AMBER Spectroscopy Program with RF Separated Beams

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Max Planck Institute for Physics

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August 31, 2022

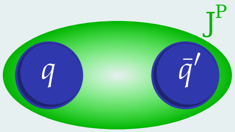
COMPASS

AMBER
Apparatus for Meson and Baryon Experimental Research

MAX PLANCK INSTITUTE FOR PHYSICS

The Strange-Meson Spectrum

Understanding the light-meson spectrum



- Completing $\mathrm{SU}(3)_{\mathrm{flavor}}$ multiplets
- Identifying supernumerous states
  - ➥ Search for exotic strange mesons

Input to other fields of physics

- Strange mesons appear as resonances in multi-body hadronic final states with kaons
- Searches for CP violation
- Searches for physics beyond SM

The Strange-Meson Spectrum

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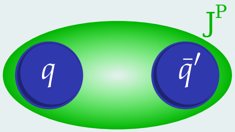

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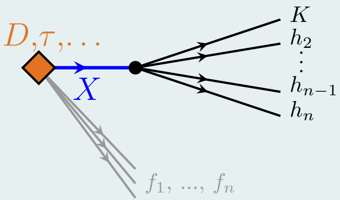

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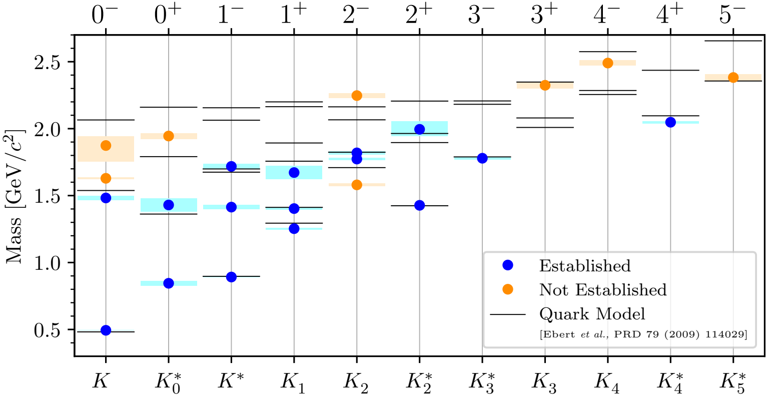
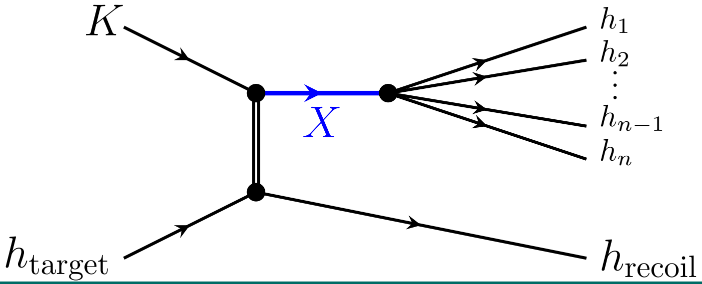
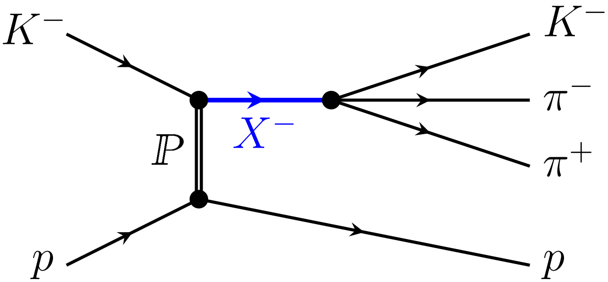


PDG lists 25 strange mesons (2021)

- 16 established states, 9 need further confirmation
- Missing states with respect to quark-model predictions
- No experimental evidence for strange exotica (except for $K_0^*(700)/\kappa$)



- Diffractive scattering of high-energy kaon beam
- Strange mesons appear as intermediate resonances X^-
- Decay to multi-body hadronic final states
- $K^-\pi^-\pi^+$ final state
  - Study in principle all strange mesons
  - Study a wide mass range
  - Study different decay modes



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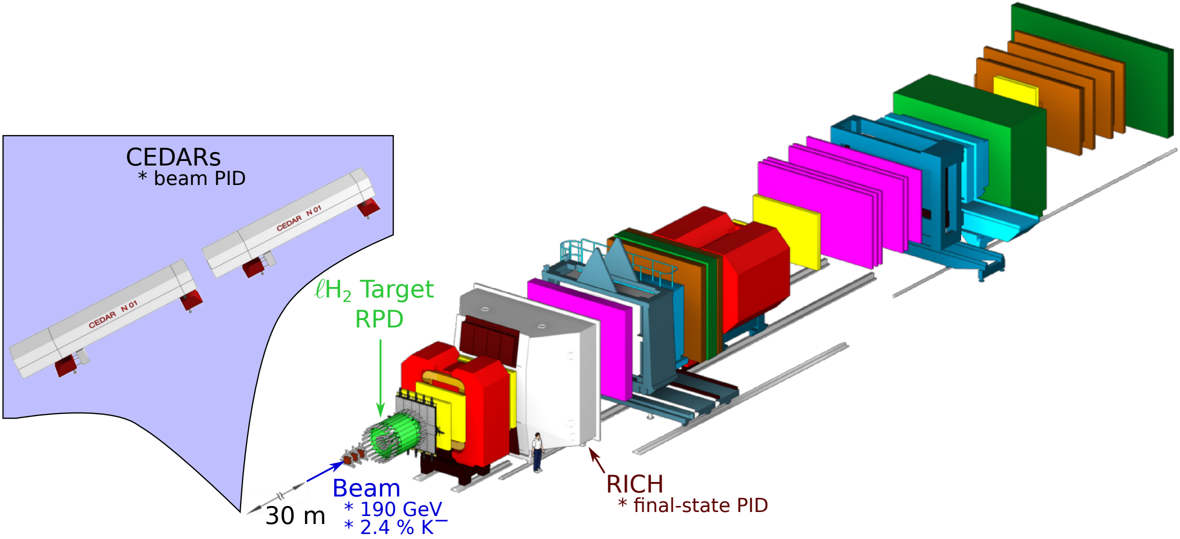
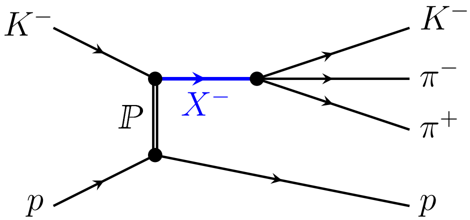
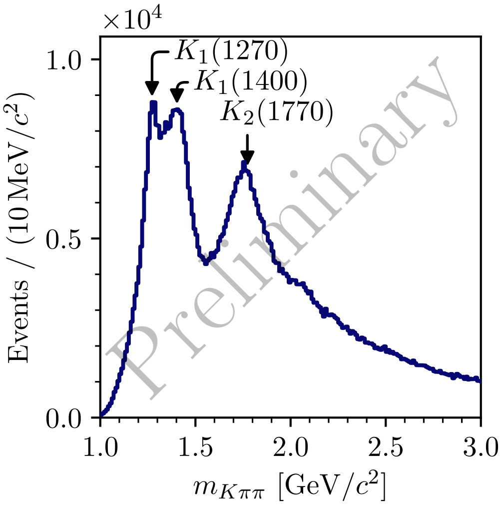
COMPASS Setup for Hadron Beams

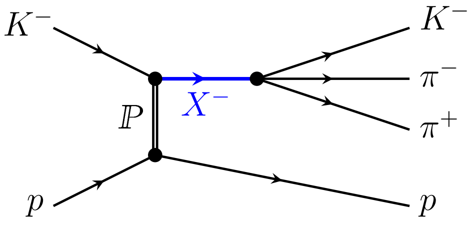
[COMPASS, Nucl. Instrum. Methods **779** (2015) 69]





- World's largest data set of about 720 k events
- Rich spectrum of overlapping and interfering X^-
  - Dominant well known states
  - States with lower intensity are "hidden"





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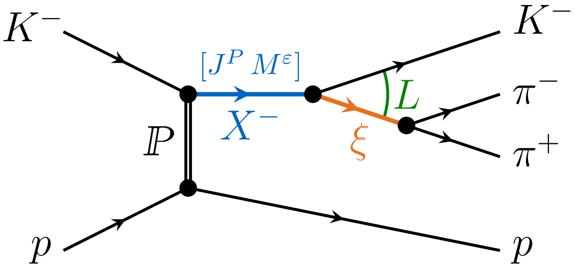
Partial wave: $J^P\, M^\varepsilon\, \xi\, b^-\, L$

- J^P spin and parity
- M^ε spin projection
- ξ isobar resonance
- b^- bachelor particle
- L orbital angular momentum



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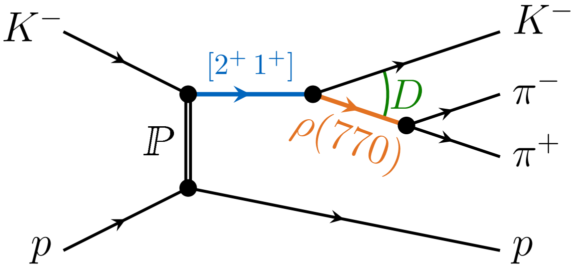
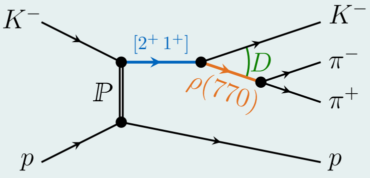
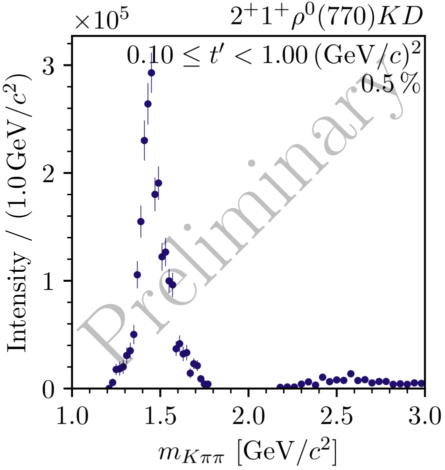
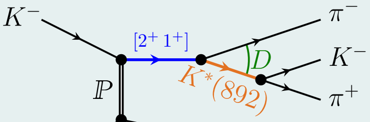




- Signal in $K_2^*(1430)$ mass region
- In different decays
  - $\rho(770)\,K\,D$
  - $K^*(892)\,\pi\,D$
- In agreement with previous measurements
- Cleaner signal in COMPASS data





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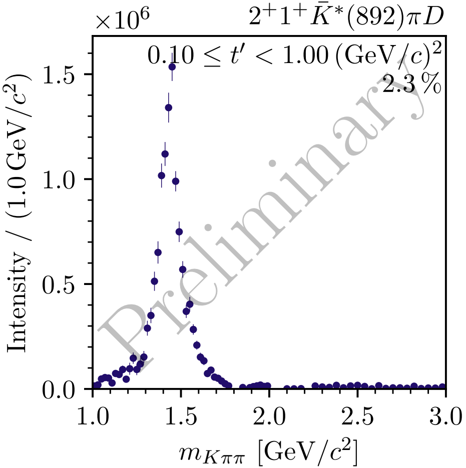

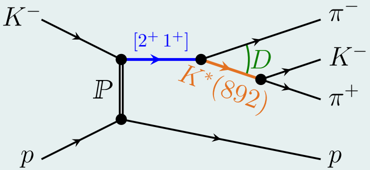

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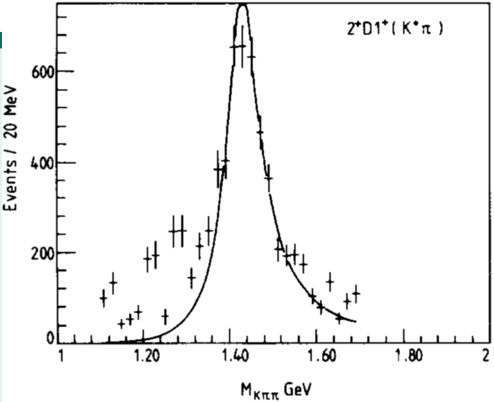


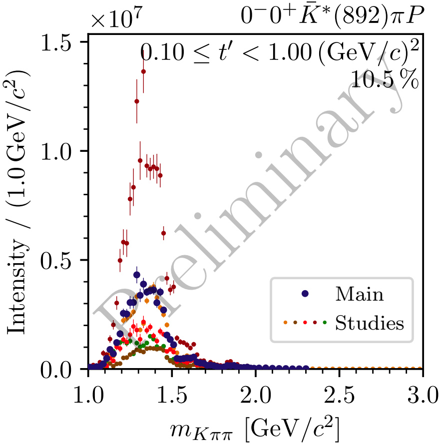
WA03 (CERN), 200 000 events, ACCMOR, Nucl. Phys. B **187** (1981)

Limited Kinematic Range of Final-State Particle Identification

- Final-state particle identification does not cover full momentum range
  - ➥ Loss of distinguishing power for some partial waves
  - ➥ Analysis artifacts in these partial waves

- Artifacts can be identified
- Mainly affects only
  - a sub-set of partial waves
  - the range $m_{K\pi\pi} \lesssim 1.6\,\mathrm{GeV}/c^2$

- Limits access to certain decay modes
- Induces non-negligible systematic uncertainties



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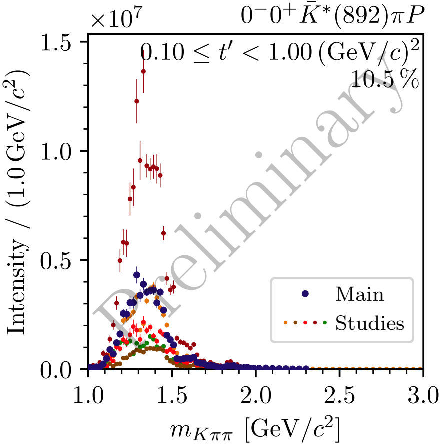

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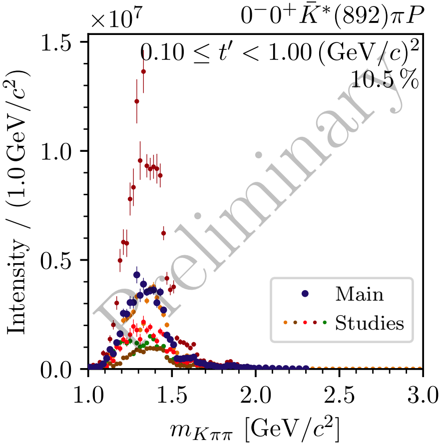


Limitations for Strange-Meson Spectroscopy at COMPASS

Main limiting factors

- Final-state particle identification
- Size of the data samples
  - Low kaon fraction in the beam ($\approx 2\,\%$)
  - Sample for strange-mesons about 150-times smaller than sample for non-strange mesons
    - 720 k $K^- + p \to K^-\pi^-\pi^+ + p$ events
    - 115 M $\pi^- + p \to \pi^-\pi^-\pi^+ + p$ events

High-Precision Strange-Meson Spectroscopy at AMBER

AMBER

Apparatus for Meson and Baryon Experimental Research

Phase I: After long shutdown 2 of LHC [CERN-SPSC-2019-022], approved by CERN RRB

- Proton charge-radius measurement
- Drell-Yan and charmonium production
- p-induced $\bar{p}$ production cross section

Phase II: After long shutdown 3 of LHC [arXiv:1808.00848]

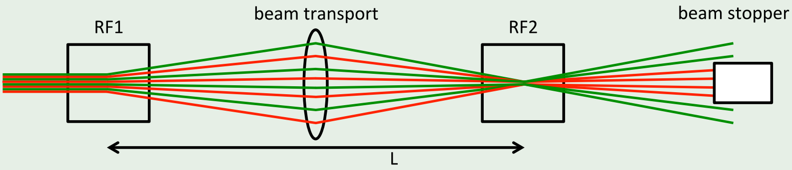
- Physics with kaon beams
  - Strange-meson spectroscopy
    goal: 10× larger data sample
  - Kaon-induced charmonium production
  - ...
- ...

- Upgrade of final-state particle identification
  - Cover full momentum range
  - Large and uniform acceptance
- Efficient beam-particle identification for high-purity sample

- Eliminate artifacts caused by limited final-state particle identification
- Increase size of the data sample by increasing acceptance

Radio-frequency separated high-energy kaon beam

- Increase size of the data sample by increasing kaon fraction in beam



- Improved precision
- Study also small signals in data
  - Identify exotic strange mesons by studying various decay modes
- Access to novel analysis methods

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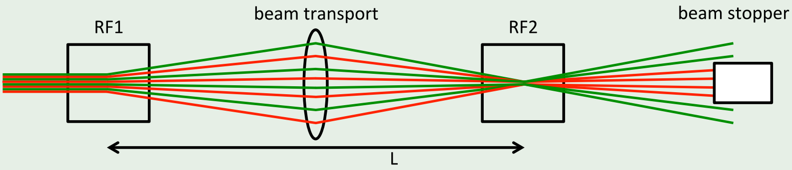

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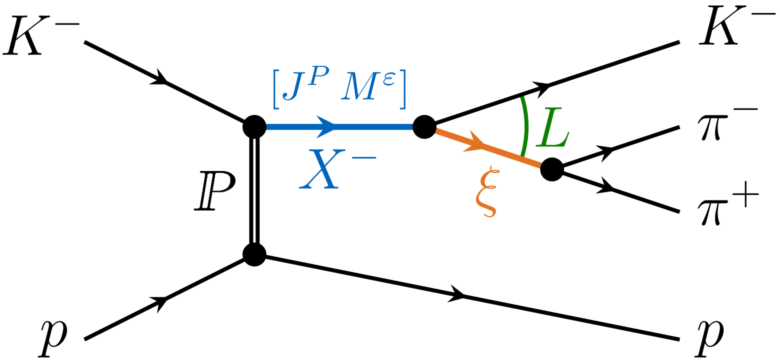


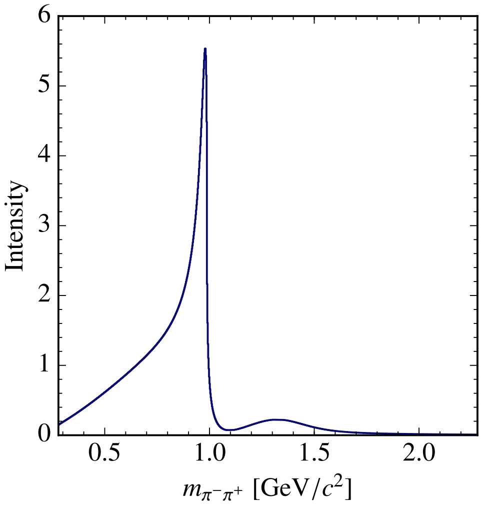
Challenge

Need knowledge of isobar amplitude in conventional partial-wave analysis

Extract isobar amplitudes from data

- Replace model for isobar amplitude with step-like amplitude
- Extract binned shape from data
- Computationally more expensive
  - Up to 100 additional parameters per wave with freed isobar
- Needs large data sets

$[\pi\pi]_S$ isobar amplitude



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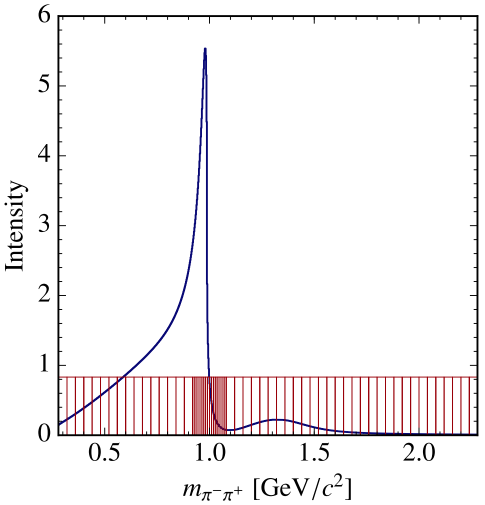

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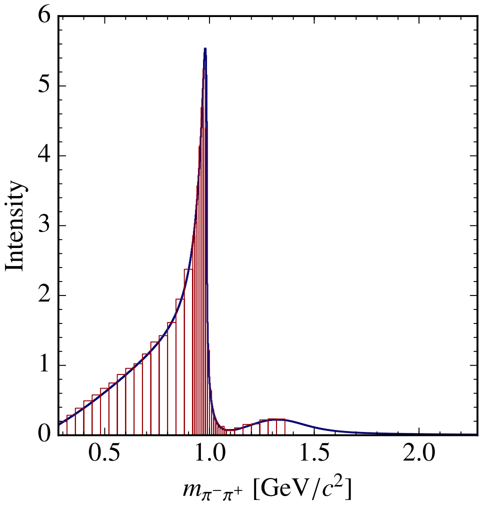

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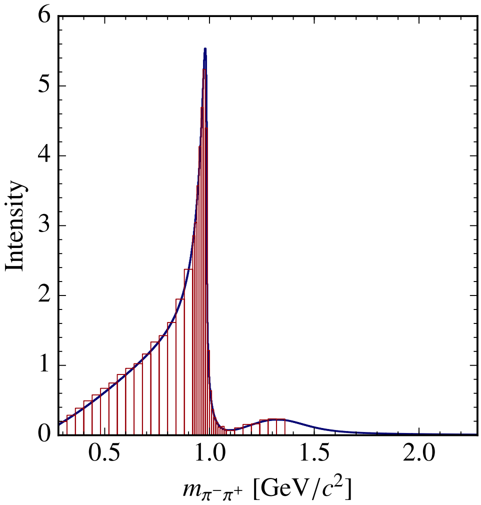
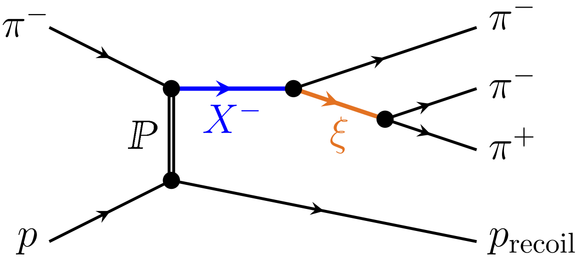
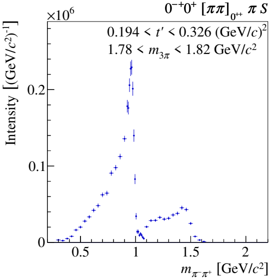




- Freed-isobar method developed for and successfully applied to COMPASS $\pi^-\pi^-\pi^+$ sample

[Adolph et al., PRD 95, 032004 (2017)]



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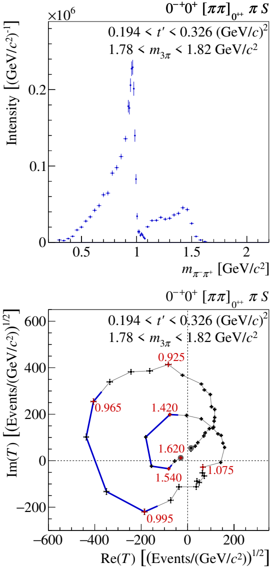
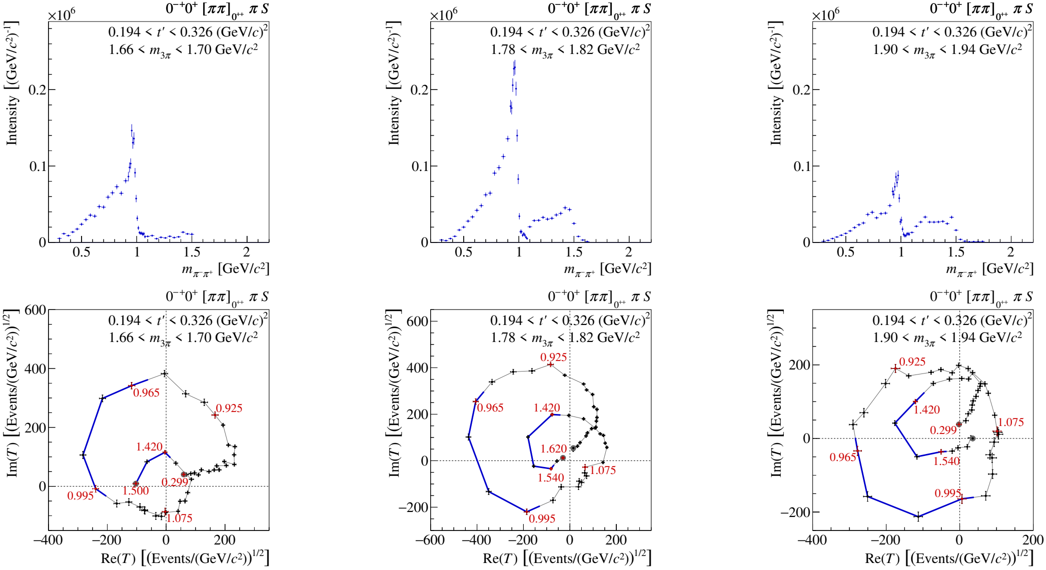
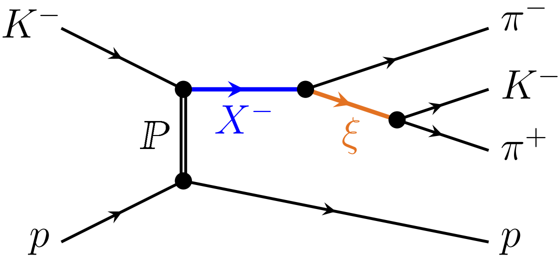


High-Precision Strange-Meson Spectroscopy at AMBER

The Virtue of Large Data Samples: Freed-Isobar Analysis

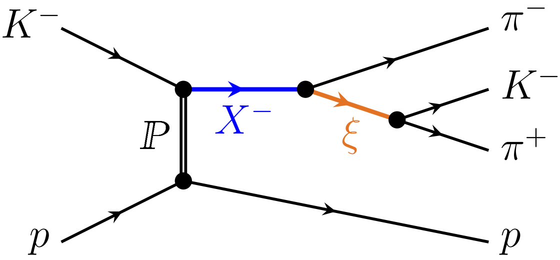
[Adolph et al., PRD 95, 032004 (2017)]





Freed-isobar partial-wave analysis for strange-meson spectroscopy

- K_0^* mesons ($J^P = 0^+$) cannot be directly produced in diffractive scattering
- K_0^* mesons appear in $K^-\pi^+$ sub-system of the $K^-\pi^-\pi^+$ final state
- Freed-isobar method allows us to study mesons in sub-systems



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- Many strange-meson candidates require further confirmation
- Search for strange partners of exotic non-strange light mesons

COMPASS

- World's largest data sample on $K^- + p \to K^-\pi^-\pi^+ + p$
  - Most detailed and comprehensive analysis of the $K^-\pi^-\pi^+$ final state so far
- Limited by final-state particle identification and small kaon fraction in beam

AMBER: High-Precision Strange-Meson Spectroscopy

- Goal: Collect 10×larger sample using high-intensity and high-energy kaon beam
- Rewrite the PDG for strange mesons, with a single and self-consistent measurement
- Requires experimental setup with uniform acceptance over wide kinematic range including particle identification and measurement of neutral particles
- AMBER is open for interested collaborators to join

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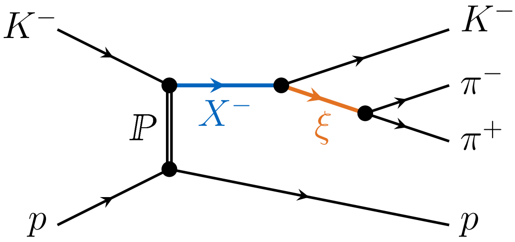
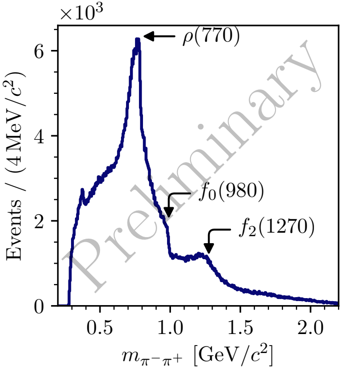
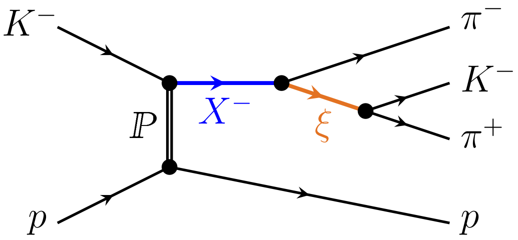
Backup

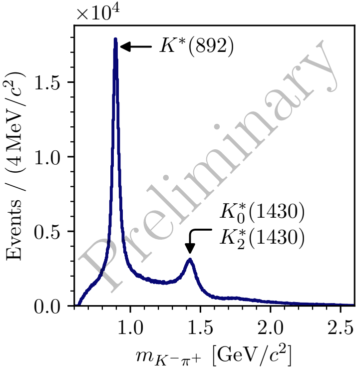
Outline





- Also structure in $\pi^-\pi^+$ and $K^-\pi^+$ subsystems
  - ➥ Successive 2-body decay via $\pi^-\pi^+$ / $K^-\pi^+$ resonance called isobar
- Also structure in angular distributions





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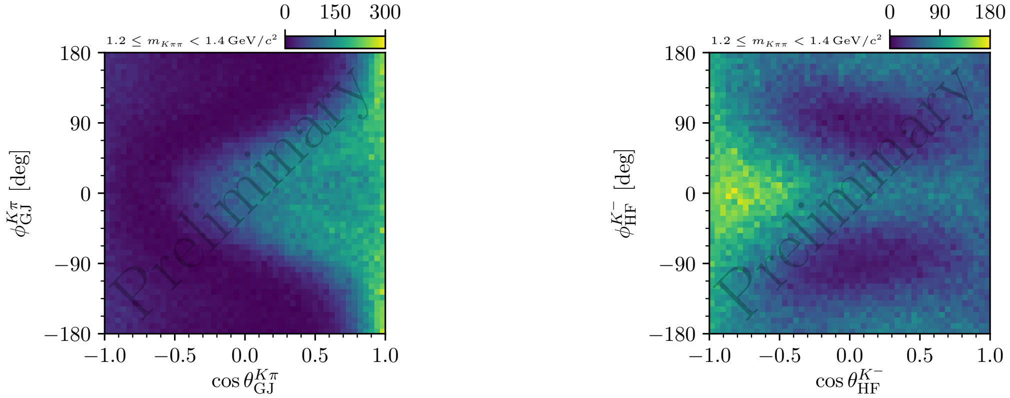
Kinematic Distribution of $K^-\pi^-\pi^+$ Events

Subsystem



- Also structure in $\pi^-\pi^+$ and $K^-\pi^+$ subsystems
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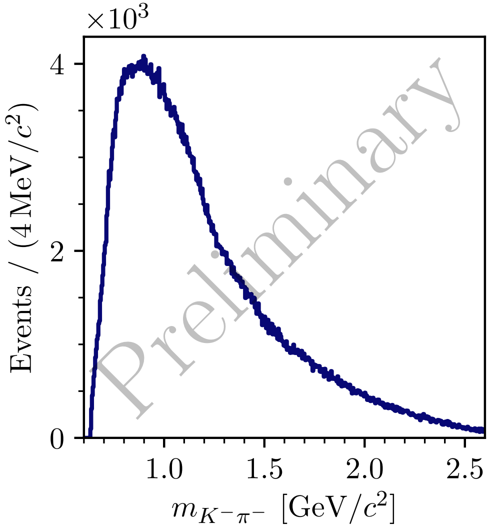
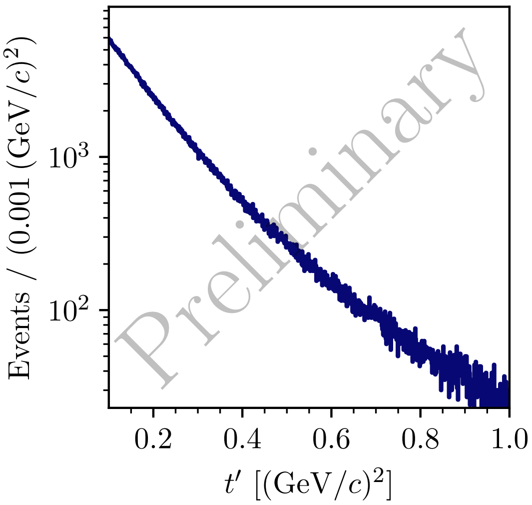
- Also structure in angular distributions

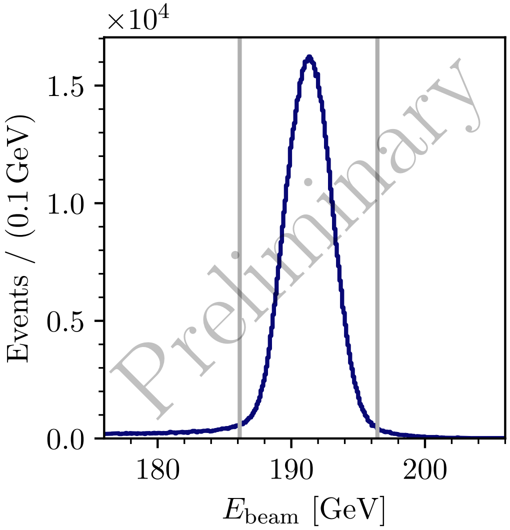
- No dominant resonant structures

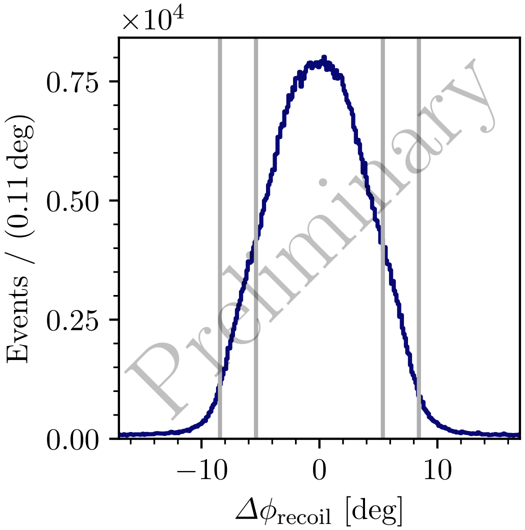


t' Spectrum

- Exponential shape
- Shallower for larger t'

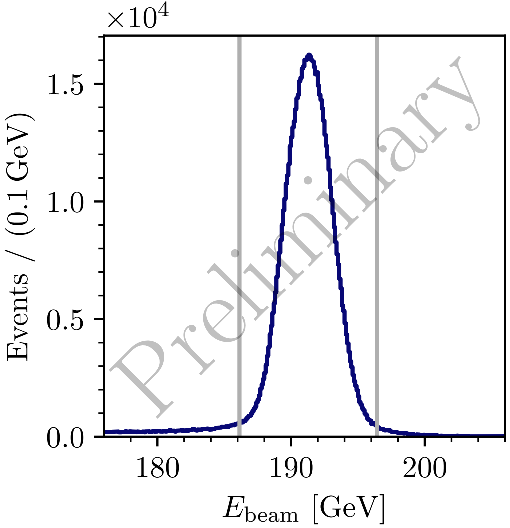


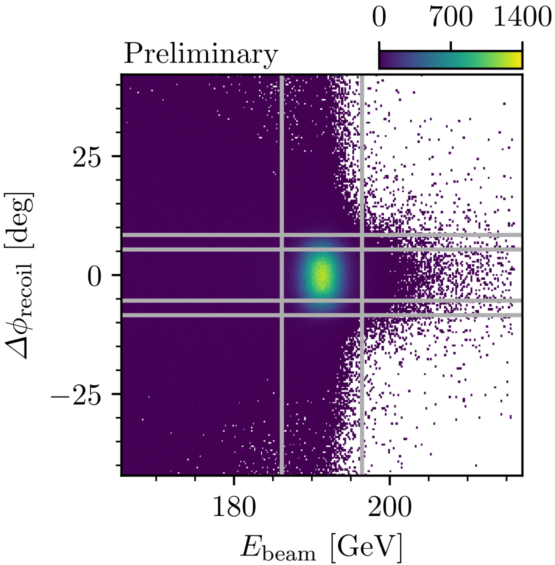

$\times 10^4$
1.5
1.0
0.5
0.0
Events / (0.1 GeV)
180
190
200
E_{beam} [GeV]
Preliminary


$\times 10^4$
0.75
0.50
0.25
0.00
Events / (0.11 deg)
−10
0
10
$\Delta\phi_{\text{recoil}}$ [deg]
Preliminary

Kinematic Distribution of $K^-\pi^-\pi^+$ Events

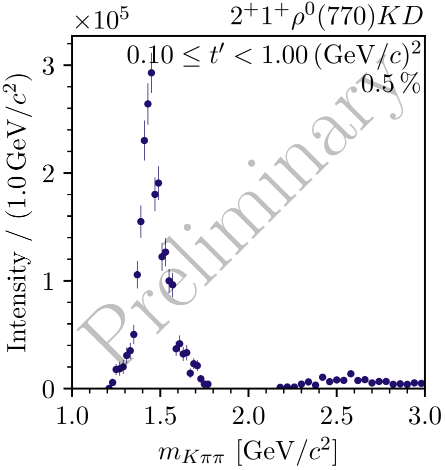
Exclusivity





Partial waves with $J^P = 2^+$

- Signal in $K_2^*(1430)$ mass region
- In Different decays
  - $\rho(770)\,K\,D$
  - $K^*(892)\,\pi\,D$
- Clear phase motion in $K_2^*(1430)$ region
  - Characteristic of narrow isolated resonances
- In agreement with previous measurement



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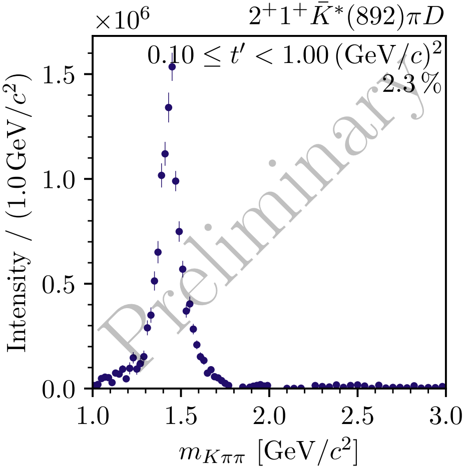

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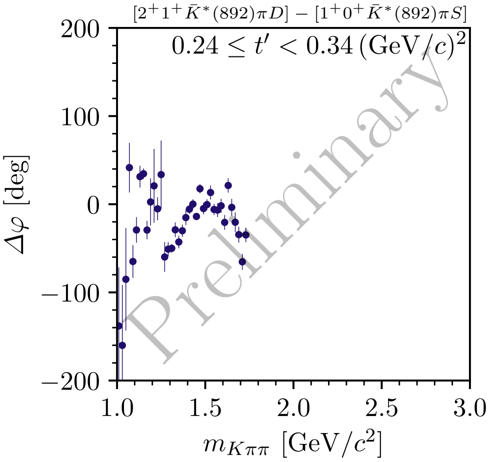

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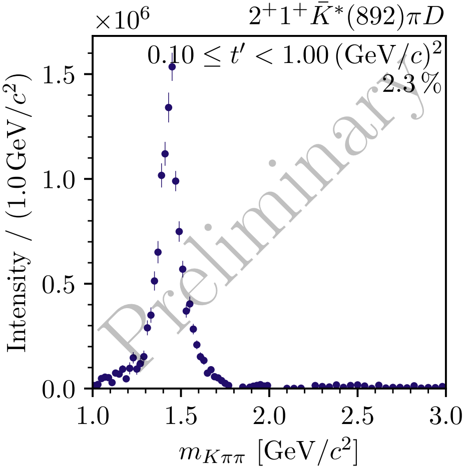

WA03 (CERN), 200 000 events, ACCMOR, Nucl. Phys. B **187** (1981)

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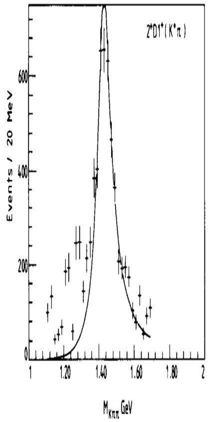

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Partial-Wave Decomposition of $K^-\pi^-\pi^+$

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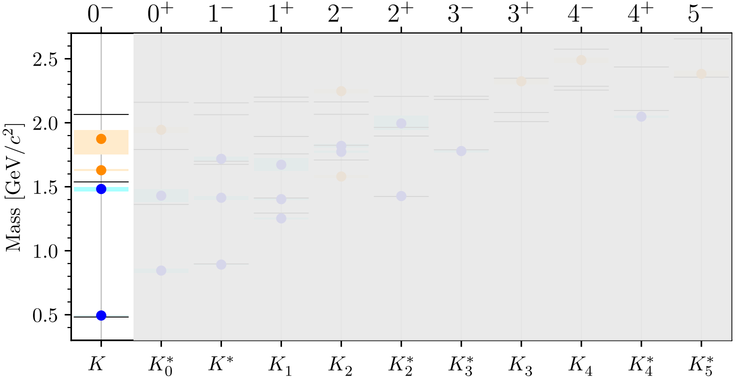

PDG (2021)

- $K(1460)$ and $K(1830)$ potentially quark-model states
- $K(1630)$ candidate for supernumerary state
  - Unexpectedly small width: 16 MeV/c^2
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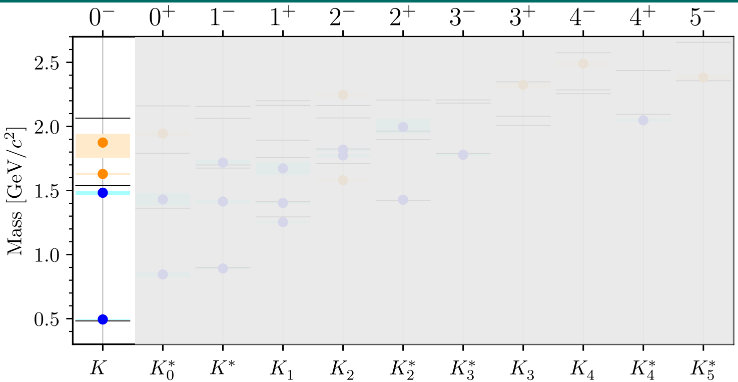

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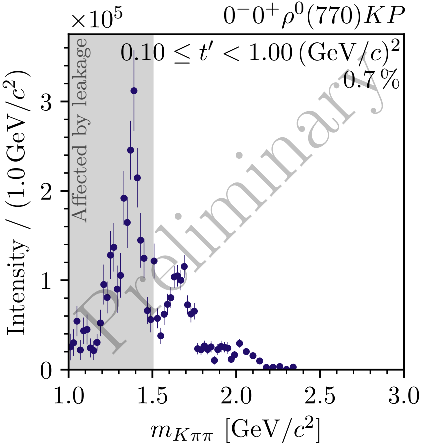

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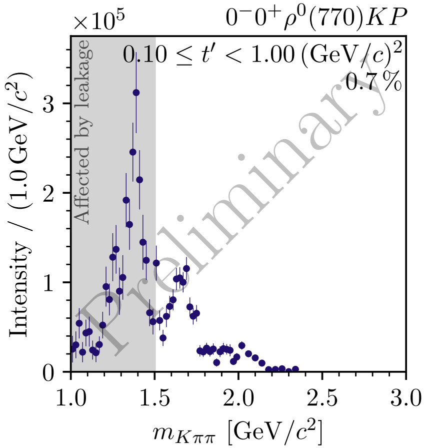

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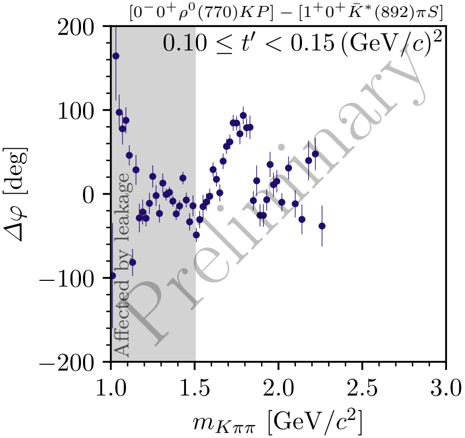

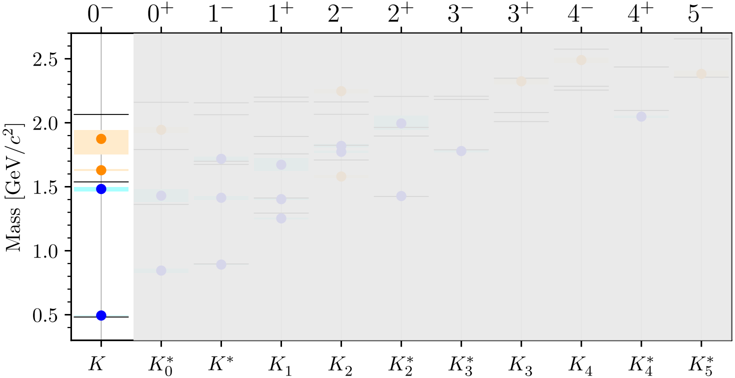

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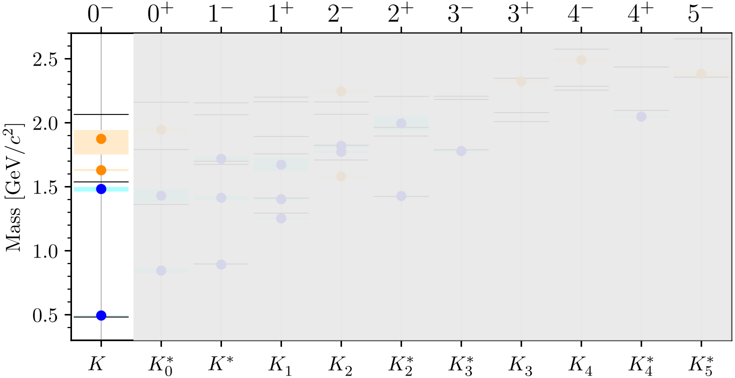

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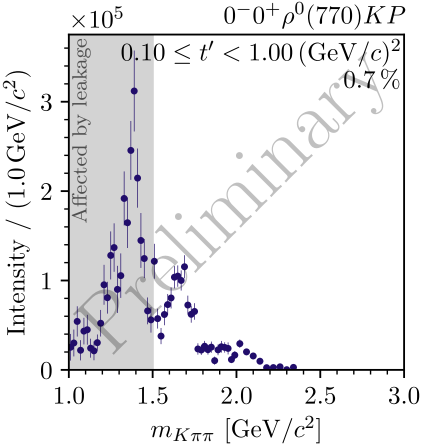

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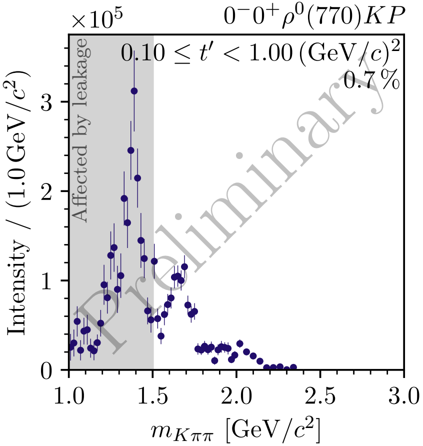

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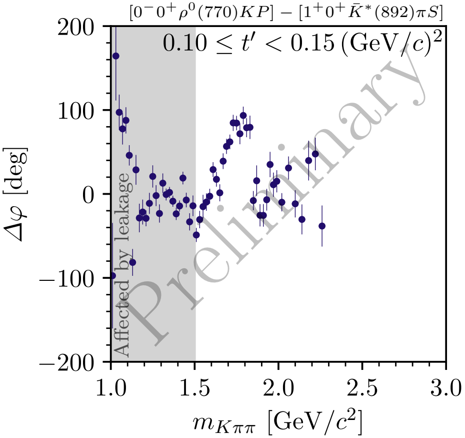

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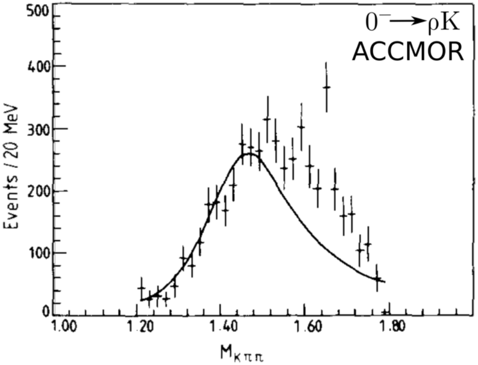

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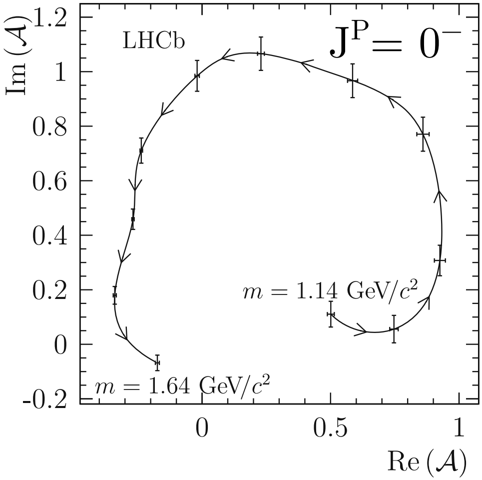

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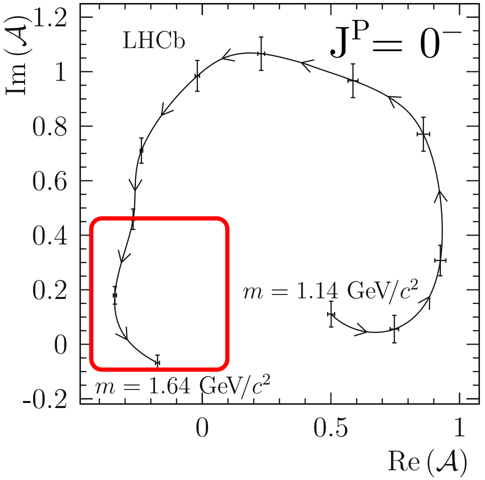

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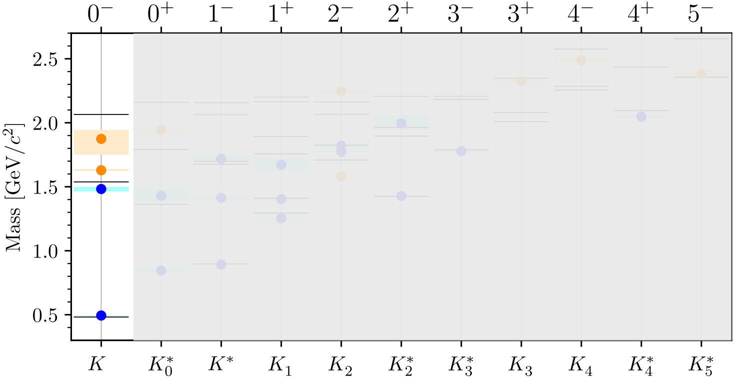

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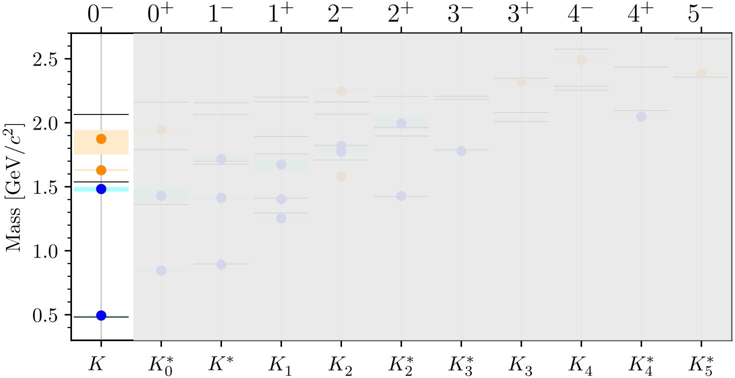

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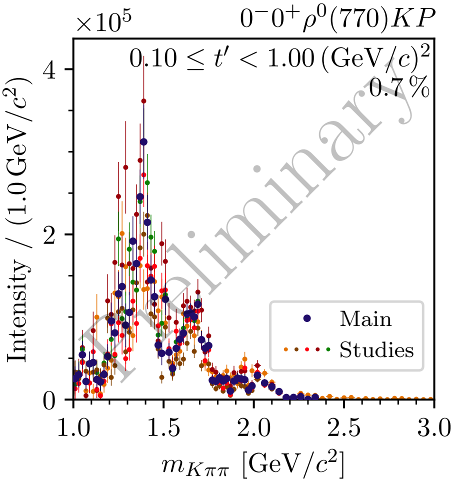
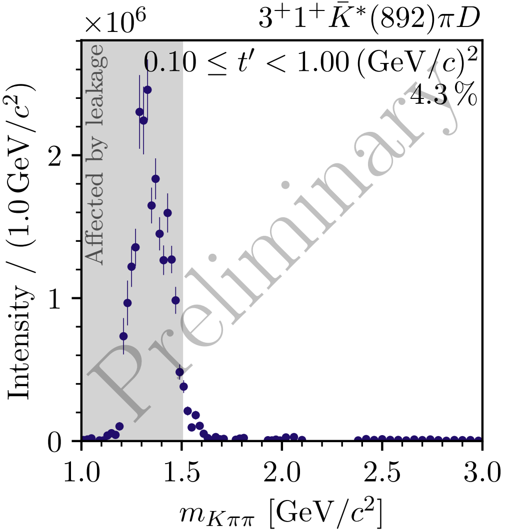


Leakage Effect

- Unexpected low-mass enhancement in $3^+\,1^+$ $K^*(892)\,\pi\,D$ wave
- Similar to dominant 1^+ wave
- Sensitive to systematic effects
- Decay amplitudes of different J^P are orthogonal
- Loss of orthogonality taking acceptance into account
- Limited acceptance due to limited kinematic range of final-state PID
- Only a small sub-set of partial waves affected



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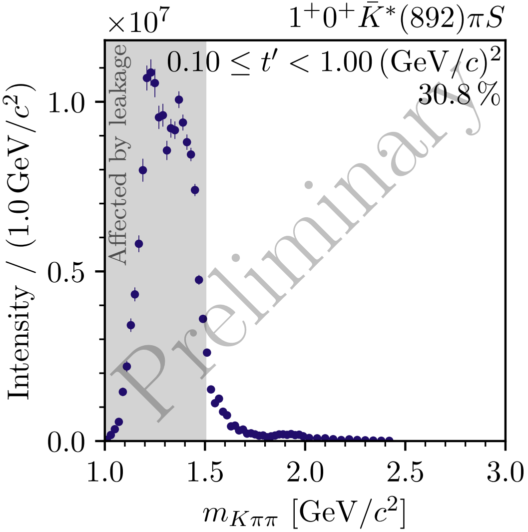

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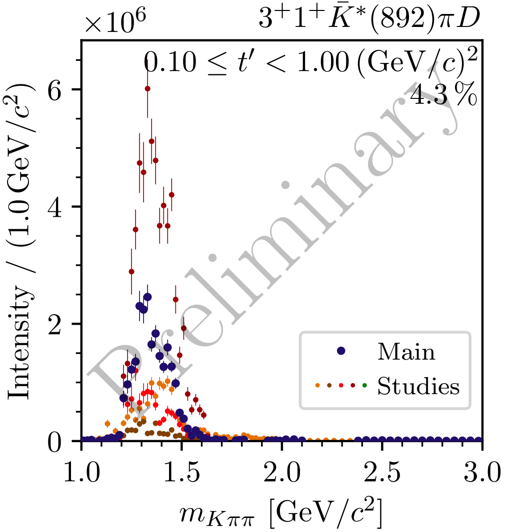

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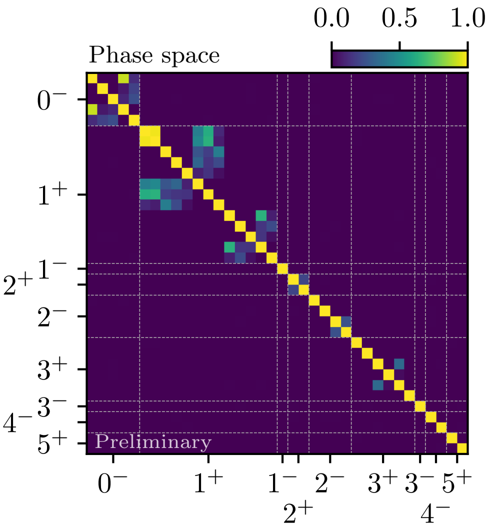

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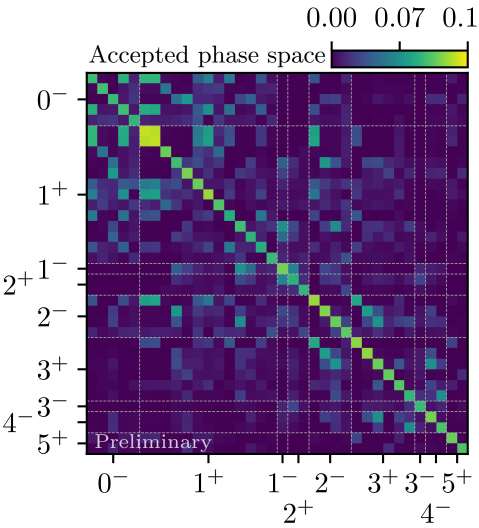

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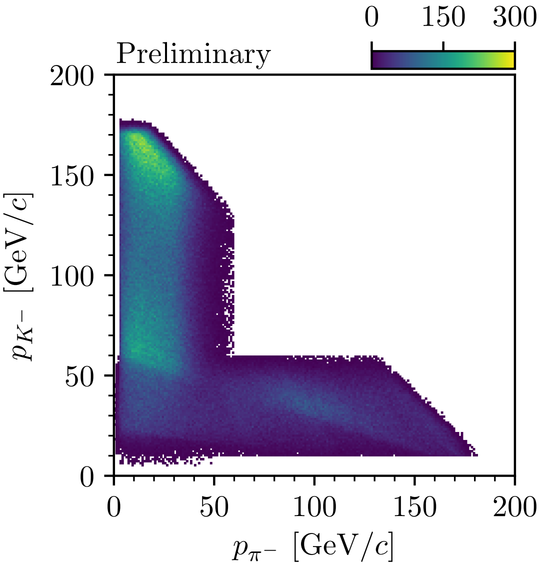

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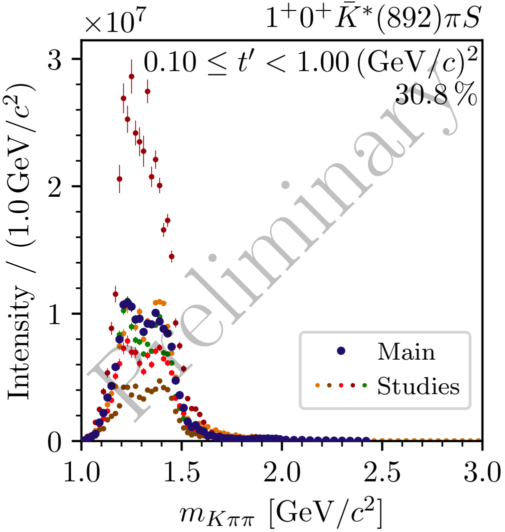

- $K^-\pi^-\pi^+$ and $\pi^-\pi^-\pi^+$ similar experimental footprint
- Distinguishable only by
  - Beam particle identification
  - Final-state particle identification
- Excellent beam PID:
  - Expect small contamination from beam π^-
- Final-state PID does not suppress $\pi^-\pi^-\pi^+$ background
  - ➥ Non-negligible $\pi^-\pi^-\pi^+$ background in $K^-\pi^-\pi^+$ sample of about 7 %
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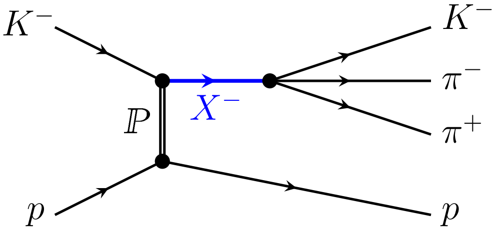

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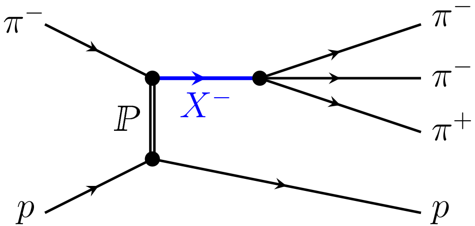

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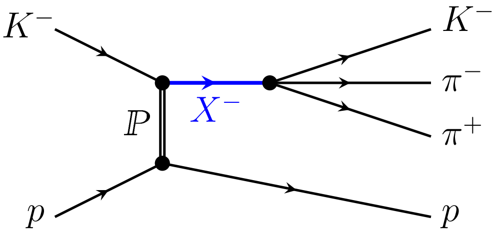

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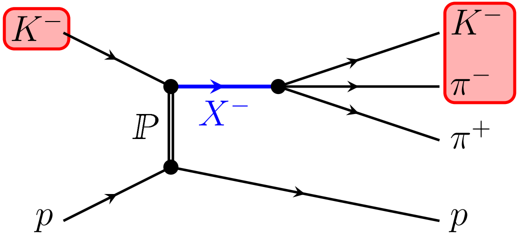

Incoherent Background

- Well established model for $\pi^- + p \to \pi^-\pi^-\pi^+ + p$
  - From very same data set
  - Measured with high precision
  - Acceptance corrected
- Generate $\pi^-\pi^-\pi^+$ Monte Carlo sample
- Mis-interpret $\pi^-\pi^-\pi^+$ Monte Carlo events as $K^-\pi^-\pi^+$
  - Apply wrong mass assumption
  - Same event reconstruction and selection as for $K^-\pi^-\pi^+$
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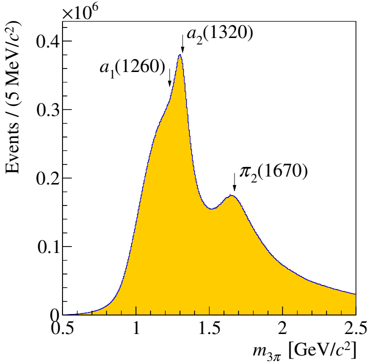

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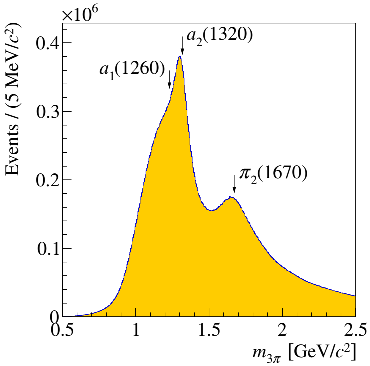

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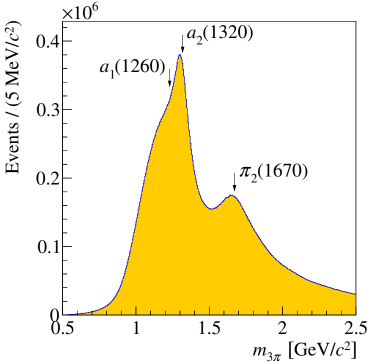

[Phys. Rev. D 95 (2017) 032004]

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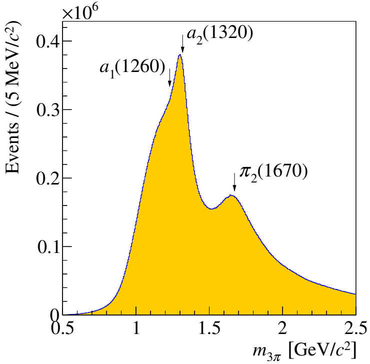
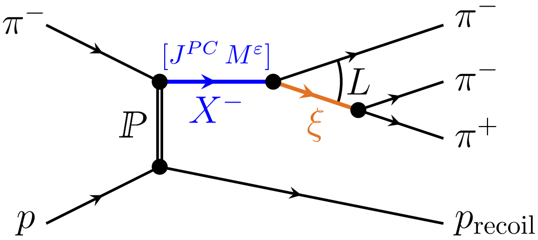
- Mis-interpret $\pi^-\pi^-\pi^+$ Monte Carlo events as $K^-\pi^-\pi^+$
  - Apply wrong mass assumption
  - Same event reconstruction and selection as for $K^-\pi^-\pi^+$
- Perform partial-wave decomposition of mis-interpreted $\pi^-\pi^-\pi^+$ Monte Carlo sample
  - Using the same PWA model as for measured $K^-\pi^-\pi^+$ sample

➥ Study $\pi^-\pi^-\pi^+$ background in individual $K^-\pi^-\pi^+$ partial waves



Freed-Isobar Method



Challenge

Need knowledge of isobar amplitude to calculate decay amplitudes $\Psi_a(\tau)$

- How good are the parameterizations?
  - Single isobar may not be approximated well by a Breit-Wigner amplitude
  - Effects of rescattering may distort the isobar shape

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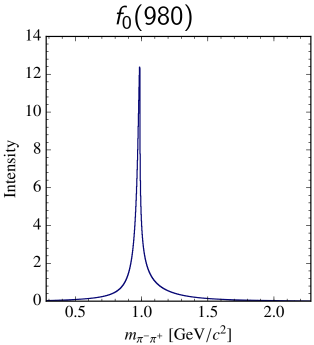

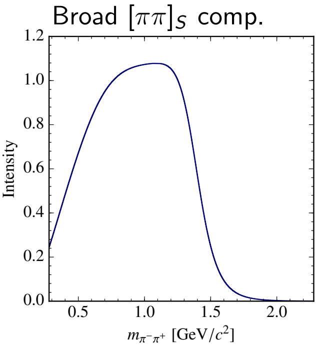

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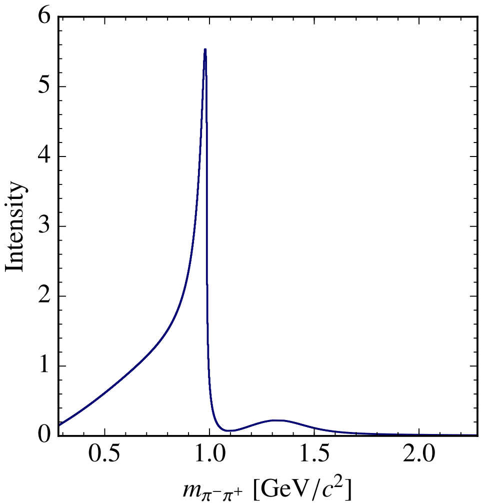
Freed-Isobar Method

$[\pi\pi]_S$ isobar amplitude

Extract isobar amplitudes from data

- Replace model for isobar amplitude with step-like amplitude
- Extract binned shape from data
- Computationally more expensive
  - Up to 100 additional parameters per wave with freed isobar
- Needs large data sets



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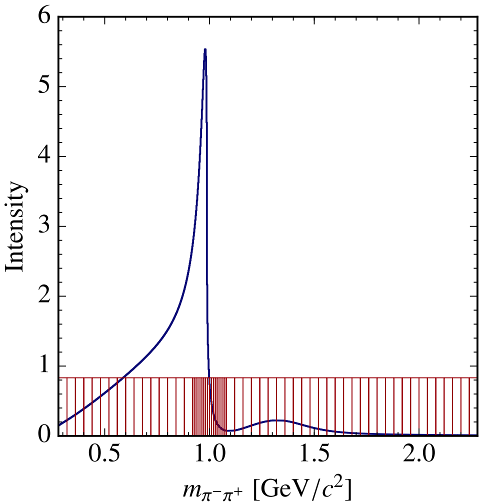

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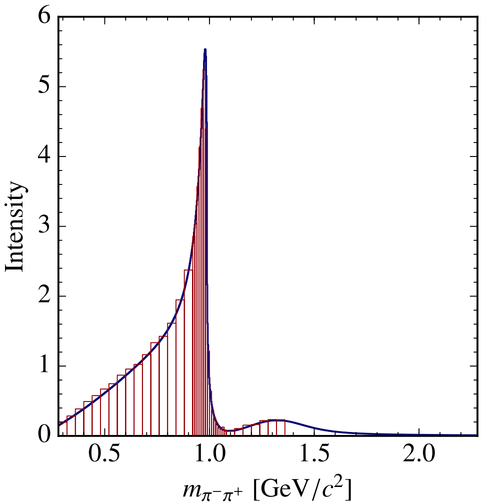

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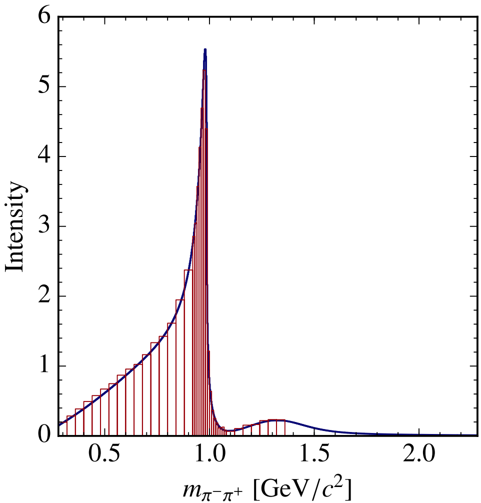

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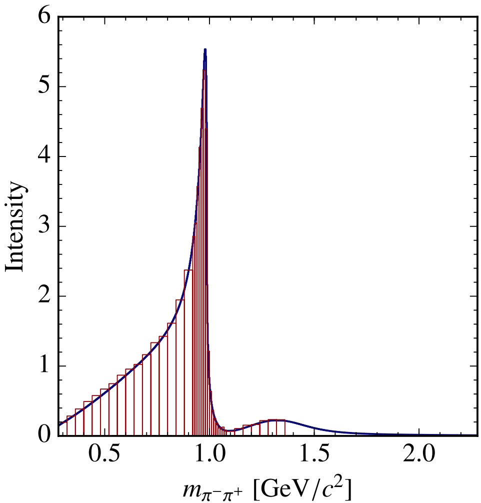

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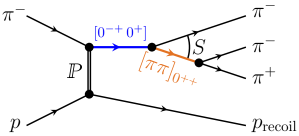

Example: $0^{-+}\,0^+\,[\pi\pi\ S\text{-wave}]\,\pi\,S$ wave

- Comparison of $0^{-+}\,0^+\,[\pi\pi\ S\text{-wave}]\,\pi\,S$ wave intensity between
  - sum of all conventional isobar waves
  - freed-isobar method
- Compatible shapes
- $\pi(1800)$ peak prominent

Freed-Isobar Method

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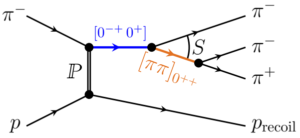
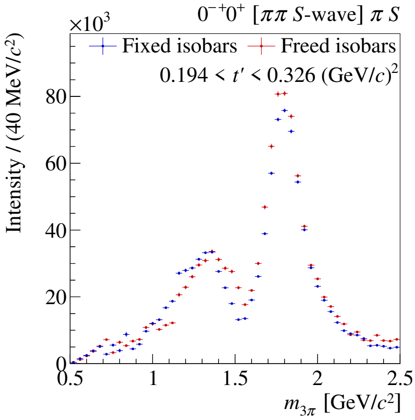
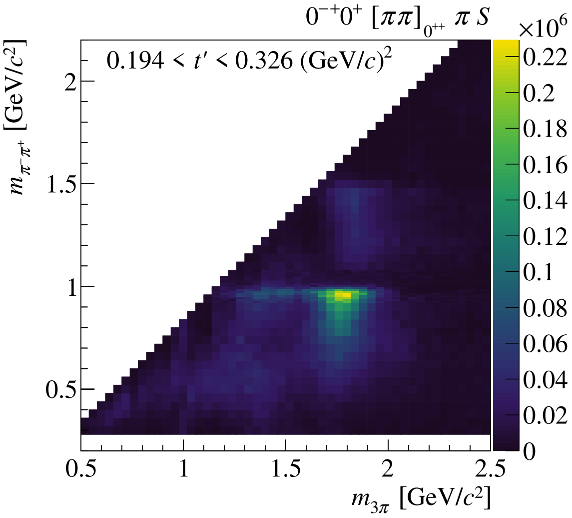




Freed-Isobar Method

[Adolph et al., PRD 95, 032004 (2017)]



This is not a Dalitz-plot

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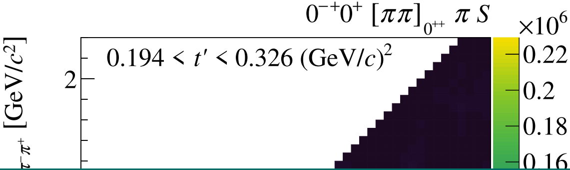
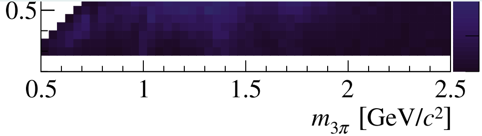


Investigate the $\pi\pi$ subsystem with $J^{PC} = 0^{-+}$

- No constrains on $\pi\pi$ resonances
- Extract $\pi\pi$ amplitude (intensity & phase)
  - Extract $\pi\pi$ resonances
- Investigate effects of rescattering



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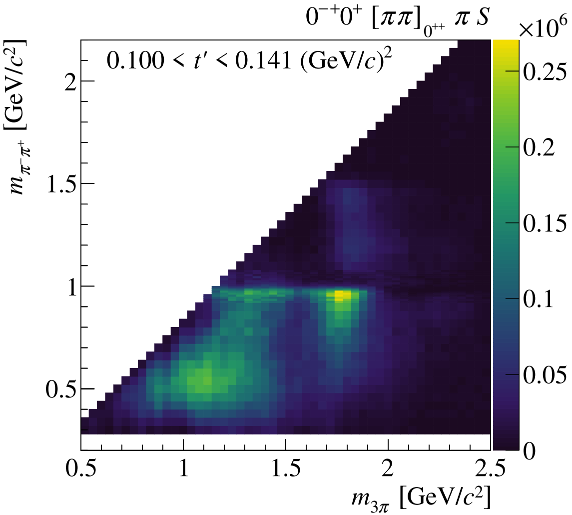

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Freed-Isobar Method: $0^{-+}\,0^{+}\,[\pi\pi]_{0^{++}}\,\pi\,S$

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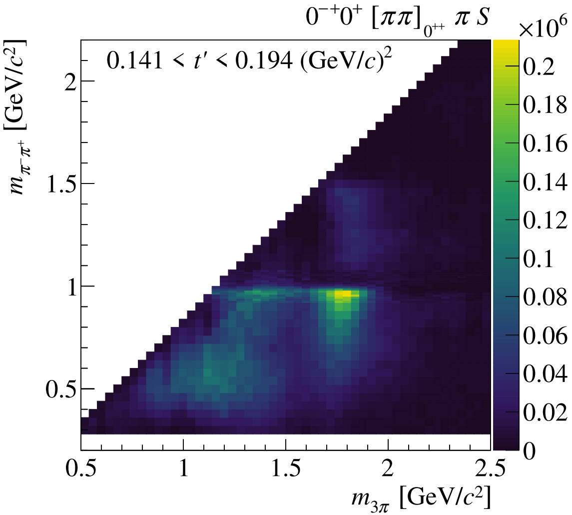

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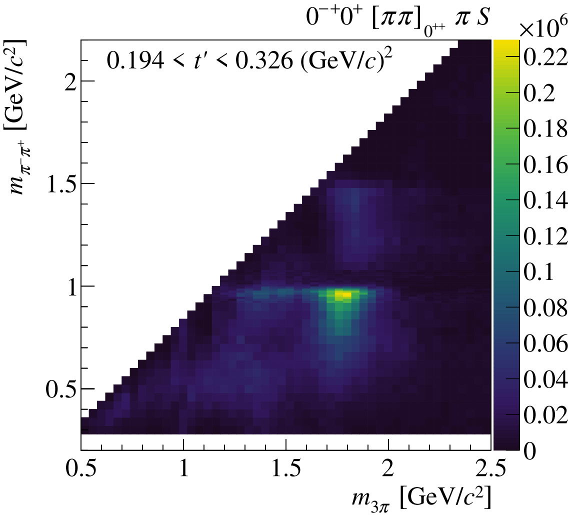

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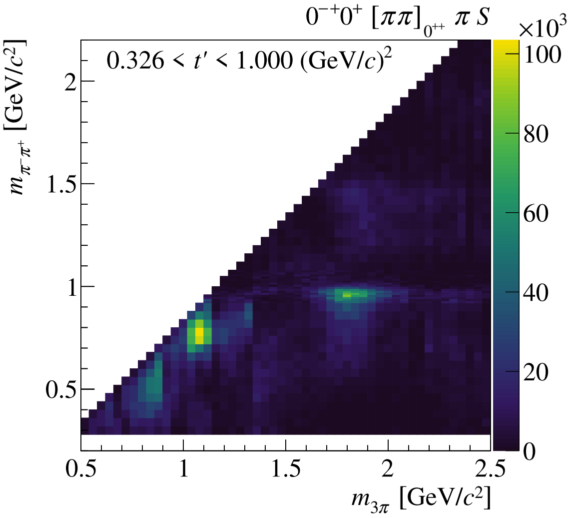

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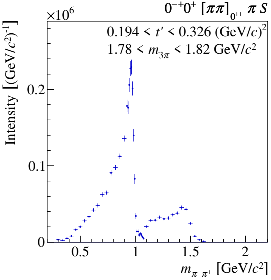

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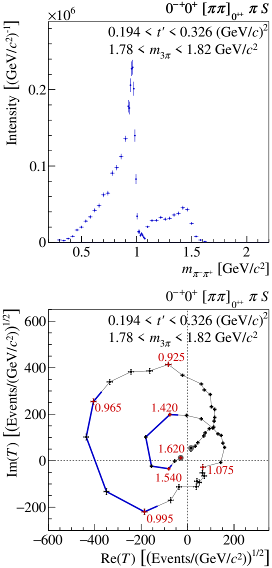

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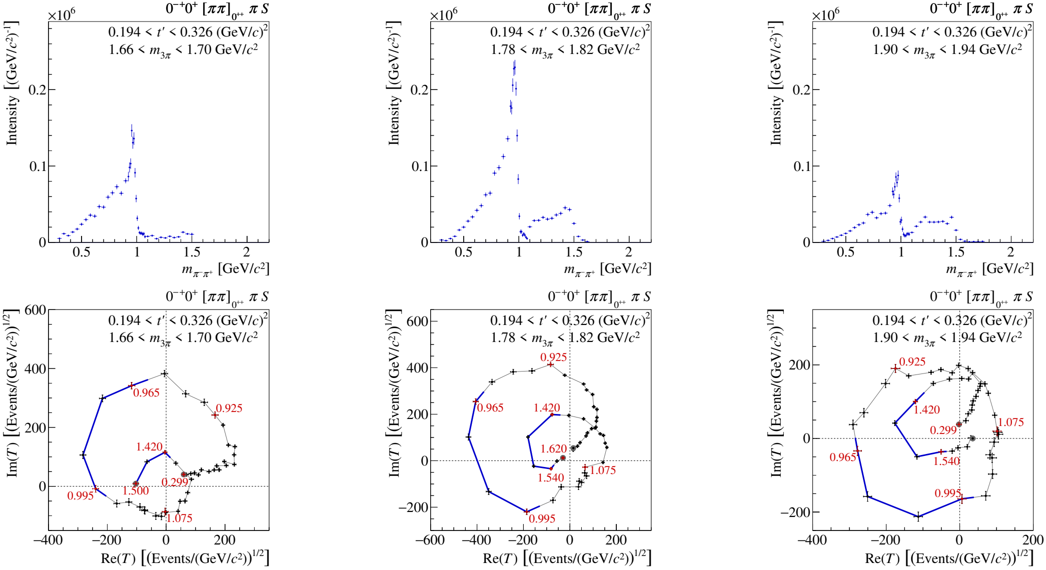


- Total intensity in one $(m_{3\pi}, t')$-bin as function of phase-space variables $\vec{\tau}$:

$$\mathcal{I}(\vec{\tau}) = \left|\sum_{i}^{\text{waves}} \mathcal{T}_i \left[\psi_i(\vec{\tau})\,\Delta_i(m_{\pi^-\pi^+}) + \text{Bose Symm.}\right]\right|^2$$

Fit parameters: Production amplitudes $\mathcal{T}_i$

Fixed: Angular distributions $\psi_i(\vec{\tau})$, dynamic isobar amplitudes $\Delta_i(m_{\pi^-\pi^+})$

- Replace fixed isobar amplitudes by piece-wise constant function:

$$\Delta_i(m_{\pi^-\pi^+}) \to \sum_{\text{bins}} \mathscr{T}_i^{\text{bin}} \Delta_i^{\text{bin}}(m_{\pi^-\pi^+}) \equiv [\pi\pi]_{J^{PC}}$$

$$\Delta_i^{\text{bin}}(m_{\pi^-\pi^+}) = \begin{cases} 1, & \text{if } m_{\pi^-\pi^+} \text{ in the bin.} \\ 0, & \text{otherwise.} \end{cases}$$

- Each $m_{\pi^-\pi^+}$ bin behaves like an independent partial wave $\mathcal{T}_i^{\text{bin}} = \mathcal{T}_i \mathscr{T}_i^{\text{bin}}$:

$$\mathcal{I}(\vec{\tau}) = \left|\sum_{i}^{\text{waves}} \sum_{\text{bin}}^{\text{bins}} \mathcal{T}_i^{\text{bin}} \left[\psi_i(\vec{\tau})\,\Delta_i^{\text{bin}}(m_{\pi^-\pi^+}) + \text{Bose Symm.}\right]\right|^2$$

- Approach similar to binning in $m_{3\pi}$

- Extend freed-isobar wave set
- Free isobar dynamic amplitudes of 11 biggest waves:
  - Minimize potential leakage

Freed-isobar wave set

$0^{-+}0^{+}[\pi\pi]_{0^{++}}\pi S$	$1^{++}1^{+}[\pi\pi]_{1^{--}}\pi S$	$2^{-+}0^{+}[\pi\pi]_{2^{++}}\pi S$
$0^{-+}0^{+}[\pi\pi]_{1^{--}}\pi P$	$2^{-+}0^{+}[\pi\pi]_{0^{++}}\pi D$	$2^{-+}1^{+}[\pi\pi]_{1^{--}}\pi P$
$1^{++}0^{+}[\pi\pi]_{0^{++}}\pi P$	$2^{-+}0^{+}[\pi\pi]_{1^{--}}\pi P$	$2^{++}1^{+}[\pi\pi]_{1^{--}}\pi D$
$1^{++}0^{+}[\pi\pi]_{1^{--}}\pi S$	$2^{-+}0^{+}[\pi\pi]_{1^{--}}\pi F$	

- Extend freed-isobar wave set
- Free isobar dynamic amplitudes of 11 biggest waves:
  - Minimize potential leakage
- Add spin exotic $1^{-+}1^{+}[\pi\pi]_{1^{--}}\pi P$ wave
  - Wave of major interest
- 12 freed-isobar waves replace 16 fixed-isobar waves
- In addition 72 fixed-isobar waves in the model
- 40 MeV wide $m_{3\pi}$ bins from 0.5 to 2.5 GeV
- 4 non-equidistant bins in t'
- 50 bins in $m_{3\pi}$, 4 bins in t': $4 \times 50 = 200$ independent bins

Zero mode in the spin-exotic wave

What is a “zero mode”?

- Freed-isobar analysis: much more freedom than fixed-isobar analysis
  - Causes continuous mathematical ambiguities in the model
- “Zero mode” = dynamic isobar amplitude $\Omega\left(m_{\pi^-\pi^+}\right)$, that does not contribute to the **total** amplitude
- Spin-exotic wave:

$$\psi\left(\vec{\tau}\right)\Omega\left(m_{\pi^-\pi^+}\right) + \text{Bose Symm.} = 0$$

at **every point** $\vec{\tau}$ in phase space

- Process: $X^- \to \xi \pi_3^- \to \pi_1^- \pi_2^+ \pi_3^-$.
- Condition for zero mode at all points $\vec{\tau}$ in phase-space:

$$\psi\left(\vec{\tau}_{123}\right)\,\Omega\left(m_{12}\right) + \mathrm{Bose\ Symm.} = 0 \qquad (1)$$

- Tensor formalism with pion momenta defined in the X^- rest frame:

$$\psi\left(\vec{\tau}_{123}\right) \propto \vec{p}_1 \times \vec{p}_3$$

- Bose symmetrization ($\pi_1^- \leftrightarrow \pi_3^-$):

$$\vec{p}_1 \times \vec{p}_3\,\Omega\left(m_{12}\right) + \vec{p}_3 \times \vec{p}_1\,\Omega\left(m_{23}\right) = \vec{p}_1 \times \vec{p}_3\left[\Omega\left(m_{12}\right) - \Omega\left(m_{23}\right)\right]$$

  - Fulfill eq. (1) at every point in phase space $\Rightarrow \Omega\left(m_\xi\right) = \mathrm{const.}$

- If $\Omega\left(m_\xi\right)$ is added to the physical dynamic isobar amplitude $\Delta^{\mathrm{phys}}\left(m_\xi\right)$, the total amplitude, and thus the intensity, is not altered:

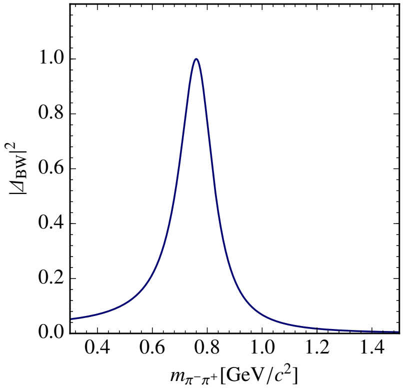
$$\left|\psi\left(\vec{\tau}\right)\Delta^{\mathrm{phys}}\left(m_\xi\right) + \mathrm{B.\,S.}\right|^2 = \left|\psi\left(\vec{\tau}\right)\left[\Delta^{\mathrm{phys}}\left(m_\xi\right) + \mathcal{C}\Omega\left(m_\xi\right)\right] + \mathrm{B.\,S.}\right|^2$$

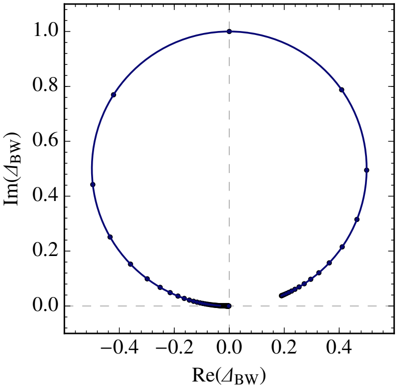
  for any complex-valued zero-mode coefficient $\mathcal{C}$
- $\mathcal{C}$: complex-valued ambiguity in the model

Zero mode in the spin-exotic wave
Effects on dynamic isobar amplitudes

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \quad (2)$$

$\mathcal{C} = 0.00 + 0.00i$

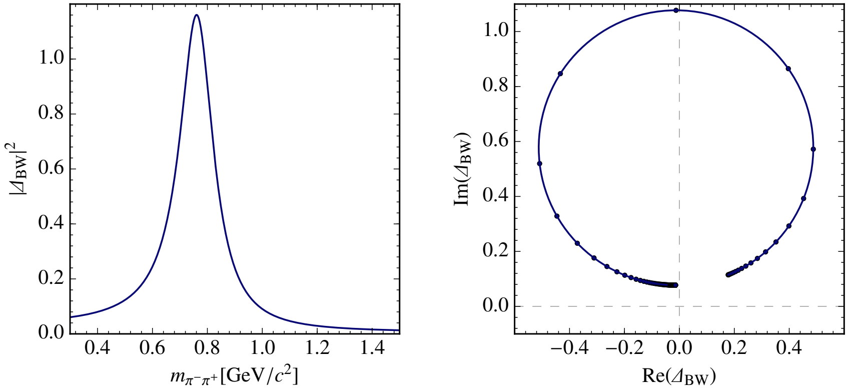




All amplitudes describe the same intensity

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \quad (2)$$

$$\mathcal{C} = -0.01 + 0.08i$$



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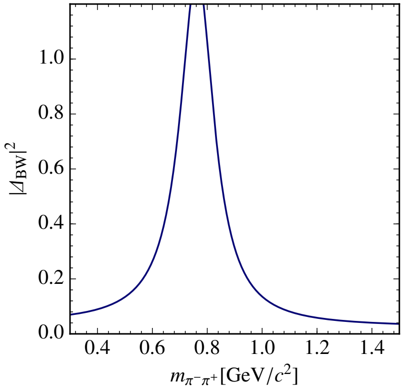
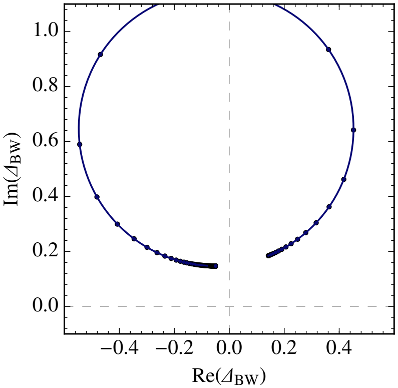
$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \tag{2}$$

$\mathcal{C} = -0.05 + 0.15i$





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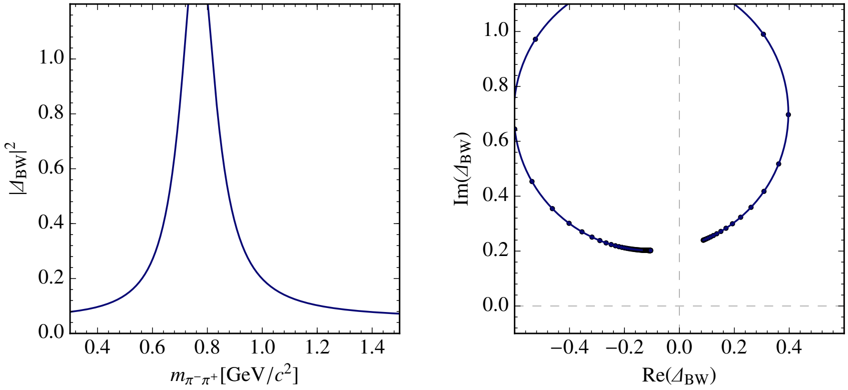
$$\Delta_{\text{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \tag{2}$$

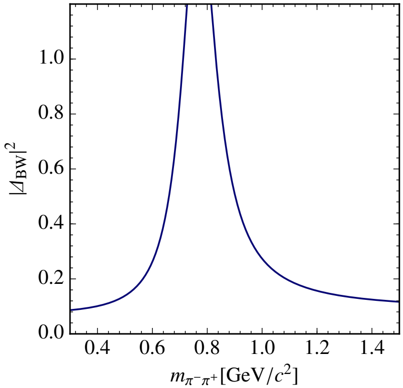
$$\mathcal{C} = -0.10 + 0.20i$$

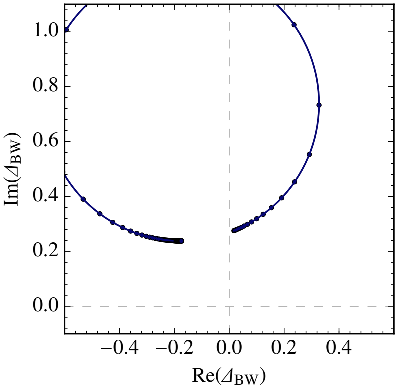


All amplitudes describe the same intensity

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \quad (2)$$

$$\mathcal{C} = -0.17 + 0.24i$$

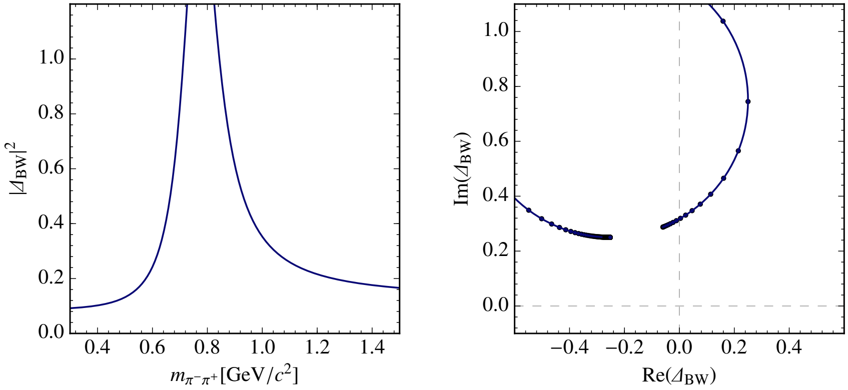




All amplitudes describe the same intensity

$$\Delta_{\text{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \tag{2}$$

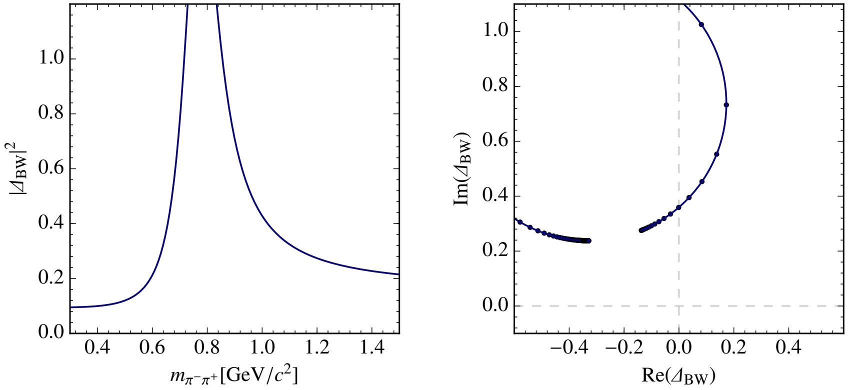
$\mathcal{C} = -0.25 + 0.25i$



All amplitudes describe the same intensity

$$\Delta_{\text{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \qquad (2)$$

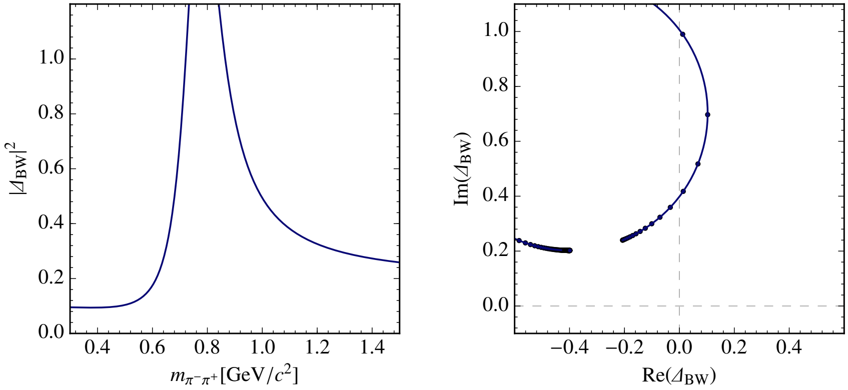
$$\mathcal{C} = -0.33 + 0.24i$$



All amplitudes describe the same intensity

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \quad (2)$$

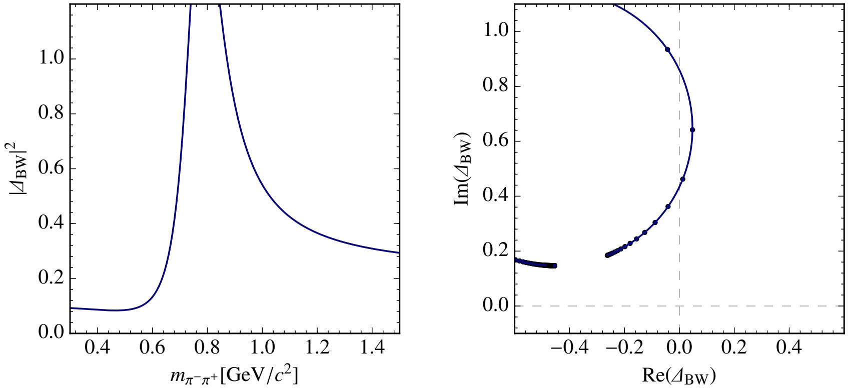
$\mathcal{C} = -0.40 + 0.20i$



All amplitudes describe the same intensity

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \quad (2)$$

$$\mathcal{C} = -0.45 + 0.15i$$



All amplitudes describe the same intensity

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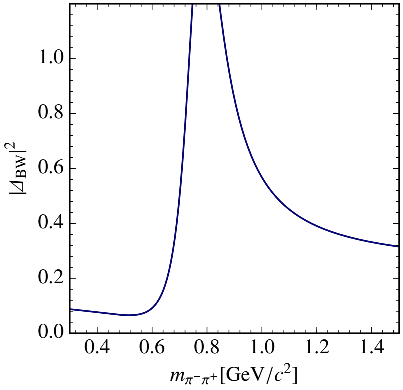
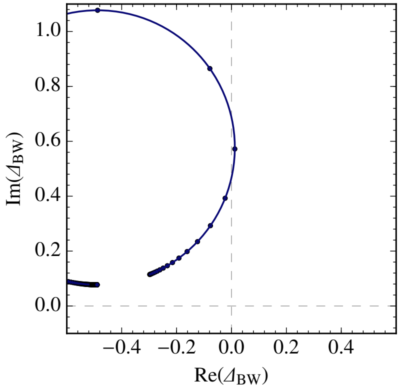
$$\mathcal{C} = -0.49 + 0.08i$$

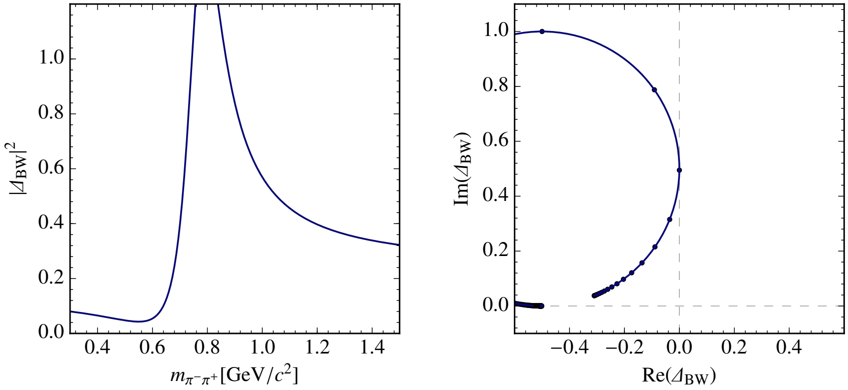




All amplitudes describe the same intensity

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \tag{2}$$

$\mathcal{C} = -0.50 + 0.00i$



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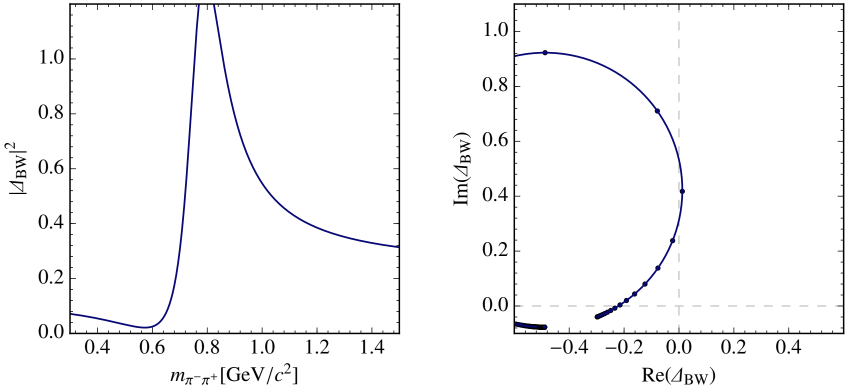
$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \tag{2}$$

$$\mathcal{C} = -0.49 - 0.08i$$



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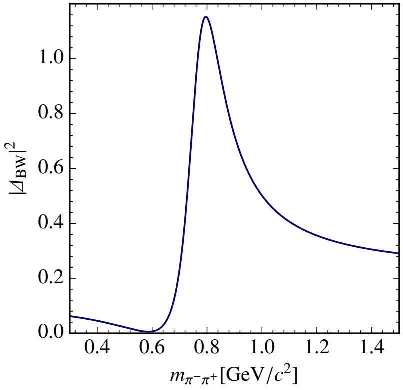
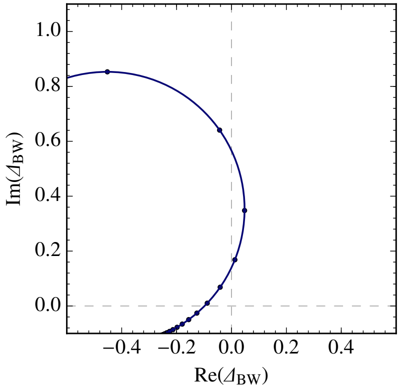
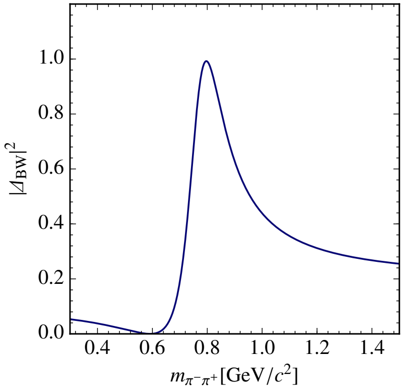


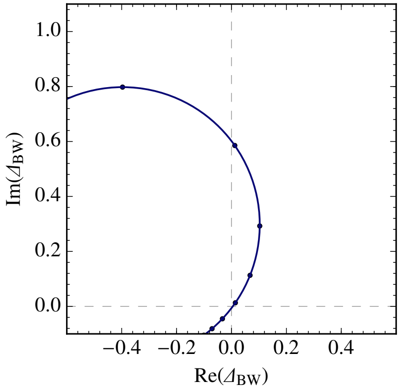


All amplitudes describe the same intensity

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \tag{2}$$

$\mathcal{C} = -0.40 - 0.20i$

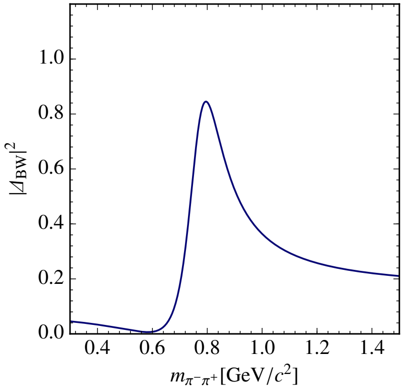


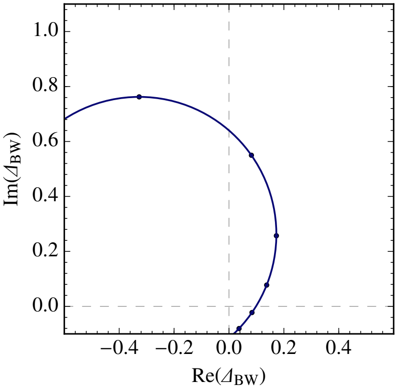


All amplitudes describe the same intensity

$$\Delta_{\text{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \tag{2}$$

$$\mathcal{C} = -0.33 - 0.24i$$

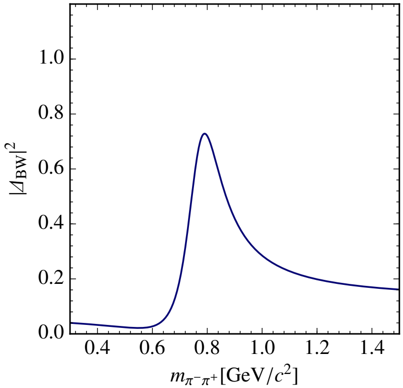


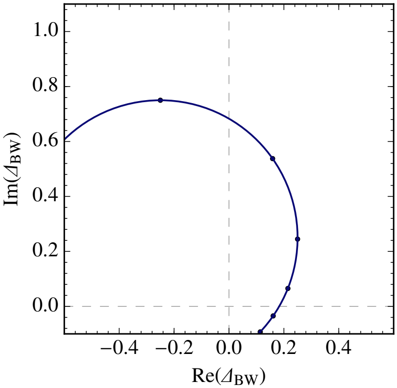


All amplitudes describe the same intensity

$$\Delta_{\text{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \tag{2}$$

$\mathcal{C} = -0.25 - 0.25i$

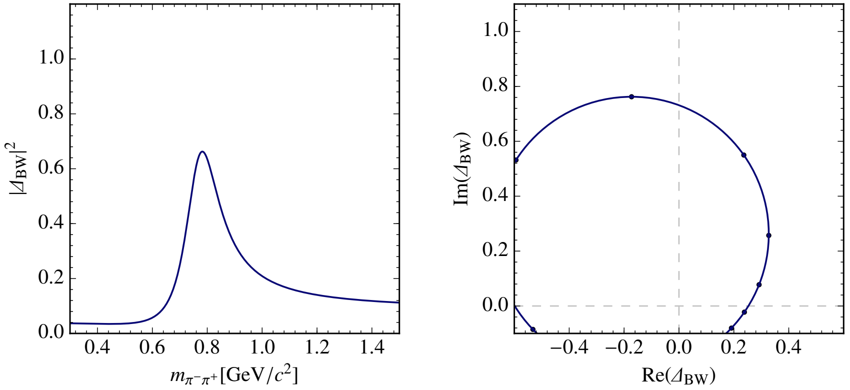




All amplitudes describe the same intensity

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \tag{2}$$

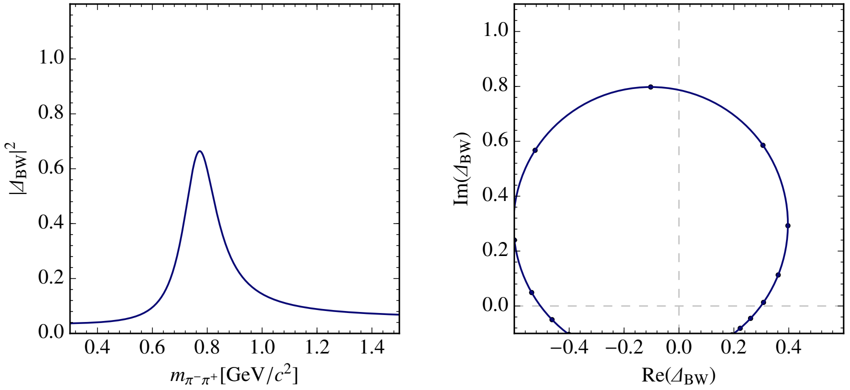
$\mathcal{C} = -0.17 - 0.24i$



All amplitudes describe the same intensity

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \tag{2}$$

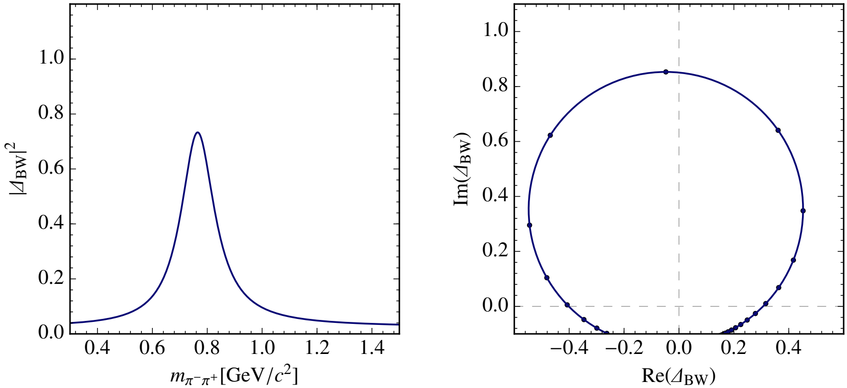
$$\mathcal{C} = -0.10 - 0.20i$$



All amplitudes describe the same intensity

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \quad (2)$$

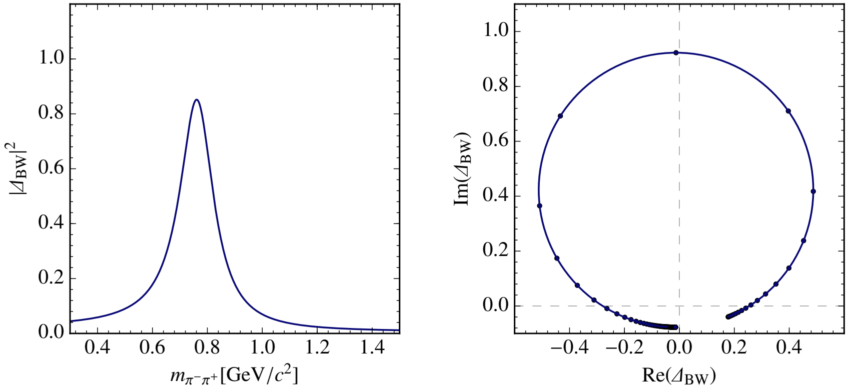
$\mathcal{C} = -0.05 - 0.15i$



All amplitudes describe the same intensity

$$\Delta_{\text{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \qquad (2)$$

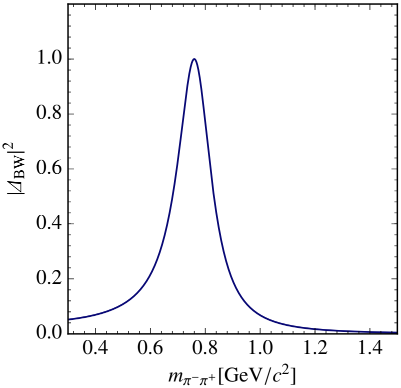
$\mathcal{C} = -0.01 - 0.08i$

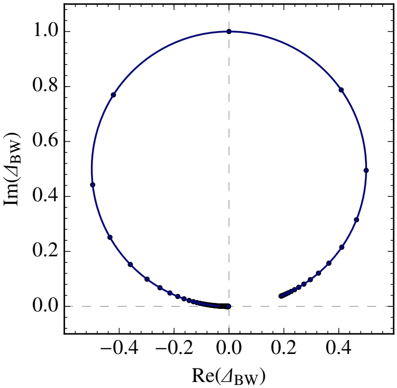


All amplitudes describe the same intensity

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \tag{2}$$

$\mathcal{C} = 0.00 + 0.00i$





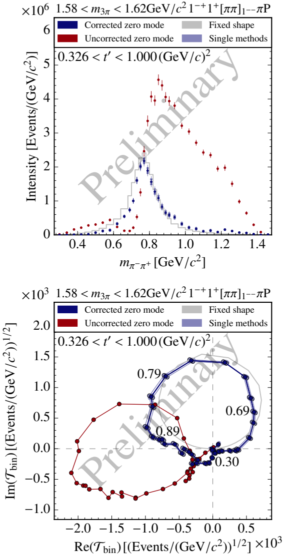
All amplitudes describe the same intensity

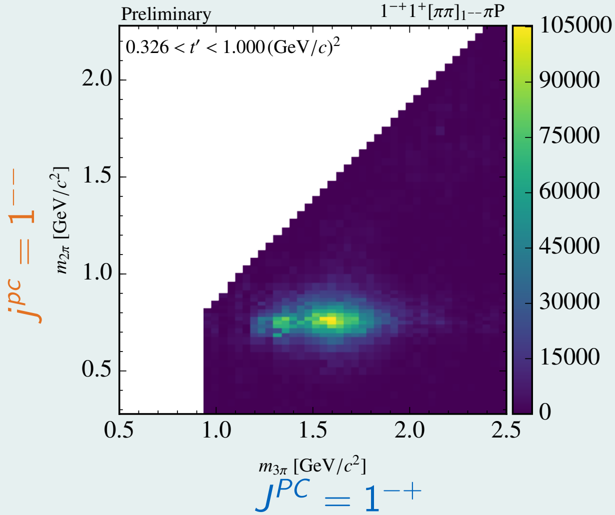
- Now for $m_{\pi^-\pi^+}$ bins: $\vec{\mathcal{T}}^0 = \{\Omega(m_{\text{bin}})\}$ for all $m_{\pi^-\pi^+}$bins
- The fitting algorithm might find a solution, shifted away from the physical solution $\vec{\mathcal{T}}^{\text{phys}}$:

$$\vec{\mathcal{T}}^{\text{phys}} = \vec{\mathcal{T}}^{\text{fit}} + \mathcal{C}\vec{\mathcal{T}}^0$$

- Obtain physical solution: constrain $\mathcal{C}$ by conditions on the resulting dynamic amplitudes $\vec{\mathcal{T}}^{\text{fit}}$
- In the case of the $1^{-+}1^{+}[\pi\pi]_{1^{--}}\pi P$ wave:
  - use the Breit-Wigner for the $\rho(770)$ resonance with fixed resonance parameters as in the fixed-isobar analysis
  - use a Breit-Wigner for the $\rho(770)$ resonance with floating resonance parameters
- Final results: weighted average of these two methods
- **Note:** Resolving the ambiguity fixes only a single complex-valued degree of freedom. $n_{\text{bins}} - 1$ complex-valued degrees of freedom remain free.

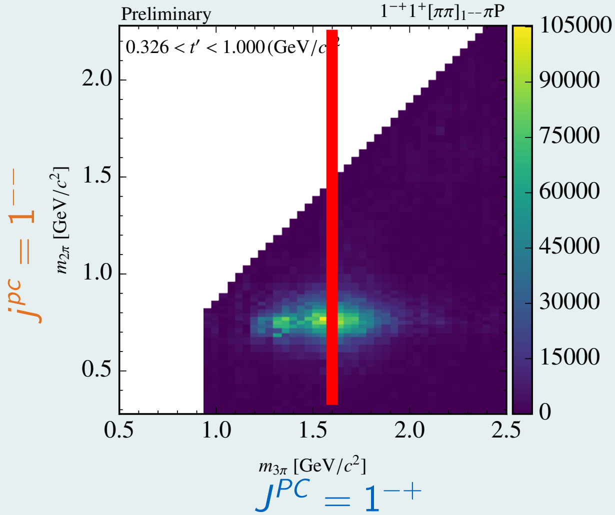
- Example: Single $(m_{3\pi}, t')$ bin
  - $1.58 < m_{3\pi} < 1.62\,\mathrm{GeV}/c^2$
  - $0.326 < t' < 1.000\ (\mathrm{GeV}/c)^2$
- Zero-mode ambiguity resolved with $\rho(770)$ used as constraint





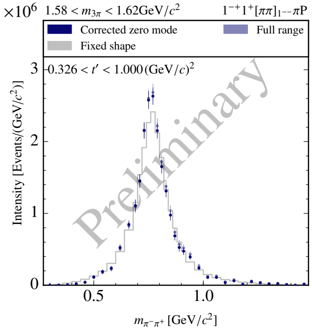
Preliminary
$1^{-+}1^{+}[\pi\pi]_{1^{--}}\pi P$
$0.326 < t' < 1.000\,(\text{GeV}/c)^2$
2.0
1.5
1.0
0.5
$m_{2\pi}$ [GeV/c^2]
$j^{pc} = 1^{--}$
0.5
1.0
1.5
2.0
2.5
$m_{3\pi}$ [GeV/c^2]
$J^{PC} = 1^{-+}$
105000
90000
75000
60000
45000
30000
15000
0

$J^{PC} = 1^{-+}$ Wave with freed $j^{pc} = 1^{--}$ Isobar Amplitude



Freed-Isobar Analysis

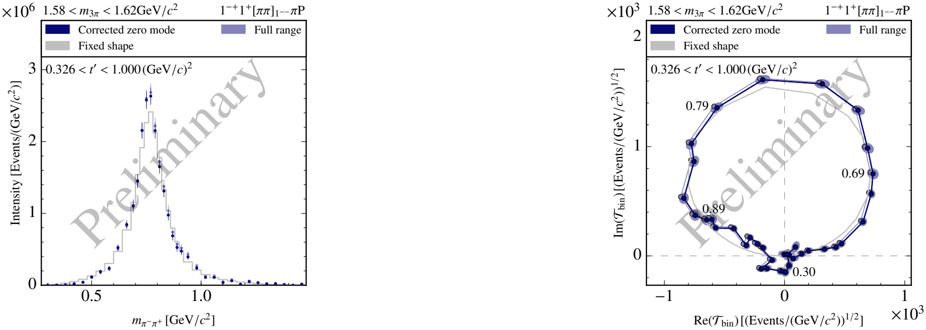
$J^{PC} = 1^{-+}$ Wave with freed $j^{pc} = 1^{--}$ Isobar Amplitude



- Study $\pi^-\pi^+$ amplitude as a function of $m_{3\pi}$
- $m_{\pi^-\pi^+}$ spectrum shows good agreement with $\rho(770)$ Breit-Wigner
- Extract $m_{\pi^-\pi^+}$ dependence of complex-valued amplitude
- Shape of $m_{3\pi}$ spectrum is in fair agreement with fixed-isobar analysis
  - ➥ $\pi_1(1600)$ signal at about 1.6 GeV/c^2 robust

Freed-Isobar Analysis

$J^{PC} = 1^{-+}$ Wave with freed $j^{pc} = 1^{--}$ Isobar Amplitude



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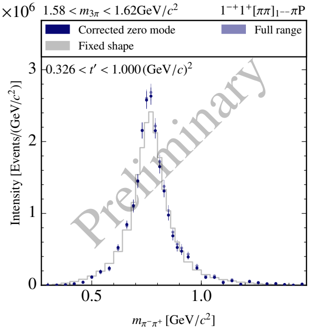
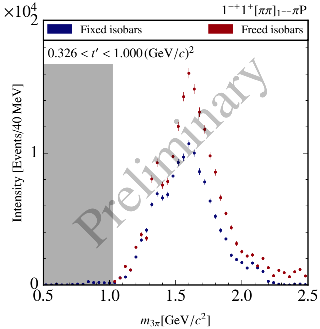
$J^{PC} = 1^{-+}$ Wave with freed $j^{pc} = 1^{--}$ Isobar Amplitude





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