Quarkonium polarization
from high to low $p_T$

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COMPASS Seminar
May 18th, 2018

1. Quarkonium production: LHC data vs NRQCD
2. Beyond the polarization puzzle: new insights from recent data
3. What about the $p_T \to 0$ limit?
4. Low-$p_T$ polarization: puzzles and opportunities
5. “Offline” appendix: polarization basics, invariants and the Lam-Tung relation
Almost all the visible matter is made of *hadrons*

The *dynamics* of hadron formation is not well understood.

How do quarks combine into a bound state?

The ideal case study is Quarkonium, bound state of a *heavy quark and its antiquark*.
Studying quarkonia we can see the bound-state formation. 

\[ c \ t_{BS} \sim \text{hadron size} \sim 1 \text{ fm} \]

Heisenberg:\n\[ \Delta t_{q\bar{q}} \sim 1/m_q \]
\[ >> t_{BS} \quad q = u, d, s \]
\[ < t_{BS} \quad q = c, b \]

With light quarks, we cannot set with sufficient precision the starting time of our experiment: the final meson may be formed much before we even realize that the \( q-\bar{q} \) interaction has started!
Theory of quarkonium production

- Nonrelativistic QCD (NRQCD) is the most complete approach.
- In the “factorization” hypothesis, cornerstone of NRQCD, a variety of production mechanisms is in principle foreseen for each quarkonium state.

1) **short-distance** partonic process produces *neutral* or *coloured* Q̅Q pair of any \(2S+1L_J\) quantum numbers:

- \(1S_0, 1S_0, 3S_1, 3P_0, 3P_2, 1D_2, 3P_1, 3S_1, 3D_2, 3D_1, 3P_1\)

2) in the **long-distance** evolution to the observed (neutral) bound state *quantum numbers change* to final

- \(\eta_c, \eta_b [1S_0]\)
- \(\Psi, \Upsilon [3S_1], \chi_{c0}, \chi_{b0} [3P_0]\)
- \(\chi_{c1}, \chi_{b1} [3P_1], \chi_{c2}, \chi_{b2} [3P_2]\)
Theory of quarkonium production

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2) in the **long-distance** evolution to the observed (neutral) bound state quantum numbers change to final.

What is produced in the hard scattering (and determines kinematics and polarization) is a **pre-resonance** $Q\bar{Q}$ state with its own quantum properties.
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1) **short-distance** partonic process produces *neutral* or *coloured* $Q\bar{Q}$ pair of any $^{2S+1}\ell_J$ quantum numbers

2) *in the long-distance* evolution to the observed (neutral) bound state *quantum numbers change* to final

1) **short-distance coefficients** (SDCs): $p_T$-dependent partonic cross sections

$$\sigma(A + B \rightarrow Q + X) = \sum_{S, L, C} S\{A + B \rightarrow (Q\bar{Q})_C^{2S+1}\ell_J + X\} \cdot \mathcal{L}\{(Q\bar{Q})_C^{2S+1}\ell_J \rightarrow Q\}$$

$Q\bar{Q}$ angular momentum and *colour* configuration

2) **long-distance matrix elements** (LDMEs): constant, *fitted from data*
NRQCD hierarchies

Approximations (*heavy-quark limit*) and calculations induce hierarchies and links between pre-resonance contributions:
NRQCD hierarchies

Approximations (*heavy-quark limit*) and calculations induce hierarchies and links between pre-resonance contributions

1) Small quark velocities $v$ in the bound state $\rightarrow$ “$v$-scaling” rules for LDMEs

\[
\begin{align*}
[3S_1] & \quad J/\psi, \Psi(2S) \\
[3P_1] & \quad \Upsilon(1S), \Upsilon(2S), \Upsilon(3S) \\
[3P_2] & \quad \chi_{c1}, \chi_{b1} \\
[1S_0] & \quad \eta_c, \eta_b
\end{align*}
\]
NRQCD hierarchies

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1) Small quark velocities $v$ in the bound state $\rightarrow$ “$v$-scaling” rules for LDMEs

\[ \begin{align*}
^3S_1 & \quad ^1S_0 & \quad ^3S_1 & \quad ^3P_{0|1|2} & \rightarrow & \quad [^3S_1] & \quad J/\psi, \, \psi(2S) \\
^3P_1 & \quad ^3S_1 & \rightarrow & \quad [^3P_1] & \quad \chi_{c1}, \, \chi_{b1} \\
^3P_2 & \quad ^3S_1 & \rightarrow & \quad [^3P_2] & \quad \chi_{c2}, \, \chi_{b2} \\
^1P_1 & \quad ^1S_0 & \quad ^3S_1 & \quad ^1S_0 & \rightarrow & \quad [^1S_0] & \quad \eta_c, \, \eta_b
\end{align*} \]
NRQCD hierarchies

Approximations (heavy-quark limit) and calculations induce hierarchies and links between pre-resonance contributions

1) Small quark velocities $v$ in the bound state $\rightarrow \text{“}v\text{-scaling” rules for LDMEs}

2) Perturbative calculations $\rightarrow$ some SDCs are negligible:

\[ ^3S_1, \; ^1S_0, \; ^3S_1, \; ^3P_0|1|2 \rightarrow [^3S_1] \text{ J/ψ, Ψ(2S)} \]
\[ \text{Υ(1S), Υ(2S), Υ(3S)} \]

\[ ^3P_1, \; ^3S_1 \rightarrow [^3P_1] \text{ χ}_{c1}, \text{ χ}_{b1} \]

\[ ^3P_2, \; ^3S_1 \rightarrow [^3P_2] \text{ χ}_{c2}, \text{ χ}_{b2} \]

\[ ^1P_1, \; ^1S_0, \; ^3S_1, \; ^1S_0 \rightarrow [^1S_0] \text{ η}_{c}, \text{ η}_{b} \]
NRQCD hierarchies

Approximations {**heavy-quark limit**} and calculations induce hierarchies and links between pre-resonance contributions

1) Small quark velocities $v$ in the bound state $\rightarrow$ "$v$-scaling" rules for LDMEs

2) **Perturbative calculations** $\rightarrow$ some SDCs are negligible:

$$
{^1S_0} \quad ^3S_1 \quad ^3P_0|1|2 \quad \rightarrow \quad [^3S_1] \quad J/\psi, \Psi(2S) \\
\gamma(1S), \gamma(2S), \gamma(3S)
$$

$$
^3P_1 \quad ^3S_1 \quad \rightarrow \quad [^3P_1] \quad \chi_{c1}, \chi_{b1}
$$

$$
^3P_2 \quad ^3S_1 \quad \rightarrow \quad [^3P_2] \quad \chi_{c2}, \chi_{b2}
$$

$$
^3S_1 \quad ^1S_0 \quad \rightarrow \quad [^1S_0] \quad \eta_{c}, \eta_{b}
$$
NRQCD hierarchies

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\begin{align*}
{^1S_0} & \quad {^3S_1} & \quad {^3P_0|1|2} & \rightarrow & \quad [^{3S_1}] & \quad J/\psi, \psi(2S) \\
{^3P_1} & \quad {^3S_1} & \rightarrow & \quad [^{3P_1}] & \quad \chi_{c1}, \chi_{b1} \\
{^3P_2} & \quad {^3S_1} & \rightarrow & \quad [^{3P_2}] & \quad \chi_{c2}, \chi_{b2} \\
{^3S_1} & \quad {^1S_0} & \rightarrow & \quad [^{1S_0}] & \quad \eta_c, \eta_b
\end{align*}
\]

3) Heavy-quark spin symmetry $\rightarrow$ relations between LDMEs of different states

\[
\begin{align*}
{^3S_1} & \rightarrow \chi_{c2} = \frac{5}{3}, & \quad {^3S_1} & \rightarrow \eta_c = \frac{1}{5} {^1S_0} \rightarrow J/\psi, & \quad {^3S_1} & \rightarrow \chi_{c1} = \frac{3}{5} \rightarrow \chi_{b2} \\
{^3S_1} & \rightarrow \chi_{b1} \\
{^3S_1} & \rightarrow \eta_b = \frac{1}{5} {^1S_0} \rightarrow \Upsilon, & \quad etc.
\end{align*}
\]
The dominant short-distance cross section contributions

Mixture of different pre-resonance contributions, with rather diversified kinematics and characteristic polarizations

→ by fitting the experimental $p_T$ distributions it is possible to determine the coefficients of all terms (LDMEs) and consequently predict the polarizations

...a delicate procedure!

Curves from
H.-S. Shao et. al., PRL 108, 242004; PRL 112, 182003;
In 1995, the CDF experiment at Fermilab observed $J/\psi$ and $\psi(2S)$ production yields ~50 times larger than predicted by the “colour-singlet model”
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$\rightarrow$ NRQCD: colour-octet terms (with data-fitted normalizations) seemed to solve the problem

$\rightarrow$ $^3S_1$ octet (not $^3S_1$ singlet) would be the main process
...puzzles

**THEORY:**
the fits of $J/\psi$ $p_T$ distributions seemingly indicated a strong dominance of the transversely polarized $^3S_1$-octet term
→ prediction: $\lambda_\theta(HX) \sim +1$ at high $p_T$

CDF measurements contradicted it

$J/\psi$, pp $\sqrt{s} = 1.96$ TeV

**EXPERIMENT:** first polarization measurements, from Tevatron, mutually excluded each other...

$\Upsilon(1S)$, pp $\sqrt{s} = 1.96$ TeV
Improved measurement techniques have been adopted by all LHC experiments.

Towards the experimental clarification of quarkonium polarization

Pietro Faccioli, Carlos Lourenço, João Seixas, Hermine K. Wöhri

The new measurements removed existing ambiguities.
... and the theory puzzle consolidated!

CERN Courier  July/August 2013
The return of quarkonia
Pietro Faccioli, LIP-Lisbon.

The physics of heavy quark–antiquark bound states is a long-standing puzzle, made more intriguing by results from the LHC.
Solution: more rigorous fits of theory to data

New fit method
- treating for the first time correlated observables and theoretical uncertainties consistently
- taking into account limitations of the theoretical ingredients

Theory finally predicts weak polarization

Fit quality improves drastically when removing low $p_T/M$ data
For $p_T/M > 3$ the fit gets stable
A closer look at fits of the past

A representative example of the theory fits which, until recently, lead to the prediction of transverse polarization at high $p_T$

Reminder:
the fit freely adjusts the normalizations (LDMEs) of the colour-octet terms
A closer look at fits of the past

A representative example of the theory fits which, until recently, lead to the prediction of transverse polarization at high $p_T$

And yet, the fit was described as very good, because the *a posteriori* theory uncertainty band (from scale variations) managed to submerge any inconsistency.

Reminder:
the fit freely adjusts the normalizations (LDMEs) of the colour-octet terms.
A closer look at fits of the past

A representative example of the theory fits which, until recently, lead to the prediction of transverse polarization at high $p_T$.

And yet, the fit was described as very good, because the *a posteriori* theory uncertainty band (from scale variations) managed to submerge any inconsistency.

This is clearly a wrong procedure, also because scale variations mainly affect the global normalization, while the fit is sensitive to the subprocess composition *only by virtue of shape* differences.

Reminder: the fit freely adjusts the normalizations (LDMEs) of the colour-octet terms.
A closer look at fits of the past

A representative example of the theory fits which, until recently, lead to the prediction of transverse polarization at high $p_T$

The result of this kind of fit was that the transversely polarized $^3S_1$ and $^3P_J$ octet terms dominated, while the unpolarized $^1S_0$ term was only a “correction.”
A closer look at fits of the past

Let’s look at the high-$p_T$ behaviors, by normalizing the curves to data points for $p_T/M > 3$

The octet $^1S_0$ term has the shape most similar to the data and must therefore be the dominating term → and this is the unpolarized contribution!

With a proper consideration of current theory limitations, the puzzle of the “unexplained” lack of polarization disappears.
The long-neglected culprit

The fact that perturbative calculations for Tevatron and LHC struggle at low $p_T$ has always been “under our eyes”. Most studies now recognize that NLO NRQCD cannot reproduce the curvature shown by data below $p_T \approx M$.

Theory-data comparisons, restricted to high $p_T$, show the “no-turn-down” theory flaw
New insights
Unexpectedly simple data patterns

P. Faccioli, C. Lourenço, M. Araújo, V. Knünz, I. Krätschmer and J. Seixas,
*Quarkonium production at the LHC: A data-driven analysis of remarkably simple experimental patterns*,

Mid-rapidity cross section measurements show a common shape pattern for $p_T/M \lesssim 2$, independent of $M$ and quantum numbers.

\[
\frac{d\sigma}{dp_T} \text{ vs } p_T/M
\]
Unexpectedly simple data patterns

Scaling all data to match the $J/\psi$ normalization

ATLAS

CMS

7 TeV

- $J/\psi$
- $\chi_{c1}$
- $\chi_{c2}$
- $\psi(2S)$
- $Y(1S)$
- $Y(2S)$
- $Y(3S)$

13 TeV

- $J/\psi$
- $\psi(2S)$
- $Y(1S)$
- $Y(2S)$
- $Y(3S)$

PRL 114 (2015) 191802
JHEP 09 (2014) 079
EPJ C 76 (2016) 283
JHEP 07 (2014) 154
PRD 87 (2013) 052004
PLB 749 (2015) 14

PLB 780 (2018) 251
A “surprising” agreement with NRQCD

The variety of kinematic behaviours in NRQCD seems redundant with respect to the observed “universal” $p_T/M$ scaling and lack of polarization.

⇒ cancellations are needed to reproduce data....

...and they actually happen!

P. Faccioli, C. Lourenço, M. Araújo, V. Knünz, I. Krätschmer and J. Seixas,
From identical S- and P-wave $p_T$ spectra to maximally distinct polarizations: probing NRQCD with $\chi_c$ states,
EPJ C 78 (2018) 268
Ultimate conspiracy or need for a better NRQCD?

The seeming success of NRQCD uncovers a strong prediction: the unmeasured $\chi_c^1$ and $\chi_c^2$ polarizations must be very different from one another.

A potentially striking exception to the uniform picture of mid-rapidity quarkonium production!

$\chi_c$ polarization analysis ongoing in the CMS quarkonium group

Will we find... a large $\chi_c^2 - \chi_c^1$ polarization difference? $\Rightarrow$ smoking gun!
... weak $\chi_c^1$ and $\chi_c^2$ polarizations as for S-wave states?
$\Rightarrow$ need of improved (simpler?) NRQCD hierarchies or better perturbative calculations
Long-distance scaling: another universal pattern?

The QQbar→bound-state “transition probabilities” show a clear correlation with binding energy, – common to charmonium and bottomonium, – identical at 7 and 13 TeV:

\[ \frac{\sigma(\psi/\Upsilon)}{\sigma_{Q\bar{Q}}} \propto E_{\text{binding}} \]

\[ \delta = 0.63 \pm 0.02 \]

\[ \delta = 0.63 \pm 0.04 \]

\( d\sigma/dp_T(M) \mid_{M=M(QQbar)} \)

\[ \frac{d\sigma/dp_T(\text{quarkonium})}{d\sigma/dp_T(M=2m_Q)} \]

Plotted ratio: measured cross sections defined by extrapolating to
\( 2m_Q = M_{\eta_c(1S)} \) or \( M_{\eta_b(1S)} \)

→ an experimental confirmation of the “factorization” ansatz of NRQCD

P. Faccioli, C. Lourenço, M. Araújo and J. Seixas,
At the current level of experimental precision, mid-rapidity LHC pp data for charmonium and bottomonium are well described by a simple parametrization reflecting a surprisingly universal (=state-independent) scaling with two variables:

1. shapes of the $p_T$ distributions in pp collisions $\rightarrow p_T/M$
2. pp cross-section scaling with mass $\rightarrow E_{binding}$

This parametrization mirrors well the general idea of factorization of NRQCD. The observed simplicity is in seeming contradiction with the variegated structure of subprocesses composing the NRQCD expansion. Coincidental cancellations ultimately enable NRQCD to succeed in describing the data...

But does this point to the existence of a simpler (more natural) hierarchy of processes?

Next:
- more precise S-wave polarization measurements: how “fine tuned” are the cancellations?
- first P-wave polarization measurements: will they upset the “simple” picture?
Thoughts about quarkonium production at low $p_T$

The fundamental physics concepts of NRQCD and the factorized short×long-distance description of quarkonium production can very well be valid at any $p_T$ and, in particular, for quarkonium production at COMPASS.

However, as the polarization puzzle taught us, **fixed-order calculations** of the short-distance QQbar production cross sections (SDCs) have a limited validity domain. In particular, SDC calculations used for LHC analyses are **unreliable below $p_T/M \approx 3$**, where they do not even reproduce the natural turn-down of $d\sigma/dp_T$ towards zero for $p_T \to 0$. This is actually not surprising: they explicitly target the case where the quarkonium $p_T$ reflects the particle’s recoil in a $2\to2$ process, $i j \to Q+X$.

Moreover, they neglect parton-$k_T$ effects, small at high quarkonium momentum.

For low-$p_T$ studies, the factorized NRQCD description, including colour octet and singlet processes, should be adopted as the most viable and general method currently known. Specific short-distance calculations including $2\to1$ processes with parton-$k_T$ effects should be considered → S. Baranov et al. **EPJ C 75, 455 (2015)**
**PRD 93, 094012 (2016)**

In any case, as the LHC experience taught us, calculations of quarkonium production must be handled with care! Because of the lack of proper theoretical uncertainties, by blindly relying on calculations we risk to incur in new “puzzles”!

→ Always maintain a **data-driven approach**: look for patterns, differences and similarities (different states, different conditions) in advance of any “theory fit”.

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→ Always maintain a **data-driven approach**: look for patterns, differences and similarities (different states, different conditions) in advance of any “theory fit”.
Thoughts about quarkonium production at low $p_T$

From high-$p_T$ data we learned that, for $\psi/\Upsilon$ production,

a) colour-singlet production is negligible

b) lack of polarization → the dominating colour-octet channel is $^1S_0$

c) the production kinematics is (also thanks to b) surprisingly independent of final state

d) the observed mass scaling of cross sections strongly indicates that the bound-state-formation process really factorizes out of short-distance QQbar production

Without detailed *measurements*, none of these very strong physical indications would have been reached!

We do not expect identical observations in low-$p_T$ data.
For example, colour-singlet production may be important in the $2\rightarrow 1$ regime, leading to a less “universal” kinematic scenario.
Detailed *measurements*, not theory, will tell us if this is the case!

Measurement of:
- $p_T$ and $x_F$ distributions
- polarizations
- for all states in the family: not only $J/\psi$, but also $\chi_{c1}$, $\chi_{c2}$ (if not $\eta_c$ and $\chi_{c0}$)

*do not exist* at low $p_T$ and theory cannot replace them or integrate missing parts.
→ an unmissable occasion to build from scratch our knowledge of an unexplored domain!
Thoughts about quarkonium production at low $p_T$

Cross section ratios and the search of mass/momentum scaling patterns provide physical indications with minimal use of theory assumptions.

Polarization remains the cleanest probe of the production mechanism. It is determined by angular momentum conservation and other fundamental symmetries (parity, chirality...). Just as energy conservation, these symmetries are not affected by perturbative “truncations”. Polarization measurements are likely to give strong physical indications without any numerically calculated (and approximated) theory input.

As an example of a possible “workflow” from measurement to understanding, I will consider the existing low-$p_T$ measurements from fixed-target experiments.

They form a rather complex picture...

[see “Appendix” for illustrations of basic polarization concepts]
A low-$p_T$ polarization puzzle?
J/ψ polarization in the CS frame

Collins-Soper

\[ \lambda_\theta \]

\[ \langle x_F \rangle \]

\[ \langle p_T \rangle \text{ (GeV/c)} \]

- E866 38.8 GeV \( p-Cu \)
- E444 20.6 GeV \( \pi^\pm-C/Cu/W \)
- NA3 22.9 GeV \( p-H_2 \)
- NA3 22.9 GeV \( p-Pt \)
- Hera-B 41.6 GeV \( p-C/Ti/W \)
$J/\psi$ polarization in the GJ frame

Gottfried-Jackson

\[ \lambda_{\theta} \]

\[ \langle x_F \rangle \]

\[ \langle p_T \rangle \text{ (GeV/c)} \]

- $p^{-}$-W/$\pi^{-}$-W
- $p^{-}$-Be/$\pi^{-}$-Be
- $p^{-}$-Cu/$\pi^{-}$-Cu
- WA11 16.8 GeV $\pi^{-}$-Be
- E672/706 31.1 GeV $\pi^{-}$-Be
- E771 31.6 GeV $p$-Be
- 38.8 GeV $p$-Be
- E615 38.8 GeV $p$-Si
- HB 41.6 GeV $p^{-}$-Ti/W
- 21.8 GeV $\pi^{\pm}$-W
- 2.5 GeV $p$-C/Ti/W
J/ψ polarization in the HX frame

centre-of-mass helicity

\[ \lambda_\theta \]

\[ \langle x_F \rangle \]

\[ \langle p_T \rangle \text{ (GeV/c)} \]
Polarization in the CS frame

\[ \lambda(1S) \]

\[ \lambda(2S) + \lambda(3S) \]

E866 p-Cu @ 38.8 GeV

\[ \langle x_F \rangle \]

\[ \langle p_T \rangle \text{ (GeV/c)} \]
Indications and motivations from existing data

Picture to be observed “with a grain of salt”:
• most of these measurements were obtained from 1D analyses (with risks discussed in [P. Faccioli, Mod. Phys. Lett. A Vol. 27, 1230022 (2012)])
• for some of them systematic uncertainties were never evaluated
• some of them exhibit suspicious fluctuations, even reaching unphysical values
• we are mixing different energies and target nuclei (nuclear effects may exist)

Provide accurate (multidimensional) measurements!

1) The magnitude of the polarizations is systematically bigger in the CS frame and follows the hierarchy CS → GJ → HX
The CS > GJ > HX hierarchy and the “natural” frame

Hierarchy clearly seen in HERA-B measurement in the three frames (where uncertainties are ~100% correlated).

It reflects the geometrical difference between the three frames: the GJ polarization axis has always an intermediate direction between CS and HX.

The CS axis shows a larger polarization effect ⇒ It more naturally reflects the alignment of the J/ψ angular momentum.

Moreover, the almost maximal \( \Upsilon(2S)+ \Upsilon(3S) \) polarization seen by E866 would appear “smeared” in the HX frame:
Physical meaning of “CS = natural frame”

As illustrated in the discussion of individual processes contributing to Drell-Yan production (→ Appendix), 2 → 2 processes (t-channel or s-channel) naturally lead to polarizations along the GJ and HX axes.

At the lowest order, DY production is a 2 → 1 process, as such characterized by a “natural” polarization along the direction of the collision, approximated by the CS axis

\[ q-\bar{q} \text{ rest frame} = V \text{ rest frame} \]

The produced object\textit{ inherits by direct angular momentum conservation}\nthe polarization state of the system of the colliding partons,\nwhich is polarized\textit{ along the direction of the collision}.\n
In 2 → 2 processes, instead, the polarization legacy of the partons is \textit{shared} between the two final objects. The angular momentum balance is more complex and depends on the coupling of the final states to the intermediate virtual particles...
**Quarkonium polarization in $2 \rightarrow 1 q\bar{q}$ scattering**

Fully analogous to the DY case, replacing the virtual photon with one or more gluons:

$q\bar{q}$ rest frame

$= Q\bar{Q}$ rest frame

helicity conservation

$\Rightarrow |Q\bar{Q}\rangle = |1, +1\rangle$ or $|1, -1\rangle$, $|1, 0\rangle$

$\Rightarrow$ transverse polarization

With only one gluon, a **coloured QQbar pair in $^3S_1$ state** is produced:

This transversely polarized state transforms to $\Psi/\Upsilon$ by emitting two low-energy gluons (to neutralize colour and maintain the $^3S_1$ quantum numbers), a process analogous to the $\Psi(2S) \rightarrow J/\Psi \pi\pi$ decay.

As indicated by BES data on $(e^+e^- \rightarrow) \Psi(2S) \rightarrow J/\Psi \pi^+\pi^- [PRD 62, 032002 (2000)]$, in such a transition the $c\bar{c}$-bar system maintains its polarization

$\Rightarrow$ the $\pi\pi$ system (= the emitted $gg$) has $J=0$: it does not “carry away” angular momentum

$\Rightarrow$ directly produced $\Psi/\Upsilon$ inherits the **fully transverse polarization** of the $^3S_1$ QQbar

- **Different octet states** (e.g. $^1S_0$, $^3P_J$) require two gluons → relatively suppressed
Quarkonium polarization in $2 \rightarrow 1$ $q$-$q\bar{q}$ scattering

Fully analogous to the DY case, replacing the virtual photon with one or more gluons:

$q$-$\bar{q}$ rest frame

= $Q\bar{Q}$ rest frame

⇒ $|Q\bar{Q}\rangle = |1, +1\rangle$ or $|1, -1\rangle$, $|1, 0\rangle$

⇒ transverse polarization

**Colour singlet** $\psi/\Upsilon$ production requires **3 gluons**

for colour neutralization and correct quantum numbers (2 gluons could produce $\chi$ states)

The $\psi/\Upsilon$ is produced as an already observable colour-neutral state, without further gluon emissions, and remains, therefore, transverse

In summary, the $\psi/\Upsilon$ produced directly in $2 \rightarrow 1$ $q$-$q\bar{q}$ scattering, via the colour octet or colour singlet mechanisms, should be transversely polarized
Quarkonium polarization in $2 \rightarrow 1$ $g$-$g$ scattering

Until recently it was believed that $2 \rightarrow 1$ $g$-$g$ quarkonium production is one of the cases forbidden by the Landau-Yang theorem preventing $(J=1) \rightarrow \gamma\gamma$ decays and, therefore, $\gamma\gamma \rightarrow (J=1)$ production (if the initial photons are on shell).

This is true for colour-singlet production: with colours “switched off”, $gg \rightarrow (J=1, \text{singlet})$ is fully analogous to the LY-forbidden $\gamma\gamma \rightarrow (J=1)$

However, in $gg \rightarrow (J=1, \text{octet})$ the additional colour degrees of freedom are not contemplated in the symmetry properties used by the LY theorem.

It has been recently shown that $2 \rightarrow 1$ production is indeed possible in QCD at NLO [W. Beenakker et al., arXiv:1508.07115], in particular through processes like these:

Here the final state comes from a gluon and is therefore a $^3S_1$ octet

These are NLO processes, while $2 \rightarrow 1$ $q$-$\bar{q}$bar production happens at LO

$\Rightarrow 2 \rightarrow 1$ $gg$ production may have some penalty

[Even before this result, the $k_T$-factorization method (S. Baranov et al.) considered off-shell scattering gluons, automatically immune to the LY theorem]
Quarkonium polarization in 2 → 1 g-g scattering

In 2 → 1 g-g scattering quarkonium is produced via the colour-octet mechanism. The transverse nature of the colliding gluons allows only for a subset of the possible angular momentum states of the produced colour-octet state:

\[ |\Omega\Omega\rangle = |2, \pm 2\rangle \text{ or } |J, 0\rangle \ (J=0,1,2) \]

\[ |J, \pm 1\rangle \text{ forbidden} \]

\[ \Rightarrow \text{polarized state if } J > 0 \]

The polarization of the observed quarkonium will depend on which octet state is produced:

\(^1S_0\), dominating high-\(p_T\) 2→2 production at the LHC, is an isotropic state producing exactly \textit{unpolarized} (directly produced) \(J/\psi\)

\(^3S_1\) with \(J_z = 0\) means \textbf{fully longitudinal} (directly produced) \(J/\psi\):

a good starting point for describing HERA-B and NA3 data.
Indications and motivations from existing data

Picture to be observed “with a grain of salt”:
• most of these measurements were obtained from 1D analyses
  (with risks discussed in [P. Faccioli, Mod. Phys. Lett. A Vol. 27 N. 23, 1230022 (2012)])
• for some of them systematic uncertainties were never evaluated
• some of them exhibit suspicious fluctuations, even reaching unphysical values
• we are mixing different energies and target nuclei (nuclear effects may exist)
  provide accurate (multidimensional) measurements!

1) The magnitude of the polarizations is systematically bigger in the CS frame and follows the hierarchy CS → GJ → HX
⇒ probable dominance of $2 \rightarrow 1 q\bar{q}$ and $g\bar{g}$ scattering processes, where the produced state is strongly polarized, directly inheriting the angular momentum state of the system of the colliding partons → we see the partons’ natural polarizations always measure in more than one frame!

2) The $J/\psi$ polarization magnitude, but not the $\Upsilon$ one, seems to decrease quickly with increasing $p_T$:
does this indicate that in HERA-B, but not in E866, we start seeing $2 \rightarrow 2$ processes “smearing” the polarization? Rather, what about the parton $k_T$?
Effect of the intrinsic $k_T$ in $2 \rightarrow 1$ processes

The intrinsic transverse momenta of the partons cause an event-by-event tilt between the “natural” polarization axis (relative direction of the colliding partons), and the polarization axis used in the experimental analysis (CS).

The tilt angle $\delta$ satisfies

$$\sin^2 \delta \approx \frac{2k_T^2}{m_T^2} \approx \frac{p_T^2}{M^2 + p_T^2}$$

and the relation between observed and natural (*) polarizations is

$$\lambda_{\vartheta} = \frac{\lambda^*_{\vartheta} - \frac{3}{2} \lambda^*_{\vartheta} \sin^2 \delta}{1 + \frac{1}{2} \lambda^*_{\vartheta} \sin^2 \delta}$$

This description approximately accounts for the $p_T$ dependence observed for the $c\bar{c}$...

...and for the lack of a corresponding observation in the $b\bar{b}$ case.
The \( \varphi \) dimension

The tilt tends to create an azimuthal anisotropy, but not in a straightforward way as in usual frame transformations (see Appendix).

We can define two extreme cases corresponding to two subcategories of events:

1) events for which \( \delta \) represents a rotation \textit{in the production plane}:

\[
\Rightarrow \lambda_\varphi = \frac{\frac{1}{2} \lambda_\vartheta^* \sin^2 \delta}{1 + \frac{1}{2} \lambda_\vartheta^* \sin^2 \delta} \quad \text{as in a usual transformation between two frames, e.g. CS} \rightarrow \text{HX}
\]

These events \textit{preserve} the invariant \( \tilde{\lambda} \) (while \( \lambda_\vartheta \) is reduced)

2) events for which \( \delta \) represents a rotation \textit{in the plane} \( \perp \) \textit{to the production plane}:

\[
\Rightarrow \lambda_\varphi = 0 \quad \text{Because the azimuthal anisotropy is created \textit{with respect to} the production plane and is therefore unobservable (smeared in the sum over all events)}
\]

As a test of the hypothesis of a natural constant polarization “tilted” by a \( k_T \) effect, we should therefore observe a \( \lambda_\varphi \) \textit{included between the two cases above}, as indeed we see:

A positive or larger negative \( \lambda_\varphi \) would tell us that the observed \( p_T \) dependence is \textit{not} due to \( k_T \) smearing (only)

At the same time, we should observe that the invariant \( \tilde{\lambda} \) is \textit{less} affected by the \( k_T \) smearing than \( \lambda_\vartheta \) and \textit{does not} show an opposite “antismearing” effect
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• we are mixing different energies and target nuclei (nuclear effects may exist)

provide accurate (multidimensional) measurements!

1) The magnitude of the polarizations is systematically bigger in the CS frame and follows the hierarchy \( \text{CS} \rightarrow \text{GJ} \rightarrow \text{HX} \)
\( \Rightarrow \) probable dominance of \( 2 \rightarrow 1 \) \( q \)-qbar and \( g \)-g scattering processes, where the produced state is strongly polarized, directly inheriting the angular momentum state of the system of the colliding partons \( \rightarrow \) we see the partons’ natural polarizations always measure in more than one frame!

2) The decrease in longitudinal \( J/\psi \) polarization with increasing \( p_T \), as observed by HERA-B, is consistent with the tilt effect induced by the parton \( k_T \) on the angular distribution (the effect is negligible for the \( \Upsilon \), as found by E866).

The \( p_T \rightarrow 0 \) limit gives, therefore, the most interesting (untilted) polarization measurement. Moreover, the invariant \( \tilde{\lambda} \) is less sensitive to the tilt effect than \( \lambda_\theta \)

measure down to the smallest possible \( p_T \)!
measure the invariant polarization!
3) We seem to recognize possible trends towards longitudinal polarization for $J/\psi$ data points approaching $x_F = 0$ and $x_F = 1$...

...but $\Upsilon$ data seem to belong to another story...
Map of data in the parton/proton x space

*u* quark chosen as representative of spin-1/2 partons (it has, by far, the largest PDF among quarks/antiquarks)

$$\lambda_\theta$$

α_s uncertainty

\[ x_1 - x_2 = x_F \]
\[ x_1 \cdot x_2 = (M_{QQ}/\sqrt{s})^2 \]
Map of data in the parton/proton $x$ space

$Q^{++2} = 10$ GeV$^{++2}$

- Blue: gluon cteq66a (as=0.125)
- Red: gluon cteq66a (as=0.112)
- Pink: gluon cteq66 (central value)
- Black: up cteq66 (central value)

**Ratio of gluon/u-quark PDFs**

$x_F \rightarrow 0 \left(\text{HERA-B and NA3}\right):$

$x_1 = x_2 = \mathcal{O}(0.1)$

$g/u > 1 \Rightarrow gg/q\bar{q} > 1$

**At small $x_F$ gluon-gluon production could be the largest contribution**

$x_1 - x_2 = x_F$

$x_1 \cdot x_2 = (M_{QQ}/\sqrt{s})^2$
Map of data in the parton/proton $x$ space

For E615 (pion-nucleus) we assume similar qualitative behaviour between pion and proton PDFs

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$g_2/u_2 >> 1$  $g_1/u_1 ~ 1$

$\Rightarrow gg/qq >> 1$

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$\Rightarrow gg/qq >> 1$
Map of data in the parton/proton x space

\[ x_F \sim 0.3 \]

(NA3, E866 and E615):
small \( x_2 \), but intermediate \( x_1 \approx x_F \)

\[ \frac{g_2}{u_2} > 1 \quad \frac{g_1}{u_1} < 1 \]

as small as it can be

\[ \Rightarrow \frac{gg}{q\bar{q}} \sim O(1) \]

For E615 (pion-nucleus) we assume similar qualitative behaviour between pion and proton PDFs

\[ x_1 - x_2 = x_F \]

\[ x_1 \cdot x_2 = (M_{QQ}/\sqrt{s})^2 \]
Map of data in the parton/proton x space

What about the “very different” \( \Upsilon(2+3S) \) result?

\[ E866, \text{ intermediate } x_F \]

\[ g_1/u_1 \sim 1 \quad g_2/u_2 \ll 1 \]

\[ \Rightarrow gg/qq \ll 1 \]

For bottomonium, quark-antiquark production wins over gluon-gluon fusion at intermediate \( x_F \)

\[ x_1 - x_2 = x_F \]

\[ x_1 \cdot x_2 = \left( \frac{M_{QQ}}{\sqrt{s}} \right)^2 \]
Map of data in the parton/proton $x$ space

What about the “very different” $\Upsilon(2+3S)$ result?

E866, lowest-$x_F$ point

$g/u < 1$ for both partons

$\Rightarrow gg/qq << 1$

for bottomonium, quark-antiquark production dominates even at small $x_F$

$x_1 - x_2 = x_F$

$x_1 \cdot x_2 = (M_{QQ}/\sqrt{s})^2$
By measuring quarkonium polarization in the low-$p_T$ $2 \rightarrow 1$ domain (and comparing with corresponding theoretical predictions for the $gg$ and $qqbar$ cases) we can probe the identity of the colliding partons!
Indications and motivations from existing data

3) A consistency can be recognized in the perplexing scenario of $J/\psi$ and $\Upsilon$ polarizations vs $x_F = x_1 - x_2$ when we correlate the observed **longitudinal** polarizations with the dominance of $gg$ processes and **transverse** polarizations with the dominance of $qqbar$ processes. This correlation is in agreement with the corresponding expectations for polarizations in $2 \rightarrow 1$ processes, hinting at $gg$ production via $^3S_1$-octet (the longitudinal case).

Polarization measurements are a unique probe of elementary partonic processes! Measure as much “differentially” as possible vs $|x_F|$!
3) A consistency can be recognized in the perplexing scenario of $J/\psi$ and $\Upsilon$ polarizations vs $x_F = x_1 - x_2$ when we correlate observed longitudinal polarizations with the dominance of $gg$ processes and transverse polarizations with the dominance of $qq\bar{q}$ processes. This correlation is in agreement with the corresponding expectations for polarizations in $2 \rightarrow 1$ processes, hinting at $gg$ production via $^3S_1$-octet (the longitudinal case) polarization measurements are a unique probe of elementary partonic processes! measure as much “differentially” as possible vs $|x_F|!$

4) In all previous considerations we neglected a crucial detail: the feed-down from $\chi$ states, ~20% for the $J/\psi$ (HERA-B*), probably larger for the $\Upsilon(1S)$. In fact, the $\Upsilon(2S)+\Upsilon(3S)$ measurement by E866 stands apart as the strongest transverse polarization, while $\Upsilon(1S)$ is almost unpolarized. Feed-down from $\chi$ states must be the culprit...

*PRD 79, 012001
The role of feed-down decays

As long as heavier and lighter S-states have the same production mechanism, feed-down from heavier S-states is “invisible” from the polarization point of view, as shown by the mentioned BES $\psi(2S) \rightarrow J/\psi \, \pi^+\pi^-$ result.

Feed-down from $\chi$ is more complex:

a) P-wave states can have different production mechanisms with respect to S waves
b) they decay to $J/\psi$ and $\Upsilon$ with the emission of a transversely polarized photon, which alters the spin-alignment of the QQbar [Faccioli et al., Phys. Rev. D 83, 096001 (2011)]

As a result, we can expect different polarizations for $\psi/\Upsilon$ from $\chi$ decays with respect to the directly produced ones
χ polarization in $2\rightarrow 1$ $qq\bar{q}$ production

Colour-octet:

The coloured $^3S_1$ QQbar has $J_z = \pm 1$. It transforms to $\chi_j$ by emitting one low-energy gluon (to neutralize colour and produce a $^3P_j$ state). The $\chi_j$ then decays radiatively to $\psi/\Upsilon$.

The resulting $\psi/\Upsilon$ has:

$$\lambda_\theta = +1/5 \quad (\chi_1)$$

$$\lambda_\theta = +21/73 \quad (\chi_2)$$

(directly produced $\psi/\Upsilon$ has $\lambda_\theta = +1$)

Colour-singlet:

Two gluons are sufficient to produce a colourless $^3P_j$ already with the $\chi_j$ quantum numbers. The $\chi_j$ has $J_z = \pm 1$ and decays radiatively to $\psi/\Upsilon$.

The resulting $\psi/\Upsilon$ has:

$$\lambda_\theta = -1/3 \quad (\chi_1)$$

$$\lambda_\theta = -1/3 \quad (\chi_2)$$

(directly produced $\psi/\Upsilon$ has $\lambda_\theta = +1$)

⇒ expect a strong reduction of the transverse $\psi/\Upsilon$ polarization from direct to prompt if $\chi$ feed-down is large
\( \chi \) polarization in 2→1 \( gg \) production

\( ^3S_1 \) is the favoured octet for \( \chi_j \) production (only requires one emitted gluon to give \( ^3P_j \))

The QQbar has \( J_z = 0 \) (transverse colliding gluons)

The \( \chi_j \) then decays radiatively to \( \psi/\Upsilon \)

The resulting \( \psi/\Upsilon \) has:

\[
\lambda_\theta = -\frac{1}{3} \quad (\chi_1) \\
\lambda_\theta = -\frac{21}{47} \quad (\chi_2)
\]

directly produced \( \psi/\Upsilon \) has

\[
\lambda_\theta = -1 \quad (\text{from } ^3S_1 \text{ octet}) \\
\lambda_\theta = 0 \quad (\text{from } ^1S_0 \text{ octet})
\]

\( \chi_2 \) (like \( \eta \) and \( \chi_0 \)) can be produced directly as colourless QQbar, with \( J_z = 0 \) or \( J_z = \pm 2 \).

This decays radiatively to \( \psi/\Upsilon \)

The resulting \( \psi/\Upsilon \) has

\[
\lambda_\theta = -\frac{3}{5} \quad \text{or} \quad +1 \quad (\chi_2)
\]

(no \( \chi_1 \) or direct \( \psi/\Upsilon \) in this channel)

(\( \chi_1 \) allowed from off-shell gluons \( \Rightarrow \lambda_\theta = +1 \)?)

⇒ longitudinal prompt-\( \psi/\Upsilon \) polarization scenario favoured with minimum smearing from \( \chi \) feed-down
1S, 2S and 3S states should have the same polarization when directly produced → \( \lambda_\theta \approx +1 \)
(or when coming from heavier \( \Upsilon \))

To justify the large difference between 2-3S and 1S, we must assume that \( \chi \) feed-down:

a) is negligible for 2-3S states and large for 1S
b) tends to be longitudinal

No measurements of how many \( \Upsilon \) come from \( \chi_b \) exist for low-\( p_T \) quarkonium production.
\( \chi \) production may be large with respect to \( \psi \) production, since, for example, it is easier to produce a singlet \( \chi \) (2 intermediate gluons) than a singlet \( \psi/\Upsilon \) (3 gluons).
**χ feed-down and the E866 “puzzle”**

1S, 2S and 3S states should have the same polarization when **directly produced** $\rightarrow \lambda_\theta \approx +1$
(or when when coming from heavier $\gamma$)

To justify the large difference between 2-3S and 1S, we must assume that **χ feed-down:**
a) is **negligible for 2-3S states** and **large for 1S**
b) tends to be **longitudinal**

No measurements of how many $\gamma$ come from $\chi_b$ exist for low-$p_T$ $qqbar$ production.
$\chi$ production may be large with respect to $\psi$ production, since, for example,
it is easier to produce a singlet $\chi$ (2 intermediate gluons) than a singlet $\psi/\gamma$ (3 gluons).

**If...**

- 50–60% of the $\gamma(1S)$ come from $\chi_b$
- $\chi$ states are produced in the colour-singlet channel $\rightarrow \lambda_\theta = -1/3$

$\Rightarrow$ the observed $\gamma(1S)$ would have $\lambda_\theta$ in the range $1/13 - 1/4 = 0.08 - 0.25$

**Note:** sum rule for polarizations of different samples:
Indications and motivations from existing data

3) A consistency can be recognized in the perplexing scenario of J/ψ and Υ polarizations vs $x_F = x_1 - x_2$ when we correlate the observed 
longitudinal polarizations with the dominance of $gg$ processes and 
transverse polarizations with the dominance of $qqbar$ processes. 
This correlation is in agreement with the corresponding expectations for polarizations 
in $2 \rightarrow 1$ processes, hinting at $gg$ production via $^3S_1$-octet (the longitudinal case) 

- polarization measurements are a unique probe of elementary partonic processes! 
- measure as much “differentially” as possible vs $|x_F|$!

4) E866 data clearly show how sensitive the observed polarizations of J/ψ and Υ can be to the feed-down from χ states. 
So much that strong hypotheses on the (unknown) feed-down fractions and polarizations 
of $\chi_b$ states are required to describe the striking difference between $\Upsilon(2S)+\Upsilon(3S)$ and 
$\Upsilon(1S)$ observations. 
$\chi$ production is not a second-order correction for J/ψ and Υ yields and polarizations 
and it is equally interesting for the understanding of the fundamental processes 

- measure production properties of $\chi$ states 
  (feed-down fractions, polarizations)!
“Offline” appendix: polarization basics, invariants and the Lam-Tung relation
Measure **polarization** of a particle = measure the (average) **angular momentum composition** in which the particle is produced, by studying the **angular distribution** of its **decay** in its rest frame.
Example: polarization of vector particles

\( J = 1 \rightarrow \) three \( J_z \) eigenstates \(|1, +1\rangle, |1, 0\rangle, |1, -1\rangle\) wrt a certain \( z \)

The decay into a fermion-antifermion pair is an especially clean case to be studied.

The shape of the observable angular distribution is determined by a few basic principles:

1) **elementary coupling properties**
   E.g.: “helicity conservation”

2) **rotational covariance**
   of angular momentum eigenstates

3) **parity properties**
1: elementary coupling properties

Relevant property for cases considered here:
EW and strong forces preserve the *chirality* (L/R) of fermions.
In the relativistic (massless) limit, *chirality* = *helicity* = spin-momentum alignment
→ the fermion spin never flips in the coupling to gauge bosons:

\[ \gamma^*, Z, g, ... \]
example: dilepton decay of $J/\psi$

$J/\psi \rightarrow \ell^+ \ell^-$

$J/\psi$ rest frame:
example: dilepton decay of $J/\psi$

$\gamma^*$

$\ell^+$

$\ell^-$

$J/\psi$ angular momentum component along the polarization axis $z$:

$$M_{J/\psi} = -1, 0, +1$$ (determined by production mechanism)
example: dilepton decay of $J/\psi$

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$J/\psi$ angular momentum component along the polarization axis $z$:

$$M_{J/\psi} = -1, 0, +1$$  (determined by production mechanism)

The two leptons can only have total angular momentum component

$$M'_{\ell^+\ell^-} = +1 \text{ or } -1$$

$0$ is forbidden
example: dilepton decay of Z (or DY)

$Z$ angular momentum component along the polarization axis $z$:

$$M_Z = -1, 0, +1$$

(determined by production mechanism)

The two leptons can only have total angular momentum component

$$M'_{\ell^+\ell^-} = +1 \text{ or } -1$$

along their common direction $z'$

0 is forbidden
2: rotation of angular momentum eigenstates

change of quantization frame:

\[ R(\vartheta, \varphi): \quad z \rightarrow z' \]
\[ y \rightarrow y' \]
\[ x \rightarrow x' \]

\[ |J, M'\rangle = \sum_{M=-J}^{+J} D_{MM'}^{J}(\vartheta, \varphi) |J, M\rangle \]

Wigner D-matrices
2: rotation of angular momentum eigenstates

change of quantization frame:

\[ R(\vartheta, \varphi): \ z \rightarrow z' \]
\[ y \rightarrow y' \]
\[ x \rightarrow x' \]

\[ | J, M' \rangle = \sum_{M = -J}^{+J} D_{M M'}^{J}(\vartheta, \varphi) \ | J, M \rangle \]

Wigner D-matrices

Example:

Classically, we would expect \[ | 1, 0 \rangle \]
2: rotation of angular momentum eigenstates

Change of quantization frame:

\[ R(\vartheta, \varphi): \ z \rightarrow z' \]
\[ y \rightarrow y' \]
\[ x \rightarrow x' \]

Wigner D-matrices

\[ | J, M' \rangle = \sum_{M = -J}^{+J} D_{M M'}^{+J}(\vartheta, \varphi) \ | J, M \rangle \]

Example:

\[ \frac{1}{2} | 1, +1 \rangle + \frac{1}{2} | 1, -1 \rangle - \frac{1}{\sqrt{2}} | 1, 0 \rangle \]
example: $M = 0$

$V (M_V = 0) \rightarrow \ell^+ \ell^- (M'_{\ell^+ \ell^-} = +1)$

$\downarrow$

$V = J/\psi | Z$

$| 1, +1 \rangle = D_{-1,+1}^1(\vartheta, \phi) | 1, -1 \rangle + D_{0,+1}^1(\vartheta, \phi) | 1, 0 \rangle + D_{+1,+1}^1(\vartheta, \phi) | 1, +1 \rangle$
example: $M = 0$

$$V (M_V = 0) \rightarrow \ell^+\ell^- (M'_\ell + \ell = +1)$$

$$V = J/\psi \mid Z$$

$V$ rest frame

$|1, 0\rangle$ → the $J_z$, eigenstate $|1, +1\rangle$ "contains" the $J_z$ eigenstate $|1, 0\rangle$

with component amplitude $D_{0, +1}^1(\vartheta, \varphi)$

→ the decay distribution is

$$|\langle 1, +1 \mid \mathcal{O} \mid 1, 0 \rangle|^2 \propto |D_{0, +1}^{1*}(\vartheta, \varphi)|^2 = \frac{1}{2} (1 - \cos^2 \vartheta)$$

$\ell^+\ell^- \leftarrow J/\psi$
$3: \text{parity}$

$$\ell^+$$

$$\ell^-$$

$$\theta$$

$$\langle 1, -1 \rangle$$

$$\propto |D_{-1, +1}^1(\theta, \phi)|^2 \propto 1 + \cos^2 \theta - 2 \cos \theta$$
3: parity

| 1, −1 \rangle and | 1, +1 \rangle distributions are mirror reflections of one another

\[ \frac{dN}{d\Omega} \propto |D_{-1,+1}^1(\vartheta, \varphi)|^2 \propto 1 + \cos^2 \vartheta - 2\cos \vartheta \]

\[ \frac{dN}{d\Omega} \propto |D_{+1,+1}^1(\vartheta, \varphi)|^2 \propto 1 + \cos^2 \vartheta + 2\cos \vartheta \]

Are they equally probable?
3: parity

| 1, −1 ⟩ and | 1, +1 ⟩ distributions are mirror reflections of one another

$$\frac{dN}{d\Omega} \propto |D^*_{-1,+1}(\theta, \phi)|^2 \propto 1 + \cos^2 \theta - 2\cos \theta$$

$$\frac{dN}{d\Omega} \propto |D^*_{1,+1}(\theta, \phi)|^2 \propto 1 + \cos^2 \theta + 2\cos \theta$$

Are they equally probable?

$$\mathcal{P}(-1) > \mathcal{P}(+1)$$

$$\mathcal{P}(-1) = \mathcal{P}(+1)$$

$$\mathcal{P}(-1) < \mathcal{P}(+1)$$

$$\frac{dN}{d\Omega} \propto 1 + \cos^2 \theta + 2[\mathcal{P}(+1) - \mathcal{P}(-1)] \cos \theta$$
3: parity

$|1, -1\rangle$ and $|1, +1\rangle$ distributions are mirror reflections of one another.

Decay distribution of $|1, 0\rangle$ state is always parity-symmetric:
General two-body decay distribution

reference plane
(= production plane,
or plane of daugher and
mother momenta for CX)

chosen polarization axis

particle
rest frame

\[
\frac{dN}{d\Omega} \propto 1 + \lambda_\theta \cos^2 \theta + \lambda_\varphi \sin^2 \theta \cos 2\varphi + \lambda_{\theta\varphi} \sin 2\theta \cos \varphi \\
+ 2A_\theta \cos \theta + 2A_\varphi \sin \theta \cos \varphi + \ldots
\]
General two-body decay distribution

reference plane
(= production plane,
or plane of daughter and
mother momenta for CX)

chosen polarization axis

particle rest frame

\[ \frac{dN}{d\Omega} \propto 1 + \lambda_\vartheta \cos^2 \vartheta + \lambda_\varphi \sin^2 \vartheta \cos 2\varphi + \lambda_\vartheta \varphi \sin 2\vartheta \cos \varphi + 2A_\vartheta \cos \vartheta + 2A_\varphi \sin \vartheta \cos \varphi \]

parity violating
General two-body decay distribution

reference plane
(= production plane,
or plane of daughter and
mother momenta for CX)

chosen polarization axis

particle rest frame

average polar anisotropy
average azimuthal anisotropy

correlation
polar - azimuthal

\[
\frac{dN}{d\Omega} \propto 1 + \lambda_\vartheta \cos^2 \vartheta + \lambda_\varphi \sin^2 \vartheta \cos 2\varphi + \lambda_{\vartheta\varphi} \sin 2\vartheta \cos \varphi + 2A_\vartheta \cos \vartheta + 2A_\varphi \sin \vartheta \cos \phi + \ldots
\]

parity violating
General two-body decay distribution

reference plane
(= production plane,
or plane of daughter and
mother momenta for CX)

chosen polarization axis

pp c.o.m. helicity (HX): particle direction wrt pp c.o.m.
Gottfried-Jackson (GJ): direction of one or the other beam
Collins-Soper (CS): average of the two beam directions
perpendicular helicity (PX): perpendicular to CS
cascade helicity (CX): (in cascade decays) decaying particle’s
momentum direction in rest frame of its mother
etc.

average polar anisotropy
average azimuthal anisotropy
correlation polar - azimuthal

\[
\frac{dN}{d\Omega} \propto 1 + \lambda_\theta \cos^2 \theta + \lambda_\varphi \sin^2 \theta \cos 2\varphi + \lambda_{\theta\varphi} \sin 2\theta \cos \varphi
\]

\[
+ 2A_\theta \cos \theta + 2A_\varphi \sin \theta \cos \phi
\]

parity violating

\[
\lambda_\theta, \lambda_\varphi, \lambda_{\theta\varphi}, \text{ etc. depend on the chosen frame} \ [\text{Faccioli et al., Eur. Phys. J C 69, 657 (2010)}],
\]

while \[
F = \frac{1 + \lambda_\theta + 2\lambda_\varphi}{3 + \lambda_\theta}
\]
(as well as its functions, e.g. \(\tilde{\lambda}\)) does not

Alternative notation

reference plane
(= production plane, or plane of daugher and mother momenta for CX)

chosen polarization axis

pp c.o.m. helicity (HX): particle direction wrt pp c.o.m.
Gottfried-Jackson (GJ): direction of one or the other beam
Collins-Soper (CS): average of the two beam directions
perpendicular helicity (PX): perpendicular to CS
cascade helicity (CX): (in cascade decays) decaying particle’s momentum direction in rest frame of its mother
e tc.

particle rest frame

\[
\frac{dN}{d\Omega} \propto \frac{1}{2} + A_0 \cos^2 \theta + \frac{A_2}{2} \sin^2 \theta \cos 2\phi + A_1 \sin 2\theta \cos \phi
\]

\[
+ A_4 \cos \theta + A_3 \sin \theta \cos \phi + ... 
\]

\[
A_0, A_1, A_2, \text{ etc. depend on the chosen frame,}
\]

\[
\mathcal{F} = \frac{1}{2} \left( 1 + \frac{A_2 - A_0}{2} \right)
\] (as well as its functions, e.g. \(A_2-A_0\)) does not
Relation to production mechanism

\[ |V \rangle = b_{+1} |1, +1 \rangle + b_0 |1, 0 \rangle + b_{-1} |1, -1 \rangle \]

V production mechanism

polarization measurement:
\[ dN/d\Omega(\cos\vartheta, \varphi) \rightarrow b_{+1}, b_0, b_{-1} \]

These amplitudes are frame-dependent!

\[ \frac{dN}{d\Omega} \propto 1 + \frac{A_0}{2} + \left(1 - \frac{3}{2}A_0\right) \cos^2 \vartheta + \frac{A_2}{2} \sin^2 \vartheta \cos 2\varphi + A_1 \sin 2\vartheta \cos \varphi + \ldots \]

for decay to $\ell^+\ell^-$

\[ \begin{align*}
A_0 &= 2|b_0|^2 \\
A_1 &= \sqrt{2} \text{Re}[b_0^*(b_{+1} - b_{-1})] \\
A_2 &= 4 \text{Re}(b_{+1}^* b_{-1})
\end{align*} \]

these relations depend on the decay channel
“Transverse” and “longitudinal”

“Transverse” polarization, like for real photons. The word refers to the alignment of the field vector, not to the spin alignment!

\[ | J/\psi \rangle = | 1, +1 \rangle \]

or \[ | 1, -1 \rangle \]

\[ \frac{dN}{d\Omega} \propto 1 + \cos^2 \theta \]

(parity-conserving case)

“Longitudinal” polarization

\[ | J/\psi \rangle = | 1, 0 \rangle \]

\[ \frac{dN}{d\Omega} \propto 1 - \cos^2 \theta \]
Why “photon-like” polarizations are common

We can apply helicity conservation at the production vertex to predict that all vector states produced in fermion-antifermion annihilations (q-\bar{q} or e^+e^-) at Born level have transverse polarization.

\[ V = \gamma^*, Z, W \]

\[ |V\rangle = |1, +1\rangle \]

The “natural” polarization axis in this case is the relative direction of the colliding fermions (Collins-Soper axis).

Drell-Yan is a paradigmatic case
But not the only one
The observed polarization depends on the frame

For $|p_L| \ll p_T$, the CS and HX frames differ by a rotation of $90^\circ$

- **Collins-Soper**:
  - longitudinal
  - $\frac{dN}{d\Omega} \propto 1 - \cos^2 \theta$
  - $|\psi\rangle = |0\rangle$
    - (pure state)

- **Helicity**:
  - "transverse"
  - $\frac{dN}{d\Omega} \propto 1 + \cos^2 \theta - \sin^2 \theta \cos 2\varphi$
  - $|\psi\rangle = \frac{1}{\sqrt{2}} |+\rangle - \frac{1}{\sqrt{2}} |-\rangle$
    - (mixed state)
The observed polarization depends on the frame.

For $|p_L| < p_T$, the CS and HX frames differ by a rotation of 90°.

$\psi = \pm \frac{1}{\sqrt{2}} (|+1\rangle + |1\rangle)$

(pure state)

$\frac{dN}{d\Omega} \propto 1 + \cos^2 \theta$

transverse

$\frac{dN}{d\Omega} \propto 1 - \frac{1}{3} \cos^2 \theta + \frac{1}{3} \sin^2 \theta \cos 2\varphi$

moderately “longitudinal”

$\psi = \frac{1}{2} |+1\rangle + \frac{1}{2} |-1\rangle m \frac{1}{\sqrt{2}} |0\rangle$

(mixed state)
All reference frames are equal... but some are more equal than others

What do different detectors measure with *arbitrary* frame choices?

Gedankenscenario:
- **dileptons are fully transversely polarized in the CS frame**
- the decay distribution is measured at the $\Upsilon(1S)$ mass by 6 detectors with different *dilepton acceptances*:

| Detector         | $|y| < $ |
|------------------|--------|
| CDF              | 0.6    |
| D0               | 1.8    |
| ATLAS & CMS      | 2.5    |
| ALICE $e^+e^-$   | 0.9    |
| ALICE $\mu^+\mu^-$ | $2.5 < y < 4$ |
| LHCb             | $2 < y < 4.5$ |
The lucky frame choice

(CS in this case)

\[
\frac{dN}{d\Omega} \propto 1 + \cos^2\vartheta
\]
Less lucky choice

(HX in this case)

\[ \lambda_\theta = +0.65 \]

\[ \lambda_\theta = -0.10 \]

ALICE \( \mu^+\mu^- \) / LHCb

ATLAS / CMS

D0

ALICE e^+e^-

CDF

artificial (experiment-dependent!)

kinematic behaviour

→ measure in more than one frame!
The azimuthal anisotropy is not a detail

Quarkonium measurements used to ignore the azimuthal component of the distribution. This is a mutilation of the measurement!

Case 1: natural transverse polarization

Case 2: natural longitudinal polarization, observation frame \( \perp \) to the natural one

• Two very different (opposite) physical cases, with same \( \lambda_\theta \)
• distinguishable only by measuring \( \lambda_\varphi \) (no integration over \( \varphi \) !)
A complementary approach: frame-independent polarization

The *shape* of the distribution is (obviously) frame-invariant (= invariant by rotation) → it can be characterized by a frame-independent parameter, writeable e.g. as

\[ \tilde{\lambda} = \frac{\lambda_\theta + 3\lambda_\phi}{1 - \lambda_\phi} \quad \text{or} \quad \mathcal{F} = \frac{1 + \lambda_\theta + 2\lambda_\phi}{3 + \lambda_\theta} \quad \left( \mathcal{F} = \frac{1 + \tilde{\lambda}}{3 + \tilde{\lambda}} \right) \]

rotations *in the production plane!*
A complementary approach: frame-independent polarization

The *shape* of the distribution is (obviously) frame-invariant (= invariant by rotation) → it can be characterized by a frame-independent parameter, writeable e.g. as

\[ \tilde{\lambda} = \frac{\lambda_\theta + 3\lambda_\varphi}{1 - \lambda_\varphi} \]

or

\[ F = \frac{1 + \lambda_\theta + 2\lambda_\varphi}{3 + \lambda_\theta} \]

\[ \left( F = \frac{1 + \tilde{\lambda}}{3 + \tilde{\lambda}} \right) \]

N.B: \( \tilde{\lambda} \) is convenient because it is “homogeneous” to \( \lambda_\theta \), but \(-1 < \tilde{\lambda} < +\infty\) (no upper bound!), while \( F \) is more conveniently normalized in the range \( 0 < F < 1 \)

rotations in the production plane!
Frames for Drell-Yan, Z and W polarizations

- polarization is always fully **transverse**...
  \[ V = \gamma^*, Z, W \]

  Due to helicity conservation at the \( q-\bar{q}-V \) (\( q-q^*-V \)) vertex, \( J_z = \pm 1 \) along the \( q-\bar{q} \) (\( q-q^* \)) scattering direction \( z \)

- ...but with respect to a **subprocess-dependent quantization axis**

  \( z = \text{relative dir. of incoming } q \text{ and } q\bar{\text{bar}} \)

  \( \sim \text{Collins-Soper frame} \)

  important only up to \( p_T = \mathcal{O}(\text{parton } k_T) \)

  \( z = \text{dir. of one incoming quark} \)

  \( \sim \text{Gottfried-Jackson frame} \)

  \( z = \text{dir. of outgoing } q \)

  \( (= \text{parton-cms-helicity} \approx \text{lab-cms-helicity}) \)
“Optimal” frames for Drell-Yan, Z and W polarizations

Different subprocesses have different “natural” quantization axes

For \textbf{s-channel processes} the \textit{natural axis} is
the direction of the outgoing quark
(= direction of dilepton momentum)

→ optimal frame (= maximizing polar anisotropy): $\text{HX}$

(neglecting parton-parton-cms vs proton-proton-cms difference!)

- $\text{HX}$
- $\text{CS}$
- $\text{PX}$
- $\text{GJ1}$ (negative beam)
- $\text{GJ2}$ (positive beam)

Graph showing $\lambda^\parallel$ vs $p_T$ [GeV/c] for different processes.
“Optimal” frames for Drell-Yan, Z and W polarizations

Different subprocesses have different “natural” quantization axes

For \( t \)- and \( u \)-channel processes the natural axis is the direction of either one or the other incoming parton (~ “Gottfried-Jackson” axes)

\[ \Rightarrow \text{optimal frame: geometrical average of GJ1 and GJ2 axes} = CS (p_T < M) \text{ and } PX (p_T > M) \]

\[ \lambda \]

\[ \begin{align*}
H_X & \quad \text{example: } Z \\
CS & \quad y = +0.5 \\
PX & \\
GJ1 & = \text{GJ2}
\end{align*} \]
Rotation-invariant Drell-Yan, $Z$ and $W$ polarizations

- polarization is always fully **transverse**...

Due to helicity conservation at the $q$-$\bar{q}$-$V$ ($q$-$q^*$-$V$) vertex, $J_z = \pm 1$ along the $q$-$\bar{q}$ ($q$-$q^*$) scattering direction $z$

- ...but with respect to a **subprocess-dependent quantization axis**

**O($\alpha_s^0$)**

\[ q \rightarrow V \rightarrow \bar{q} \]

"natural" $z$ = relative dir. of $q$ and $q\bar{q}$

$\rightarrow \lambda_9(\text{"CS"}) = +1$

wrt any axis: $\tilde{\lambda} = +1$

**O($\alpha_s^1$)**

(LO) QCD corrections

\[ q \rightarrow V \rightarrow q^* \quad \bar{q} \rightarrow V \rightarrow q^* \quad q^* \rightarrow V \rightarrow g \quad g \rightarrow V \rightarrow q \]

$z$ = dir. of one incoming quark

$\rightarrow \lambda_9(\text{"GJ"}) = +1$

$\tilde{\lambda} = +1$

\[ \tilde{\lambda} = +1 \quad \text{any frame} \]

$z = \text{dir. of outgoing } q$

$\rightarrow \lambda_9(\text{"HX"}) = +1$

$\tilde{\lambda} = +1$

N.B.: $\tilde{\lambda} = +1$ in both pp-HX and qg-HX frames!

In all these cases the $q$-$q$-$V$ lines are in the production plane ("**planar**" processes)

The CS, GJ, pp-HX and qg-HX axes only differ by a rotation in the production plane
Example: $Z/\gamma^*/W$ polarization (CS frame) as a function of contribution of LO QCD corrections:

Case 1: dominating $q\bar{q}$ QCD corrections

$\lambda_{\gamma^*}$ vs $\tilde{\lambda}$

- $\lambda = +1$
- $f_{QCD}$ (indep. of $y$)
- $M = 150 \text{ GeV}/c^2$
- $p_T = 50 \text{ GeV}/c$
- $p_T = 200 \text{ GeV}/c$ (indep. of $y$)
Example: $Z/\gamma^*/W$ polarization (CS frame) as a function of contribution of LO QCD corrections:

Case 1: dominating $q\bar{q}$ QCD corrections

- $\lambda = +1$ 
- $f_{QCD}$ (indep. of $y$)
- $M = 150 \text{ GeV}/c^2$
- $p_T = 50 \text{ GeV}/c$
- $p_T = 200 \text{ GeV}/c$

Case 2: dominating $q-g$ QCD corrections

- $\lambda = +1$ 
- $y = 2$
- $y = 0$
- $M = 150 \text{ GeV}/c^2$
- $p_T = 50 \text{ GeV}/c$
- $p_T = 200 \text{ GeV}/c$
Example: $Z/\gamma^*/W$ polarization (CS frame) as a function of contribution of LO QCD corrections:

Case 1: dominating $q\bar{q}$ QCD corrections

Case 2: dominating $q-g$ QCD corrections

Mass dependent!
Example: $Z/\gamma^*W$ polarization (CS frame) as a function of contribution of LO QCD corrections:

Case 1: dominating $q\bar{q}$ QCD corrections

Case 2: dominating $qg$ QCD corrections

W by CDF&D0
Example: $Z/\gamma^*/W$ polarization (CS frame) as a function of contribution of LO QCD corrections:

### Case 1: dominating $q\bar{q}$ QCD corrections

- $\lambda_\gamma = +1$
- $f_{QCD}$
- $M = 80 \text{ GeV}/c^2$
- $p_T = 50 \text{ GeV}/c$
- $p_T = 200 \text{ GeV}/c$
- $f_{QCD}$ (indep. of $y$)

### Case 2: dominating $qg$ QCD corrections

- $\lambda_\gamma = +1$
- $f_{QCD}$
- $M = 80 \text{ GeV}/c^2$
- $p_T = 50 \text{ GeV}/c$
- $p_T = 200 \text{ GeV}/c$
- $y = 0, 2$
- $f_{QCD}$ (mass dependent!)

W by CDF&D0

"unpolarized"?
No, $\tilde{\lambda} = +1$!
Example: $Z/\gamma^*/W$ polarization (CS frame) as a function of contribution of LO QCD corrections:

Case 1: dominating $q\bar{q}$ QCD corrections

- $\lambda = +1$
- $\tilde{\lambda} = +1$
- $f_{QCD}$ (indep. of $y$)
- $M = 80 \text{ GeV}/c^2$
- $p_T = 50 \text{ GeV}/c$
- $p_T = 200 \text{ GeV}/c$

Case 2: dominating $q-g$ QCD corrections

- $\lambda = +1$
- $\tilde{\lambda} = +1$
- $f_{QCD}$ (indep. of $y$)
- $M = 80 \text{ GeV}/c^2$
- $p_T = 50 \text{ GeV}/c$
- $y = 2$
- $y = 0$
- $p_T = 200 \text{ GeV}/c$
- $y = 2$
- $y = 0$

- $W$ by CDF&D0
- $\lambda$ depends on $p_T$, $y$ and mass
- by integrating we lose significance
- is far from being maximal
- depends on process admixture
- need pQCD and PDFs

$\tilde{\lambda}$ is constant, maximal and independent of process admixture
Example: $Z/\gamma^*/W$ polarization (CS frame) as a function of contribution of LO QCD corrections:

Case 1: dominating $q\bar{q}$ QCD corrections

Case 2: dominating $qg$ QCD corrections

On the other hand, $\tilde{\lambda}$ forgets about the direction of the quantization axis.
This information is crucial if we want to disentangle the $qg$ contribution, the only one resulting in a rapidity-dependent $\lambda_\theta$

Measuring $\lambda_\theta$ (CS) as a function of rapidity gives information on the gluon content of the proton
The Lam-Tung relation

PHYSICAL REVIEW D  VOLUME 21, NUMBER 9  1 MAY 1980

Parton-model relation without quantum-chromodynamic modifications in lepton pair production

C. S. Lam
Wu-Ki Tung

Parton-model relations provide much more to the quark-parton model than just integrated Drell-Yan cross-section formula. Lepton angular distributions are controlled by structure functions which obey parton-model relations similar to those between $F_1$ and $F_2$ in deep-inelastic scattering (DIS). How are these relations affected by perturbative QCD corrections? The answer to this question is quite surprising: At least one of these relations—the exact counterpart of the Callan-Gross relations—is not modified at all by first-order QCD corrections, although individual terms in this relation may be subject to large corrections. In the rest of this note, we spell out explicitly the parton model as the contrast between this relation and the cross-section formula [essentially $W_L$, Eq. (2)]. This appears to be an especially remarkable result; we are not aware of any other parton-model result which is not affected by QCD corrections. For this reason, we sketch in the appendix a derivation of Eq. (5) from the diagram which is more direct.

Relation (for LPP) takes the form $W_L = 2W_{AA}$, Eq. (7). Although for LPP, the helicity structure functions depend on the choice of coordinate axes (e.g., Gottfried-Jackson, Collins-Soper, etc.), this relation remains frame independent—i.e., if the QCD-quark-parton model is correct, the two structure functions $W_L$ and $W_{AA}$ must be related by Eq. (7), for any choice of axes in the lepton-pair center-of-mass frame. This strong result again demonstrates the significance of this relation.

We know the angular distribution of the lepton...
The Lam-Tung relation

PHYSICAL REVIEW D 76, 074006 (2007)

Transverse momentum dependence of the angular distribution of the Drell-Yan process

Edmond L. Berger,¹,* Jian-Wei Qiu,¹,2,† and Ricardo A. Rodriguez-Pedraza²,‡

We calculate the transverse momentum $Q_\perp$ dependence of the helicity structure functions for the hadroproduction of a massive pair of leptons with pair invariant mass $Q$. These structure functions determine the angular distribution of the leptons in the pair rest frame. Unphysical behavior in the region $Q_\perp \to 0$ is seen in the results of calculations done at fixed order in QCD perturbation theory. We use current conservation to demonstrate that the unphysical inverse-power and $\ln(Q/Q_\perp)$ logarithmic divergences in three of the four independent helicity structure functions share the same origin as the divergent terms in fixed-order calculations of the angular-integrated cross section. We show that the resummation of these divergences to all orders in the strong coupling strength $\alpha_s$ can be reduced to the solved problem of the resummation of the divergences in the angular-integrated cross section, resulting in well-behaved predictions in the small $Q_\perp$ region. Among other results, we show the resummed part of the helicity structure functions preserves the Lam-Tung relation between the longitudinal and double spin-flip structure functions as a function of $Q_\perp$ to all orders in $\alpha_s$. 
The Lam-Tung relation

A fundamental result of the theory of vector-boson polarizations (Drell-Yan, directly produced $Z$ and $W$) is that, at leading order in perturbative QCD,

$$\lambda_g + 4\lambda_\varphi = 1$$

independently of the polarization frame

*Lam-Tung relation*

This identity was considered as a surprising result

Today we know that it is only a *special* case of general frame-independent polarization relations, corresponding to a *transverse* intrinsic polarization:

$$\tilde{\lambda} = \frac{\lambda_g + 3\lambda_\varphi}{1 - \lambda_\varphi} = +1 \quad \Rightarrow \quad \lambda_g + 4\lambda_\varphi = 1$$

It is, therefore, *not properly a “QCD” relation*, but a consequence of

1) rotational invariance
2) properties of the quark-photon/$Z$/W couplings (helicity conservation)
Beyond the Lam-Tung relation

Even when the Lam-Tung relation is violated, \( \tilde{\lambda} \) can always be defined and is always frame-independent

\[ \rightarrow \text{any violation, } \tilde{\lambda} - 1 \neq 0, \text{ is quantitatively frame-independent} \]

N.B.:
the quantity \( 4\lambda_\varphi - (1 - \lambda_\theta) \), or \( 2
\nu - (1 - \lambda) \) in an alternative notation, often adopted to estimate violations of the LT relation, is not frame independent!

\[ A_2 - A_0 \text{ is frame independent:} \]

\[ A_2 - A_0 = 4F - 2 = 2 \frac{\tilde{\lambda} - 1}{3 + \tilde{\lambda}} \]
Beyond the Lam-Tung relation

Even when the Lam-Tung relation is violated, 
\( \tilde{\lambda} \) can always be defined and is *always frame-independent*

\[ \rightarrow \] any violation, \( \tilde{\lambda} - 1 \neq 0 \), is quantitatively frame-independent

\[ \tilde{\lambda} = +1 \quad (\mathcal{F} = 1/2) \rightarrow \text{Lam-Tung. New interpretation: only vector boson} \rightarrow \text{quark} \rightarrow \text{quark couplings (in planar processes)} \rightarrow \text{automatically verified in DY at QED \\ & \text{& LO QCD levels and in several higher-order QCD contributions}} \]

\[ \tilde{\lambda} = +1 - \mathcal{O}(0.1) \rightarrow \text{same, “ordinary” vector-boson} \rightarrow \text{quark} \rightarrow \text{quark couplings, but in non-planar processes (higher-order contributions)} \]

\[ \begin{array}{c}
\mathcal{F} = 1/2 - \mathcal{O}(0.1) \\
\text{OR} \\
\text{smearing due to intrinsic parton } k_T
\end{array} \]

\[ \begin{array}{c}
\tilde{\lambda} < 1 \ll +1 \\
(0 < \mathcal{F} << 1/2)
\end{array} \rightarrow \text{contribution of different/new couplings or processes} \\
\text{(e.g.: } Z \text{ from Higgs, } W \text{ from top, triple } ZZ\gamma \text{ coupling,} \\
\text{higher-twist effects in DY production, etc...)}
\]

\[ \begin{array}{c}
\tilde{\lambda} > +1 \\
(1/2 < \mathcal{F} < 1)
\end{array} \rightarrow \text{experimental mistake} \]
Example 1

\[ +1 < \tilde{\lambda} < +\infty \]
\[ (1/2 < \mathcal{F} < 1) \] → higher-twist effects in DY production

very large effect, progressively approaching the physical limit \( \mathcal{F} = 1 \)

\([\mathcal{F} = 1 \) represents a very peculiar case: fully longitudinal polarization along the axis perpendicular to the production plane]→ not a mere “higher-order correction”!

See discussion in
Example 2

CMS measurement of $Z \rightarrow \ell^+\ell^-$ [PLB 750, 154 (2015)], using parametrization of slide 11

$$A_2 = \frac{8\lambda_\varphi}{3 + \lambda_\varphi} \quad A_0 = 2 \frac{1 - \lambda_\varphi}{3 + \lambda_\varphi}$$

$$A_0 - A_2 = 2 - 4F$$

$A_0 - A_2 > 0 \quad \rightarrow \quad F < 1/2$

This is the case

$F = 1/2 - O(0.1)$

with $F \rightarrow 1/2$ for $p_T \rightarrow 0$

Parton-$k_T$ effects are negligible for Z production: partons in the Z rest frame have $k_L > 50$ GeV [see quantitative description in EPJ C 69, 657 (2010)]

The formulation of the LT-violation in terms of $F$, with its known physical limits, allows us to quantify the magnitude of the effect:

$A_0 - A_2 \approx 0.1$ corresponds to $F - 1/2 \approx 0.025$

relatively small!

$\rightarrow$ we are “seeing” the contribution of higher-order, non-planar processes
Further reading


