June 27th, 2023
Prague

International Workshop on Hadron Structure and Spectroscopy 2023

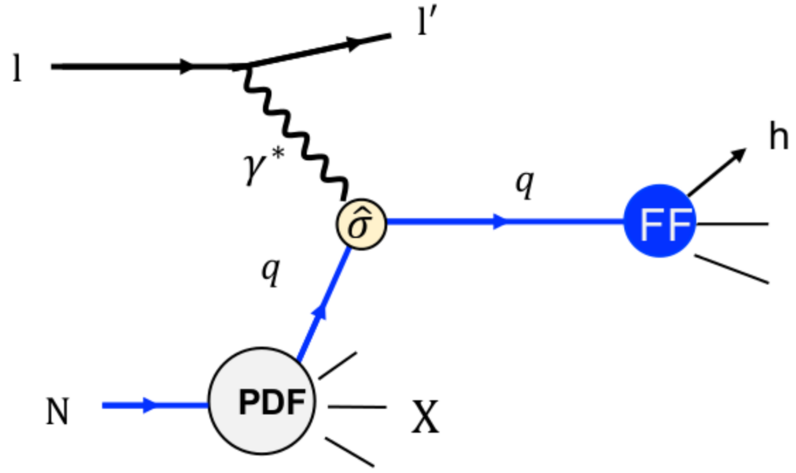
Modeling spin effects in electron-positron annihilation to hadrons

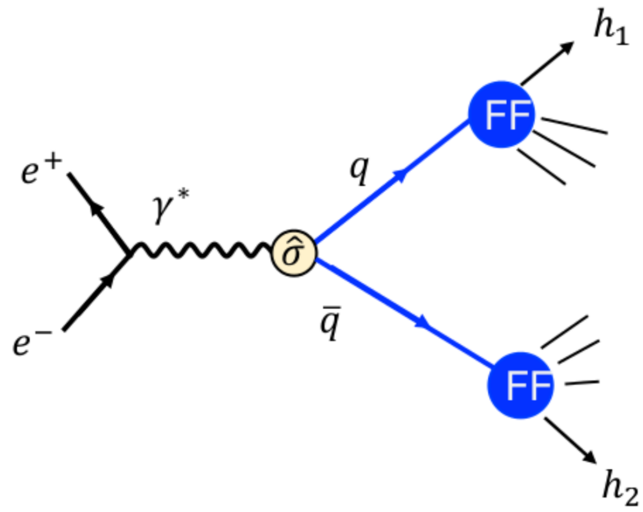
Albi Kerbizi
University of Trieste and INFN Trieste

work done in the context of the POLFRAG project

Ministero dell'Università e della Ricerca
Istituto Nazionale di Fisica Nucleare

Introduction: nucleon structure and hadronization

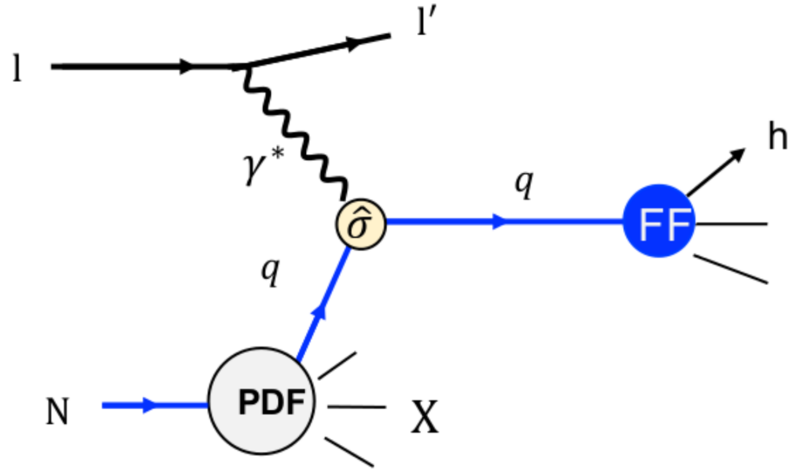


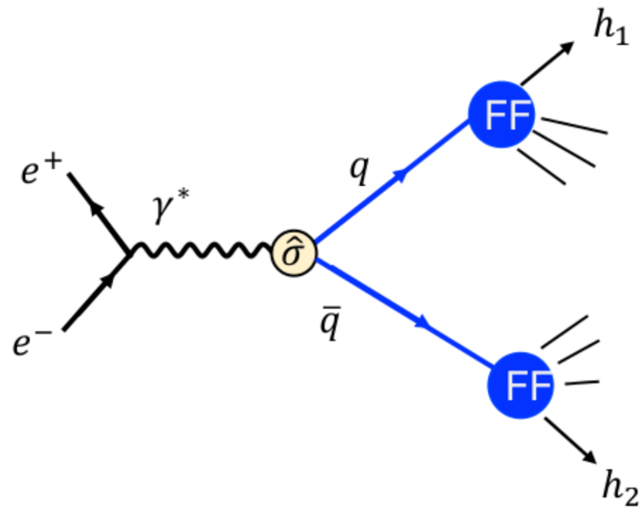


SIDIS → access to nucleon structure
via convolutions of PDFs and FFs

e^+e^- annihilation to hadrons → access to FFs

Introduction: nucleon structure and hadronization





SIDIS → access to nucleon structure
via convolutions of PDFs and FFs

e^+e^- annihilation to hadrons → access to FFs

Combined to extract information on the transverse spin structure of nucleons
e.g., transversity (but also Boer-Mulders TMD,..)

2h channel –> talk of A. Metz

Example: Collins asymmetry in SIDIS

transversity Collins FF

Collins, NPB 396, 161 (1993).

$$A_{UT}^{\sin\phi_h+\phi_S-\pi} = \frac{\sum_q e_q^2\, h_1^q \otimes H_{1q}^{\perp h}}{\sum_q e_q^2\, f_1^q \otimes D_{1q}^h}$$

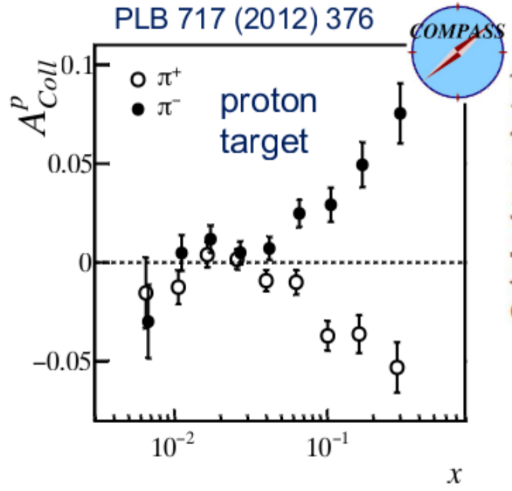
Collins asymmetry in e^+e^-

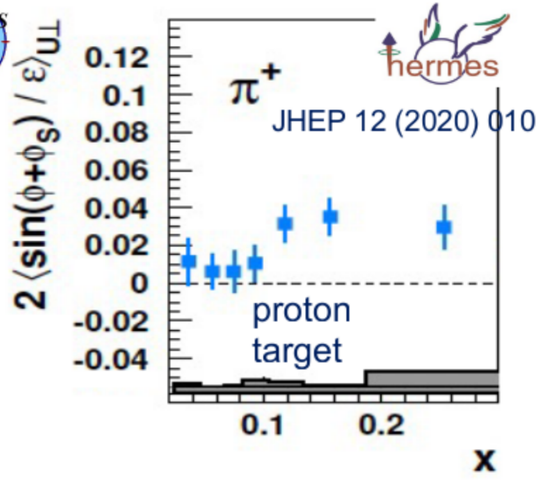
$$A_{12}^{UL} = \frac{\sum_q e_q^2\ H_{1q}^{\perp h_1}\ H_{1\bar{q}}^{\perp h_2}}{\sum_q e_q^2\ D_{1q}^{h_1}\ D_{1\bar{q}}^{h_2}}$$

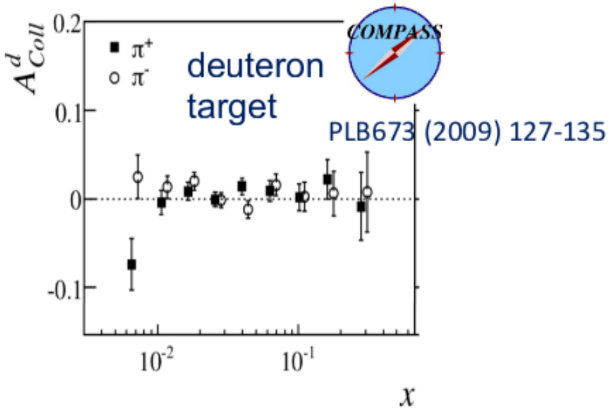
Boer, NPB, 806:23–67, 2009
D'Alesio et al., JHEP 10 (2021) 078

Introduction: nucleon structure and hadronization

PLB 717 (2012) 376







Many measurements in SIDIS

HERMES (p), COMPASS (p,d), Jlab (n)

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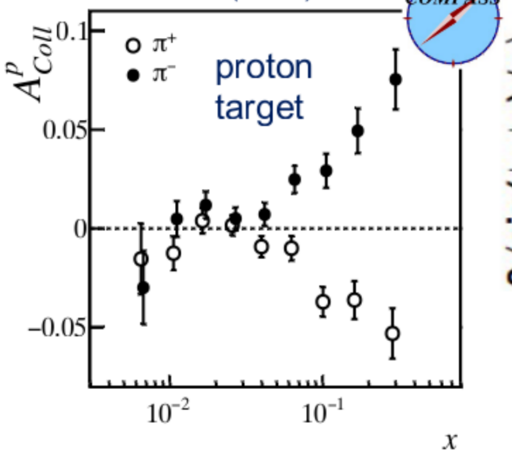
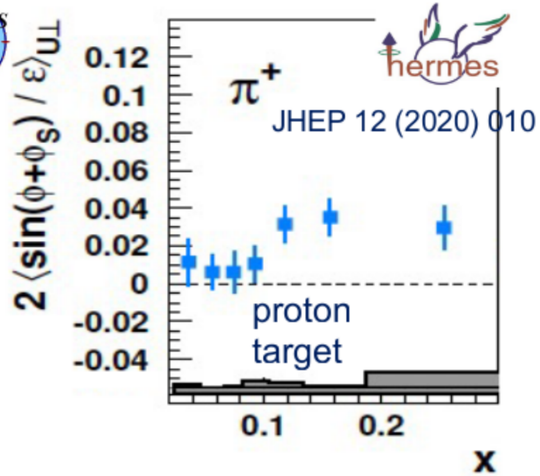
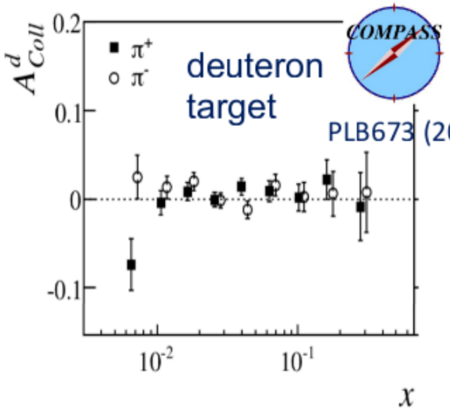


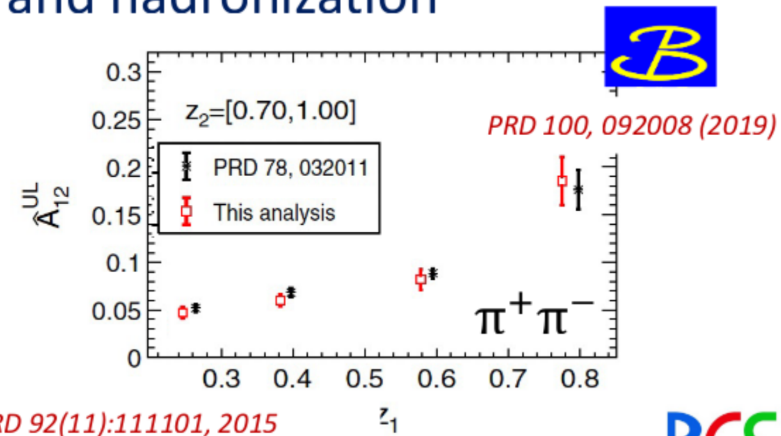




PLB673 (2009) 127-135

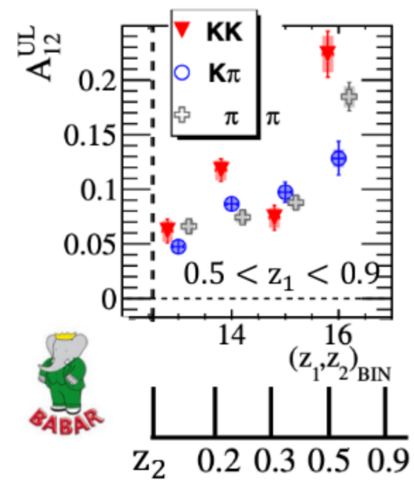
Many measurements in SIDIS

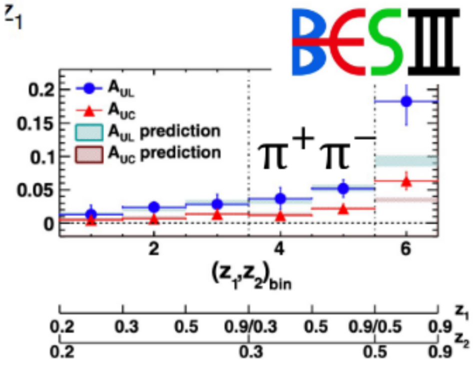
HERMES (p), COMPASS (p,d), Jlab (n)



PRD 100, 092008 (2019)

PRD 92(11):111101, 2015





PRL 116(4):042001, 2016

Different measurements in e^+e^-

Belle, BaBar, BESIII

Used to extract transversity (and Collins FF) by different groups

Anselmino et al, PRD 92 (11) (2015) 114023

Martin et al., PRD 91(1):014034, 2015

Kang et al., PRD 93 (1) (2016) 014009

...

But also a benchmark for hadronization models!

Modeling hadronization: the string+$^3P_0$ model

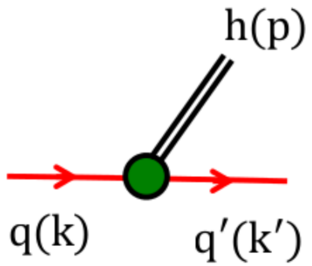
We have developed a model for the simulation of the fragmentation polarized quarks

→ string+$^3P_0$ model

AK, Artru, Belghobsi, Bradamante, Martin, PRD 97, 074010 (2018) PS

AK, Artru, Belghobsi, Martin, PRD 100, 014003 (2019) PS

AK, Artru, Martin, PRD 104, 114038 (2021) PS + VM



$\mathbf{k}_T = \mathbf{p}_T + \mathbf{k}'_T$

$Z_+ = p^+/k^+$

Quark splitting described by a 2x2 splitting amplitude

$$T_{q',h,q} \propto \left[F^{Lund}_{q',h,q}(Z_+, \mathbf{p}_T; \mathbf{k}_T)\right]^{1/2} \left[\mu + \sigma_z \boldsymbol{\sigma}_T \cdot \mathbf{k}'_T\right] \Gamma_{h,s_h}$$

$^3P_0$ mechanism
μ complex mass paramter

Coupling
e.g.
$\Gamma_{h=PS} = \sigma_z$

$\mathrm{Im}(\mu)$ → T spin effects (Collins, dihadron)

$\mathrm{Im}(\mu)$ → L spin effects (jet handedness)

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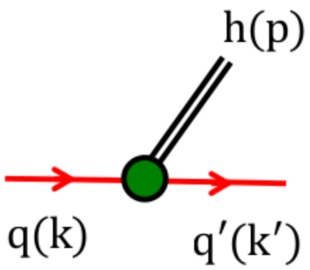

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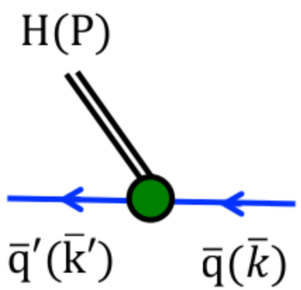

$\bar{\mathbf{k}}_T = \mathbf{P}_T + \bar{\mathbf{k}}'_T$
$Z_- = P^-/\bar{k}^-$

For anti-quark splitting
$\{q, h, q'\} \to \{\bar{q}, H, \bar{q}'\}$, $Z_+ \to Z_-$, $\{\mathbf{k}_T, \mathbf{p}_T, \mathbf{k}'_T\} \to \{\bar{\mathbf{k}}_T, \mathbf{P}_T, \bar{\mathbf{k}}'_T\}$

Modeling hadronization: the string+$^3P_0$ model

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→ string+$^3P_0$ model

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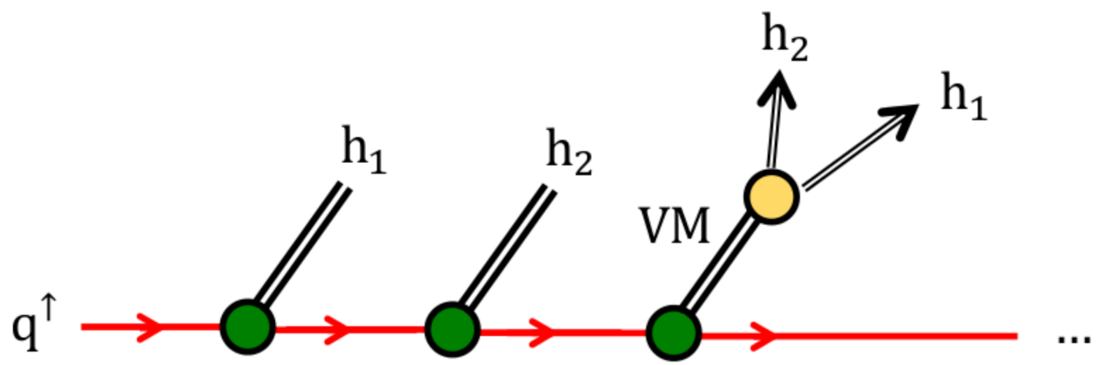
AK, Artru, Belghobsi, Martin, PRD 100, 014003 (2019) PS
AK, Artru, Martin, PRD 104, 114038 (2021) PS + VM

Used to describe the fragmentation of (transversely) polarized quarks in SIDIS



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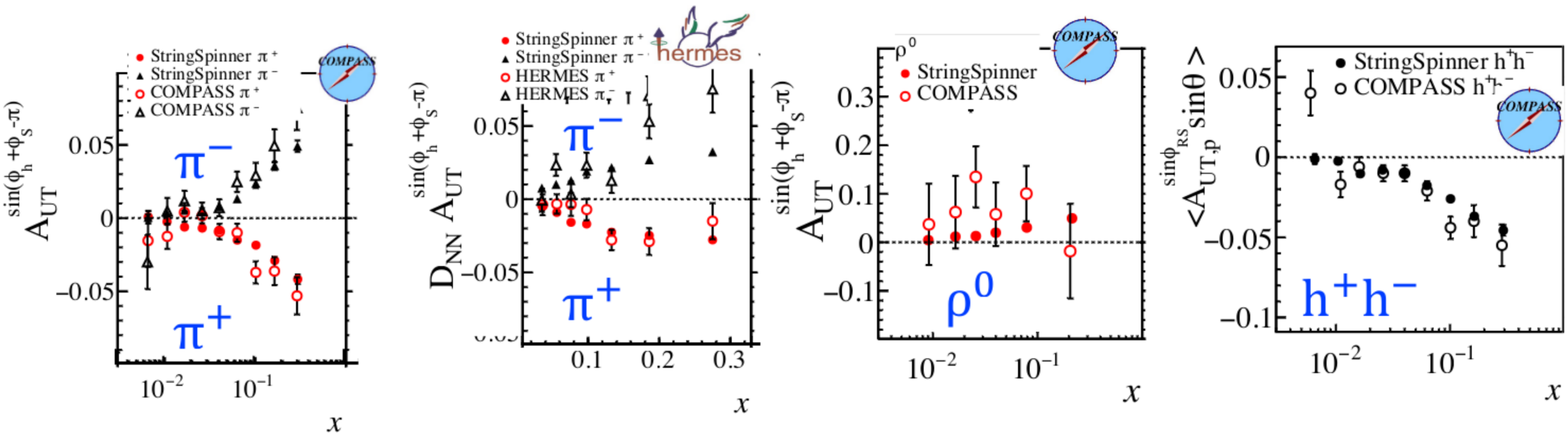
AK, Artru, Belghobsi, Martin, PRD 100, 014003 (2019) PS

AK, Artru, Martin, PRD 104, 114038 (2021) PS + VM

Implemented in Pythia for SIDIS → StringSpinner

AK, L. Lönnblad, CPC **272** (2022) 108234 PS, Pythia 8.2

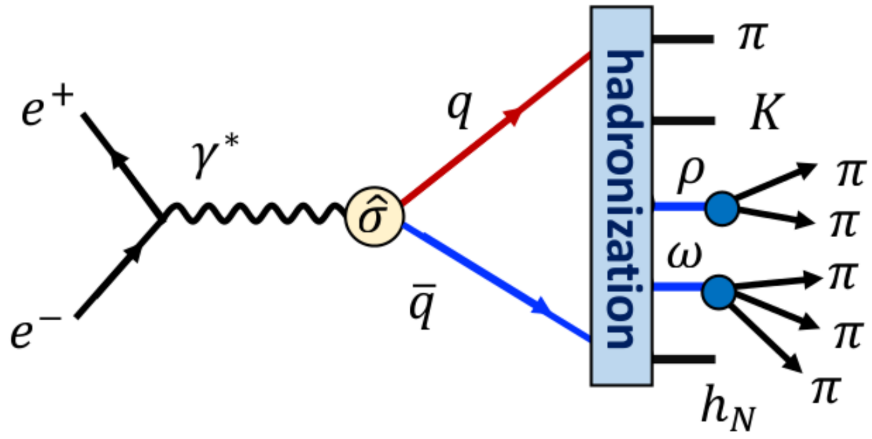
AK, L. Lönnblad, arXiv: 2305.05058 PS + VM, Pythia 8.3



Promising results for SIDIS! (more in AK, L. Lönnblad, arXiv: 2305.05058)

Can we use the model for e^+e^-?

The recursive recipe for simulating e^+e^- annihilation



Steps:
1. Hard scattering
2. Joint spin density matrix
3. Hadron emission from q
4. Update density matrix
5. Hadron emission from $\bar{q}$
6. Exit condition

The goal is to hadronize the $q\bar{q}$ system by using the string+$^3P_0$ model and accounting for

i) **correlated spin states** of q and $\bar{q}$

ii) **quantum mechanical spin-correlations** in the fragmentation chain

in collaboration with X. Artru

The recursive recipe for simulating e^+e^- annihilation

Steps:

1. Hard scattering

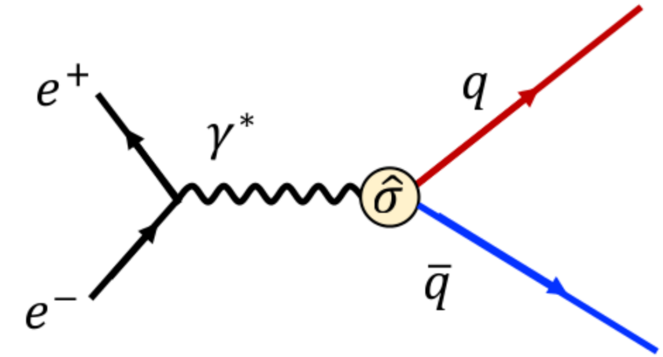
2. Joint spin density matrix

3. Hadron emission from q

4. Update density matrix

5. Hadron emission from $\bar{q}$

6. Exit condition



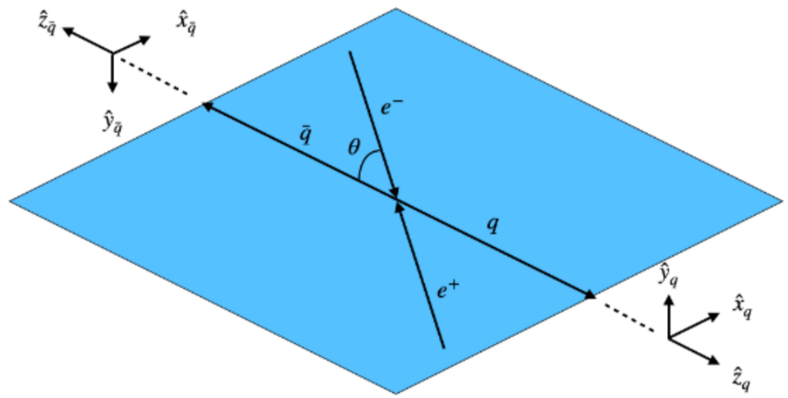
Set up the scattering $e^+e^- \to q\bar{q}$ in the c.m.s

generate the quark flavors and kinematics using

$$d\hat{\sigma}(q\bar{q})/d\cos\theta \propto \left\langle \left|\hat{M}\right|^2 \right\rangle$$

antiquark helicity frame (AHF)

$$\hat{\mathbf{z}}_q \propto \mathbf{k}_q$$
$$\hat{\mathbf{y}}_q \propto \mathbf{p}_- \times \hat{\mathbf{z}}_q$$
$$\hat{\mathbf{x}}_q = \hat{\mathbf{z}}_q \times \hat{\mathbf{y}}_q$$



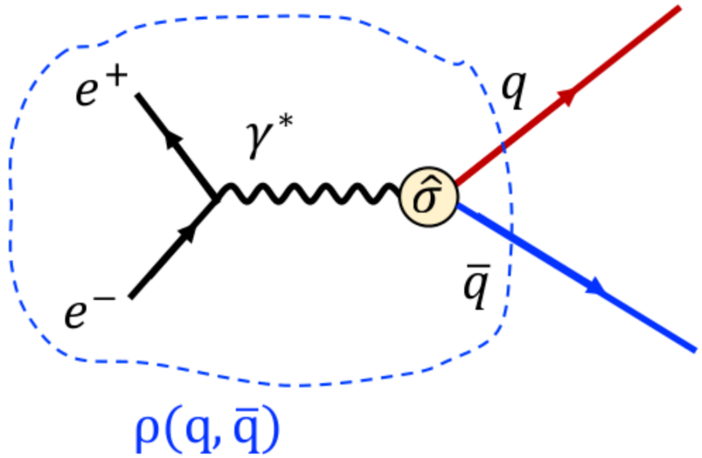
quark helicity frame (QHF)

$$\hat{\mathbf{z}}_q \propto \mathbf{k}_q$$
$$\hat{\mathbf{y}}_q \propto \mathbf{p}_- \times \hat{\mathbf{z}}_q$$
$$\hat{\mathbf{x}}_q = \hat{\mathbf{z}}_q \times \hat{\mathbf{y}}_q$$

The recursive recipe for simulating e^+e^- annihilation

Steps:

1. Hard scattering
2. **Joint spin density matrix**
3. Hadron emission from q
4. Update density matrix
5. Hadron emission from $\bar{q}$
6. Exit condition



Set up the joint spin density matrix of the $q\bar{q}$ pair

$$\rho(q,\bar{q}) = C^{q\bar{q}}_{\alpha\beta} \, \sigma^{\alpha}_{q} \otimes \sigma^{\beta}_{\bar{q}}$$

correlation coefficients

Pauli matrices along QHF and AHF

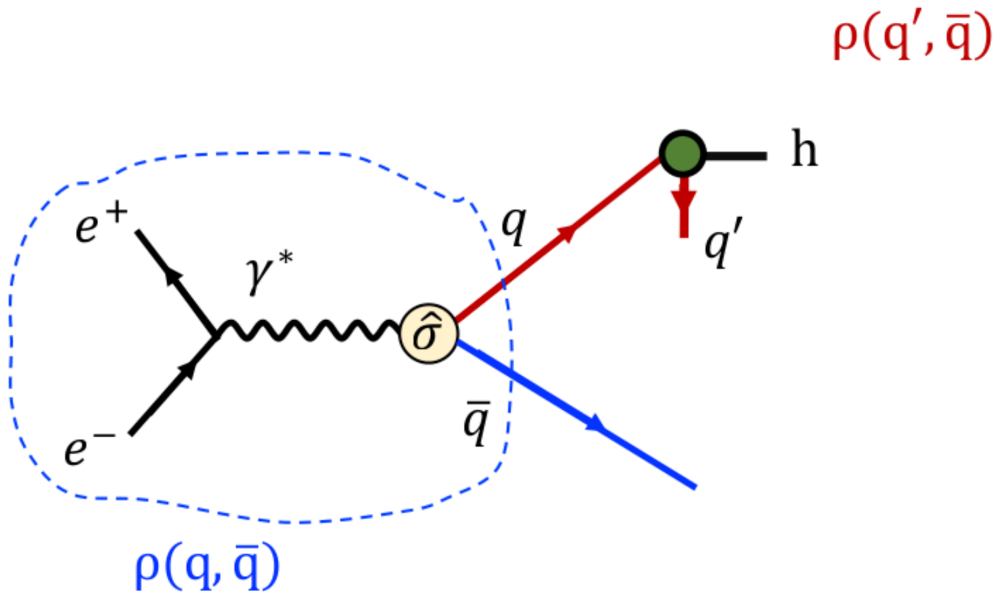
$\alpha = 0, x_q, y_q, z_q$

$\beta = 0, x_{\bar{q}}, y_{\bar{q}}, z_{\bar{q}}$

For γ^* exchange

$$\rho(q,\bar{q}) \propto 1_q \otimes 1_{\bar{q}} - \sigma^z_q \otimes \sigma^z_{\bar{q}} + \frac{\sin^2\theta}{1+\cos^2\theta}\left[\sigma^x_q \otimes \sigma^x_{\bar{q}} + \sigma^y_q \otimes \sigma^y_{\bar{q}}\right]$$

The recursive recipe for simulating e^+e^- annihilation



Steps:
1. Hard scattering
2. Joint spin density matrix
3. **Hadron emission from *q***
4. Update density matrix
5. Hadron emission from $\bar{q}$
6. Exit condition

Emit the first hadron using the splitting matrix of the string+$^3P_0$ model

splitting function (emission probability density)

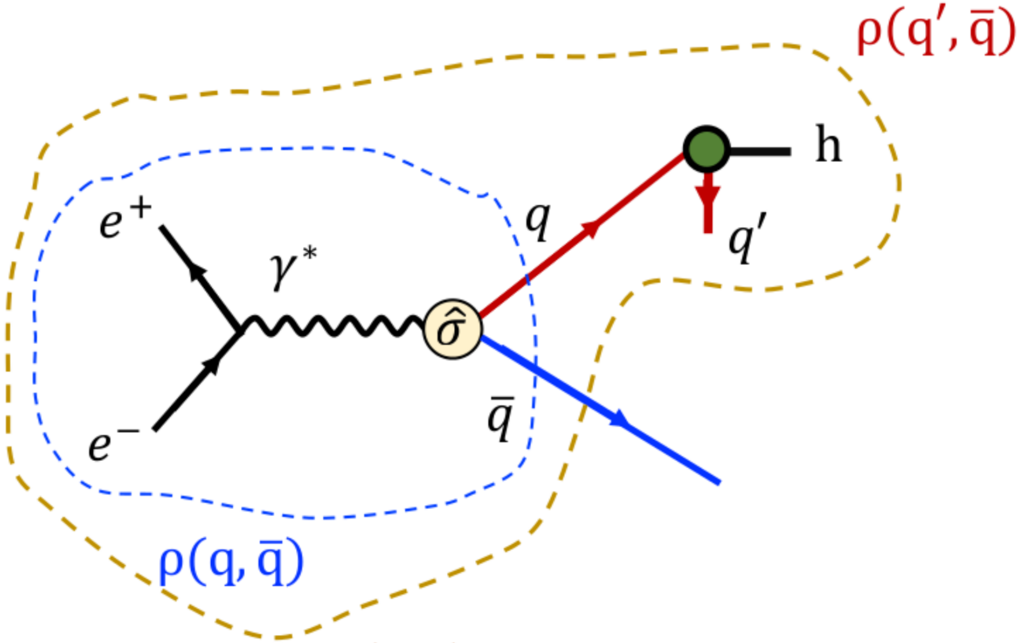
$$\frac{dP(q \to h + q'; q\bar{q})}{dZ_+ Z_+^{-1} d^2 p_T} = \mathrm{Tr}_{q'\bar{q}} \mathbf{T}_{q',h,q}\, \rho(q,\bar{q})\, \mathbf{T}^{\dagger}_{q',h,q} = F_{q',h,q}(Z_+, \mathbf{p}_T; \mathbf{k}_T, C^{q\bar{q}})$$

$$\mathbf{T}_{q',h,q} \equiv T_{q',h,q} \otimes 1_{\bar{q}}$$

in the QHF

For VM emission see backup

The recursive recipe for simulating e^+e^- annihilation



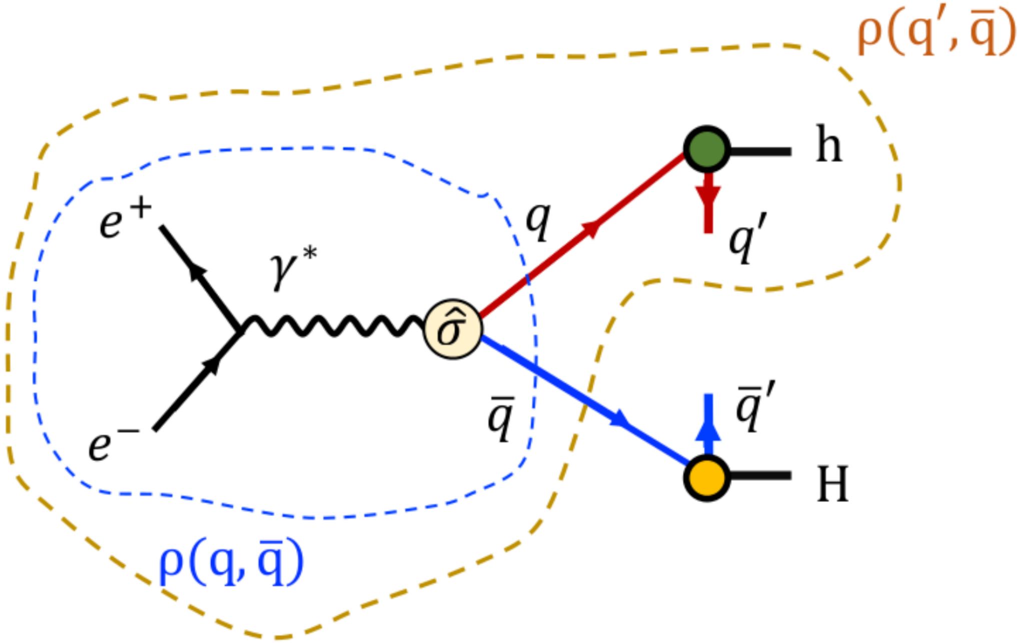
Steps:
1. Hard scattering
2. Joint spin density matrix
3. Hadron emission from q
4. **Update density matrix**
5. Hadron emission from $\bar{q}$
6. Exit condition

Evaluate the spin density matrix $\rho(q'\bar{q})$

$$\rho(q', \bar{q}) = \mathbf{T}_{q',h,q} \ \ \rho(q, \bar{q}) \ \ \mathbf{T}^{\dagger}_{q',h,q}$$

includes the information on the emission of h

The recursive recipe for simulating e^+e^- annihilation



Steps:
1. Hard scattering
2. Joint spin density matrix
3. Hadron emission from q
4. Update density matrix
5. **Hadron emission from $\bar{q}$**
6. Exit condition

Emit a hadron from the $\bar{q}$ side using the splitting function (emission probability density)

Depend on the azimuthal angle h

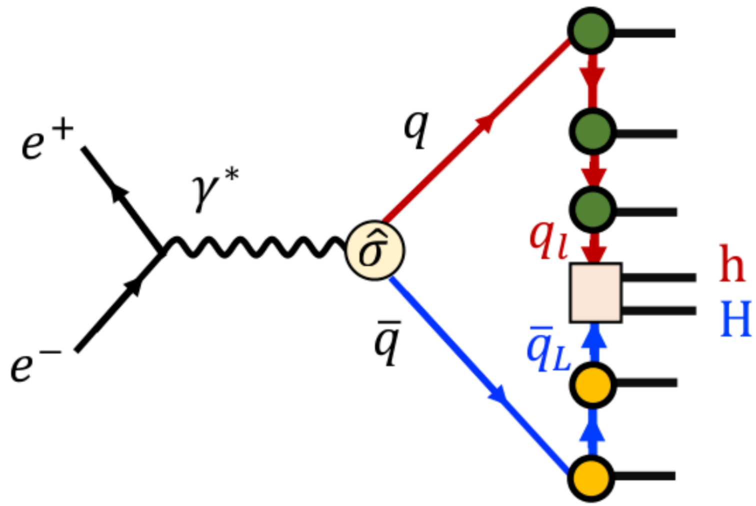
$$\frac{dP(\bar{q} \to H + \bar{q}'; q'\bar{q})}{dZ_- Z_-^{-1} d^2 P_T} = \mathrm{Tr}_{q'\bar{q}'} \mathbf{T}_{\bar{q}',H,\bar{q}}\, \rho(q',\bar{q})\, \mathbf{T}^{\dagger}_{\bar{q}',H,\bar{q}} = F_{\bar{q}',H,\bar{q}}(Z_-, P_T; \bar{k}_T, C^{q'\bar{q}})$$

Expressed in the AHF

conditional probability of emitting H, having emitted h
→ correlations between their transverse momenta

[Collins NPB, 304:794–804, 1988, Knowles NPB, 310:571–588, 1988]

The recursive recipe for simulating e^+e^- annihilation



Steps:
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4. Update density matrix
5. Hadron emission from $\bar{q}$
6. **Exit condition**

After several emissions, hadronize the last pair $q_l\bar{q}_L$

emit the hadron $h = q_l\bar{q}'$ from q_l and project $\bar{q}_L q'$ to the state H

$$dP(q_l \to h + q'; q_l\bar{q}_L) = \mathrm{Tr}_{q'\bar{q}_L}\left[T_{q',h,q_l} \otimes \Gamma_{H,s_H}\right] \quad \rho(q_l, \bar{q}_L) \quad \left[T^\dagger_{q',h,q_l} \otimes \Gamma^\dagger_{H,s_H}\right]$$

or emit the hadron $H = q'\bar{q}_L$ from $\bar{q}_L$ and project $q_l\bar{q}'$ to the state h

The model can be shown analytically to reproduce the expected form of the $e^+e^- \to h_1 h_2 X$ cross section

see backup

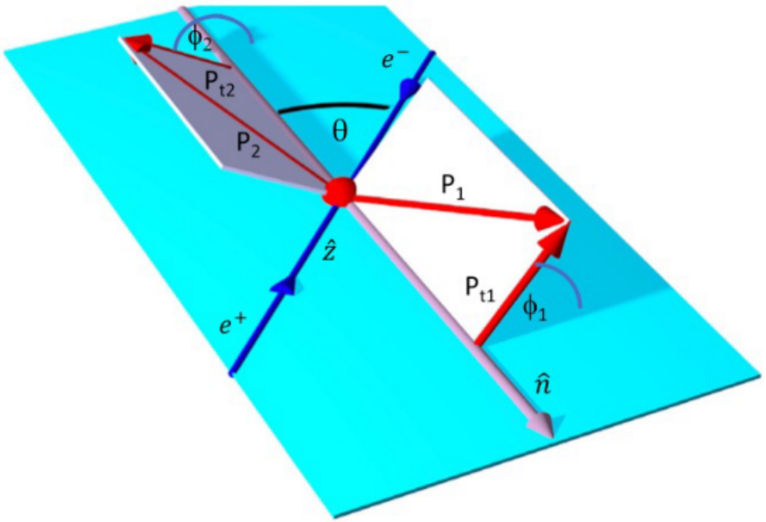
For quantitative results and phenomenology → implementation in Pythia 8.3 for e^+e^-

ongoing work, in collaboration with A. Martin and L. Lönnblad

Simulations of e^+e^- with Pythia 8.3

We are extending the StringSpinner package to e^+e^-

Next slides → preliminary results on Collins asymmetries for back-to-back hadrons



Simulations (caveats)

$e^+e^- \to q\bar{q} \to h_1h_2X$ @ $\sqrt{s} = 10$ GeV, $\theta = \frac{\pi}{2}$

only PS meson production

$q\bar{q} = u\bar{u}$

$q\bar{q}$ axis instead of the thrust axis

complex mass as in CPC **272** (2022) 108234
(to account for the absence of VMs)

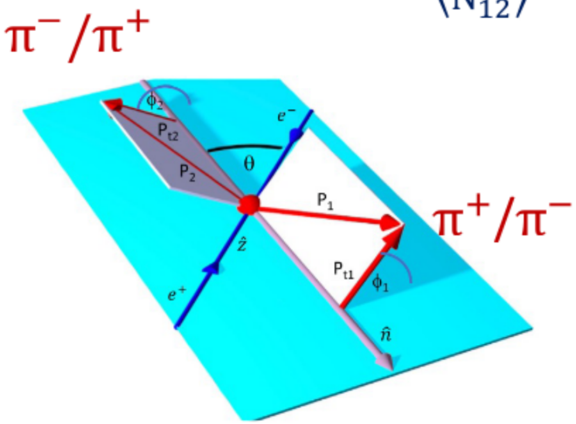
Steps for the extraction of Collins asymmetries

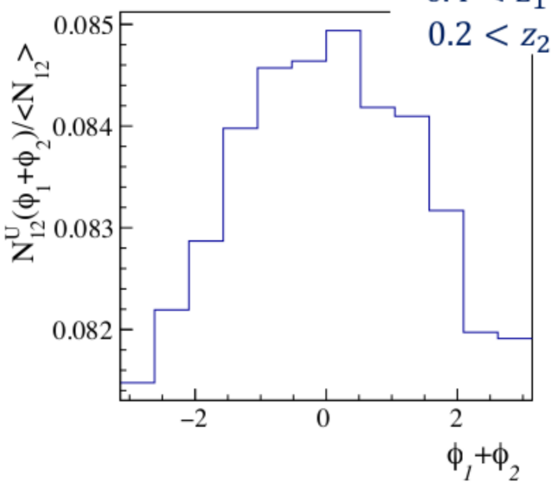
Example of $e^+e^- \to \pi^+\pi^- X$

i) Evaluate normalized yields for

$\pi^{\pm} - \pi^{\mp}$ "Unlike pairs"

$$R_{12}^{U} = \frac{N_{12}^{U}(\phi_1 + \phi_2)}{\langle N_{12} \rangle}$$





Steps for the extraction of Collins asymmetries

Example of $e^+e^- \to \pi^+\pi^- X$

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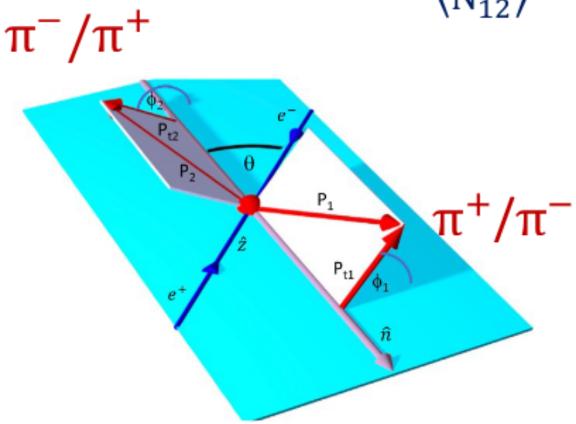
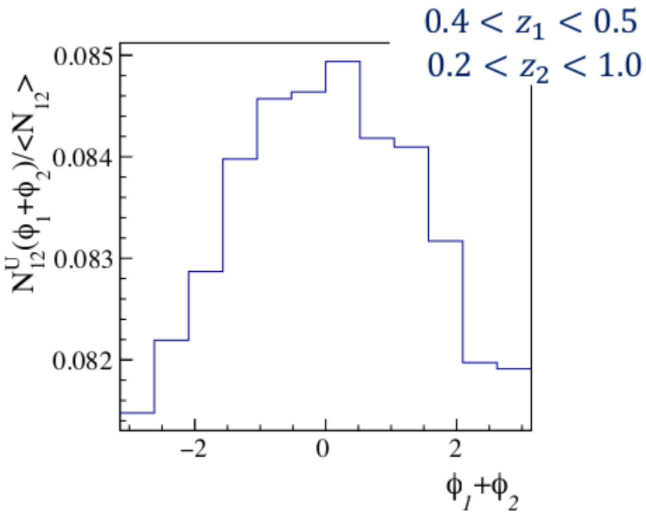
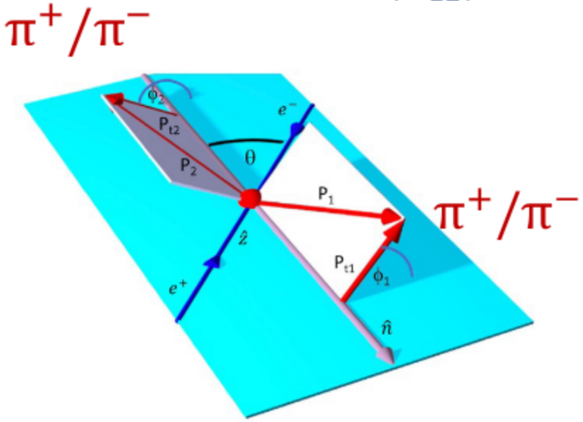
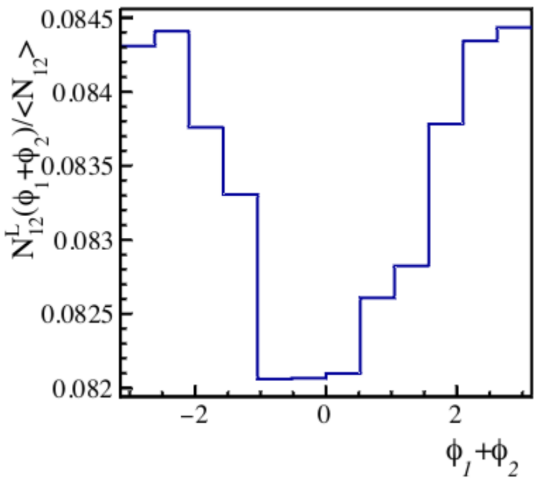




ii) Evaluate normalized yields for $\pi^\pm - \pi^\pm$ "Like pairs"

$$R_{12}^{L} = \frac{N_{12}^{L}(\phi_1 + \phi_2)}{\langle N_{12} \rangle}$$



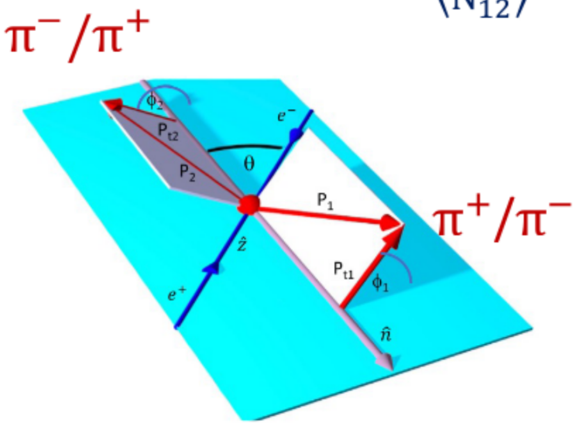


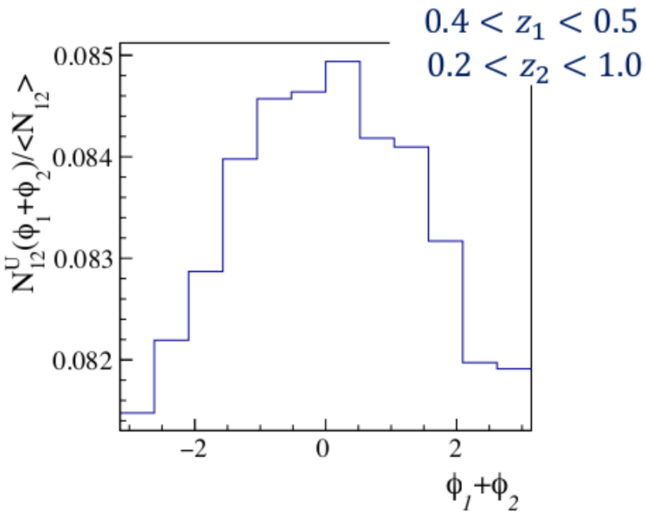
Steps for the extraction of Collins asymmetries

Example of $e^+e^- \to \pi^+\pi^- X$

i) Evaluate normalized yields for $\pi^\pm - \pi^\mp$ "Unlike pairs"

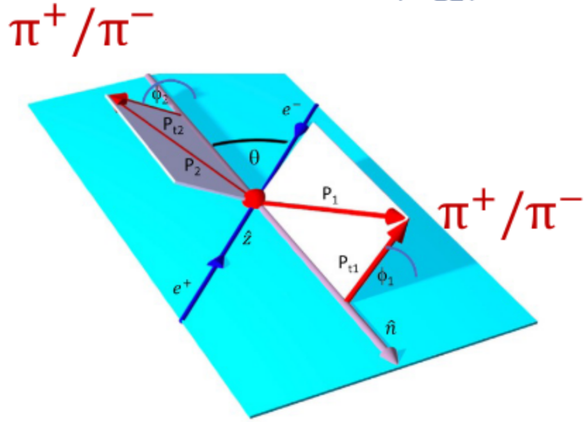
$$R^U_{12} = \frac{N^U_{12}(\phi_1+\phi_2)}{\langle N_{12}\rangle}$$

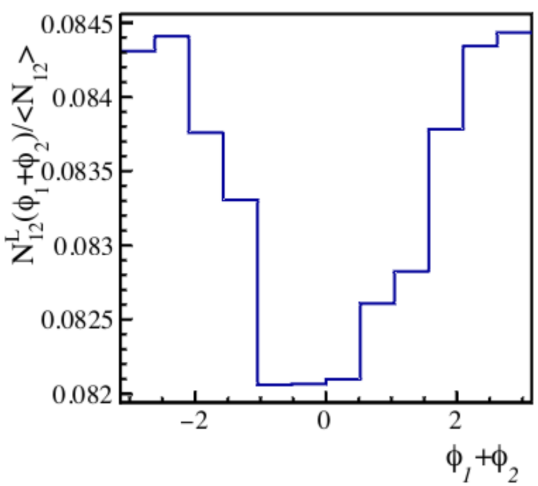




ii) Evaluate normalized yields for $\pi^\pm - \pi^\pm$ "Like pairs"

$$R^L_{12} = \frac{N^L_{12}(\phi_1+\phi_2)}{\langle N_{12}\rangle}$$



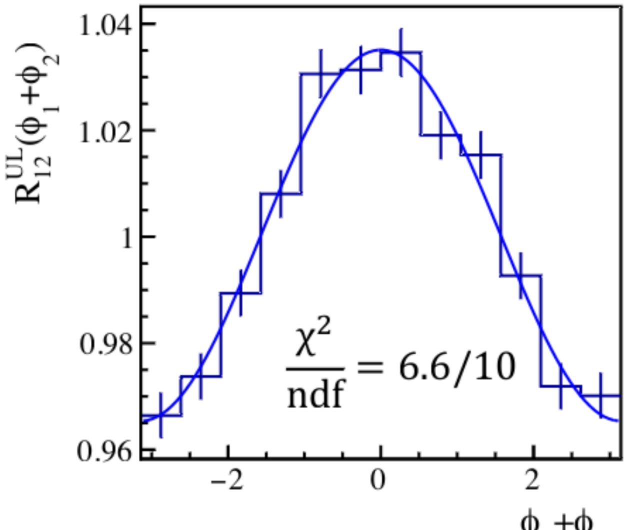


ii) Evaluate the ratio $\frac{R^U_{12}}{R^L_{12}}$ and fit the asymmetry

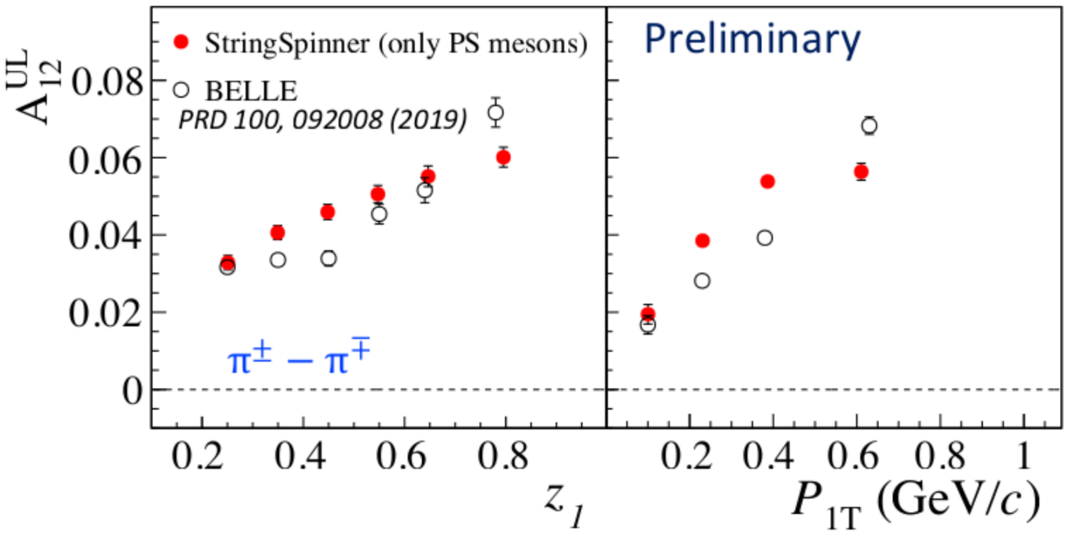
$$R^{UL}_{12} = \frac{R^U_{12}}{R^L_{12}} \approx 1 + A^{UL}_{12}\cos(\phi_1+\phi_2)$$

Fit function

$$f(\phi_1+\phi_2) = p_0 + p_1\cos(\phi_1+\phi_2)$$



Preliminary results from simulations with Pythia 8.3



The asymmetry reproduces the qualitative features of the data

- positive sign
- rising trend with z and PT
- comparable size

Introduction of VMs and decays is expected to change trends at small z and in PT

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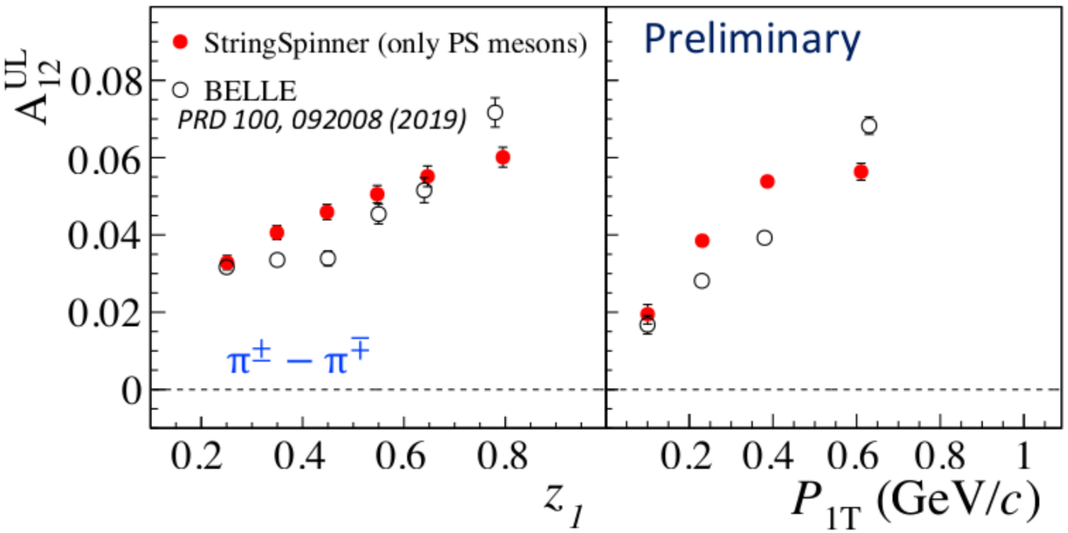

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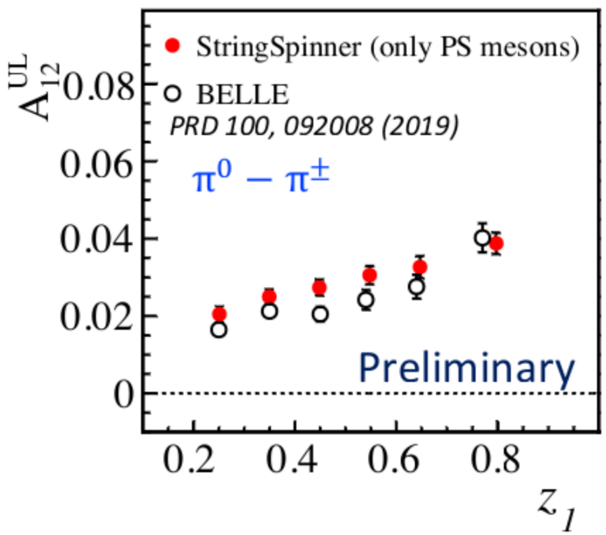
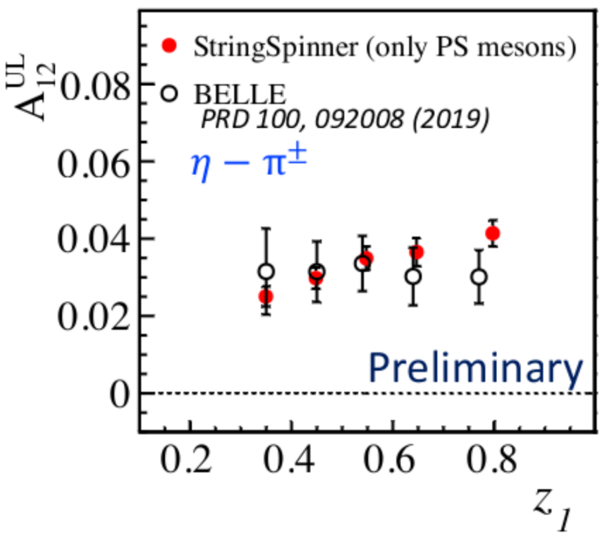




Conclusions

We generalized the string+$^3P_0$ model of hadronization to $e^+e^- \to q\bar{q} \to$ hadrons

recursive quantum mechanical recipe

The recipe is general

can be applied to other production channels of the $q\bar{q}$ pair

The implementation in Pythia 8.3 is ongoing

preliminary Collins asymmetry for back-to-back pions promising

(More) phenomenological studies ongoing

the goal is to publish the results in few months..

Backup

Relevant free parameters for string fragmentation used in simulations

(see AK, L. Lönnblad, arXiv: 2305.05058)

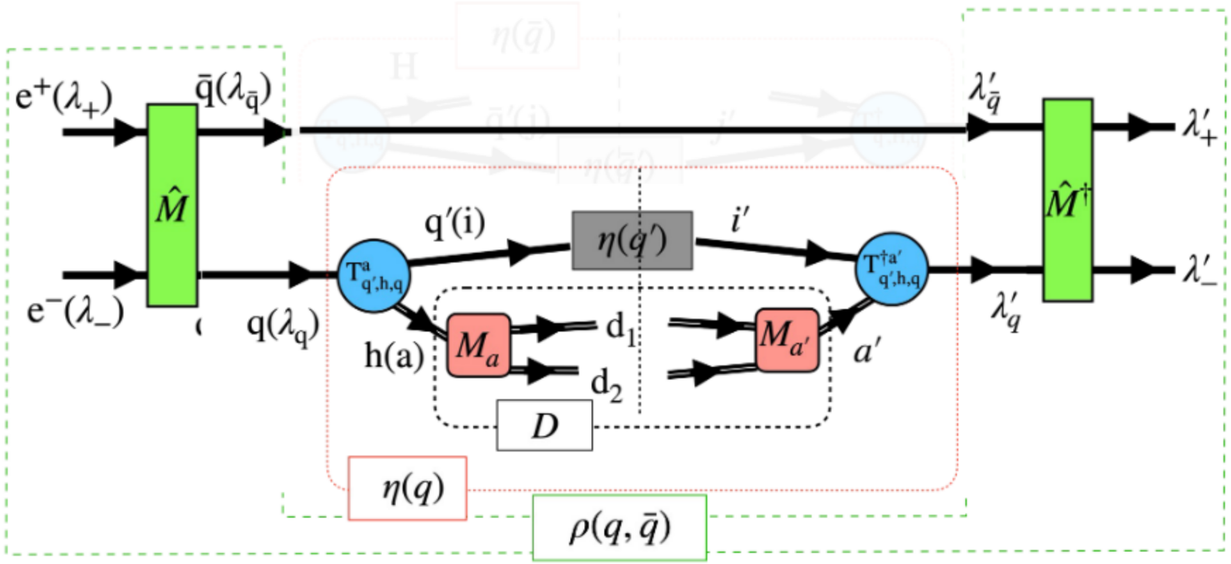
Pythia parameters

StringZ:aLund	default	
StringZ:bLund	default	
StringPT:sigma	default	
StringPT:enhancedFraction	0.0	
StringPT:enhancedWidth	0.0	GeV/c

String+$^3P_0$ parameters

$\mathrm{Re}(\mu)$	0.42	$\mathrm{GeV/c^2}$
$\mathrm{Im}(\mu)$	0.76	$\mathrm{GeV/c^2}$
f_L	0.93	
θ_{LT}	0	

The recursive recipe for simulating e^+e^- annihilation: VM emission



For a vector meson h=VM

$$\rightarrow \eta(q) = \mathbf{T}^{a'\dagger}_{q',h=VM,q}\, \eta(q')\, \mathbf{T}^{a}_{q',h=VM,q} D_{a'a},\ \eta(q') = 1_{q'} \text{ and } \eta(\bar{q}) = 1_{\bar{q}}$$

Steps:

i) Emission probability density (summing over decay information, i.e. $D_{a'a} = \delta_{a'a}$)

$$\frac{dP(q \to h = VM + q'; q\bar{q})}{dM^2 dZ_+ Z_+^{-1} d^2p_T} = \mathrm{Tr}_{q'\bar{q}}\, \mathbf{T}^{a}_{q',h,q}\ \rho(q,\bar{q})\ \mathbf{T}^{a\dagger}_{q',h,q} = F_{q',h,q}(M^2, Z_+, p_T; k_T, C^{q\bar{q}})$$

ii) Calculate the spin density matrix of h=VM, and decay the meson

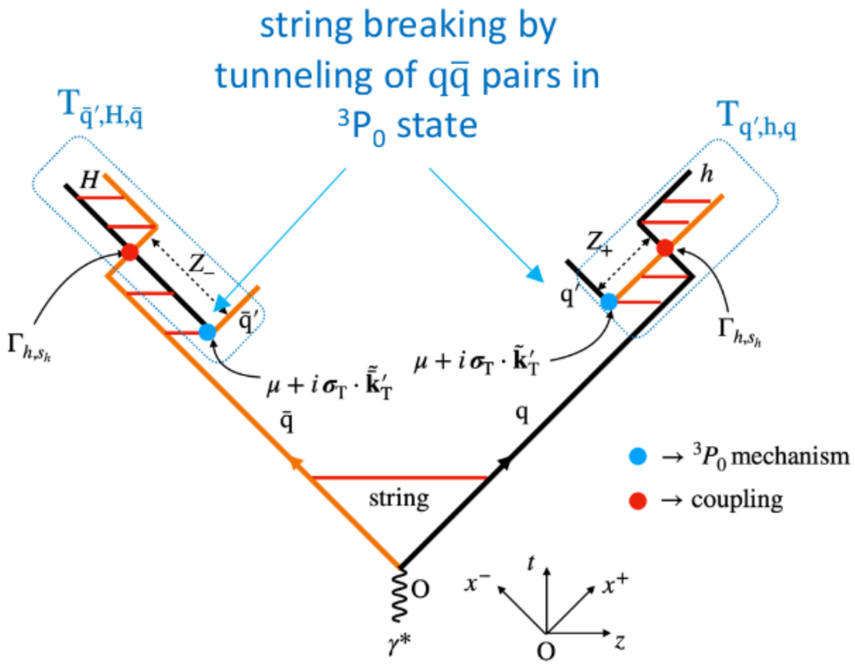
$$\rho_{aa'}(h) = \mathrm{Tr}_{q'\bar{q}}\, \mathbf{T}^{a}_{q',h,q}\, \rho(q,\bar{q})\, \mathbf{T}^{a'\dagger}_{q',h,q}$$

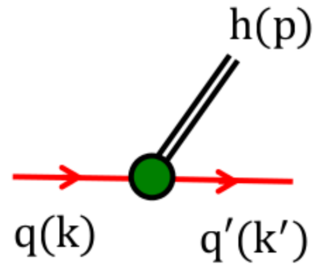
iii) Decay the meson $p \to p_1\, p_2 . .$

$$dN(p_1, p_2 . .)/d\Omega \propto M^{a}_{dec.}(p \to p_1 p_2 . .)\, \rho_{aa'}(h) M^{\dagger a'}_{dec.}(p \to p_1 p_2 . .)$$

iv) Build the decay matrix $D_{a'a}(p_1, p_2, . .) = M^{\dagger a'}_{dec.}(p \to p_1 p_2 . .)\, M^{a}_{dec.}(p \to p_1 p_2 . .)$

The hadronization model: string+$^3P_0$





quark splitting $q \to h + q'$

Relevant variables:
$\mathbf{k}_T = \mathbf{p}_T + \mathbf{k}'_T$
$Z_+ = p^+/k^+$
$\varepsilon_h^2 = M^2 + p_T^2$

Transverse vectors defined w.r.t. string axis

Quark splitting amplitude in the string+$^3P_0$ model

$$T_{q',h,q} \propto C_{q',h,q}\; D_h(M^2) \left(\frac{1-Z_+}{\varepsilon_h^2}\right)^{\frac{a}{2}} \exp\left[-\frac{b_L \varepsilon_h^2}{2Z_+}\right] N_a^{-\frac{1}{2}}(\varepsilon_h^2)\, e^{-\frac{b_T k'^2_T}{2}} \quad [\mu + \sigma_z \boldsymbol{\sigma}_T \cdot \mathbf{k}'_T] \quad \Gamma_{h,s_h}$$

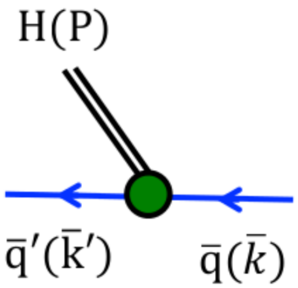
flavor — mass — longitudinal momentum — transverse momentum (w.r.t string axis) — $^3P_0$ mechanism [μ complex mass paramter] — Coupling e.g. $\Gamma_{h=PS} = \sigma_z$

Free param. Lund

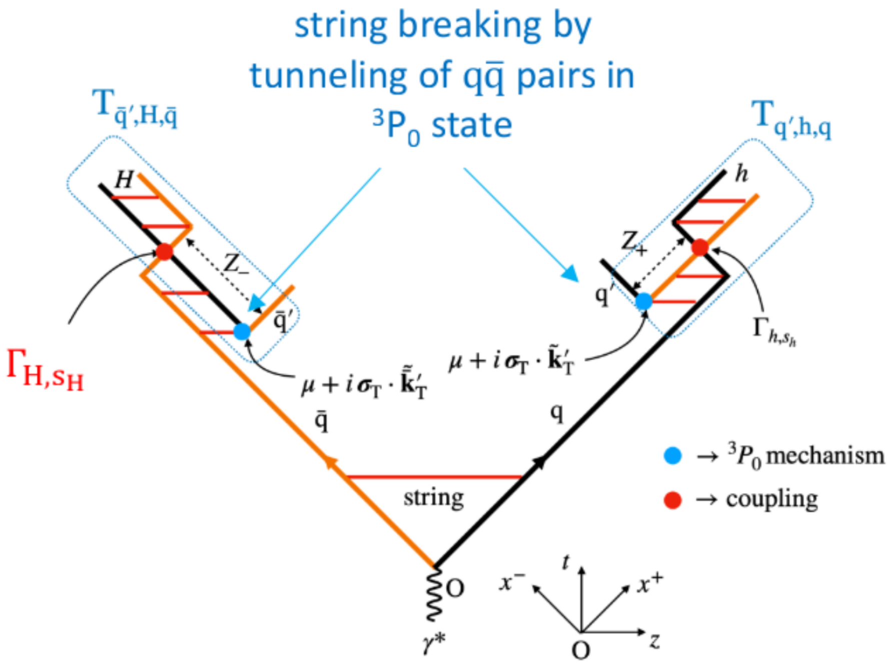
Free param. string+$^3P_0$

AK, Artru, Martin, PRD 104, 114038 (2021)

The hadronization model: **string+$^3P_0$**



antiquark splitting
$\bar{q} \rightarrow H + \bar{q}'$



Relevant variables:
$\bar{\mathbf{k}}_T = \mathbf{P}_T + \bar{\mathbf{k}}'_T$
$Z_- = P^-/\bar{k}^-$
$\varepsilon_H^2 = M^2 + P_T^2$

Antiquark splitting amplitude in the string+$^3P_0$ model obtained by the quark one by

$$\{q, h, q'\} \rightarrow \{\bar{q}, H, \bar{q}'\}, \qquad Z_+ \rightarrow Z_-, \qquad \{\mathbf{k}_T, \mathbf{p}_T \mathbf{k}'_T\} \rightarrow \{\bar{\mathbf{k}}_T, \mathbf{P}_T, \bar{\mathbf{k}}'_T\}$$

Application of the recipe to the first two hadrons produced

Application of the recipe to $e^+e^- \to h\,H\,X$

h = PS and H = PS being the first two hadrons produced

$$dP(e^+e^- \to h\,H\,X) = \hat{\sigma}^{-1}\frac{d\hat{\sigma}}{d\cos\theta} \times F_{q',h,q}(Z_+,\mathbf{p}_T;\mathbf{k}_T,C^{q\bar{q}}) \times F_{\bar{q}',H,\bar{q}}(Z_-,\bar{\mathbf{p}}_T;\bar{\mathbf{k}}_T,C^{q'\bar{q}})$$

$\mathrm{Prob}(e^+e^- \to q\bar{q})$ $\quad$ $\mathrm{Prob}(q \to h + q')$ $\quad$ $\mathrm{Prob}(\bar{q} \to H + \bar{q}'; q \to h + q')$

$$\propto (1+\cos^2\theta) \times (\dots) \times \left[1 + \frac{\sin^2\theta}{1+\cos^2\theta}\,\frac{2\mathrm{Im}(\mu)p_T}{|\mu|^2+p_T^2}\,\frac{2\mathrm{Im}(\mu)P_T}{|\mu|^2+P_T^2}\cos(\phi_h+\phi_H)\right]$$

expected form for the azimuthal distribution of back-to-back hadrons!

For quantitative results and phenomenology
→ implementation of the model in Pythia 8.3 for of $e^+e^- \to$ hadrons