Understanding the Ambiguities in the Partial-Wave Decomposition of the $K_S^0 K^-$ Final State

Julien Beckers for the COMPASS Collaboration

HK 7.3: Hadron Structure and Spectroscopy I

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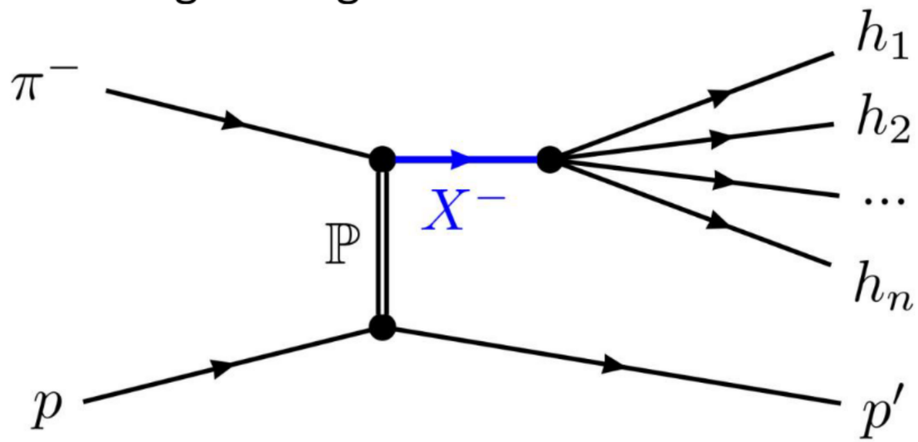
Excited Light Mesons at COMPASS

- Inelastic scattering reactions of high-energetic meson beam



- Strong interaction (Pomeron exchange) between beam meson and target proton
- Intermediate hadronic resonances X^- are created, then decay into n-body final state

  → wide range of allowed (spin) quantum numbers

- Final-state particles measured in the spectrometer

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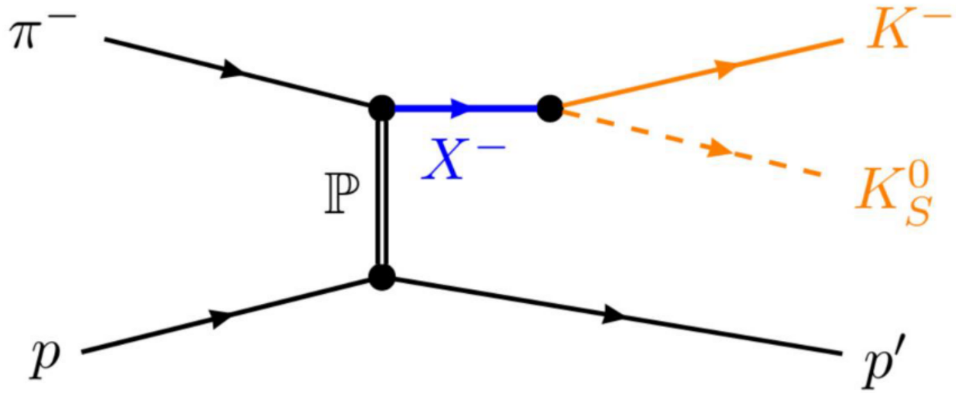

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The COMPASS Experiment

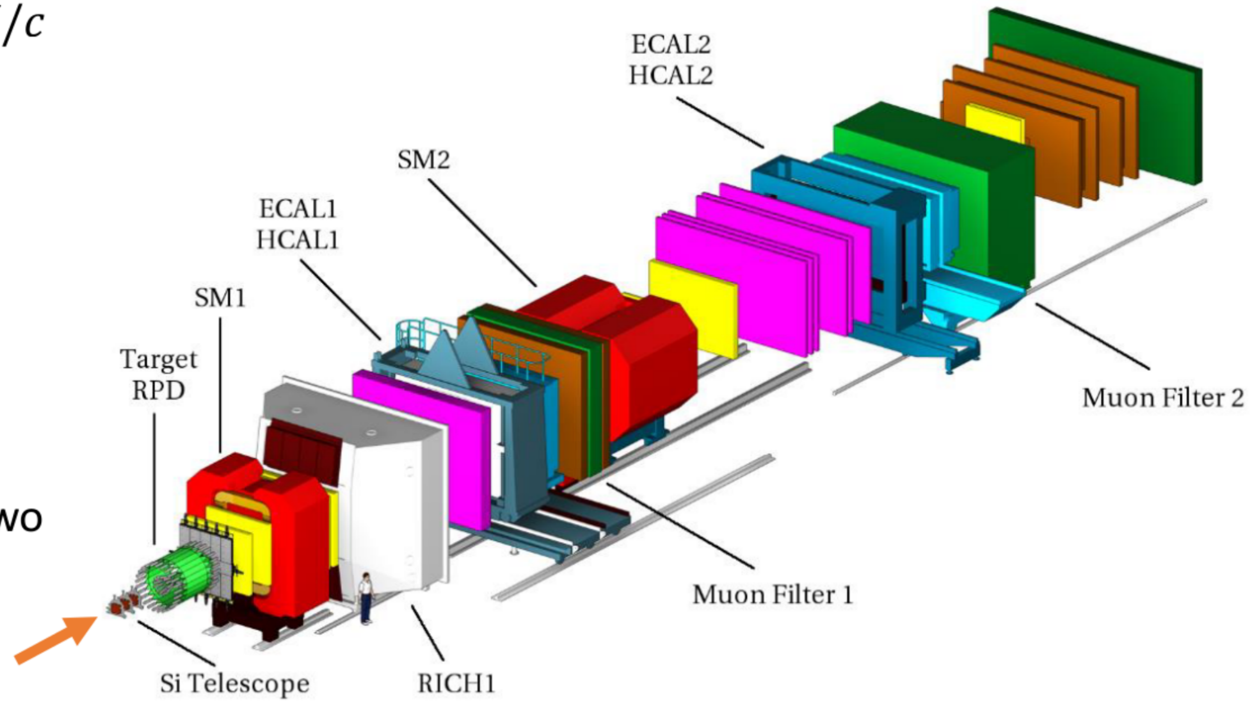
Large-acceptance magnetic spectrometer @ CERN-SPS

Beam:

- Secondary hadrons (π^-, K^-) at $190\ \mathrm{GeV}/c$
- produced via primary proton beam from SPS impinging on Be target
- Cherenkov detectors identify beam particles

Spectrometer:

- Liquid-hydrogen target
- Two-stage spectrometer setup around two dipole magnets SM1/2 :
- Cherenkov detector (RICH-1) identifies final-state particles in momentum range
- EM and hadronic calorimeters, tracking

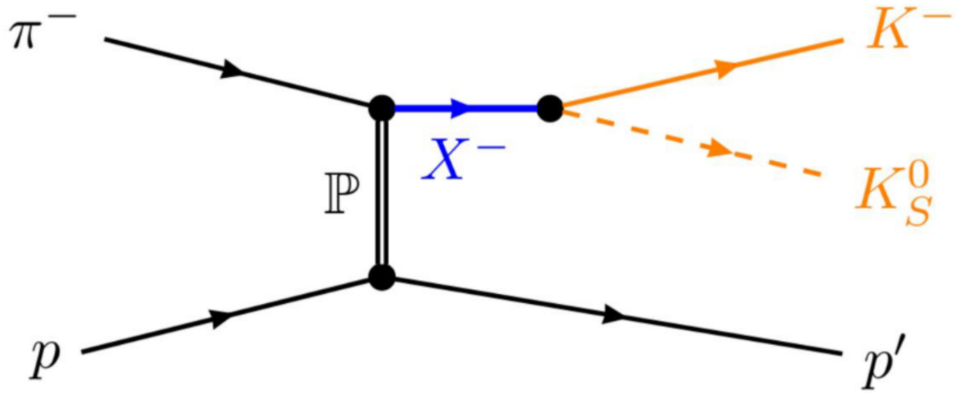


From COMPASS Collab., The COMPASS Setup for Physics with Hadron Beams (Nucl. Instrum. Methods Phys. Res. A 779 (2014), pp. 69–115)

Excited Light Mesons at COMPASS

We study the diffractive dissociation reaction

$$\boldsymbol{\pi^- + p \to K_S^0 K^- + p}$$



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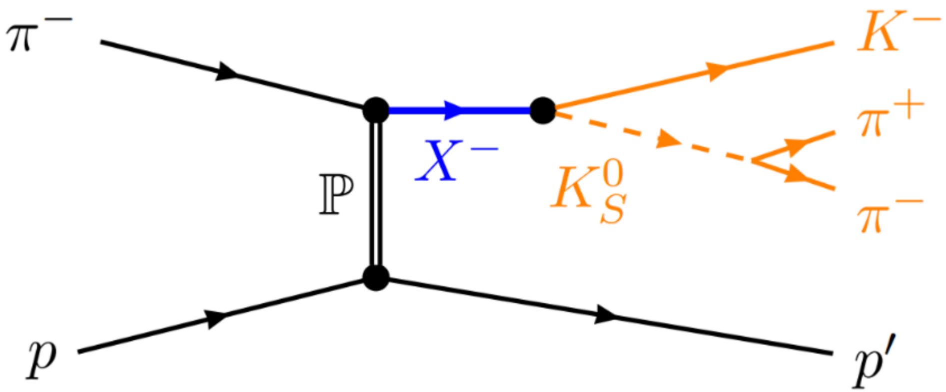

$$\boldsymbol{\pi^- + p \rightarrow K_S^0 K^- + p}$$

$$K_S^0 \rightarrow \pi^-\pi^+$$

- Can be analyzed as two-body state, by reconstructing the K_S^0 from decay daughters

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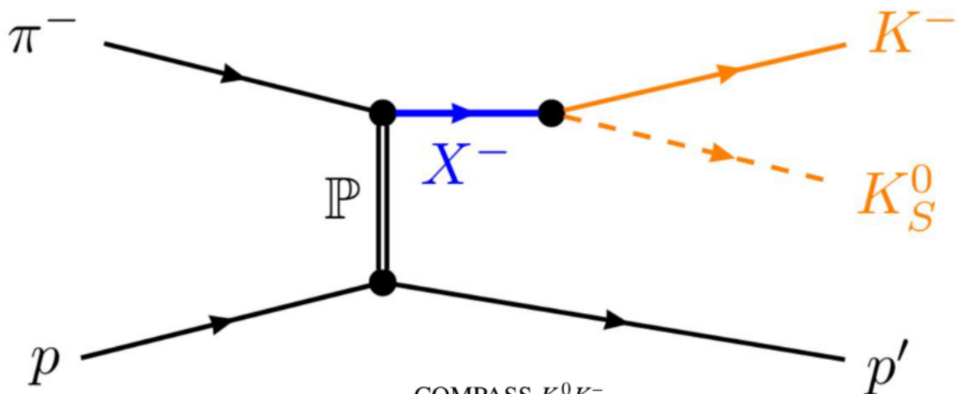

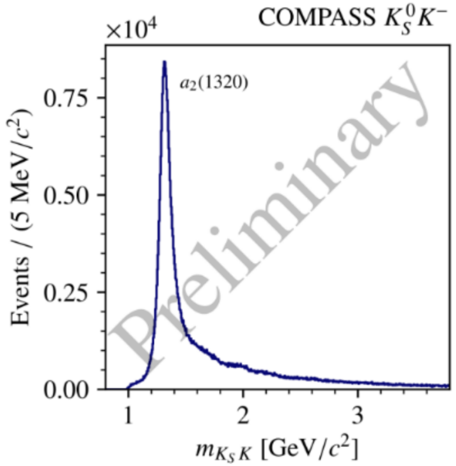

- Can be analyzed as two-body state, by reconstructing the K_S^0 from decay daughters

- X^- resonance content:

  $J^{PC} = 1^{--}, 2^{++}, 3^{--}, \ldots$ from decay

  $$\to \begin{cases} \boldsymbol{\rho_J} & \text{odd } J \quad \text{(suppressed)} \\ \boldsymbol{a_J} & \text{even } J \end{cases}$$

- Study excited ρ states, search for a_4', a_6

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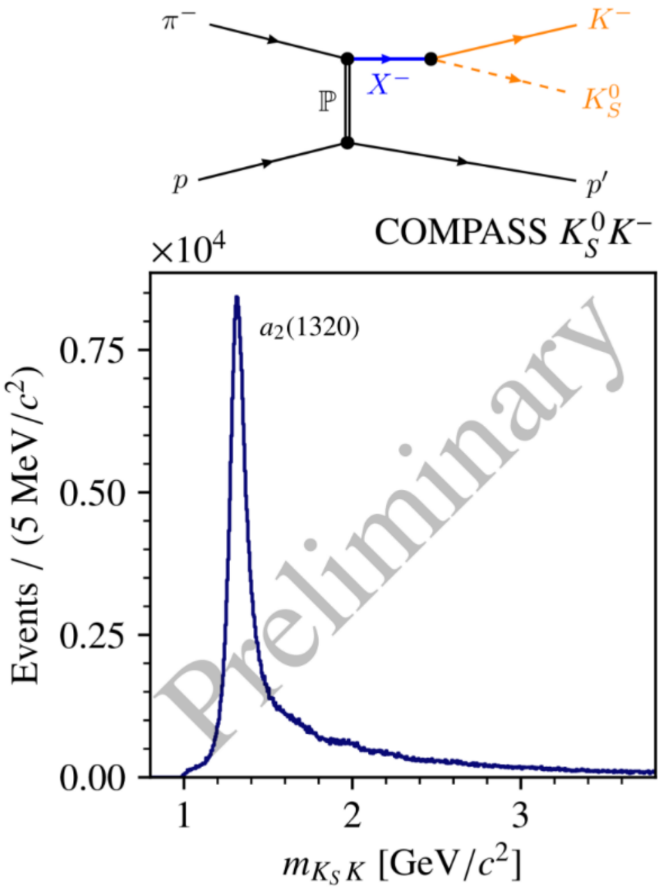

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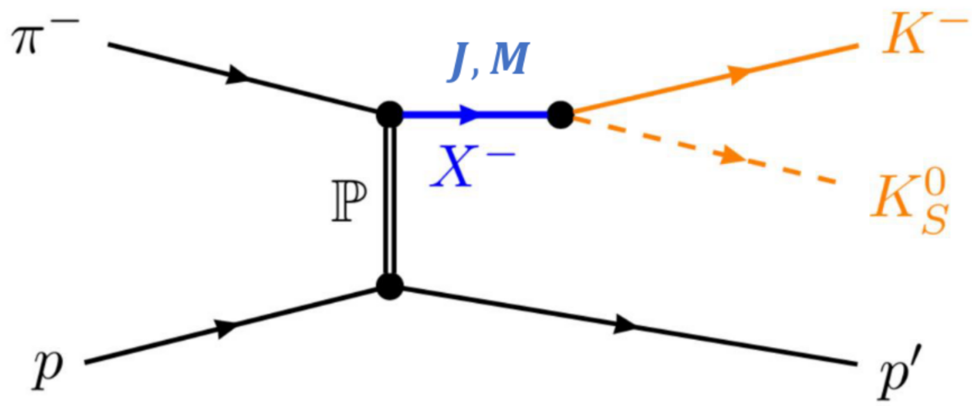
- Study excited ρ states, search for a_4', a_6

Partial-Wave Decomposition

$$I(m_X, t'; \theta, \phi) = \left|M_{fi}\right|^2$$

- Separate process amplitude into **partial waves**
  - Spin $\boldsymbol{J}$ and spin-projection $\boldsymbol{M}$

    $$\boldsymbol{P} = \boldsymbol{C} = (-\mathbf{1})^{\boldsymbol{J}}$$



Partial wave:
specific $(\boldsymbol{J}^{\boldsymbol{PC}}\boldsymbol{M})$

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$$I(m_X, t'; \theta, \phi) = \left| \sum_{JM} T_{JM}(m_X, t')\, \psi_{JM}(\theta, \phi) \right|^2$$

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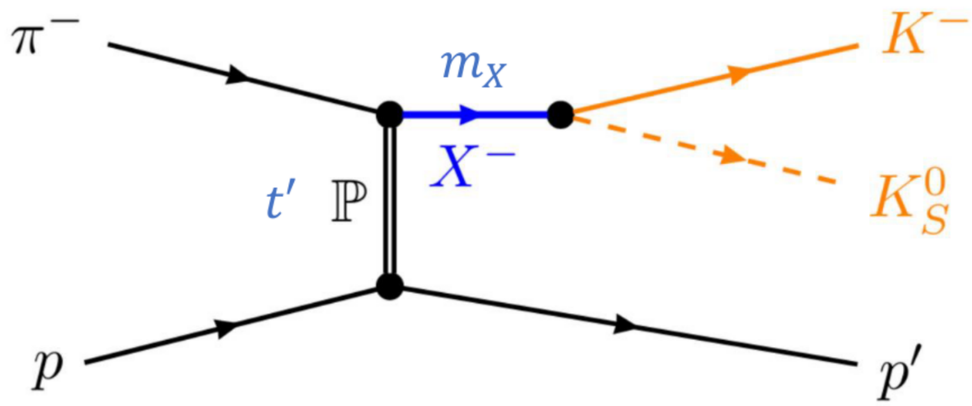
$$\boldsymbol{T_{JM}(m_X, t')}$$



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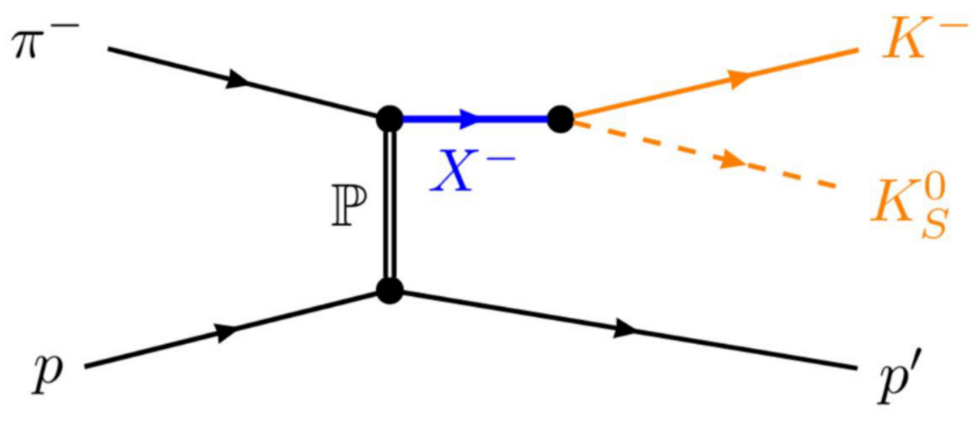
$$\boldsymbol{T_{JM}(m_X, t')}$$

$$\boldsymbol{\psi_{JM}(\theta, \phi) = Y_J^M(\theta, \phi)}$$

$$|\boldsymbol{M}| \geq \mathbf{1}$$



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(unnat. parity exchange ($\varepsilon = -1$) suppressed $\rightarrow$ M $\neq$ 0)

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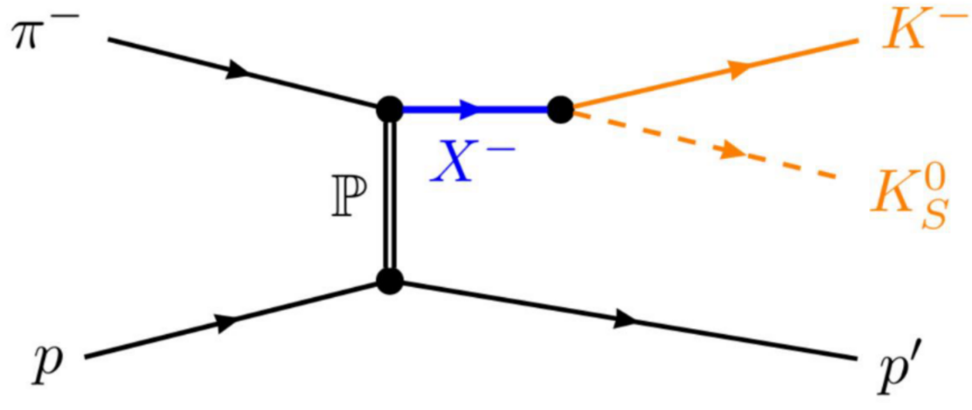

- Production, propagation and decay of X^-

$$\boldsymbol{T_{JM}(m_X, t')}$$

$$\boldsymbol{\psi_J(\theta, \phi) = Y_J^1(\theta, \phi)}$$

- Assume strong dominance of $\boldsymbol{|M| = 1}$

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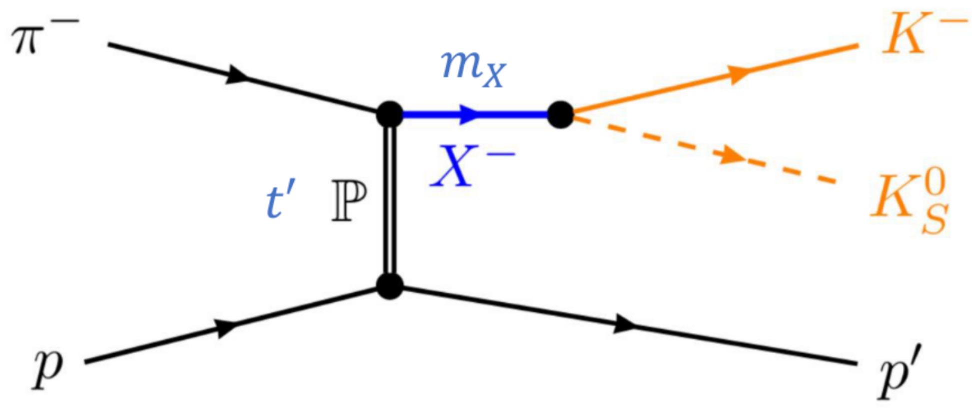

- Production, propagation and decay of X^-

$$\boldsymbol{T_{JM}(m_X, t')}$$
$$\boldsymbol{\psi_J(\theta, \phi) = Y_J^1(\theta, \phi)}$$

Partial wave:
specific (J^{PC})

- Assume strong dominance of $|\boldsymbol{M}| = \boldsymbol{1}$

- Fit $I(m_X, t'; \theta, \phi)$ to data **in $(\boldsymbol{m_X}, \boldsymbol{t'})$ bins**

Ambiguities in the Partial-Wave Decomposition

Decomposition of intensity into $\{T_J\}$ is not **unique** (for any final state with two spin-0 particles)

Several sets of $\{T_J\}$ lead to the **same** $\boldsymbol{I(\theta, \phi)}$ in each (m_X, t') bin

$$I(\theta, \phi) = \left| \sum_J T_J^{(1)} \psi_J(\theta, \phi) \right|^2 = \left| \sum_J T_J^{(2)} \psi_J(\theta, \phi) \right|^2$$

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$$I(\theta,\phi) = \left|\sum_J T_J\, \psi_J(\theta,\phi)\right|^2 = \underbrace{\left|\sum_J T_J\, Y_J^1(\theta,0)\right|^2}_{\boldsymbol{a(\theta)}} |\sin\phi|^2$$

$$Y_J^1(\theta,0) = \sum_{j=0}^{J-1} y_j \tan^{2J}\theta$$

$$a(\theta) = \sum_{j=0}^{\boldsymbol{J_{\max}-1}} c_J(\{T_J\}) \tan^{2J}(\theta) = c(\{T_J\}) \prod_{k=1}^{J_{\max-1}} \left(\tan^2(\theta) - \boldsymbol{r_k}(\{T_J\})\right)$$

root decomposition

$a(\tan^2\theta = r_k) = 0$

"Barrelet zeros"

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$$= \left|\sum_{j=0}^{\boldsymbol{J_{\max}-1}} c_j(\{T_J\})\ \tan^{2J}(\theta)\right|^2 |\sin\phi|^2$$

$$= c^2 \prod_{k=1}^{J_{\max-1}} |\tan^2(\theta) - \boldsymbol{r_k}|^2\ |\sin\phi|^2 \quad = c^2 \prod_{k=1}^{J_{\max-1}} |\tan^2(\theta) - \boldsymbol{r_k^*}|^2\ |\sin\phi|^2$$

$\{T_J'\} \neq \{T_J\}$

↑

$\{c_j'\}$

↑

$\boldsymbol{r_k^*}$

Continuous Amplitude Model

J^{PC}	Resonances
$\mathbf{1}^{--}$	$\rho(1450)$
$\mathbf{2}^{++}$	$a_2(1320), a_2'(1700)$
$\mathbf{3}^{--}$	None
$\mathbf{4}^{++}$	$a_4(1970)$

I. Continuous intensity model

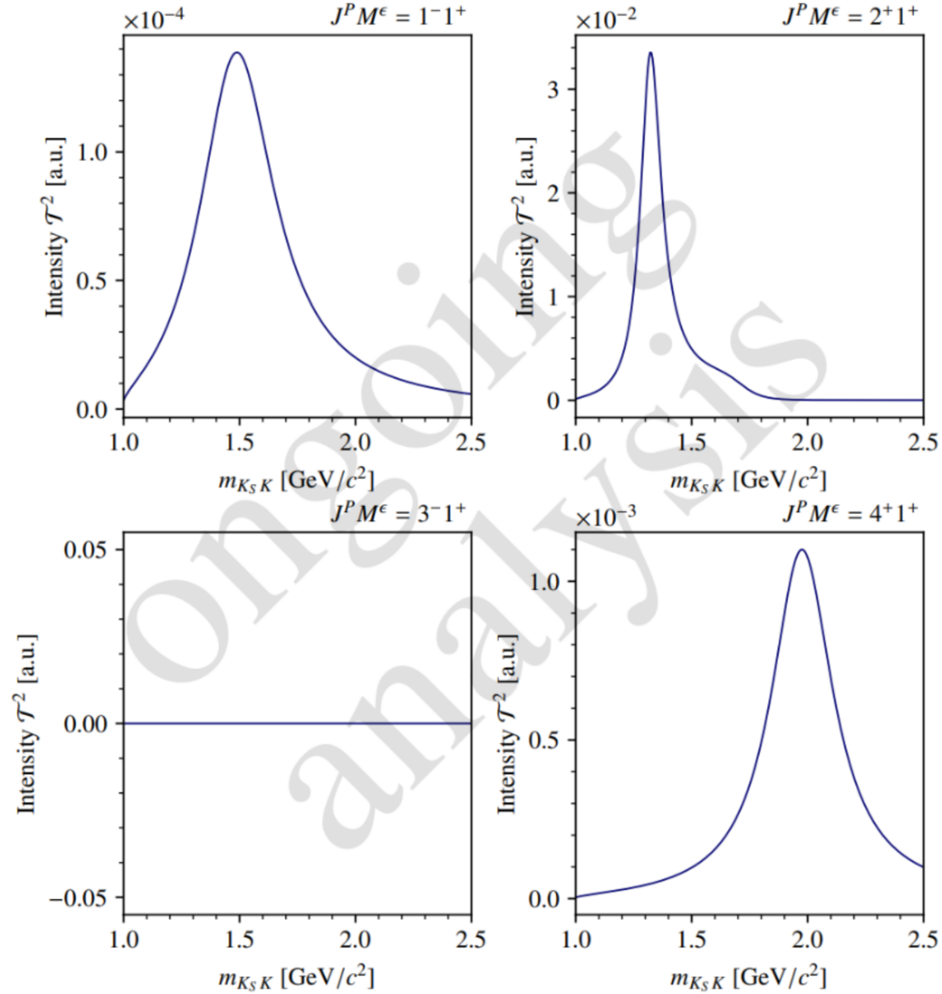
- create a model for the amplitudes in selected waves in m_X range
- m_X-dependence by Breit-Wigner amplitudes (PDG parameters)

$$T(m_X) = \underbrace{\sqrt{m_X}\,\sqrt{\rho_2(m_X)}}_{\text{phase-space factor}} \cdot \underbrace{Ce^{i\phi}}_{\text{complex scale}} \cdot \underbrace{D_{BW}(m_X; M_0, \Gamma_0)}_{\frac{M_0\,\Gamma_0}{M_0^2 - m_X^2 - iM_0\Gamma_0}}$$

Continuous Amplitude Model

I. Continuous intensity model

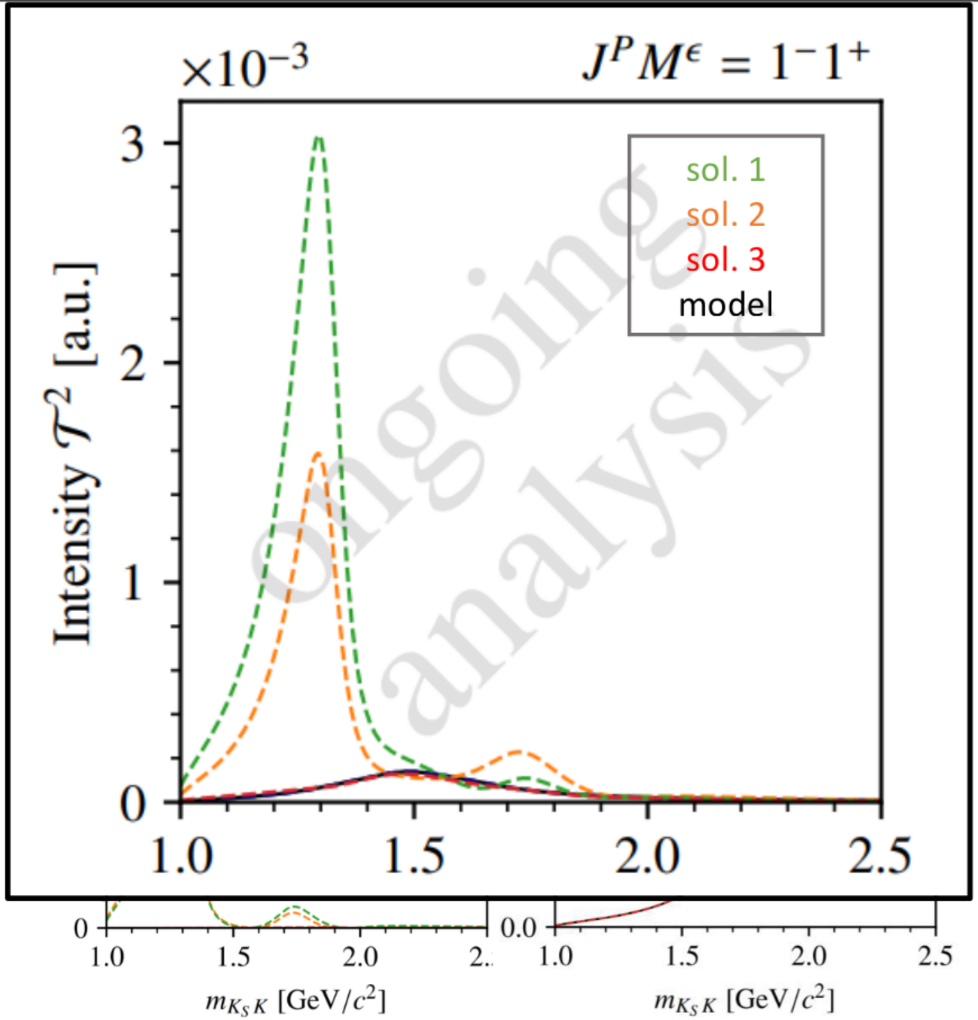
- create a model for the amplitudes in four waves
- m_X-dependence by Breit-Wigner amplitudes



Continuous Amplitude Model

I. Continuous intensity model $N_a = 3$

- Sample points in m_X and calculate ambiguous solutions
- Ambiguous intensities are also continuous
- Not all solutions are different from each other!
- 4^{++} intensity is invariant!



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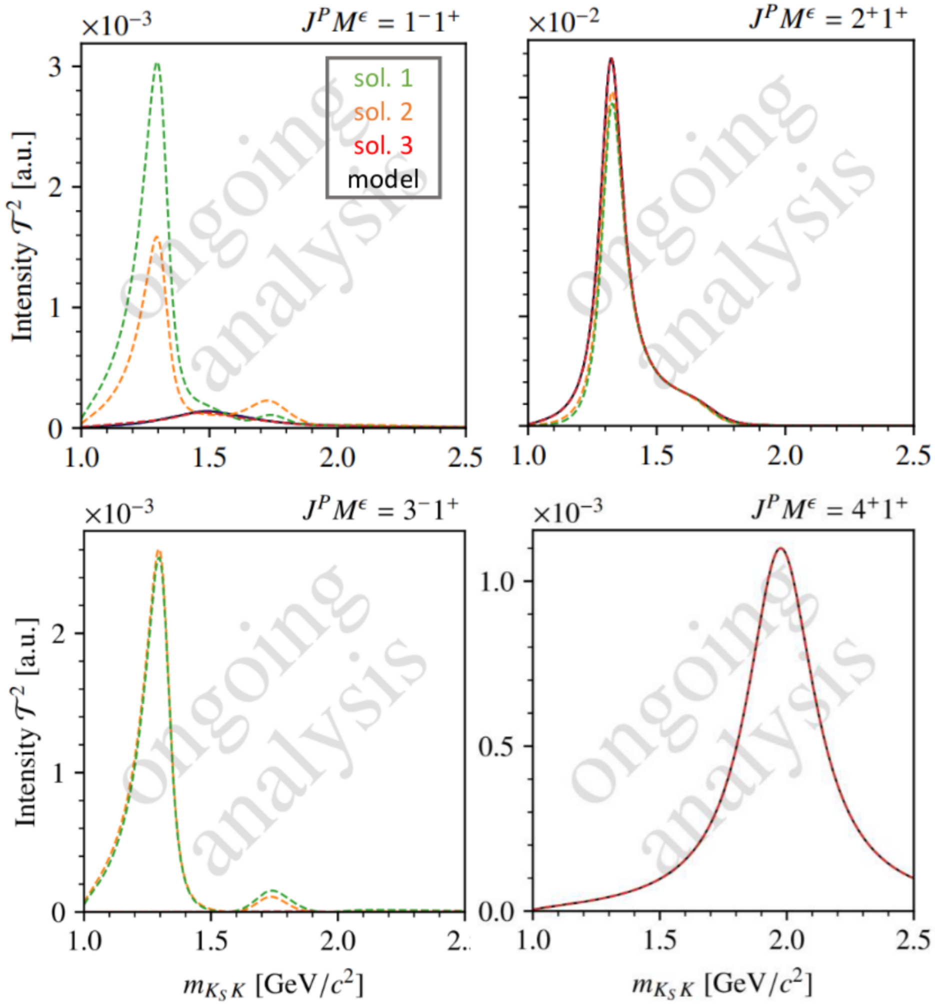

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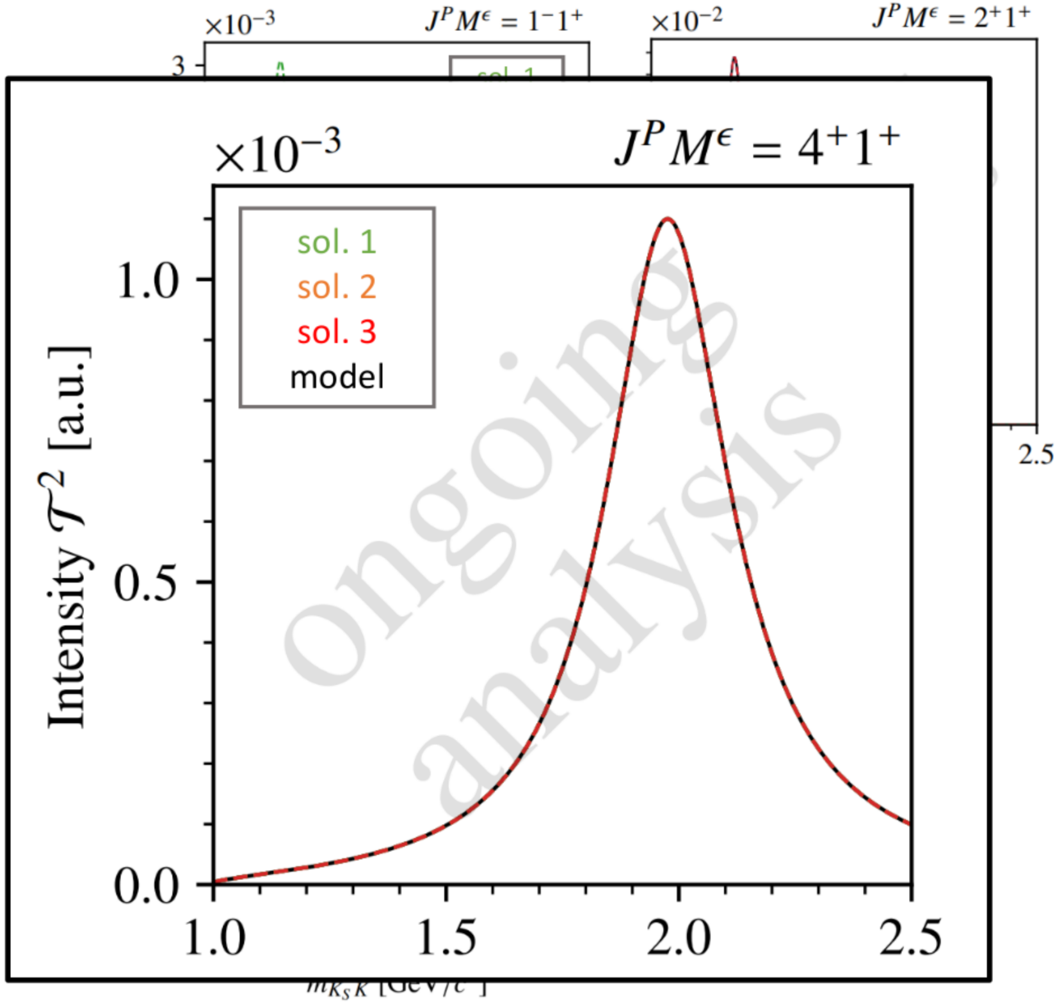

Study of the Ambiguities

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II. Finite pseudo-data

- reality: finite data and amplitudes unknown
- generate pseudo-data according to model
- perform a partial-wave decomposition fit

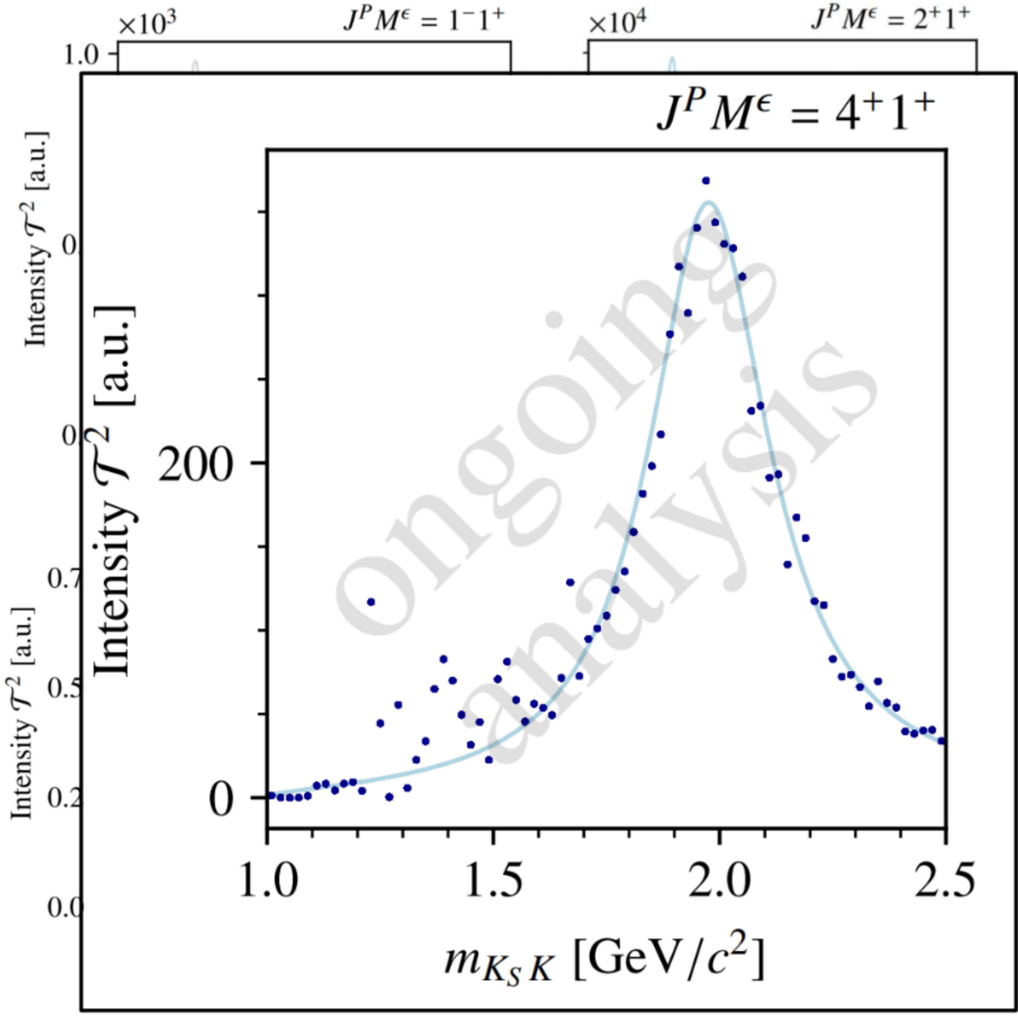
→ 3000 attempts with random start values

Partial-Wave Decomposition Fits on Pseudodata

II. Finite pseudo-data

- 4^{++} intensity is still invariant!
- Overall, amplitude values found by the fit follow the calculated distributions
- Not all solutions are found in each m_X bin

→ PWD fit distorts the intensity distribution!



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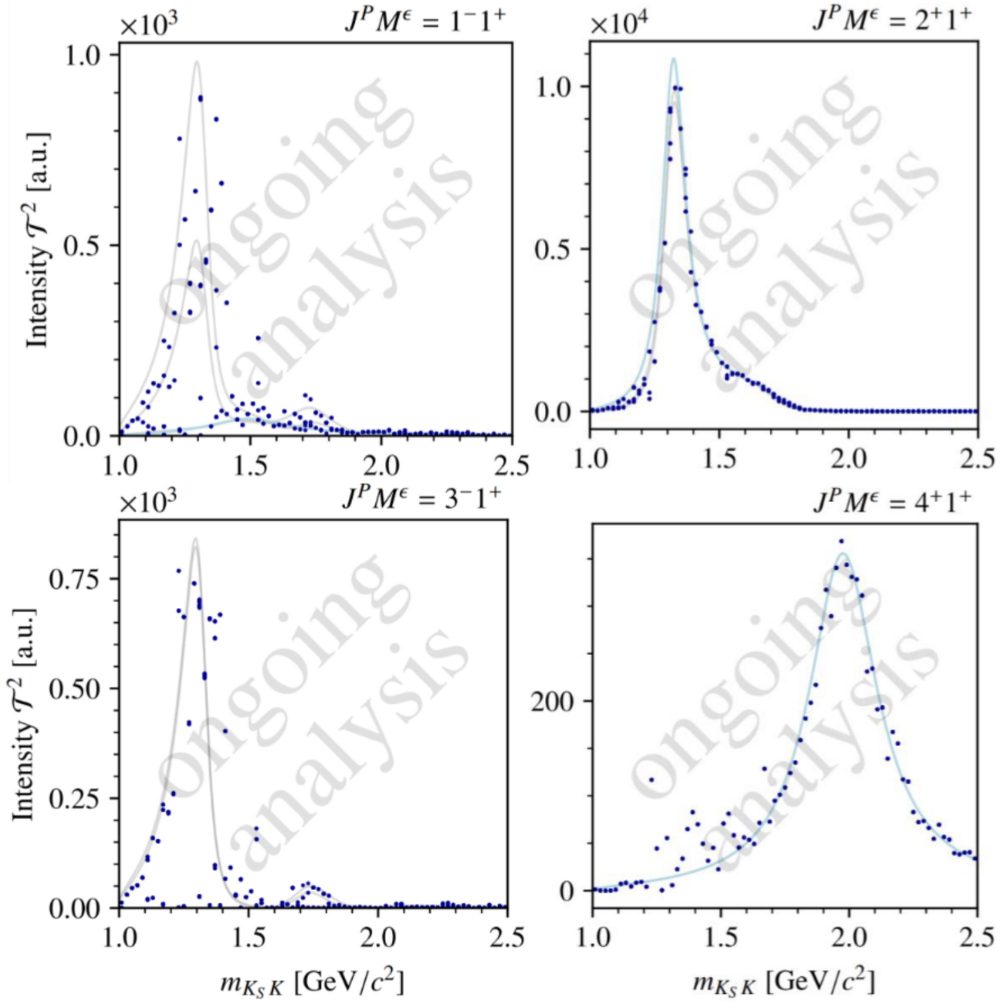

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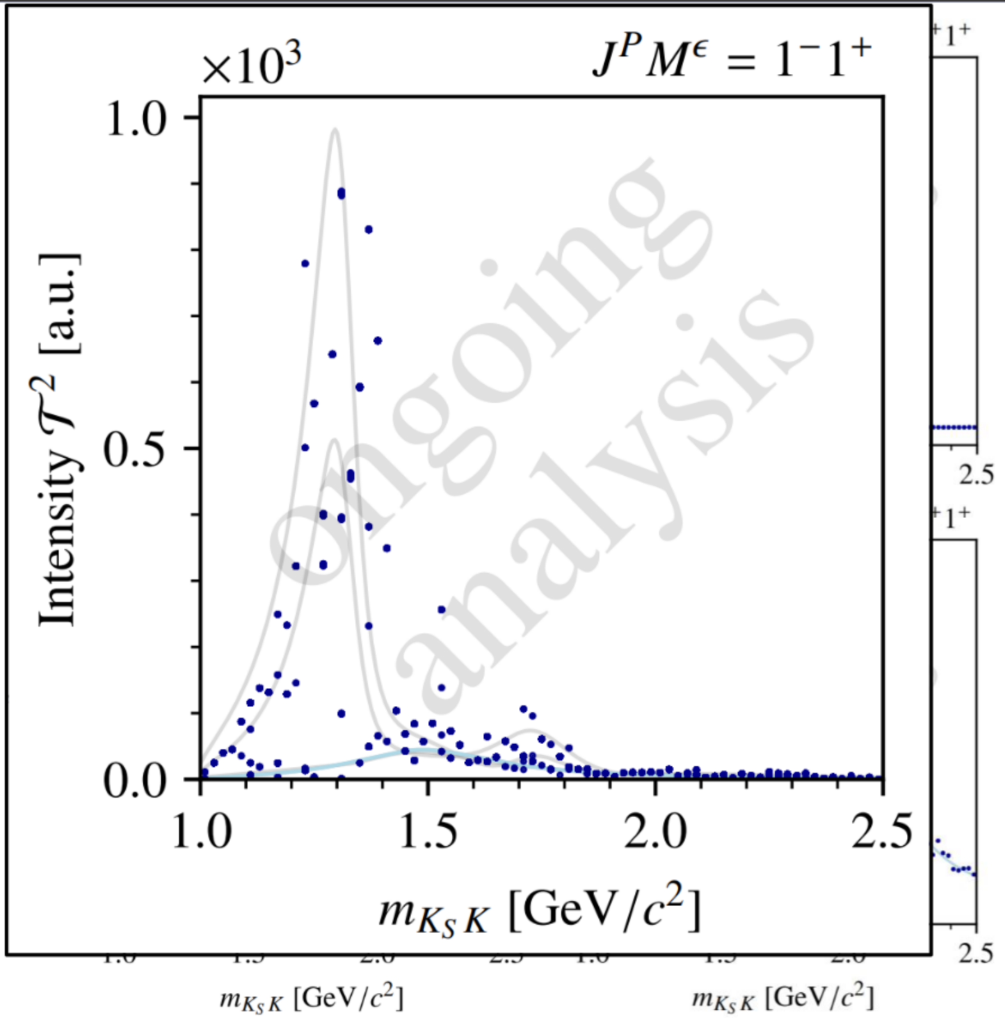
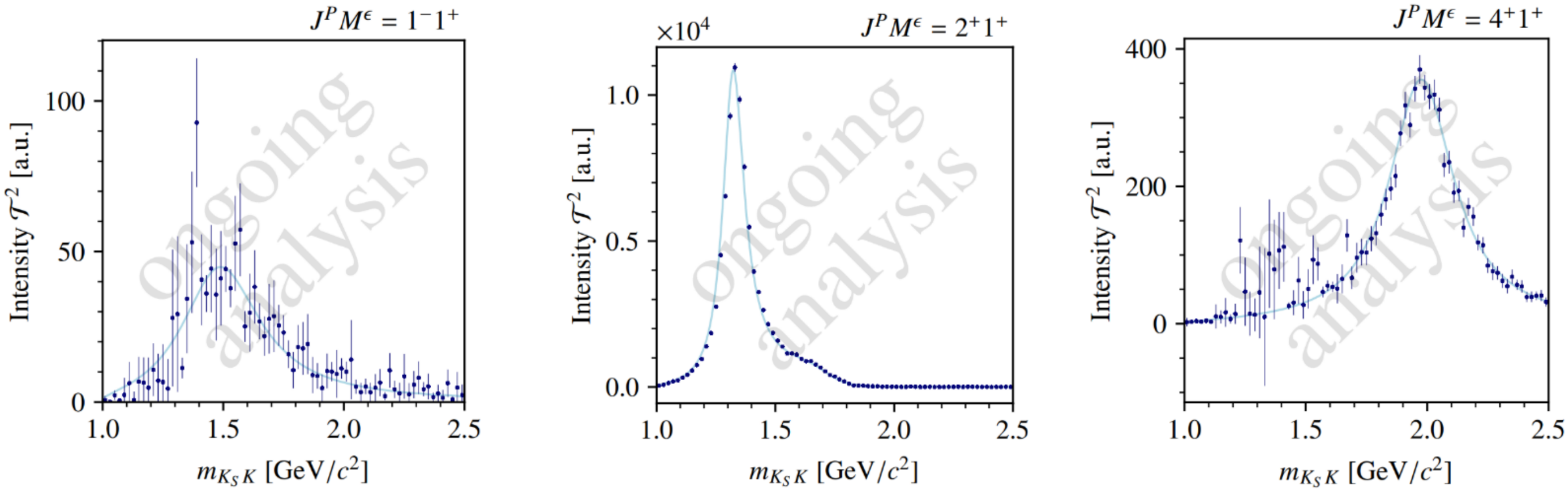


Reducing the Ambiguities

- Intensity of highest-spin wave is unaffected by ambiguities
- Including $M \geq 2$ → additional angular information → **resolves ambiguities**
- Remove wave with $J < J_{\max}$ → **resolves ambiguities**



Conclusions

- $\pi^- + p \to K_S^0 K^- + p$ allows selective study of a_J states

- **Ambiguities** appear in the partial-wave decomposition
- Complex-conjugation of roots $r_k = r_k(\{T_J\})$ of $a(\theta) \to$ **ambiguous solutions** for $\{T_J\}$

- We have shown that the values $\{T_J\}$ can be calculated.

Our study shows that:

- Ambiguous amplitudes are **continuous**
- PWD fit seems to distort intensity and **removes certain solutions**
- **Choice of included partial waves** may suppress the ambiguities

Outlook

- Improve reliability of PWD fit

- Perform partial-wave analysis of measured COMPASS $K_S^0 K^-$ data

- Expect contributions with $M = 2$ from dominant $J = 2$ partial wave in some m_X

→ ambiguities may be resolved in these bins

→ investigate influence of $M = 2$ in pseudodata

- Due to production mechanisms, odd J suppressed

→ ambiguities may be resolved

→ investigate influence of removing these waves from the PWD fit

Thank you for your attention!

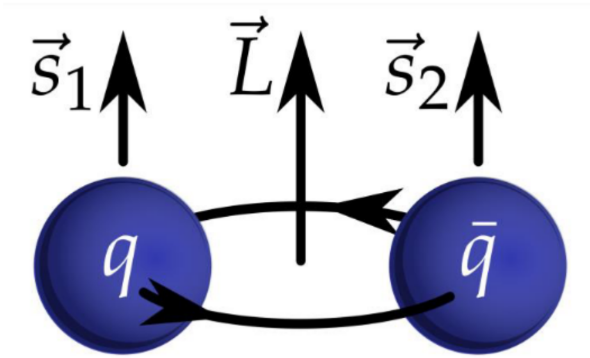
BACKUP

QCD in the Resonance Region

- At low energies (hadron regime): **QCD not solvable perturbatively**

- Theory: rely on models and effective theories, e.g. **quark model** (hadrons as bound states of **valence quarks**)

- Experimentally: **precision measurements** of hadronic states and search for so-called **exotic states** (forbidden in the quark model)



From B. Ketzer et al., Prog.Part.Nucl.Phys. 113 (2020) 103755, Light-Meson Spectroscopy with COMPASS

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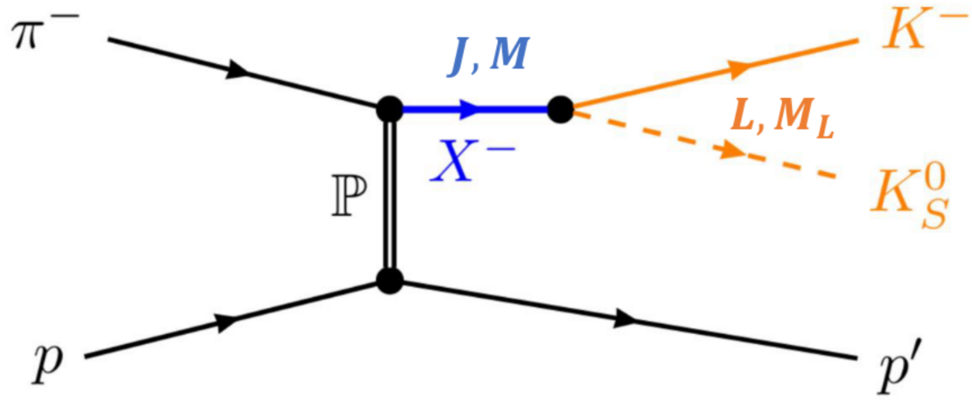
- Separate process amplitude into **partial waves**
  - Spin $\boldsymbol{J}$ and spin-projection $\boldsymbol{M}$

$$\boldsymbol{J = L, M = M_L}$$

$$\boldsymbol{P = C = (-1)^J}$$



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$$\boldsymbol{T_{JM}(m_X, t')}$$

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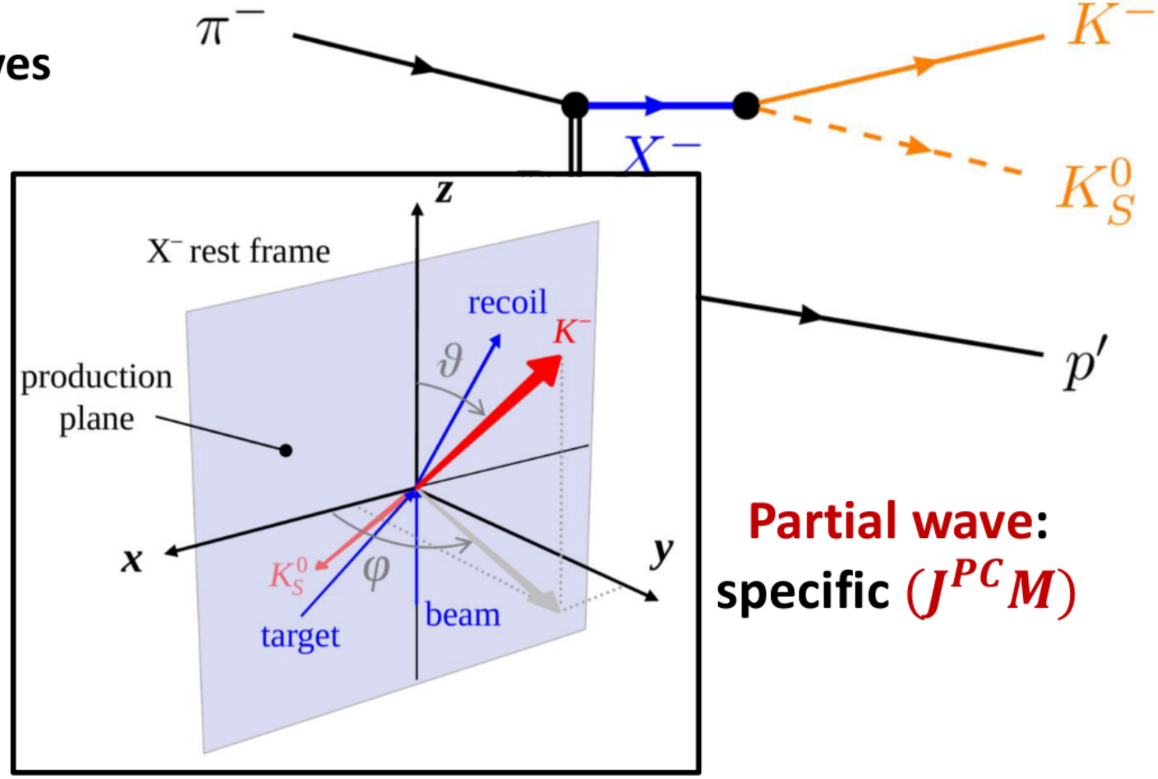
$$\boldsymbol{M = 1}$$

(reflectivity basis, $\varepsilon = -1$ suppressed $\rightarrow$ $M \neq 0$)



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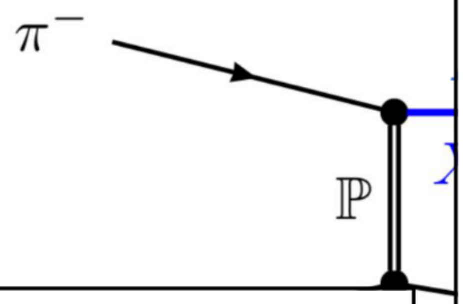
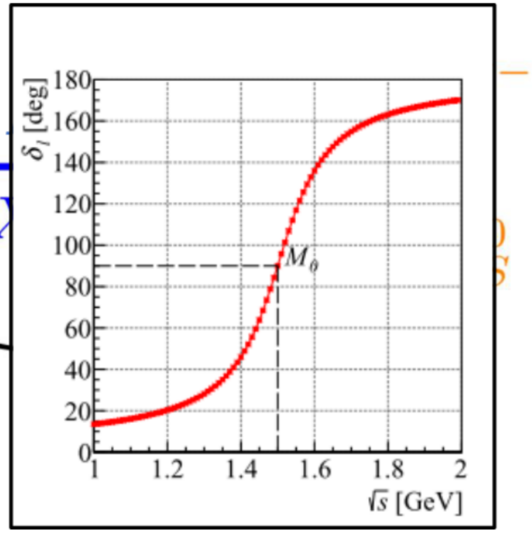
$$\boldsymbol{T_J(m_X, t') = P(m_X, t')\, D(m_X)}$$

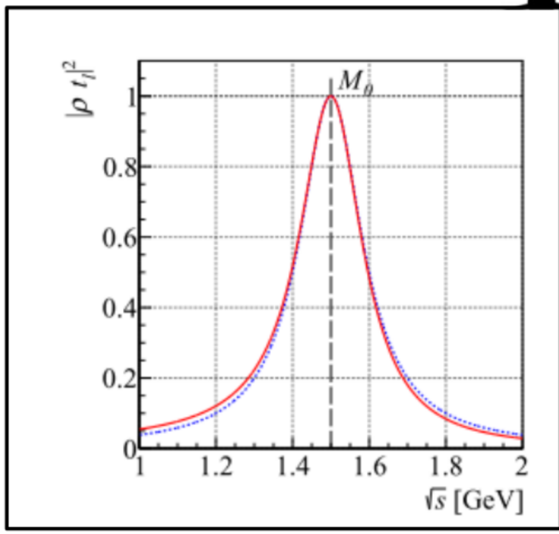
$$\boldsymbol{\psi_J(\theta, \phi) = Y_J^M(\theta, \phi)}$$

$$\boldsymbol{M = 1}$$

- Fit $I(m_X, t'; \theta, \phi)$ to data **in** $(\mathbf{m_X}, \mathbf{t'})$ **bins**:
- Choose finite set of $\{J^{PC}\}$







Partial wave: specific (J^{PC})

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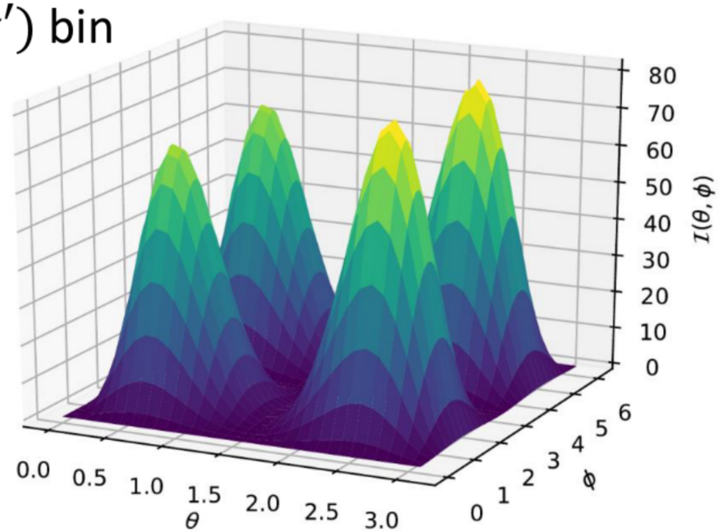

$$\{T_J\} = \begin{cases} 1+2i,\, 14+3i,\, 0+0i,\, 2+0i \\ \text{or} \\ 7+0i,\, 2-7i,\, 10+2i,\, 0+2i \end{cases}$$

Ambiguities in Incoherent Sectors

$$\boldsymbol{\varepsilon = \pm 1}: \quad I(m_X, t'; \theta, \phi) = \left|\sum_{JM} T_{JM}^{+}(m_X, t')\, \psi_{JM}^{+}(\theta, \phi)\right|^2 + \left|\sum_{JM} T_{JM}^{-}(m_X, t')\, \psi_{JM}^{-}(\theta, \phi)\right|^2$$

$$a_0^- = \sum_{J=0}^{J_{\max}^-} T_{J0}^- \, Y_J^0(\theta, 0) \qquad \varepsilon = -1, M = 0$$

$$a_1^- = \sum_{J=1}^{J_{\max}^-} T_{J1}^- \, Y_J^1(\theta, 0) \qquad \varepsilon = -1, M = 1$$

$$a_1^+ = \sum_{J=1}^{J_{\max}^+} T_{J1}^+ \, Y_J^1(\theta, 0) \qquad \varepsilon = +1, M = 1$$

- $a_s^- = a_0^- + a_1^-$, then same procedure as for a single sector

- New amplitudes for $\varepsilon = +1$: $|a_1^+|^2 = |a_1^-|^2 - \text{const.}$ → **positivity requirement**!

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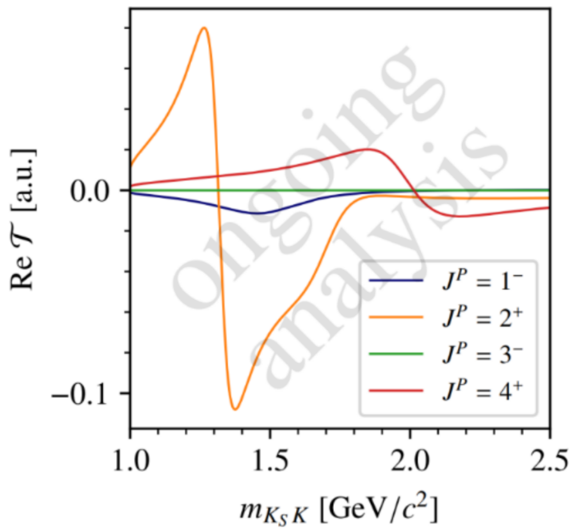
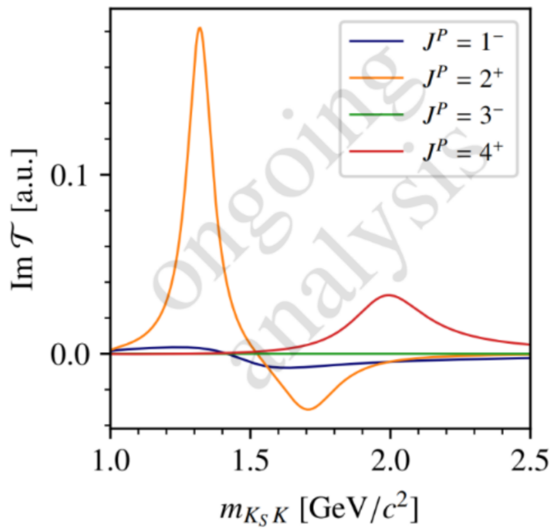
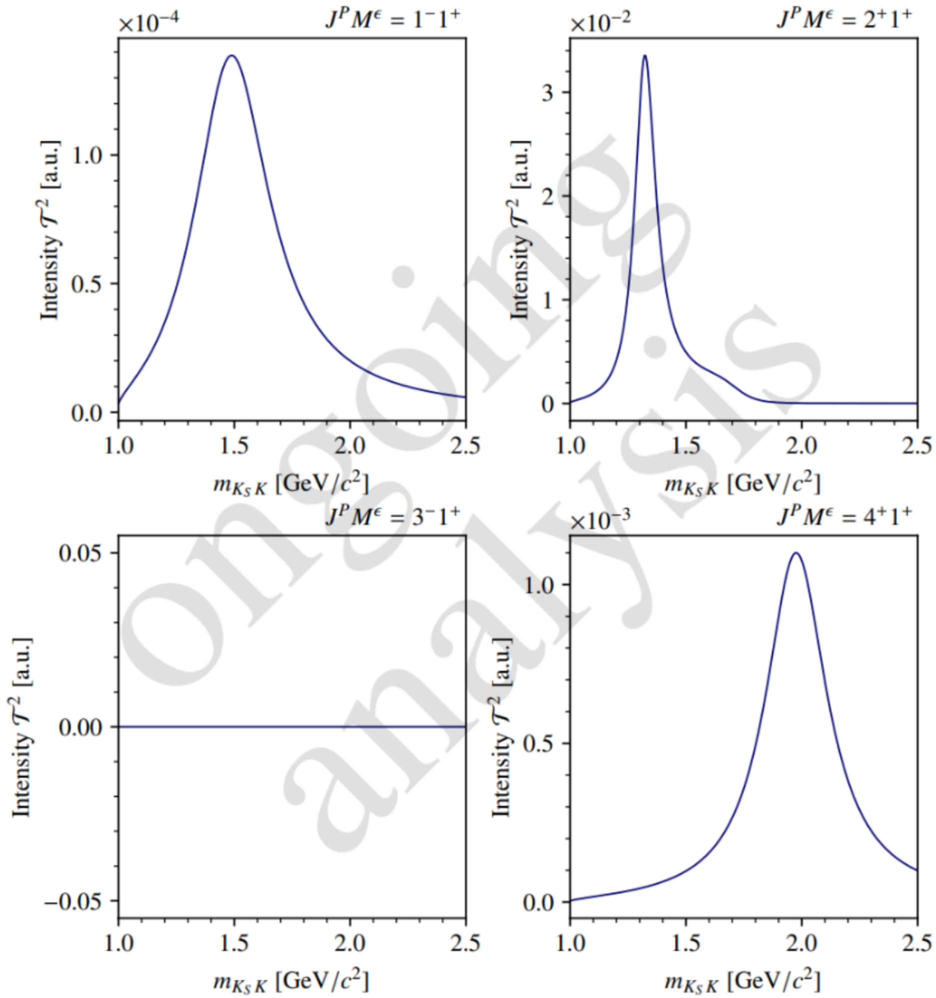
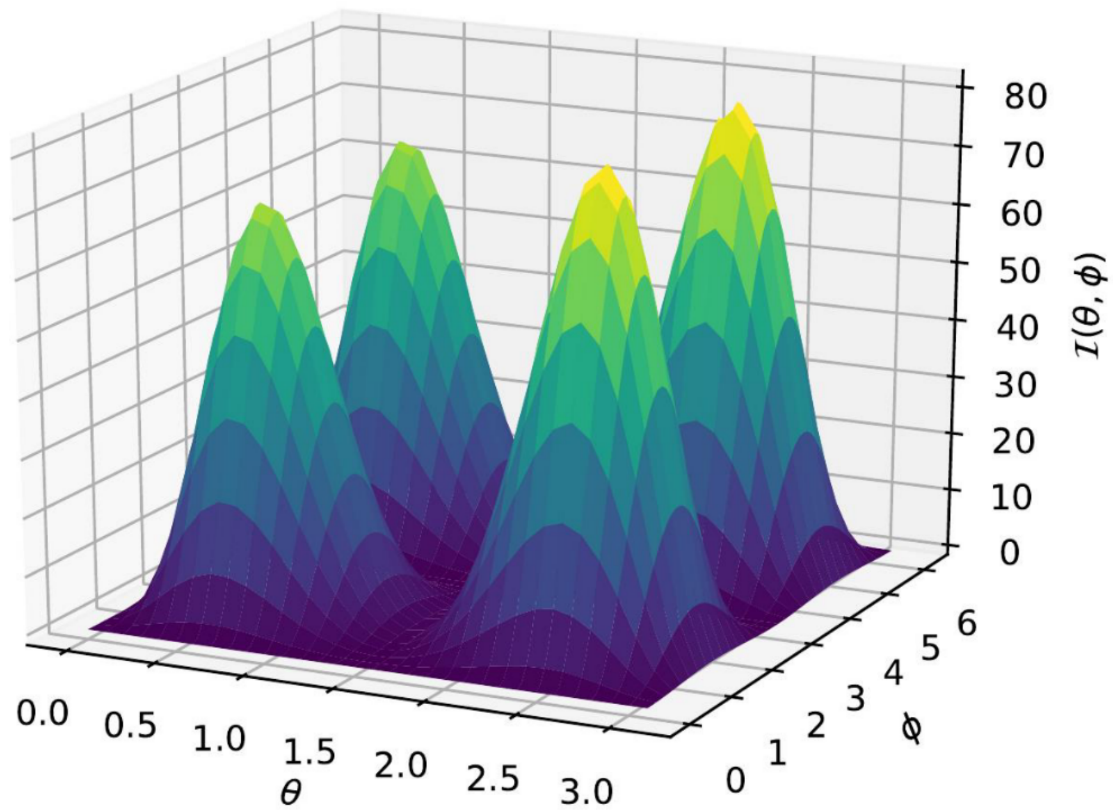
- m_X-dependence by Breit-Wigner amplitudes

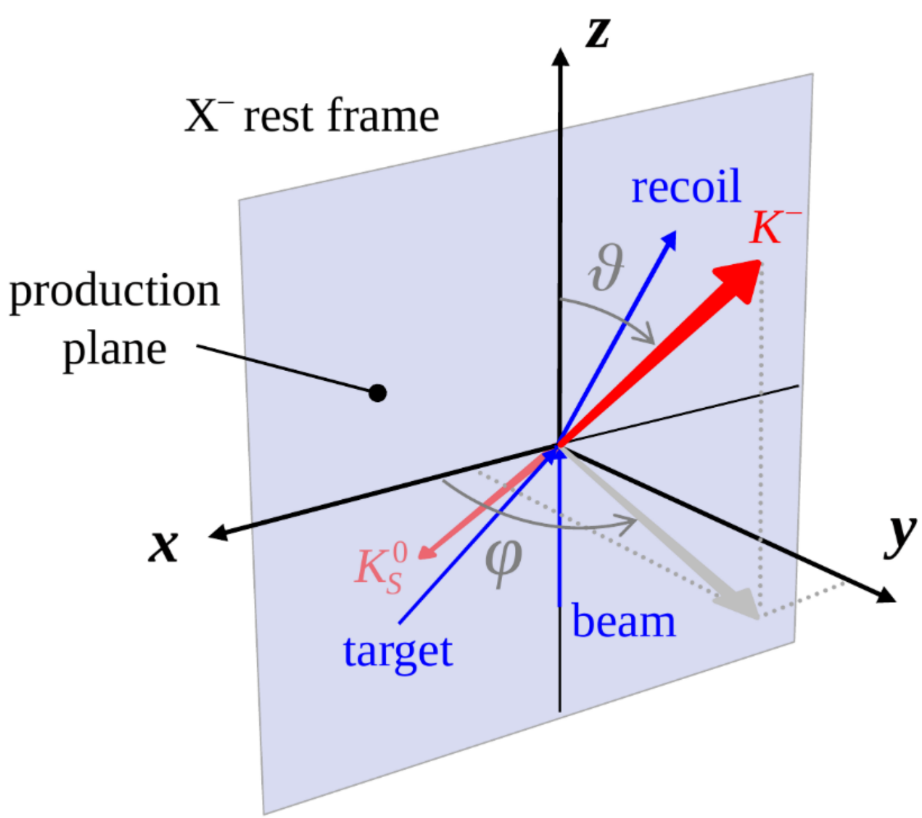








80
70
60
50
40
30
20
10
0
$\mathcal{I}(\theta, \phi)$
0.0
0.5
1.0
1.5
2.0
2.5
3.0
θ
0
1
2
3
4
5
6
ϕ



z
X^- rest frame
recoil
K^-
production
plane
ϑ
x
y
K_S^0
φ
beam
target