A Triangle Singularity as the Origin of the $a_1(1420)$

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on behalf of the COMPASS collaboration

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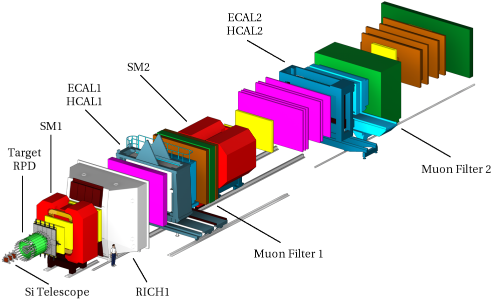
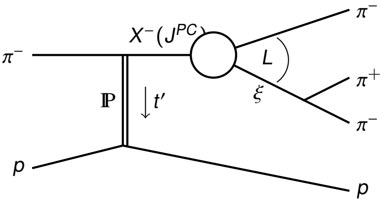
supported by BMBF

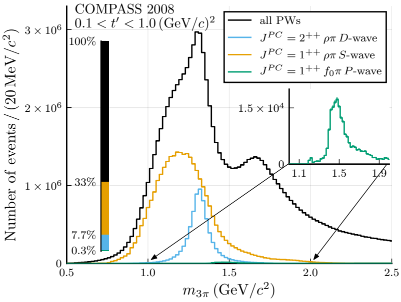
- Secondary hadron beam, mostly π^- ($\sim 97\,\%$)
- $E_{\mathrm{beam}} = 190\,\mathrm{GeV}$
- Fixed liquid hydrogen target (40 cm)



[COMPASS, NIM A**779**, 69-115 (2015)]

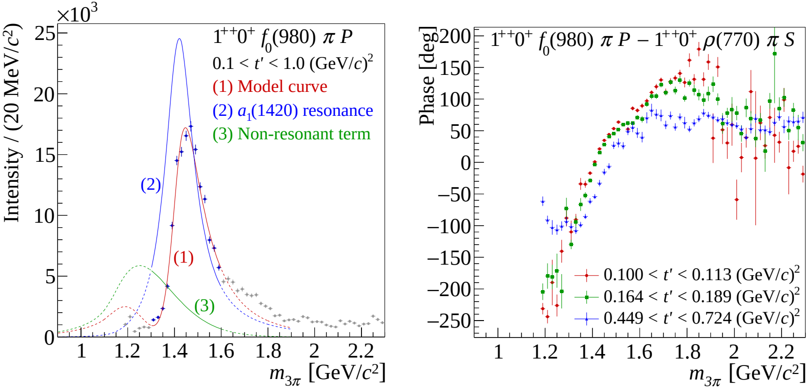
- Secondary hadron beam, mostly π^- ($\sim 97\,\%$)
- $E_{\text{beam}} = 190\,\text{GeV}$
- Fixed liquid hydrogen target (40 cm)
- $\pi^- + p \quad \rightarrow \quad \pi^- + \pi^- + \pi^+ + p$
- PWA with 88 waves binned in $m_{3\pi}$, t'





[COMPASS, accepted PRL,
arXiv: 2006.05342]

BW-fit to resonance-like signal in 1^{++} partial wave

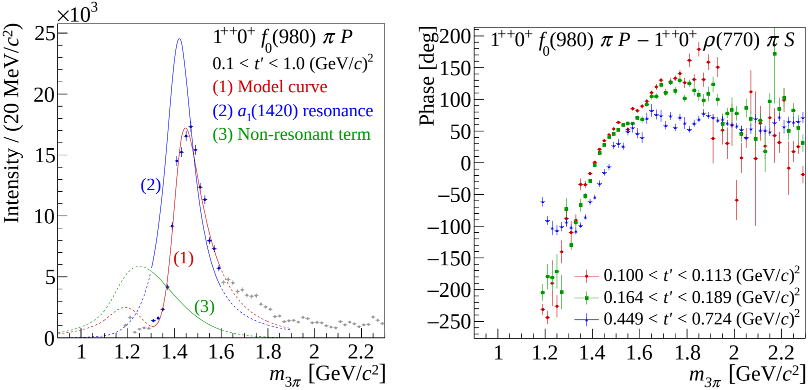


[COMPASS, PRL **115**, 082001 (2015)]

- $a_1(1420)$ narrow peak, strong phase motion
- Very close to ground state $a_1(1260)$
- Narrower than ground state

$\Rightarrow$ No ordinary radial excitation

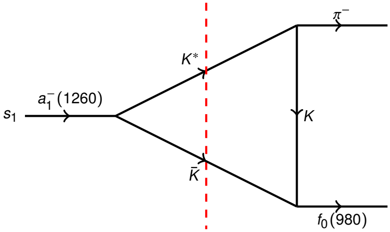
BW-fit to resonance-like signal in 1^{++} partial wave



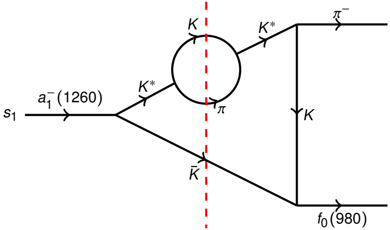
[COMPASS, PRL **115**, 082001 (2015)]

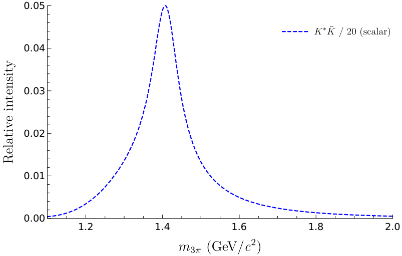
- 4-quark state [H.-X. Chen et al. (2015)], [T. Gutsche et al. (2017)]
- $K^{*}\,\bar{K}$ molecule (similar to $X(3872)$) [T. Gutsche et al. (2017)]
- Dynamic effect of interference with Deck-amplitude [Basdevant & Berger, PRL **114**, 192001 (2015)]
- **Triangle singularity (TS)** [Mikhasenko et al., PRD **91**, 094015 (2015)] [Aceti et al., PRD **94**, 096015 (2016)]

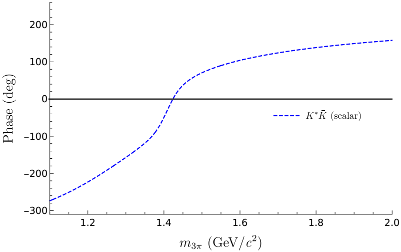
- Dispersive approach



- Dispersive approach
- Include finite width of K^*

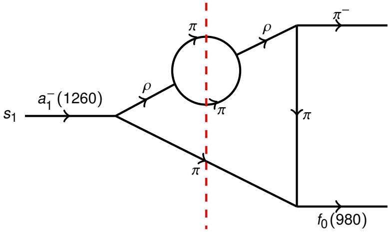


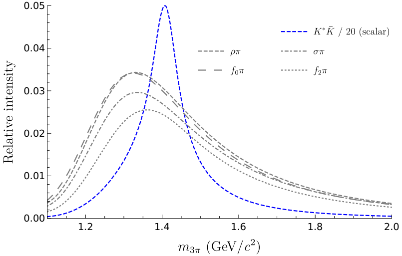


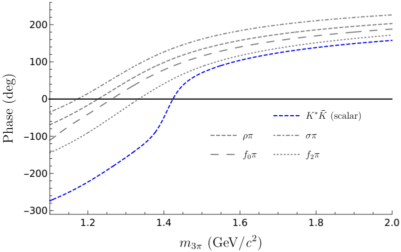


[COMPASS, accepted PRL, arXiv: 2006.05342]

- Dispersive approach
- Include finite width of K^*
- Negligible contribution from other triangles







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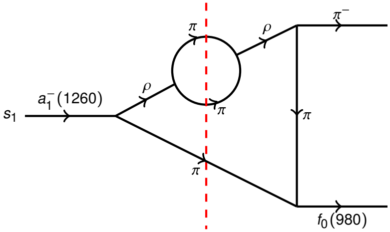

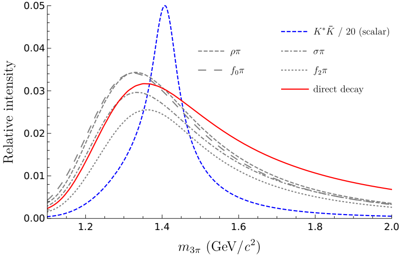

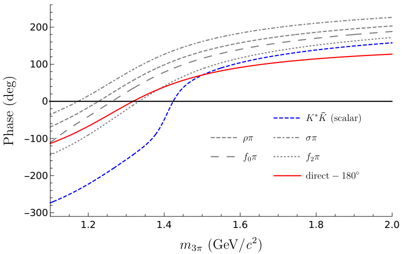

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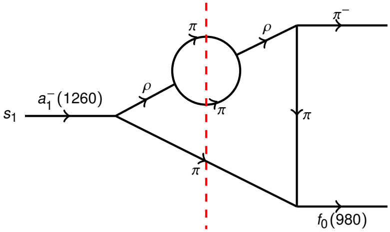
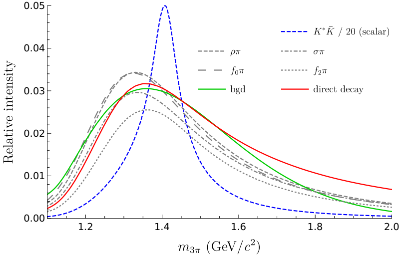
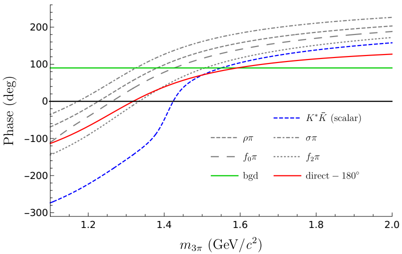
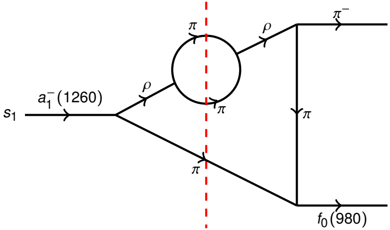


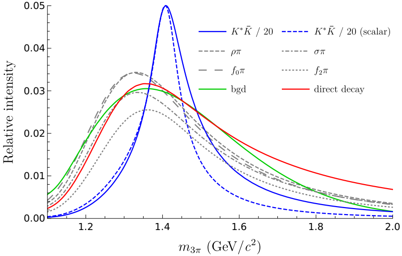


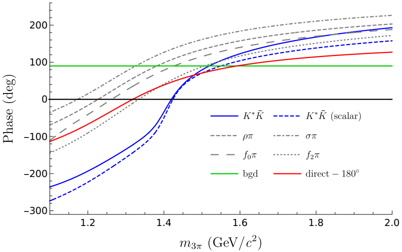


[COMPASS, accepted PRL, arXiv: 2006.05342]

- Dispersive approach
- Include finite width of K^*
- Negligible contribution from other triangles
- Inclusion of spin distorts shape



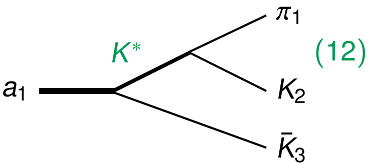




[COMPASS, accepted PRL, arXiv: 2006.05342]

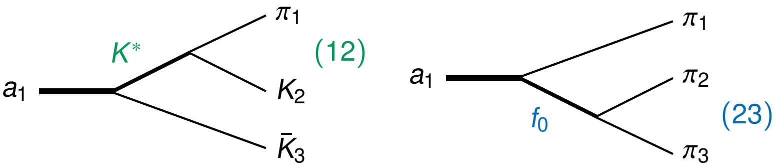
Include spin via partial-wave projection:

1. Look at the partial wave for $a_1(1260) \to K\bar{K}\pi$ with isobar K^*



Include spin via partial-wave projection:

1. Look at the partial wave for $a_1(1260) \to K\bar{K}\pi$ with isobar K^*
2. Project it onto the 3π final state with isobar $f_0(980)$



Include Spin

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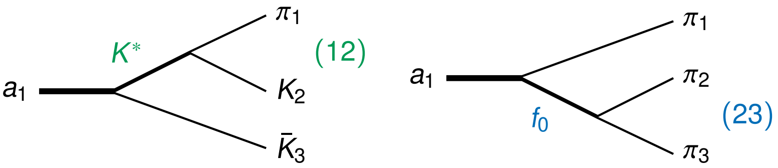
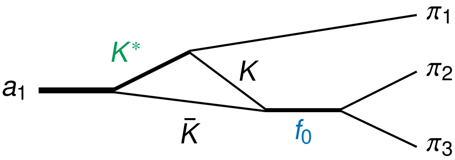
3. Obtain the first order approximation of the Khuri-Treiman approach

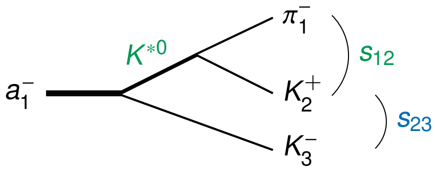




$$A(\tau) = \sum_{w=(JMLS)} \left[F_w(s_{12}) Z_w^*(\Omega_{3,12}) + F_w(s_{23}) Z_w^*(\Omega_{1,23}) \right]$$

Simple model: $F_w(s_{12}) = C_{a_1} \cdot t_{K^*}(s_{12})$

- $A(\tau)$: full amplitude of kinematic variables τ
- $F_w(s_{ij})$: isobar amplitude of decay with isobar in (ij)-channel
- $Z_w(\Omega_{k,ij})$: angular dependence of amplitude in (ij)-channel



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Projection to channel (23):

$$\begin{aligned} A_w(s_{23}) &= \int \mathrm{d}\,\Omega_{1,23} Z_w(\Omega_{1,23}) A(\tau) \\ &= F_w(s_{23}) + \hat{F}_w(s_{23}) \end{aligned}$$

with $\hat{F}_w(s_{23}) := \int \mathrm{d}\, Z_w(s_{23}) \sum_{w'} F_{w'}(s_{12}) Z_{w'}^*(\Omega_{3,12})$

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unitarity for PW amplitude A_w:

$$\Rightarrow \quad F_w(s_{23}) = t_\xi(s_{23}) \left[C_w + \frac{1}{2\pi} \int_{s_{\mathrm{th}}}^{\infty} \frac{\rho(\tilde{s}_{23}) \hat{F}_w(\tilde{s}_{23})}{\tilde{s}_{23} - s_{23}} \mathrm{d}\,\tilde{s}_{23} \right]$$

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Problem: $\hat{F}$ depends on F as well! $\rightsquigarrow$ solve iteratively

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KT:

- calculate effects of rescattering on the 2-body subsystem invariant-mass dependence s_{ij}
- Iterative framework to include rescattering to any order

$$F_w(s, s_{23}) = t_\xi(s_{23}) \left[C_w(s) + \frac{1}{2\pi} \int_{s_{\text{th}}}^{\infty} \frac{\rho(\tilde{s}_{23}) \hat{F}_w(s, \tilde{s}_{23})}{\tilde{s}_{23} - s_{23}} \mathrm{d}\,\tilde{s}_{23} \right]$$

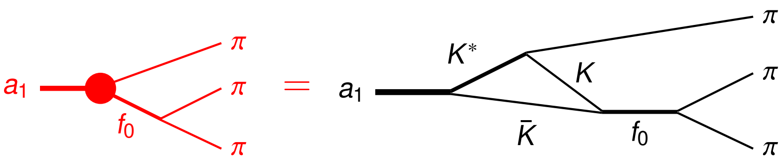
KT:

- calculate effects of rescattering on the 2-body subsystem invariant-mass dependence s_{ij}
- Iterative framework to include rescattering to any order

Our method:

- calculate effects of rescattering on the 3-body invariant-mass dependence $s = m_{3\pi}^2$
- Stop after the first iteration

$$F(s_{23}) = t_{f_0}(s_{23}) \frac{1}{2\pi} \int_{4m_K^2}^{\infty} \mathrm{d}\,\tilde{s}_{23} \frac{\rho(\tilde{s}_{23}) \int \mathrm{d}\, Z_{f_0}(\tilde{s}_{23})\; C_{a_1}\, t_{K^*}(s_{12}) Z^*_{K^*}(\Omega_{3,12})}{\tilde{s}_{23} - s_{23}}$$



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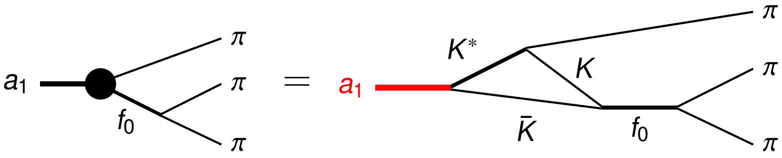

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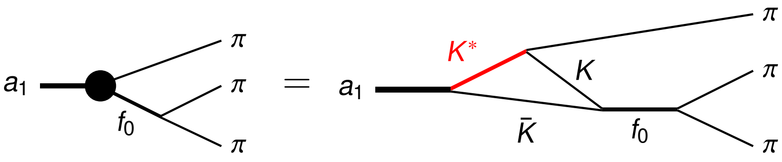

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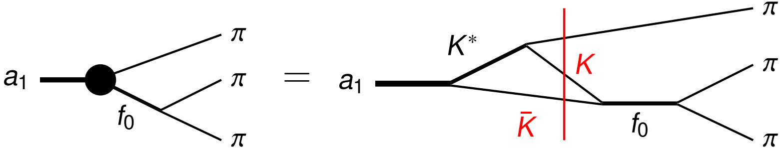

Partial-Wave Projection

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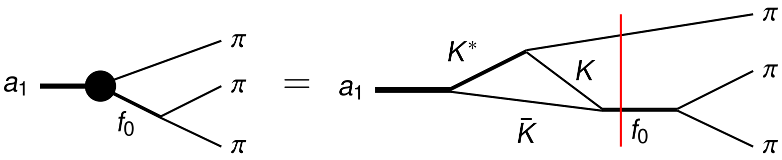

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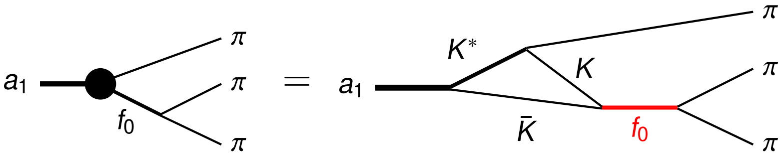

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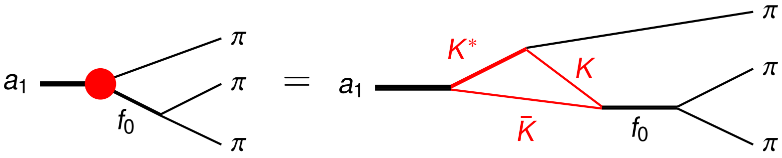
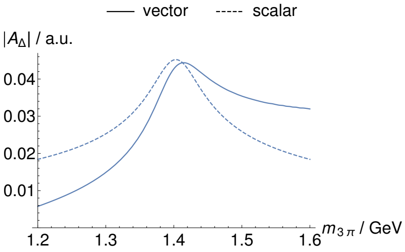




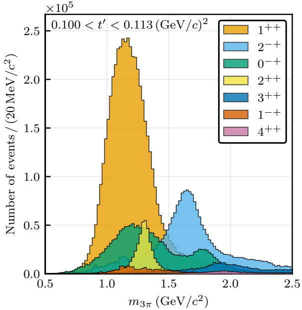
- Shape distorted, but similar
- Peak and phase motion at the same position

⇒ Scalar approximation reproduces main features

Minimal fit model $\rightsquigarrow$ choose 3 of the 88 waves of the PWA

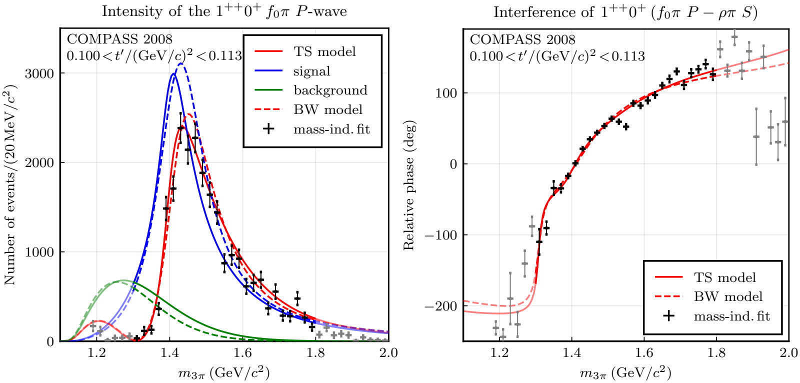
Notation: $J^{PC}\, M^{\varepsilon}\, \xi\pi\, L$

- $1^{++}\, 0^{+}\, \rho\pi$ S-wave:
  Contains source $a_1(1260)$, but huge non-res. background
- $1^{++}\, 0^{+}\, f_0(980)\pi$ P-wave:
  Signal of interest $a_1(1420)$
- $2^{++}\, 1^{+}\, \rho\pi$ D-wave:
  Clean $a_2(1320)$ with almost no non-res. background



[B. Ketzer, B. Grube, D. Ryabchikov, PPNP **113**, 0146-6410 (2020)]

Note: Fit all t'-slices with common resonance parameters. Show only fit of first slice.

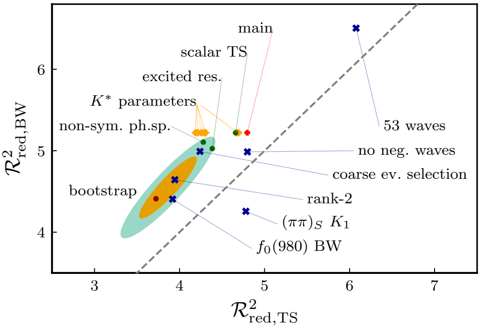


[COMPASS, accepted PRL, arXiv: 2006.05342]

- Comparison between TS model (solid) and BW model (dashed)
- Similar fit quality

Compare $\mathcal{R}^2_{\text{red}} = \sum_i \left(\frac{y_i - f(x_i)}{\sigma_i}\right)^2$,

but sum only over $f_0\pi$ P-intensity and its rel. phase to $\rho\pi$ S



- - - $\mathcal{R}^2_{\text{red, TS}} = \mathcal{R}^2_{\text{red, BW}}$

● main fit

× syst. studies of PWA

● syst. studies of model

◆ changing K^* resonance parameters

1σ and 2σ ellipses for bootstrap of data points

[COMPASS, accepted PRL, arXiv: 2006.05342]

(Almost) all studies show a better fit quality for the TS model.

Conclusion:

- Reproduce features with scalar approximation

$\Rightarrow$ Good starting point for first investigation

- $a_1(1420)$ fully explainable with rescattering
- Similar fit quality as with Breit-Wigner
- No free parameters needed to fix the position!
- Triangle singularity expected to be present
- Systematic studies also prefer the TS model
- Occam's razor: No need for a new genuine resonance

$\Rightarrow$ First complete analysis in the light sector with a TS model

Outlook:

- Look into $\tau \rightarrow 3\pi$, no Deck-like background
- Investigate $K\bar{K}\pi$ spectrum

Thank you for your attention!