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Spin structure function g_1 at small Q^2 and the recent COMPASS data

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talk based on results obtained in collaboration with

M. Greco and **S.I. Troyan**

Outlook of theoretical results on g_1

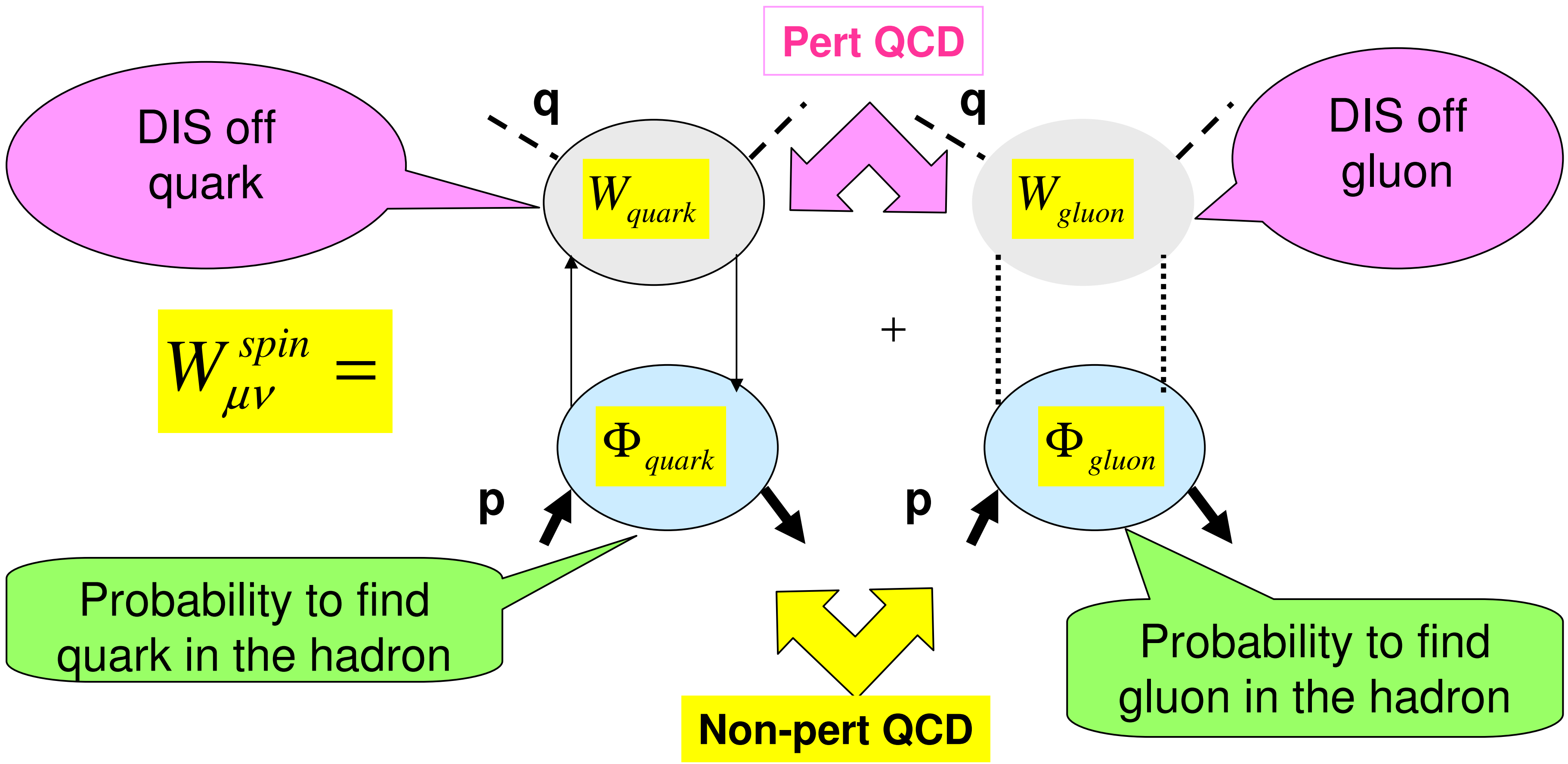
The spin-dependent part of the hadronic tensor is parameterized by two structure functions:

$$W_{\mu\nu}^{spin} = \frac{m}{pq} i\varepsilon_{\mu\nu\lambda\rho} q_\lambda \left[S_\rho g_1(x,Q^2) + \left(S_\rho - \frac{Sq}{pq} p_\rho \right) g_2(x,Q^2) \right]$$

Structure functions

where **m, p** and **S** are the hadron mass, momentum and spin;
q is the virtual photon momentum ($Q^2 = -q^2 > 0$). Both of the functions depend on Q^2 and $x = Q^2/2pq$, $0 < x < 1$.

When 2pq is large compared to the mass scale, one can use the **factorization:**



DIS off quark and gluon can be studied with perturbative QCD.

Probabilities to find a quark/gluon involve non-perturbaive QCD. There is no a regular analytic way to calculate them. Instead, they are substituted by initial quark and gluon densities defined from experimental data at x~1 and $Q^2 \sim 1$ GeV^2

So, the conventional form of the hadronic tensor is:

$$W_{\mu\nu} = W_{\mu\nu}^{quark} \otimes \delta q + W_{\mu\nu}^{gluon} \otimes \delta g$$

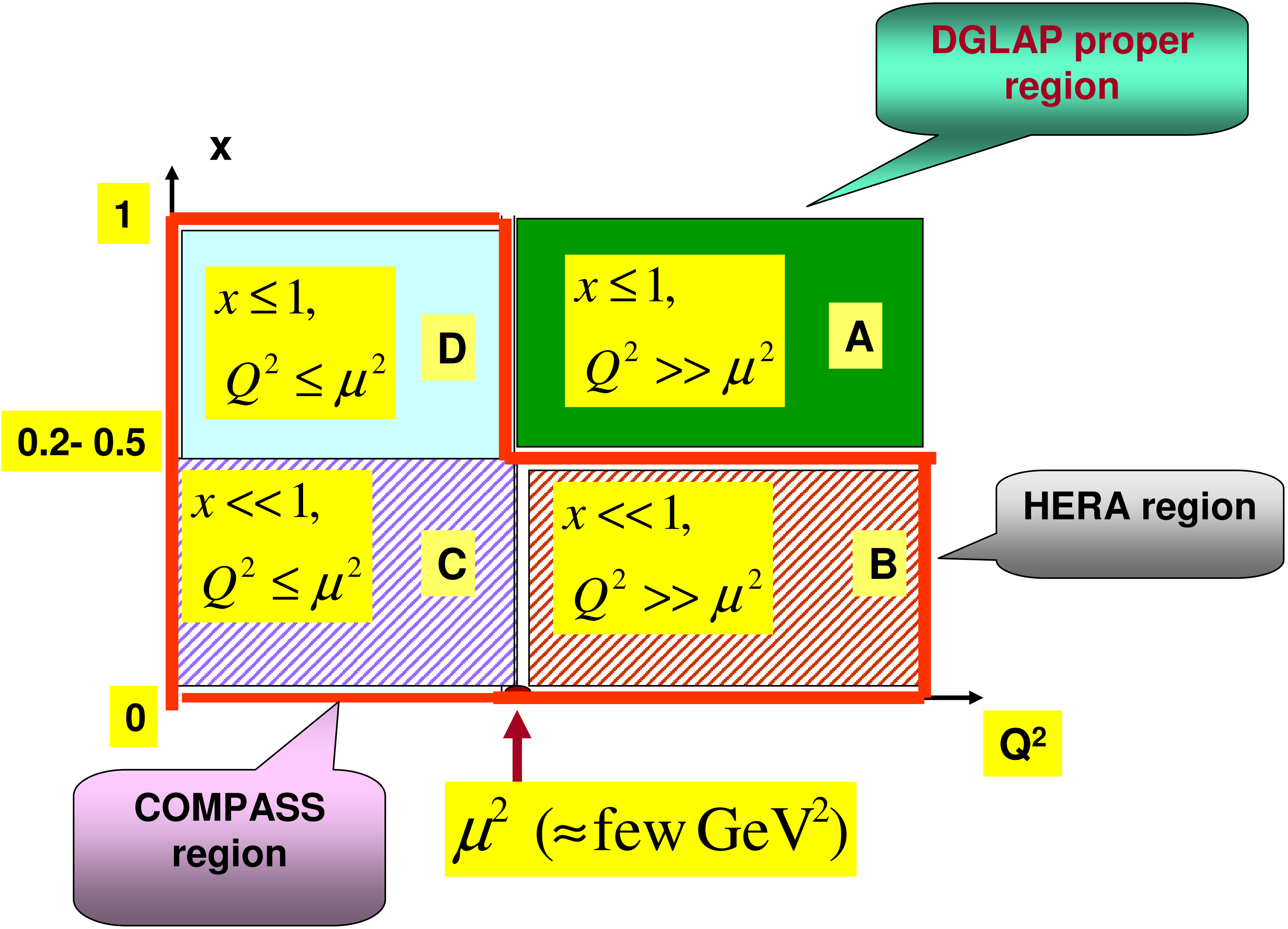
Initial quark distribution

Initial gluon distribution

DIS off the quark

DIS off the gluon

Kinematical regions to cover:



The Standard instrument for theoretical investigation of DIS is DGLAP

Altarelli-Parisi, Gribov-Lipatov, Dokshitzer

In particular,

$$g_1^{NS}(x,Q^2) = (e_q^2/2)C_{NS}(x,y) \otimes \Delta q(y,Q^2)$$

Coefficient function

Evolved quark distribution

where

$$\frac{d\Delta q}{d\ln Q^2} = \frac{\alpha_s}{2\pi} P_{qq} \otimes \Delta q$$

Splitting function

Expression for the singlet g_1 is similar, though more involved. It includes two coefficient functions, four anomalous dimensions and, in addition to the quark distribution, the gluon distribution

DGLAP evolution equations for the singlets

$$\frac{d\Delta q}{d\ln Q^2} = \frac{\alpha_s}{2\pi} P_{qq} \otimes \Delta q \;+ \frac{\alpha_s}{2\pi} P_{qg} \otimes \Delta g$$

$$\frac{d\Delta g}{d\ln Q^2} = \frac{\alpha_s}{2\pi} P_{gq} \otimes \Delta q \;+ \frac{\alpha_s}{2\pi} P_{gg} \otimes \Delta g$$

$P_{qq},\ P_{qg},\ P_{gq}, P_{gg}$ splitting functions

$\Delta q, \Delta g$ evolved quark and gluon distributions

Mellin transform of the splitting functions = anomalous dimensions

Applying the Mellin transform, obtain a simpler expression for g_1^{NS} :

Initial quark density

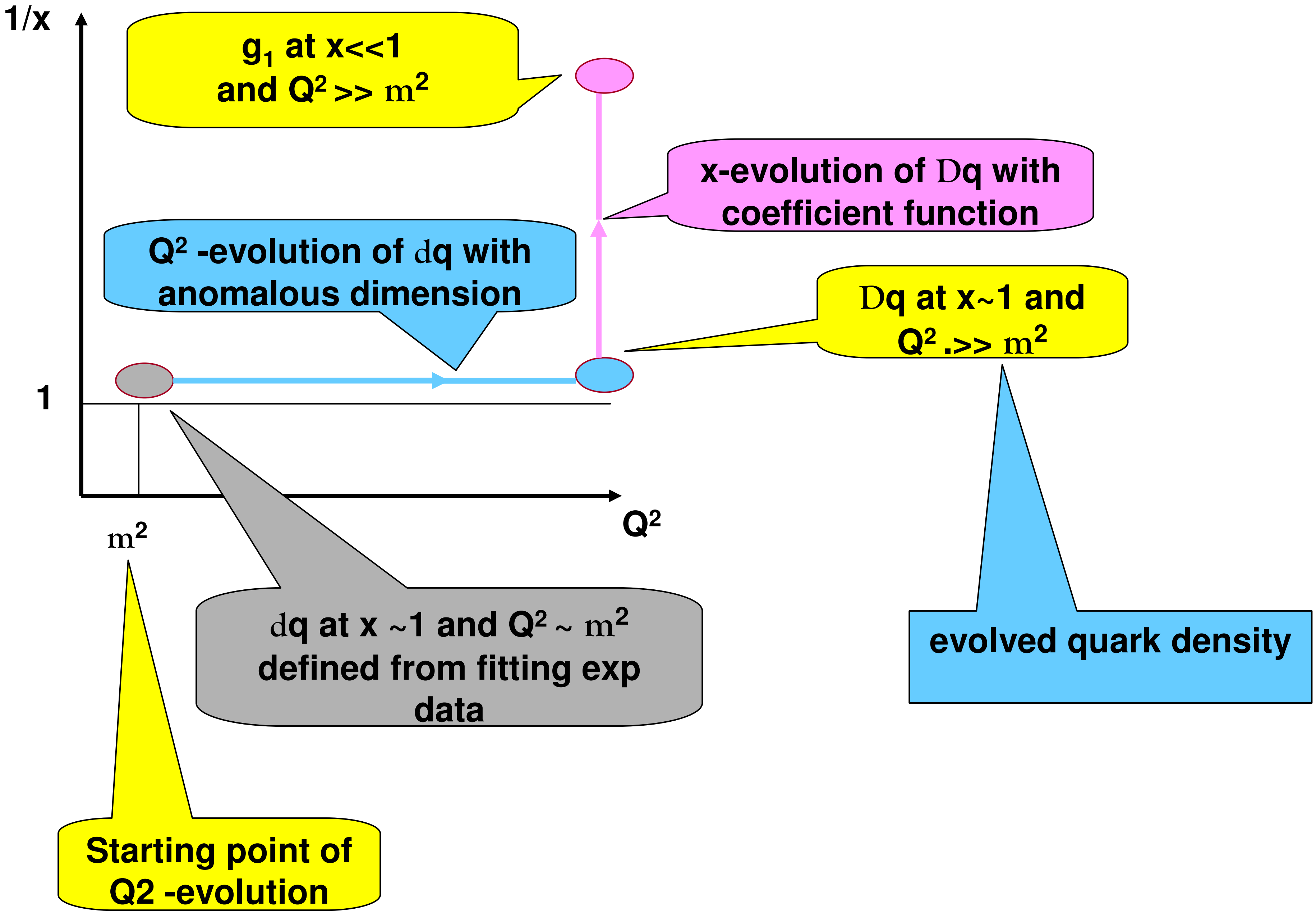
$$g_1^{NS}(x,Q^2) = (e_q^2/2)\int_{-i\infty}^{i\infty}\frac{d\omega}{2\pi i}\left(\frac{1}{x}\right)^{\omega} C(\omega)\,\delta q(\omega)\exp\left[\int_{\mu^2}^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\gamma(\omega,\alpha_s(k_\perp^2))\right]$$

Coefficient function

Anomalous dimension

Non-Pert QCD

Pert QCD


1/x
g_1 at x<<1
and Q^2 >> m^2
x-evolution of Δq with
coefficient function
Q^2 -evolution of δq with
anomalous dimension
Δq at x~1 and
Q^2 .>> m^2
1
Q^2
m^2
δq at x ~1 and Q^2 ~ m^2
defined from fitting exp
data
evolved quark density
Starting point of
Q2 -evolution

In DGLAP, coefficient functions and anomalous dimensions are known with LO and NLO accuracy, often at integer $w = n$

LO

NLO

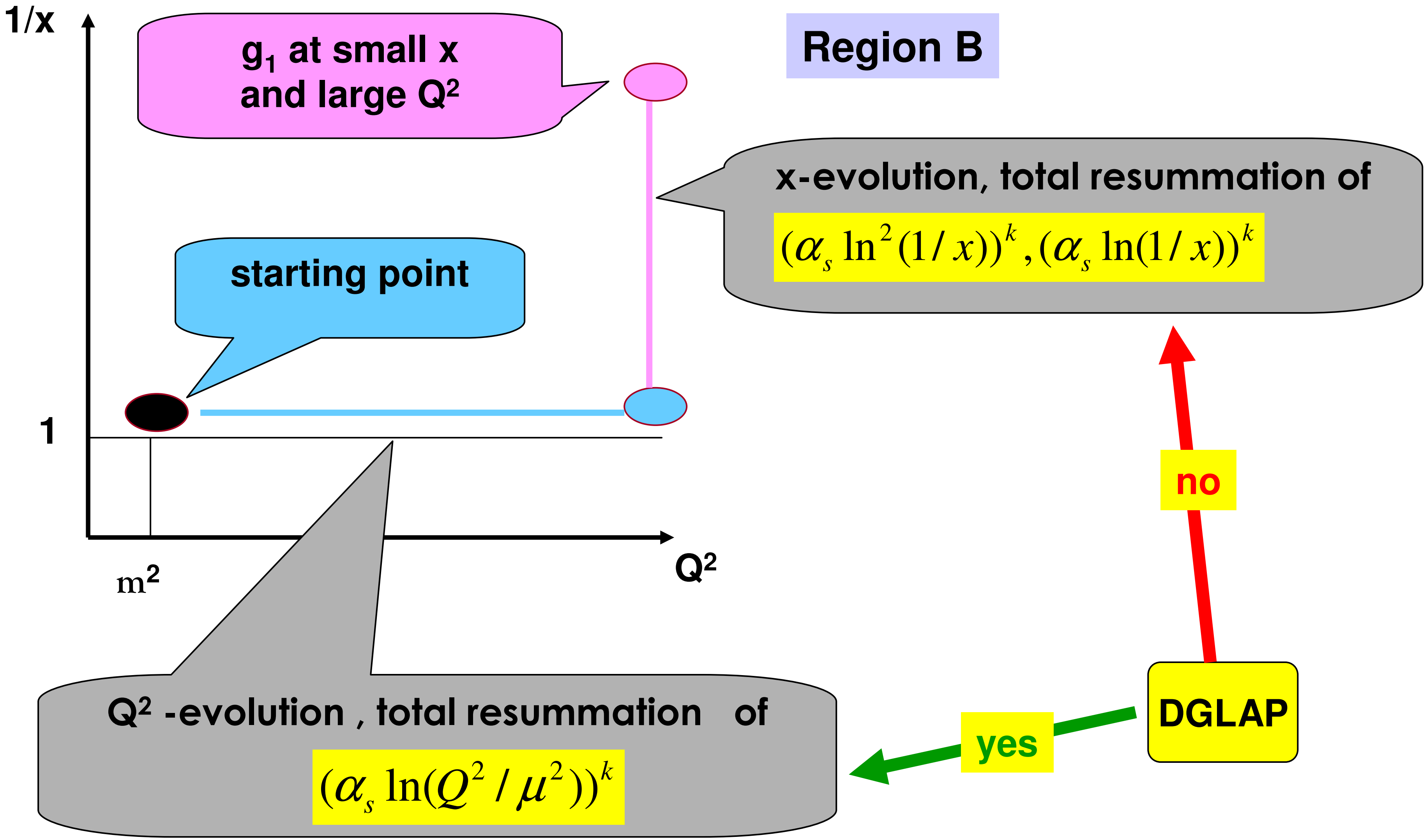
$$C\,(\omega) = 1 + (\alpha_s(Q^2)/2\pi)\, C^{(1)}\,(\omega) + \ldots$$

$$\gamma\,(\omega) = (\alpha_s(Q^2)/4\pi)\gamma^{(0)}\,(\omega) + (\alpha_s(Q^2)/2\pi)^2 \gamma^{(1)}\,(\omega) + \ldots$$

LO

NLO

One can say that DGLAP includes both Science and Art :



DGLAP does not account for the total resummation of logs of x and from theoretical grounds cannot be used in Region B

In practice SA solves this problem through introducing singular fits for initial parton densities, they cause a fast growth at small x and thereby mimic the resummation **Week point:** no theoretical grounds

Altarelli-Ball-Forte-Ridolfi, Blumlein- Botcher, Leader- Sidorov-Stamenov, Hirai et al

In the literature, there are different fits for initial parton densities. For example,

$$\delta q = N x^{-\alpha} [(1-x)^{\beta}(1+\gamma x^{\delta})]$$

$$\delta q = N [\ln^{\alpha}(1/x) + \gamma x \ln^{\beta}(1/x)]$$

Altarelli-Ball-Forte-Ridolfi,

Parameters $N, \alpha, \beta, \gamma, \delta$ should be fixed from experiment

Alternative, Straightforward Way: Total resummation of leading logs of x

As value of the cut-off is not fixed, one can evolve the structure functions with respect to μ the name of the method:

Infra-Red Evolution Equations (IREE)

Highlights of the history of the method

- Analyses of two-particle cuts in Regge kinematics — Gribov
- Factorization of photons with small transverse momenta — Gribov
- Infrared cut-off in the transverse momentum space — Lipatov
- Quark-quark scattering amplitudes — Kirschner-Lipatov
- Generalization of Gribov bremsstrahlung theorem to QCD , inelastic quark form factors — Ermolaev-Fadin-Lipatov
- QCD inelastic processes in Regge kinematics — Ermolaev-Lipatov
- Applications to Polarized Deep-Inelastic scattering — Bartels-Ermolaev-Manaenkov-Ryskin- Greco-Troyan

Expression for the singlet g_1 at large Q^2: $Q^2 > \mu^2;\ \mu \approx 5\ \mathrm{GeV}$

$$g_1^S = \frac{< e_q^2 >}{2} \int \frac{d\omega}{2\pi i}\left(\frac{1}{x}\right)^{\omega}\left[F_q \delta q + F_g \delta g\right]$$

$$F_q = C_q^{(+)}\left(\frac{Q^2}{\mu^2}\right)^{\Omega^{(+)}} + C_q^{(-)}\left(\frac{Q^2}{\mu^2}\right)^{\Omega^{(-)}}$$

$$F_g = C_g^{(+)}\left(\frac{Q^2}{\mu^2}\right)^{\Omega^{(+)}} + C_g^{(-)}\left(\frac{Q^2}{\mu^2}\right)^{\Omega^{(+)}}$$

$C_{q,g}^{(+)}(\omega), C_{q,g}^{(-)}(\omega), \quad \Omega^{(+)}(\omega), \Omega^{(+)}(\omega)$ contain leading logs to all orders

coefficient functions

anomalous dimensions

Description of g_1 in Region C: small Q^2 and small x:

Generalization of our previous results through the shift

$$Q^2 \to Q^2 + \mu^2 \quad \Rightarrow \quad x \to \bar{x} = (Q^2 + \mu^2)/2pq = x + z$$

Infrared cut-off

Similar shifts have been used for DIS structure functions by many authors, however from phenomenological considerations. We do It from analysis of the involved Feynman graphs

Expression for the singlet g_1 at small Q^2: $Q^2 < \mu^2; \ \mu \approx 5 \text{ GeV}$

$$g_1^S = \frac{<e_q^2>}{2} \int \frac{d\omega}{2\pi i} \left(\frac{1}{x+z} \right)^{\omega} \left[F_q \delta q + F_g \delta g \right]$$

$$F_q = C_q^{(+)} \left(\frac{Q^2 + \mu^2}{\mu^2} \right)^{\Omega^{(+)}} + C_q^{(-)} \left(\frac{Q^2 + \mu^2}{\mu^2} \right)^{\Omega^{(-)}}$$

$$F_g = C_g^{(+)} \left(\frac{Q^2 + \mu^2}{\mu^2} \right)^{\Omega^{(+)}} + C_g^{(-)} \left(\frac{Q^2 + \mu^2}{\mu^2} \right)^{\Omega^{(+)}}$$

$x = Q^2 / 2pq, \ \ z = \mu^2 / 2pq$

COMPASS: 10^{-1} GeV^2 < Q^2 < 3 GeV^2 ⟶ DGLAP cannot be used:

$$\ln\left[\frac{\ln\left(Q^2/\Lambda^2\right)}{\ln\left(\mu^2/\Lambda^2\right)}\right] > 1 \Rightarrow \quad Q^2 >> \mu^2$$

Our approach is not sensitive to values of Q^2, so we can use it

Prediction 1: very weak dependence g_1 on x at the COMPASS range of Q^2 even at very small x

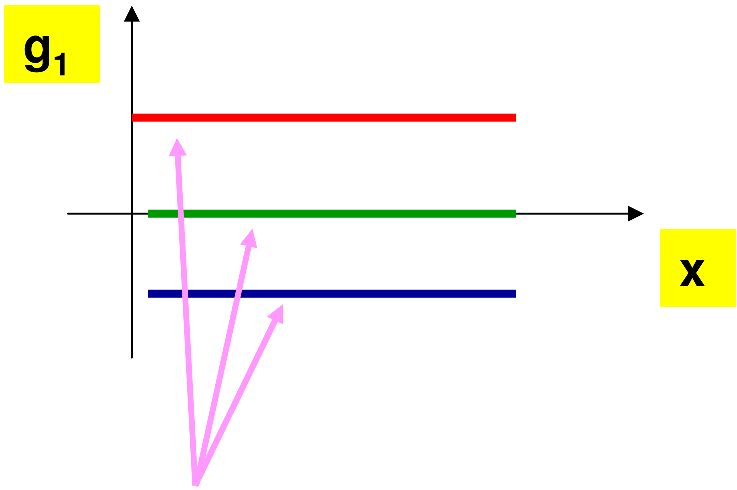
when $Q^2 << \mu^2$

$$x << z \Rightarrow g_1(x+z) \approx g_1(z) + x d g_1(z)/dz + \ldots$$

$$z = \mu^2/(2pq)$$

$$g_1(z) = \left(\frac{<e_q^2>}{2}\right)\int_{-i\infty}^{i\infty}\frac{d\omega}{2\pi i}\left(\frac{1}{z}\right)^{\omega}\left[C_q(\omega)\delta q + C_g(\omega)\delta g\right]$$

so Q^2- dependences is flat, even for $x<<1$.



Status of this prediction:

Confirmed by COMPASS

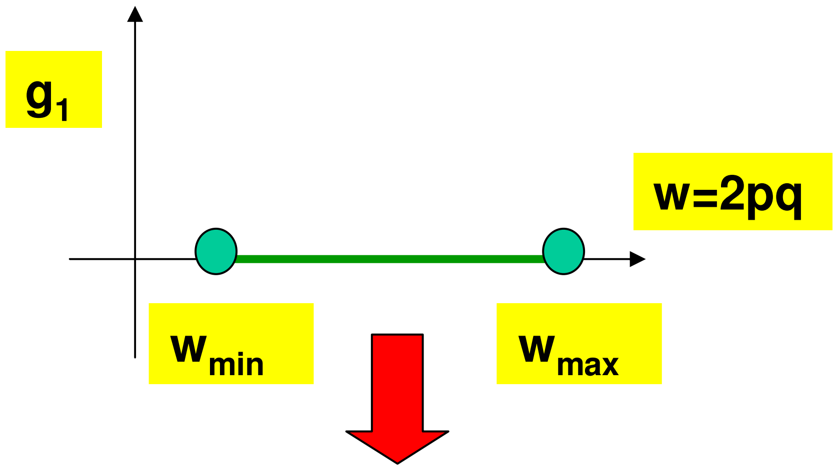
Location of the line cannot be predicted from theoretical grounds because depends on the interplay between the unknown initial quark and gluon densities: at small Q^2

$$g_1(z) = \left(\frac{<e_q^2>}{2}\right)\int_{-i\infty}^{i\infty}\frac{d\omega}{2\pi i}\left(\frac{1}{z}\right)^{\omega}\left[C_q(\omega)\delta q + C_g(\omega)\delta g\right]$$

are calculated

COMPASS data: g_1 =0 at small Q^2 but values of 2pq are not specified. It leaves two possibilities:

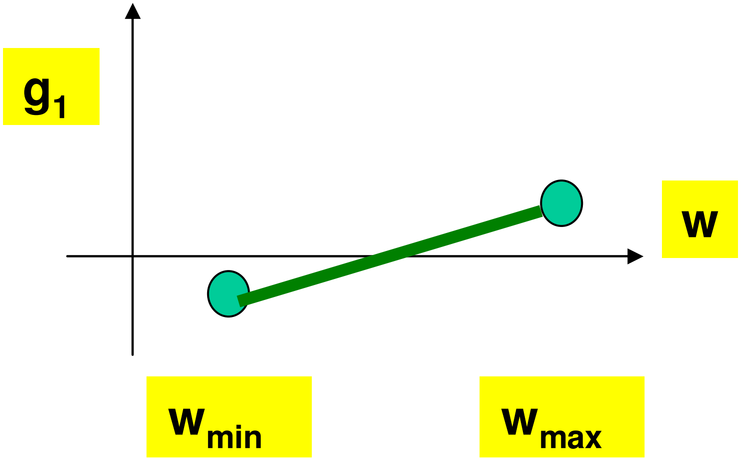
Case A: g_1=0 at any value of 2pq (=w) in the COMPASS 2pq- range: 30 GeV^2 < w <270 GeV^2

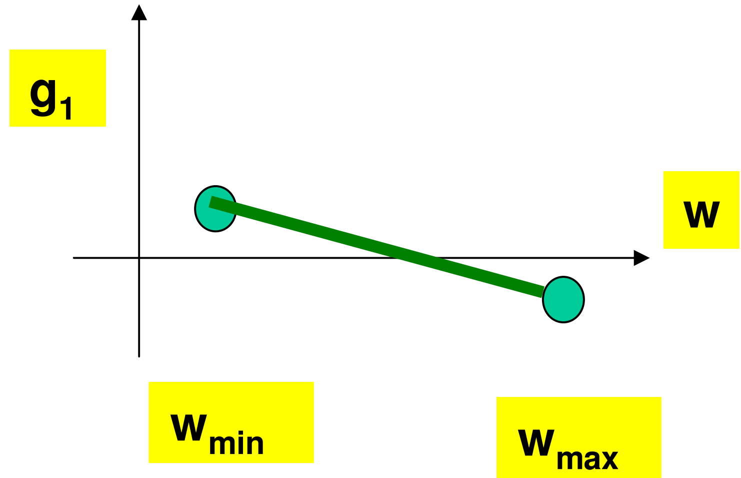


$$\int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i}\left(\frac{\mu^2}{w}\right)^{\omega}\left[C_q(\omega)\delta q + C_g(\omega)\delta g\right] \approx 0$$

for all w from the interval $w_{\min} \le w \le w_{\max}$

Case B: g1=0 only after averaging over w





Problem: What expressions for the Initial parton densities to use?

Usually they are fixed from phenomenological considerations

For example, in DGLAP

$$\delta q = 0.4 z^{0.5}(1-z)^3(1+3z), \quad \delta g = 1.7 z^{-0.5}(1-z)^4(1+3z)$$

(Altarelli-Ball-Forte-Ridolfi)

singular at z->0

DGLAP needs singular factors to mimic the total resummation of logs. When the resummation is accounted for, they should be dropped, so the fits can be chosen as

$$\delta q = N_q z^{a_q} (1-z)^{b_q}, \quad \delta g = N_g z^{a_g} (1-z)^{b_g},$$

with all parameters >0, so there are no singularities in the fits. They are supposed to mimic the hadron structure

PROBLEM: in the COMPASS range of w, z is not small: 0.5 > z > 0.17, so non-logarithmic contributions to the coefficient functions are essential and must be accounted for. We do it in the one-loop approximation, like it is in NLO DGLAP

SUGGESTION: Let us choose the DGLAP-like fits for the initial parton densities, then play with parameters in them

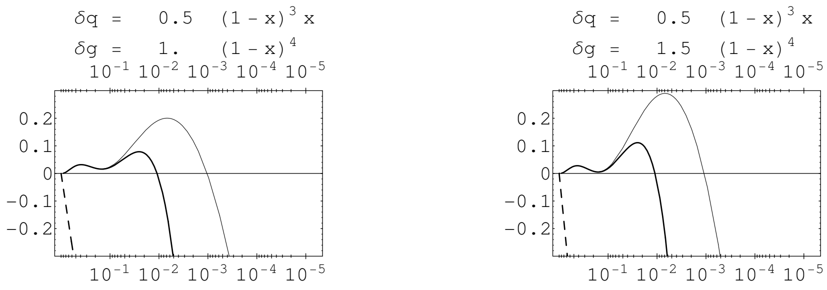


FIG. 1: $G(z)$ for $\delta q = 0.5z(1-z)^3$, $\delta g = N_g(1-z)^4$ and for $N_g = 1, 1.5$. Thin curve – pure AP. Thick curve – BE+ . Dashed curve displays 1-loop DL.

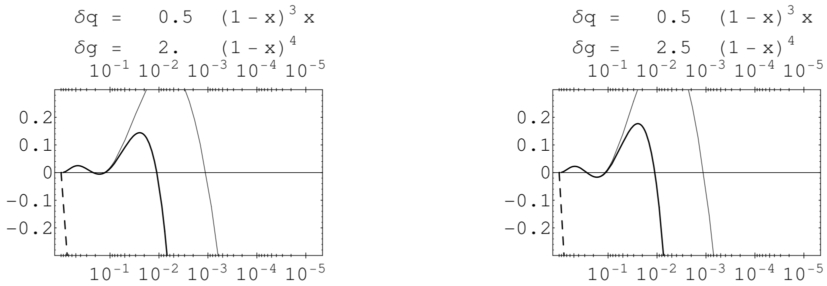


FIG. 2: $G(z)$ for $\delta q = 0.5z(1-z)^3$, $\delta g = N_g(1-z)^4$ and for $N_g = 2, 2.5$. Thin curve – pure AP. Thick curve – BE+ . Dashed curve displays 1-loop DL.

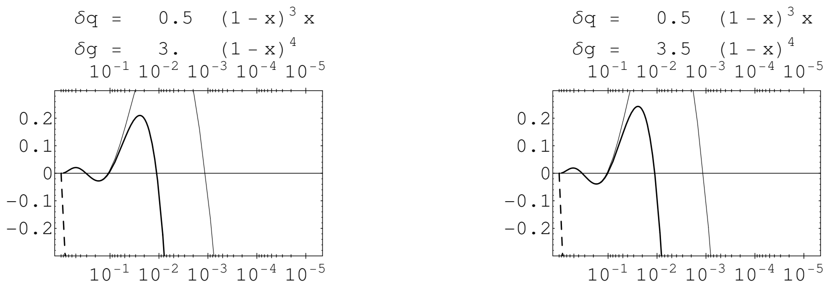


FIG. 3: $G(z)$ for $\delta q = 0.5z(1-z)^3$, $\delta g = N_g(1-z)^4$ and for $N_g = 3, 3.5$. Thin curve – pure AP. Thick curve – BE+ . Dashed curve displays 1-loop DL.

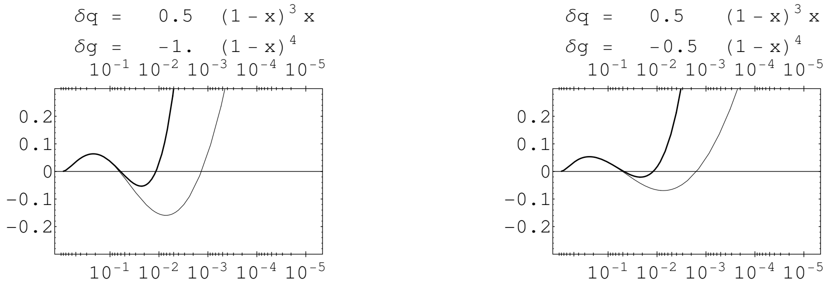


FIG. 1: $G(z)$ for $\delta q = 0.5z(1-z)^3$, $\delta g = N_g(1-z)^4$ and for $N_g = -1, -0.5$. Thin curve – pure AP. Thick curve – BE+ .

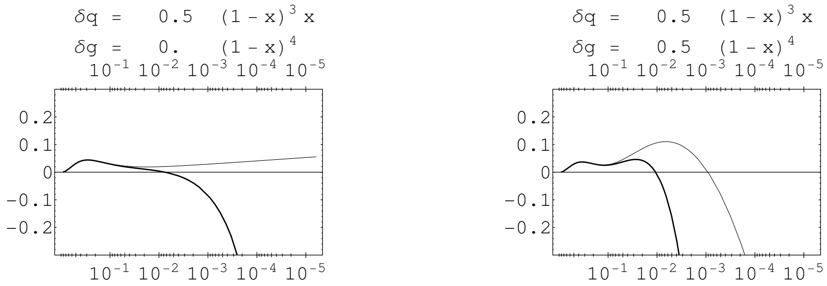


FIG. 2: $G(z)$ for $\delta q = 0.5z(1-z)^3$, $\delta g = N_g(1-z)^4$ and for $N_g = 0, 0.5$. Thin curve – pure AP. Thick curve – BE+ .

Positive Ng, with Nq/Ng = 0.15 approx, (Fig 3) are in more agreement with the COMPASS data than negative Ng when DGLAP-like fits for the initial parton densities are used

Alternatively, let us use the fits with the proton structure neglected. Such fits are proportional to the delta-function:

$$\delta q = N_q \delta(1 - z), \quad \delta g = N_g \delta(1 - z)$$

Using them brings opposite results:

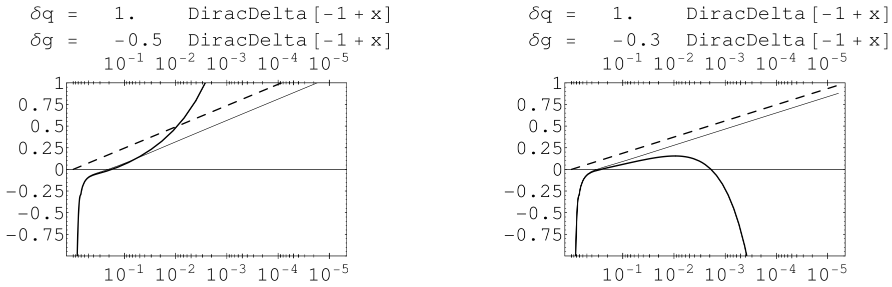


FIG. 1: $G(z)$ for $\{\delta q, \delta g\} = \{N_q, N_g\}\delta(1-z)$ and for $N_g = -0.5, -0.3$. Thin curve – pure AP. Thick curve – BE+ . Dashed curve displays $G(z) = \delta(1-z) + (\frac{\alpha_s(\mu)}{2\pi}\ln(1/z))[C_F - n_f N_g]$.

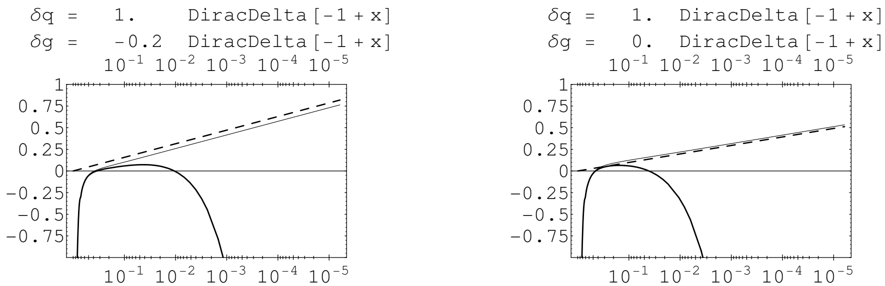


FIG. 2: $G(z)$ for $\{\delta q, \delta g\} = \{N_q, N_g\}\delta(1-z)$ and for $N_g = -0.2, 0.$. Thin curve – pure AP. Thick curve – BE+ . Dashed curve displays $G(z) = \delta(1-z) + (\frac{\alpha_s(\mu)}{2\pi}\ln(1/z))[C_F - n_f N_g]$.

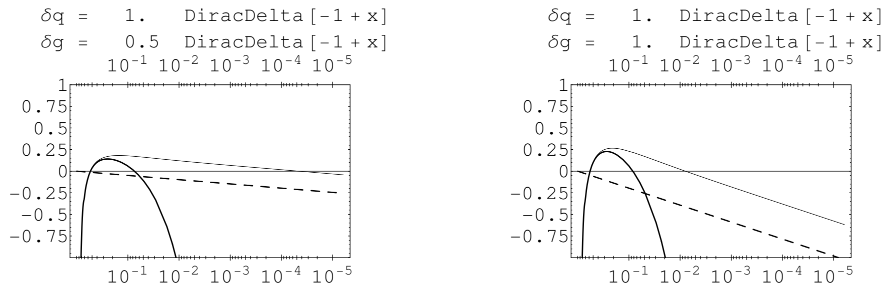


FIG. 3: $G(z)$ for $\{\delta q, \delta g\} = \{N_q, N_g\}\delta(1-z)$ and for $N_g = 0.5, 1.0$. Thin curve – pure AP. Thick curve – BE+ . Dashed curve displays $G(z) = \delta(1-z) + (\frac{\alpha_s(\mu)}{2\pi}\ln(1/z))[C_F - n_f N_g]$.

The plots show that negative Ng are more in agreement with the COMPASS data than positive Ng

SUMMARY AND OUTLOOK

1. Studying the x-dependence of g_1 at small Q^2 is not of much interest because g_1 does not depend on x in this region. Instead, it would be interesting to study the w-dependence.

2. In absence of info on the w-dependence, the COMPASS result $g_1=0$ can be interpreted in two ways:

(a) $g_1=0$ for any w in the COMPASS w-range – incompatible with the both fits we have used

(b) g_1 , averaged over w is zero- compatible with the both fits, however using different fits leads to opposite conclusions

3. Recovering the w-dependence would allow to specify the fits and answer the questions about the sign of the gluon density and to fix the ratio Ng/Nq

4. Using another target would allow to measure the non-singlet g1 and define Nq