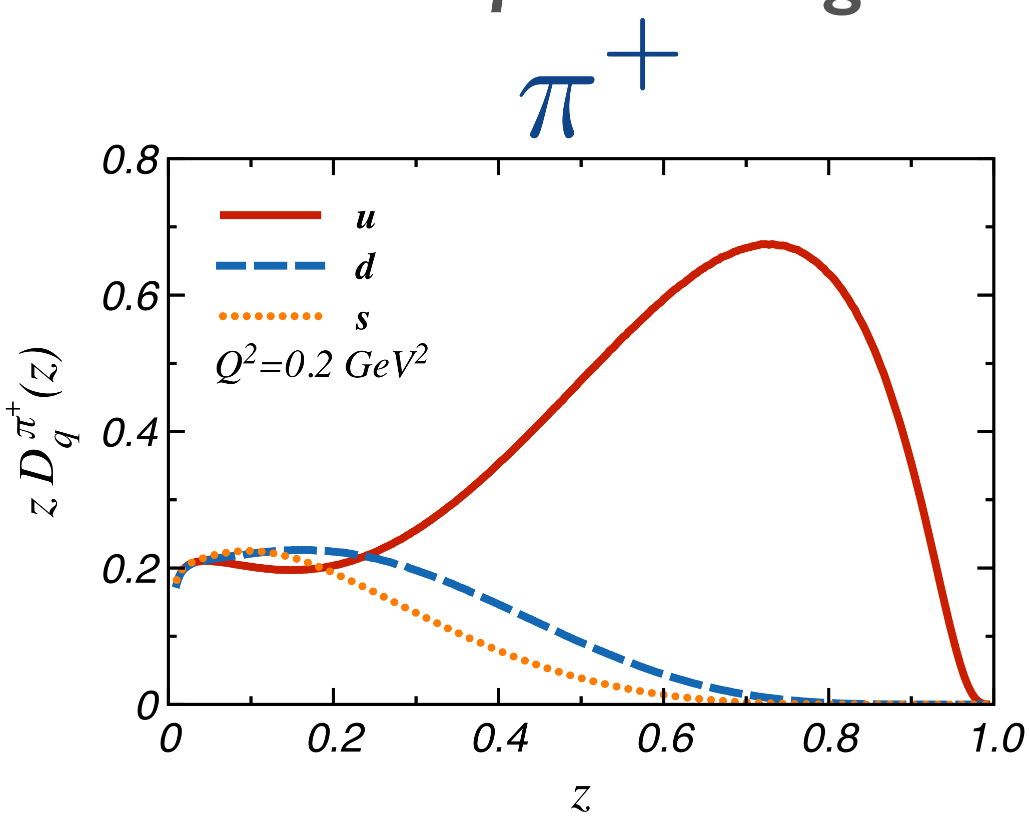
H.M., Thomas, Bentz, PRD. 83:074003, 2011

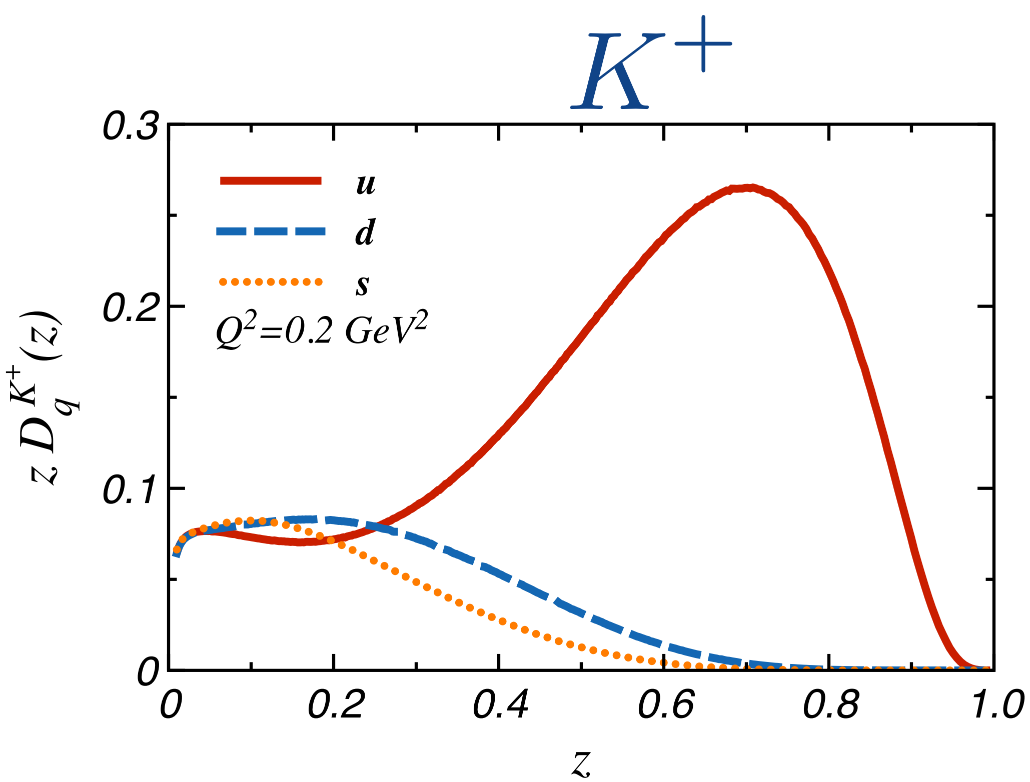
✦ *Input elementary probabilities from NJL:*





✦ *Solutions of the integral equations:*

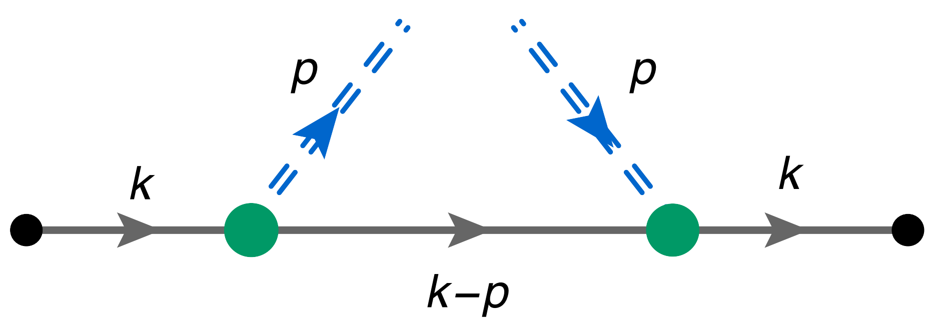


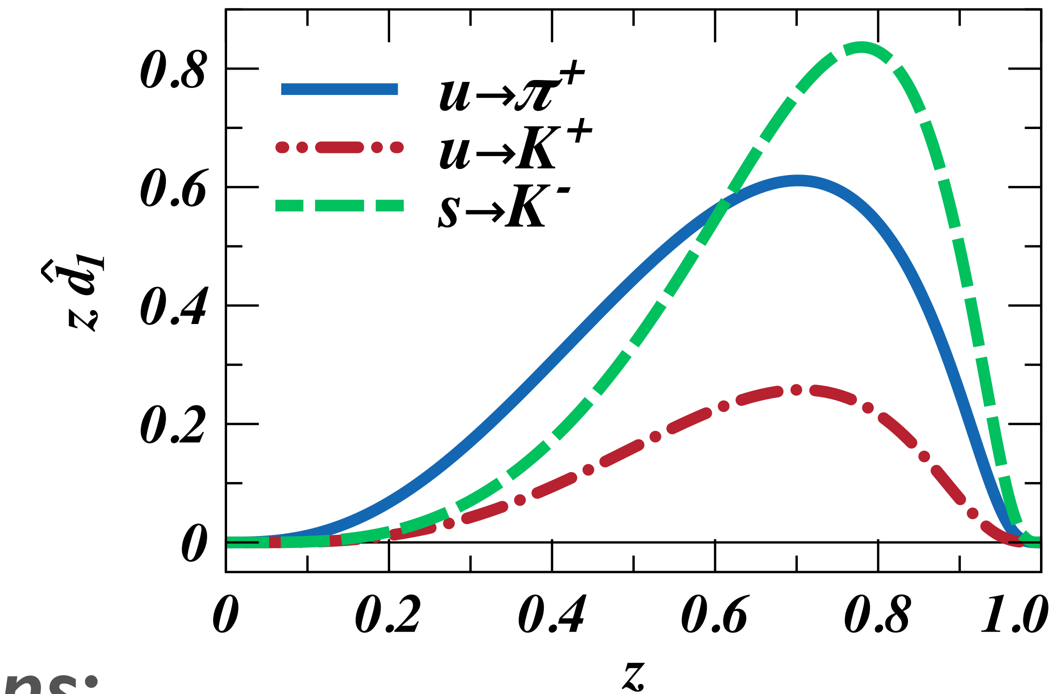


SOLUTIONS OF THE INTEGRAL EQUATIONS

H.M., Thomas, Bentz, PRD. 83:074003, 2011

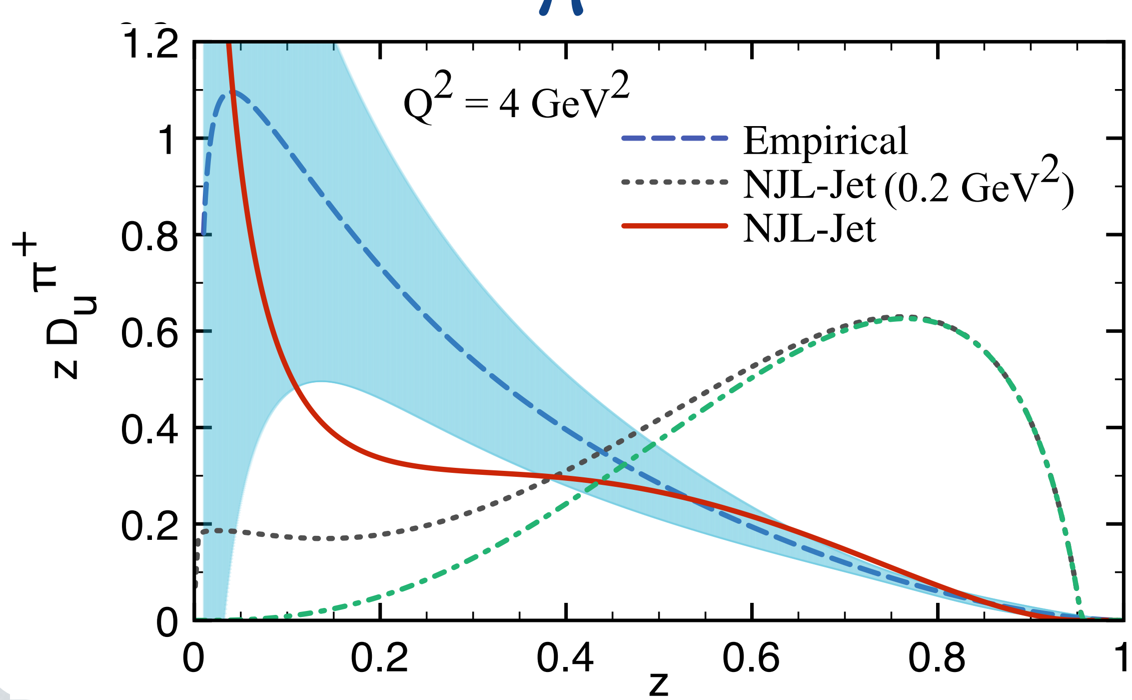
✦ *Input elementary probabilities from NJL:*



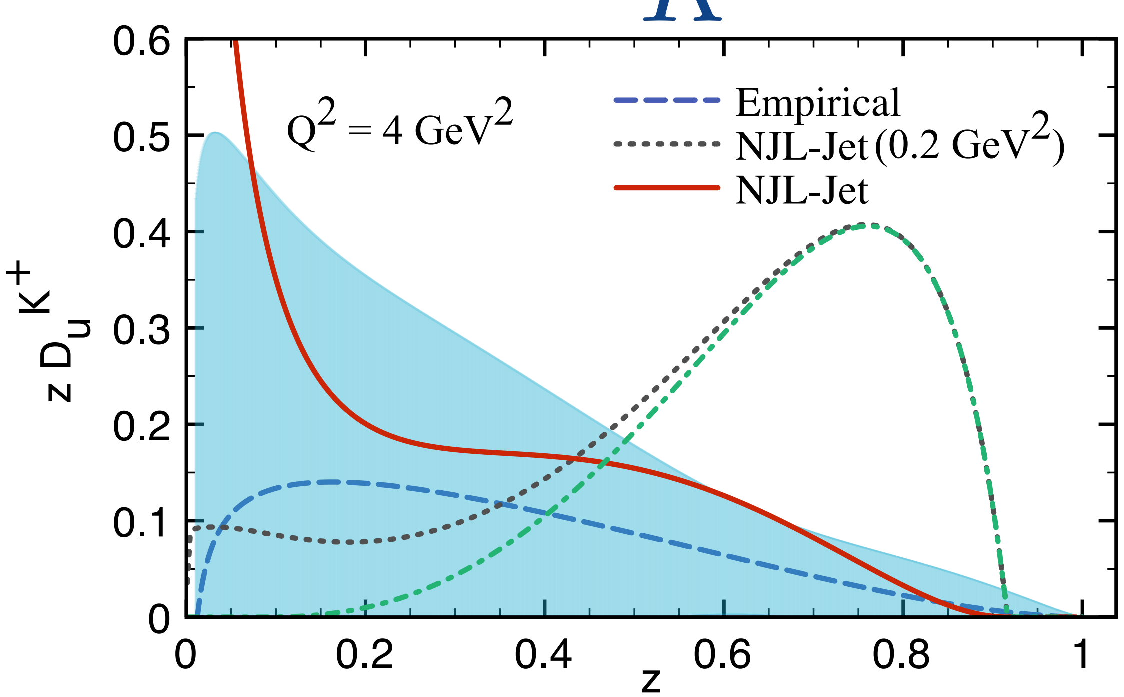


✦ *Solutions of the integral equations:*

π^+



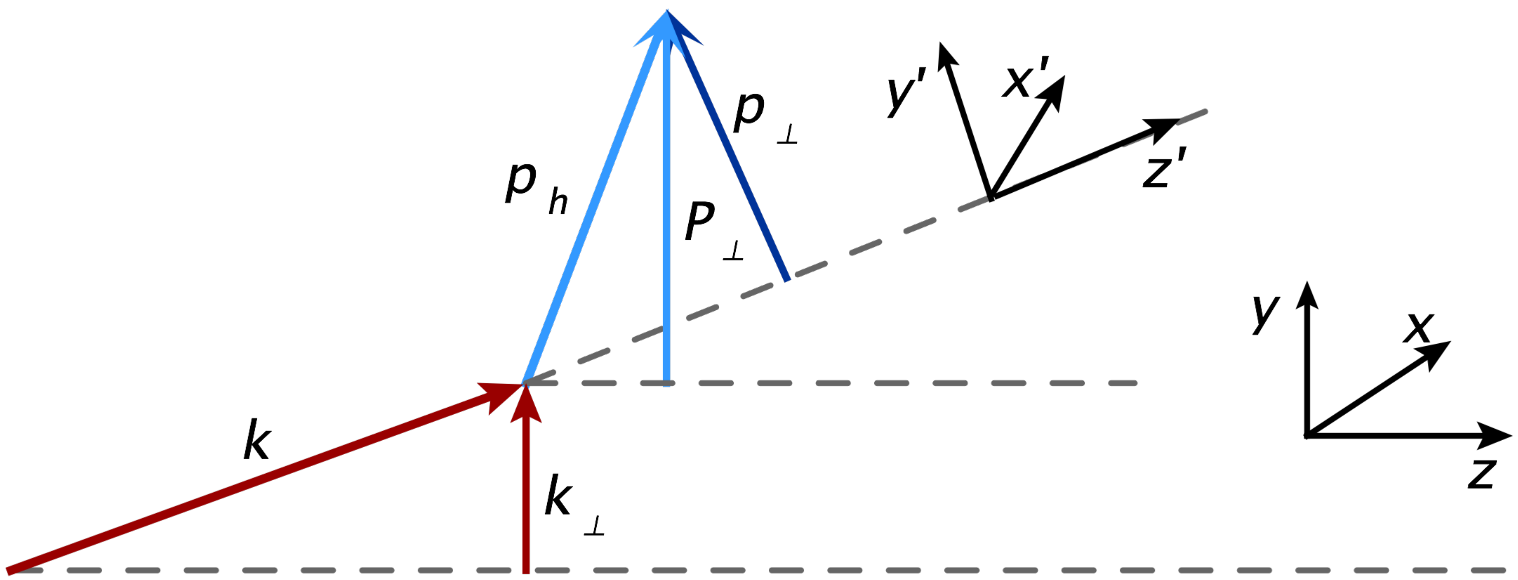
K^+



Lorentz Transforms of TM

Diehl: NPB 596, 33 (2001)(2015)

- **Boosts from *0* TM frame that preserve "-" *component.***



$$\begin{pmatrix} 1 & \frac{\boldsymbol{k}_\perp^2}{2(k^-)^2} & \frac{k_1}{k^-} & \frac{k_2}{k^-} \\ 0 & 1 & 0 & 0 \\ 0 & \frac{k_1}{k^-} & 1 & 0 \\ 0 & \frac{k_2}{k^-} & 0 & 1 \end{pmatrix}$$

	q	h
$\mathcal{L}'$	$(k'^+, k'^-, \boldsymbol{k}'_\perp = 0)$	$(p^+, p^-, \boldsymbol{p}_\perp)$
$\mathcal{L}$	$(k^+, k^- = k'^-, \boldsymbol{k}_\perp)$	$(P^+, P^- = p^-, \boldsymbol{P}_\perp = \boldsymbol{p}_\perp + z\boldsymbol{k}_\perp)$

$$\boxed{z \equiv \frac{p^-}{k^-} = \frac{p'^-}{k'^-}}$$

$$\boxed{\mathbf{P}_\perp = \mathbf{p}_\perp + z\mathbf{k}_\perp}$$

- **In case of two (or more) hadrons: same story!**

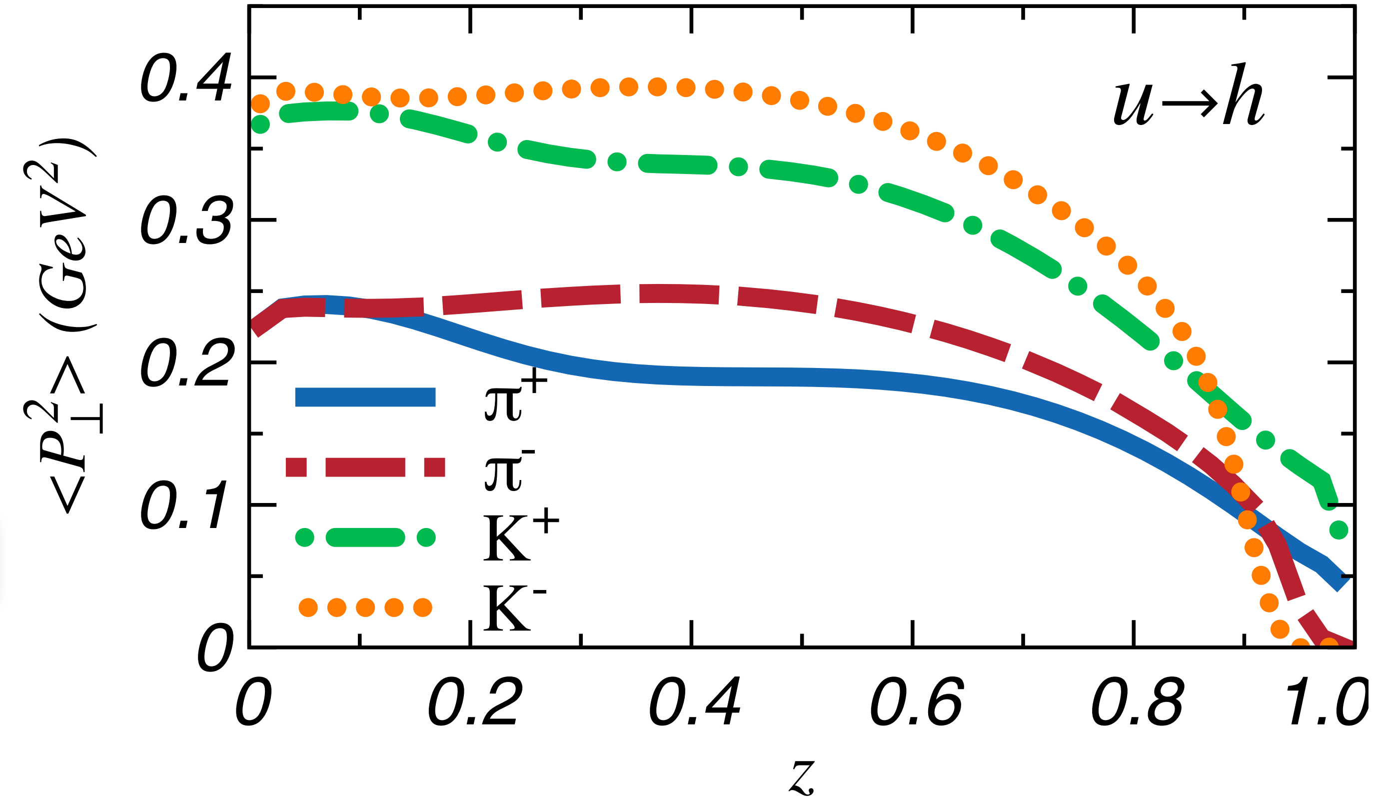
$$\boxed{\boldsymbol{P}_{1\perp} = \boldsymbol{p}_{1\perp} + z_1\boldsymbol{k}_\perp \qquad \boldsymbol{P}_{2\perp} = \boldsymbol{p}_{2\perp} + z_2\boldsymbol{k}_\perp}$$

AVERAGE Transverse Momenta vs z

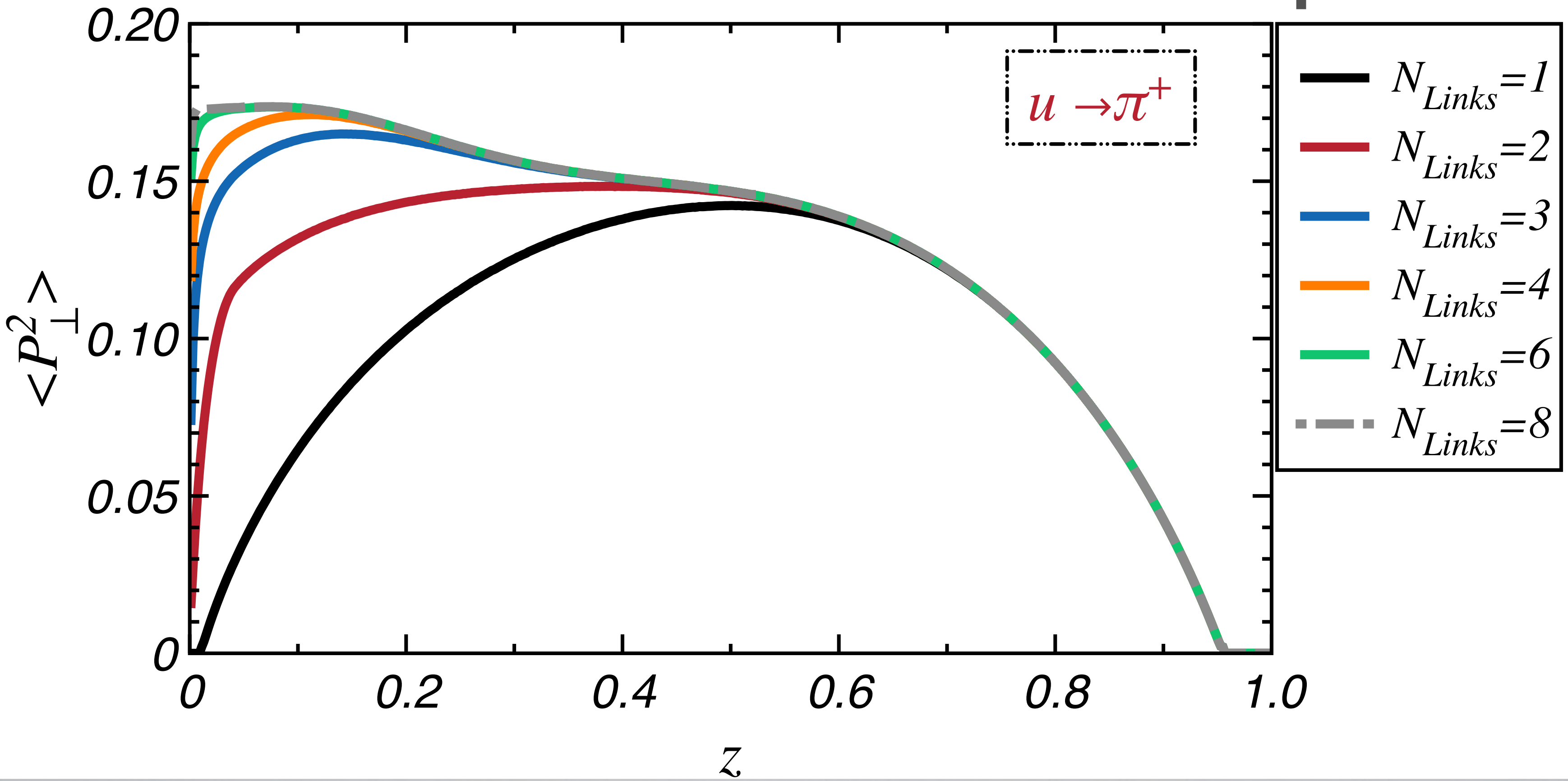
FRAGMENTATION

$$\langle P_\perp^2 \rangle_{unf} > \langle P_\perp^2 \rangle_f$$

✦Indications from HERMES data: A. Signori, et al: JHEP 1311, 194 (2013)

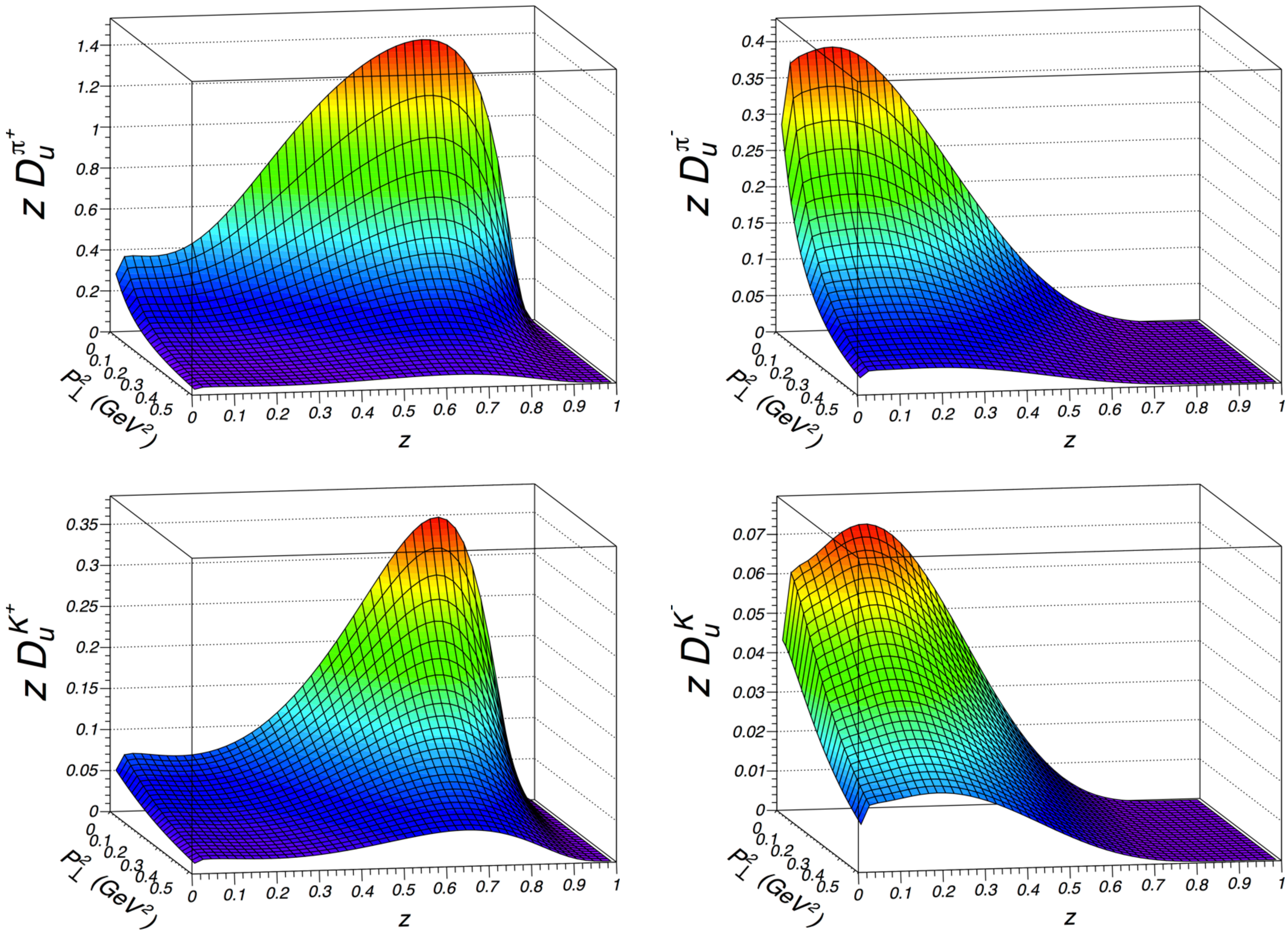


✓Multiple hadron emissions: ***broaden*** the TM dependence at ***low z***!



TMD FRAGMENTATION FUNCTIONS

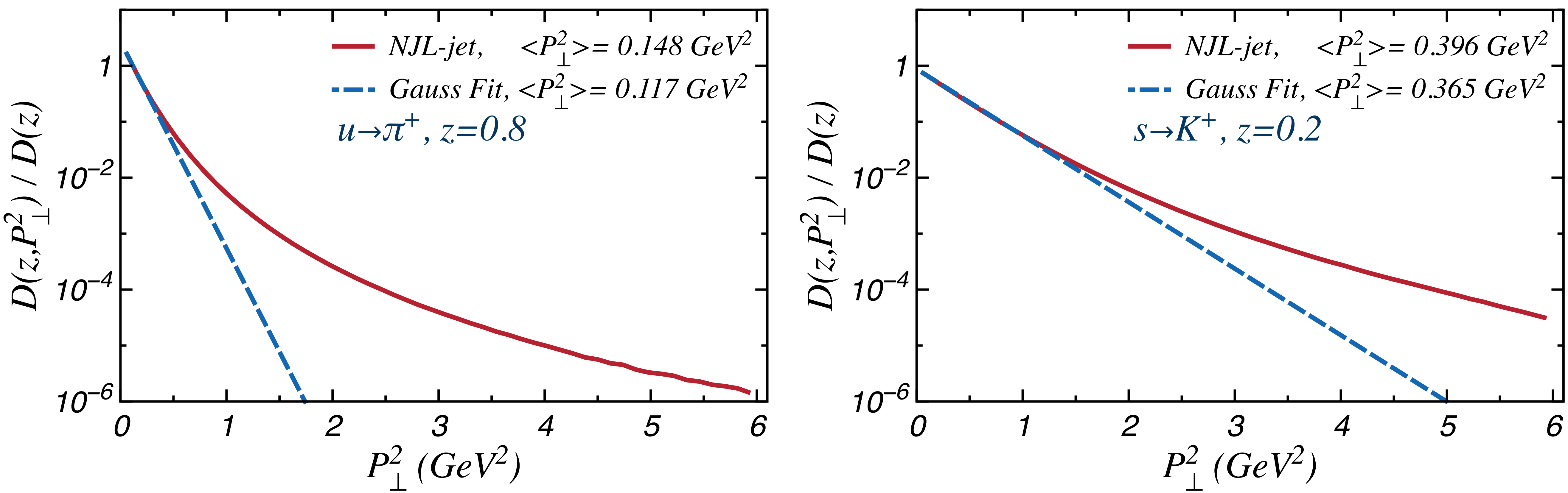
FAVORED • UNFAVORED



π

K

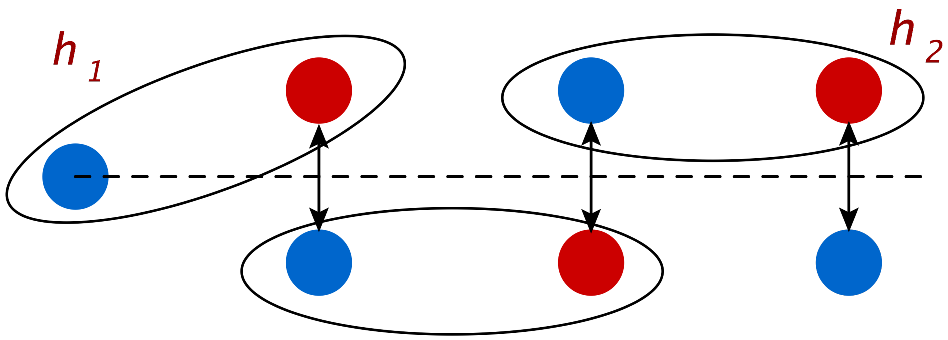
COMPARISON WITH GAUSSIAN ANSATZ

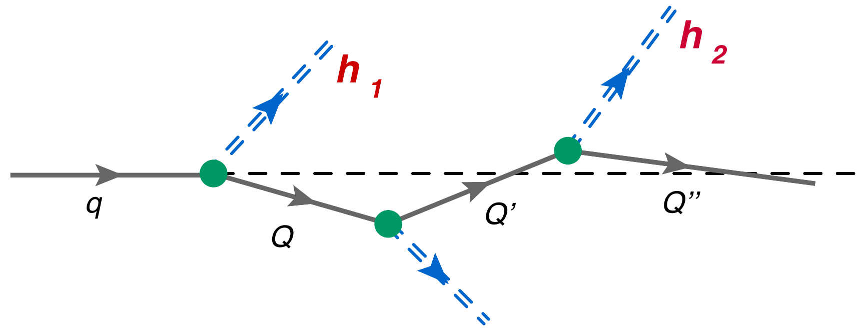


- Average TM: $\langle P_\perp^2 \rangle \equiv \dfrac{\int d^2\mathbf{P}_\perp \; P_\perp^2 \, D(z, P_\perp^2)}{\int d^2\mathbf{P}_\perp \; D(z, P_\perp^2)}$
- Gaussian ansatz assumes: $D(z, P_\perp^2) = D(z) \dfrac{e^{-P_\perp^2/\langle P_\perp^2 \rangle}}{\pi \langle P_\perp^2 \rangle}$

Different Hadronization Mechanisms.

LUND Model	Quark-Jet
✦ Fragmentation of $q\bar{q}$ pair: break-up of the string.	✦ Fragmentation of q, similar to QFT definition of FFs.
✦ Independent breaking of the string.	✦ Time-ordered hadron emissions.
✦ Quark TM indep. of hadron type.	✦ $q \to Qh$ depends on **h** (spin, mass).
$u \to u + s\bar{s}, \quad s \to s + s\bar{s}$	$u \to K^+ + s, \quad s \to \phi + s$ $u \to K^{*+} + s$
✦ No correlation in TM: **h₁** and **h₂**.	✦ Recoil TM of **h₁** affects **h₂**





❖ Can we find a signature in polarized FFs? Perhaps Dihadron FFs?

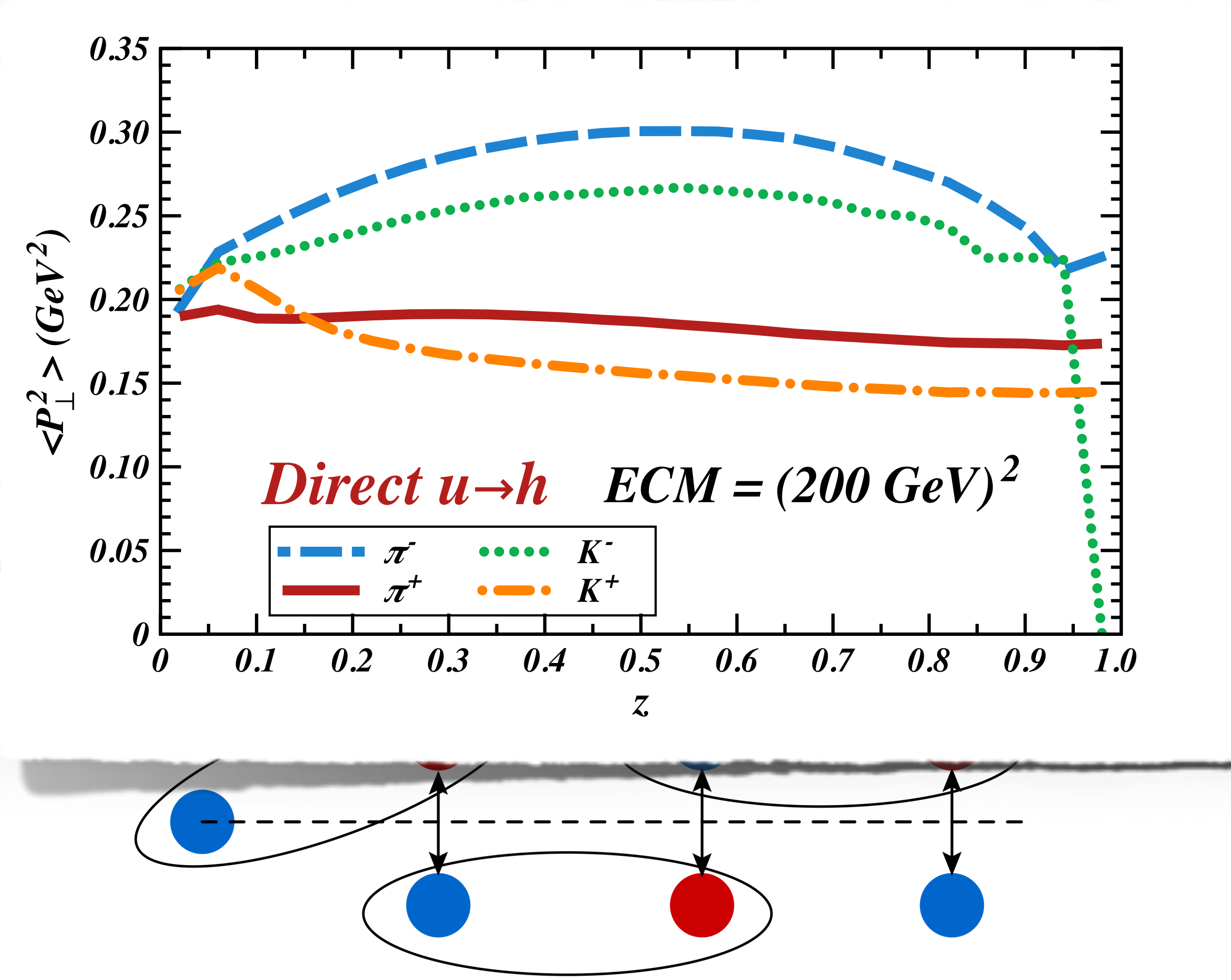
Different Hadronization Mechanisms.

LUND Model

✦ Fragmentation of $q\bar{q}$ pair: break-up of the string.

✦ Independent breaking of the string.

✦ Quark TM indep. of hadron type.

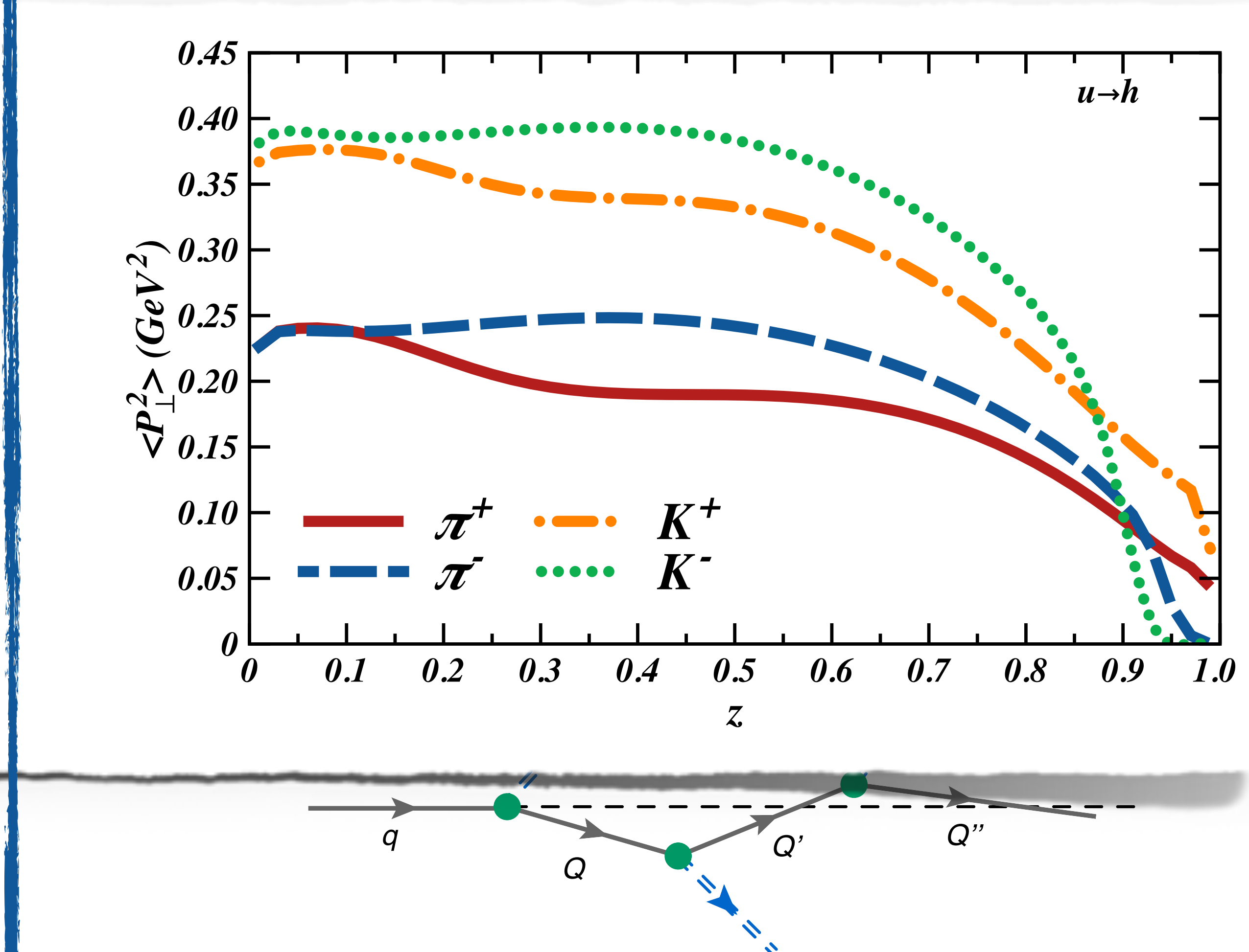


Quark-Jet

✦ Fragmentation of q, similar to QFT definition of FFs.

✦ Time-ordered hadron emissions.

✦ $q \to Qh$ depends on **h** (spin, mass).



❖ Can we find a signature in polarized FFs? Perhaps Dihadron FFs?

Different Hadronization Mechanisms.

LUND Model

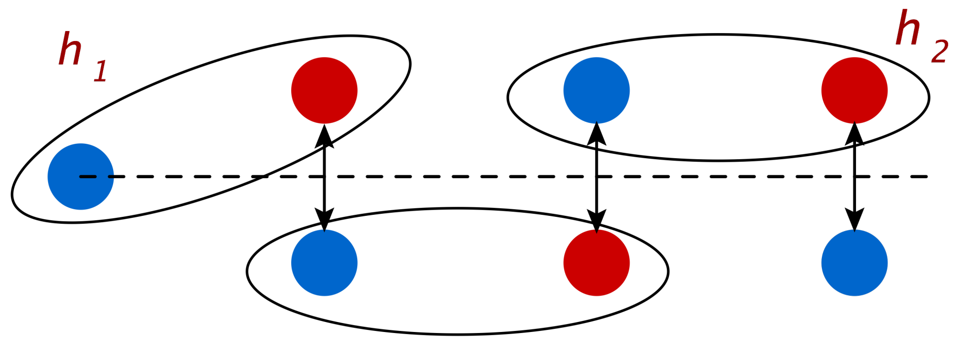
✦ Fragmentation of $q\bar{q}$ pair: break-up of the string.

✦ Independent breaking of the string.

✦ Quark TM indep. of hadron type.

$$u \to u + s\bar{s}, \quad s \to s + s\bar{s}$$

✦ No correlation in TM: h_1 and h_2.



Quark-Jet

✦ Fragmentation of q, similar to QFT definition of FFs.

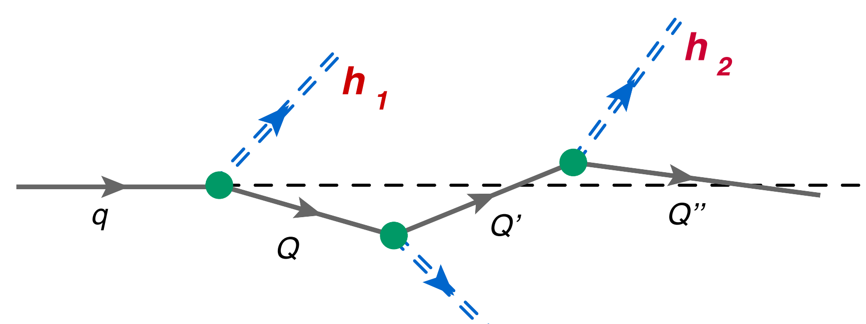
✦ Time-ordered hadron emissions.

✦ $q \to Qh$ depends on **h** (spin, mass).

$$u \to K^+ + s, \quad s \to \phi + s$$

$$u \to K^{*+} + s$$

✦ Recoil TM of h_1 affects h_2



❖ Can we find a signature in polarized FFs? Perhaps Dihadron FFs?

TMD FFs for Spin-0 and Spin-1/2 Hadrons

❖ ***The transverse momentum (TM) of the hadron can couple with both its own spin and the spin of the quark!***

$$F^{q\to\pi}(z, \boldsymbol{p}_\perp; \boldsymbol{s})$$

π/q	U	L	T
U	D_1		$H_1^\perp$

$$F^{q\to h^\uparrow}(z, \mathbf{p}_\perp; \mathbf{s}, \mathbf{S})$$

h/q	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	$H_{1T} H_{1T}^\perp$

✦ **TMD Polarized Fragmentation Functions at LO.**

- *Only two for unpolarised final state hadrons.*
- *8 for spin 1/2 final state (including quark). Similar to TMD PDFs.*

Field-Theoretical Definitions

- ***The quark-quark correlator.***

$$\Delta^{[\Gamma]}(z,\vec{p}_T) \equiv \frac{1}{4}\int \frac{dp^+}{(2\pi)^4} Tr[\Delta\Gamma]|_{p^-=zk^-}$$

$$= \frac{1}{4z}\sum_X \int \frac{d\xi^+ d^2\vec{\xi}_T}{2(2\pi)^3} e^{i(p^-\xi^+/z-\vec{\xi}_T\cdot\vec{p}_T)} \langle 0|\psi(\xi^+,0,\vec{\xi}_T)|p,S_h,X\rangle\langle p,S_h,X|\bar{\psi}(0)\Gamma|0\rangle$$

- ***The definitions of FFs from the quark correlator***

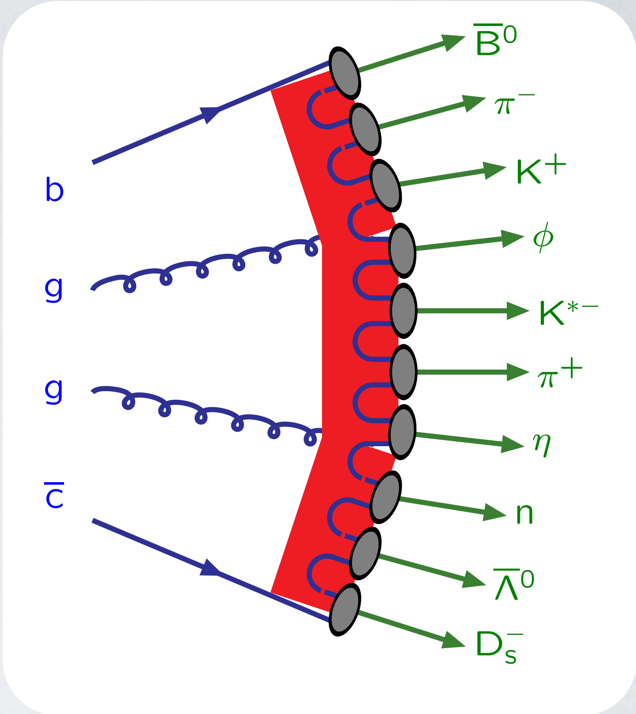
$$\Delta^{[\gamma^+]} = D(z,p_\perp^2) - \frac{1}{M}\epsilon^{ij}k_{Ti}S_{Tj}D_T^\perp(z,p_\perp^2)$$

$$\Delta^{[\gamma^+\gamma_5]} = S_L G_L(z,p_\perp^2) + \frac{\boldsymbol{k}_T\cdot S_T}{M}\, G_T(z,p_\perp^2)$$

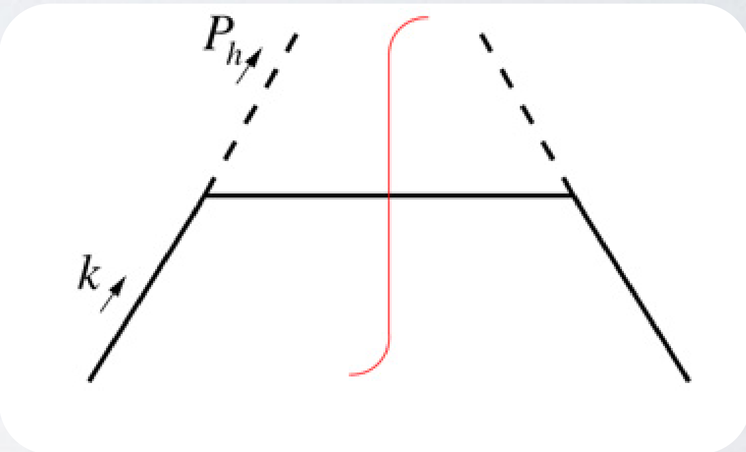
$$\Delta^{[i\sigma^{i+}\gamma_5]} = S_T^i H_T(z,p_\perp^2) + \frac{S_L}{M}k_T^i H_L^\perp(z,p_\perp^2) + \frac{k_T^i(\boldsymbol{k}_T\cdot S_T)}{M^2}H_T^\perp(z,p_\perp^2) - \frac{\epsilon^{ij}k_{Tj}}{M}H^\perp(z,p_\perp^2)$$

(SOME of the) MODELS FOR FRAGMENTATION

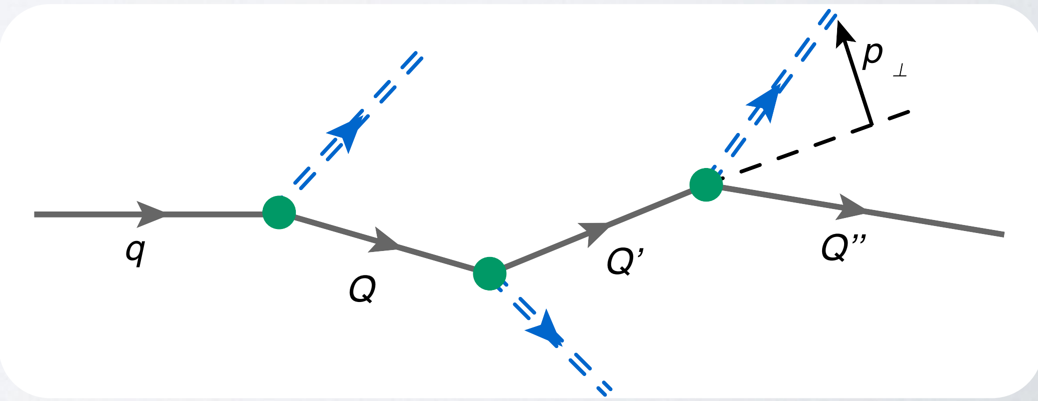
- *Lund String Model*
  - *Very Successful* implementation in *JETSET, PYTHIA*.
  - Highly Tunable.
  - No Spin Effects - Formal developments by X. Artru et al, *very recent unpublished results*.



- *Spectator Model*
  - Quark model calculations with empirical form factors.
  - No unfavored fragmentations.
  - Need to tune parameters for small z dependence.



- *NJL-jet Model*
  - *Multi-hadron* emission framework with effective quark model input.
  - *Monte-Carlo framework* allows flexibility in including the transverse momentum, spin effects, two-hadron correlations, etc.



Positivity and Polarisation of Quark

Bacchetta et al, PRL 85, 712 (2000) .

✦ The probability density is Positive Definite: constraints on FFs.

✦ Leading-order T-Even functions FULLY Saturate these bounds!

✦ For non-vanishing $H^{\perp}$ and $D_T^{\perp}$, need to calculate T-Even FFs at next order!

✦ Average value of remnant quark's spin.

$$\langle \boldsymbol{S}_T \rangle_Q = \boldsymbol{s}_T \frac{\int dz \left[h_T^{(q\to Q)}(z) + \frac{1}{2z^2 M_Q^2} \; h_T^{\perp[1](q\to Q)}(z) \right]}{\int dz \; d^{(q\to Q)}(z)}$$

✦ In spectator model, at leading order: $h_T(z) = -d(z)$

✦ Non-zero $h_T^{\perp}$ means $\langle \boldsymbol{S}_T \rangle_Q \neq -\boldsymbol{s}_T$ (full flip of the spin)!

SPECTATOR MODELS

E.G. - Bacchetta et al, PLB 659:234, 2008

- *Use Field-theoretical definition of FFs from a Correlator.*

$$\boxed{\Delta(z,k_T) = \frac{1}{2z}\int dk^+\,\Delta(k,P_h) = \frac{1}{2z}\sum_X\int\frac{d\xi^+\,d^2\boldsymbol{\xi}_T}{(2\pi)^3}\,e^{ik\cdot\xi}\,\langle 0|\mathcal{U}^{n_+}_{(+\infty,\xi)}\psi(\xi)|h,X\rangle\langle h,X|\bar\psi(0)\mathcal{U}^{n_+}_{(0,+\infty)}|0\rangle\big|_{\xi^-=0}}$$

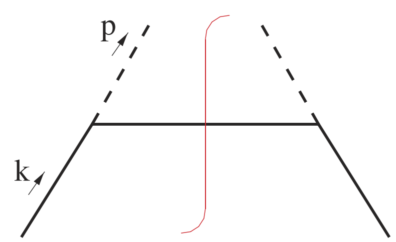
$$D_1(z,z^2\vec{k}_T^2) = \mathrm{Tr}[\Delta(z,\vec{k}_T)\gamma^-].$$

$$\frac{\epsilon_T^{ij}k_{Tj}}{M_h}H_1^\perp(z,k_T^2) = \frac{1}{2}\mathrm{Tr}[\Delta(z,k_T)i\sigma^{i-}\gamma_5]$$

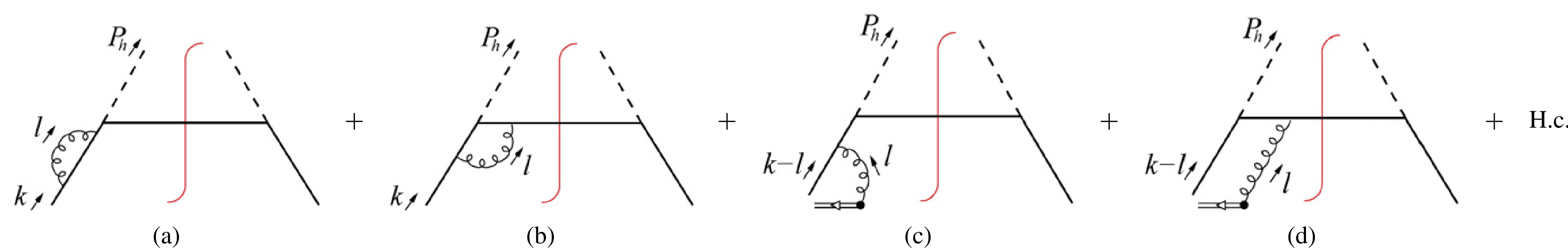
- *Approximate the remnant X as a "spectator" (quark).*

- *Calculate the FFs at leading-order in favourite quark model.*

$D_1(z,p_\perp^2)$



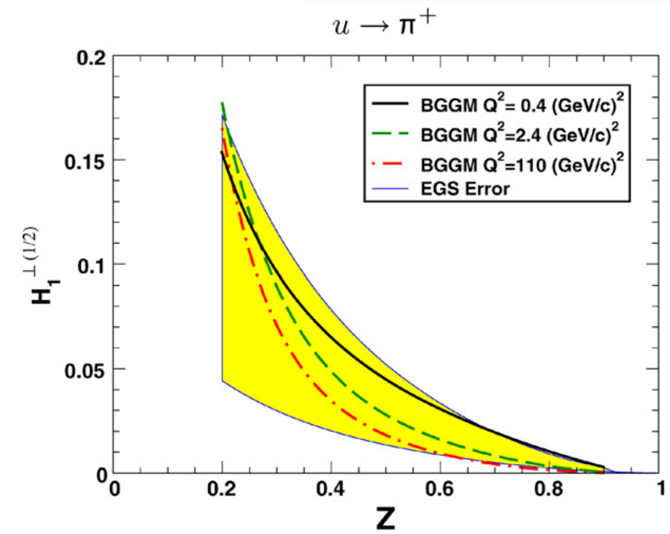
$H_1^\perp(z,p_\perp^2)$

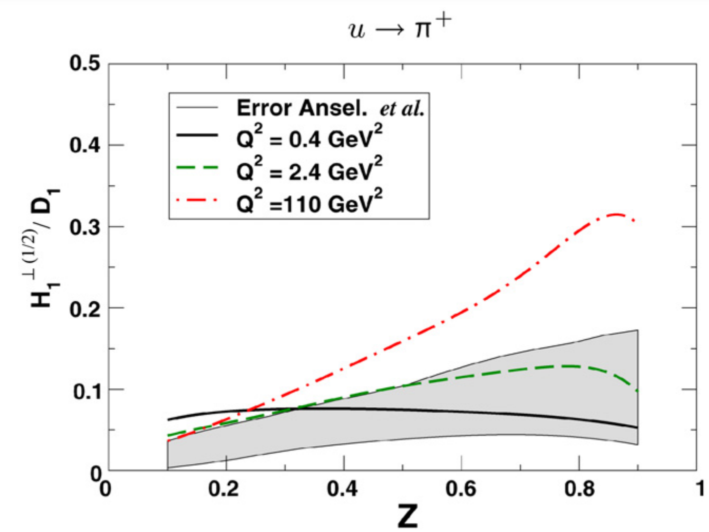


SPECTATOR MODELS

E.G. - Bacchetta et al, PLB 659:234, 2008

✦ *Calculated Collins FF.*





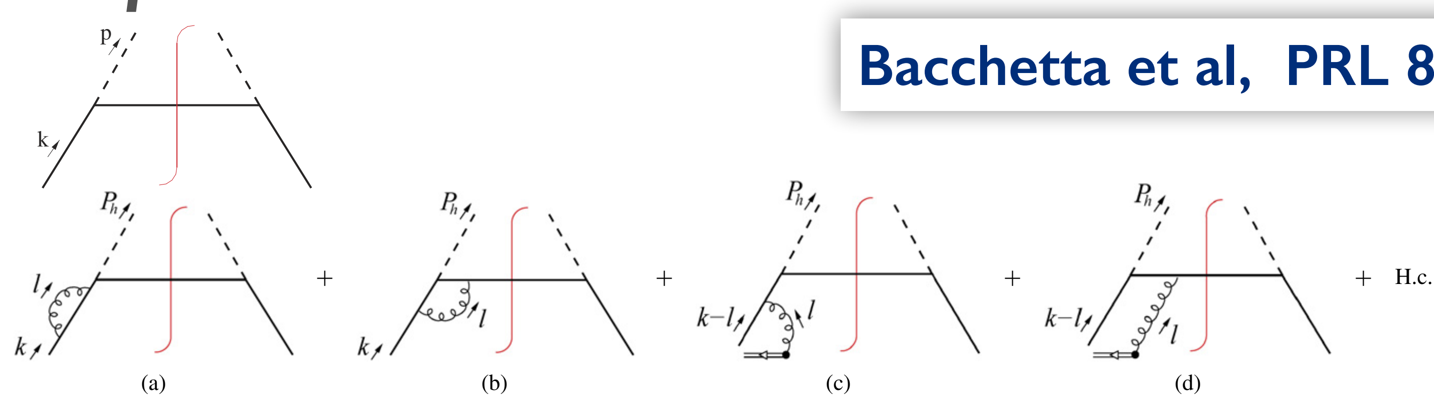
Issues with ALL the model calculations *to date*:

✦ *Mismatch in orders of calculations : VIOLATION OF POSITIVITY*

Bacchetta et al, PRL 85, 712 (2000) .

$D_1(z, p_\perp^2)$

$H_1^\perp(z, p_\perp^2)$



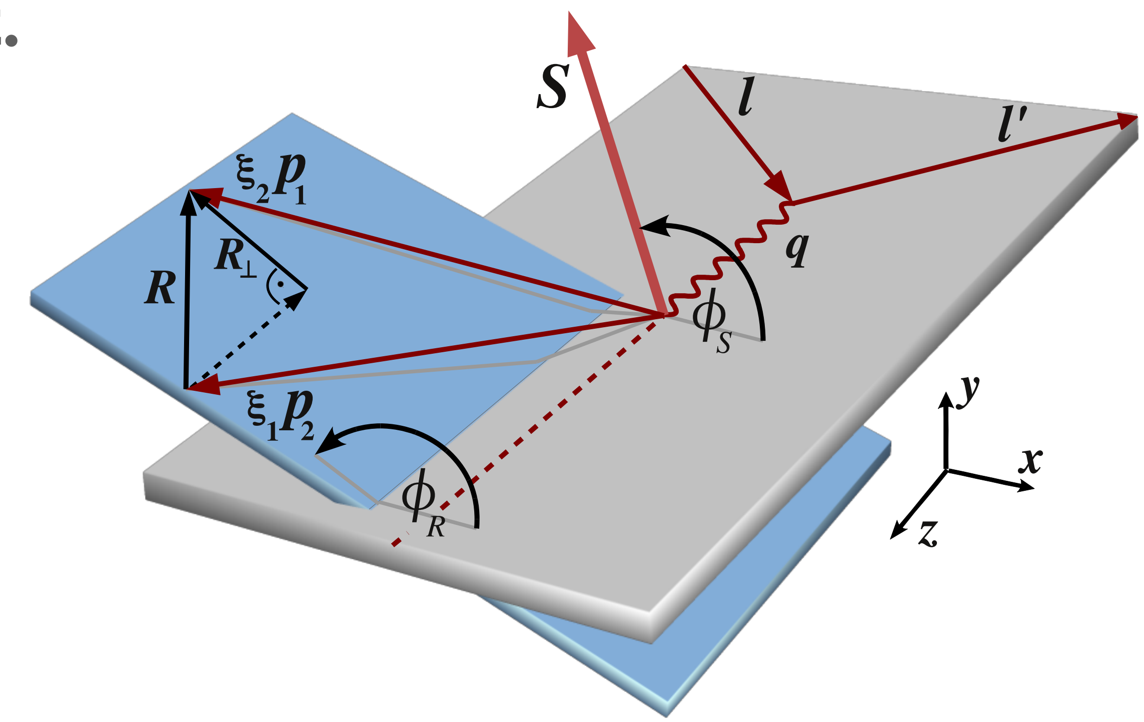
✦ *Missing **multi-hadron** emission effect:*

- *No direct access to unfavored FFs.*
- *Description of small-z region.*

RECENT COMPASS RESULTS

COMPASS, PLB736, 124-131 (2014).

✦ SIDIS with transversely polarized target.



✦ Collins single spin asymmetry:

$$A_{Coll} = \frac{\sum_q e_q^2 \; h_1^q \otimes H_1^{\perp h/q}}{\sum_q e_q^2 \; f_1^q \otimes D_1^{h/q}}$$

✦ Two hadron single spin asymmetry:

$$A_{UT}^{\sin\phi_{RS}} = \frac{|\boldsymbol{p}_1 - \boldsymbol{p}_2|}{2M_{h^+h^-}} \frac{\sum_q e_q^2 \cdot h_1^q(x) \cdot H_{1,q}^{\sphericalangle}(z,\, M_{h^+h^-}^2,\, \cos\theta)}{\sum_q e_q^2 \cdot f_1^q(x) \cdot D_{1,q}(z,\, M_{h^+h^-}^2,\, \cos\theta)}$$

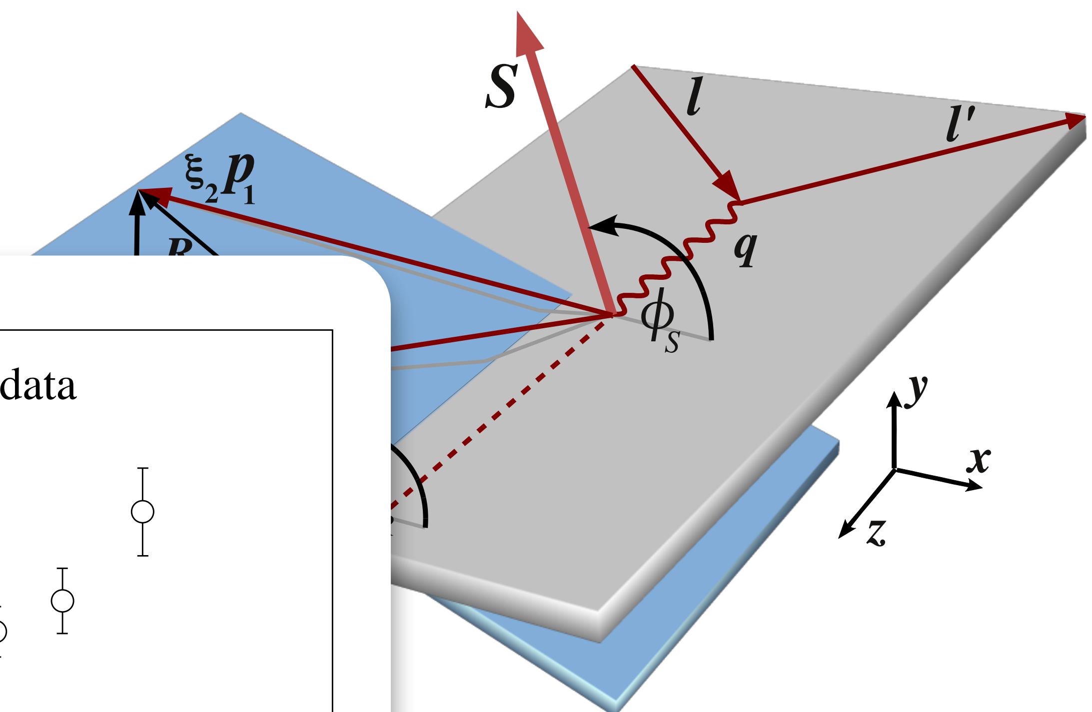
✦ Note the choice of the vector

$$\boldsymbol{R}_{Artru} = \frac{z_2 \boldsymbol{P}_1 - z_1 \boldsymbol{P}_2}{z_1 + z_2}$$

RECENT COMPASS RESULTS

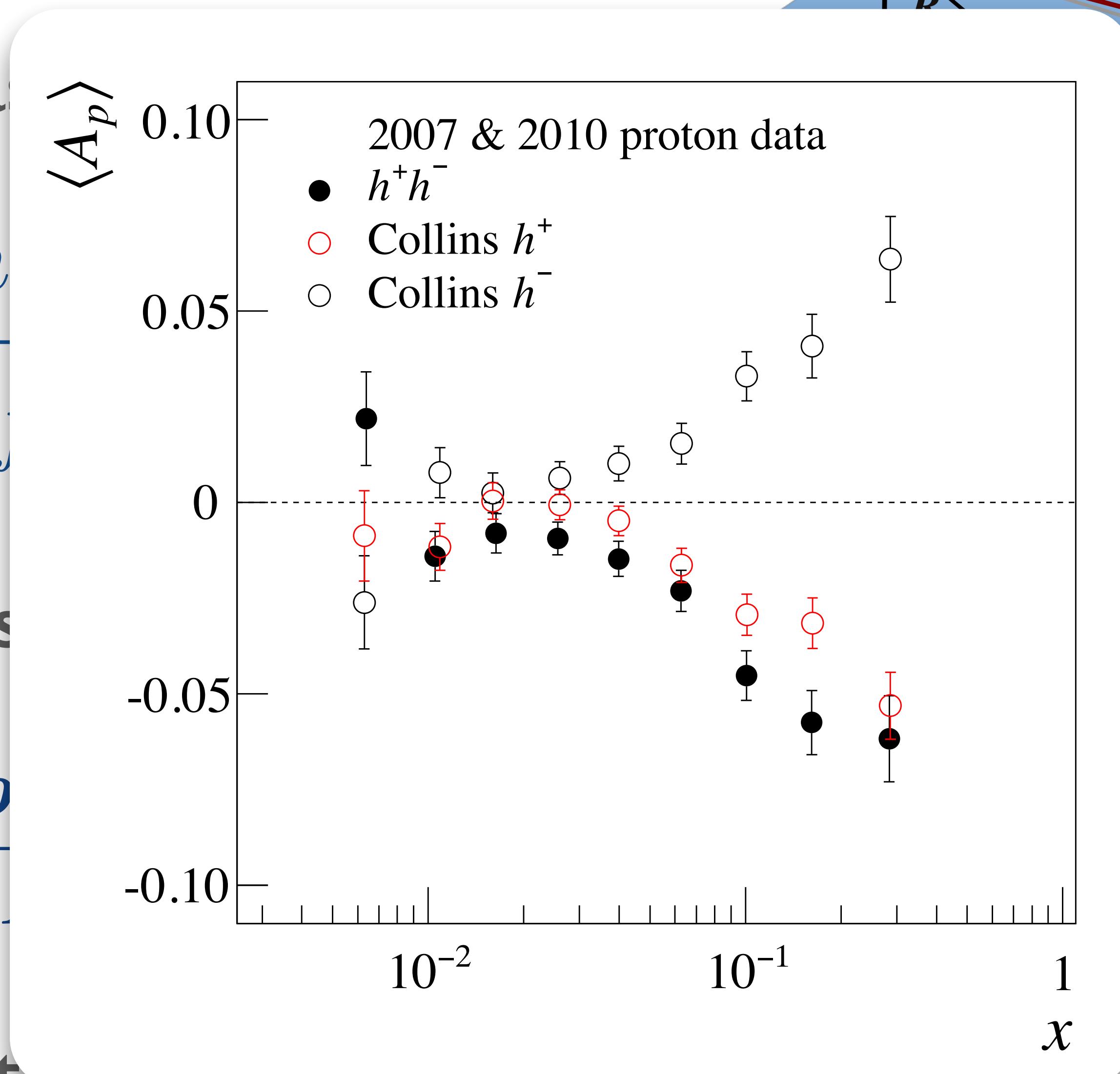
COMPASS, PLB736, 124-131 (2014).

✦SIDIS with transversely polarized target.



✦Collins single spin a

$$A_{Coll} = \frac{\sum_q e_q^2\ h}{\sum_q e_q^2}$$



✦Two hadron single s

$$A_{UT}^{\sin\phi_{RS}} = \frac{|p}{2}$$

✦Note the choice of the vector

$$\boldsymbol{R}_{Artru} = \frac{z_2 \boldsymbol{P}_1 - z_1 \boldsymbol{P}_2}{z_1 + z_2}$$