Transverse spin-dependent asymmetries at COMPASS experiment

International Workshop DIS 2024

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on behalf of COMPASS Collaboration
University of Warsaw

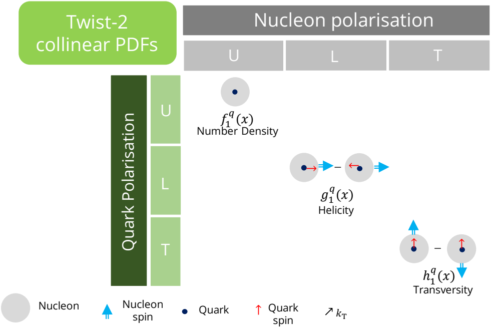
10 IV 2024, Grenoble

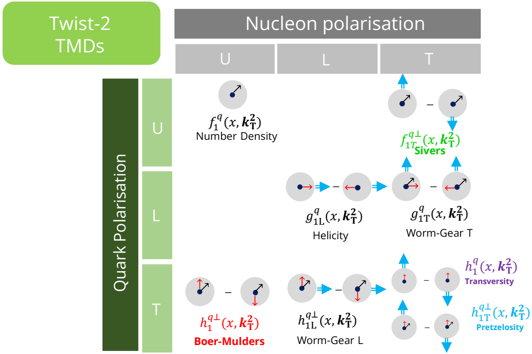
Transverse Momentum Dependent Parton Distribution Functions

“Well begun is half done.”

Old Proverb

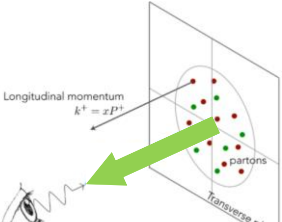
Nucleon spin structure



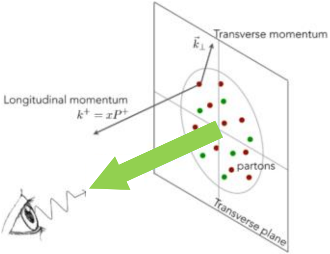


TMD PDFs can be accessed through measurement of target spin dependent azimuthal asymmetries both in SIDIS and Drell-Yan processes.

$$h_1^{q\perp}(\mathrm{SIDIS}) = -h_1^{q\perp}(\mathrm{DY})$$
$$f_{1T}^{q\perp}(\mathrm{SIDIS}) = -f_{1T}^{q\perp}(\mathrm{DY})$$



Courtesy of A. Bacchetta

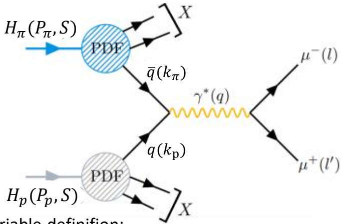


Single polarised Drell-Yan process

“Polarisation data has often been the graveyard of fashionable theories. If theorists had their way, they might well ban such measurements altogether out of self-protection.”

James Bjorken

Single polarised Drell-Yan process



Variable definifion:

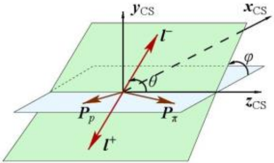
$$M_{\mu\mu} = (p_{\mu^+} + p_{\mu^-})^2$$

q_L^* - γ^* longitudinal momentum in $\pi - \mathrm{p}$ rest frame

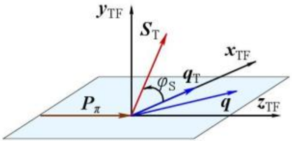
$$x_F = \frac{2q_L^*}{\sqrt{s}}, x_{\pi,p} = \frac{1}{2}\left(\sqrt{x_F^2 + 4\frac{M_{\mu\mu}^2}{s}} \pm x_F\right)$$

Cross-section, LO TMD approach for transversely polarised target:

$$\frac{d\sigma^{LO}}{dx_p dx_\pi d^2 q_T d\varphi d(\cos\theta) d\varphi_S} = C_0 \left\{ \begin{array}{c} (1+\cos^2\theta) F_U^1 + \sin^2\theta \cos 2\varphi F_U^{\cos 2\varphi} \\ + |S_T| \left[\begin{array}{c} (1+\cos^2\theta)\sin(\varphi_S) F_T^{\sin(\varphi)} + \\ \sin^2\theta \left(\begin{array}{c} \sin(2\varphi+\varphi_S) F_T^{\sin(2\varphi+\varphi_S)} \\ + \\ \sin(2\varphi-\varphi_S) F_T^{\sin(2\varphi-\varphi_S)} \end{array} \right) \end{array} \right] \end{array} \right\}$$



Collin-Soper frame



Target frame

Boer-Mulders

$$F_U^{\cos(2\varphi)} \propto h_{1,\pi}^{q\perp} \otimes h_{1,N}^{q\perp}$$

Sivers

$$F_T^{\sin(\varphi_S)} \propto f_{1,\pi}^{q} \otimes f_{1T,N}^{q\perp}$$

Pretzelosity

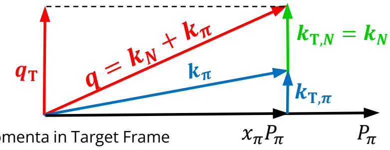
$$F_T^{\sin(2\varphi+\varphi_S)} \propto h_{1,\pi}^{q\perp} \otimes h_{1T,N}^{q\perp}$$

Transversity

$$F_T^{\sin(2\varphi-\varphi_S)} \propto h_{1,\pi}^{q\perp} \otimes h_{1,N}^{q}$$

Single polarised Drell-Yan process

The convolution of TMD PDFs runs aver the intrinsic transverse momenta $\boldsymbol{k}_{\mathrm{T}}$.



TMD PDFs are accessed through measurement of target spin dependent azimuthal asymmetries TSAs.

Sivers Asymmetries

$$A_{\mathrm{T}}^{\sin\varphi_S} = \frac{F_{\mathrm{T}}^{\sin\varphi_S}}{F_{\mathrm{U}}^{1}}$$

Sivers for nucleon, number density for π^-

Pretzelosity Asymmetries

$$A_{\mathrm{T}}^{\sin(2\varphi_{CS}+\varphi_S)} = \frac{F_{\mathrm{T}}^{\sin(2\varphi_{CS}+\varphi_S)}}{2F_{\mathrm{U}}^{1}}$$

Pretzelosity for nucleon, Boer–Mulders for π^-

Transversity Asymmetries

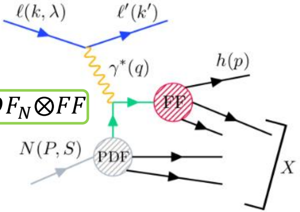
$$A_{\mathrm{T}}^{\sin(2\varphi_{CS}-\varphi_S)} = \frac{F_{\mathrm{T}}^{\sin(2\varphi_{CS}-\varphi_S)}}{2F_{\mathrm{U}}^{1}}$$

Transversity for nucleon, Boer–Mulders for π^-

SIDIS vs. DY

SIDIS

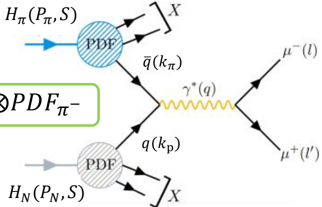
$$A_{SIDIS} \propto PDF_N \otimes FF$$



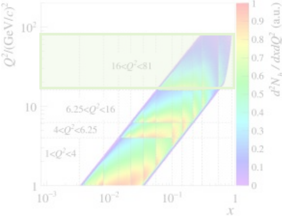
$$\frac{d\sigma^{LO}_{SIDIS}}{dx\,dy\,dz\,d^2p_T\,d\phi_h\,d\phi_S} \propto \left\{ \begin{array}{c} 1 + \cos(2\phi_h)\,\varepsilon A_{UU}^{\cos(2\phi_h)} \\ +|S_T| \left[\begin{array}{c} \sin(\phi_h - \phi_S) A_{UT}^{\sin(\phi_h-\phi_S)} + \\ \sin(\phi_h + \phi_S)\,\varepsilon A_{UT}^{\sin(\phi_h+\phi_S)} + \\ \sin(3\phi_h - \phi_S)\,\varepsilon A_{UT}^{\sin(3\phi_h-\phi_S)} + \end{array} \right] \end{array} \right\}$$

Drell-Yan

$$A_{DY} \propto PDF_N \otimes PDF_{\pi^-}$$

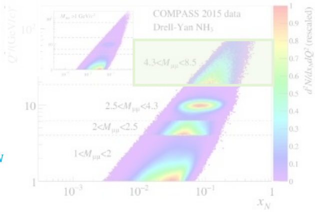


$$\frac{d\sigma^{LO}_{DY}}{d\Omega\,d^4q} \propto \left\{ \begin{array}{c} 1 + D_{\sin^2\theta} \cos(2\varphi) A_U^{\cos(2\varphi)} \\ +|S_T| \left[\begin{array}{c} \sin(\varphi_S) A_T^{\sin(\varphi_S)} + \\ D_{\sin^2\theta} \left(\begin{array}{c} \sin(2\varphi + \varphi_S) A_T^{\sin(2\varphi+\varphi_S)} \\ + \\ \sin(2\varphi - \varphi_S) A_T^{\sin(2\varphi-\varphi_S)} \end{array} \right) \end{array} \right] \end{array} \right\}$$



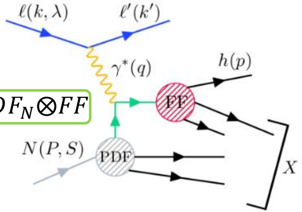
Comparable $Q^2 - x$ kinematic coverage
Minimization of possible Q^2 evolution effects

SIDIS		DY
$A_{UU}^{\cos(2\phi_h)} \propto h_{1,N}^{q\perp} \otimes H_{1q}^{\perp h}$	**Boer-Mulders**	$A_U^{\cos(2\varphi)} \propto h_{1,\pi}^{q\perp} \otimes h_{1,N}^{q\perp}$
$A_{UT}^{\sin(\phi_h-\phi_S)} \propto f_{1T,N}^{q\perp} \otimes D_{1q}^{h}$	**Sivers**	$A_T^{\sin(\varphi_S)} \propto f_{1,\pi}^{q} \otimes f_{1T,N}^{q\perp}$
$A_{UT}^{\sin(3\phi_h-\phi_S)} \propto h_{1T,N}^{q\perp} \otimes H_{1q}^{\perp h}$	**Pretzelosity**	$A_T^{\sin(2\varphi+\varphi_S)} \propto h_{1,\pi}^{q\perp} \otimes h_{1T,N}^{q\perp}$
$A_{UT}^{\sin(\phi_h+\phi_S)} \propto h_{1T,N}^{q} \otimes H_{1q}^{\perp h}$	**Transversity**	$A_T^{\sin(2\varphi-\varphi_S)} \propto h_{1,\pi}^{q\perp} \otimes h_{1,N}^{q}$



SIDIS vs. DY

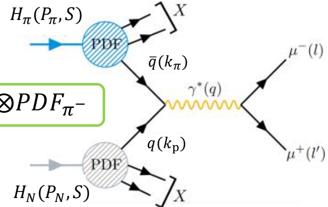
SIDIS



$A_{SIDIS} \propto PDF_N \otimes FF$

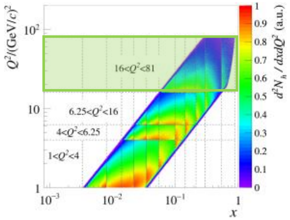
$$\frac{d\sigma^{LO}_{SIDIS}}{dx\,dy\,dz\,d^2p_T\,d\phi_h\,d\phi_S} \propto \left\{ \begin{array}{c} 1 + \cos(2\phi_h)\,\varepsilon A_{UU}^{\cos(2\phi_h)} \\ +|S_T| \left[\begin{array}{c} \sin(\phi_h - \phi_S) A_{UT}^{\sin(\phi_h-\phi_S)} + \\ \sin(\phi_h + \phi_S)\,\varepsilon A_{UT}^{\sin(\phi_h+\phi_S)} + \\ \sin(3\phi_h - \phi_S)\,\varepsilon A_{UT}^{\sin(3\phi_h-\phi_S)} + \end{array} \right] \end{array} \right\}$$

Drell-Yan



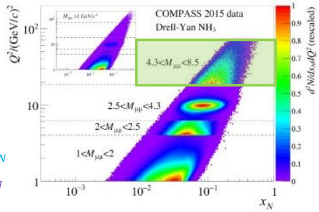
$A_{DY} \propto PDF_N \otimes PDF_{\pi^-}$

$$\frac{d\sigma^{LO}_{DY}}{d\Omega\,d^4q} \propto \left\{ \begin{array}{c} 1 + D_{\sin^2\theta} \cos(2\varphi) A_U^{\cos(2\varphi)} \\ +|S_T| \left[\begin{array}{c} \sin(\varphi_S) A_T^{\sin(\varphi_S)} + \\ D_{\sin^2\theta} \left(\begin{array}{c} \sin(2\varphi + \varphi_S) A_T^{\sin(2\varphi+\varphi_S)} \\ + \\ \sin(2\varphi - \varphi_S) A_T^{\sin(2\varphi-\varphi_S)} \end{array} \right) \end{array} \right] \end{array} \right\}$$



Comparable $Q^2 - x$ kinematic coverage
Minimization of possible Q^2 evolution effects

$A_{UU}^{\cos(2\phi_h)} \propto h_{1,N}^{q\perp} \otimes H_{1q}^{\perp h}$	← **Boer-Mulders** →	$A_U^{\cos(2\varphi)} \propto h_{1,\pi}^{q\perp} \otimes h_{1,N}^{q\perp}$
$A_{UT}^{\sin(\phi_h-\phi_S)} \propto f_{1T,N}^{q\perp} \otimes D_{1q}^{h}$	← **Sivers** →	$A_T^{\sin(\varphi_S)} \propto f_{1,\pi}^{q} \otimes f_{1T,N}^{q\perp}$
$A_{UT}^{\sin(3\phi_h-\phi_S)} \propto h_{1T,N}^{q\perp} \otimes H_{1q}^{\perp h}$	← **Pretzelosity** →	$A_T^{\sin(2\varphi+\varphi_S)} \propto h_{1,\pi}^{q\perp} \otimes h_{1T,N}^{q\perp}$
$A_{UT}^{\sin(\phi_h+\phi_S)} \propto h_{1T,N}^{q} \otimes H_{1q}^{\perp h}$	← **Transversity** →	$A_T^{\sin(2\varphi-\varphi_S)} \propto h_{1,\pi}^{q\perp} \otimes h_{1,N}^{q}$



COMPASS Experiment

“Knowledge is of no value unless you put it into practice.”

Anton Chekhov

COMPASS Collaboration

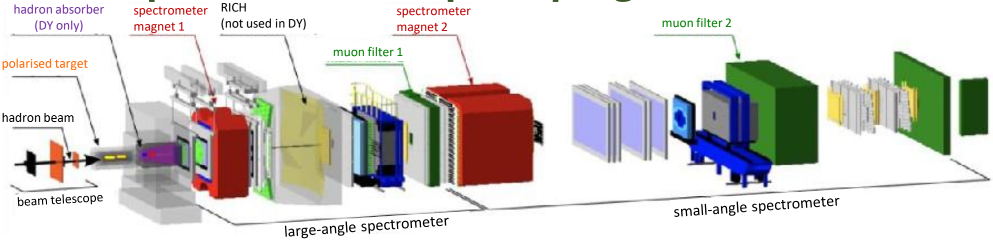
Common **M**uon and **P**roton **A**pparatus for **S**tructure and **S**pectroscopy

- 24 institutions from 13 countries (approximately 220 physicists)
- CERN SPS North Area
- Fixed target experiment

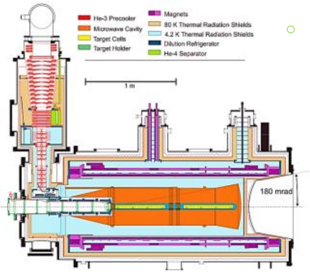
An extensive research programme on the structure of nucleons, including spin and on hadron spectroscopy

Drell Yan data taking 2015 + 2018

COMPASS experimental setup: DY programme

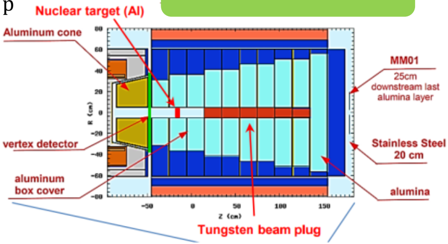


Polarised target



- High energy negative hadron beam: 190 GeV
- Beam composition: **97% π^-,** 2% K^-, 1% p^-
- Targets:
  - polarised NH_3
  - Al, W
- Hadron absorber
- Muon filters
- Triggering systems
- 2 spectrometer stages for a wide phase space coverage

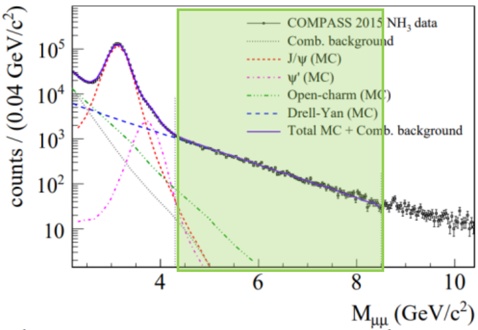
Hadron absorber

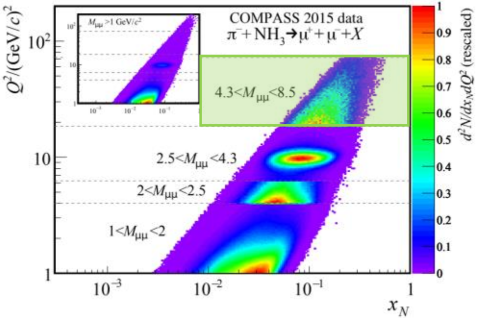


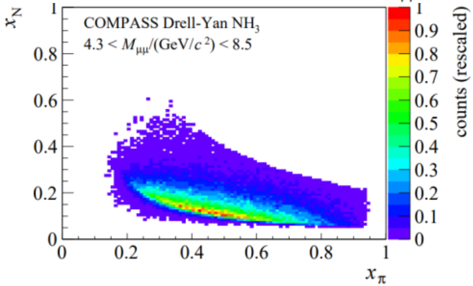
Drell-Yan measurement at COMPASS

The dimuon mass range $4.3 < M_{\mu\mu}/(\mathrm{GeV}/c^2) < 8.5$ **is 96% pure Drell-Yan.**

- Low background
- Valence region



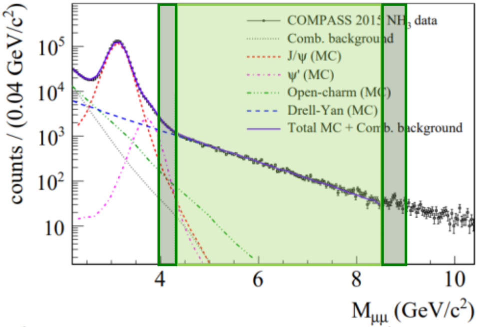


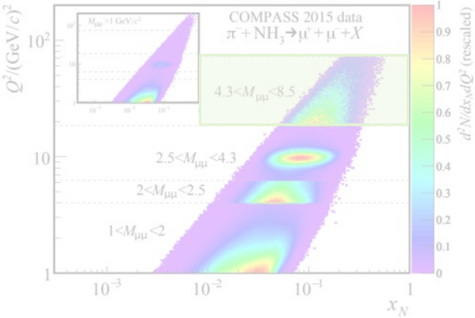


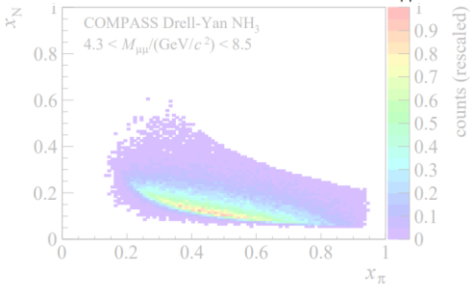
Drell-Yan measurement at COMPASS

$4.3 < M_{\mu\mu}/(\mathrm{GeV}/c^2) < 8.5$ ⟷ **96% pure Drell-Yan**

Applying an appropriate background correction, we can enlarge mass range to $4 < M_{\mu\mu}/(GeV/c^2) < 9$







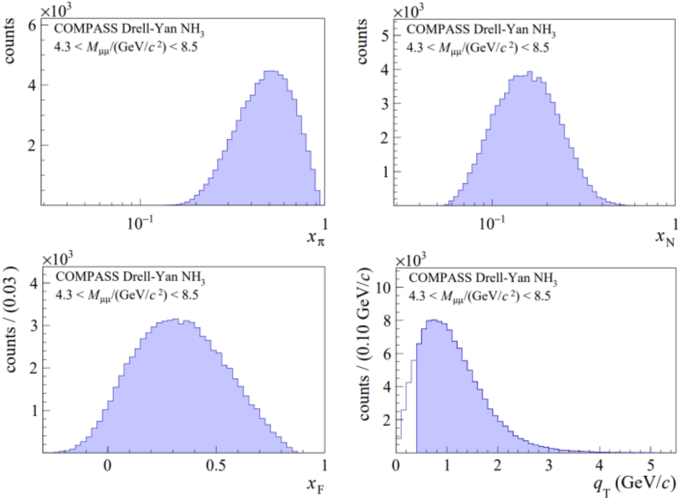
Drell-Yan measurement at COMPASS

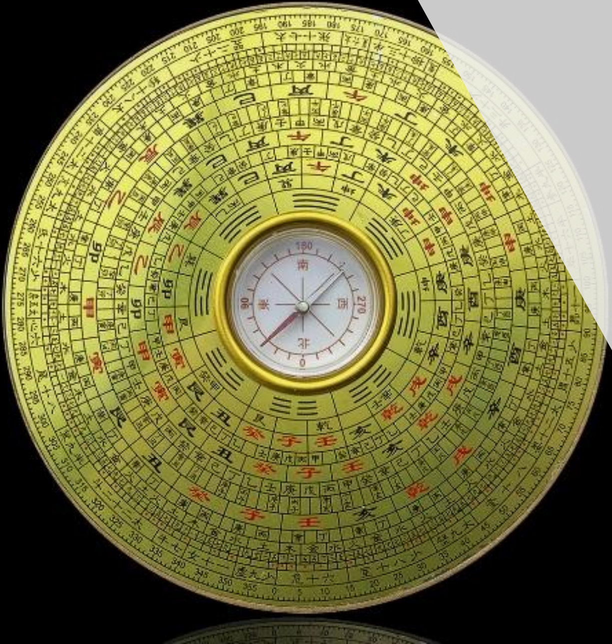
High mass region ⟷ $4.3 < M_{\mu\mu}/(\mathrm{GeV}/c^2) < 8.5$

$\langle x_\pi \rangle$	0.5
$\langle x_N \rangle$	0.17
$\langle q_\mathrm{T} \rangle$	$1.17\ \mathrm{GeV}/c^2$
$\langle M_{\mu\mu} \rangle$	$5.3\ \mathrm{GeV}/c^2$

Statistics

2015	~40 000 events
2018	~43 000 events
Total	~83 000 events



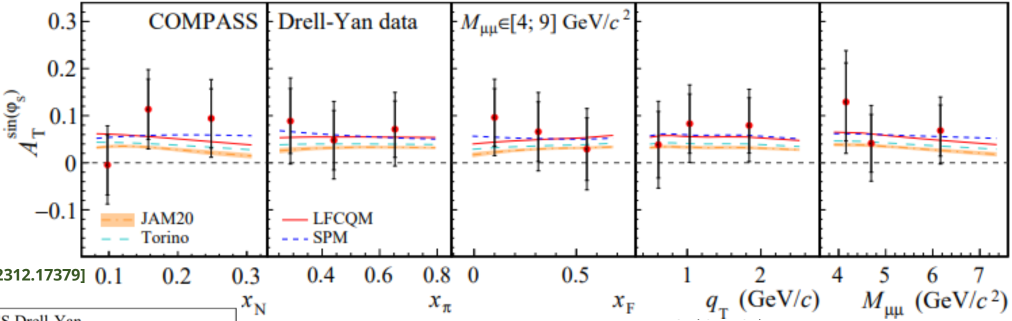


Standard TSAs

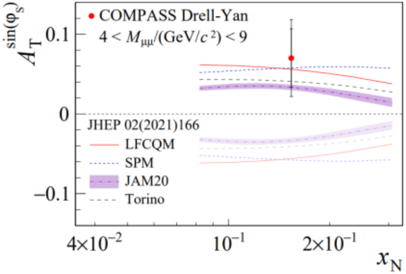
TSA Results: Sivers (SIDIS and DY)

$A_{\mathrm{T}}^{\sin(\varphi_S)} \propto f_{1,\pi}^{q} \otimes f_{1\mathrm{T},N}^{q\perp}$

Sivers DY TSA

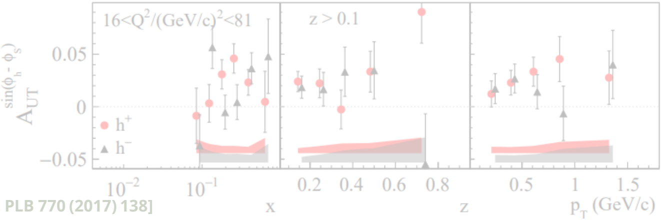


[COMPASS, hep-ex/2312.17379]



SIDIS Sivers TSA

$A_{UT}^{\sin(\phi_h - \phi_S)} \propto f_{1\mathrm{T},N}^{q\perp} \otimes D_{1q}^{h}$

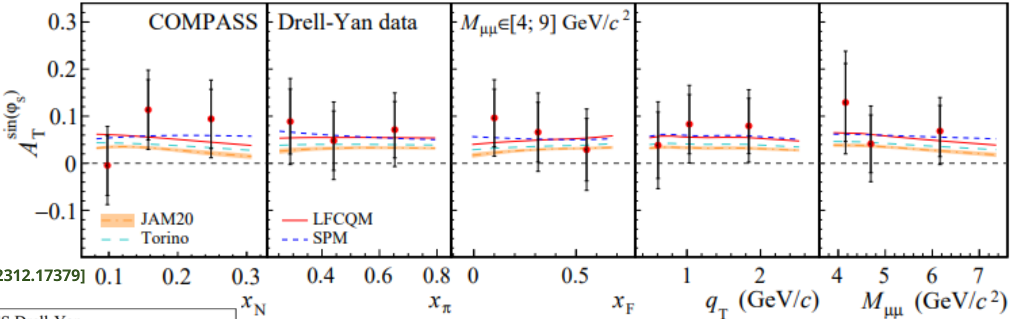


[COMPASS, PLB 770 (2017) 138]

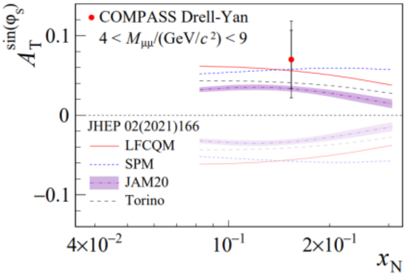
$$A_{\mathrm{T}}^{\sin(\varphi_S)} \propto f_{1,\pi}^{q} \otimes f_{1T,N}^{q\perp}$$

TSA Results: Sivers (SIDIS and DY)

Sivers DY TSA

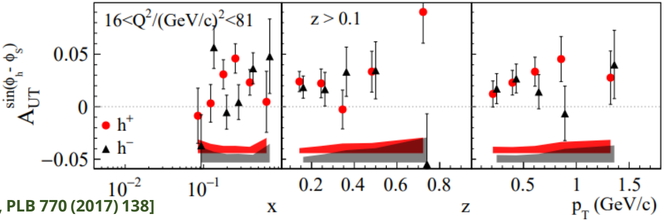


[COMPASS, hep-ex/2312.17379]



SIDIS Sivers TSA

$$A_{UT}^{\sin(\phi_h-\phi_S)} \propto f_{1T,N}^{q\perp} \otimes D_{1q}^{h}$$

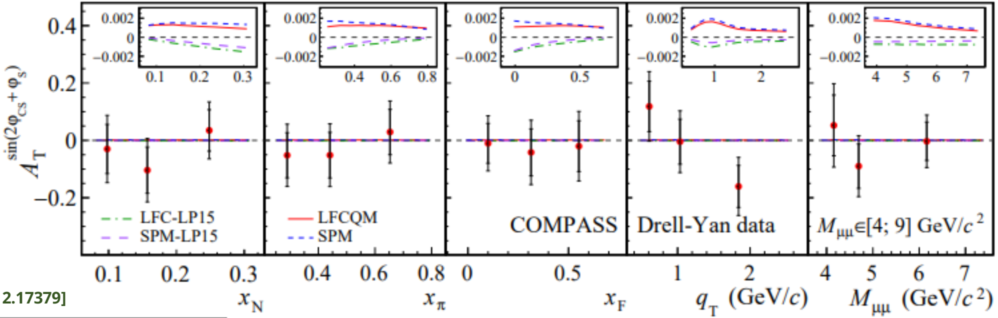


[COMPASS, PLB 770 (2017) 138]

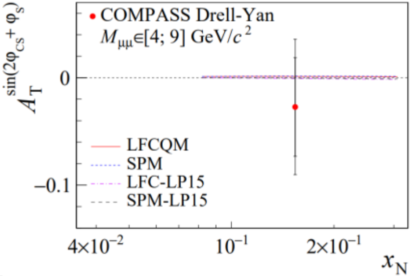
$A_T^{\sin(2\varphi+\varphi_S)} \propto h_{1,\pi}^{q\perp} \otimes h_{1T,N}^{q\perp}$

TSA Results: Pretzelosity (SIDIS and DY)

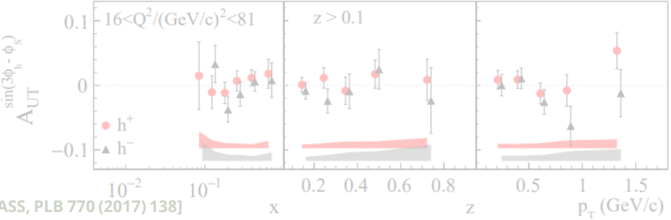
Pretzelosity DY TSA



[COMPASS, hep-ex/2312.17379]



Pretzelosity SIDIS TSA $A_{UT}^{\sin(3\phi_h-\phi_S)} \propto h_{1T,N}^{q\perp} \otimes H_{1q}^{\perp h}$

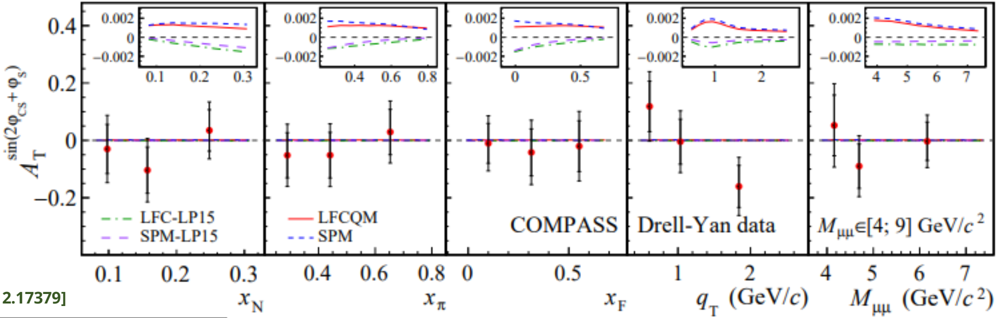


[COMPASS, PLB 770 (2017) 138]

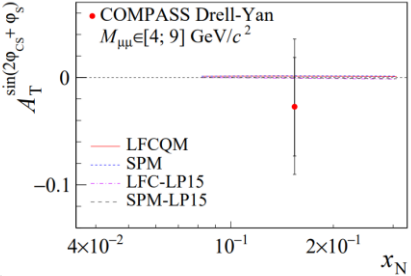
$A_T^{\sin(2\varphi+\varphi_S)} \propto h_{1,\pi}^{q\perp} \otimes h_{1T,N}^{q\perp}$

TSA Results: Pretzelosity (SIDIS and DY)

Pretzelosity DY TSA

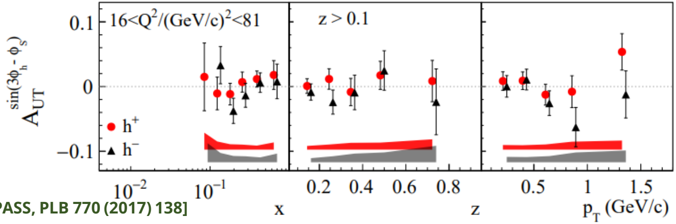


[COMPASS, hep-ex/2312.17379]



Pretzelosity SIDIS TSA

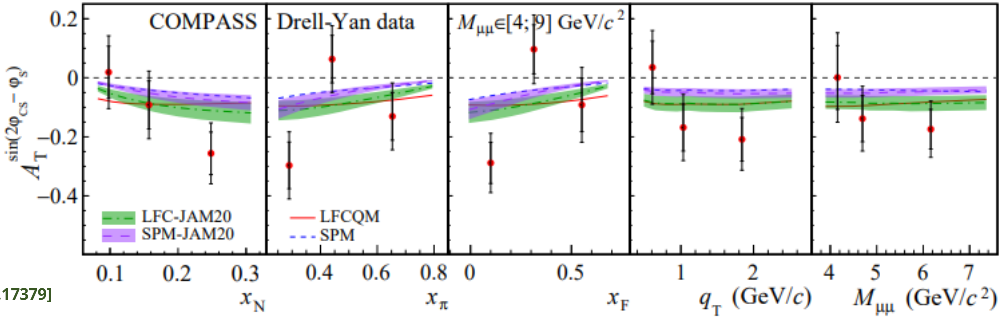
$A_{UT}^{\sin(3\phi_h-\phi_S)} \propto h_{1T,N}^{q\perp} \otimes H_{1q}^{\perp h}$



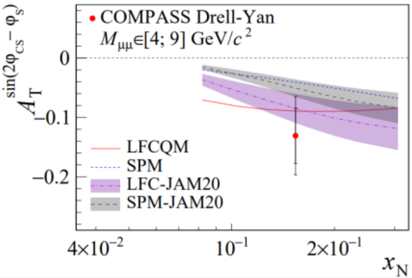
[COMPASS, PLB 770 (2017) 138]

$A_T^{\sin(2\varphi-\varphi_S)} \propto h_{1,\pi}^{q\perp} \otimes h_{1,N}^{q}$ **TSA Results: Transversity (SIDIS and DY)**

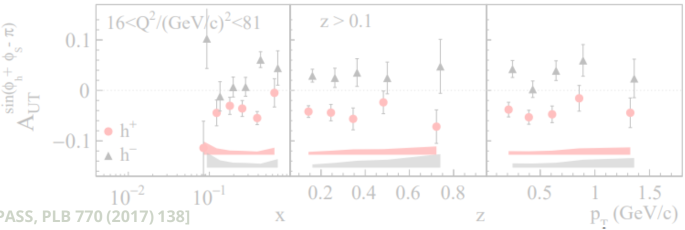
Transversity DY TSA



[COMPASS, hep-ex/2312.17379]



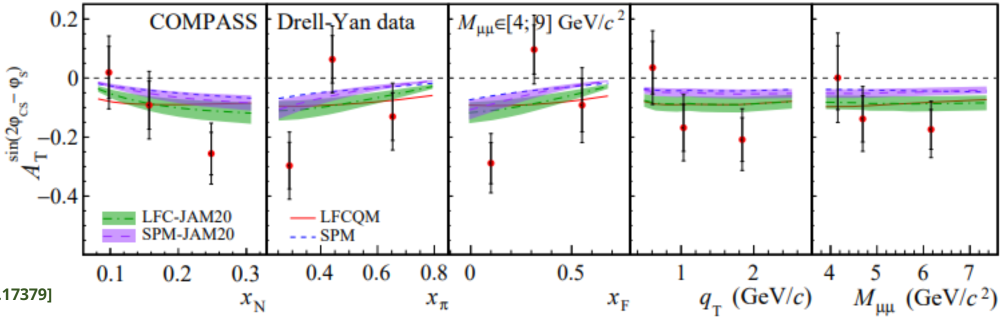
Transversity SIDIS TSA $A_{UT}^{\sin(\phi_h+\phi_S)} \propto h_{1,N}^{q} \otimes H_{1q}^{\perp h}$



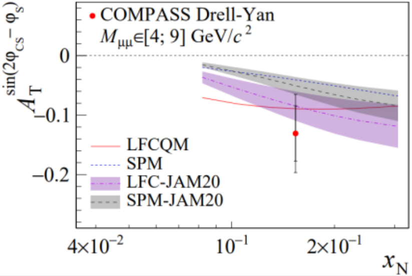
[COMPASS, PLB 770 (2017) 138]

$A_T^{\sin(2\varphi-\varphi_S)} \propto h_{1,\pi}^{q\perp} \otimes h_{1,N}^{q}$ TSA Results: Transversity (SIDIS and DY)

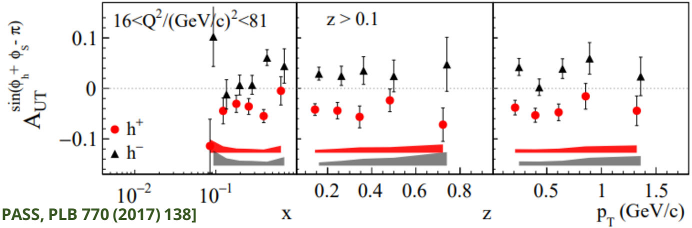
Transversity DY TSA



[COMPASS, hep-ex/2312.17379]

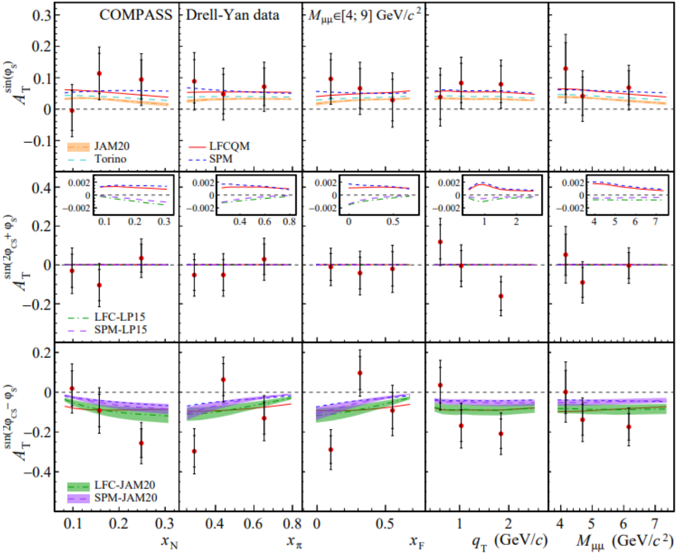


Transversity SIDIS TSA $A_{UT}^{\sin(\phi_h+\phi_S)} \propto h_{1,N}^{q} \otimes H_{1q}^{\perp h}$

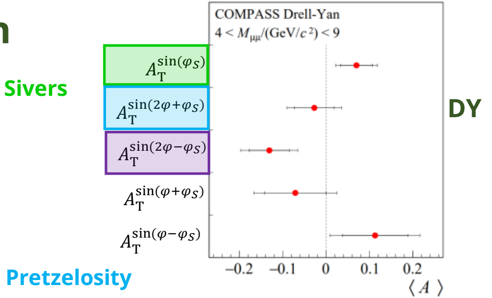


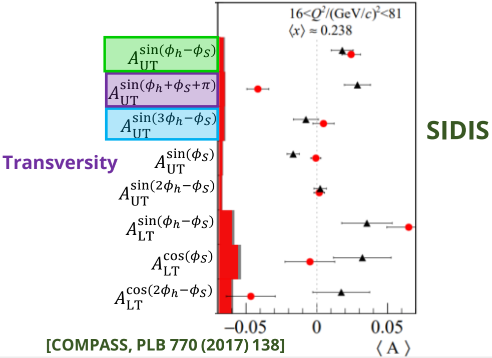
[COMPASS, PLB 770 (2017) 138]

TSA Results: SIDIS and DY comparison



[COMPASS, hep-ex/2312.17379]





[COMPASS, PLB 770 (2017) 138]

Weighted TSAs

“With four parameters I can fit an elephant, and with five I can make him wiggle his trunk.”

John von Neumann

Weighted TSAs in Drell–Yan

The convolution cannot be resolved without assumptions about the dependence of the TMD PDF on the intrinsic transverse momentum.

Weighting with powers of the transverse momentum allows to avoid assumptions on k_{T}.

Asymmetries in terms of structure functions:

$$A_{\mathrm{T}}^{\sin\Phi W_{\Phi}} = \frac{\int \mathrm{d}^2 \boldsymbol{q_T} W_{\Phi} F_{\mathrm{T}}^{\sin\Phi}}{\int \mathrm{d}^2 \boldsymbol{q}_{\mathrm{T}} F_{\mathrm{U}}^{1}},$$

where W_{Φ} is weight for $\Phi = \varphi_S, 2\varphi + \varphi_S, 2\varphi - \varphi_S$.

The n-th moment of a TMD PDF of a pion or proton:

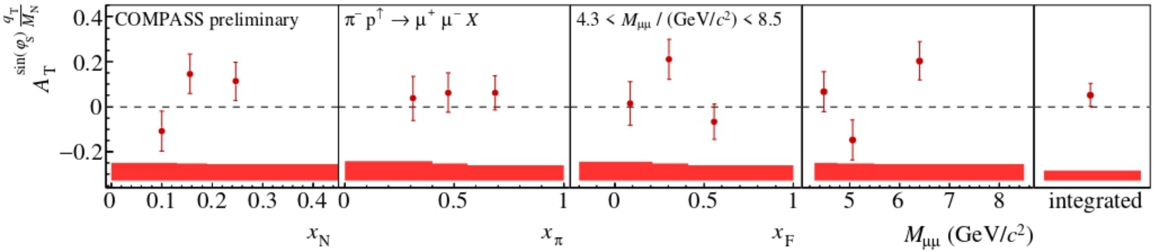
$$f^{(n)}(x) = \int \mathrm{d}^2 \boldsymbol{k}_{\mathrm{T}} \left(\frac{k_{\mathrm{T}}^2}{2M^2}\right)^n f\left(x, k_{\mathrm{T}}^2\right).$$

TSA		WTSA
$\mathrm{A}_{\mathrm{T}}^{\sin(\varphi_S)} \propto f_{1,\pi}^{q} \otimes f_{1\mathrm{T},N}^{q\perp}$	⟵ **Sivers** ⟶	$\mathrm{A}_{\mathrm{T}}^{\sin(\varphi_S)\frac{q_T}{M_N}} \propto f_{1,\pi}^{q} \times f_{1\mathrm{T},N}^{q\perp(1)}$
$\mathrm{A}_{\mathrm{T}}^{\sin(2\varphi+\varphi_S)} \propto h_{1,\pi}^{q\perp} \otimes h_{1\mathrm{T},N}^{q\perp}$	⟵ **Pretzelosity** ⟶	$\mathrm{A}_{\mathrm{T}}^{\sin(2\varphi+\varphi_S)\frac{q_T^3}{2M_N^2 M_\pi}} \propto h_{1,\pi}^{q\perp(1)} \times h_{1\mathrm{T},N}^{q\perp(2)}$
$\mathrm{A}_{\mathrm{T}}^{\sin(2\varphi-\varphi_S)} \propto h_{1,\pi}^{q\perp} \otimes h_{1,N}^{q}$	⟵ **Transversity** ⟶	$\mathrm{A}_{\mathrm{T}}^{\sin(2\varphi-\varphi_S)\frac{q_T}{M_\pi}} \propto h_{1,\pi}^{q\perp(1)} \times h_{1,N}^{q}$

WTSA Results: Sivers (SIDIS and DY)

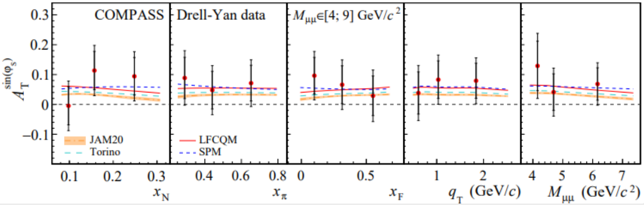
$$A_{\mathrm{T}}^{\sin(\varphi_S)\frac{q_T}{M_N}} \propto f_{1,\pi}^{q} \times f_{1\mathrm{T},N}^{q\perp(1)}$$

Sivers DY WTSA



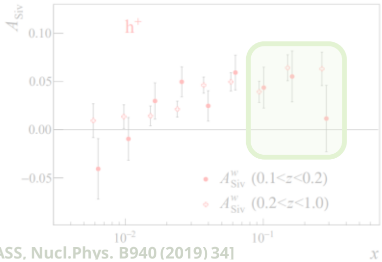
1σ positive **Sivers WTSA** compatible with **Sivers TSA**

Sivers DY TSA $A_{\mathrm{T}}^{\sin(\varphi_S)} \propto f_{1,\pi}^{q} \otimes f_{1\mathrm{T},N}^{q\perp}$



$$A_{UT}^{\sin(\phi_h-\phi_S)\frac{P_T}{zM_N}} \propto f_{1\mathrm{T},N}^{q\perp(1)} \times D_{1q}^{h}$$

Sivers SIDIS WTSA

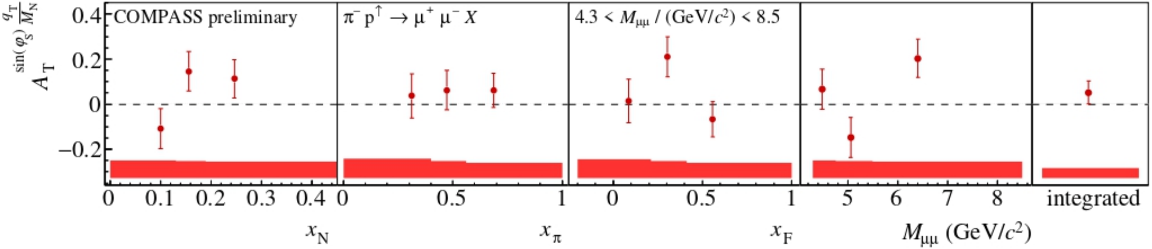


[COMPASS, Nucl.Phys. B940 (2019) 34]

WTSA Results: Sivers (SIDIS and DY)

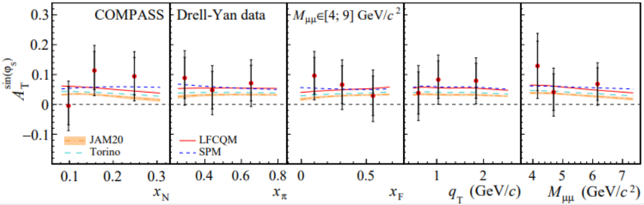
$A_{\mathrm{T}}^{\sin(\varphi_S)\frac{q_T}{M_N}} \propto f_{1,\pi}^{q} \times f_{1T,N}^{q\perp(1)}$

Sivers DY WTSA



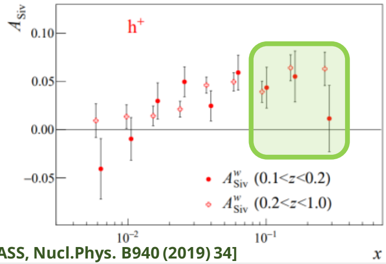
1σ positive **Sivers WTSA** compatible with **Sivers TSA**

Sivers DY TSA $A_{\mathrm{T}}^{\sin(\varphi_S)} \propto f_{1,\pi}^{q} \otimes f_{1T,N}^{q\perp}$



$A_{UT}^{\sin(\phi_h-\phi_S)\frac{P_T}{zM_N}} \propto f_{1T,N}^{q\perp(1)} \times D_{1q}^{h}$

Sivers SIDIS WTSA



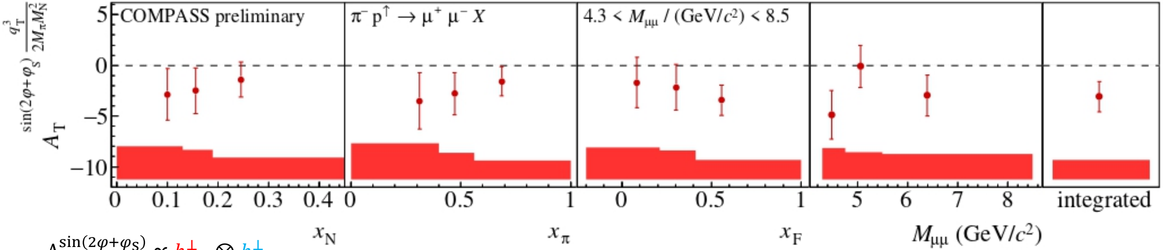
[COMPASS, Nucl.Phys. B940 (2019) 34]

WTSA Results: Pretzelosity (DY)

Comparison of WTSA and TSA for **Pretzelosity** asymmetry

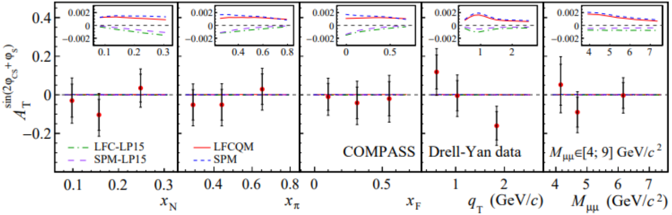
$$A_{\mathrm{T}}^{\sin(2\varphi+\varphi_S)\frac{q_{\mathrm{T}}^3}{2M_N^2 M_\pi}} \propto h_{1,\pi}^{q\perp(1)} \times h_{1\mathrm{T},N}^{q\perp(2)}$$

DY Pretzelosity WTSA



$$A_{\mathrm{T}}^{\sin(2\varphi+\varphi_S)} \propto h_{1,\pi}^{\perp} \otimes h_{1\mathrm{T},N}^{\perp}$$

DY Pretzelosity TSA



Pretzelosity is expected to be zero

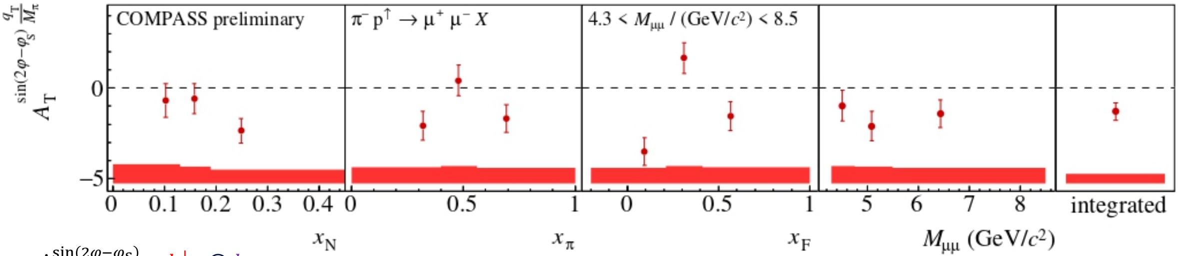
2σ negative **Pretzelosity WTSA**

WTSA Results: Transversity (DY)

Comparison of WTSA and TSA for **Transversity** asymmetry

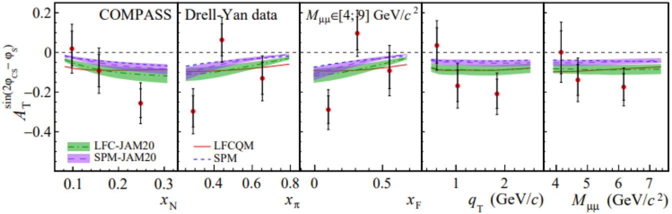
DY Transversity WTSA

$A_T^{\sin(2\varphi-\varphi_S)\frac{q_T}{M_\pi}} \propto h_{1,\pi}^{q\perp(1)} \times h_{1,N}^{q}$



$A_T^{\sin(2\varphi-\varphi_S)} \propto h_{1,\pi}^{\perp} \otimes h_{1,N}$

DY Transversity TSA



2σ negative **Transversity**

Results compatible with **Transversity TSA**

Conclusions

- **COMPASS probes 3-dimensional structure of nucleon**
- COMPASS SIDIS and Drell-Yan TSAs measurements represent a unique experimental input to study the universality of TMD PDFs

Drell-Yan TSAs

$$A_{DY} \propto PDF_N \otimes PDF_{\pi^-}$$

- 1σ positive **Sivers TSA**
- **Pretzelosity TSA** found to be small and compatiblewith zero
- 2σ negative **Transversity TSA**
- Results agree with theoretical predictions and consistent with analogous measurements for SIDIS

Transverse momentum weighted Drell-Yan TSA

$$A^{W}_{DY} \propto PDF_N \times PDF_{\pi^-}$$

- A way to overcome the convolution over intrinsic k_{T}
- A direct access to the k_{T}^2-moments of TMD PDFs
- $\sim 1\sigma$ positive **Sivers WTSA** compatible with DY TSA and SIDIS P_{T}-weighted TSA
- $\sim 2\sigma$ negative **Pretzelosity WTSA** effect
- $\sim 2\sigma$ negative **Transversity WTSA** consistent with TSAs

Prospects

- Analysis of a WTSA ongoing, paper in preparation

Thank you for attention!

“All of physics is either impossible or trivial. It is impossible until you understand it, and then it becomes trivial.”

Ernest Rutherford

Backup slides

Backup: Single Polarised Drell-Yan Process

Each structure function can be written as a TMD PDF convolution over the instrinsic transverse momenta.

$$F_{\mathrm{T}}^{\sin\varphi_S} = \mathcal{C}\left[\frac{\boldsymbol{q_T k_{T,N}}}{q_T M_N} f_{1,\pi}\, f_{1T,p}^{\perp}\right]$$

Sivers for proton, number density for π^-

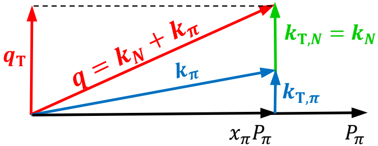
$$F_{\mathrm{T}}^{\sin(2\varphi_{CS}+\varphi_S)} = -\mathcal{C}\left[\frac{2(\boldsymbol{q_T k_{T,N}})[2(\boldsymbol{q_T k_{T,N}})(\boldsymbol{q_T k_{T,\pi}}) - q_T^2(\boldsymbol{k_{T,N} k_{T,\pi}})] - q_T^2 k_{T,N}^2(\boldsymbol{q_T k_{T,\pi}})}{2 q_T^3 M_N^2 M_\pi} h_{1,\pi}^{\perp} h_{1T,N}^{\perp}\right]$$

Pretzelosity for proton, Boer–Mulders for π^-

$$F_{\mathrm{T}}^{\sin(2\varphi_{CS}-\varphi_S)} = -\mathcal{C}\left[\frac{\boldsymbol{q_T k_{T,\pi}}}{q_T M_\pi} h_{1,\pi}^{\perp} h_{1.N}\right]$$

Transversity for proton, Boer–Mulders for π^-

$$\mathcal{C}[w(\boldsymbol{k_{T,\pi}};\boldsymbol{k_{T,N}};\boldsymbol{q_T})\mathrm{f}_\pi \mathrm{f}_N] = \frac{1}{\mathrm{N_c}}\sum_q \left\{ \begin{array}{c} e_q^2 \int \mathrm{d}^2\boldsymbol{k_{T,\pi}}\mathrm{d}^2\boldsymbol{k_{T,N}}\delta^{(2)}(\boldsymbol{q_T}-\boldsymbol{k_{T,\pi}}-\boldsymbol{k_{T,N}}) \\ \times\, w(\boldsymbol{k_{T,\pi}};\boldsymbol{k_{T,N}};\boldsymbol{q_T}) \left[\begin{array}{c} f_\pi^{\bar{q}}(x_\pi,k_{T,\pi}^2) f_N^q(x_N,k_{T,N}^2) \\ + \\ f_\pi^{q}(x_\pi,k_{T,\pi}^2) f_p^{\bar{q}}(x_p,k_{T,p}^2) \end{array}\right] \end{array} \right\}$$



Momenta in Target Frame

TMD PDFs can be accessed through measurement of target spin (in)dependent azimuthal asymmetries

$$A_{\mathrm{U}}^{\cos2\varphi} = \frac{F_{\mathrm{U}}^{\cos2\varphi}}{F_{\mathrm{U}}^1} \qquad A_{\mathrm{T}}^{\sin\varphi_S} = \frac{F_{\mathrm{T}}^{\sin\varphi_S}}{F_{\mathrm{U}}^1} \qquad A_{\mathrm{T}}^{\sin(2\varphi_{CS}+\varphi_S)} = \frac{F_{\mathrm{T}}^{\sin(2\varphi_{CS}+\varphi_S)}}{2F_{\mathrm{U}}^1} \qquad A_{\mathrm{T}}^{\sin(2\varphi_{CS}-\varphi_S)} = \frac{F_{\mathrm{T}}^{\sin(2\varphi_{CS}-\varphi_S)}}{2F_{\mathrm{U}}^1}$$