New developments in the statistical approach of parton distributions

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- A Statistical Approach for Polarized Parton Distributions

- Recent Tests for the Statistical Parton Distributions

- The Statistical Parton Distributions: status and prospects

- The extension to the transverse momentum of the statistical parton distributions

- Stangeness asymmetry of the nucleon in the statistical parton model

- New tests and predictions from the statistical parton distributions
  (in preparation)
Basic procedure to construct the statistical polarized parton distributions

Essential features from the Deep Inelastic Scattering data

First predictions tested against new data DIS and hadronic processes

Very recent tests from DIS COMPASS, H1

Conclusions and future
Use a simple description of the PDF, at input scale $Q_0^2$, proportional to $[\exp[(x - X_{0p})/\bar{x}] \pm 1]^{-1}$, plus sign for quarks and antiquarks, corresponds to a Fermi-Dirac distribution and minus sign for gluons, corresponds to a Bose-Einstein distribution. $X_{0p}$ is a constant which plays the role of the thermodynamical potential of the parton $p$ and $\bar{x}$ is the universal temperature, which is the same for all partons.
We recall that from the chiral structure of QCD, we have **two important properties**, allowing to relate quark and antiquark distributions and to restrict the gluon distribution:

- The potential of a quark $q^h$ of helicity $h$ is opposite to the potential of the corresponding antiquark $\bar{q}^{-h}$ of helicity $-h$

\[ X_{0q}^h = -X_{0\bar{q}}^{-h} \, . \]

- The potential of the gluon $G$ is zero

\[ X_{0G} = 0 \, . \]
For light quarks $q = u, d$ of helicity $h = \pm$, we take

$$xq^{(h)}(x, Q_0^2) = \frac{AX^h_{0q}x^b}{\exp[(x - X^h_{0q})/\bar{x}]} + 1 + \frac{\tilde{A}x^\tilde{b}}{\exp(x/\bar{x}) + 1},$$
The polarized PDF at $Q_0^2 = 4\text{GeV}^2$

For light quarks $q = u, d$ of helicity $h = \pm$, we take

$$
qx^{(h)}(x, Q_0^2) = \frac{AX_{0q}^h x^b}{\exp[(x - X_{0q}^h)/\bar{x}] + 1} + \frac{\tilde{A}x^{\tilde{b}}}{\exp(x/\bar{x}) + 1},
$$

consequently for antiquarks of helicity $h = \mp$

$$
qx^{(-h)}(x, Q_0^2) = \frac{\tilde{A}(X_{0q}^h)^{-1} x^{2\tilde{b}}}{\exp[(x + X_{0q}^h)/\bar{x}] + 1} + \frac{\tilde{A}x^{\tilde{b}}}{\exp(x/\bar{x}) + 1}.
$$

Note: $q = q^+ + q^-$ and $\Delta q = q^+ - q^-$ (idem for $\bar{q}$)
For the strange quarks and antiquarks, $s$ and $\bar{s}$, given our poor knowledge on both unpolarized and polarized distributions, we take

$$xs(x, Q_0^2) = x\bar{s}(x, Q_0^2) = \frac{1}{4} [x\bar{u}(x, Q_0^2) + x\bar{d}(x, Q_0^2)] ,$$

and

$$x\Delta s(x, Q_0^2) = x\Delta \bar{s}(x, Q_0^2) = \frac{1}{3} [x\Delta \bar{d}(x, Q_0^2) - x\Delta \bar{u}(x, Q_0^2)] .$$

This is the original version, which was improved recently (see BBS, Phys. Lett. B648, 39 (2007))
We use a **Bose-Einstein** expression given by

\[ xG(x, Q^2_0) = \frac{A_G x^{b_G}}{\exp(x/\bar{x}) - 1}, \]

with a **vanishing potential** and the same temperature \(\bar{x}\). We also need to specify the polarized gluon distribution and we take the particular choice \(x \Delta G(x, Q^2_0) = 0\). (Note that \(b_G = \tilde{b} + 1\))
From well established features of $u$ and $d$ extracted from DIS data, we anticipate some simple relations between the potentials:

- $u(x)$ dominates over $d(x)$, therefore we should have $X_{0u}^+ + X_{0u}^- > X_{0d}^+ + X_{0d}^-$
- $\Delta u(x) > 0$, therefore $X_{0u}^+ > X_{0u}^-$
- $\Delta d(x) < 0$, therefore $X_{0d}^- > X_{0d}^+$. 
So we expect $X_{0u}^+$ to be the largest potential and $X_{0d}^+$ the smallest one. In fact, we have the following ordering (see below)

$$X_{0u}^+ > X_{0d}^- \sim X_{0u}^- > X_{0d}^+.$$ 

This ordering has important consequences for $\bar{u}$ and $\bar{d}$, namely
\[ \bar{d}(x) > \bar{u}(x), \quad \text{flavor symmetry breaking expected from Pauli exclusion principle.} \]

This was already confirmed by the violation of the Gottfried sum rule (NMC).

\[ \Delta \bar{u}(x) > 0 \quad \text{and} \quad \Delta \bar{d}(x) < 0, \quad \text{which remain to be checked and this will be done at RHIC-BNL from } W^\pm \text{ production.} \]
Note that since \( u^-(x) \sim d^-(x) \), it follows that \( \bar{u}^+(x) \sim \bar{d}^+(x) \), so we have

\[
\Delta \bar{u}(x) - \Delta \bar{d}(x) \sim \bar{d}(x) - \bar{u}(x)
\]

i.e. the flavor symmetry breaking is almost the same for unpolarized and polarized distributions (\( \bar{u} \) and \( \bar{d} \) polarizations contribute to about 10\% to the Bjorken sum rule).
An interesting observation (Will come back to this)
Only eight free parameters

By performing a NLO QCD evolution of these PDF in 2002, we were able to obtain a good description of a large set of very precise data on $F_2^p(x, Q^2)$, $F_2^n(x, Q^2)$, $xF_3^\nu N(x, Q^2)$ and $g_1^{p,d,n}(x, Q^2)$, in correspondence with eight free parameters: the four potentials $X_{0u}^+ = 0.46128$, $X_{0u}^- = 0.29766$, $X_{0d}^- = 0.30174$, $X_{0d}^+ = 0.22775$, and

$\bar{x} = 0.09907$, $b = 0.40962$, $\tilde{b} = -0.25347$, $\tilde{A} = 0.08318$. 
We also have three additional parameters, which are fixed by two normalization conditions

\[ u - \bar{u} = 2, \quad d - \bar{d} = 1, \]

and the momentum sum rule.

\[ A = 1.74938, \quad \bar{A} = 1.90801, \quad A_G = 14.27535. \]
Ealier results (Will come back to this)
Earlier results
Earlier results

![Graph showing data points with error bars for different values of $x$. The graph plots $g_1(x, Q^2) + c(x)$ against $Q^2$. The x-axis ranges from 1 to $10^2$, and the y-axis ranges from 5 to 20. The graph includes data from E143, E154, E155, HERMES, and SMC.]
Pol. Light quarks distri. (vs. x)

\[ Q^2 = 20 \text{GeV}^2 \]

\[ x f(x, Q^2) \]

\[ x f(x, Q^2) \]
PDF (vs. x) and d/u ratio: 3 regions

\[ Q^2 = 20 \text{GeV}^2 \]

\[ Q^2 = 4 \text{GeV}^2 \]

\[ Q^2 = 100 \text{GeV}^2 \]
First predictions tested against data

- Deep Inelastic Scattering
- Hadronic Collisions
Light quarks distributions (vs. x)

\[ Q^2 = 3000 \text{ GeV}^2 \]
- H1 NC e^\pm, CC e^-

\[ Q^2 = 8000 \text{ GeV}^2 \]
- Statistical PDF

\[ Q^2 = 3000 \text{ GeV}^2 \]
- H1 CC e^+

\[ Q^2 = 8000 \text{ GeV}^2 \]
- Statistical PDF
Light quarks distribution (vs. $Q^2$)

New developments in the statistical approach of parton distributions – p. 24/40
but final data move down!!!
Neutral current in $e^\pm p$ collisions (H1)
Neut. current in $e^\pm p$ collisions (ZEUS)
Charged current in $e^\pm p$ collisions

**H1**

- $e^- p \rightarrow \nu X$, $\sqrt{s} = 320$ GeV
- $e^+ p \rightarrow \bar{\nu} X$, $\sqrt{s} = 300$ GeV

**ZEUS**

- $e^- p \rightarrow \nu X$, $\sqrt{s} = 320$ GeV
- $e^+ p \rightarrow \bar{\nu} X$, $\sqrt{s} = 300$ GeV

$\sigma(x, Q^2)$ vs. $Q^2$ (GeV$^2$) plots for different values of $x$.
Charged current neutrino c. sections

[Graphs showing data for CCFR/NuTeV at 85 GeV for different values of x: 0.045, 0.08, 0.125, 0.175, 0.275, 0.35, 0.45, 0.55, 0.65. The graphs compare Neutrino and Anti-Neutrino data.]
Inclusive $\pi^0$ production in $pp$ collisions

$p + p \rightarrow \pi^0 + X$
- STAR
- $\sqrt{s} = 200$ GeV
- $\eta = 3.6$

$p + p \rightarrow \pi^0 + X$
- PHENIX
- $\sqrt{s} = 200$ GeV
- $|\eta| < 0.35$
Single-jet production in $\bar{p}p$ collisions

\[
\frac{d^2\sigma}{dE_T \, d\eta} \text{ [nb/GeV]} \\
\eta \text{ range: } 0.1 < \eta < 0.7
\]

![Graph showing single-jet production in $\bar{p}p$ collisions](image-url)
Very recent tests from DIS (COMPASS)

Non-Singlet part of $g_1^p - g_1^n$: red BBS $\Delta \bar{u} > 0$, $\Delta \bar{d} < 0$ and blue AAC $\Delta \bar{u} = \Delta \bar{d}$
Very recent tests from DIS (COMPASS)

Comparison of various parametrizations with the data
Very recent tests from DIS (COMPASS)

A flavor symmetric $\Delta \bar{q}$ leads to a much higher value $\sim 0.6$
Accurate determination of $G(x)$ at HERA

Two ways to determine the gluon distribution: measure $F_L$ or $dF_2/d\ln Q^2$

At LO $dF_2/d\ln Q^2 \sim \alpha_s G(2x) + \ldots$
Accurate determination of $G(x)$ at HERA

Two ways to determine the gluon distribution: measure $F_L$ or $dF_2/dlnQ^2$

$F_L$ is the last measurement at HERA
Conclusions and Future

- A new set of PDF is constructed in the framework of a statistical approach of the nucleon
- All *unpolarized and polarized* distributions depend upon *eight* free parameters
- New tests against experimental (unpolarized and polarized) data are very satisfactory
- Special features need to be confirmed