# Fit of the $a_1(1420)$ as a Triangle Singularity

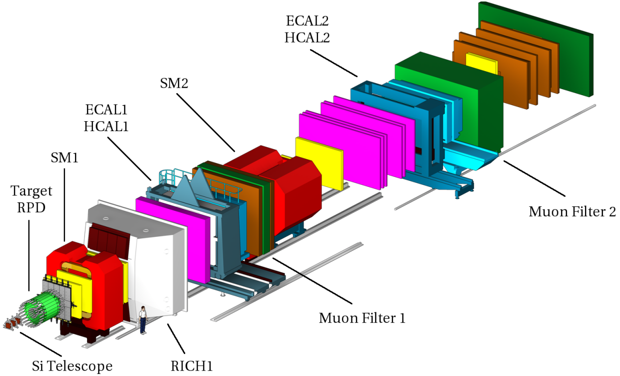

**Mathias Wagner**
on behalf of the COMPASS collaboration

HISKP, Bonn University


August 19, 2022

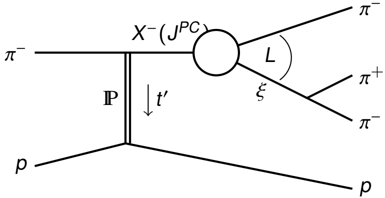
at the Workshop
$e^+e^-$ Collisions From Phi to Psi 2022

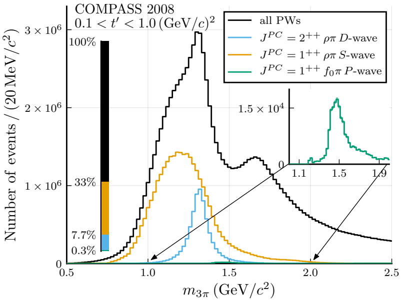
- Secondary hadron beam, mostly $\pi^-$ ($\sim 97\,\%$)
- $E_{\text{beam}} = 190\,\text{GeV}$
- Fixed liquid-hydrogen target (40 cm)




[COMPASS, NIM A**779**, 69-115 (2015)]

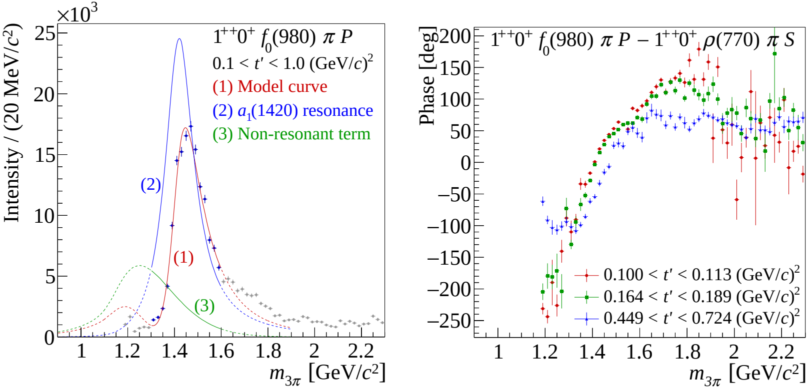
- Secondary hadron beam, mostly $\pi^-$ ($\sim 97\,\%$)
- $E_{\text{beam}} = 190\,\text{GeV}$
- Fixed liquid-hydrogen target (40 cm)
- $\pi^- + p \quad \to \quad \pi^- + \pi^- + \pi^+ + p$
- PWA with 88 waves binned in $m_{3\pi}$, $t'$




[COMPASS, PRL **127**, 082501]

# BW-fit to resonance-like signal in $1^{++}$ partial wave

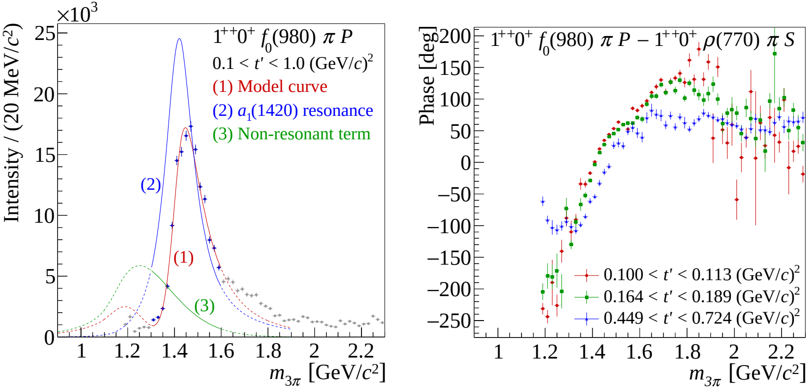



[COMPASS, PRL **115**, 082001 (2015)]

- $a_1(1420)$ narrow peak, strong phase motion
- Very close to ground state $a_1(1260)$
- Narrower than ground state

⇒ No ordinary radial excitation

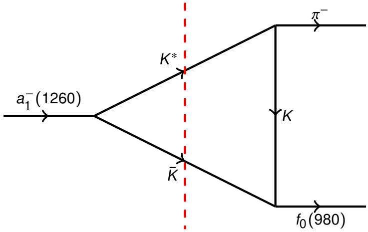
# BW-fit to resonance-like signal in $1^{++}$ partial wave



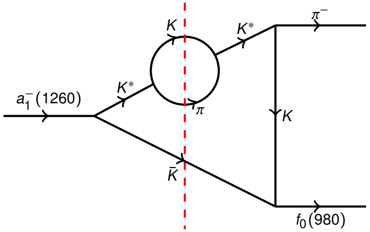

[COMPASS, PRL **115**, 082001 (2015)]

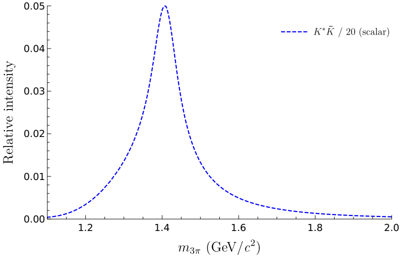
- 4-quark state [H.-X. Chen et al. (2015)], [T. Gutsche et al. (2017)]
- $K^* \bar{K}$ molecule (similar to $X(3872)$) [T. Gutsche et al. (2017)]
- Dynamic effect of interference with Deck-amplitude [Basdevant & Berger, PRL **114**, 192001 (2015)]
- **Triangle singularity (TS)** [Mikhasenko et al., PRD **91**, 094015 (2015)] [Aceti et al., PRD **94**, 096015 (2016)]

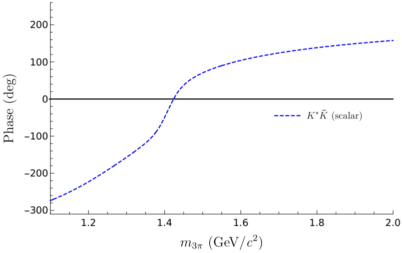
- Dispersive approach



- Dispersive approach
- Include finite width of $K^*$

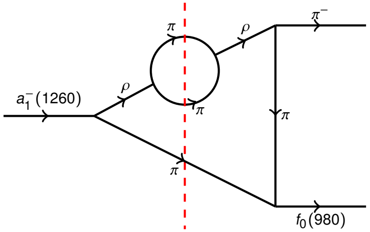


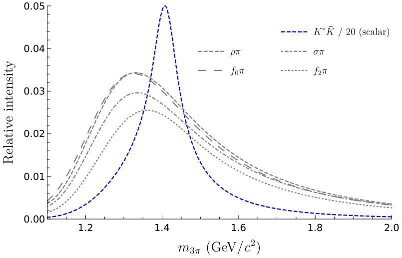



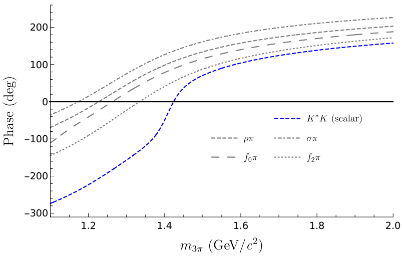




[COMPASS, PRL **127**, 082501]

- Dispersive approach
- Include finite width of $K^*$
- Negligible contribution from other triangles



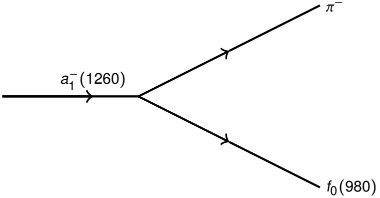



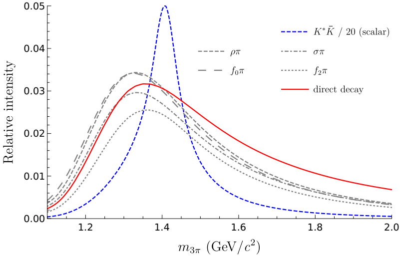



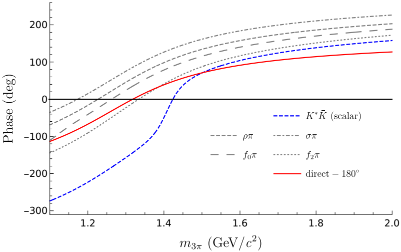

[COMPASS, PRL **127**, 082501]

- Dispersive approach
- Include finite width of $K^*$
- Negligible contribution from other triangles

direct decay:



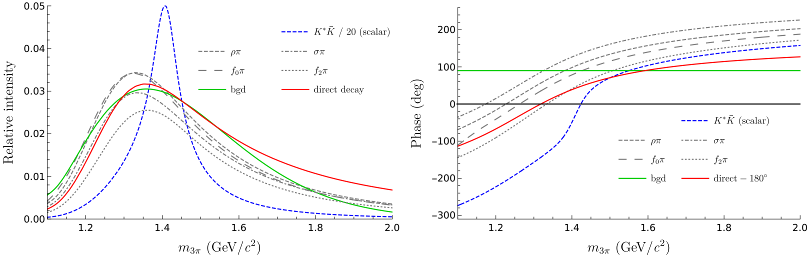




[COMPASS, PRL **127**, 082501]

- Dispersive approach
- Include finite width of $K^*$
- Negligible contribution from other triangles

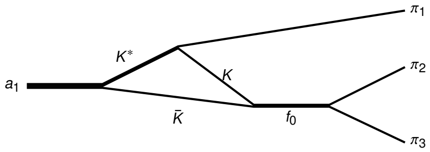
phenomenological background

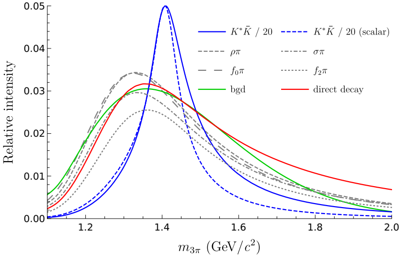


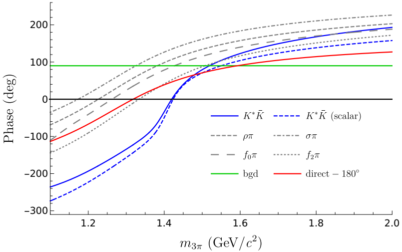

[COMPASS, PRL **127**, 082501]

- Dispersive approach
- Include finite width of $K^*$
- Negligible contribution from other triangles
- Inclusion of spin distorts shape

partial-wave projection:



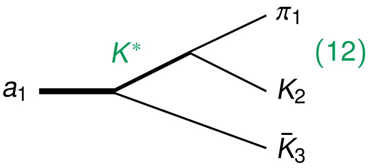




[COMPASS, PRL **127**, 082501]

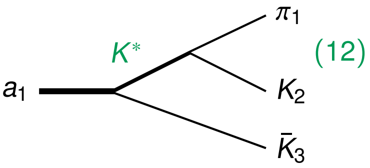
Include spin via partial-wave projection:

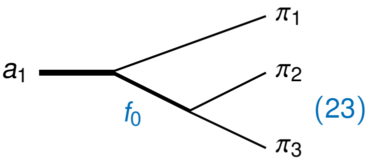
1. Look at the partial wave for $a_1(1260) \to K\bar{K}\pi$ with isobar $K^*$



Include spin via partial-wave projection:

1. Look at the partial wave for $a_1(1260) \to K\bar{K}\pi$ with isobar $K^*$
2. Project it onto the $3\pi$ final state with isobar $f_0(980)$



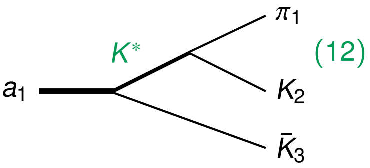



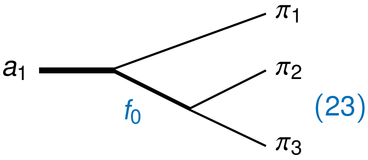
# Include Spin

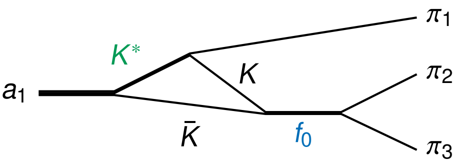


Include spin via partial-wave projection:

1. Look at the partial wave for $a_1(1260) \to K\bar{K}\pi$ with isobar $K^*$
2. Project it onto the $3\pi$ final state with isobar $f_0(980)$
3. Obtain the first order approximation of the Khuri-Treiman approach



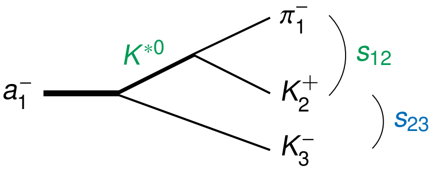



$$A(\tau) = \sum_{w=(JMLS)} \left[ F_w(s_{12}) Z_w^*(\Omega_{3,12}) + F_w(s_{23}) Z_w^*(\Omega_{1,23}) \right]$$

Simple model: $F_w(s_{12}) = C_{a_1} \cdot t_{K^*}(s_{12})$

- $A(\tau)$: full amplitude of kinematic variables $\tau$
- $F_w(s_{ij})$: isobar amplitude of decay with isobar in $(ij)$-channel
- $Z_w(\Omega_{k,ij})$: angular dependence of amplitude in $(ij)$-channel



$$A(\tau) = \sum_{w=(JMLS)} \left[ F_w(s_{12}) Z_w^*(\Omega_{3,12}) + F_w(s_{23}) Z_w^*(\Omega_{1,23}) \right]$$

Projection to channel (23):

$$\begin{aligned} A_w(s_{23}) &= \int \mathrm{d}\,\Omega_{1,23} Z_w(\Omega_{1,23}) A(\tau) \\ &= F_w(s_{23}) + \hat{F}_w(s_{23}) \end{aligned}$$

with $\hat{F}_w(s_{23}) := \int \mathrm{d}\, Z_w(s_{23}) \sum_{w'} F_{w'}(s_{12}) Z_{w'}^*(\Omega_{3,12})$

$$A(\tau) = \sum_{w=(JMLS)} \left[ F_w(s_{12}) Z_w^*(\Omega_{3,12}) + F_w(s_{23}) Z_w^*(\Omega_{1,23}) \right]$$

Projection to channel (23):

$$\begin{aligned} A_w(s_{23}) &= \int \mathrm{d}\,\Omega_{1,23} Z_w(\Omega_{1,23}) A(\tau) \\ &= F_w(s_{23}) + \hat{F}_w(s_{23}) \end{aligned}$$

with $\hat{F}_w(s_{23}) := \int \mathrm{d}\, Z_w(s_{23}) \sum_{w'} F_{w'}(s_{12}) Z_{w'}^*(\Omega_{3,12})$

unitarity for PW amplitude $A_w$:

$$\Rightarrow \quad F_w(s_{23}) = t_\xi(s_{23}) \left[ C_w + \frac{1}{2\pi} \int_{s_{\mathrm{th}}}^{\infty} \frac{\rho(\tilde{s}_{23}) \hat{F}_w(\tilde{s}_{23})}{\tilde{s}_{23} - s_{23}} \mathrm{d}\,\tilde{s}_{23} \right]$$

$$A(\tau) = \sum_{w=(JMLS)} \left[ F_w(s_{12}) Z_w^*(\Omega_{3,12}) + F_w(s_{23}) Z_w^*(\Omega_{1,23}) \right]$$

Projection to channel (23):

$$\begin{aligned} A_w(s_{23}) &= \int \mathrm{d}\,\Omega_{1,23} Z_w(\Omega_{1,23}) A(\tau) \\ &= F_w(s_{23}) + \hat{F}_w(s_{23}) \end{aligned}$$

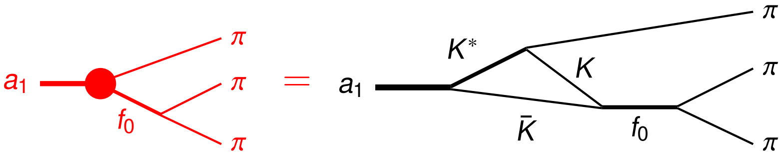
with $\hat{F}_w(s_{23}) := \int \mathrm{d}\, Z_w(s_{23}) \sum_{w'} F_{w'}(s_{12}) Z_{w'}^*(\Omega_{3,12})$

unitarity for PW amplitude $A_w$:

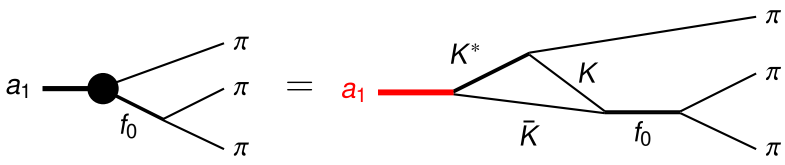
$$\Rightarrow \quad F_w(s_{23}) = t_\xi(s_{23}) \left[ C_w + \frac{1}{2\pi} \int_{s_{\text{th}}}^{\infty} \frac{\rho(\tilde{s}_{23}) \hat{F}_w(\tilde{s}_{23})}{\tilde{s}_{23} - s_{23}} \mathrm{d}\,\tilde{s}_{23} \right]$$

**Problem:** $\hat{F}$ depends on $F$ as well! $\rightsquigarrow$ solve iteratively

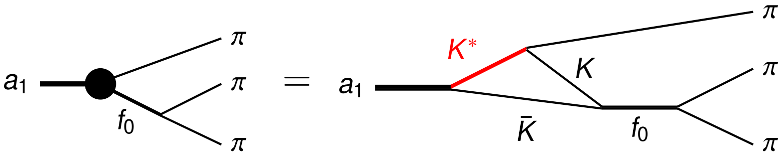
$$F(s_{23}) = t_{f_0}(s_{23}) \frac{1}{2\pi} \int_{4m_K^2}^{\infty} \mathrm{d}\,\tilde{s}_{23} \frac{\rho(\tilde{s}_{23}) \int \mathrm{d}\, Z_{f_0}(\tilde{s}_{23})\; C_{a_1}\, t_{K^*}(s_{12}) Z^*_{K^*}(\Omega_{3,12})}{\tilde{s}_{23} - s_{23}}$$



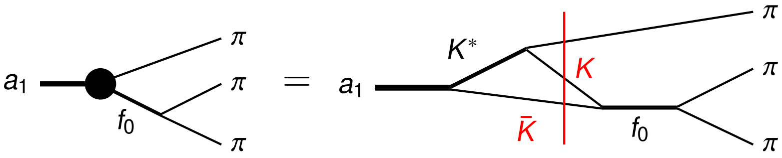
$$F(s_{23}) = t_{f_0}(s_{23})\frac{1}{2\pi}\int_{4m_K^2}^{\infty} \mathrm{d}\,\tilde{s}_{23}\frac{\rho(\tilde{s}_{23})\int \mathrm{d}\,Z_{f_0}(\tilde{s}_{23})\ {\color{red}C_{a_1}}\,t_{K^*}(s_{12})Z^*_{K^*}(\Omega_{3,12})}{\tilde{s}_{23}-s_{23}}$$



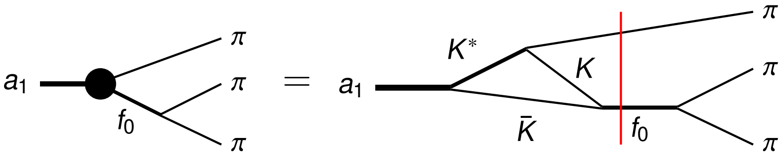
$$F(s_{23}) = t_{f_0}(s_{23}) \frac{1}{2\pi} \int_{4m_K^2}^{\infty} \mathrm{d}\,\tilde{s}_{23} \frac{\rho(\tilde{s}_{23}) \int \mathrm{d}\, Z_{f_0}(\tilde{s}_{23})\; C_{a_1}\, t_{K^*}(s_{12}) Z_{K^*}^{*}(\Omega_{3,12})}{\tilde{s}_{23} - s_{23}}$$



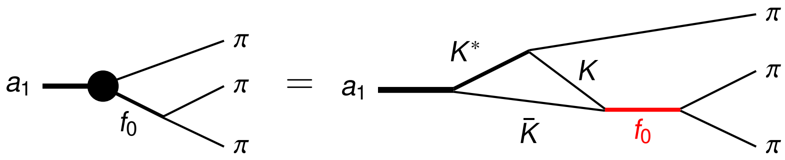
$$F(s_{23}) = t_{f_0}(s_{23}) \frac{1}{2\pi} \int_{4m_K^2}^{\infty} \mathrm{d}\,\tilde{s}_{23} \frac{\rho(\tilde{s}_{23}) \int \mathrm{d}\, Z_{f_0}(\tilde{s}_{23})\; C_{a_1}\, t_{K^*}(s_{12}) Z^*_{K^*}(\Omega_{3,12})}{\tilde{s}_{23} - s_{23}}$$



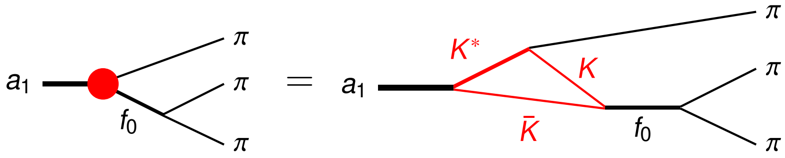
$$F(s_{23}) = t_{f_0}(s_{23}) \frac{1}{2\pi} \int_{4m_K^2}^{\infty} \mathrm{d}\,\tilde{s}_{23} \frac{\rho(\tilde{s}_{23}) \int \mathrm{d}\, Z_{f_0}(\tilde{s}_{23})\; C_{a_1}\, t_{K^*}(s_{12}) Z^*_{K^*}(\Omega_{3,12})}{\tilde{s}_{23} - s_{23}}$$



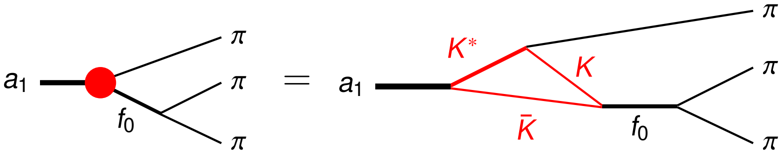
$$F(s_{23}) = t_{f_0}(s_{23}) \frac{1}{2\pi} \int_{4m_K^2}^{\infty} \mathrm{d}\,\tilde{s}_{23} \frac{\rho(\tilde{s}_{23}) \int \mathrm{d}\, Z_{f_0}(\tilde{s}_{23})\; C_{a_1}\, t_{K^*}(s_{12}) Z_{K^*}^{*}(\Omega_{3,12})}{\tilde{s}_{23} - s_{23}}$$

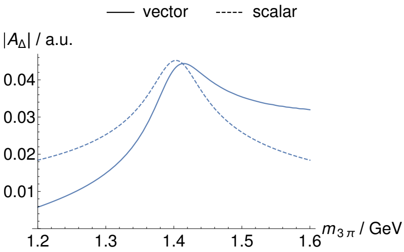


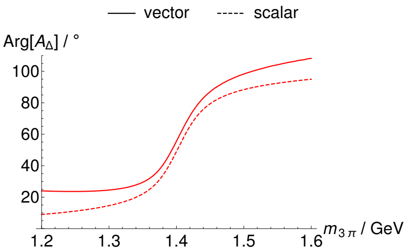
$$F(s_{23}) = C_{a_1}\, t_{f_0}(s_{23}) \frac{1}{2\pi} \int_{4m_K^2}^{\infty} \mathrm{d}\,\tilde{s}_{23} \frac{\rho(\tilde{s}_{23}) \int \mathrm{d}\, Z_{f_0}(\tilde{s}_{23})\ t_{K^*}(s_{12}) Z_{K^*}^{*}(\Omega_{3,12})}{\tilde{s}_{23} - s_{23}}$$



# Result — Partial-Wave Projection



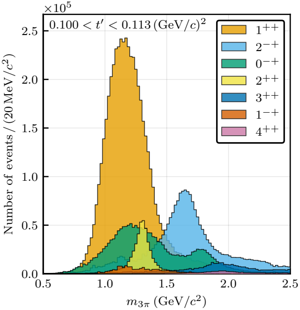

- Shape distorted, but similar
- Peak and phase motion at the same position

$\Rightarrow$ Scalar approximation reproduces main features

Minimal fit model ⇝ choose 3 of the 88 waves of the PWA

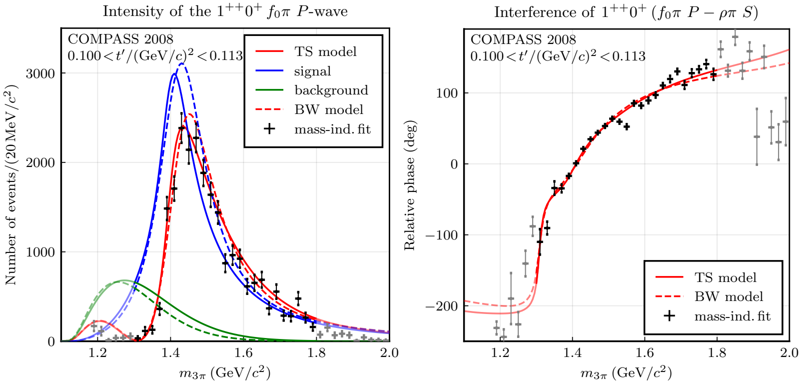
**Notation:** $J^{PC}\, M^{\varepsilon}\, \xi\pi\, L$

- $1^{++}\, 0^{+}\, \rho\pi$ $S$-wave:
  Contains source $a_1(1260)$, but huge non-res. background
- $1^{++}\, 0^{+}\, f_0(980)\pi$ $P$-wave:
  Signal of interest $a_1(1420)$
- $2^{++}\, 1^{+}\, \rho\pi$ $D$-wave:
  Clean $a_2(1320)$ with almost no non-res. background




[B. Ketzer, B. Grube, D. Ryabchikov, PPNP **113**, 0146-6410 (2020)]

**Note:** Fit all $t'$-slices with common resonance parameters. Show only fit of first slice.

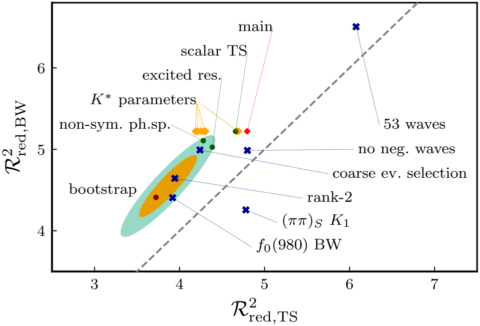



[COMPASS, PRL **127**, 082501]

- Comparison between TS model (solid) and BW model (dashed)
- Similar fit quality

Compare $\mathcal{R}^2_{\text{red}} = \sum_i \left(\frac{y_i - f(x_i)}{\sigma_i}\right)^2$,

but sum only over $f_0\pi$ $P$-intensity and its rel. phase to $\rho\pi$ $S$




[COMPASS, PRL **127**, 082501]

- - - $\mathcal{R}^2_{\text{red, TS}} = \mathcal{R}^2_{\text{red, BW}}$

● main fit

× syst. studies of PWA

● syst. studies of model

◆ changing $K^*$ resonance parameters

$1\sigma$ and $2\sigma$ ellipses for bootstrap of data points

(Almost) all studies show a better fit quality for the TS model.

- Scalar approximation already matches the data well

⇒ Good starting point for first investigation

- $a_1(1420)$ fully explainable with rescattering
- Similar fit quality as with Breit-Wigner
- No free parameters needed to fix the position!
- Triangle singularity expected to be present
- Systematic studies also prefer the TS model
- Occam's razor: No need for a new genuine resonance

⇒ First complete analysis in the light sector with a TS model

- Investigate $K\bar{K}\pi$ spectrum at COMPASS
- Look into $\tau \to 3\pi + \nu_\tau$ at BELLE II, no Deck-like background

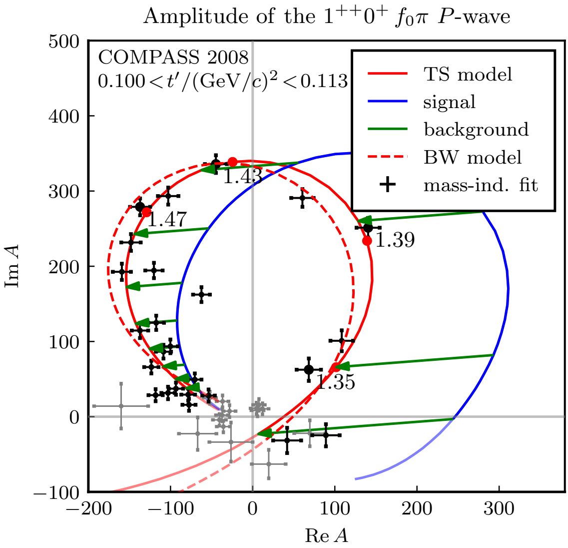
- Heavy quarks: XYZ states
  - Most peaks in data too narrow to be only TS
  - *"Observation of a Narrow Pentaquark State, $P_c(4312)^+$, and of the Two-Peak Structure of the $P_c(4450)^+$"* [LHCb, PRL **122**, 222001 (2019)]
  - *"Amplitude analysis and the nature of the $Z_c(3900)$"* [JPAC, Mod. Phys. Lett. B **772** (2017)]

- Baryon sector
  - *"Photoproduction of $K^+\Lambda(1405) \to K^+\pi^0\Sigma^0$ extending to forward angles and low momentum transfer"* [BGOOD, arXiv:2108.12235 (2021)]
  - *"Observation of a structure in the $M_{p\eta}$ invariant mass distribution near 1700 MeV/c$^2$ in the $\gamma p \to p\pi^0\eta$ reaction"* [CBELSA/TABS, Eur. Phys. J. A **57**, 325 (2021)]

Thank you for your attention!

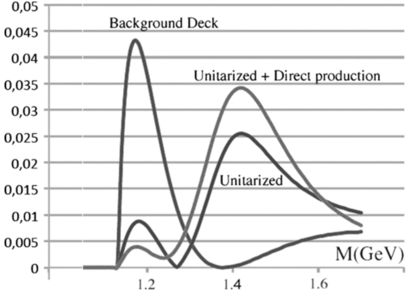
# Back-up

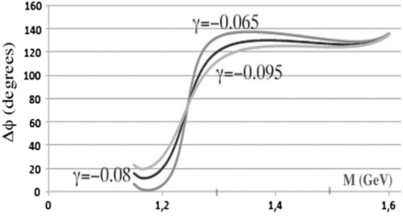
# Argand Diagram

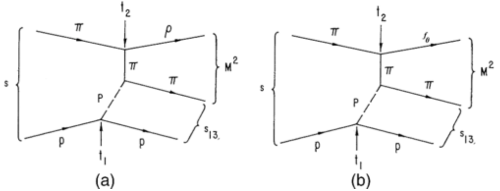



[COMPASS, PRL **127**, 082501]

# Basdevant-Berger Model

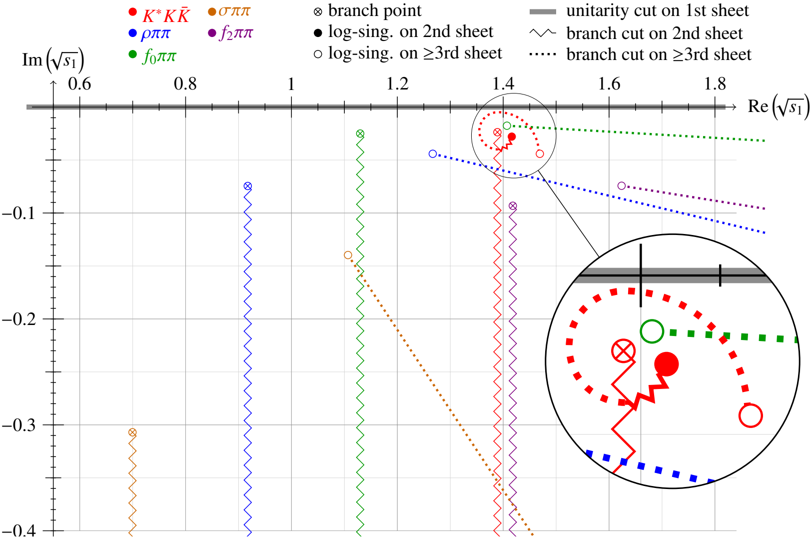


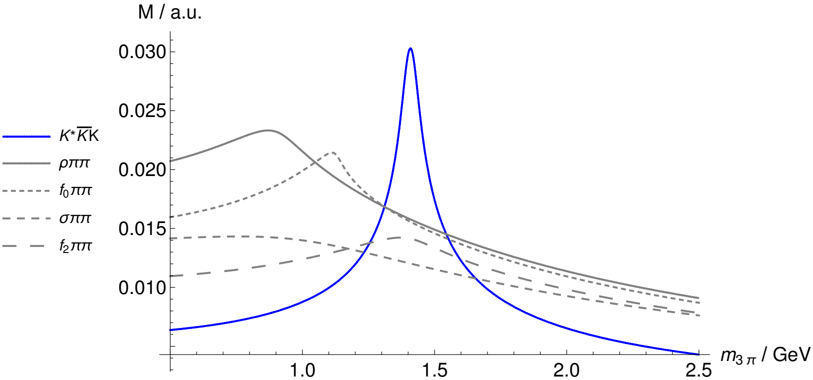




[Basdevant & Berger, PRL **114**, 192001 (2015)]

# Pole Positions



# Other Amplitudes



# Comparison to KT

$$F_w(s_{23}) = t_\xi(s_{23})\left[C_w + \frac{1}{2\pi}\int_{s_{\text{th}}}^{\infty} \frac{\rho(\tilde{s}_{23})\hat{F}_w(\tilde{s}_{23})}{\tilde{s}_{23} - s_{23}} \mathrm{d}\,\tilde{s}_{23}\right]$$

**KT**:

- calculate effects of rescattering on the 2-body subsystem invariant-mass dependence $s_{ij}$
- Iterative framework to include rescattering to any order

# Comparison to KT

$$F_w(s, s_{23}) = t_\xi(s_{23}) \left[ C_w(s) + \frac{1}{2\pi} \int_{s_{\text{th}}}^{\infty} \frac{\rho(\tilde{s}_{23}) \hat{F}_w(s, \tilde{s}_{23})}{\tilde{s}_{23} - s_{23}} \mathrm{d}\, \tilde{s}_{23} \right]$$

**KT**:

- calculate effects of rescattering on the 2-body subsystem invariant-mass dependence $s_{ij}$
- Iterative framework to include rescattering to any order

**Our method**:

- calculate effects of rescattering on the 3-body invariant-mass dependence $s = m^2_{3\pi}$
- Stop after the first iteration

# Unitarity & Iterative Procedure

$$A_{LS}^{JM}(\sigma) = F_{LS}^{JM}(\sigma) + \hat{F}_{LS}^{JM}(\sigma)$$

Unitarity:

$$\text{Disc}_\sigma A_{LS}^{JM}(\sigma) = i t_S^\dagger(\sigma)\rho(\sigma) A_{LS}^{JM}(\sigma)$$

$$\text{Disc}_\sigma F_{LS}^{JM}(\sigma) = i t_S^\dagger(\sigma)\rho(\sigma)\Big(F_{LS}^{JM}(\sigma) + \hat{F}_{LS}^{JM}(\sigma)\Big)$$

From unitarity relation:

$$F_{LS}^{JM}(\sigma) = t_S(\sigma)\left[C_{LS}^{JM}(\sigma) + \frac{1}{2\pi}\int_{\sigma_{\text{th}}}^{\infty} \frac{\rho(\sigma')\hat{F}_{LS}^{JM}(\sigma')}{\sigma' - \sigma} \mathrm{d}\,\sigma'\right]$$

$$\hat{F}_{LS}^{JM}(\sigma) = \int \mathrm{d}\,Z(\sigma) F_{L'S'}^{J'M'}(\sigma') Z_{L'S'}^{J'M'*}(\Omega')$$

Solve iteratively:

$$\begin{aligned}
\hat{F}^{(i+1)}(\sigma) &= \int \mathrm{d}\,Z^{(i+1)}(\sigma)\, F^{(i)}(\sigma')\Big(Z^{(i)}(\sigma')\Big)^* \\
F^{(i+1)}(\sigma) &= t_S^{(i+1)}(\sigma)\left[C^{(i+1)} + \frac{1}{2\pi}\int_{\sigma_{\text{thr}}}^{\infty} \mathrm{d}\,\sigma' \frac{\rho(\sigma')\hat{F}^{(i+1)}(\sigma')}{\sigma' - \sigma}\right]
\end{aligned}$$

# Angular Dependence of the Amplitudes

$$Z_{LS}^{JM}(\Omega_1, \Omega_{23}) = \sqrt{(2L+1)(2S+1)} \sum_{\lambda} \langle L0; S\lambda | J\lambda \rangle \, D_{M\lambda}^{J}(\Omega_1) D_{\lambda 0}^{S}(\Omega_{23})$$