Studies of GPDs and TMDs at JLab

Harut Avakian (JLab)

COMPASS Seminar, March 29 2012
Outline

Transverse structure of the nucleon and partonic correlations

- Introduction
- $k_T$-effects with unpolarized and polarized target data
- SSA measurements and “puzzles”
- Studies of 3D PDFs at JLab at 6 GeV
- Hard exclusive processes and correlations between transverse degrees of freedom
- Studies of 3D structure of the nucleon at JLab12 and beyond
- Extracting transversity from di-hadron production
- Summary
Wide kinematic coverage of large acceptance detectors allows studies of exclusive (GPDs) and semi-inclusive (TMDs) processes providing complementary information on transverse structure of the nucleon.
TMD Distributions: Theory

- Classification of TMDs and SIDIS and DY x-sections (Ralston, Soper 1979, Mulders, Tangerman, Kotzinian 1995)

- The role of final state interactions in SSA (Brodsky et al, Collins 2002)

- Universality of $k_T$-dependent distribution and fragmentation functions. Sign flip for $f_{1T}$, $h_{1T}$ from DY to SIDIS predicted. (Collins, Metz 2003)

- Gauge invariant definition of $k_T$-dependent PDFs (Belitsky, Ji, Yuan 2003)

- Factorization proven for small $k_T$ (Ji, Ma, Yuan 2005)

- Complete definition of TMDs (Collins 2011 “Foundation of Perturbative QCD”)

- Evolution of TMDs, (Collins, Aybat, Rogers 2011)

- TMDs on Lattice, (Musch, Haegler et al. 2011)

- Fracture Functions and SIDIS x-sections (Trentadue, Veneziano 1974, Anselmino, Barone, Kotzinian 2011)

- $k_T$-dependent flavor decomposition (BGMP procedure, 2011)
Generalized Parton Distributions: Theory

- Decomposition of the OAM (X. Ji 1997)
- Factorization for hard exclusive electroproduction of mesons in QCD (Collins, Frankfurt & Strikman 1996)
- Transversity GPDs (Hoodbhoy & Ji 1998, M. Diehl 2001)
- GPDs from DVCS and DVMP (Ji 1996, Mueller 2001, VGG 1999)
- GPDs on Lattice, (QCDSF, Haegler et al. 2006)
- GPD (CFF extraction from DVCS data (D. Mueller, M. Guidal, H. Mutarde, …)
- Accessing transversity GPDs in DVMP (Liuti & Goldstein 2008, Kroll & Goloskokov 2010)
**k_\perp**-dependence of TMDs

- Transition from low p_\perp to high p_\perp

Directly obtained ETQS functions are opposite in sign to those from k_\perp moments “sign mismatch”

Sivers function extracted assuming k_\perp distribution is gaussian

- With orbital angular momentum TMD can’t be gaussian
- How to measure k_\perp-dependences of TMDs

(Z. Kang et al, 2011)
TMD evolution

Q^2 evolution of Sivers asymmetry may be very significant

SIDIS: partonic cross sections

\[ \nu = \frac{(qP)}{M} \]
\[ Q^2 = (k - k')^2 \]
\[ y = \frac{(qP)}{(kP)} \]
\[ x = \frac{Q^2}{2(qP)} \]
\[ z = \frac{(qP_{h})}{(qP)} \]

\[ \sigma = F_{UU} + P_t F_{UL}^{\sin \phi} \sin 2\phi + P_b F_{LU}^{\sin \phi} \sin \phi \ldots \]

Transverse momentum of hadrons in SIDIS provides access to orbital motion of quarks

\[ P_T = p_\perp + z \, k_T \]

Ji, Ma, Yuan Phys. Rev. D71: 034005, 2005

\[ d\sigma^h \propto \sum e_q^2 \int d^2 k_T d^2 p_T d^2 l_T f^{H^+q(x, k_T)} D^{q^+ h}(z, p_\perp) S(l_T) H(Q) \delta(z k_T + p_T + l_T - P_T) \]
Azimuthal moments in SIDIS

\[
\frac{d\sigma}{dx \, dy \, d\psi \, dz \, d\phi_h \, dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos \phi_h F_{UU}^{\cos \phi_h} + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin \phi_h F_{LU}^{\sin \phi_h} \right. \\
+ S_{||} \left[ \sqrt{2\varepsilon(1+\varepsilon)} \sin \phi_h F_{UL}^{\sin \phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \\
+ S_{\perp} \left[ \sin(\phi_h - \phi_S) \left( F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)} \right) \right. \\
+ \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h-\phi_S)} \\
+ \sqrt{2\varepsilon(1+\varepsilon)} \sin \phi_S F_{UT}^{\sin \phi_S} + \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h-\phi_S)} \\
+ \left. \left. S_{\perp} \lambda_e \left[ \sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)} \cos \phi_S F_{LT}^{\cos \phi_S} \right. \right. \\
+ \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h-\phi_S)} \right\}, \right.
\]

Experiment for a given target polarization measures all moments simultaneously

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Extracting the moments

Moments mix in experimental azimuthal distributions

**Azimuthal acceptance**

\[ \text{Acc}_k(\phi_h) = \frac{R_{k}^{\text{mc}}(\phi_h)}{G_{k}^{\text{mc}}(\phi_h)} \]

Acceptance:

Virtual photon angle:

\[ \sin \theta_\gamma = \sqrt{\frac{4M^2x^2}{Q^2 + 4M^2x^2} \left(1 - y - \frac{M^2x^2y^2}{Q^2}\right)} \]

**Simplest acceptance** -> \( 1 + A \cos \phi \)

**Correction to normalization**

\[
(1 + \alpha \cos \phi)(1 + A \cos \phi) \to 1 + A\alpha/2
\]

\[
(1 + \beta\lambda \Lambda + \gamma\lambda \Lambda \cos \phi)(1 + A \cos \phi) \to 1 + (\beta + \gamma A/2)\lambda \Lambda
\]

**Correction to DSA**

\[
(1 + S_T\delta \sin \phi_S)(1 + A \cos \phi) \to 1 + S_T/2\delta A(\sin \phi - \phi_S) + \ldots
\]

**Correction to SSA**

\[
\frac{1 + \beta\lambda \Lambda}{1 + a \cos \phi} \to 1 - a\beta\lambda \Lambda \cos \phi
\]

**Fake DSA cos**

Simultaneous extraction of moments is important also because of correlations!
TMD Distributions: First experiments

\[ H_1 \]

Spin structure

FermiLab E-764

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Kaon $\langle \cos 2\phi \rangle$ @ HERMES

"Kaon puzzle" in spin-orbit correlations

$u$ – dominance

$K^+ \{ u\bar{s} \}$  $\pi^+ \{ u\bar{d} \}$

$\frac{H_{1, u \rightarrow K^+}}{D_{1,u \rightarrow K^+}} > \frac{H_{1, u \rightarrow \pi^+}}{D_{1,u \rightarrow \pi^+}}$

Relative sign $H_{1, \text{fav}} / H_{1, \text{unfav}}$ for $\pi$ and $K$ inconsistent
Is there a link between HERMES and BRAHMS Kaon vs pion moments (K- has the same sign as K+ and \( \pi^+ \), comparable with K+)?
JLab Experimental Halls

Hall A

\[ ^3\text{He}^+(e, e' h), \ h = \pi^+, \pi^- \]

IC

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$P_T$-dependence studies at Hall-C

H. Mkrtchyan (DIS2011)

Experiment E00-108

Beam energy 5.5 GeV

4 cm LH2 and LD2 targets

\[ \sigma_d^{\pi^+} \propto (4D^+ + D^-)(u+d) \]

\[ \sigma_d^{\pi^-} \propto (4D^- + D^+)(u+d) \]

\[ \frac{\sigma_d^{\pi^+}}{\sigma_d^{\pi^-}} = \frac{4D^+ + D^-}{4D^- + D^+} \]

\[ D^-/D^+ = (4 - r) / (4r - 1) \]

\[ r = \sigma_d(\pi^+)/\sigma_d(\pi^-) \]

x-dependence of $p^+/p^-$ ratio is good agreement with the quark parton model predictions (lines CTEQ5M+BKK).
$P_T$-dependence studies at Hall-C

H. Mkrtchyan (DIS2011)
Experiment E00-108
Beam energy 5.5 GeV
4 cm LH2 and LD2 targets

Data (assuming only valence quarks and only two fragmentation functions contribute) indicate that $k_T$-width of u-quarks is larger than for d-quarks
HT-distributions in SIDIS

Factorization of higher twists in SIDIS not proved
To study HT pdfs with dihadron SIDIS (replace $H_1^\perp$ with IFF PRD69 (2004))
Forces and binding effects in the partonic medium

\[ xe = x\tilde{e} + \frac{m}{M} f_1 \]
\[ xh_L = x\tilde{h}_L + \frac{p_T^2}{M^2} h_{1L} + \frac{m}{M} g_{1L} \]

“Wandzura-Wilczek approximation” is equivalent to setting functions with a tilde to zero.

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Boer-Mulders Force on the active quark right after scattering (t=0)

\[ F_{yy}(0) = \frac{M^2}{2} e_2 \]

Interpreting HT (quark-gluon-quark correlations) as force on the quarks (Burkardt hep-ph:0810.3589)
$A_1(\pi) \propto \frac{\sum_q e_q^2 g_1^q(x) D_1^q \rightarrow \pi^+(z)}{\sum_q e_q^2 f_1^q(x) D_1^q \rightarrow \pi^-(z)}$

$A_1(x, z, P_T) = A_1(x, z) \left( \frac{P_T^{2, \text{unp}}}{P_T^{2, \text{pol}}} \right)^{ \alpha_{LL} } \exp\left( -\frac{P_T^2}{\langle P_T^{2, \text{pol}} \rangle} - \frac{P_T^2}{\langle P_T^{2, \text{unp}} \rangle} \right)$

CLAS data suggests that width of $g_{1}$ is less than the width of $f_{1}$

New CLAS data would allow multidimensional binning to study $k_T$-dependence for fixed $x$
Kotzinian-Mulders Asymmetries

\[ A_{UL}^{\sin 2\phi} \sim h_{1L}^L H_{1}^L \sin 2\phi \]

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<th>N-q</th>
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<tr>
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<td>( h_{1}^1 )</td>
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<tr>
<td>L</td>
<td>g_1</td>
<td>( h_{1T} )</td>
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<tr>
<td>T</td>
<td>f_{1T}</td>
<td>g_{1T}</td>
<td>( h_{1}^1 ), ( h_{1T} )</td>
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J. Huang (DIS2011)

Hall-A E06-010

Neutron \( A_{LT}^{\cos(\phi_h, \phi_s)} \)

Worm gear TMDs are unique (no analog in GPDs)

B. Musch arXiv:0907.2381
B. Pasquini et al, arXiv:0910.1677
$^3\text{He}^\uparrow(e, e' h), \ h = \pi^+, \pi^-$

$^3\text{He}^\uparrow = 0.865 \cdot n^\uparrow - 2 \times 0.028 \cdot p^\uparrow$

**Collins** asymmetries for neutron are not large, except at $x=0.34$

**Sivers** agree with global fit, and light-cone quark model.
Deeply Virtual Compton Scattering

The DVCS amplitude is expressed in terms of Compton Form Factors (CFF) at LO:

\[ \mathcal{H}(\xi, t) = \sum_q e_q^2 \left\{ i\pi \left[ H^q(\xi, \xi, x) - H^q(-\xi, \xi, x) \right] + \mathcal{P} \int_{-1}^{+1} dx \left[ \frac{1}{\xi - x} - \frac{1}{\xi + x} \right] H^q(\xi, x, t) \right\} \]

(similarly for other GPDs)

\( \xi = \frac{x_B}{2-x_B} \quad k = -t/4M^2 \)

Polarized beam, unpolarized target (BSA):

\[ \Delta \sigma_{LU} \sim \sin \phi \text{Im}\{F_1 \mathcal{H} + \xi(F_1+F_2)\tilde{\mathcal{H}} - kF_2E\} \text{d}\phi \]

\[ \text{Im}\{\mathcal{H}_p, \tilde{\mathcal{H}}_p, E_p\} \]

Unpolarized beam, longitudinal target (ITSA):

\[ \Delta \sigma_{UL} \sim \sin \phi \text{Im}\{F_1 \tilde{\mathcal{H}} + \xi(F_1+F_2)(\mathcal{H} + x_B/2E) - \xi kF_2 \tilde{E} + \ldots \} \text{d}\phi \]

\[ \text{Im}\{\mathcal{H}_p, \tilde{\mathcal{H}}_p\} \]

Polarized beam, longitudinal target (BITSA):

\[ \Delta \sigma_{LL} \sim (A+B\cos \phi) \text{Re}\{F_1 \tilde{\mathcal{H}} + \xi(F_1+F_2)(\mathcal{H} + x_B/2E)\ldots\} \text{d}\phi \]

\[ \text{Re}\{\mathcal{H}_p, \tilde{\mathcal{H}}_p\} \]

Unpolarized beam, transverse target (tTSA):

\[ \Delta \sigma_{UT} \sim \cos \phi \text{Im}\{k(F_2 \mathcal{H} - F_1 \mathcal{E}) + \ldots \} \text{d}\phi \]

\[ \text{Im}\{\mathcal{H}_p, E_p\} \]
DVCS x-sections from CLAS

\[ e p_0 \rightarrow e' p_f \gamma \quad t = (p_0 - p_f)^2 \]

Radiative corrections and \( \pi^0 \) contamination accounted qualitatively agrees with Hall-A

- In certain region of azimuthal angles the x-section is higher than BH calculations indicating data may be sensitive to DVCS already in JLab kinematics.
SSAs in exclusive pion production

\[ M_{0-,++}^{\text{twist-3}} \approx e_0 \sqrt{1 - \xi^2} \int_1^{+1} d\bar{x} H_{0-,++}[H_T^{(3)} + \ldots] \]

<table>
<thead>
<tr>
<th>observable</th>
<th>dominant interf. term</th>
<th>amplitudes</th>
<th>low ( t' ) behavior</th>
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<tbody>
<tr>
<td>( A_{UT}^{\sin(\phi-\phi_s)} )</td>
<td>LL</td>
<td>( \text{Im}[M_{0-,0+}^* M_{0-,0+}] )</td>
<td>( \propto \sqrt{-t'} )</td>
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<td>TT</td>
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<td>( A_{UT}^{\sin(3\phi-\phi_s)} )</td>
<td>TT</td>
<td>( \text{Im}[M_{0-,+-}^* M_{0+,+-}] )</td>
<td>( \propto (-t')^{3/2} )</td>
</tr>
<tr>
<td>( A_{UT}^{\sin \phi_s} )</td>
<td>LT</td>
<td>( \text{Im}[M_{0-,++}^* M_{0+,0+}] )</td>
<td>const.</td>
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<tr>
<td>( A_{UL}^{\sin \phi} )</td>
<td>LT</td>
<td>( \text{Im}[M_{0-,++}^* M_{0-,0+}] )</td>
<td>( \propto \sqrt{-t'} )</td>
</tr>
<tr>
<td>( A_{LU}^{\sin \phi} )</td>
<td>LT</td>
<td>( \text{Im}[M_{0-,++}^* M_{0-,0+}] )</td>
<td>( \propto \sqrt{-t'} )</td>
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<tr>
<td>( A_{LL}^{\cos \phi} )</td>
<td>LT</td>
<td>( \text{Re}[M_{0-,++}^* M_{0-,0+}] )</td>
<td>( \propto \sqrt{-t'} )</td>
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\[ A_{LU}^{\sin \phi} / A_{UL}^{\sin \phi} \approx \sqrt{(1 - \epsilon)/(1 + \epsilon)} \]

- HT SSAs are expected to be very significant
- Wider coverage (CLAS12, EIC) would allow measurements of \( Q^2 \) dependence of HT SSAs
Exclusive $\pi^0$ production

Recent progress with GPD-based description
- Goloskokov & Kroll, Goldstein & Liuti. Include **transversity GPDs** $H_T$ and $E_T = 2\tilde{H}_T + E_T$
  - Dominate in CLAS kinematics. Successfully described data.

Structure Functions

- $x_b = 0.34$
- $Q^2 = 2.3$ GeV$^2$
- $\sigma_T + \varepsilon \sigma_L$
- $\sigma_{LT}$
- $\sigma_{TT}$

$\eta/\pi^0$ Ratio

- Goloskokov and Kroll: GPD including transversity $H_T$
- W = 3.83 GeV
- $Q^2 = 3.44$ GeV$^2$
- Eides, Frankfurt, Strikman GPD $\sim 1$

Goloskokov & Kroll
- $H_T$ and $E_T$ dominate
SIDIS at JLab12

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GPDs in DVCS experiments at JLab12

Nucleon polarization

- UP
- LP
- TP

Sensitivity to GPDs

- $H$, $\tilde{H}$, $E$
- $\tilde{H}$, $H$, $E$
- $E$, $H$

- E12-06-114 : $\gamma$, $\pi^0$ (A) proton
- E12-06-119 : $\gamma$, $\pi^0$ (B) proton
- E12-11-003: $\gamma$, $\pi^0$ (B) neutron
- E12-06-119 : $\gamma$, $\pi^0$ (NH$_3$) (B) proton
- LOI12-11-105 : $\gamma$, $\pi^0$ (HD) (B) proton

GPD $H$ only contribution

$$q(x, b_\perp) = \int d^2 \Delta_\perp \left( \frac{2\pi}{\Delta_\perp} \right)^2 e^{i b_\perp \cdot \Delta_\perp} H(x, \xi = 0, -\Delta_\perp^2)$$

extrapolate to $\xi=0$
In general, 8 GPD quantities accessible

(Compton Form Factors)

\[ H_{Re} = P \int_{0}^{1} dx \left[ H(x, \xi, t) - H(-x, \xi, t) \right] C^+(x, \xi) \]  
\[ E_{Re} = P \int_{0}^{1} dx \left[ E(x, \xi, t) - E(-x, \xi, t) \right] C^+(x, \xi) \]  
\[ \tilde{H}_{Re} = P \int_{0}^{1} dx \left[ \tilde{H}(x, \xi, t) + \tilde{H}(-x, \xi, t) \right] C^-(x, \xi) \]  
\[ \tilde{E}_{Re} = P \int_{0}^{1} dx \left[ \tilde{E}(x, \xi, t) + \tilde{E}(-x, \xi, t) \right] C^-(x, \xi) \]  
\[ H_{Im} = H(\xi, \xi, t) - H(-\xi, \xi, t), \]  
\[ E_{Im} = E(\xi, \xi, t) - E(-\xi, \xi, t), \]  
\[ \tilde{H}_{Im} = \tilde{H}(\xi, \xi, t) + \tilde{H}(-\xi, \xi, t) \quad \text{and} \]  
\[ \tilde{E}_{Im} = \tilde{E}(\xi, \xi, t) + \tilde{E}(-\xi, \xi, t) \]  

with

\[ C^\pm(x, \xi) = \frac{1}{x - \xi} \pm \frac{1}{x + \xi}. \]
The $Q^2$, $xB$, and $t$ dependences of the DVCS single and double asymmetries will be studied in a wide range of kinematics. Demonstrate capabilities to reconstruct protons.
Extraction of GPDS from CLAS12 data

The full set of Compton Form Factors (CFFs) can be reconstructed, using the full set of single and double spin asymmetries.

Obs = \text{Amp}(\text{DVCS} + \text{BH}) \otimes \text{CFFs}
## SIDIS at JLab12

**Proton**

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<td>$g_{1T}$</td>
<td>$h_1$</td>
<td>$h_{1T}$</td>
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**Hall C**

- E12-09-017: π⁺, π⁻, K⁺, K⁻
- C12-11-102: π⁺

**Hall A**

- E12-09-008: K⁺, K⁻, K⁰
- C12-11-102: π⁺, π⁻

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**D₂**

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<td>$h_{1T}$</td>
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**Hall C**

- E09-008: π⁺, π⁻, K⁺, K⁻
- E07-107: π⁺, π⁻, π⁰
- E09-009: K⁺, K⁻, K⁰

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**³He**

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<td>$h_1$</td>
<td>$h_{1T}$</td>
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**Hall A**

- E12-07-007: π⁺, π⁻
- E10-006: π⁺, π⁻
- E12-09-018: π⁺, π⁻, K⁺, K⁻

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H. Avakian, CERN, March 29

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**Jefferson Lab**
$A_1 P_T$-dependence in SIDIS (CLAS12)

\[ A_1(\pi) \propto \frac{\sum_q e_q^2 g_1^q(x) D_{1\pi}(z)}{\sum_q e_q^2 f_1^q(x) D_{1\pi}(z)} \exp \left( -z^2 P_T^2 \frac{\left(\mu_0^2 - \mu_D^2\right)}{\left(\mu_0^2 + z^2 \mu_0^2\right)\left(\mu_D^2 + z^2 \mu_D^2\right)} \right) \]

\[ \pi^+ \quad \pi^- \quad \pi^0 \]

\begin{align*}
\Delta_{\text{CLAS 5.7 GeV}} & \quad \mu_0^2 = 0.10 \\
\Delta_{\text{CLAS12}} & \quad \mu_0^2 = 0.17 \\
\bullet_{\text{EIC 4x60 GeV}} & \quad \mu_0^2 = 0.25
\end{align*}

$\mu_0^2 = 0.25 \text{GeV}^2$

$\mu_D^2 = 0.2 \text{GeV}^2$

Perturbative limit calculations available for $g_1^q(x, k_T)$, $f_1(x, k_T)$:

\[ f_1^q(x, k_T) = f_1(x) \exp \left( -\frac{k_T^2}{\mu_0^2} \right) \]

\[ g_1^q(x, k_T) = g_1(x) \exp \left( -\frac{k_T^2}{\mu_0^2} \right) \]

\[ D_1^q(z, p_T) = D_1(z) \exp \left( -\frac{p_T^2}{\mu_D^2} \right) \]

\[ \bullet A_{LL}(\pi) \text{ sensitive to difference in } k_T \text{ distributions for } f_1 \text{ and } g_1 \]

\[ \bullet \text{ Wide range in } P_T \text{ allows studies of transition from TMD to perturbative approach} \]
Quark distributions at large $k_T$: lattice

$g_1^q = \Delta q = (q^+ - q^-)/2$

$< x > \approx 0.3$

$g_1^q = \Delta q = (q^+ - q^-)/2$

$B. Musch et al arXiv:1011.1213$

$B. Pasquini et al$

Significant correlations of spin and transverse degrees of freedom predicted

$u^+(k_T)/u^-(k_T)$

$u(k_T)/d(k_T)$

$u^+/u^-$

$u^+/d^+$

$B. Musch et al$

$B. Pasquini et al$

$g_1^q (x, k_T) = g_1(x) \frac{1}{\pi \mu_2^2} \exp \left( -\frac{k_T^2}{\mu_2^2} \right)$

$f_1^q (x, k_T) = f_1(x) \frac{1}{\pi \mu_0^2} \exp \left( -\frac{k_T^2}{\mu_0^2} \right)$

Higher probability to find a quark anti-aligned with proton spin at large $k_T$ and $b_T$
**SOLID** $A_{UL}$ on $^3\text{He}$

$$e^3\text{He} \rightarrow e\pi^+\chi$$

$E12-11-007$

5 GeV Data

$2 \text{ GeV}^2 < Q^2 < 3 \text{ GeV}^2$

$0.40 < z < 0.45$

First Neutron $^3\text{He}$ Data

$2 \text{ GeV}^2 < Q^2 < 3 \text{ GeV}^2$

$0.40 < z < 0.45$

H. Avakian, CERN, March 29
Studies of Spin-Orbit Correlations with Longitudinally Polarized Target

\[ \frac{d\sigma}{dxdy\phi_S d\phi_h dP_{h\perp}^2} \propto S_L \left[ \sqrt{2\epsilon(1+\epsilon)\sin\phi_h F_{UL}^{\sin\phi_h} + \epsilon\sin(2\phi_h) F_{UL}^{\cos(2\phi_h)}} \right] \\
+ S_L \lambda e \left[ \sqrt{1 - \epsilon^2 F_{LL}^{\cos\phi_h} + \sqrt{2\epsilon(1-\epsilon)\cos(\phi_h) F_{LL}^{\cos\phi_h}}} \right] \\
+ h.t. \]

\[ h_{1L} \otimes H_i \]

\[ \text{h.t.} \]

\[ g_{1L} \otimes D_1 \]

\[ \text{h.t.} \]

\[ \text{A}_1 \text{ P}_{T}\text{-dependence provides access to helicity dependence of } k_T \text{-distributions of quarks} \]

\[ \text{p & d data required for } P_T \text{-dependence flavor decomposition} \]
CLAS12 $A_{UT}$ with transverse proton target

Stat. error for a 4D analysis of the $\pi^+$ Sivers asymmetry on proton (x1.5 on D) target

From JLab12 to EIC

- Study of high $x$ domain requires high luminosity, low $x$ higher energies
- Wide range in $Q^2$ is crucial to study the evolution
- Overlap of EIC and JLab12 in the valence region will be crucial for the TMD program

JLab@12GeV (25/50/75)  
$0.1 < x_B < 0.7$: valence quarks  
EIC $\sqrt{s} = 140, 50, 15$ GeV  
$10^{-4} < x_B < 0.3$: gluons and quarks, higher $P_T$ and $Q^2$.  

$A_{UT}^{\sin(\phi - \phi_S)} = \frac{\sum_q e_q^2 f^{q}_1 T D^{q}_1}{\sum_q e_q^2 f^{q}_1 T D^{q}_1}$
For a given lumi (30min of runtime) and given bin in hadron z and $P_T$, higher energy provides higher counts and wider coverage in $Q^2$, allowing studies of $Q^2$ evolution of 3D partonic distributions in a wide $Q^2$ range.
Extracting Sivers function from asymmetries

\[ A_{UT}^{\sin(\phi-\phi_S)} = \frac{\sum_q e_q^2 f_{1T}^{q} D_1^q}{\sum_q e_q^2 f_{1T}^{q} D_1^q} \]

EIC with energy setting of \( \sqrt{s} = 45 \text{ GeV} \) and an integrated lumi of 4 fb\(^{-1}\)

Extraction based on Gaussian Sivers, generated and then extracted with assumption of the same shape as used in generation (unclear systematics)

- It is crucial to have a well defined, model-independent procedure for extraction of \( k_T \)-dependent PDFs.
SIDIS with Bessel weighting

\[ F_{UU,T} = x_B \sum_a e_a^2 \int \frac{d^2 p_T \, d^2 k_T}{(2\pi)^2} \, \delta^{(2)}(p_T - k_T - P_{h\perp}/z) \, w(p_T, k_T) \, f^a(x, p_T^2) \, D^a(z, k_T^2). \]

\[ \delta^{(2)}(z p_T + K_T - P_{h\perp}) = \int \frac{d^2 b_T}{(2\pi)^2} e^{i b_T (z p_T + K_T - P_{h\perp})} \]

\[ F_{UU,T} = x_B \sum_a e_a^2 \int \frac{d|b_T|}{(2\pi)} |b_T| J_0(|b_T| |P_{h\perp}|) \, \tilde{f}_1(x, z^2 b_T^2) \, \tilde{D}_1(z, b_T^2) \]

\[ \int_0^\infty d|P_{h\perp}| \, |P_{h\perp}| J_n(|P_{h\perp}| |b_T|) J_n(|P_{h\perp}| B_T) = \frac{1}{B_T} \delta(|b_T| - B_T) \]

\[ \tilde{f}_1(x, z^2 b_T^2) \tilde{D}_1^{\rightarrow \pi}(z, b_T^2) \]

\[ \tilde{f}(x, b_T^2) = \int d^2 p_T e^{i b_T \cdot p_T} \, f(x, p_T^2) = 2\pi \int d|p_T| |p_T| \, J_0(|b_T| |p_T|) \, f(x, p_T^2) \]

\[ F_{LL} = x_B \sum_a e_a^2 \int \frac{d|b_T|}{(2\pi)} |b_T| J_0(|b_T| |P_{h\perp}|) \, \tilde{g}_{1L}(x, z^2 b_T^2) \, \tilde{D}_1(z, b_T^2) \]

- the formalism in \( b_T \)-space avoids convolutions
- provides a model independent way to study kinematical dependences of TMD
SIDIS with Bessel weighting

$$\tilde{f}(x, b_T^2) \equiv \int d^2 p_T e^{i b_T \cdot p_T} \ f(x, p_T^2) = 2\pi \int d|p_T||p_T| \ J_0(|b_T||p_T|) \ f(x, p_T^2)$$

$$f_1(x, k_T) = \frac{N}{\pi \ < k_T^2 >} e^{-\frac{k_T^2}{< k_T^2 >}}$$

$$\tilde{f}_1(x, b_T^2) = \frac{1}{2} \ < k_T^2 > \ N e^{-\frac{< k_T^2 > b_T^2}{4}}$$

-the data analysis can be performed in the $b_T$-space.
Lattice calculations and b⁻⁻-space

(PDFs in terms of Lorenz invariant amplitudes
Musch et al, arXiv:1011.1213)

\[ f_1^{[1]}(k_{\perp}^2) = \frac{c_2 \sigma_2^2}{4\pi} e^{-\frac{k_{\perp}^2}{(c_2/\sigma_2)^2}} \]
FAST-MC for CLAS12

SIDIS MC in 8D \((x, y, z, \phi, \phi_S, p_T, \lambda, \pi)\)

Simple model with 10\% difference between \(f_1\) (0.2\(\text{GeV}^2\)) and \(g_1\) widths with a fixed width for \(D_1\) (0.14\(\text{GeV}^2\))

\[
f_q(x, k_\perp) = f_q(x) \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-k_\perp^2 / \langle k_\perp^2 \rangle}
\]

CLAS12 acceptance & resolutions

Events in CLAS12

Reasonable agreement of kinematic distributions with realistic LUND simulation

H. Avakian, CERN, March 29
BGMP: extraction of $k_T$-dependent PDFs

Need: project x-section onto Fourier mods in $b_T$-space to avoid convolution

\[
\int_0^\infty d|P_{h\perp}| |P_{h\perp}| J_0(|P_{h\perp}|b_T) \left[ \frac{d\sigma}{dx_B \, dy \, d\phi_S \, dz_h \, d\phi_h |P_{h\perp}|d|P_{h\perp}|} \right]
\]

\[
S^{\text{unp}}_{\pi} (x_i, z_i, b_{Tj}) = \sum_{i=1}^{N_{+}/N_{-}} J_0 (b_{Tj} P_{Ti}) / \eta_i / A(x_i, y_i)
\]

acceptance

\[
A(x, y) = \frac{\alpha^2}{x_B y Q^2} \frac{y^2}{(1 - \varepsilon)} \left( 1 + \frac{\gamma^2}{2 x_B} \right)
\]

\[
\tilde{f}_1^q (x, z^2 b_T^2) \tilde{D}_{1}^{q \rightarrow \pi} (z, b_T^2)
\]

\[
\Delta u(x, b_T) / u(x, b_T) = \frac{S^{\text{pol+}}_\pi - S^{\text{pol-}}_\pi}{S^{\text{unp+}}_\pi + S^{\text{unp-}}_\pi}
\]

• the formalism in $b_T$-space avoids convolutions → easier to perform a model independent analysis
• provides a model independent way to study kinematical dependences of TMD
• requires wide range in hadron $P_T$
BGMP: extraction of $k_T$-dependent PDFs

Need: project x-section onto Fourier mods in $b_T$-space to avoid convolution

$$\int_0^{2\pi} d\phi_h \sin \phi_h \int_0^\infty d|\mathbf{P}_{h\perp}| |\mathbf{P}_{h\perp}| \frac{2J_1(|\mathbf{P}_{h\perp}| |\mathbf{b}_{T}|)}{z M_h |\mathbf{b}_{T}|} \left[ \frac{d\sigma}{dx dy dz d\phi_h |\mathbf{P}_{h\perp}| d|\mathbf{P}_{h\perp}|} \right]$$

$$\sum_a e_{\alpha}^2 \tilde{e}_{\alpha}^2 (x, z^2 b_{T}^2) \hat{H}_1(1)_{\alpha} (z, b_{T}^2) + \ldots$$

- With different Bessel weights BGMP provides a model independent way to extract $k_T$-dependences for all TMDs
- requires wide range in hadron $P_T$
Transversity: single-hadron vs di-hadron

No simple way to extract transversity

\[ F_{UT}^{\sin(\phi_h + \phi_S)} = C \left[ -\frac{\hat{h} \cdot k_T}{M_h} h_1 H_1^\perp \right] \]

\[ C[w f D] = x \sum_a e_a^2 \int d^2 p_T \, d^2 k_T \, \delta^{(2)}(p_T - k_T - P_{h\perp}/z) \, w(p_T, k_T) \, f^a(x, p_T^2) \, D^a(z, k_T^2) \]

Compare single hadron and dihadron SSAs

\[ F_{UT}^{\sin(\phi_R + \phi_S)} = B(y) \sin(\phi_R + \phi_S) \frac{|\vec{R}_T|}{M_h} h_1(x) H_1^\perp(z, \zeta, M_h^2) \]

In dihadron production we deal with the product of functions instead of convolution
Dihadron Fragmentation

\[ z_1, z_2 \text{- fractions of energy carried by a hadrons} \]

- Factorization proven
- Evolution known
- Extracted at BELLE for \( \pi\pi \) pairs, planned for \( \pi K \) pairs

Dihadron productions offers exciting possibility to access transversity distribution

\[ R_T = R \equiv (P_1 - P_2)/2 \]
Dihadron production with transversely polarized target

Significant asymmetries observed at HERMES and COMPASS
Large acceptance of CLAS12 makes dihadron production a perfect tool to extract transversity
Dihadron Fragmentation

• Evolution effects small for DiFF/D₁
• DiFF represent the easiest way to measure the polarization of a fragmenting quark
• DiFF contain information on interferences between different channels (e.g., rho and continuum), which cannot be encoded in MC generators based on the Lund model
CLAS12 will provide precision measurements of single target asymmetries in dihadron pair production in SIDIS

$X_F$ - momentum in the CM frame

100 days of transversely polarized HD (proton)
Dihadron production with CLAS12

\[ \frac{x h_1^u(x)}{x f_1^u(x)} = -\frac{A(y)}{B(y)} \left( \frac{|R|}{M_h} \right)^{-1} \frac{D_1^u(z, M_h)}{H_1^{u, u}(z, M_h)} A_{UT}^{\sin(\phi_R + \phi_S) \sin \theta}(x, y, z, M_h, Q) \]

\[ \frac{17}{18} \frac{x h_1^u(x)}{x f_1^u(x)} = -\frac{A(y)}{B(y)} \left( \frac{|R|}{M_h} \right)^{-1} \frac{D_1^u(z, M_h)}{H_1^{u, u}(z, M_h)} A_{UT}^{\sin(\phi_R + \phi_S) \sin \theta} \]

\[ D_1^u(z, M_h) \approx D_1^d(z, M_h) \]

100 days of transversely polarized HD will allow precision measurement of the transversity distribution.
Summary

- Current JLab data are consistent with a partonic picture:
  - The data consistent with factorization (no x/z-dependence observed in single and double spin asymmetry measurements).
  - Measured spin and azimuthal asymmetries asymmetries ($<\sin\phi>$, $<\sin(\phi+/\phi_S)>$, $<\sin2\phi>$, ...), are in agreement with theory predictions and measurements at higher energies.

- Measurements of azimuthal dependences of double and single spin asymmetries in SIDIS indicate that there are significant correlations between spin and transverse distribution of quarks.

- Sizable higher twist asymmetries measured both in SIDIS and exclusive production indicate the quark-gluon correlations may be significant at moderate $Q^2$.

- $k_T$-dependent flavor decomposition is required to extract the PDFs in multidimensional space in a model independent way.

Measurements of TMDs at JLab & JLab12 in the valence region will provide important input into the global analysis of Transverse Momentum Distributions (involving HERMES, COMPASS, RHIC, BELLE, BABAR)
Support slides....
TMDs from different experiments

Data suggests $Q^2$ evolution of Sivers function may be significant
Studies of the Sivers asymmetry with CLAS12

Higher luminosity with 4.2T magnet will provide comparable to full acceptance coverage up to 6 GeV in $Q^2$.

Replace the central detector with the transverse target.
Model predictions: transverse target

- Models agree on a large target SSA for $\pi\pi$ pair production
- Deuteron target measurements provide complementary information on flavor dependence

\[ A_{UT}^{\sin(\phi_R+\phi_S)\sin\theta} (x, y, z, M_h, Q) = -\frac{B(y)}{A(y)} \frac{|R|}{M_h} x \sum_q e_q^2 h_{1}^q(x) H_{1,s,p}^{<,q}(z, M_h) \]

\[ \times \sum_q e_q^2 f_1^q(x) D_{1,ss+pp}^q(z, M_h) \]
Longitudinal Target SSA measurements at CLAS

\[ A_{UL}(\phi) = \frac{1}{P_t} \frac{N^+ - N^-}{N^+ + N^-} \]

- \( W^2 > 4 \text{ GeV}^2 \)
- \( Q^2 > 1.1 \text{ GeV}^2 \)
- \( y < 0.85 \)
- \( M_X > 1.4 \text{ GeV} \)
- \( P_T < 1 \text{ GeV} \)
- \( 0.12 < x < 0.48 \)
- \( 0.4 < z < 0.7 \)

\(~10\% \text{ of E05-113 data}\)

Data consistent with negative \( \sin^2 \phi \) for \( \pi^+ \)

**CLAS-2009 (E05-113)**

- \( p_1 = 0.059 \pm 0.010 \)
- \( p_2 = -0.041 \pm 0.010 \)

**CLAS-2000**

- \( p_1 = -0.042 \pm 0.015 \)
- \( p_2 = -0.052 \pm 0.016 \)
- \( p_1 = 0.082 \pm 0.018 \)
- \( p_2 = 0.012 \pm 0.019 \)

\( p^+ e^- \pi^+ X \)
TMD Correlation Functions in other experiments

<table>
<thead>
<tr>
<th>N</th>
<th>q/h</th>
<th>U</th>
<th>L</th>
<th>T</th>
</tr>
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<tbody>
<tr>
<td>U</td>
<td>q/h</td>
<td>f₁</td>
<td></td>
<td>h₁ \perp</td>
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<tr>
<td>L</td>
<td>q/h</td>
<td>g₁</td>
<td>h₁L</td>
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<tr>
<td>T</td>
<td>q/h</td>
<td>f₁⊥</td>
<td>g₁⊥</td>
<td>h₁h₁⊥</td>
</tr>
</tbody>
</table>

BOER-MULDERS Spin Orbit effect

ν ≈ h₁𝑞 × h₁\overline{q}

Fragmentation Functions (FF)

<table>
<thead>
<tr>
<th>q/h</th>
<th>U</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>D₁</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>H₁⊥</td>
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</tr>
</tbody>
</table>

Collins Quark spin probe

A₁₂ ≈ H₁ 𝑞 × H₁\overline{q}

In di-hadron case H₁⊥

Interference Fragmentation Function (IFF)

hp → µµX

ee → ππX
Hard Scattering Processes: Kinematics Coverage

- Study of high $x$ domain requires high luminosity, low $x$ higher energies
- Wide range in $Q^2$ is crucial to study the evolution
- Overlap of EIC and JLab12 in the valence region will be crucial for the TMD program

$J\text{Lab@12 GeV (25/50/75)}$

$\Rightarrow 0.1 < x_B < 0.7$: valence quarks

$EIC \quad \sqrt{s} = 140, 50, 15$ GeV

$\Rightarrow 10^{-4} < x_B < 0.3$: gluons and quarks, higher $P_T$ and $Q^2$.
Model predictions: transverse target

\[ A_{UT}^{\sin(\phi_R + \phi_S) \sin \theta}(x, y, z, M_h, Q) = \frac{1}{|S_T|} \frac{\frac{8}{\pi} \int d\phi_R \, d \cos \theta \, \sin(\phi_R + \phi_S) (d\sigma^\uparrow - d\sigma^\downarrow)}{\int d\phi_R \, d \cos \theta (d\sigma^\uparrow + d\sigma^\downarrow)} \]

\[ = \frac{\frac{4}{\pi} \varepsilon \int d \cos \theta \, F_{UT}^{\sin(\phi_R + \phi_S)}}{\int d \cos \theta (F_{UU,T} + \epsilon F_{UU,L})} \].

Leading twist

\[ A_{UT}^{\sin(\phi_R + \phi_S) \sin \theta}(x, y, z, M_h, Q) = \frac{B(y)}{A(y)} \frac{|R|}{M_h} \frac{x}{x} \sum_q e_q^2 h_1^q(x) H_{1,s,p}^{\xi,s}(z, M_h) \]

\[ \sum_q e_q^2 h_1^q(x) D_{1,s,s+p}^q(z, M_h) \]

- Models agree on a large target SSA for ππ pair production
- Deuteron target measurements provide complementary information on flavor dependence
Transverse Momentum Dependent (TMD) Distributions

Transverse Momentum Distributions (TMDs) of partons describe the distribution of quarks and gluons in a nucleon with respect to $x$ and the intrinsic transverse momentum $k_T$ carried by the quarks.
BM TMD (1998) describes correlation between the transverse momentum and transverse spin of quarks, requires FSI or ISI

\[ f_{q/p}(x, k_T^2) = \frac{1}{2} \left[ f_{1}^q(x, k_T^2) - h_{1}^{q}(x, k_T^2) \right] \left( \hat{P} \times k_T \right) \cdot S_q \]

\[ h_{1}^{q}(SIDIS) = -h_{1}^{q}(DY) \]

BM TMD under intensive studies worldwide, including SIDIS and DY experiments, model calculations, lattice simulations.
Studies of the Sivers asymmetry with CLAS12

Higher luminosity with 4.2T magnet will provide comparable to full acceptance coverage up to 6 GeV in $Q^2$.

Replace the central detector with the transverse target.

$A_{UT}^{\sin(\phi-\phi_S)} = \frac{\sum_q e_q^2 f_{1T}^q D_1^q}{\sum_q e_q^2 f_T^q D_1^q}$
SSA at large $x_F$

0 moves to lower $x_F$ with energy?
For a given lumi (30 min of runtime with $10^{35}$) and given bin in hadron $z$ and $P_T$, higher energy provides higher counts and wider coverage in $x$ and $P_T$ to allow studies of correlations between longitudinal and transverse degrees of freedom.
Transverse Momentum Distributions (TMDs) of partons describe the distribution of quarks and gluons in a nucleon with respect to $x$ and the intrinsic transverse momentum $k_T$ (or the Fourier transform $b$) carried by the quarks.

$$F_{f/P}(x, b; \mu; \zeta_F) =$$

"Unsubtracted"

*Implements Subtractions/Cancellations*

*Foundations of Perturbative QCD, J.C. Collins.*

*(available May 2011)*
Evolving TMD PDFs

Up Quark TMD PDF, $x = 0.09$

$F_{\uparrow/p}(x=0.09,k_T) (\text{GeV}^2)$

$B_{T,max} = 0.5 \text{ GeV}^{-1}$

- $Q = \sqrt{2.4} \text{ GeV}$
- $Q = 5.0 \text{ GeV}$
- $Q = 91.19 \text{ GeV}$

$F_{\uparrow/p}(x=0.09,k_T)$

(Schweitzer, Teckentrup, Metz (2010))
SIDIS

(Landry et al, (2003))
Drell-Yan

H. Avakian, CERN, March 29
Dihadron production with CLAS12

Use the clasDIS (LUND based) generator + FASTMC to study $\pi\pi$ pairs

$X_F$ - momentum in the CM frame

Dihadron sample defined by SIDIS cuts+$x_F>0$ (CFR) for both hadrons
Sivers and Boer-Mulders with Lattice QCD

first exploratory lattice studies
... employ(ed) a straight gauge link
[Hägler, BM, et al. EPL (’09) and arXiv:1011.1213]

⇒ No T-odd TMDs
⇒ probably only qualitatively related to TMDs for SIDIS and Drell-Yan

now: staple-shaped links

spacelike, finite length
⇒ look for plateau at large \( \eta \)

limitations: \( \hat{\xi}_{\text{max}} = \frac{|P_{\text{lat}}|}{m_N}, \sqrt{-b^2} \gtrsim 3a \)

Lattice studies for TMDs as in SIDIS or Drell-Yan are possible
- for ratios of Fourier-transformed TMDs
- using space-like Wilson lines
  as in [Aybat, Rogers arXiv:1101.5057 (2011)]
  and J. Collins’ book (to be published)
Semi-Inclusive processes and transverse momentum distributions

\begin{align*}
&f_1^q(x, k_T) \quad g_1^q \quad f_{1T}^q \quad g_{1T}^q \\
&h_1^q \quad \tilde{h}_{1T}^q \quad h_{1L}^q \quad \tilde{h}_{1L}^q
\end{align*}

Hard exclusive processes and spatial distributions of partons

\begin{align*}
&H^q(x, \xi, t) \quad \tilde{H}^q \quad E^q \quad \tilde{E}^q \\
&H_T^q \quad \tilde{H}_T^q \quad E_T^q \quad \tilde{E}_T^q
\end{align*}

Wide kinematic coverage of large acceptance detectors allows studies of exclusive (GPDs) and semi-inclusive (TMDs) processes providing complementary information on transverse structure of nucleon.
3D structure of the nucleon

Semi-Inclusive processes and transverse momentum distributions

Hard exclusive processes and spatial distributions of partons

Wide kinematic coverage of large acceptance detectors allows studies of exclusive (GPDs) and semi-inclusive (TMDs) processes providing complementary information on transverse structure of nucleon

Pasquini & Yuan (QCDSF)

H. Avakian, CERN, March 29
**Bessel Weighted Asymmetries**

(Leonard Gamberg- DIS2011)

- Propose generalize Bessel Weights—"BW"
- BW procedure has advantages
- Introduces a free parameter $B_T [\text{GeV}^{-1}]$ that is Fourier conjugate to $P_{h\perp}$

\[
W_{\text{Sivers}} = \frac{P_{h\perp}}{M} \sin(\phi_h - \phi_S)
\]

\[
\frac{|P_{h\perp}| \sin(\phi_h - \phi_S)}{A_{UT}} = -2 \sum_a \frac{e_a^2}{\alpha_s} \frac{f_1^{(1)}(x)}{D_1^{a}(z)} \frac{D_1^{a}(z)}{z}
\]

\[
w_1 = 2J_1(|P_{h\perp}|B_T) / zMB_T
\]

\[
\frac{2J_1(|P_{h\perp}|B_T)}{zMB_T} \sin(\phi_h - \phi_S) = -2 \sum_a \frac{e_a^2}{\alpha_s} \frac{\tilde{f}_1^{(1)}(a)(x, z^2 B_T^2)}{\tilde{D}_1^{a}(z, B_T^2)} \frac{\tilde{D}_1^{a}(z, B_T^2)}{z}
\]

$\tilde{f}_1$, $\tilde{f}_1^{(1)}$, and $\tilde{D}_1$ are Fourier Transf. of TMDs/FFs

Provide access to $k_T$-dependence of TMDs
CLAS configuration with longitudinally pol. target

Polarizations:
- Beam: ~70%
- NH3 proton ~70%
- Target position -55cm
- Torus +/-2250
- Beam energy ~5.7 GeV

H. Avakian, CERN, March 29
Beam SSA: $A_{LU}^\sin\phi$ from CLAS @ JLab

$$A_{UL}^{\sin\phi} \sim g^\perp D_1(z)$$

Photon Sivers Effect Afanasev & Carlson, Metz & Schlegel

Beam SSA from initial distribution (Boer-Mulders TMD) F.Yuan using $h_1^\perp$ from MIT bag model

No leading twist contributions: provides access to quark-gluon correlations
Quark distributions at large $k_T$: models

\[ \frac{u^+(x, k_T^2)}{u^-(x, k_T^2)} / \Delta u/u \]

JMR model

\[ u^+(x, k_T^2) \propto \frac{(x M + m)^2}{(k_T^2 + \lambda_R^2)^{2\alpha}} \]
\[ u^-(x, k_T^2) \propto \frac{k_T^2}{(k_T^2 + \lambda_R^2)^{2\alpha}} \]

Sign change of $\Delta u/u$ consistent between lattice and diquark model

B. Musch et al arXiv:1011.1213

$\langle x \rangle \approx 0.3$
Flavor-dependent azimuthal modulations in unpolarized SIDIS cross section at HERMES

Marco Contalbrigo
Collins effect

Simple string fragmentation (Artru model)

If unfavored Collins fragmentation dominates measured $\pi^-$ vs $\pi^+$, why $K^-$ vs $K^+$ is different?
Boer-Mulders function extraction by:

2008 COMPASS results!
SIDIS ($\gamma^* p \rightarrow \pi X$) : Transversely polarized target

- Azimuthal moments in pion production in SIDIS
  - $\sin(\phi - \phi_S)$ (Sivers function $f_{1T}^\perp$) and relation with GPDs
  - $\sin(\phi + \phi_S)$ (Collins function $H_{1\perp}$ and transversity $h_1$)
  - $\sin(3\phi - \phi_S)$ (Collins function $H_{1\perp}$ and pretzelosity $h_{1T}^\perp$)

Pasquini and Yuan, Phys.Rev.D81:114013,2010
\[ A_1(x, z, P_T) = A_1(x, z) \frac{\langle P_{T,\text{unp}}^2 \rangle}{\langle P_{T,\text{pol}}^2 \rangle} \exp\left(-P_T^2/\langle P_{T,\text{pol}}^2 \rangle - P_T^2/\langle P_{T,\text{unp}}^2 \rangle\right) \]

\[ 0.4 < z < 0.7 \]

\[ \mu_0^2 = 0.25 \text{GeV}^2 \]
\[ \mu_D^2 = 0.2 \text{GeV}^2 \]

\[ f_1^q(x, k_T) = f_1(x) \frac{1}{\pi \mu_0^2} \exp\left(-\frac{k_T^2}{\mu_0^2}\right) \]
\[ g_1^q(x, k_T) = g_1(x) \frac{1}{\pi \mu_2^2} \exp\left(-\frac{k_T^2}{\mu_2^2}\right) \]

\[ < P_T^2(z) > = z^2 \mu_0^2/2 + \mu_D^2 \]
SIDIS in target fragmentation region

Aram Kotzinian


**SIDIS: TFR**

**Hadronization in SIDIS**

**LO cross-section in TFR**

\[
\frac{d\sigma^{\ell(l \lambda) + N(P_N, S) \rightarrow \ell(l' \lambda) + h(P) + X}}{dx dQ^2 d\phi_S d\zeta d^2P_T} \bigg|_{x_F < 0} = \frac{\alpha^2 x}{yQ^4 (1 + (1 - y)^2)} \sum_q e_q^2 \times
\]

\[
\begin{bmatrix}
M(x, \zeta, P_T^2) - S_Tr \frac{P_T}{m_h} M_T^h(x, \zeta, P_T^2) \sin(\phi_h - \phi_S) + \\
\lambda D_{ll}(y) \left( S_L \Delta M_L(x, \zeta, P_T^2) + S_T \frac{P_T}{m_h} \Delta M_T^h(x, \zeta, P_T^2) \cos(\phi_h - \phi_S) \right)
\end{bmatrix}
\]

The ideal place to test the fracture functions factorization and measure these new functions are JLab12 and EIC facilities with full coverage of phase space.
HT-distributions and dihadron SIDIS

Compare single hadron and dihadron SSAs

\[ \frac{M}{M_h} x e(x) H_1^2 (z, \zeta, M_h^2) + \frac{1}{z} f_1(x) \tilde{G}^\perp(z, \zeta, M_h^2) \]

\[ \frac{M}{M_h} x h_L(x) H_1^2 (z, \zeta, M_h^2) + \frac{1}{z} g_1(x) \tilde{G}^\perp(z, \zeta, M_h^2) \]

Only 2 terms with common unknown HT G~ term!

Aurore Courtoy/Anselm Voosen - Spin session

Higher twists in dihadron SIDIS collinear (no problem with factorization)
Bell can measure \( K^+ \pi^- \) dihadron fragmentation functions

Projections for \( (\pi^+K^-)(K^+\pi^-) \) for \( 580 \text{ fb}^{-1} \)

[Graph showing projections for different variables]

arXiv:1104.2425
Transverse momentum distributions of partons

\[ \langle P_T^2 \rangle \approx z^2 \langle k_{\perp}^2 \rangle + \langle p_{\perp}^2 \rangle \]

Transverse momentum distributions in hadronization may be flavor dependent
=> measurements of different final state hadrons required

![Graph showing the dependence of vector polarization on transverse momentum for proton and other particles.](image)
Collins effect: from asymmetries to distributions

\[ F \equiv \sigma_{UL} \sin 2\phi, \sigma_{UU} \cos 2\phi, \ldots \]

\[ \frac{H_1^{u/K^+} - H_1^{u/K^-}}{H_1^{u/\pi^+} - H_1^{u/\pi^-}} = \frac{15}{4} \frac{F_p^{K^+} - F_p^{K^-}}{3(F_p^{\pi^+} - F_p^{\pi^-}) + (F_d^{\pi^+} - F_d^{\pi^-})} \]

Combined analysis of Collins fragmentation asymmetries from proton and deuteron and for π and K may provide independent to e^+e^- (BELLE/BABAR) information on the underlying Collins function.
Chiral odd HT-distribution

How can we separate the HT contributions?

\[ F^\sin \phi_{LU} \quad F^\sin \phi_{UL} \]

\[ y \quad \phi_h \quad \phi_s = \pi \]

\[ e H_1^{-1} \quad h_L H_1^{-1} \quad \sin \phi_h \]

HT function related to force on the quark. M.Burkardt (2008)

Compare single hadron and dihadron SSAs

\[ \frac{M}{M_h} x e(x) H_1^\zeta (z, \zeta, M_h^2) + \frac{1}{z} f_1(x) \tilde{G}_\zeta (z, \zeta, M_h^2) \]

\[ \frac{M}{M_h} x h_L(x) H_1^\zeta (z, \zeta, M_h^2) + \frac{1}{z} g_1(x) \tilde{G}_\zeta (z, \zeta, M_h^2) \]

Only 2 terms with common unknown HT G~ term!
Nuclear broadening Hadronic PT-distributions

Large PT may have significant nuclear contribution
Azimuthal moments with unpolarized target

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\[ A^{\cos \phi}_{UU} \propto \frac{M_h}{M} f_1 \frac{D\_U}{z} - \frac{M}{M_h} x f_1 D_1 \]

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\[ g_1 \]

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quark polarization
Azimuthal moments with unpolarized target

quark polarization

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$A_{UU}^{\cos \phi} \sim -h_1 \frac{H}{z} + xhH_1^\perp$

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SSA with unpolarized target

### Quark Polarization

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\[ A_{LU}^{\sin \phi} \propto \frac{M_h}{M} f_1 \frac{G^\perp}{z} - \frac{x}{M_h} g^\perp D_1 \]

### Momentum Flow

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#### Cross-section

$$A_{LU}^{\sin \phi} \sim h_1 \frac{E}{z} + xeH_1^\perp$$

## Charged Current

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### SSA with long. polarized target

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#### Magnetic Form Factors

$$A_{UL}^\sin \phi \propto \frac{M_h}{M} g_1 \frac{G_{L}^\perp}{z} + \frac{M}{M_h} x f^\perp L D_1$$

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#### Amplitude

$$A_{UL}^\sin\phi \sim h_{1L}^\perp \frac{H}{z} + x h_L H_{1T}^\perp$$

#### Hadron Decompositions

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SSA with unpolarized target

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### SSA Expression

$$A_{\text{LL}}^{\cos \phi} \sim \frac{M_h}{M} g_{1L} \frac{D^\perp}{z} + x e_L H^\perp_1$$

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SSA with unpolarized target

quark polarization

\[ A_{LL}^{\cos \phi} \sim \frac{M_h}{M} \frac{h_{1L}^\perp}{z} E + xg_L^\perp D_1 \]

H. Avakian, CERN, March 29
Twist-3 PDFs : “new testament”
Quark distributions at large $k_T$

Higher probability to find a hadron at large $P_T$ in nuclei

$k_T$-distributions may be wider in nuclei?
SIDIS ($\gamma^* p \rightarrow \pi X$) x-section at leading twist

$$\frac{d\sigma}{dx dy dz d^2 \vec{P}_h} = \frac{4\pi \alpha^2 s}{Q^4} [x(1 - y + y^2/2)F_{UU} - x(1 - y)\cos(2\phi)F_{UU}^{\cos 2\phi}]$$

- Measure Boer-Mulders distribution functions and probe the polarized fragmentation function
- Measurements from different experiments consistent

TMD PDFs $f_1 D_1$, $h_1^+ H_1^-$

• Measure Boer-Mulders distribution functions and probe the polarized fragmentation function
• Measurements from different experiments consistent
Single hadron production in hard scattering

$x_F < 0$ (target fragmentation)

$x_F > 0$ (current fragmentation)

$X_F$ - momentum in the CM frame

Target fragmentation

Current fragmentation

semi-inclusive

semi-exclusive

exclusive

Fracture Functions

$k_T$-dependent PDFs

Generalized PDFs

Wide kinematic coverage of large acceptance detectors allows studies of hadronization both in the target and current fragmentation regions
Collins effect

Simple string fragmentation for pions (Artru model)

ρ production may produce an opposite sign $A_{UT}$

<table>
<thead>
<tr>
<th>Fraction of ρ in e$\pi X$</th>
<th>% left from e$\pi X$ asm</th>
</tr>
</thead>
<tbody>
<tr>
<td>20%</td>
<td>~75%</td>
</tr>
<tr>
<td>40%</td>
<td>~50%</td>
</tr>
</tbody>
</table>

Fraction of direct kaons may be significantly higher than the fraction of direct pions.

Leading pion out of page

$H_1 u \rightarrow \pi^+$

Leading ρ opposite to leading π (into page)

$H_1 u \rightarrow \rho \sim -\frac{1}{3} H_1 u \rightarrow \pi$

Fraction of ρ in e$\pi X$ asm

20%
40%

~75%
~50%

$\mu^+ \rightarrow e^+ \bar{\nu}_e \nu_\mu$

LUND-MC

H. Avakian, CERN, March 29
Sivers effect in the target fragmentation

High statistics of CLAS12 will allow studies of kinematic dependences of the Sivers effect in target fragmentation region
The $k_\perp$-even TMD quark distribution functions, $f_1(x, k_\perp)$, $g_{1L}(x, k_\perp)$, and $h_1(x, k_\perp)$ be calculated from the associated integrated quark distributions [23]3. For the non-s contributions, they are expressed as [23],

$$f_1(x_B, k_\perp) = \frac{\alpha_s}{2\pi^2} \frac{1}{k_\perp^2} C_F \int \frac{dx}{x} f_1(x) \left[ \frac{1 + \xi^2}{(1 - \xi)_+} + \delta(1 - \xi) \left( \ln \frac{x_B^2 \xi^2}{k_\perp^2} - 1 \right) \right],$$

$$g_{1L}(x_B, k_\perp) = \frac{\alpha_s}{2\pi^2} \frac{1}{k_\perp^2} C_F \int \frac{dx}{x} g_{1L}(x) \left[ \frac{1 + \xi^2}{(1 - \xi)_+} + \delta(1 - \xi) \left( \ln \frac{x_B^2 \xi^2}{k_\perp^2} - 1 \right) \right],$$

$$h_1(x_B, k_\perp) = \frac{\alpha_s}{2\pi^2} \frac{1}{k_\perp^2} C_F \int \frac{dx}{x} f_1(x) \left[ \frac{2\xi}{(1 - \xi)_+} + \delta(1 - \xi) \left( \ln \frac{x_B^2 \xi^2}{k_\perp^2} - 1 \right) \right],$$

where the color factor $C_F = (N_c^2 - 1)/2N_c$ with $N_c = 3$, $\xi = x_B/x$ and $\zeta^2 = (2v\cdot P)^2/v^2$. 

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H. Avakian, CERN, March 29
TMDs: QCD based predictions

**Large-x limit**

<table>
<thead>
<tr>
<th>N</th>
<th>q</th>
<th>U</th>
<th>L</th>
<th>T</th>
</tr>
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<tbody>
<tr>
<td>U</td>
<td></td>
<td>(f_1)</td>
<td></td>
<td>(h_1)</td>
</tr>
<tr>
<td>L</td>
<td></td>
<td>(g_1)</td>
<td>(h_{1L})</td>
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<tr>
<td>T</td>
<td></td>
<td>(f_{1T})</td>
<td>(g_{1T})</td>
<td>(h_{1T})</td>
</tr>
</tbody>
</table>

Brodsky & Yuan (2006)

\[ f_{1T} \sim (1 - x)^4 \quad g_{1T} \sim (1 - x)^4 \quad h_1 \sim (1 - x)^3 \]

Burkardt (2007)

\[ h_{1T} \sim (1 - x)^5 \]

**Large-Nc limit (Pobilitsa)**

\[ f_{1u}^\perp > 0, \ f_{1d}^\perp > 0 \quad h_1^\perp u < 0, \ h_1^\perp d < 0 \]

Do not change sign (isoscalar)

\[ f_{1u}^\perp < 0, \ f_{1d}^\perp > 0 \]

All others change sign

\(u \rightarrow d\) (isovector)
The Multi-Hall SIDIS Program at 12 GeV


for the Jlab SIDIS working group

• Inclusive and semi-inclusive deep inelastic scattering (DIS and SIDIS) are important tools for understanding the structure of nucleons and nuclei.

• Spin asymmetries in polarized SIDIS are directly related to transverse momentum dependent parton distributions (TMDs) and fragmentation functions, and are the subject of intense theoretical and experimental study.

• The TMDs, which depend also on the intrinsic transverse momentum of the parton, \( k_T \), provide a three-dimensional partonic picture of the nucleon in momentum space.

• Measurements with pions and kaons in the final state will provide important information on the hadronization mechanism in general and on the role of spin-orbit correlations in the fragmentation in particular.

H. Avakian, CERN, March 29