Meson Charge Radii and AMBER

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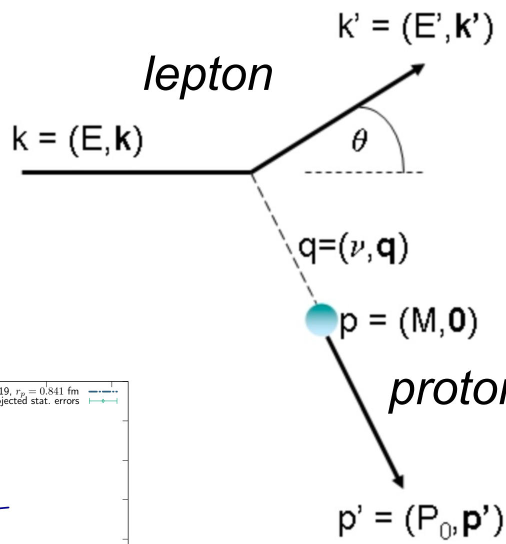
zoom

Hadron charge radii through elastic lepton scattering at low Q^2

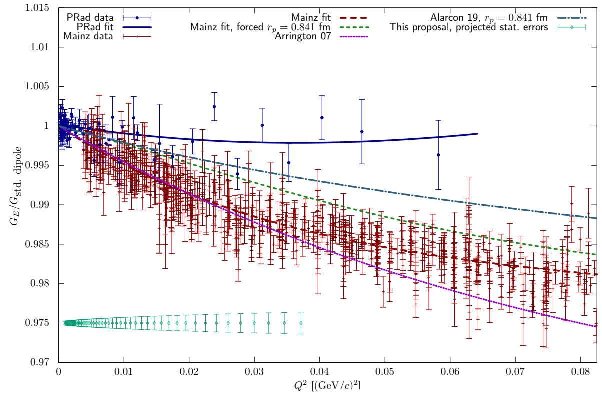
Protons in hydrogen target (or other stable nuclei):
Measurement via elastic electron or muon scattering
Cross section:

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} R \left(\varepsilon G_E^2 + \tau G_M^2\right)$$

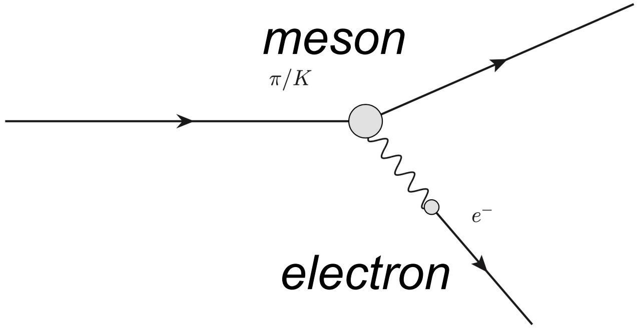


Charge radius from the slope of G_E

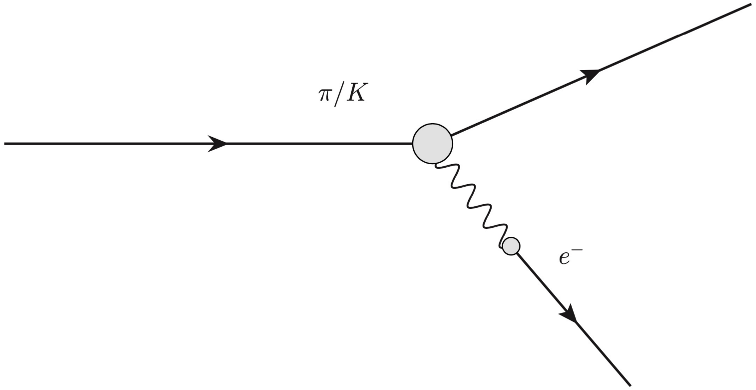
$$\langle r_E^2 \rangle = -6\hbar^2 \left. \frac{\mathrm{d}G_E(Q^2)}{\mathrm{d}Q^2} \right|_{Q^2 \to 0}$$



For unstable particles, electron scattering can be realised in *inverse kinematics*



Kinematics



$$K^- \; e^-_{target} \rightarrow \; K^- \; e^-$$

$$s = 2E_b m_e + m_b^2 + m_e^2$$

$$Q^2_{max} = \frac{4p_b^2 \, m_e^2}{s}$$

Beam	E_b [GeV]	Q^2_{max} [GeV²]	$E'_{b,min}$ [GeV]	Relative charge-radius effect on c.s. at Q^2_{max}
π	190	0.176	17.3	$\sim$40%
K	190	0.086	105.7	$\sim$20%
	80	0.066	59.9	$\sim$15%
	50	0.037	41.3	$\sim$8%

Hadron radius measurements

From: EPJC 8 (**1999**) 59, The WA89 Collaboration (measurement of Σ^- charge radius)

Measured $\langle r^2_{ch} \rangle$ in fm^2 of various hadrons

	Experiment	Soliton [7]	Skyrme [8]	non-relat. quark [12]	Skyrme [9]	Cloudy Bag [11]
p	0.74 ± 0.02 [1]	0.78	1.20	0.67	0.775	0.714
	0.67 ± 0.02 [2]					
	0.79 ± 0.03 [3]					
n	-0.11 ± 0.03 [4]	-0.09	-0.15		-0.308	-0.121
Σ^-	0.91 ± 0.32 ±0.4	0.75	1.21	0.55	0.751	0.582
π^-	0.439 ± 0.008 [5]	S. R. Amendolia, et al. , Nucl. Phys. B **277**, 168 (1986)				
K^-	0.34 ± 0.02 [6]	S. R. Amendolia, et al. , Phys. Lett. B **178**, 435 (1986)				

comparatively good accuracies (pion radius ~1%) stem from assuming a theoretical shape of the form factor

Pion and Kaon form factor measurements by NA7

S.R. Amendolia et al. / Pion electromagnetic form factor 193

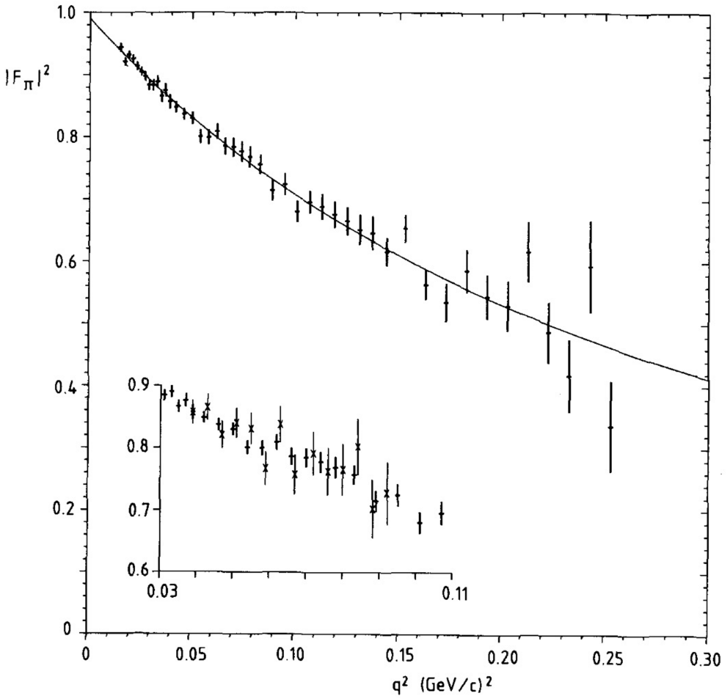


Fig. 17. The square of the pion form factor, $|F_\pi|^2$ versus q^2, with statistical error bars only. The line

S. R. Amendolia, et al. , Phys. Lett. B **178**, 435 (1986)

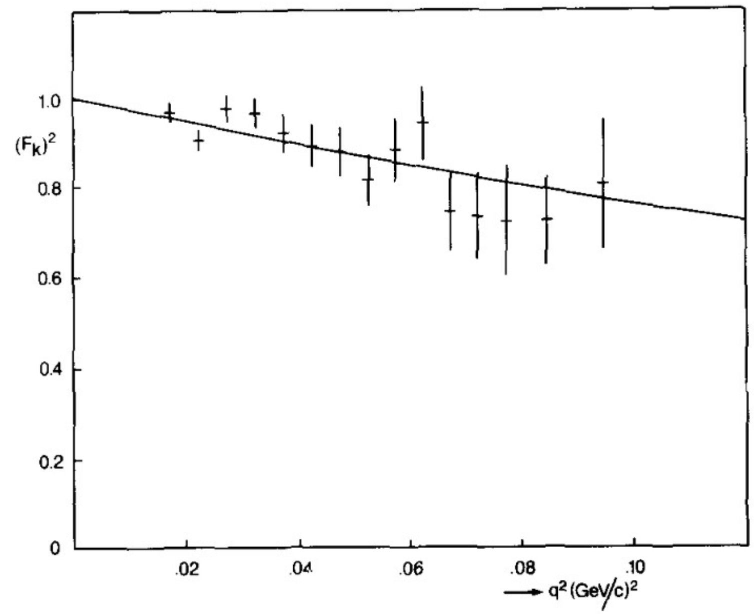


Fig. 3. The measured kaon form factor squared. The line corresponds to the pole fit with $\langle r^2 \rangle = 0.34$ fm^2.

~380,000 pion-electron scattering events

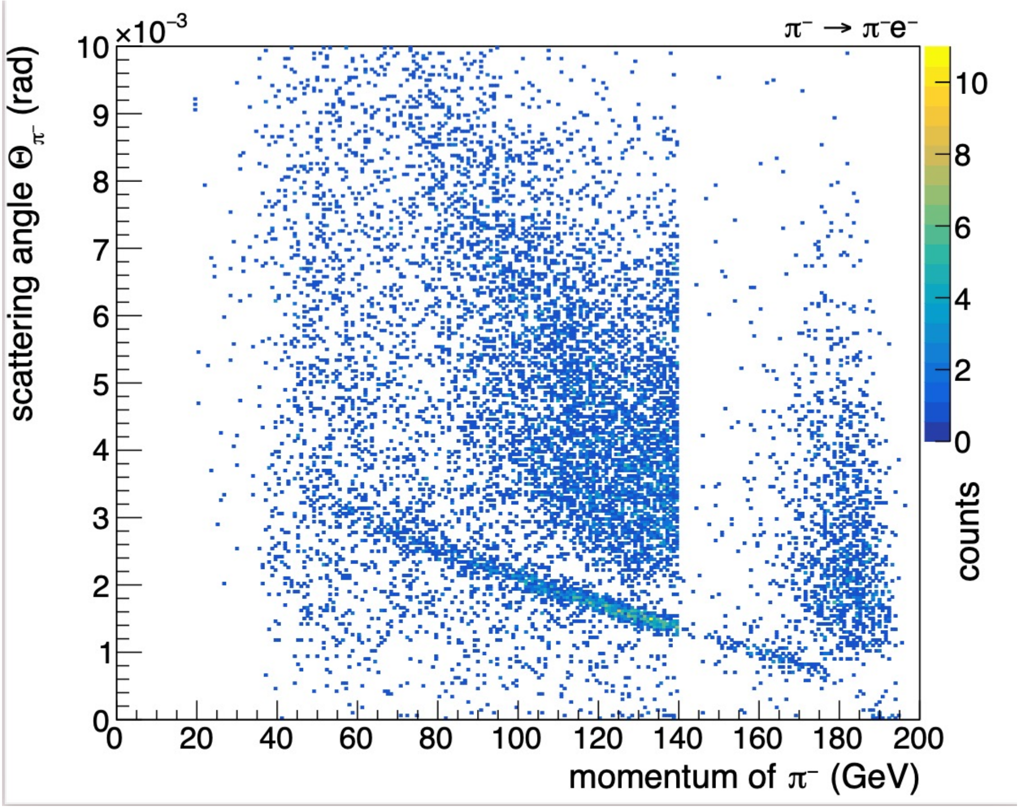
~400,000 kaon triggers (~30,000 kaon-electron scatterings?)

Extrapolation from COMPASS analysis: Count rate estimate for AMBER

By-product of COMPASS 2009 “Primakoff” analysis (constrained by cuts)

- Electrons identified in ECAL2, trigger on E_e>40 GeV

Plot prepared by Dominik Steffen (TUM)



- 190 GeV pion beam
- 9 days of beam time
- 1,500 identified elastic pion-electron scattering events in cut range $50 < E'_\pi < 140$
- i.e. $0.05 < Q^2 < 0.14$
- naïve estimate: for reproducing the NA7 result, roughly a factor 30 larger data sample would be needed

yet a similar measurement with separated kaon beam may become competitive

Summary

- Meson radii are of key interest in understanding their inner structure and the emergence of hadron mass

- For pions, some deeper investigations would be needed to see whether and how the data of previous experiments can be challenged

- For kaons, a significant increase of the form factor knowledge in the range $0.001 < Q^2 < 0.07$ appears in reach with an 80 GeV *rf-separated kaon beam*

Backup

Charge radius: definition and model dependence

Determination of the rms radius from a form factor measurement

- the rms radius of a charge distribution seen in lepton scattering is *defined* as the slope of the electric form factor at vanishing momentum transfer Q^2

$$\langle r_E^2 \rangle = -6\hbar^2 \frac{dG_E(Q^2)}{dQ^2}\bigg|_{Q^2 \to 0}$$

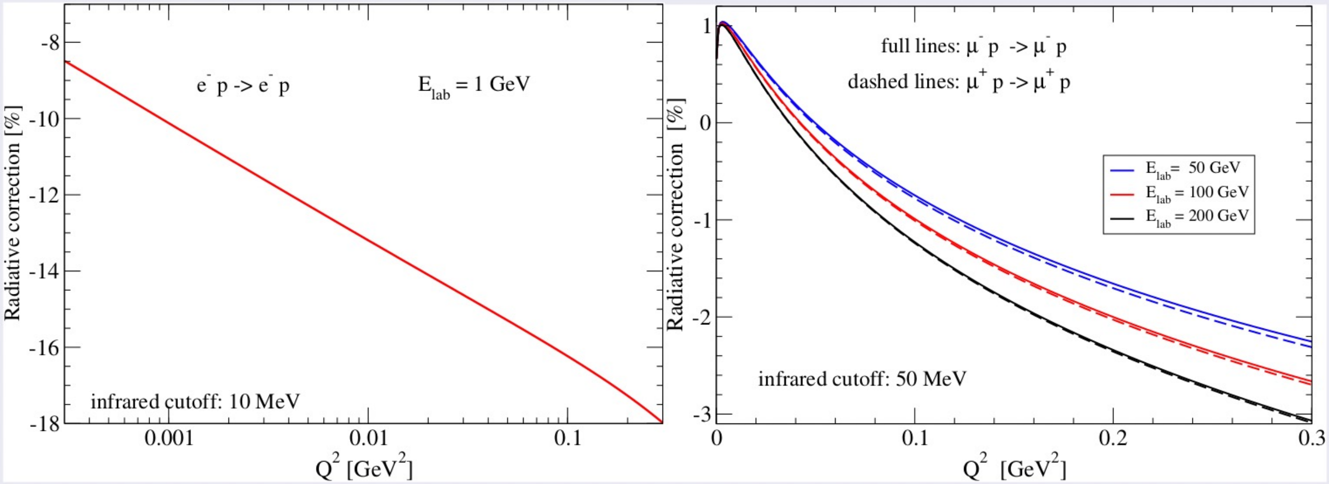
- elastic scattering experiments provide data for G_E at non-vanishing Q^2 and thus require an extrapolation procedure towards zero
  $\to$ mathematical ansatz may take more or less bounds into account (physics/theory/whatever motivated)
- Any approach (Padé, CF, DI, CM,...) *must* boil down to a series expansion

$$G_E(Q^2) = 1 + c_2 Q^2 + c_4 Q^4 + \ldots$$

  introducing possibly very different assumptions on the coefficients c_i
- recipe for experimenters: measure a sufficiently large range of Q^2 down to values as small as possible and as precise as possible

Radiative corrections for electron and muon scattering

QED radiative corrections



- for soft bremsstrahlung photon energies ($E_\gamma / E_{beam} \sim 0.01$), QED radiative corrections amount to $\sim$ 15-20% for electrons, and to $\sim$ 1.5% for muons
- important contribution to the uncertainty of elastic scattering intensities: *change* of this correction over the kinematic range of interest
- check: impact of exponantiation procedure (stricty valid only for vanishing photon energies): e^-: $2-4\%$, μ^-: 0.1%
- integrating the radiative tail out to large fraction of beam energy: shifts the correction to smaller values, but only *increases* the uncertainty