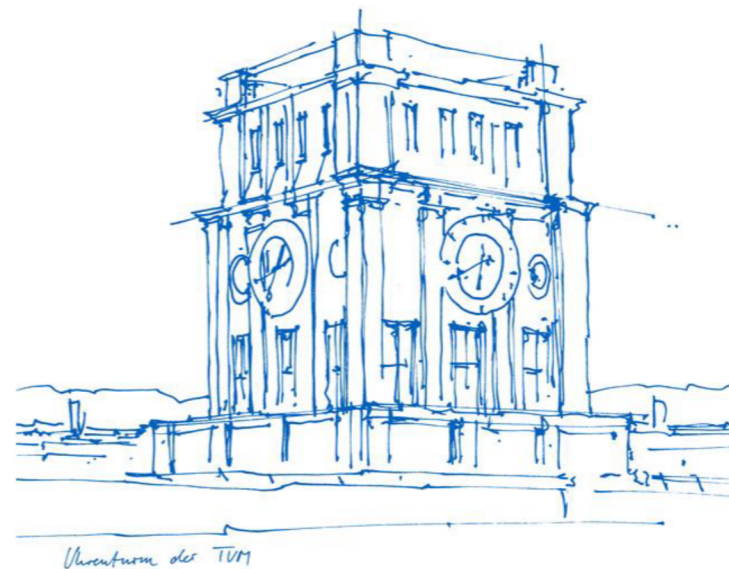


Testing Predictions of the Chiral Anomaly in Primakoff Reactions at COMPASS

Dominik Ecker, Andrii Maltsev on behalf of the COMPASS collaboration



- Lagrange density of QCD:

$$\mathcal{L}_{QCD} = \sum_{f=\substack{u,d,s, \\ c,b,t}} \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

- Lagrange density of QCD:

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→ Symmetry breaking term: $m_f = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$

Chiral limit: $m_u, m_d, m_s = 0$

- Lagrange density of QCD:

$$\mathcal{L}_{QCD} = \sum_{f=\substack{u,d,s, \\ c,b,t}} \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

- Features *axial* $U(1)$ -symmetry in chiral limit:

$$q(x) \rightarrow e^{i\theta\gamma_5} q(x)$$

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- Features *axial* $U(1)$ -symmetry in chiral limit:

$$q(x) \rightarrow e^{i\theta\gamma_5} q(x) \xrightarrow{\text{Noether}} \partial_\mu A_0^\mu = \sum_{f=u,d,s} i2m_f \bar{q}_f \gamma_5 q^f$$

The chiral anomaly

- Lagrange density of QCD:

$$\mathcal{L}_{QCD} = \sum_{f=\substack{u,d,s, \\ c,b,t}} \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

- Features *axial* $U(1)$ -symmetry in chiral limit:

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Noether

violates color symmetry

$$\partial_\mu A_0^\mu = \sum_{f=u,d,s} i2m_f \bar{q}_f \gamma_5 q_f$$

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- Features *axial* $U(1)$ -symmetry in chiral limit:

$$q(x) \rightarrow e^{i\theta\gamma_5} q(x)$$

~~Noether~~


$$\partial_\mu A_0^\mu = \sum_{f=u,d,s} i2m_f \bar{q}_f \gamma_5 q^f + \frac{3\alpha_s}{4\pi} \epsilon_{\mu\nu\rho\sigma} G^{\mu\nu} G_{\rho\sigma}$$

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- **Anomaly:** Symmetry of classical Lagrangian violated at quantum level (by renormalization choice)

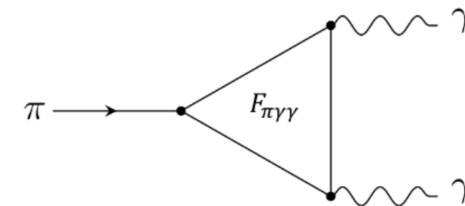
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- **Anomaly:** Symmetry of classical Lagrangian violated at quantum level (by renormalization choice)



- Adler, Bell, Jackiw 1969: $\tau_{\text{anom}}(\pi^0) = (9.5 \pm 1.5) \cdot 10^{-17} \text{s} \neq \tau_{\text{theory}}(\pi^0) \approx 10^{-13} \text{s}$

The chiral anomaly

- Lagrange density of QCD:

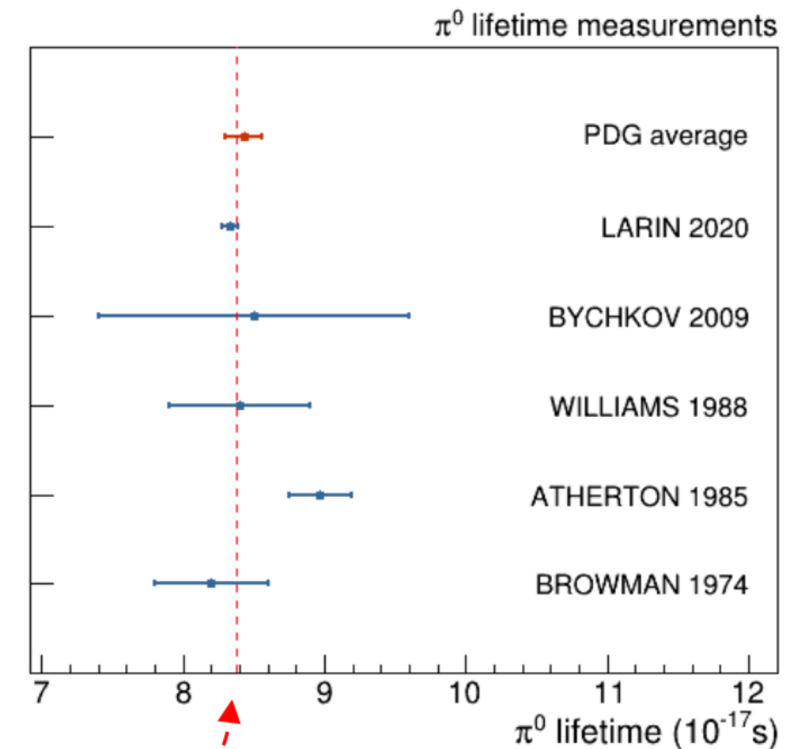
$$\mathcal{L}_{QCD} = \sum_{f=u,d,s,c,b,t} \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

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well tested in π^0 decay

- Chiral anomaly governs couplings of odd number of Goldstone bosons:

$SU(2)$ flavor	$SU(3)$ flavor
$\pi^0 \rightarrow \gamma\gamma$	$K^+K^- \rightarrow \pi^+\pi^-\pi^0$
$\gamma\pi^- \rightarrow \pi^-\pi^0$	$\eta \rightarrow \pi^+\pi^-\gamma$
$\pi^+ \rightarrow e^+\nu_e\gamma$	$K^+ \rightarrow \pi^+\pi^-e^+\nu_e$
etc.	etc.

- On tree-level: low-energy theorems with few parameters, e.g. pion decay constant F_π measured from leptonic decays of the charged pion ($\pi^\pm \rightarrow \mu^\pm + \nu$)
- Higher order corrections via Chiral Perturbation Theory (ChPT)

$F_{\pi\gamma\gamma}$

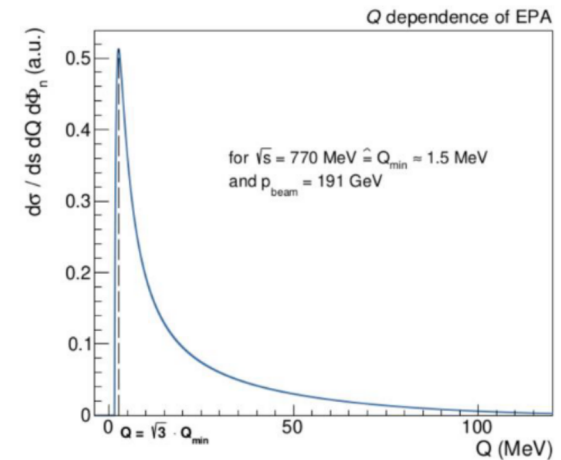
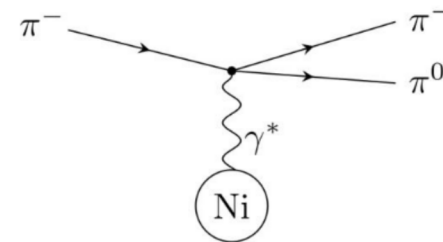
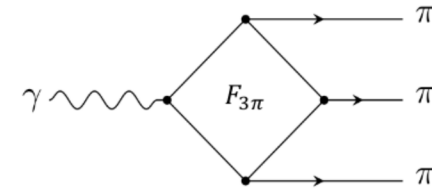
• $F_{\pi\gamma\gamma} = \frac{e^2 N_C}{12\pi^2 F_\pi} = 2.52 \cdot 10^{-2} \text{GeV}^{-1}$

$F_{3\pi}$

• $F_{3\pi} = \frac{e N_C}{12\pi^2 F_\pi^3} = (9.78 \pm 0.05) \text{GeV}^{-3}$

Testing the chiral anomaly - $F_{3\pi}$

- $F_{3\pi}$: Direct coupling of γ to 3π - process proceeds primarily via the chiral anomaly
- Accessible in Primakoff reactions via: $\pi^- \gamma^* \rightarrow \pi^- \pi^0$ ultra-relativistic pion scatters in e.m. field of nucleus (characterized by very low momentum transfer)



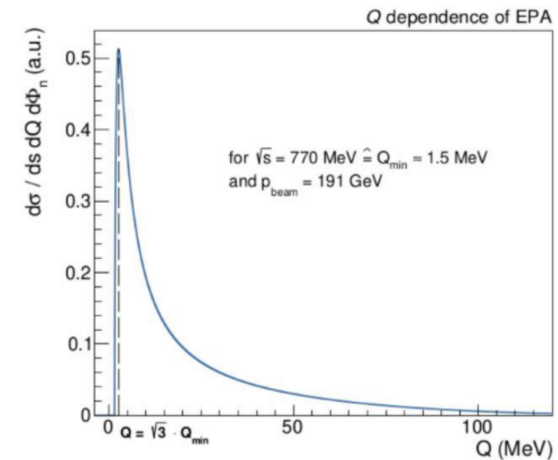
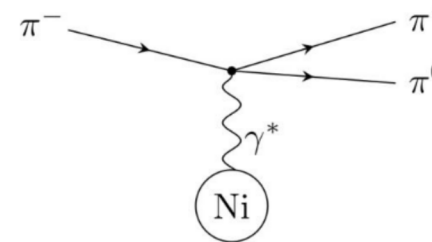
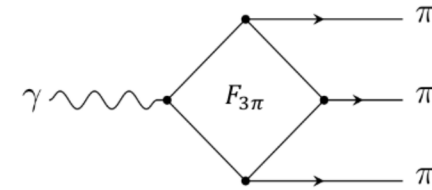
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- Accessible in Primakoff reactions via: $\pi^- \gamma^* \rightarrow \pi^- \pi^0$ ultra-relativistic pion scatters in e.m. field of nucleus (characterized by very low momentum transfer)
- Problem of explicit chiral symmetry breaking:

$$F_{3\pi} = \frac{eN_C}{12\pi^2 F_\pi^3} = (9.78 \pm 0.05) \text{GeV}^{-3} = F(s = t = u = 0)$$

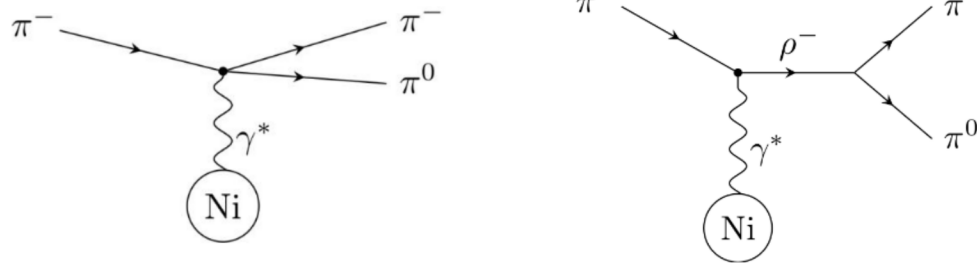
We measure at $s > (2m_\pi)^2$: use ChPT to bridge „gap“

$$F_{3\pi}(s, t, u) = F_{3\pi} (f^{(0)}(s, t, u) + f^{(1)}(s, t, u) + f^{(2)}(s, t, u) + \dots)$$



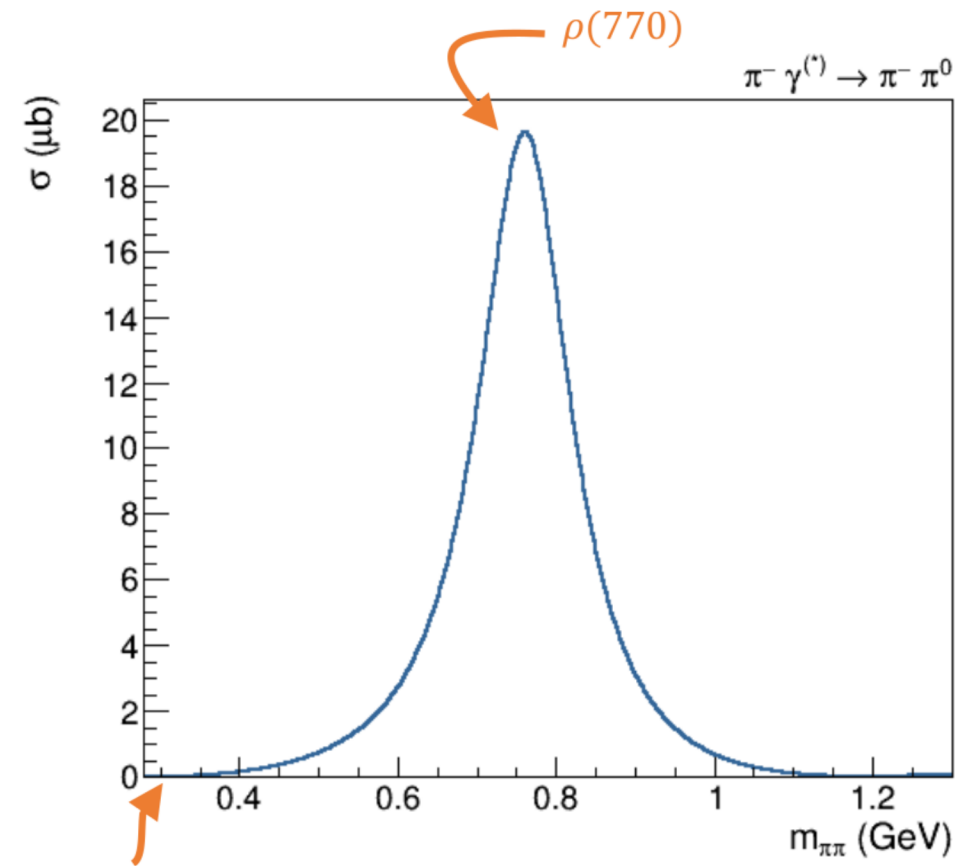
Radiative width of ρ -meson

- Cross section of $\pi^- \pi^0$ final state result of two coherent processes:



- At kinematic threshold: dominated by chiral anomaly
- Interference between Chiral Anomaly and ρ gives additional information

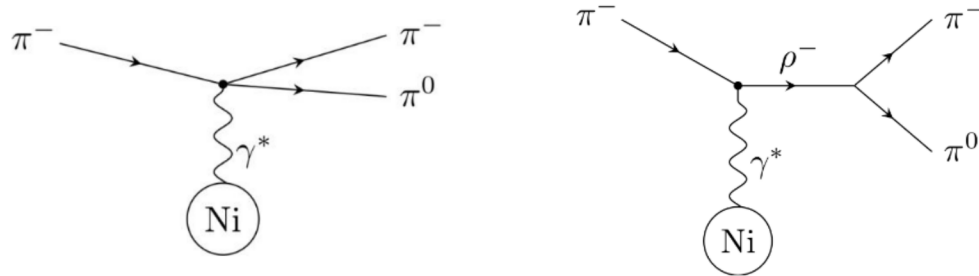
⇒ possibility of extraction of radiative width of ρ -meson:
 $\Gamma_{(\rho \rightarrow \pi\gamma)} / \Gamma_{\text{tot}} \approx 4.5 \cdot 10^{-4}$



Low-mass tail:
 mainly driven by $F_{3\pi}$

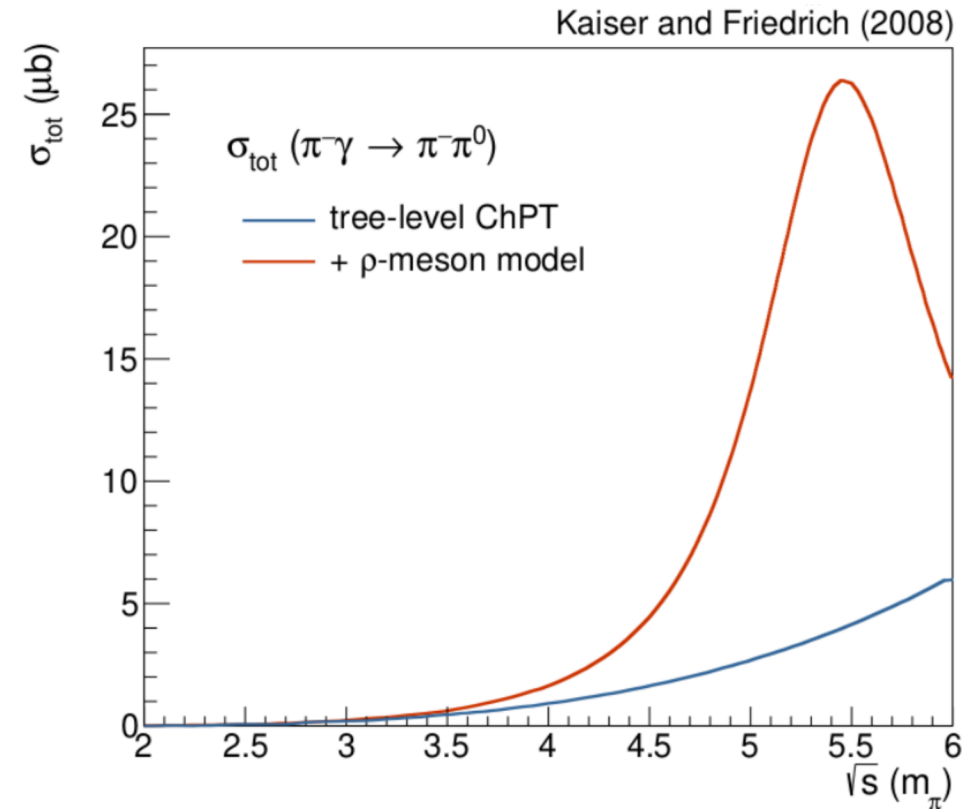
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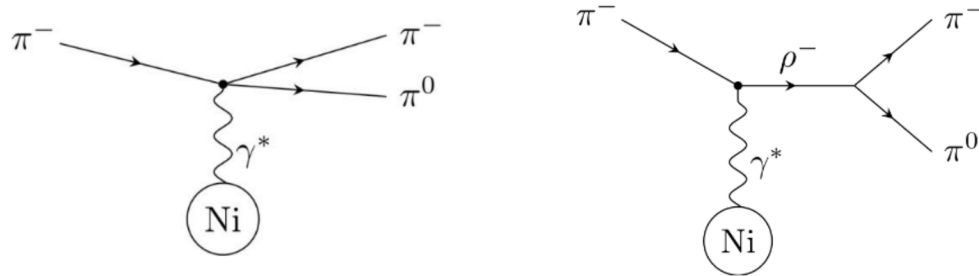
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[Kaiser, N. and Friedrich, J. M., EPJA 36 no. 2, \(2008\) 181–188](#)

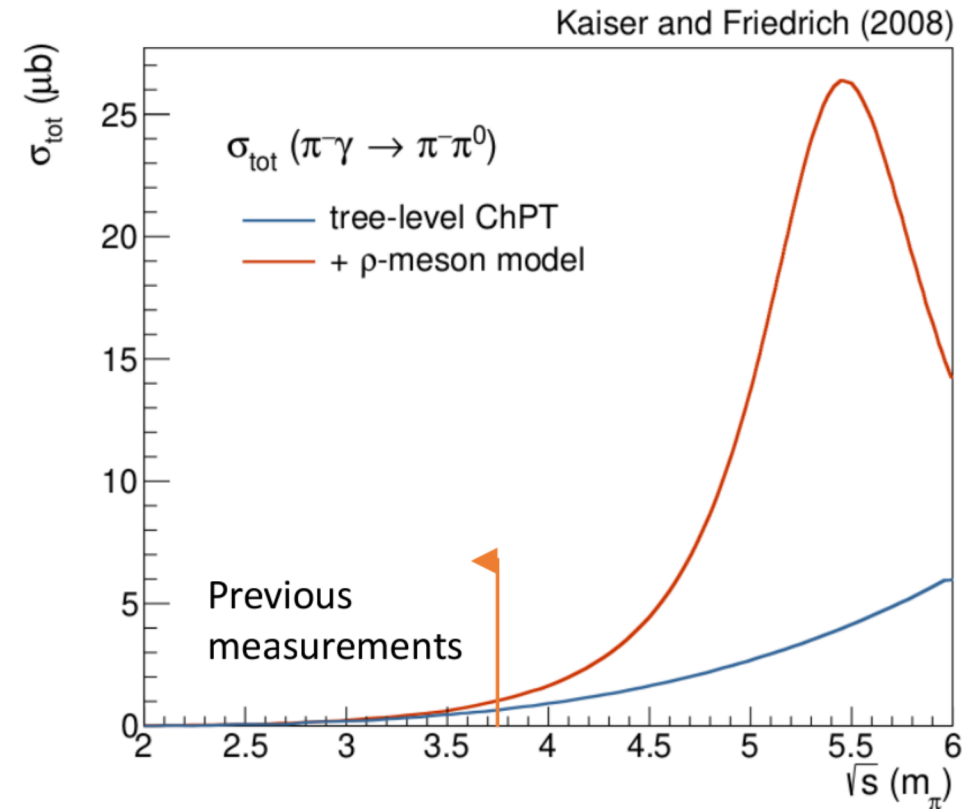
Radiative width of ρ -meson

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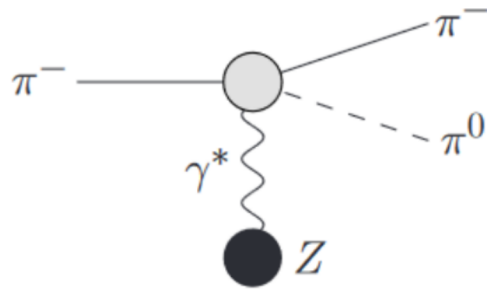
- At kinematic threshold: dominated by chiral anomaly
- Interference between Chiral Anomaly and ρ gives additional information

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[Kaiser, N. and Friedrich, J. M., EPJA 36 no. 2, \(2008\) 181–188](#)

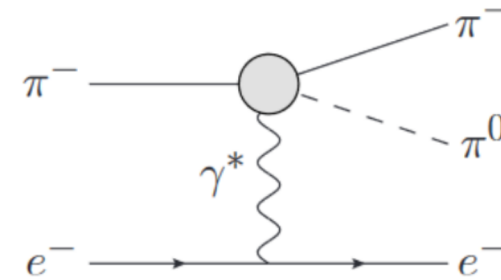
Previous measurements – $F_{3\pi}$



[Antipov, Y. et al. PRD 36 \(1987\) 101103](#)
and reanalyzed by
[Ametller, L. et al. PRD 64 \(2001\) 094009](#)

$$F_{3\pi} = (10.7 \pm 1.2) \text{ GeV}^{-3}$$

- Neglecting s -channel production of ρ meson
- No proper consideration of systematics



[Giller, I. et al. EPJ. A25 \(2005\) 229-240](#)
from cross-section data of
[Amendolia, S.R. et al., PLB 155, 457 \(1985\)](#)

$$F_{3\pi} = (9.6 \pm 1.1) \text{ GeV}^{-3}$$

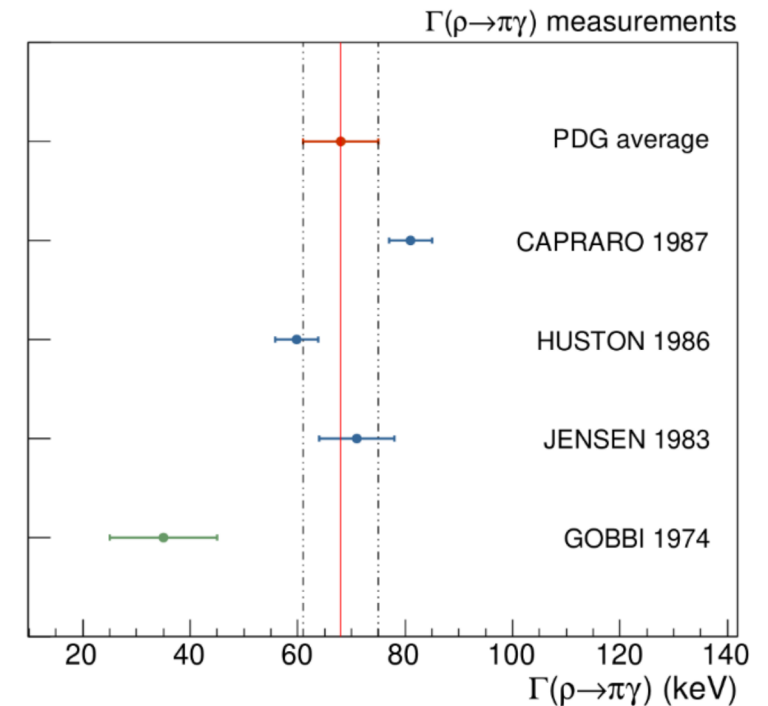
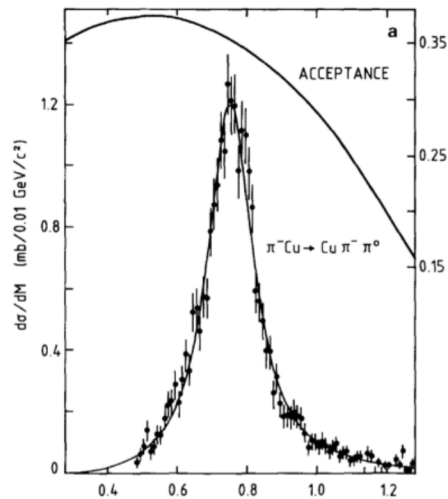
- Neglecting s -channel production of ρ meson
- No proper consideration of systematics
- Dominant background of elastically scattered pions

Previous measurements – $\Gamma_{\rho \rightarrow \pi\gamma}$

Radiative width of ρ -meson:

[Capraro, L. et al. Nucl.Phys. B288 \(1987\) 659-680](#)
at CERN (SPS):

- From fit to cross section (BW shape):
 $\Gamma(\rho \rightarrow \pi\gamma) = (81 \pm 4 \pm 4) \text{ keV}$



$\pi\gamma \rightarrow \pi\pi$ from dispersion relations

- Cross section:

$$\sigma(s) = \frac{(s - 4m_\pi^2)^{\frac{3}{2}}(s - m_\pi^2)}{1024\pi\sqrt{s}} \int_{-1}^{+1} dz (1 - z^2) |F_{3\pi}(s, t, u)|^2$$

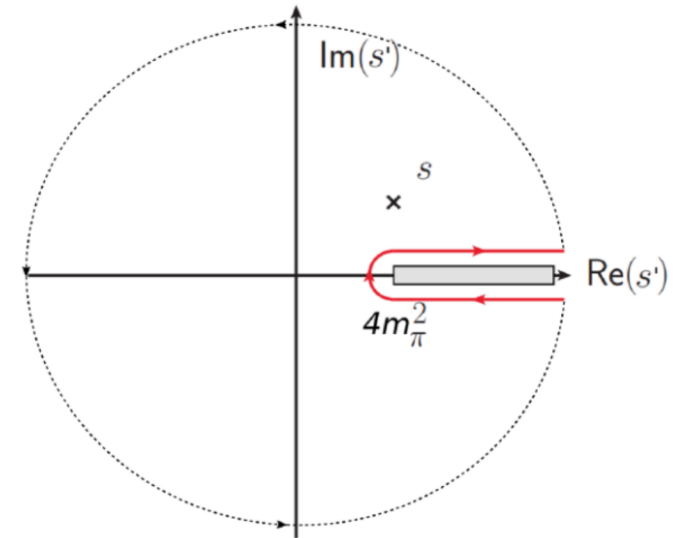
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- Dispersive framework to deduce $F_{3\pi}$ from a fit to the full data set up to 1.0 GeV including the $\rho(770)$ -resonance:

$$F_{3\pi}^{\text{DR}}(s) = \frac{1}{3} (C_2^{(1)} + C_2^{(2)} s) + \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^2} \frac{s^2}{s' - s} \times (C_2^{(1)} \text{Im}\mathcal{F}_2^{(1)}(s') + C_2^{(2)} \text{Im}\mathcal{F}_2^{(2)}(s'))$$



$\pi\gamma \rightarrow \pi\pi$ from dispersion relations

- Cross section:

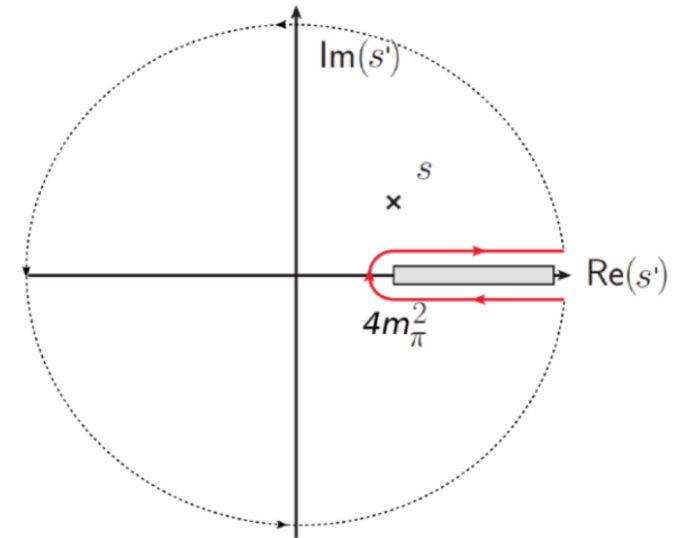
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Fit parameters

Basis functions provided in:



[M. Hoferichter, B. Kubis, and D. Sakkas, *PRD* **86** \(2012\) 116009](#)

[M. Hoferichter, B. Kubis, and M. Zanke, *PRD* **96** \(2017\) 114016](#)

$\pi\gamma \rightarrow \pi\pi$ from dispersion relations

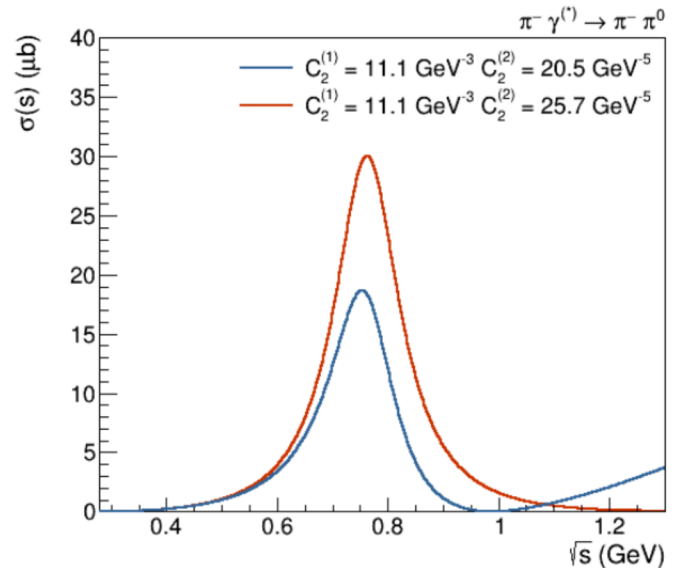
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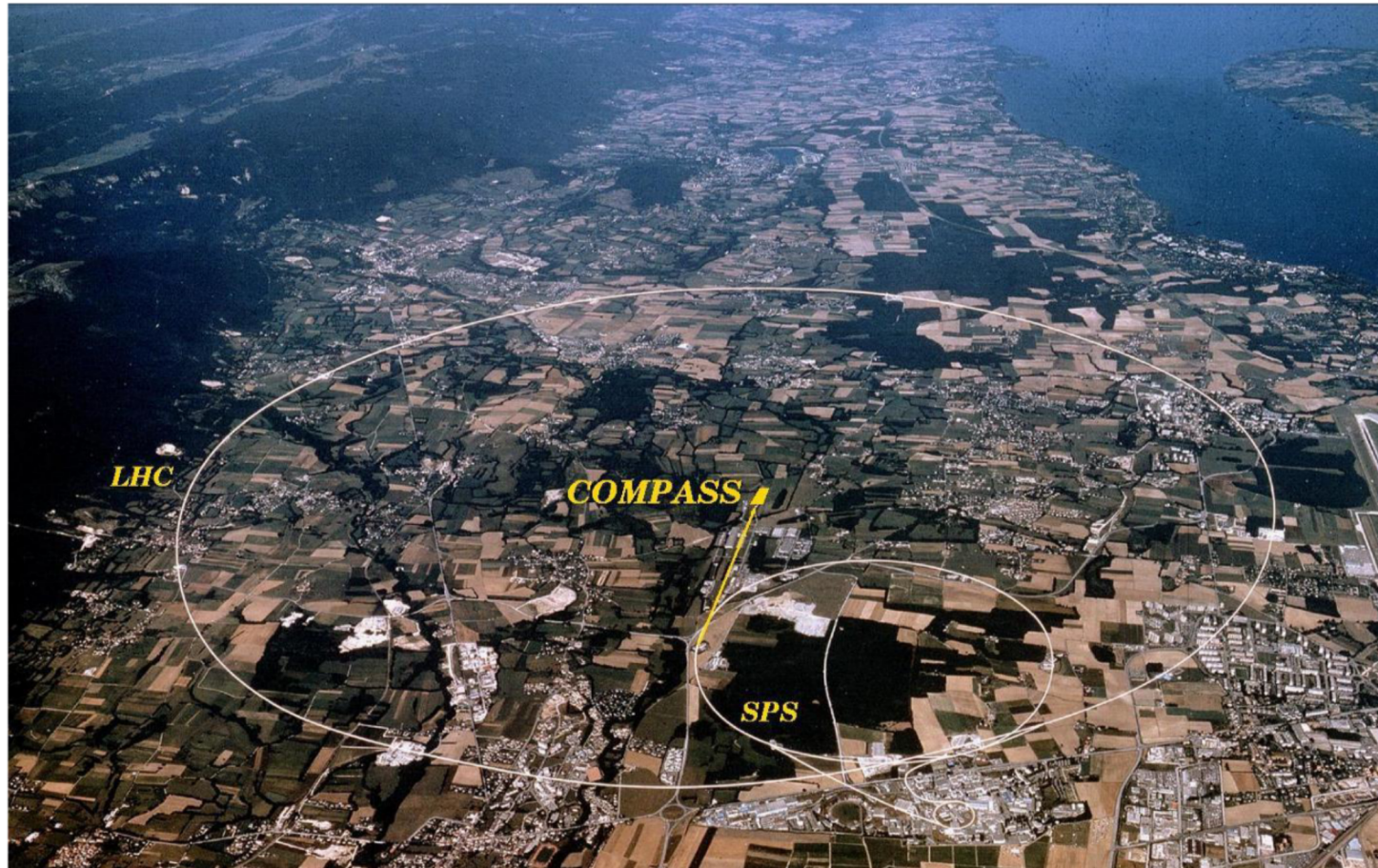


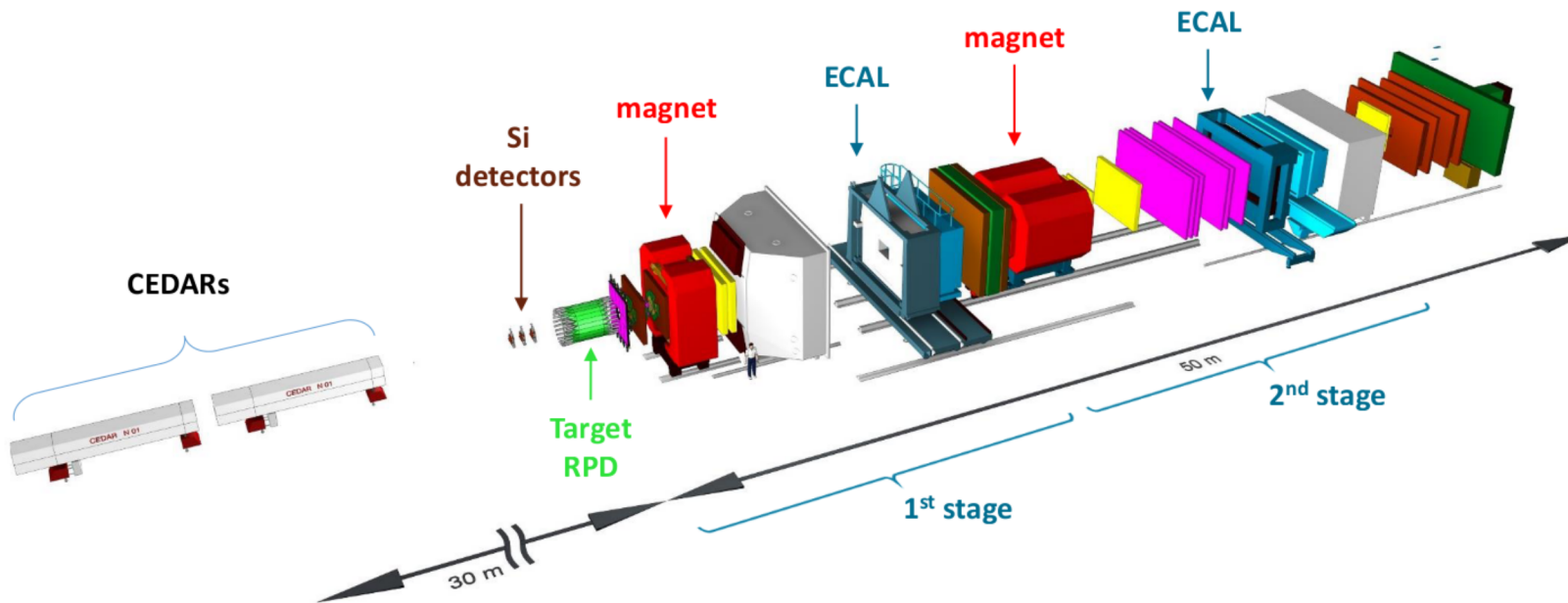
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Common Muon and Proton Apparatus for Structure and Spectroscopy

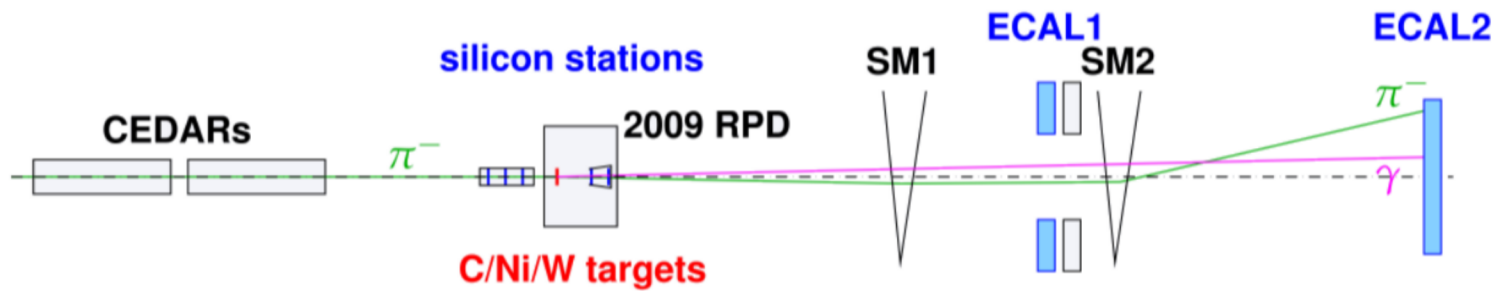




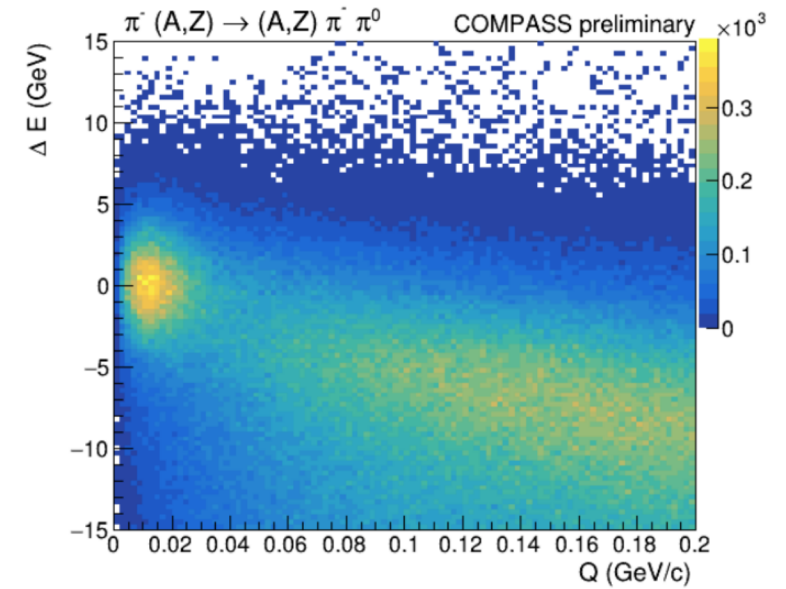
- 190 GeV negative hadron beam:
96.8% π^- , 2.4% K^- , 0.8% \bar{p}
- Beam PID by Cherenkov detectors
- Two stage magnetic spectrometer
- 4mm Ni target disk ($\approx 25\% X/X_0$)
- Calorimetric trigger on photons

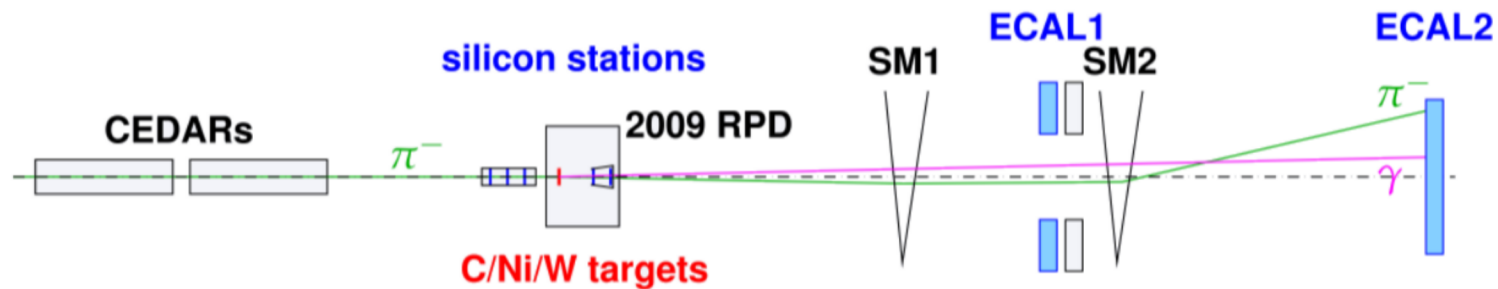
[Abbon, P. et al. NIM A 779 \(2014\) 69–115](#)

Principle of measurement



- Measure scattered π^- and photons of π^0 decay
- Select exclusive events at very low Q^2

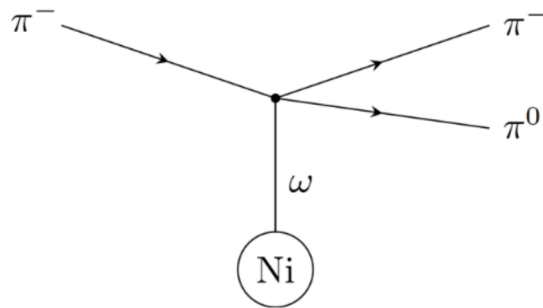




- Measure scattered π^- and photons of π^0 decay
- Select exclusive events at very low Q^2
- For absolute cross-section measurements: Luminosity
Indirect determination of luminosity via free Kaon decays

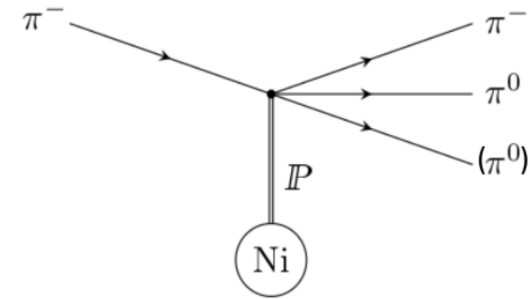


$$\int L dt = (5.21 \pm 0.04_{\text{stat}} \pm 0.48_{\text{syst}}) \text{ nb}^{-1}$$



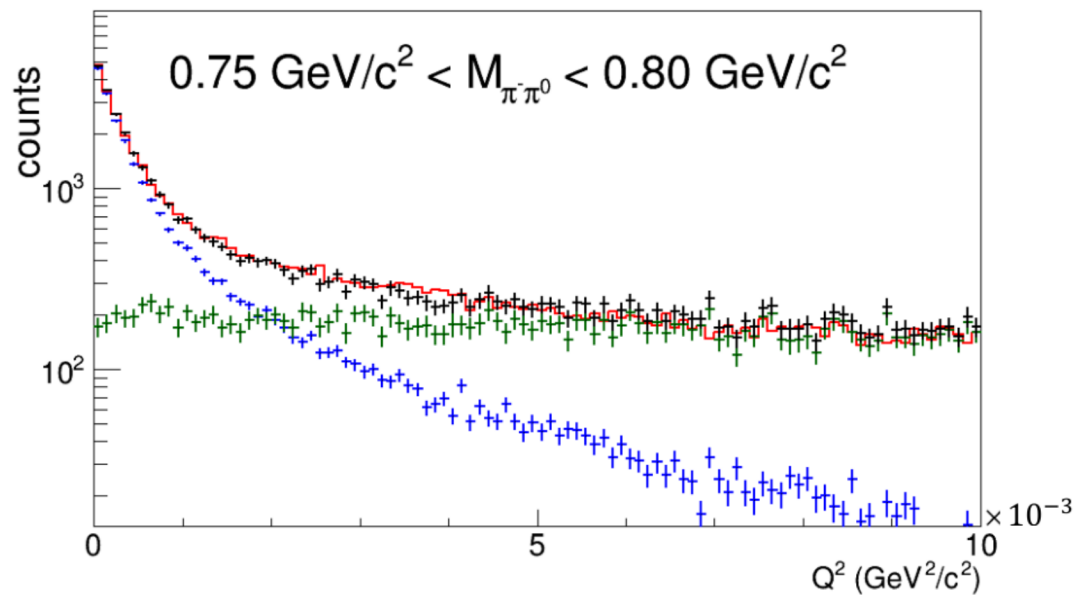
$\pi^- \pi^0$ via strong interaction

- Pomeron exchange: forbidden by G -parity conservation
- π and ω exchange: low cross section at COMPASS beam energies



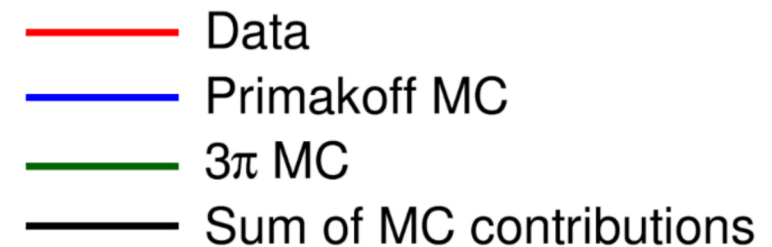
$\pi^- \pi^0 \pi^0$ via Pomeron exchange

- Large cross section
- Main background: loss of one (soft) π^0
- Approach:
 - Using the model from COMPASS $\pi^- \pi^0 \pi^0$ data
 - Apply $\pi^- \pi^0$ event selection -> realistic distributions of leakage in $\pi^- \pi^0$



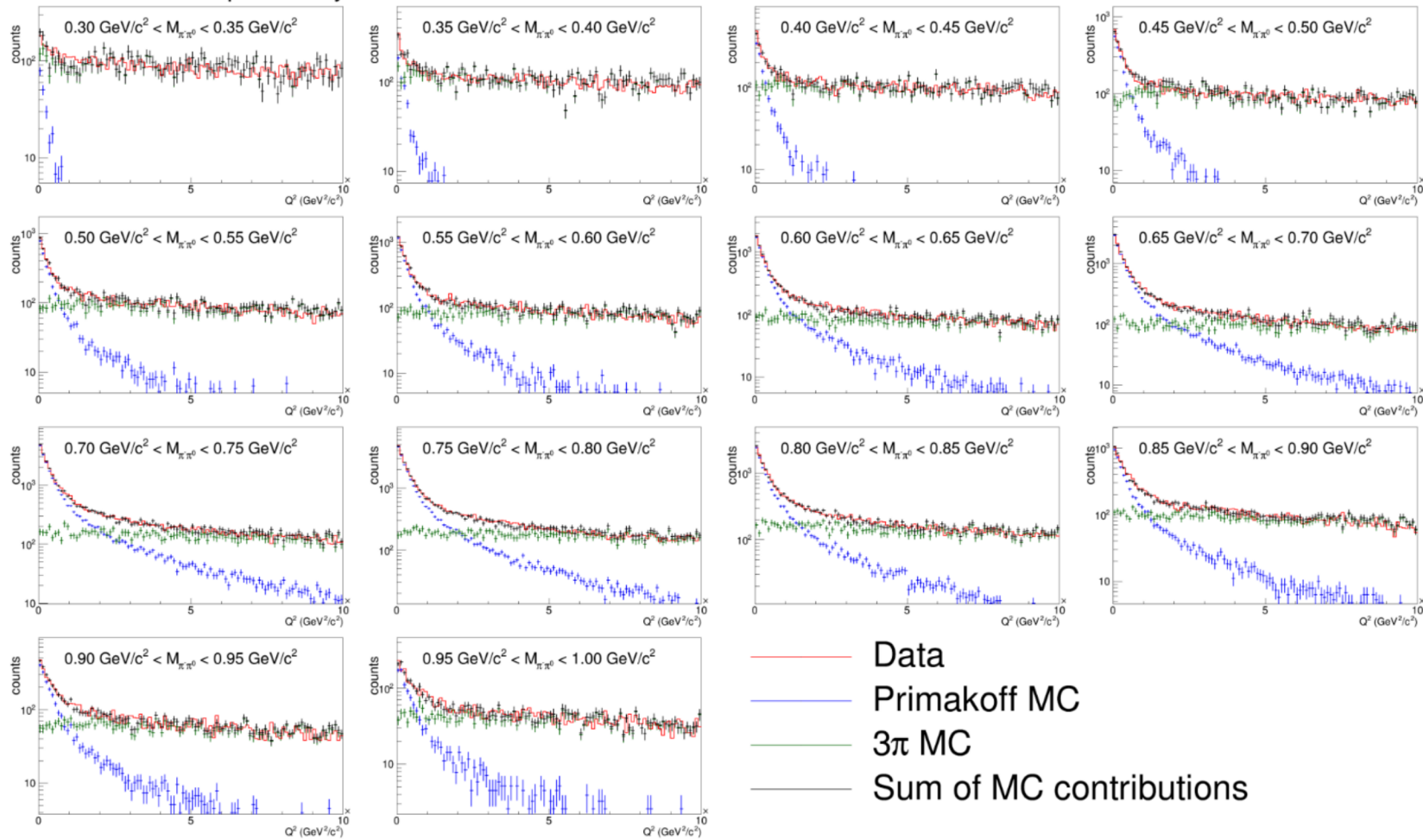
Model from COMPASS $\pi^-\pi^0\pi^0$ data:

- Realistic shapes for signal and background contributions
- Fit yields (signal vs background) to match observed momentum transfer distribution



Subtraction of 3π background

COMPASS preliminary



- Determine subtraction constants from fit
 - Use data up to $1 \text{ GeV}/c^2$
 - Exclude data around $500 \text{ MeV}/c^2$ due to background of free kaon decay

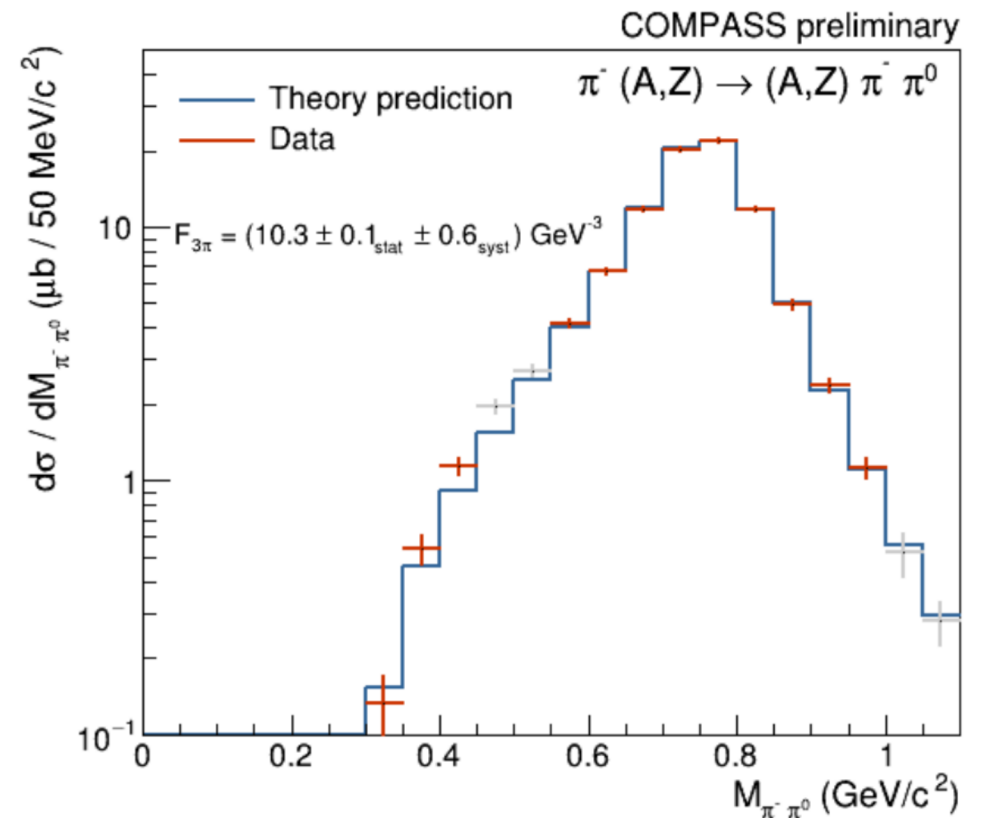
$$C_2^{(1)} = (10.5 \pm 0.1_{\text{stat}} \pm 0.6_{\text{syst}}) \text{GeV}^{-3}$$

$$C_2^{(2)} = (24.5 \pm 0.1_{\text{stat}}^{+1.6} \pm 1.4_{\text{syst}}) \text{GeV}^{-5}$$

- Use ChPT expansion (NLO) to determine $F_{3\pi}(0,0,0)$:

$$F_{3\pi} = (10.3 \pm 0.1_{\text{stat}} \pm 0.6_{\text{syst}}) \text{GeV}^{-3}$$

$$\Gamma_{\rho \rightarrow \pi\gamma} = (76 \pm 1_{\text{stat}}^{+10} \pm 8_{\text{syst}}) \text{keV}$$



Comparison to previous measurements

- COMPASS: **First combined** measurement of $F_{3\pi}$ and $\Gamma_{\rho \rightarrow \pi\gamma}$

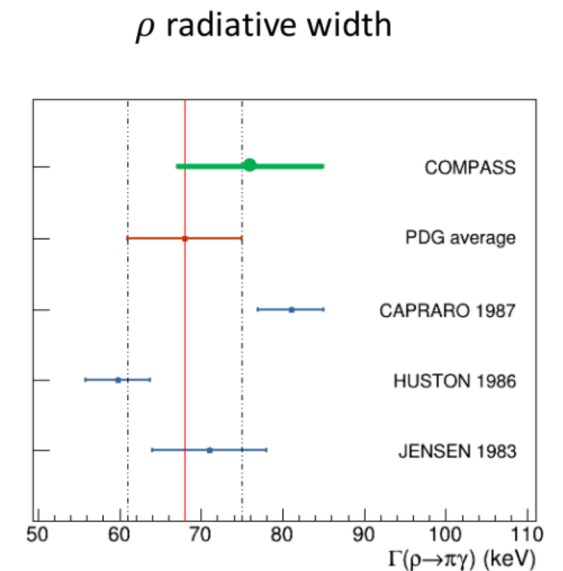
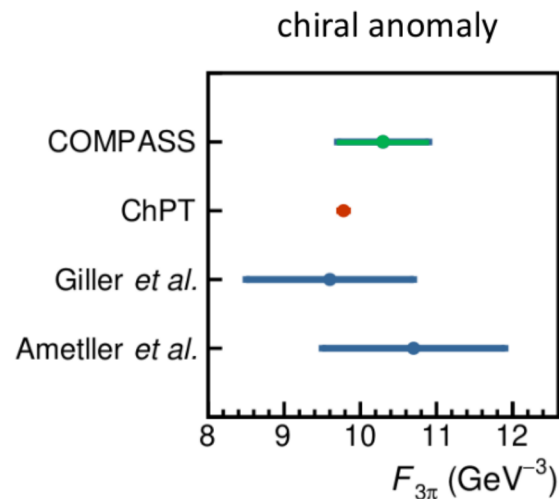
$$F_{3\pi} = (10.3 \pm 0.1_{stat} \pm 0.6_{syst}) \text{GeV}^{-3}$$

$$\Gamma_{\rho \rightarrow \pi\gamma} = (76 \pm 1_{stat}^{+10}_{-8} \text{ }_{syst}) \text{keV}$$

- Intensive test of systematics (dominant contributions):

- Luminosity
- Radiative corrections
- Background of ω , π exchange
- Background from $\pi\gamma$ final state

- Accompanied with intensive analysis of $\pi^- \text{Ni} \rightarrow \pi^- \pi^0 \pi^0 \text{Ni}$ for background estimation

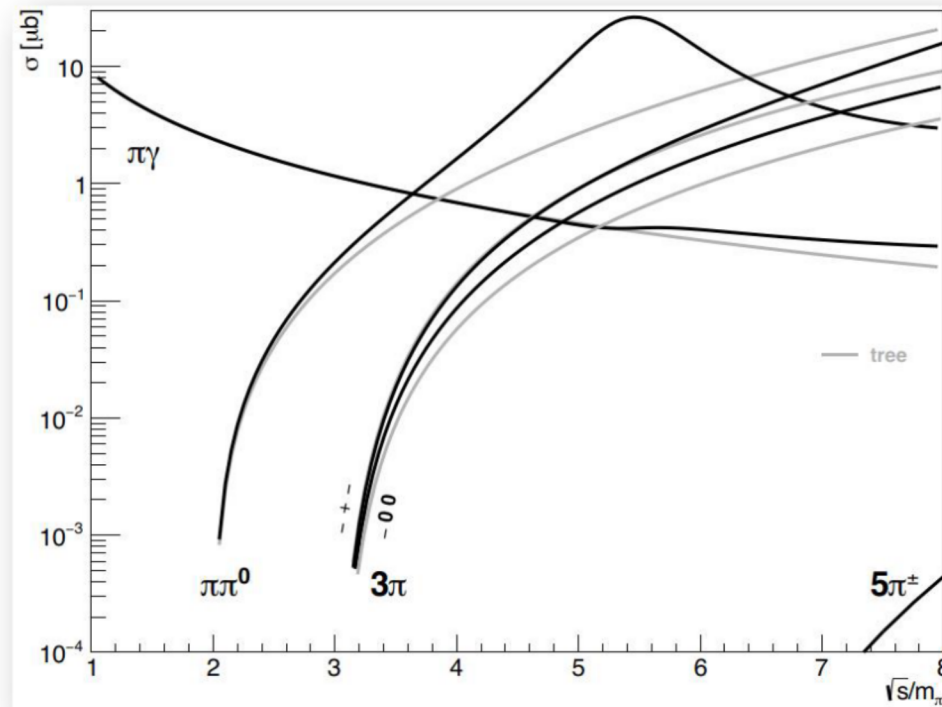


- Measurement of $F_{3\pi}$ - fundamental test of low-energy QCD
- COMPASS did first combined measurement of $F_{3\pi}$ and $\Gamma_{\rho \rightarrow \pi\gamma}$
- Result for $F_{3\pi}$ is in agreement with prediction from ChPT
- Results dominated by systematic uncertainties -> improvement expected
 - Background prediction
 - Luminosity determination
- On the future program of successor experiment AMBER: similar program on kaon sector (see talk by Oleg Denisov, “From COMPASS to AMBER”, Fri 14:00)

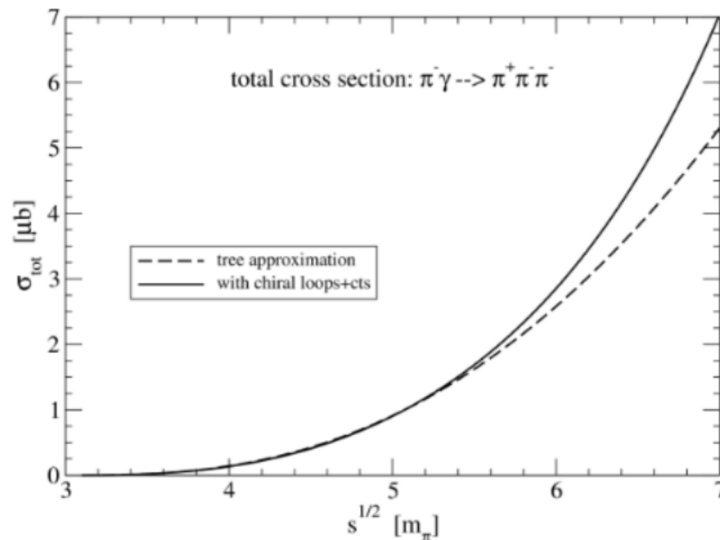
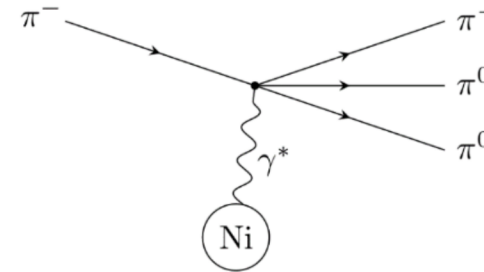
Thank you for your attention

Backup

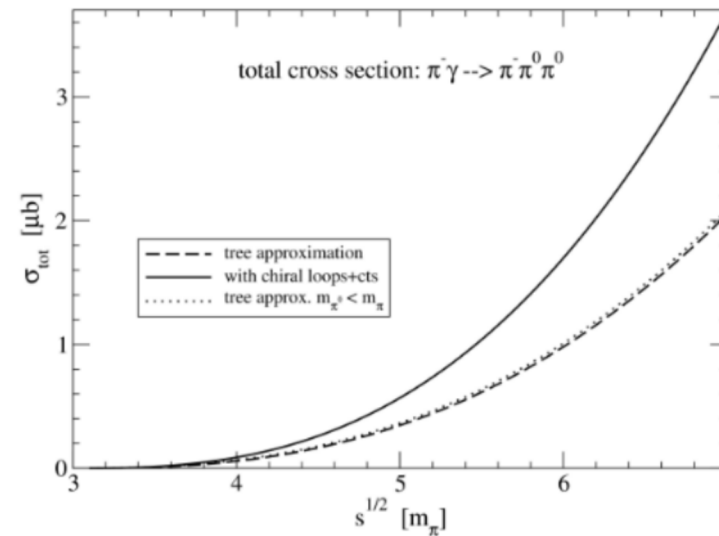
Cross sections for Primakoff effect



- Direct (point-like) coupling of photon to 4 pions
- Prediction from ChPT at tree- and loop-level available

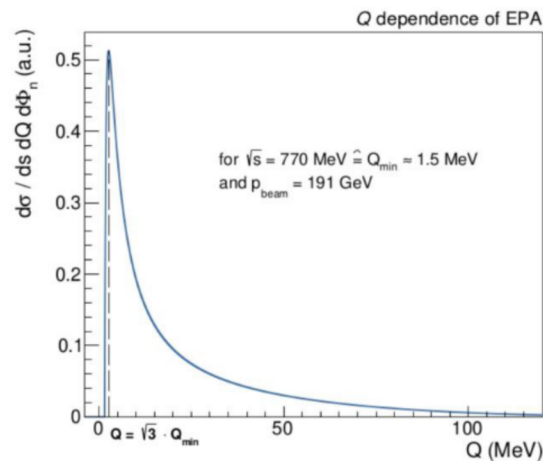
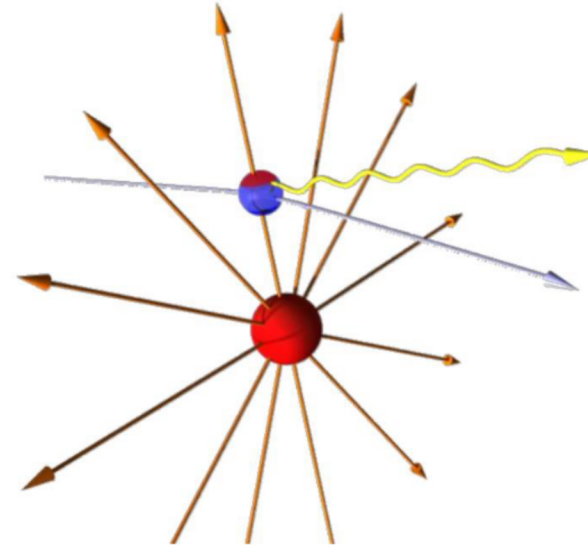


[Grabmüller S. \(2012\). Cryogenic Silicon Detectors and Analysis of Primakoff Contributions to the Reaction \$\pi^- Pb \rightarrow\$](#)



[Krämer M. \(2016\) Evaluation and Optimization of a digital calorimetric trigger and analysis of \$\pi^- Ni \rightarrow\$](#)

- Photon is provided by the strong Coulomb field of a nucleus (typical field strength at $d = 5R_{Ni}$: $E \approx 300$ kV/fm)
- Coulomb field of nucleus is a source of quasi-real ($P_\gamma^2 \ll m_\pi^2$) photons
- Large impact parameters (ultra-peripheral scattering)



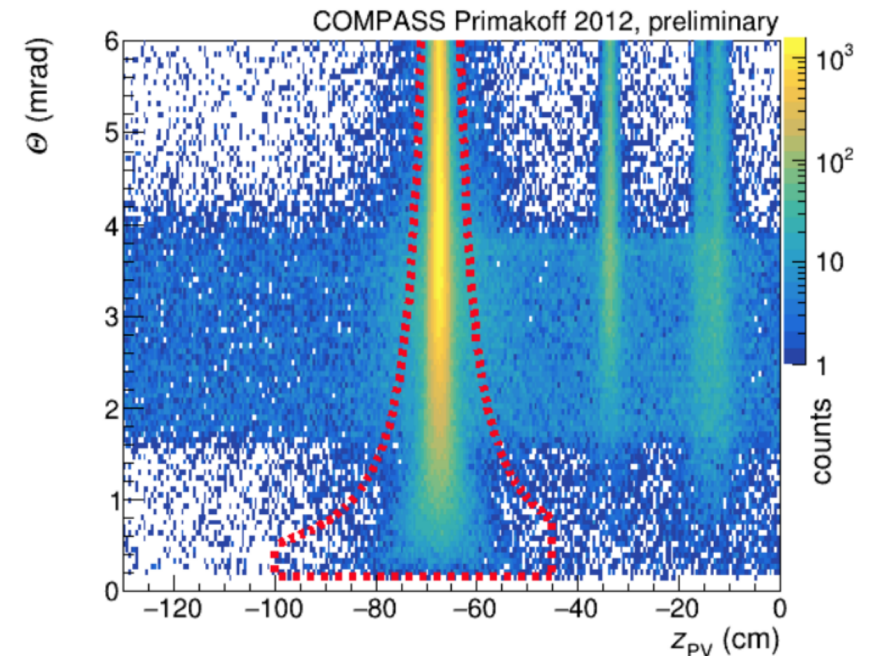
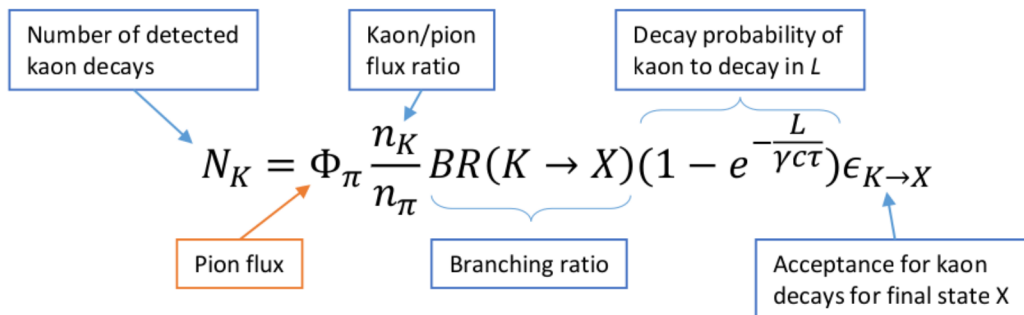
$$\frac{d\sigma}{ds dQ^2 d\Phi_n} = \underbrace{\frac{Z^2 \alpha}{\pi(s - m_\pi^2)} F^2(Q^2)}_{\text{Flux of quasi-real photons}} \frac{Q^2 - Q_{\min}^2}{Q^4} \cdot \underbrace{\frac{d\sigma_{\pi\gamma \rightarrow X}}{d\Phi_n}}_{\pi\gamma \text{ scattering cross section}}$$

Flux of quasi-real photons $\pi\gamma$ scattering cross section

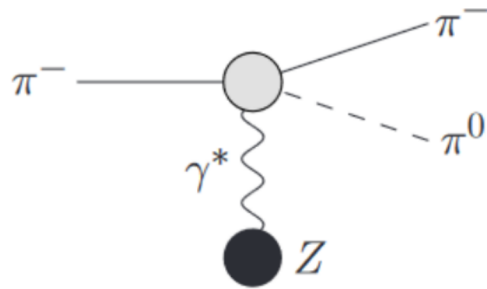
- Needed for absolute cross section measurement: effective integrated luminosity

$$L_{\text{eff}} = L \cdot (1 - \epsilon_{\text{DAQ}})$$

- Can be determined via free kaon decays:
 - Use CEDAR detectors for beam PID
 - Free decays where no material
 - Exclusive events with no momentum transfer



Previous measurements – $F_{3\pi}$



[Antipov, Y. et al. PRD 36 \(1987\) 101103](#)

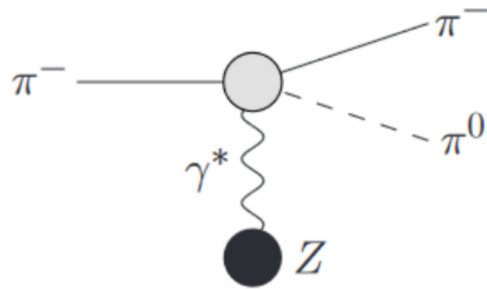
$$F_{3\pi} = (12.9 \pm 0.9) \text{ GeV}^{-3}$$

tension

$$F_{3\pi} = \frac{eN_c}{12\pi^2 F_\pi^3} = (9.78 \pm 0.05) \text{ GeV}^{-3}$$

- Assuming $F_{3\pi} = \bar{F}_{3\pi}(s, t, u)$
- Neglecting s -channel production of ρ meson
- No proper consideration of systematics

Previous measurements – $F_{3\pi}$



[Antipov, Y. et al. PRD 36 \(1987\) 101103](#)

and reanalyzed by

[Ametller, L. et al. PRD 64 \(2001\) 094009](#)

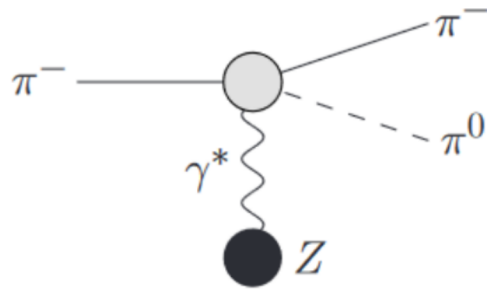
$$F_{3\pi} = (11.4 \pm 1.3) \text{ GeV}^{-3}$$

tension

$$F_{3\pi} = \frac{eN_c}{12\pi^2 F_\pi^3} = (9.78 \pm 0.05) \text{ GeV}^{-3}$$

- Neglecting s -channel production of ρ meson
- No proper consideration of systematics
- Using ChPT to extrapolate to chiral limit (NNLO)

Previous measurements – $F_{3\pi}$



[Antipov, Y. et al. PRD 36 \(1987\) 101103](#)
 and reanalyzed by
[Ametller, L. et al. PRD 64 \(2001\) 094009](#)

$$F_{3\pi} = (10.7 \pm 1.2) \text{ GeV}^{-3}$$

- Neglecting s -channel production of ρ meson
- No proper consideration of systematics
- Using ChPT to extrapolate to chiral limit (NNLO)
- Considering dominant correction

~~10.7 ± 1.2~~

$$F_{3\pi} = \frac{eN_c}{12\pi^2 F_\pi^3} = (9.78 \pm 0.05) \text{ GeV}^{-3}$$

