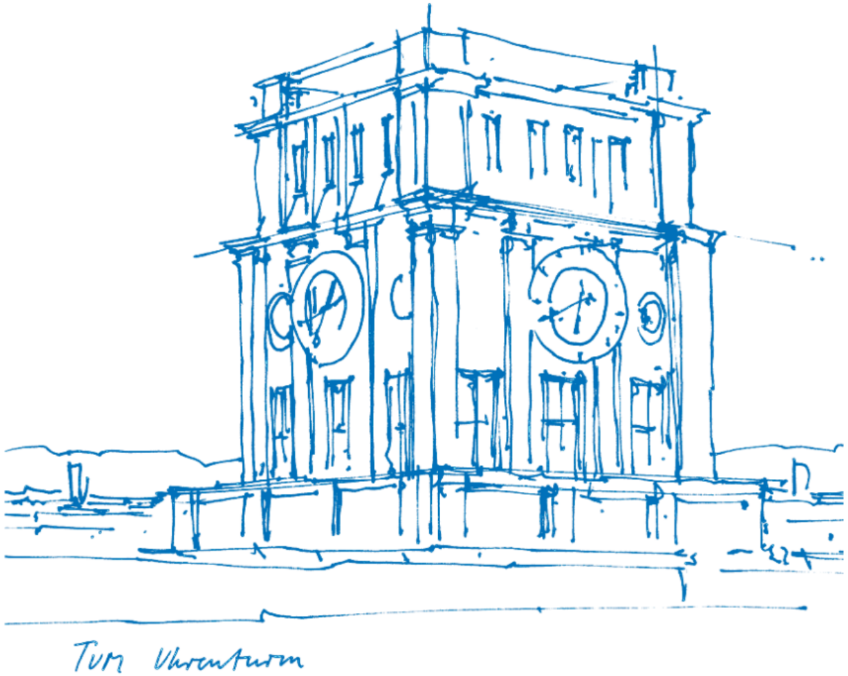
Continuity Constraints for Partial-Wave Analyses*

for the COMPASS Collaboration

Florian Markus Kaspar [1,2] (florian.kaspar@tum.de)
in collaboration with Jakob Knollmüller [1,2]

DPG SMuK (Dresden) HK 7.2
20th March 2023 17:00

[1] Technische Universität München (TUM)
[2] Excellence Cluster Origins



Bundesministerium für Bildung und Forschung

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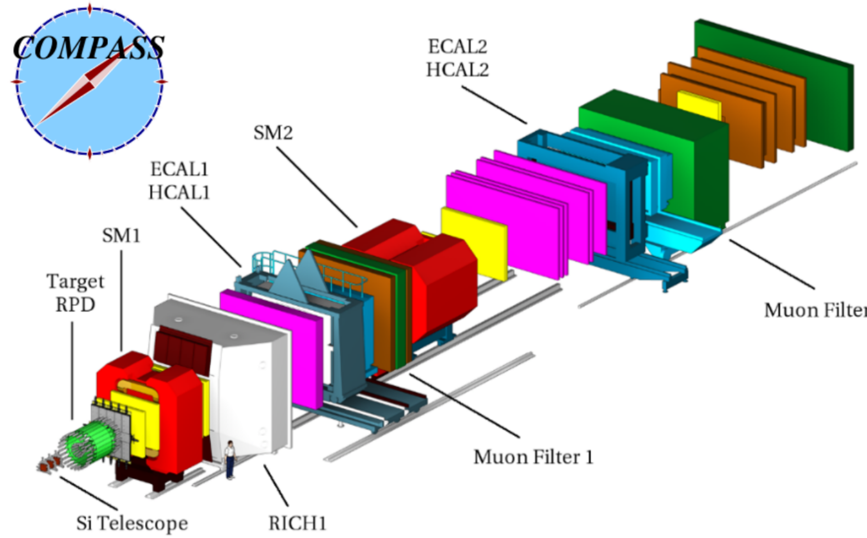
Light-Meson Resonances at COMPASS

& Partial-Wave Analysis

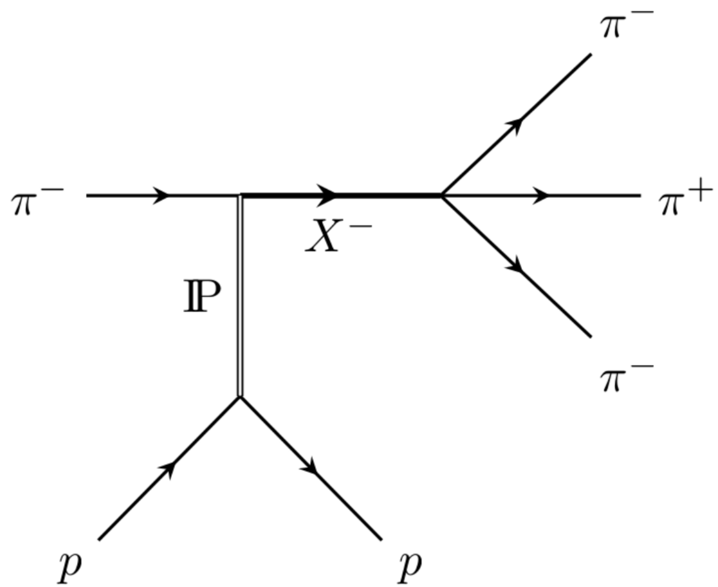
Light-Meson Resonances at COMPASS

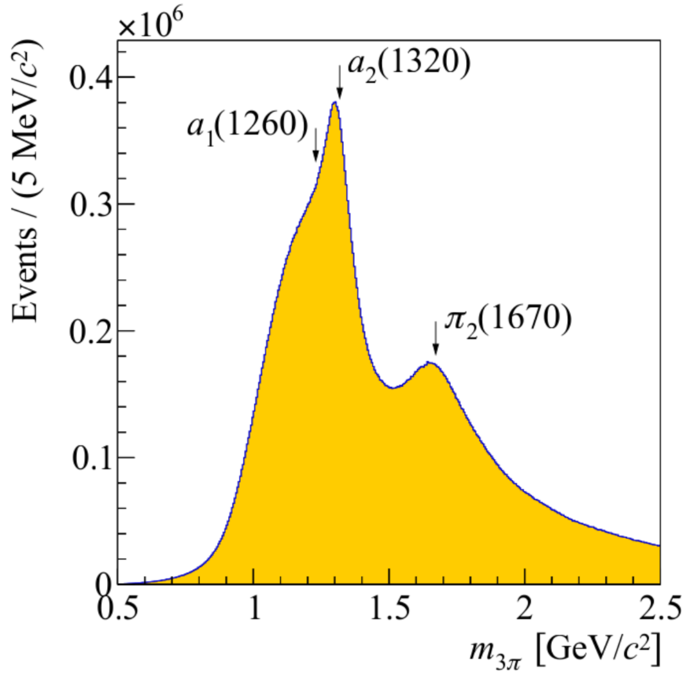
- Fixed Target Experiment at the SPS at CERN (M2 beam line)
- π^- beam $190\ \mathrm{GeV/c}$ → production of light isovector mesons via diffractive reactions
- beam excited to meson resonance X^- (π_J-like and a_J-like)
- Example: $\pi^-\pi^-\pi^+$ final state
- X^- decays into $\pi^-\pi^-\pi^+$ final-state

→ we are interested in X^-



COMPASS Phys. Res. A 779 (2014), pp. 69–115)





PRD 95 (2017) 032004

Partial-Wave Analysis: Model

How to disentangle different X^- contributions?

Model the full final-state intensity distribution:

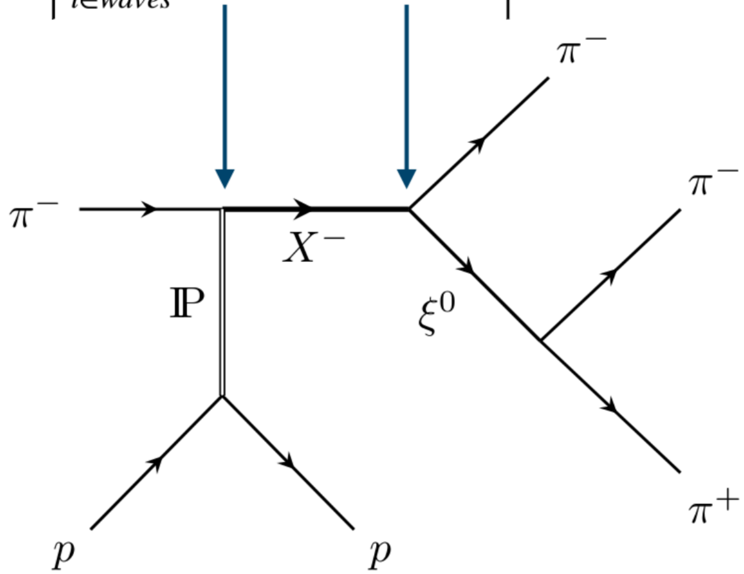
- sum over X^- quantum numbers and decays:

  $i = (J^{PC}, M, \xi^0, L)$

- partial wave i
  - decay: $\psi_i(\tau; m_{3\pi})$ calculate from data ("basis function")
  - unknown transition: $T_i(m_{3\pi}, t')$
- series truncated (more later)

Information about X_i^- in $T_i(m_{3\pi}, t')$
→ Fit to data!

$$I(\tau; m_{3pi}, t') = \left| \sum_{i \in waves} T_i(m_{3\pi}, t')\, \psi_i(\tau; m_{3\pi}) \right|^2$$



Isobars ξ^0:
$\sigma(500)$, $\rho(770)$, $f_0(980)$, $f_2(1270)$, $f_0(1500)$, $\rho_3(1690)$

Details of the Partial-Wave Decomposition in: COMPASS Collaboration, Phys. Rev. D 95 (2017) 032004, arXiv:1509.00992 [hep-ex].

Partial-Wave Analysis: Conventional Approach

Unknown $T_i(m_{3\pi}, t')$ → fit in two steps:

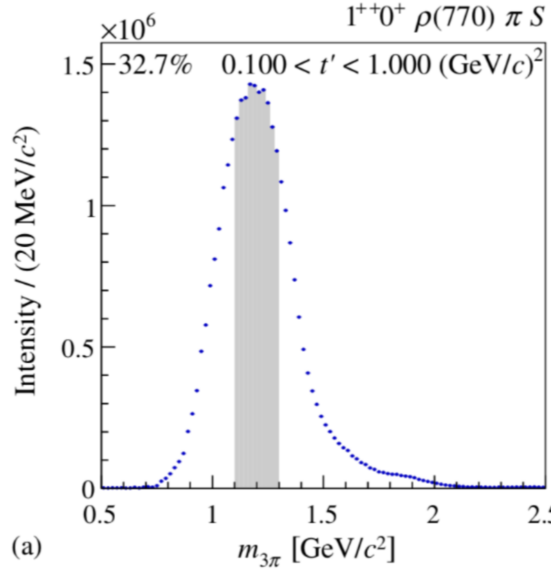
1) mass-independent fit - no assumption about resonances

1. select set of partial-waves $\{i\}$ (e.g. 88 waves)
2. complex-valued step-function for T_i → analysis in individual bins
3. fit constant $T_i(m_{3\pi}, t')$ in each bin: intensities $|T_i|^2$ & rel. phases $\Delta\phi = \arg(T_i T_j^*)$
4. estimate uncertainties as Gaussian

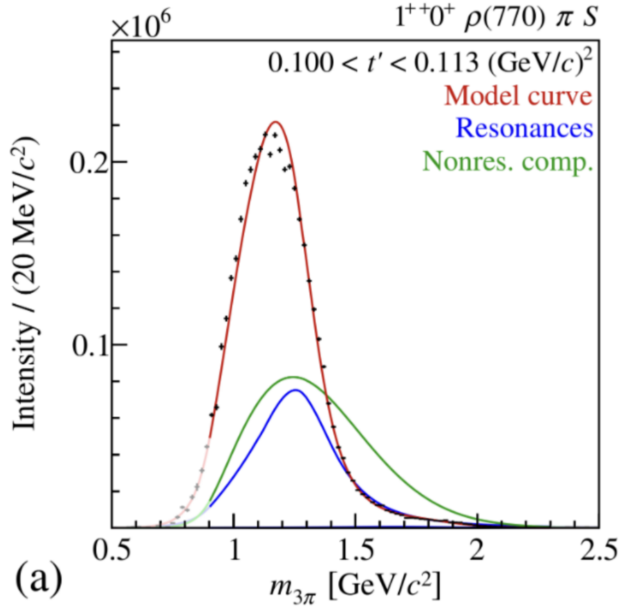
2) mass-dependent fit: - model resonances

5. results of first step: input
6. χ^2 fit of resonant + background parameterization to subset of $T_i(m_{3\pi}, t')$

→ resonance parameters = physics



[1]



[2]

[1] PRD 95 (2017) 032004, [2] PRD 98 (2018) 092003

Partial-Wave Analysis: Limitations

mass-independent fit:

- select set of partial-waves $\{i\}$ → partial-wave model
- in principle: infinitely many waves
- in practice: finite data → select relevant waves
  - truncate high spins: large wavepool (several hundred waves)
  - select subset (otherwise unstable inference)

→ partial-wave model is a large systematic uncertainty

mass-dependent fit:

- fit to mass-independent result
- approximate uncertainties as Gaussian

→ source of systematic uncertainty

→ How can we improve the extraction?

Continuity

& Single-Stage Resonance Fits

Continuous Non-Parametric Fits

Make use of prior information to stabilize mass-independent fit:

- use full wavepool but do not select subset
- physics should be continuous:

  solutions in close-by bins should be similar → correlation
- still do not assume resonances

→ replace step-functions with smooth non-parametric curves

How to implement?

Continuous Non-Parametric Fits

→ replace step-functions with smooth non-parametric curves

How to implement?

Profit form work of our colleagues at Max-Planck for Astrophysics:
→ NIFTy framework for numerical information field theory

NIFTy: https://ift.pages.mpcdf.de/nifty/

NIFTy for Partial-Wave Analysis:

- provides continuous non-parametric models
- combine with PWA model

→ extract $T_i(m_{3\pi}, t')$ as smooth curves & stabilize solutions

M87* Black Hole: https://www.mpa-garching.mpg.de/1029092/hl202201

Single-Stage Resonance Fits

We can go one step further:

Instead of mass-indep. & mass-dep. fits → combine

1. replace step-functions with smooth model (NIFTy)
   - non-parametric but incorporates smoothness
2. for selected waves add resonant part
   - flexible non-res. background
   - resonant signal sum of Breit-Wigners
   - coherent sum describes $T_i(m_{3\pi}, t')$

Goal: overcome limitations of the conventional approach

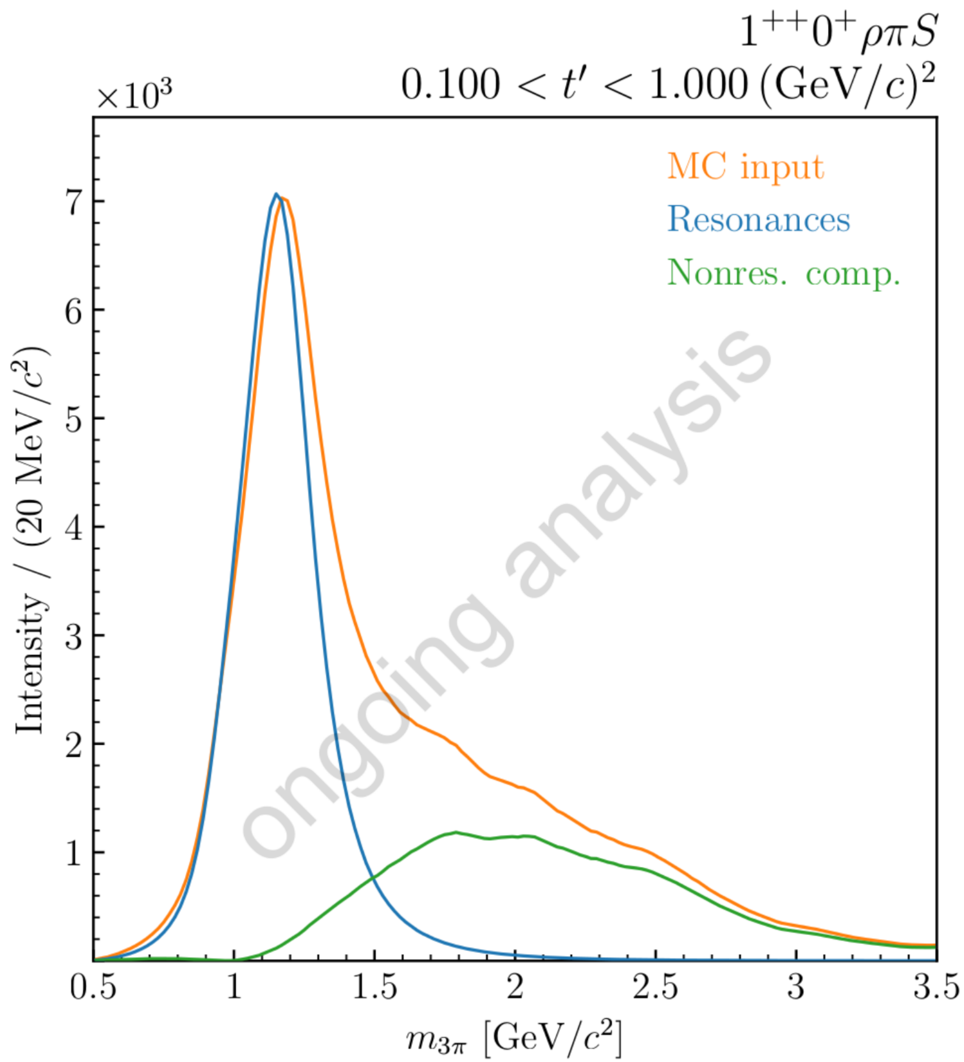
Verification on Monte Carlo Simulation

Verification on MC

Create Pseudo-Data and try to recover!

Input-Output Study:

1. generate MC data according to:
   - smooth NIFTy model
   - 81 partial-waves
   - 5 resonances
2. try to recover input



Right: intensity $|T_i|^2$ of a wave:

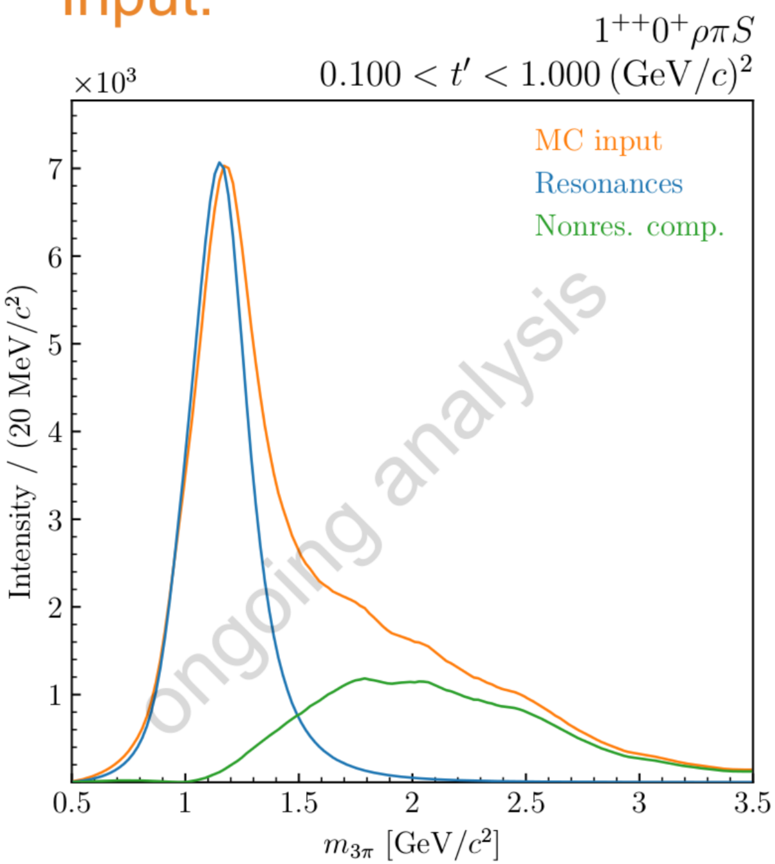
- nonres. comp. (NIFTy)
- resonance
- combined signal → input model

Verification on MC: Input-Output Study

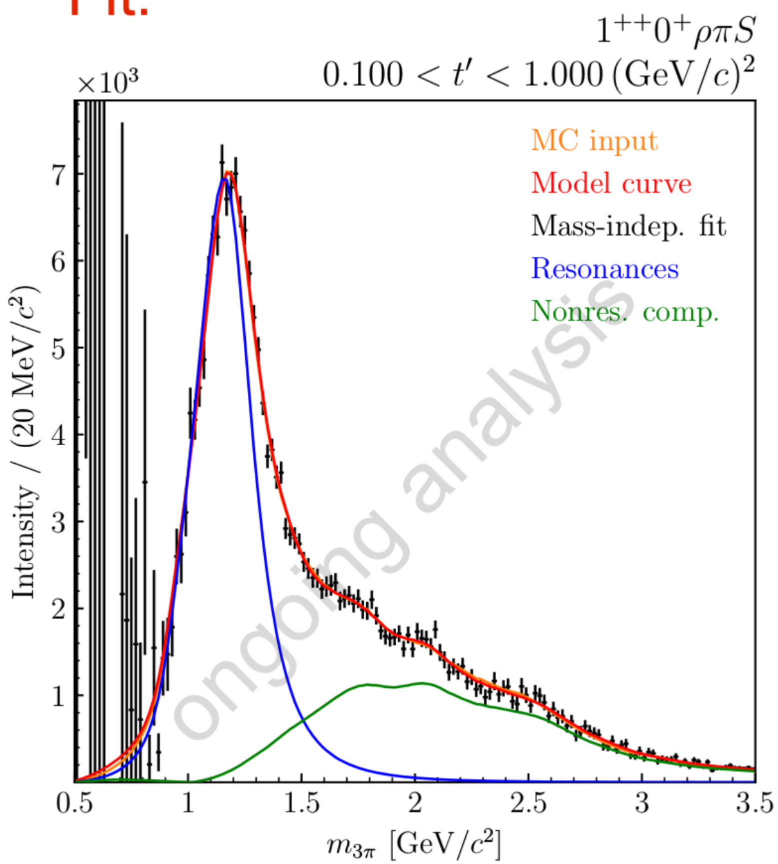
fit same 81 waves as used for input:

- mass-indep. fit: works well above ≈ 1 GeV
- single-stage fit: perfectly recovers input
- able to separate non-res. and resonant components

Input:



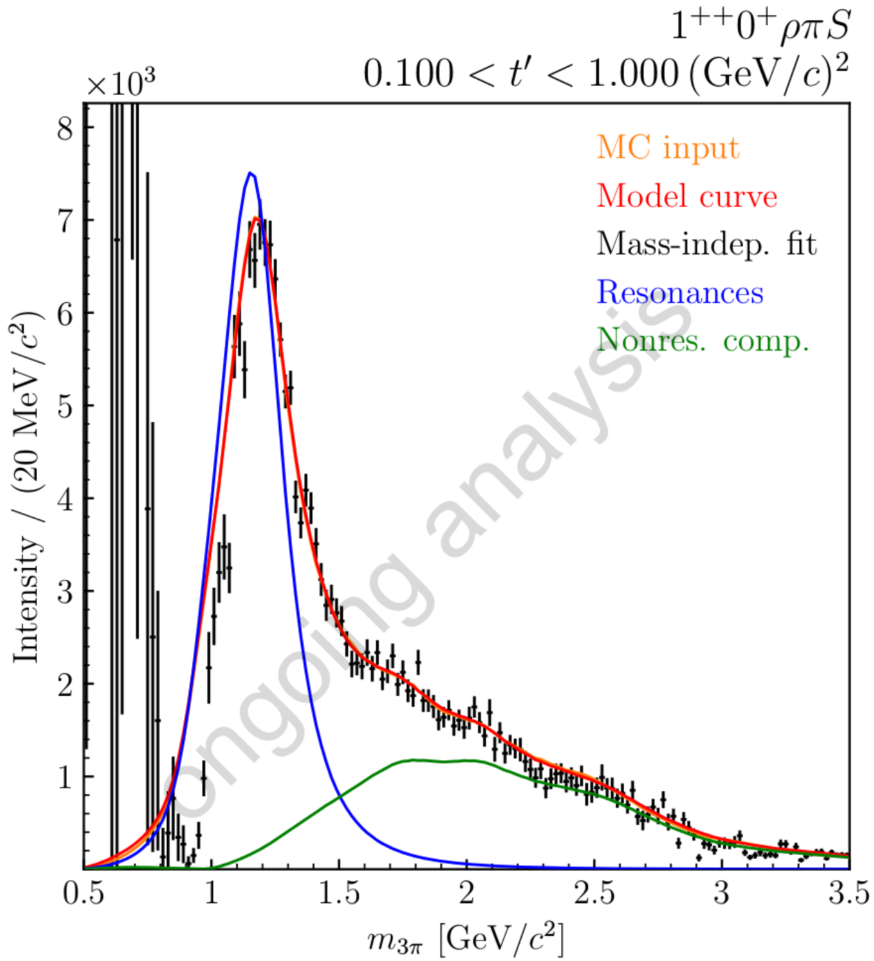
Fit:



Verification on MC: Extended Model

More realistic: consider 332 waves for fit

- mass-indep. fit: signs of overfitting bias
- single-stage fit: prior informations stabilizes fit
- still able to recover input & to separate non-res. and resonant components



Real Data: Single-Stage Resonance Fits

Real Data: Single-Stage Resonance Fits

Apply method to COMPASS 3π data

Fit model:

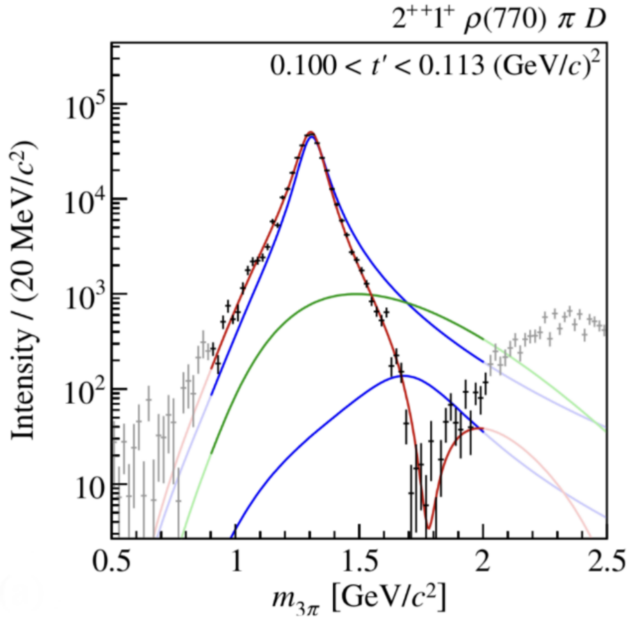
- 332 waves (vs 88 in conv.)
- fit in both $m_{3\pi}$ and t'
- 15 resonances

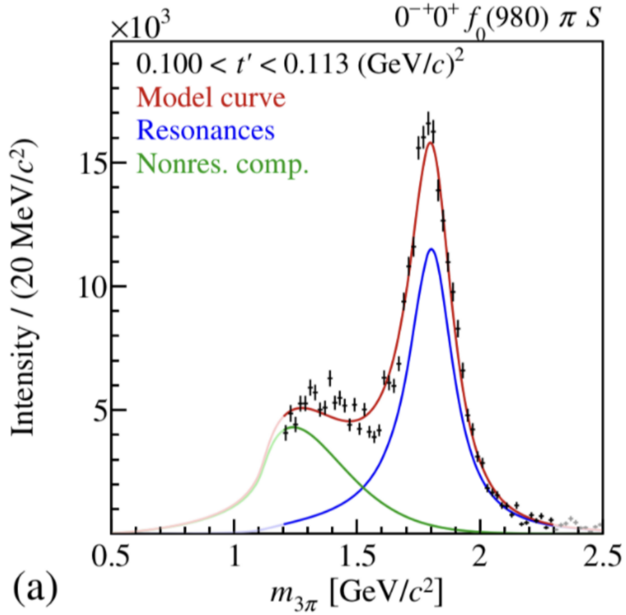
Comparison:

- resonances and background
- flexible background follows expected behavior

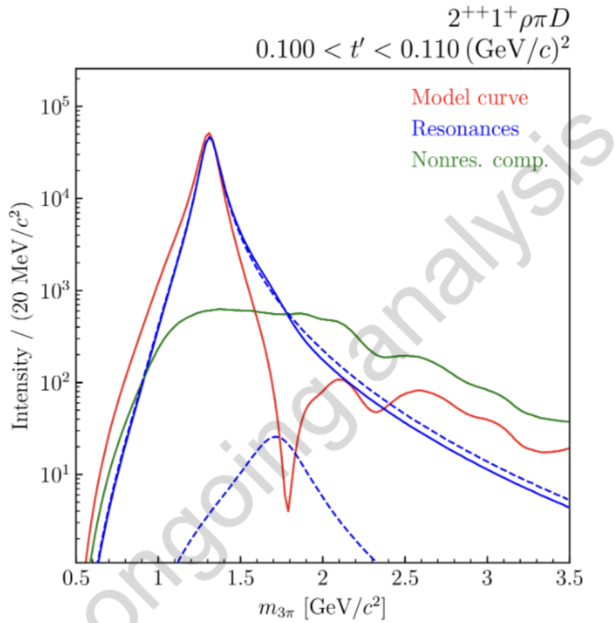
→ separation similar

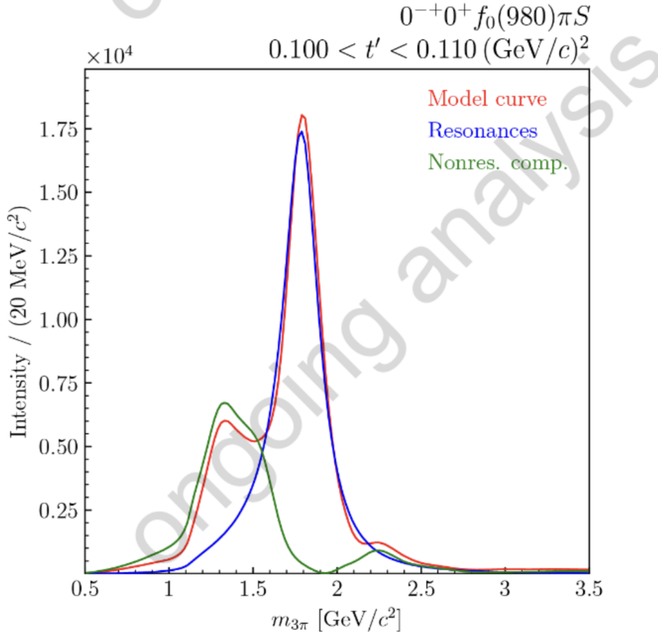
conventional:





new:



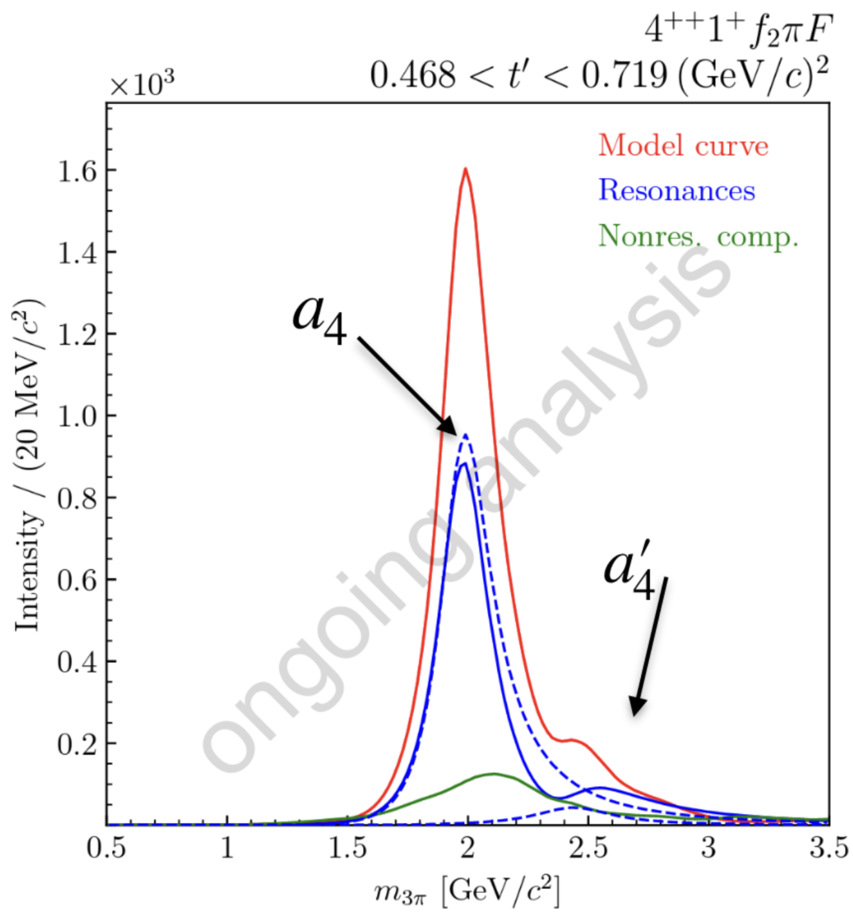


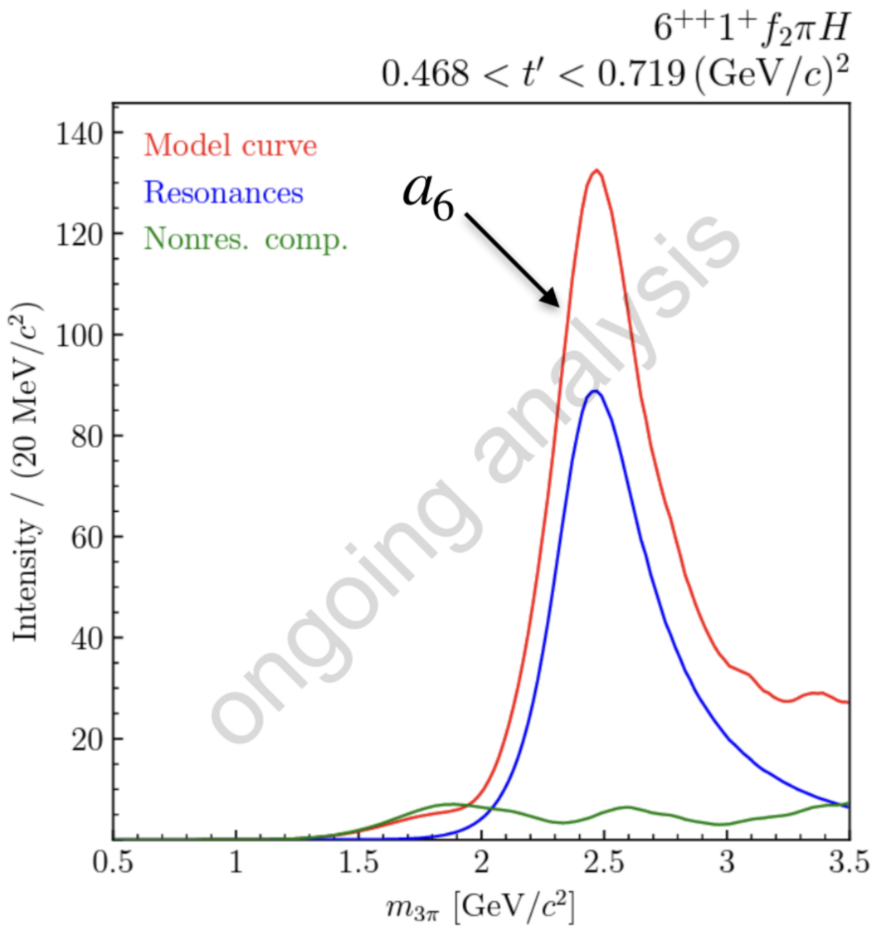
COMPASS: PRD 98, 092003 (2018)

Real data fits: Towards small signals!

Our goal is to study and small ($< 1\,\%$) signals:

Attempts of fitting a_4' and a_6 resonances!





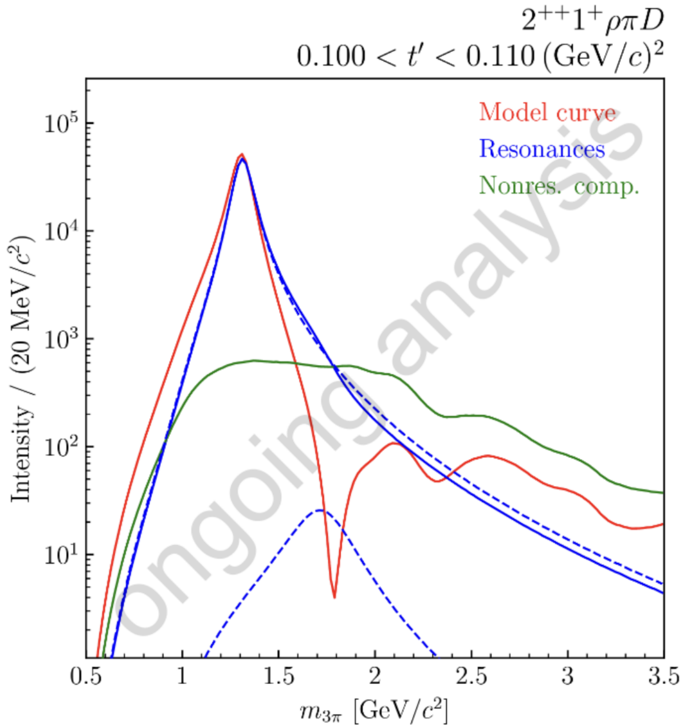
Conclusions & Outlook

Conclusions and Outlook

We have demonstrated a new approach to PWA

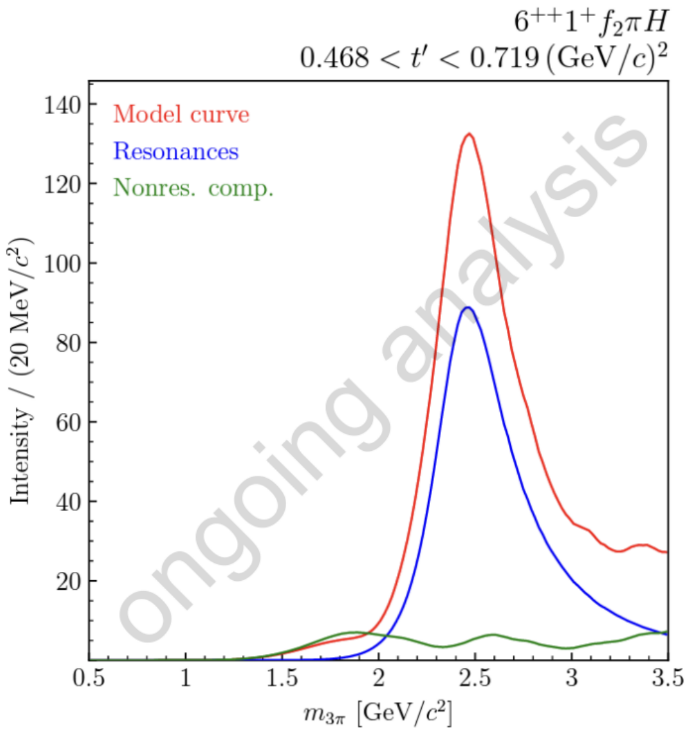
- solves limitations of conventional approach:
  - model selection
  - uncertainty propagation
- MC study and first real data fits

→ proof of principle



Next Steps:

- get uncertainties (ongoing)
- improve model
- systematic studies
- run large scale fits!



Acknowledgements

Acknowledgements

Thank you for your attention!

I would like to thank Jakob Knollmüller who helped me develop the NIFTy model

I would also like to thank Stefan Wallner and Philipp Frank with whom I worked on a first version of the NIFTy fit. The current work is partially based on this.

Questions?

Backup Slides

Likelihood & Thresholds

Likelihood

$$\mathscr{L} = \frac{\bar{n}^n}{n!} e^{-\bar{n}} \prod_j^n P(\tau^j; m_{3pi}^j, t'^j) = \frac{1}{n!} e^{-\bar{n}} \prod_j^n I(\tau^j; m_{3pi}^j, t'^j)$$

with expected number of events $\bar{n} = \int_\Omega I(\tau; m_{3pi}, t')$d LIPS$(\tau) \approx \vec{T}^\dagger M \vec{T}$ within one bin

$\rightarrow$ maximize $\log(\mathscr{L}) \rightarrow$ transition amplitudes in bin $\vec{T} \in \mathbb{C}^n$

Integral Matrix $\tilde{M}_{ij} = \int_\Omega \psi(\tau)_i \psi(\tau)_j^*$d LIPS$(\tau)$ and $M_{ij} = \dfrac{\tilde{M}_{ij}}{\sqrt{\tilde{M}_{ii}\tilde{M}_{jj}}}$

This way:

- within one bin the phase-space information is moved to the transition amplitudes $\vec{T} \in \mathbb{C}^n$ or in other words: the fit chooses the value
- $|T_i|^2$ normalized to nmb. events
- $\tilde{M}_{ii}$ contains information of the wave opening with phase-space
- M_{ii} = 1
- M_{ij} are overlaps of decay amplitudes

Real data fits

Eigenvector Thresholds:

- usually threshold in mass per wave
- threshold eigenvectors with eigenvalue smaller than 0.1 of integral matrix → set destructive interference of 10x or larger to 0

$$\tilde{M}_{ij} = \int_\Omega \psi(\tau)_i \psi(\tau)_j^* \mathrm{d}\,\mathrm{LIPS}(\tau) \text{ and } M_{ij} = \frac{\tilde{M}_{ij}}{\sqrt{\tilde{M}_{ii}\tilde{M}_{jj}}}$$

- let NIFTy move the combinations to get a smooth curve →similar behavior to ‘normal’ thresholds

- use Bowler Parameterization for $a_1(1260)$
- Production factor as in COMPASS Mass-Dep. Paper: $\left(\frac{s}{m_{3\pi}^2}\right)^{2\alpha(t)-1}$ with

$s \approx (19\,\mathrm{GeV})^2$

Generative Model

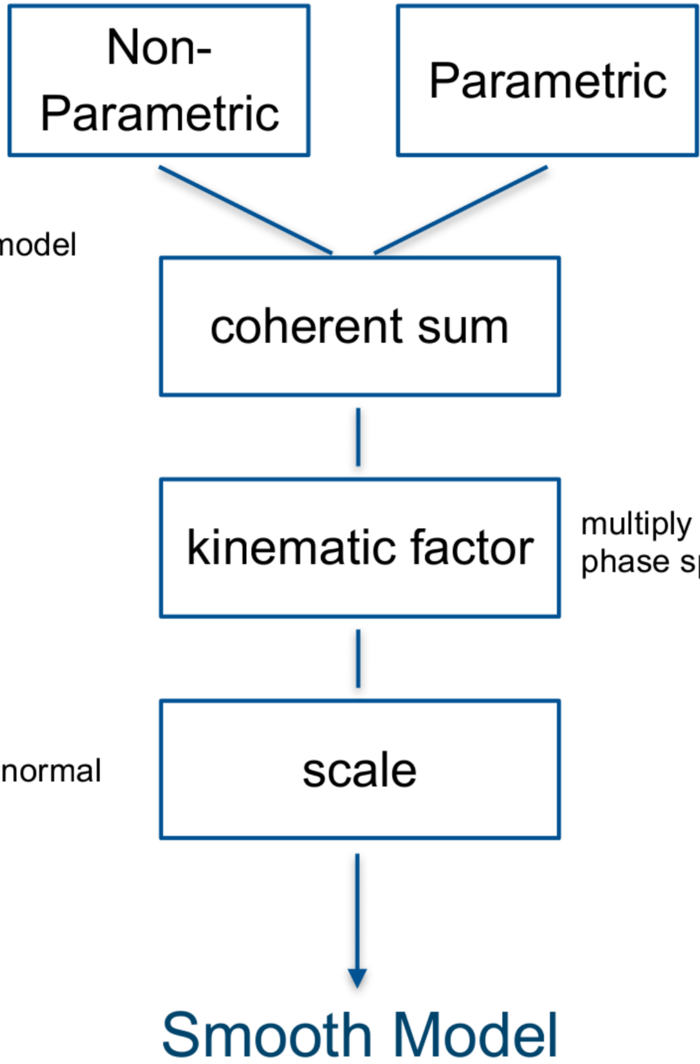
Generative Model (per wave):

Modified NIFTy correlated field maker:
→ fixed fluctuations to 1
→ loglog average slope -4
→ flexibility
→ offset

for real and imag part indiv.

functions as:
→ coh. background if there is a parametric model
→ description of transition amplitude

e.g. nothing or sum of Breit-Wigners:
Priors in masses and width
→ Lognormal
Prior on complex-valued scale:
→ 2d-normal
(scale to set relative prior strength)



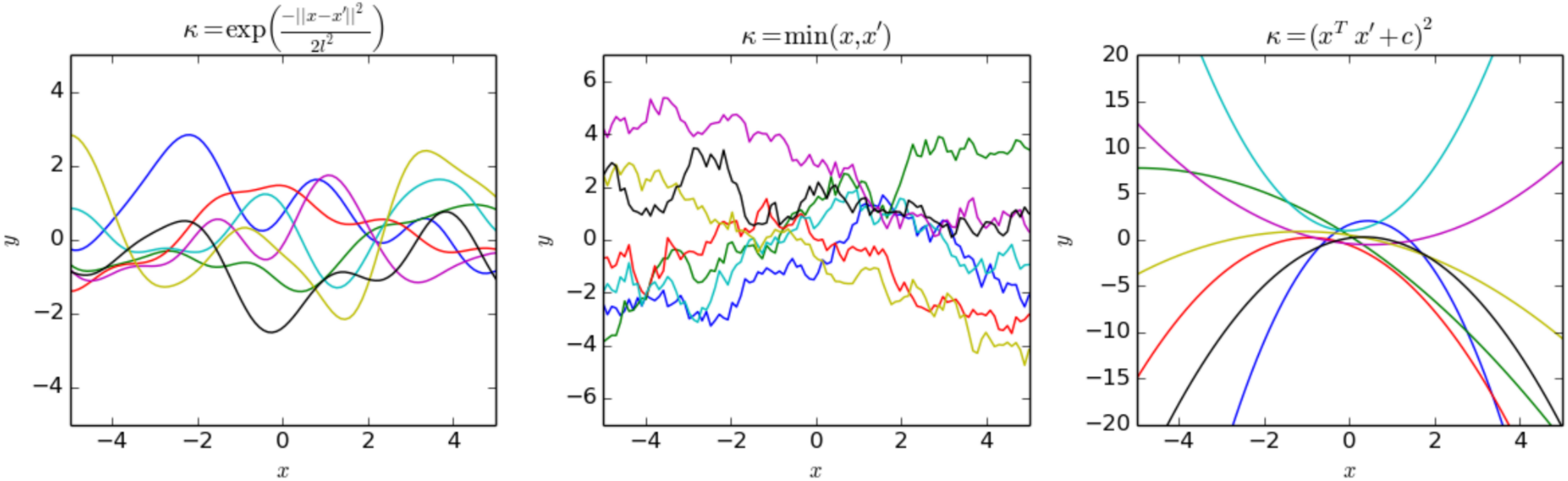
multiply with kinematic factor:
phase space of wave times production factor

scale for combined signal: 1d normal

Gaussian Processes

Formalize continuity:

- Gaussian Process: Infinite dimensional multivariate normal distribution
- Continuity given by covariance function: $\kappa(x, x')$
- encode our prior knowledge within choice of $\kappa(x, x')$



https://upload.wikimedia.org/wikipedia/commons/b/b4/Gaussian_process_draws_from_prior_distribution.png

How to chose $\kappa(x, x')$? → learn from data → NIFTy software framework

Model & Fit:

Bayes Theorem:

$$P(\{\theta_i\}\,|\,D) = \frac{P(D\,|\,\{\theta_i\})P(\{\theta_i\})}{P(D)}$$

- Prior: NIFTy: Generative Model → encodes:
  - smoothness
  - kinematic factor
  - prior on resonance parameters

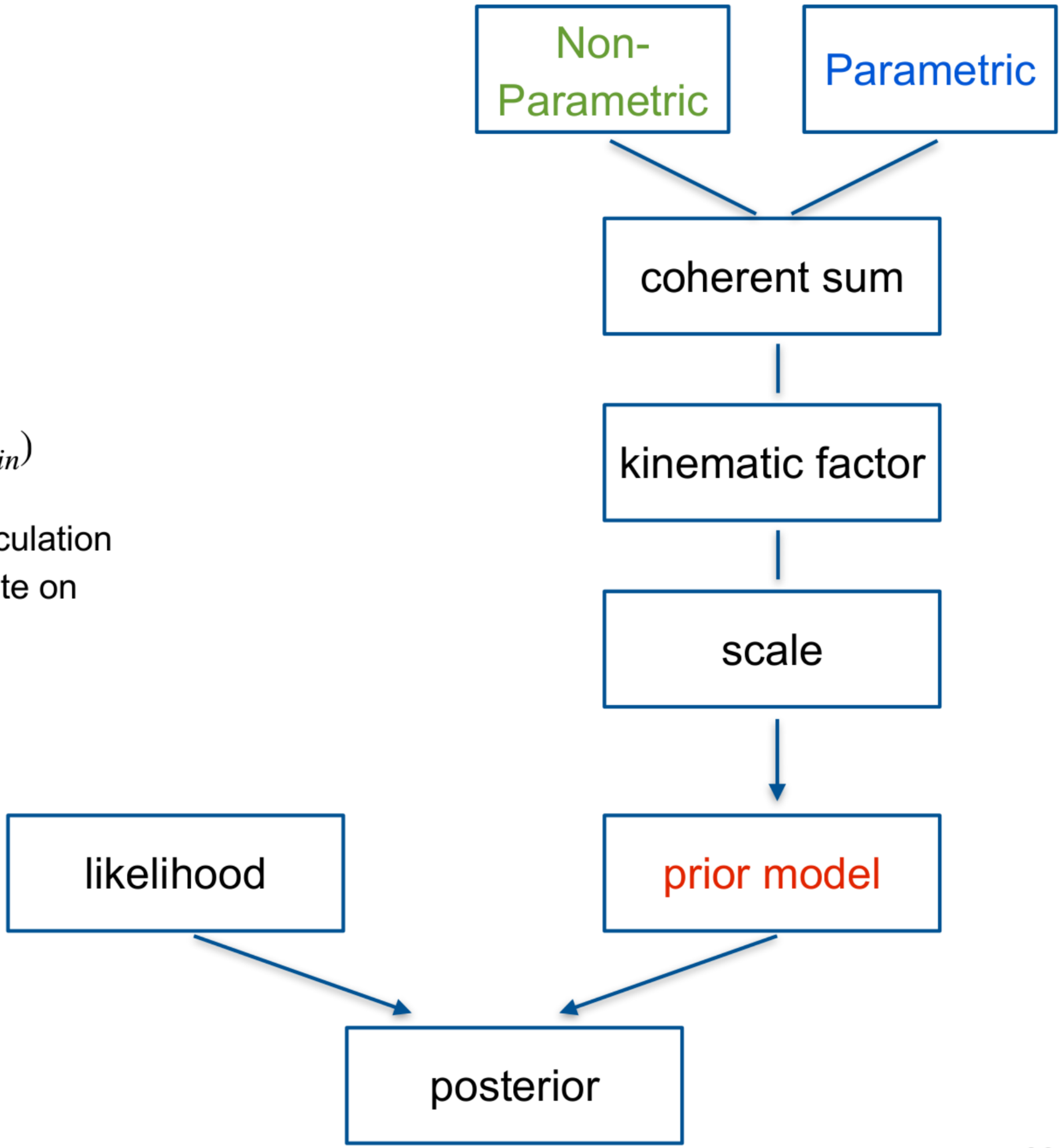
- Likelihood: From PWA framework:

$$\log \mathscr{L}(T_i\,|\,D) = \sum_{iBin} \log \mathscr{L}(T_i\,|\,D_{iBin})$$

  - cannot fit bins individually → likelihood calculation needs all bins at the same time! → distribute on multiple CPUs / machines with MPI
  - needs tens to hundreds of GB of memory

- Posterior: NIFTy Model & Likelihood

→ Fit to posterior



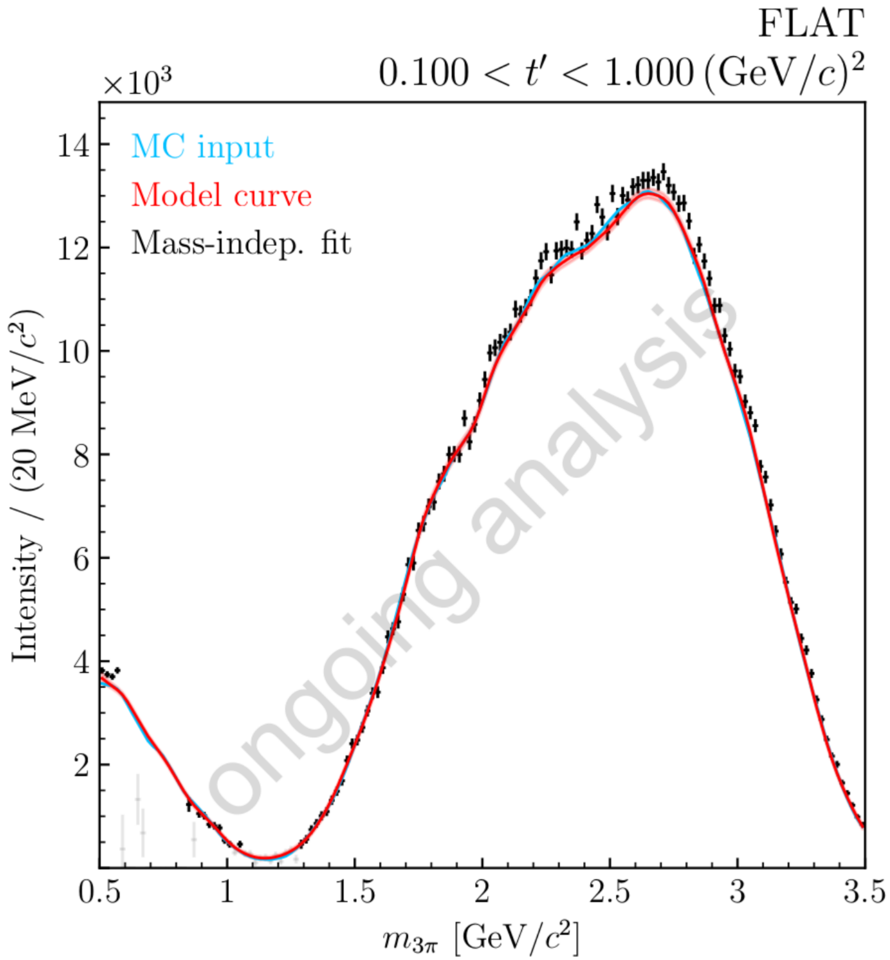
Regularized Fit

MC Model: Larger Fit Model

Non-Parametric (NIFTy) + Breit-Wigner resonance = model curve

Hybrid and mass-indep. fit reconstruction (NOW: 330 waves):

Mass-Indep. Fit with regularization:



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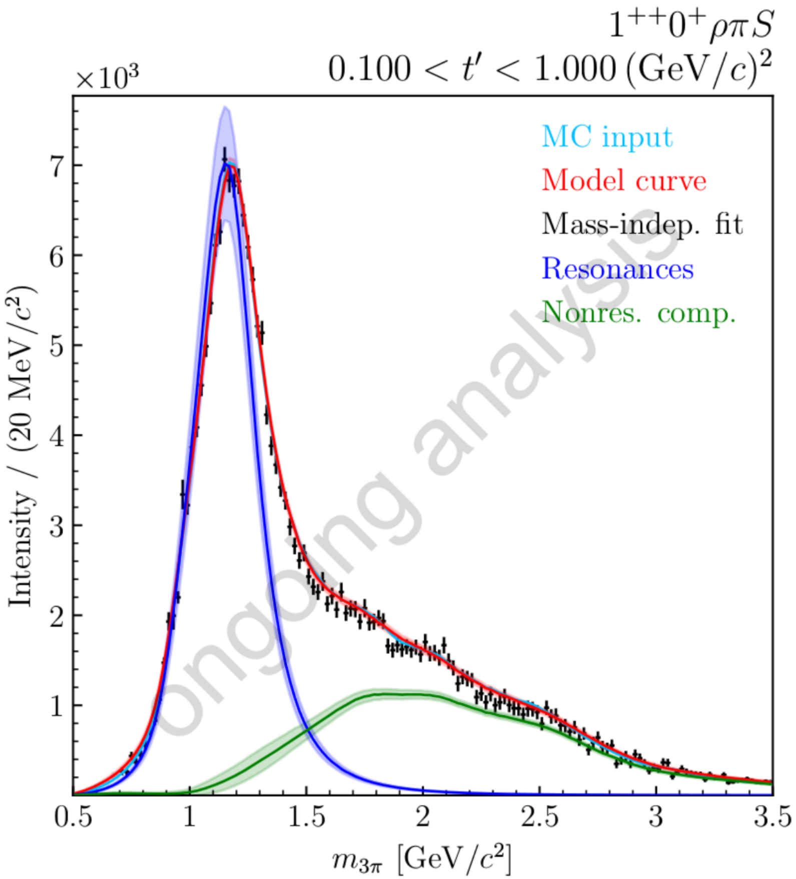

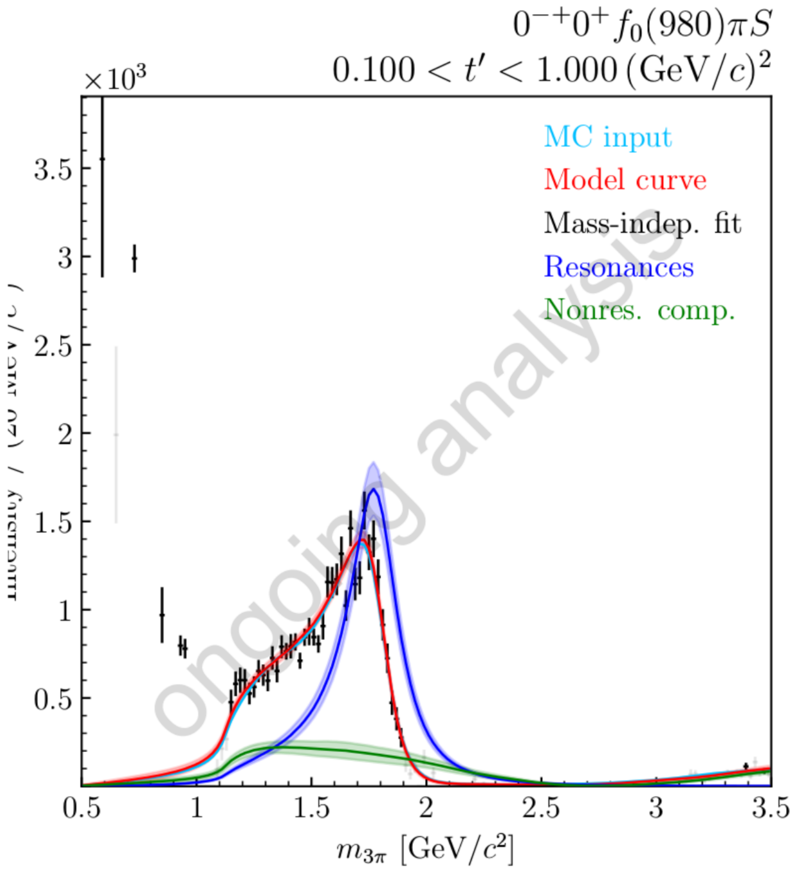

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