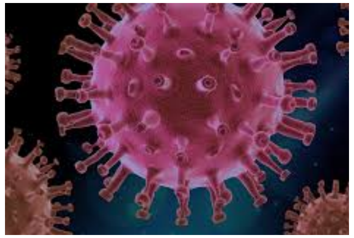
Chiral Perturbation Theory: Recent Experimental Results and Perspectives

Norbert Kaiser

Stephan Paul

TU Munich

COVID Video contribution

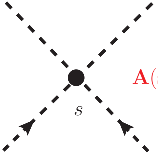


Measuring χPT sensitive processes

- π^0 lifetime
- Chiral anomaly
  - $\pi^- \gamma^* \rightarrow \pi^-\pi^0$
- Polarizability
  - $\pi\gamma^* \rightarrow \pi\gamma$
  - $K\gamma^* \rightarrow K\gamma$
- $\pi\pi$ scattering
  - K decays
  - $\pi^-\gamma^* \rightarrow \pi^-\pi^+\pi^-, \pi^-\pi^0\pi^0$
  - $\pi^-\pi^* \rightarrow \pi^-\pi$
- $K\pi$ scattering
  - $K^-\gamma^* \rightarrow K^-\pi^+\pi^-, K^-\pi^0\pi^0$
  - $K^-\pi^* \rightarrow K^-\pi$
- $\eta \rightarrow \pi\pi$, $\eta \rightarrow \pi\pi\pi$ (not covered here)
- Chiral symmetry restauration – parity doubling of mesons

Some highlights of ChPT

- Pions $\pi^{\pm 0}$: Goldstone bosons of spontaneous chiral symmetry breaking in QCD, $SU(2)_L \times SU(2)_R \to SU(2)_V$
- Their low-energy dynamics: systematically (and accurately) calculable in Chiral Perturbation Theory (= loop-expansion with effective Lagrangian)
- Leading-order pion-pion scattering amplitude in ChPT: involves as scale parameter the pion decay constant $f_\pi = 92.2\,\text{MeV}$



$$\mathbf{A(s,t,u)_{LO}} = \frac{\mathbf{s - m_\pi^2}}{\mathbf{f_\pi^2}}$$

- 2-loop prediction for $I = 0$ $\pi\pi$-scattering length: $a_0 m_\pi = 0.220 \pm 0.005$ confirmed by E865@BNL: $K^+ \to \pi^+\pi^- e^+\nu_e$ ($\pi^+\pi^-$ mass distribution)
- Cusp effect in $2\pi^0$ mass spectrum of $K^+ \to \pi^+\pi^0\pi^0$ at $\pi^+\pi^-$ threshold: $(a_0 - a_2)m_\pi = 0.257 \pm 0.006$ $(0.265 \pm 0.005)_{ChPT}$
- Conclusion: quark condensate $\langle 0|\bar{q}q|0\rangle$ is large, linear term dominates quark mass expansion of m_π^2: $m_\pi^2 f_\pi^2 = -\langle 0|\bar{q}q|0\rangle m_q + \mathcal{O}(m_q^2 \ln m_q)$

Beyond perturbation theory

- Moderately attractive isospin-0 S-wave $\pi\pi$-interaction can produce upon iteration to all orders (or unitarization) a complex resonance-pole

$$T_0^0(s) = A_0^0(s) + A_0^0(s)G(s)A_0^0(s) + A_0^0(s)G(s)A_0^0(s)G(s)A_0^0(s) + \cdots = \frac{A_0^0(s)}{1 - G(s)A_0^0(s)},$$

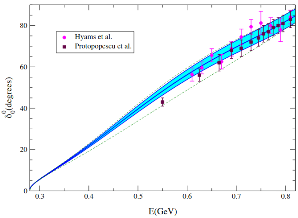
$$A_0^0(s) = \frac{2s - m_\pi^2}{f_\pi^2}, \qquad G(s) = \frac{1}{16\pi^2}\left\{\frac{1}{2} - \sqrt{1 - \frac{4m_\pi^2}{s}}\ \ln\frac{\sqrt{-s} + \sqrt{4m_\pi^2 - s}}{2m_\pi} - \ln\frac{m_\pi}{\lambda}\right\}$$

- Inverse amplitude method in chiral limit $m_\pi = 0$ gives

  $T_0^0 = \frac{2s}{f_\pi^2}\left[1 - i\frac{s}{16\pi f_\pi^2}\right]^{-1}$ has complex pole at $\sqrt{s} = (1-i)462\,\mathrm{MeV}$
- Scrutinized by dispersion relation analysis and Roy-Steiner equations

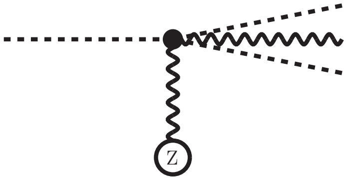
  [I. Caprini, G. Colangelo, J. Gasser, H. Leutwyler, B. Moussallam]

  σ-pole at $\sqrt{s} = m_\sigma - i\Gamma_\sigma/2 = [(450 \pm 20) - i(280 \pm 10)]\,\mathrm{MeV}$
- More to physics of $\pi\pi$ S-wave than occurrence of pole far from real axis



- Further dynamically generated poles from mesonic final-state interactions:
- $\kappa(700)$ in πK S-wave with $I = 1/2$
- $f_0(980)$ in $\pi\pi$-$K\bar{K}$ coupled channels
- $a_0(980)$ in $\pi\eta$-$K\bar{K}$ coupled channels
- $\Lambda(1405)$ in $\pi\Sigma$-$\bar{K}N$ coupled channels

COMPASS: Measurement of low-energy pion-photon reactions



- Scattering high-energy pions in nuclear Coulomb field (charge Z) allows to extract cross sections for $\pi^-\gamma$ reactions (equivalent-photon method)

$$\frac{d\sigma}{ds\,dQ^2} = \frac{Z^2\alpha}{\pi(s-m_\pi^2)}\,\frac{Q^2-Q^2_{min}}{Q^4}\,\sigma_{\pi^-\gamma}(s)\,, \qquad Q_{min} = \frac{s-m_\pi^2}{2E_{beam}}$$

- $s = (\pi^-\gamma$ invariant mass$)^2$, $Q \to 0$ momentum transfer by virtual photon
- Isolate Coulomb peak from strong interaction background
- Different final-states $\pi^-\gamma$, $\pi^-\pi^0$, $\pi^-\pi^0\pi^0, \pi^+\pi^-\pi^-$ allow to test different aspects of chiral dynamics (low-energy QCD)
- Polarizabilities, chiral anomaly, $\pi\pi$-scattering in el-magn. environment

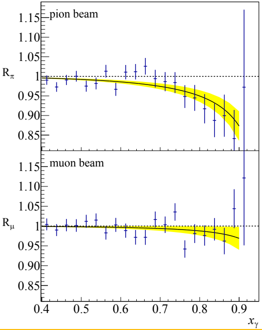
Pion electromagnetic polarizabilities

- Two-loop prediction of ChPT: $\bar{\ell}_6 - \bar{\ell}_5 = 3.0 \pm 0.3$ from radiative pion decay $\pi^+ \to e^+ \nu_e \gamma$, PIBETA@PSI: axial-to-vector ratio $F_A/F_V = 0.44$

$$\alpha_\pi - \beta_\pi = \frac{\alpha(\bar{\ell}_6 - \bar{\ell}_5)}{24\pi^2 f_\pi^2 m_\pi} + \frac{\alpha m_\pi}{(4\pi f_\pi)^4}\Big\{c^r + \frac{8}{3}\Big(\bar{\ell}_2 - \bar{\ell}_1 + \bar{\ell}_5 - \bar{\ell}_6 + \frac{65}{12}\Big)\ln\frac{m_\pi}{m_\rho} + \frac{4}{9}(\bar{\ell}_1 + \bar{\ell}_2) - \frac{\bar{\ell}_3}{3} + \frac{4\bar{\ell}_4}{3}(\bar{\ell}_6 - \bar{\ell}_5) - \frac{187}{81} + \Big(\frac{53\pi^2}{48} - \frac{41}{324}\Big)\Big\}$$

$$\alpha_\pi - \beta_\pi = (5.7 \pm 1.0)\cdot 10^{-4}\,\mathrm{fm}^3\,, \qquad \alpha_\pi + \beta_\pi = 0.16\cdot 10^{-4}\,\mathrm{fm}^3$$

- COMPASS result: $\alpha_\pi - \beta_\pi = (4.0 \pm 1.8)\cdot 10^{-4}\,\mathrm{fm}^3$ [PRL 114, 062002 ('15)]



$x_\gamma = E_\gamma/E_\pi$ in lab, $\cos\theta_{\rm cm} = 1 - 2x_\gamma s/(s - m_\pi^2)$

Analysis of data includes:

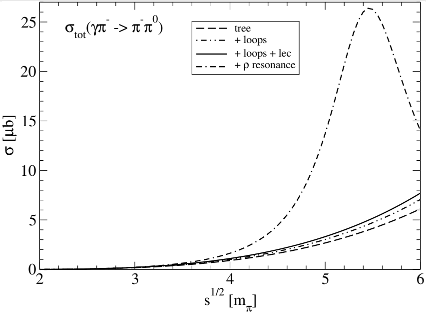
- chiral pion-loop corrections $A(s,t) \sim \ln^2(.t.)$
- radiative corrections [NPA 812, 186 ('08)]
- isospin-breaking correction $\sim (m_\pi^2 - m_{\pi^0}^2)\ln^2(.t.)$
- previous results from Mainz and Serpukhov: $\alpha_\pi - \beta_\pi = (12 - 16)\cdot 10^{-4}\,\mathrm{fm}^3$

Extracting the chiral anomaly

- $\pi^0 \to 2\gamma$ and $\gamma \to 3\pi$ couplings determined by chiral anomaly of QCD
- Amplitude and cross section for $\pi^-(p_1) + \gamma(k, \epsilon) \to \pi^-(p_2) + \pi^0(p_0)$:

$$T_{\gamma 3\pi} = \frac{e}{4\pi^2 f_\pi^3} \epsilon_{\mu\nu\kappa\lambda} \epsilon^\mu p_1^\nu p_2^\kappa p_0^\lambda \, M(s,t) \,, \qquad F_{3\pi} = 9.8\,\mathrm{GeV}^{-3}$$

$$\sigma_{\text{tot}}(s) = \frac{\alpha(s - m_\pi^2)(s - 4m_\pi^2)^{3/2}}{(4f_\pi)^6 \pi^4 \sqrt{s}} \int_{-1}^{1} dz\,(1 - z^2)\,|M(s,t)|^2$$



- $\rho(770)$-resonance must be included:

$$M(s,t)^{(\rho)} = 1 + 0.46 \left\{ \frac{s}{m_\rho^2 - s - i\sqrt{s}\,\Gamma_\rho(s)} + \frac{t}{m_\rho^2 - t} + \frac{u}{m_\rho^2 - u} \right\}$$

Extracting the chiral anomaly

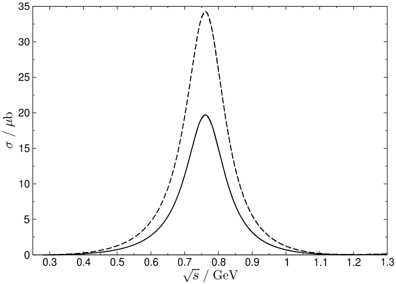
- Dispersive representation of $\pi\gamma \to \pi\pi$ with p-wave phase shifts as input [M. Hoferichter, B. Kubis, D. Sakkas, PRD 86, 116009 ('12)]

$$\frac{e}{4\pi^2 f_\pi^3} M(s,t) = F(s) + F(t) + F(u)\,, \qquad u = 3m_\pi^2 - s - t\,,$$

$$F(s) = a + b\,s + \frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im}F(s')}{s'^2(s'-s)}, \quad \mathrm{Im}F(s) = [F(s) + \hat{F}(s)]\sin\delta_1^1(s) e^{-i\delta_1^1(s)}$$

- Relevant subtraction constant $C = 3(a + b\,m_\pi^2)$ is fitted to data and matched via the chiral representation to $F_{3\pi}$

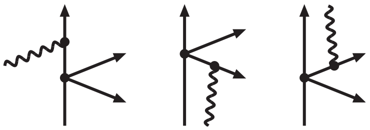
$$C = \left\{1 + \frac{m_\pi^2}{(4\pi f_\pi)^2}\left(2.9 - \ln\frac{m_\pi}{m_\rho}\right)\right\} F_{3\pi} = 1.067\, F_{3\pi}$$

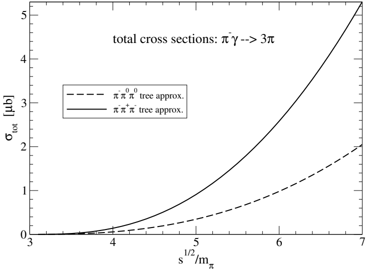


- solid line: $C = 9.78\,\mathrm{GeV}^{-3}$
  dashed line: $C = 12.9\,\mathrm{GeV}^{-3}$
- close to threshold, one-photon exchange an important correction: $1 \to 1 - 2e^2 f_\pi^2/t$
- Good theory waiting for good data
- Extension to $K^-\gamma \to \pi K$ [B. Kubis et al.]

Tree level cross sections for $\pi^-\gamma \to 3\pi$

- Coulomb gauge $\epsilon \cdot p_1 = \epsilon \cdot k = 0$, photon does not couple to incoming π^-
- No $\gamma 4\pi$ vertex at leading order





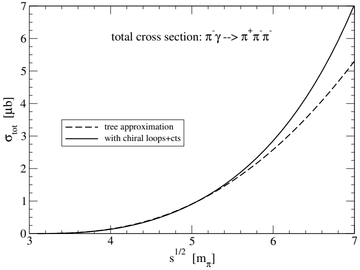
- Example: total cross section for $\pi^-(p_1) + \gamma(k,\epsilon) \to \pi^-\pi^0\pi^0$

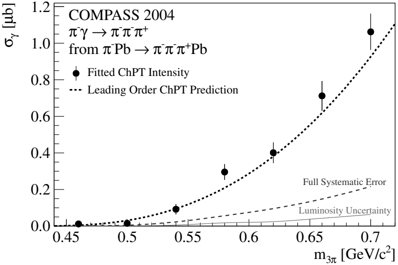
$$\sigma_{tot}(s) = \frac{\alpha}{16\pi^2(s-m_\pi^2)^3}\int_{2m_\pi}^{\sqrt{s}-m_\pi} d\mu \sqrt{\mu^2-4m_\pi^2}\left[\frac{\mu^2-m_\pi^2}{f_\pi^2}\right]^2$$
$$\times\left\{(s+m_\pi^2-\mu^2)\ln\frac{s+m_\pi^2-\mu^2+\lambda^{1/2}(s,\mu^2,m_\pi^2)}{2m_\pi\sqrt{s}} - \lambda^{1/2}(s,\mu^2,m_\pi^2)\right\}$$

- $(\mu^2 - m_\pi^2)/f_\pi^2$ is LO chiral $\pi\pi$-interaction, rest from 3-body phase space
- How large are next-to-leading order corrections from chiral loops + cts?

Charged pion-pair production

- Total cross section for $\pi^-\gamma \to \pi^+\pi^-\pi^-$





- $\sigma_{\text{tot}}(s)$ for $\sqrt{s} < 6m_\pi$ almost unchanged in comparison to tree approx.
- Suggestive explanation: $\pi^-\pi^- \to \pi^-\pi^-$ final state interaction $(1 - 0.02)^2$

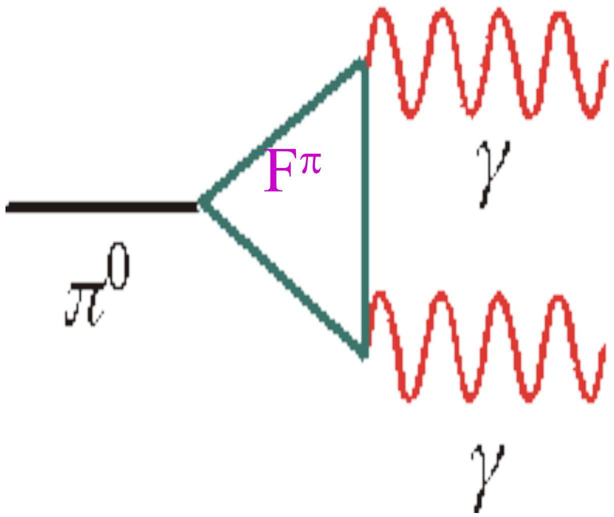
$$a_2 = -\frac{m_\pi}{16\pi f_\pi^2}\left[1 - \frac{m_\pi^2}{12\pi^2 f_\pi^2}\left(\bar{\ell}_1 + 2\bar{\ell}_2 - \frac{3\bar{\ell}_3}{8} - \frac{3\bar{\ell}_4}{2} + \frac{3}{8}\right)\right]$$

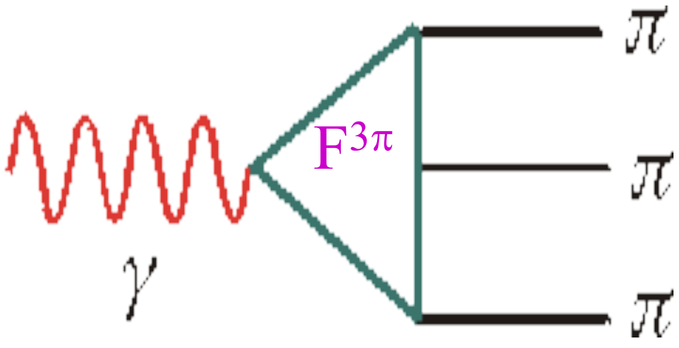
- Analysis of COMPASS data for $\sqrt{s} \leq 5m_\pi$ agrees with ChPT prediction
  First measurement of chiral dynamics in $\pi^-\gamma \to \pi^-\pi^-\pi^+$, PRL108, 192001 ('12)
- Agreement on level of full 5-dimensional phase space distribution
- For $\pi^-\gamma \to \pi^-\pi^0\pi^0$ chiral corrections are substantially larger $\sim (1+0.2)^2$

Coupling of Mesons to Photons

$\pi^- \gamma^* \to \pi^- \pi^0$

$\pi^0 \to \gamma\gamma$ and $\gamma \to 3\pi$ $(\pi^-\gamma \to \pi^-\pi^0)$ determined by chiral anomaly of QCD





$\pi^-\gamma^* \rightarrow \pi^-\pi^0$

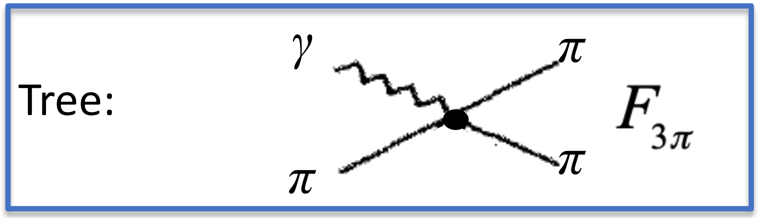
$\pi^0 \rightarrow \gamma\gamma$ and $\gamma \rightarrow 3\pi$ $(\pi^-\gamma \rightarrow \pi^-\pi^0)$ determined by chiral anomaly of QCD

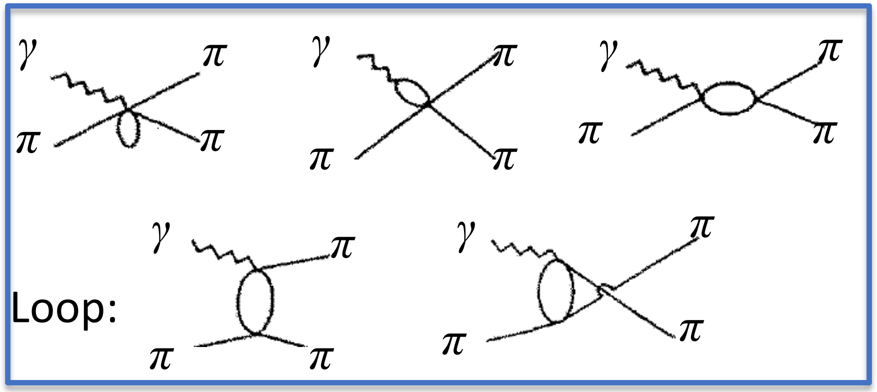
$$T_{\gamma 3\pi} \propto \frac{e}{4\pi^2 f_\pi^2} = F_{3\pi}$$

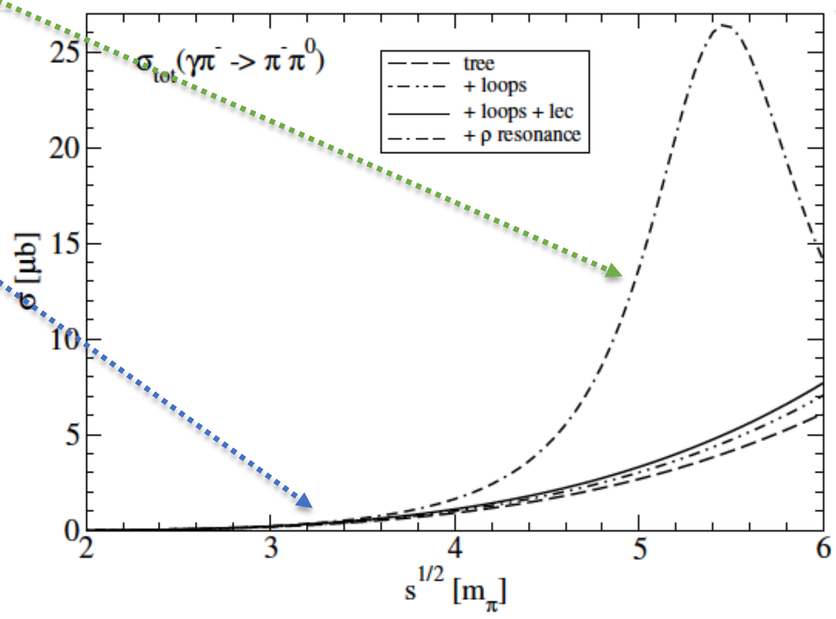
Antipov et al in Serpukhov:

$F^{3\pi} = 12.9 \pm 0.9 \pm 0.5$ GeV^{-3}

- strong final state interaction: $\pi^-\pi^0 \rightarrow \rho \rightarrow \pi^-\pi^0$



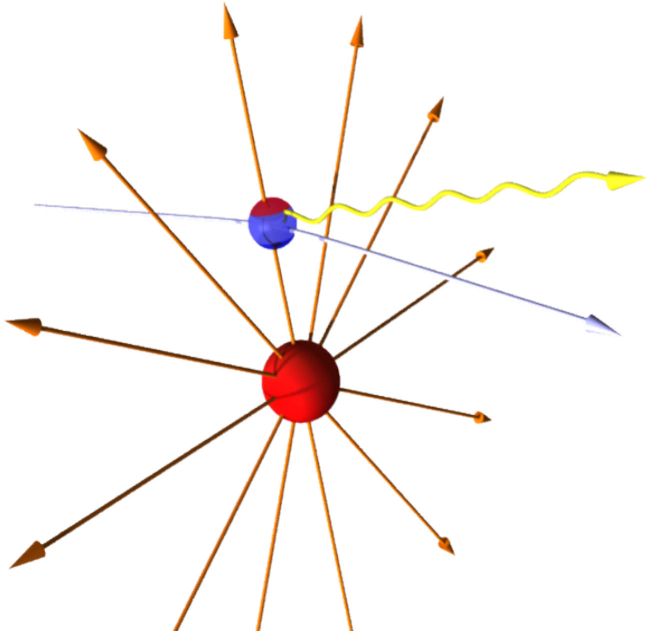




COMPASS @ CERN

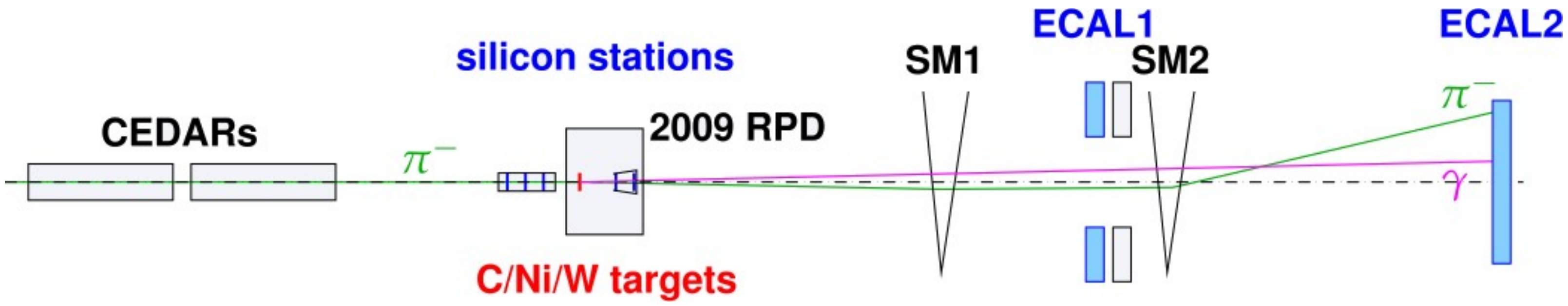
- Use (quasireal) photons as target
- Scattering off Coulomb field in heavy nucleus → Primakoff cross-section: $\propto Z^2 \Rightarrow$ large Z
- Very high Z (e.g. Pb): large corrections from 2γ processes and from screening, conversions

Optimum choice: medium heavy Ni target



Artistic depiction of $\pi^- + \mathrm{Ni} \to \pi^- + \gamma + \mathrm{Ni}$ via Primakoff process

COMPASS @ CERN



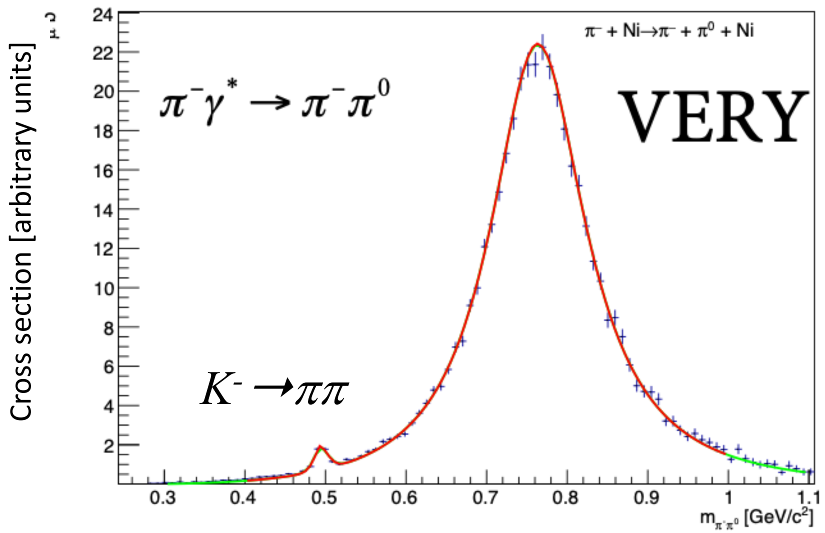
- 190 GeV negative hadron beam: 96.8% π^-, 2.4% K^-, 0.8% $\bar{p}$
- beam particle identification by Cherenkov detectors
- 4 mm Ni target disk (≈ 25% X_0)
- Measure scattered π^- and produced photons (number depends on final state)
- Select exclusive events at lowest momentum transfers
- Small scattering angles require high resolution
  - spatial resolution of tracking ≈ 10 μm
  - angular resolution of ECAL ≈ 30 μrad

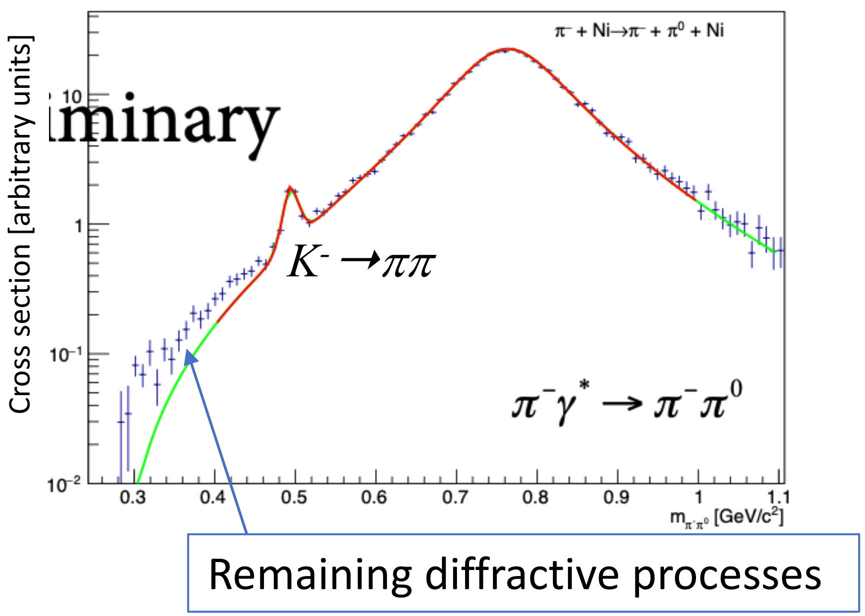
Intermediate COMPASS Results

Theory: use χPT and dispersion theory to accommodate for FSI

M. Hoferichter, B. Kubis, D. Sakkas, PRD 86, 116009 ('12)

- allows to address higher masses
- extract $F_{3\pi}$ and radiative width of ρ meson (only 2 free parameters)
- possibly access radiative width of excited ρ mesons
- Experimental issues: subtraction of diffractive production, luminosity



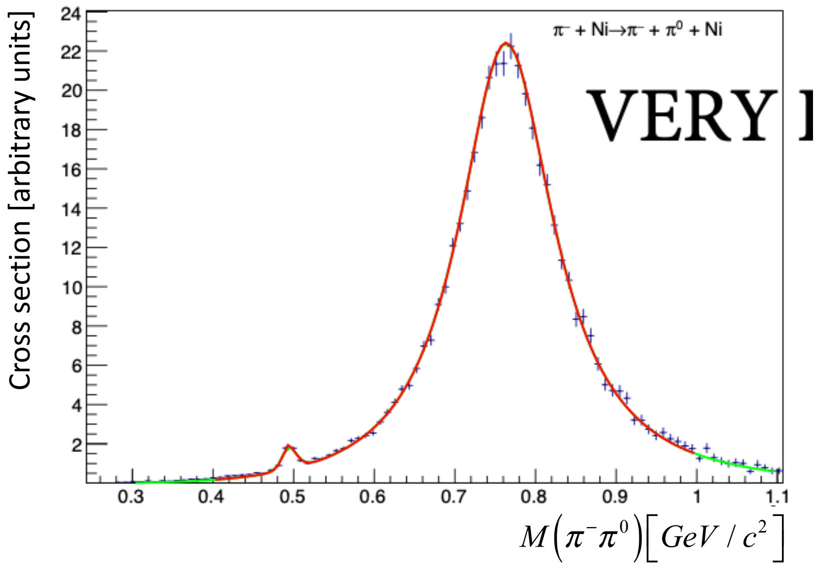


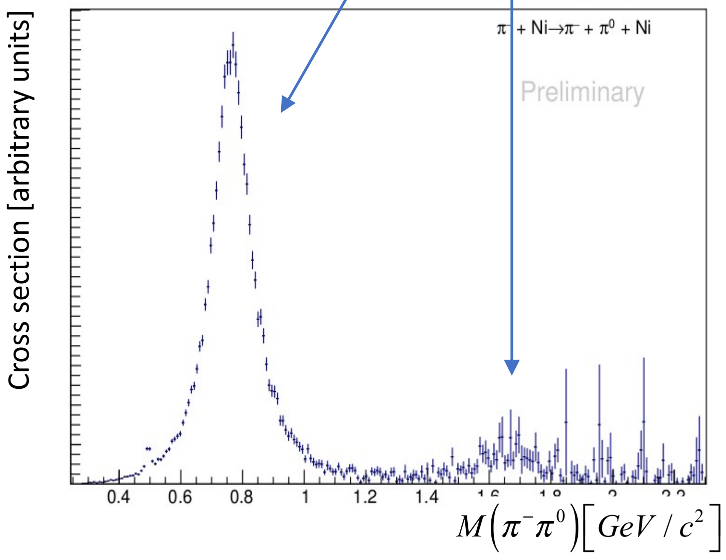
Experimental Challenges

- Measure absolute cross sections
  - Target thickness (geometry)
  - Beam flux including PID
    - Known ratio of π/K in beam (prervious measurements at CERN)
    - Count kaons
    - $K^- \to \pi^- \pi^0$ (20,7%) , $K^- \to \pi^- \pi^+ \pi^-$ (5.6%)
    - Consider backgrounds for these decays
      - $K^- \to \pi^0 e^- \bar{\nu}_e$ (5%) identification of electrons not unambiguous owing to Bremsstrahlung before detection
      - $K^- \to \pi^0 \mu^- \bar{\nu}_\mu$ (3.4%)
    - In principle: all decays can be used – X-check of normalization
    - In fact: requires excellent understanding of efficiencies and cross talk
- Understand underlying strong interaction background

Reward: Enlarged Physics Program

- Radiative width of excited mesons $\rho(770)$, $\rho_3(1670)$

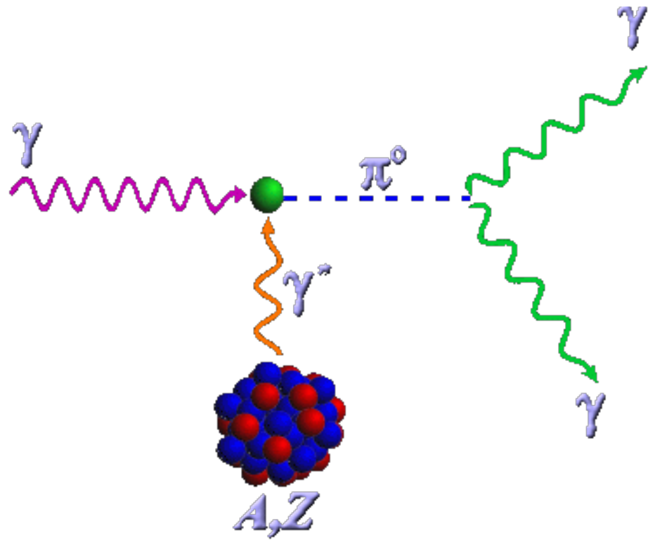




- Perform angular analysis for quantum numbers and yield (radiative width)

π^0 lifetime

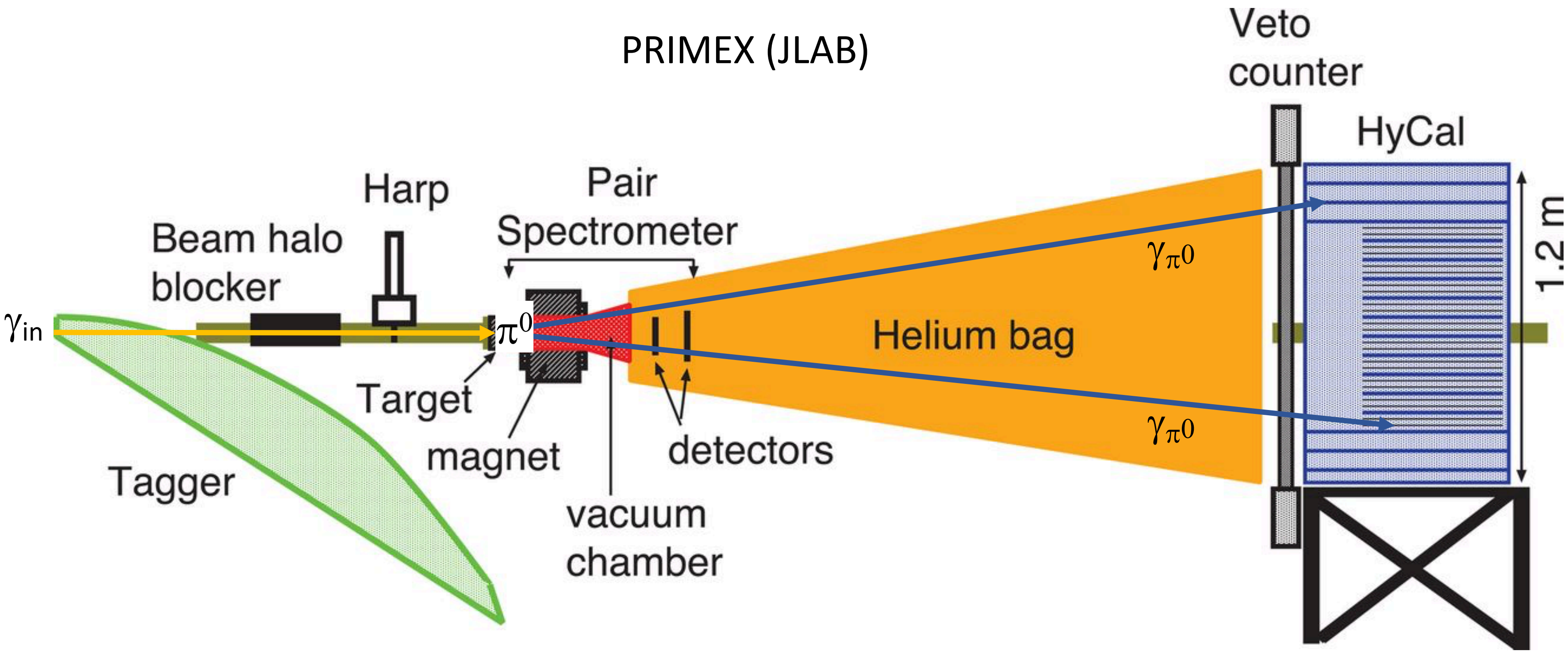
- Use Primakoff reaction to produce π^0
- Coherent photoproduction
- Measure $\gamma\gamma$ width through differential production cross section



Theory: $\Gamma\left(\pi^0 \to \gamma\gamma\right) = \frac{m_{\pi^0}^3 N_c^2 \alpha^2}{576\pi^3 F_{\pi^0}^2} = 7.750 \pm 0.016\ eV$

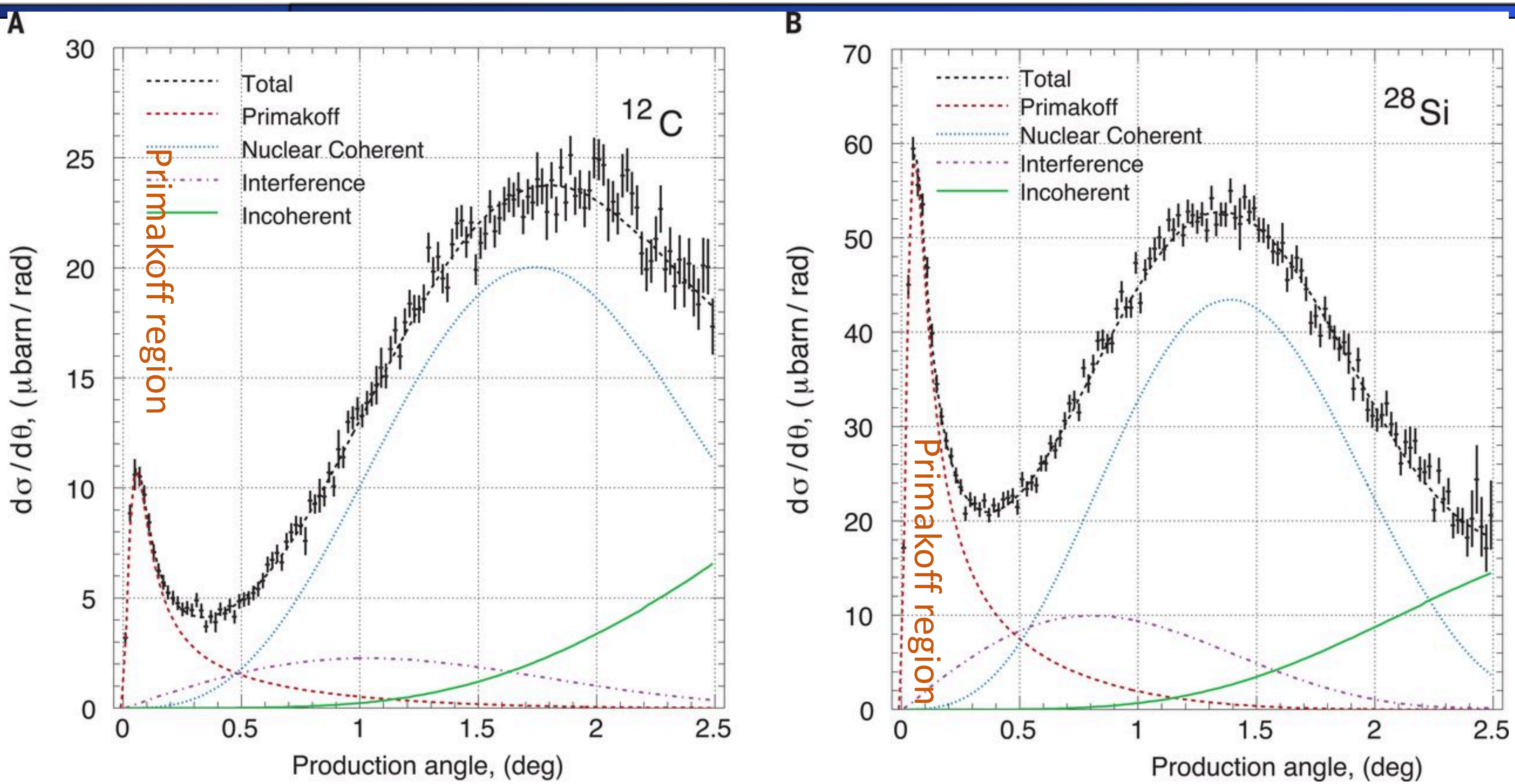
PrimEx @ JLAB

- Use real photons as beam



$$\gamma\gamma^* \to \pi^0 \to \gamma\gamma$$

Differential cross section for π^0 production



I. Larin et al. Science 2020;368:506-509

$$\Gamma\left(\pi^0 \to \gamma\gamma\right) = 7.798 \pm 0.056(stat) \pm 0.109(sys)\ eV$$

- World average: $\Gamma\left(\pi^0 \to \gamma\gamma\right) = 7.802 \pm 0.052(stat) \pm 0.105(sys)\ eV$

χPT Physics with Kaons

- Polarizability
  - $K\gamma^* \rightarrow K\gamma$
- Kπ scattering
  - $K^-\gamma^* \rightarrow K^-\pi^+\pi^-, K^-\pi^0\pi^0$
  - $K^-\pi^* \rightarrow K^-\pi$

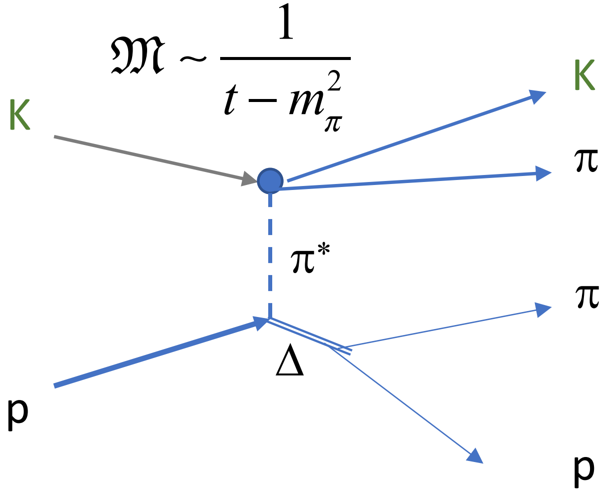
Goals:

1. Determine Kπ scattering length
2. Access Kπ resonances and their radiative width
3. Chiral anomaly for kaons - SU(3) symmetry breaking (dispersive analysis prepared)
4. Vector formfactor for kaon $K^- \rightarrow \ell \nu_\ell \gamma$ (analogon to $\pi^0 \rightarrow \gamma\gamma$)
5. Corrections larger than for ππ („large“ strange quark mass)

Special case: $\kappa - K_0(700)$

Kπ S-wave

- perform scattering experiment: use „pion cloud“ as virtual π target
  - π exchange dominates at pion pole (in scattering experiments: t < 0)
    - extrapolate to pion pole
    - enhance π exchange through Δ production (recoil particle detection)
    - examples: SLAC experiments, K_L beam at JLAB (planned)
  - theoretically complex



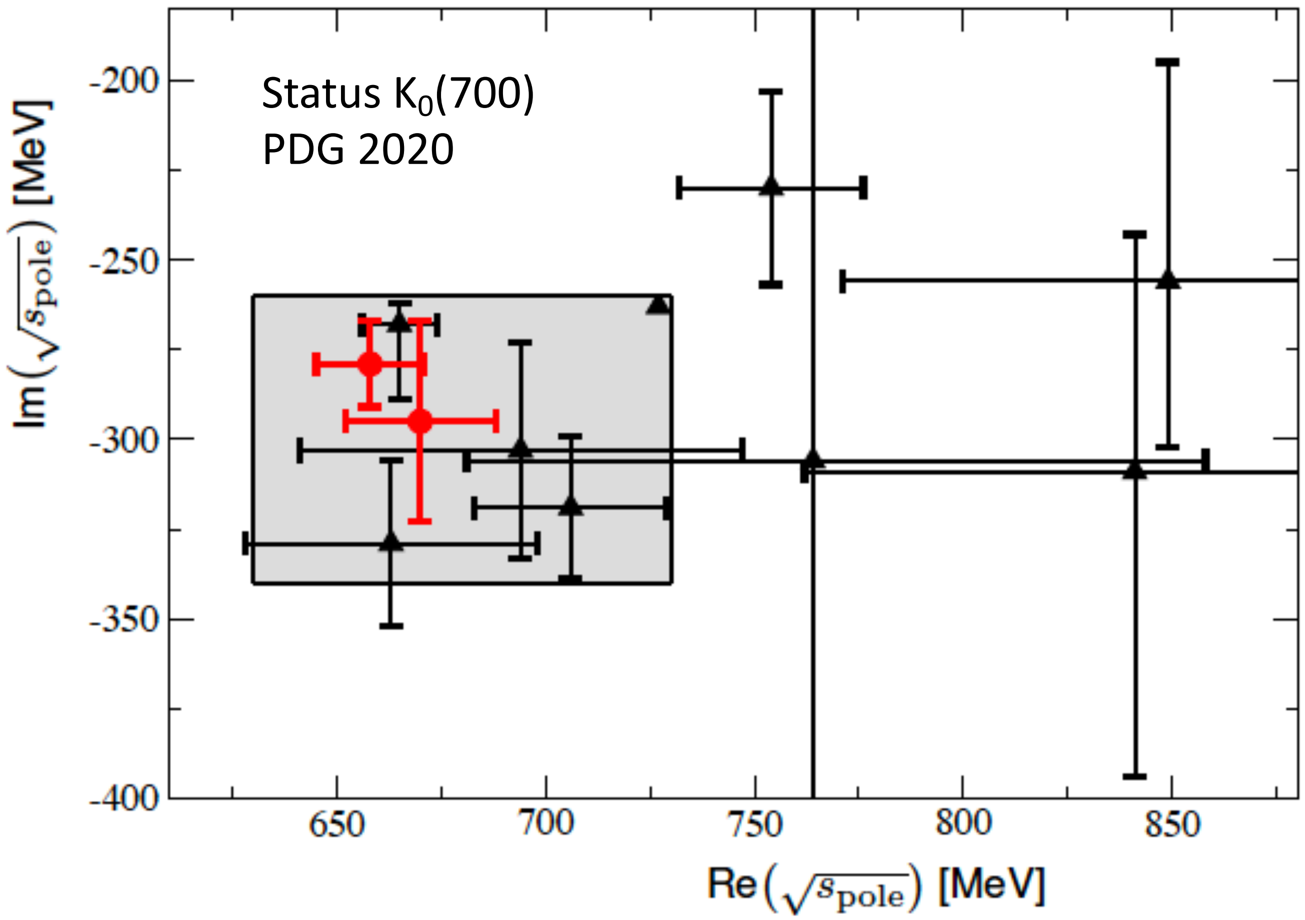
- extract Kπ S-wave from decays (Watson theorem)
  - weak: $D \to K\pi\pi \quad \tau \to K\pi\nu_\tau$
  - strong: $K^* \to K\pi\pi \quad J/\psi \to K^*(892)K\pi$

Model Free Analysis of Hadronic Decays

- Example: CLEO

$D \rightarrow K\pi\pi$

$\tau \rightarrow K\pi\nu_\tau$



Existing analyses suffer from unrecognized ambiguities (zero modes)

- $\tau \rightarrow K\pi\nu_\tau$ diffult to perform PWA (missing ν_τ) - new technique developed
  analyze: $\tau^+\tau^- \rightarrow K\pi\nu_\tau + \pi\pi\pi$

Example Hadronic Decay (Kππ) System

- Analyse Kπ subsystem in Kππ final states
- Preliminary: Kπ P-Wave
- To come: Kπ S-Wave

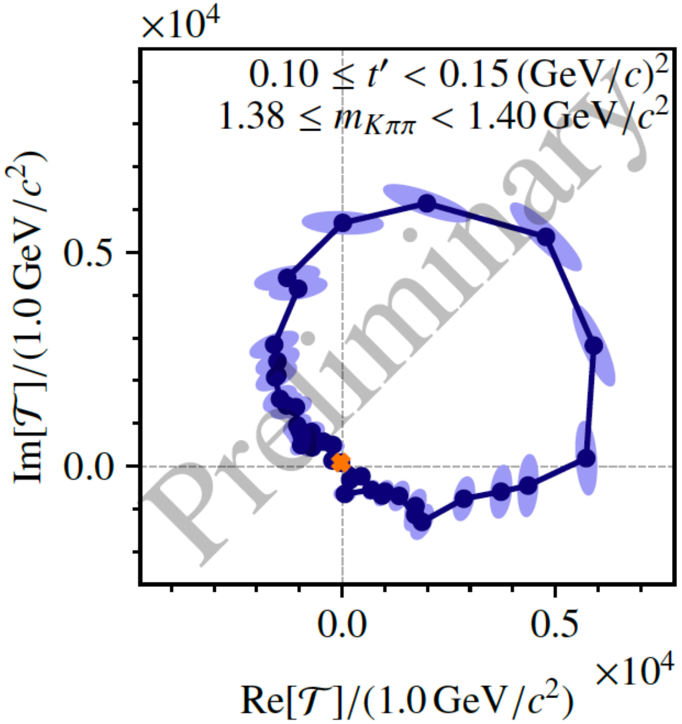
Freed of ambiguities !!

Kπ S-Wave :

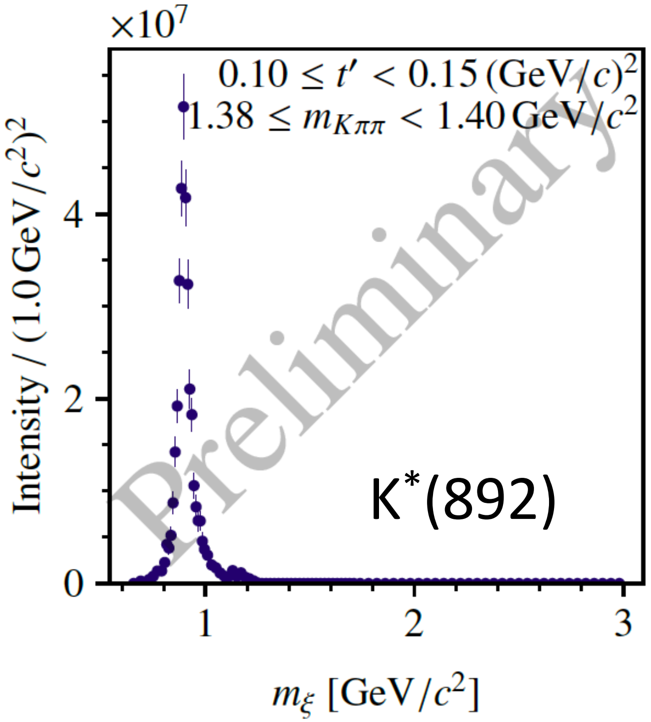
from threshold into

resonance region

Real/Imaginary part of amplitude for Kπ in relative P-wave

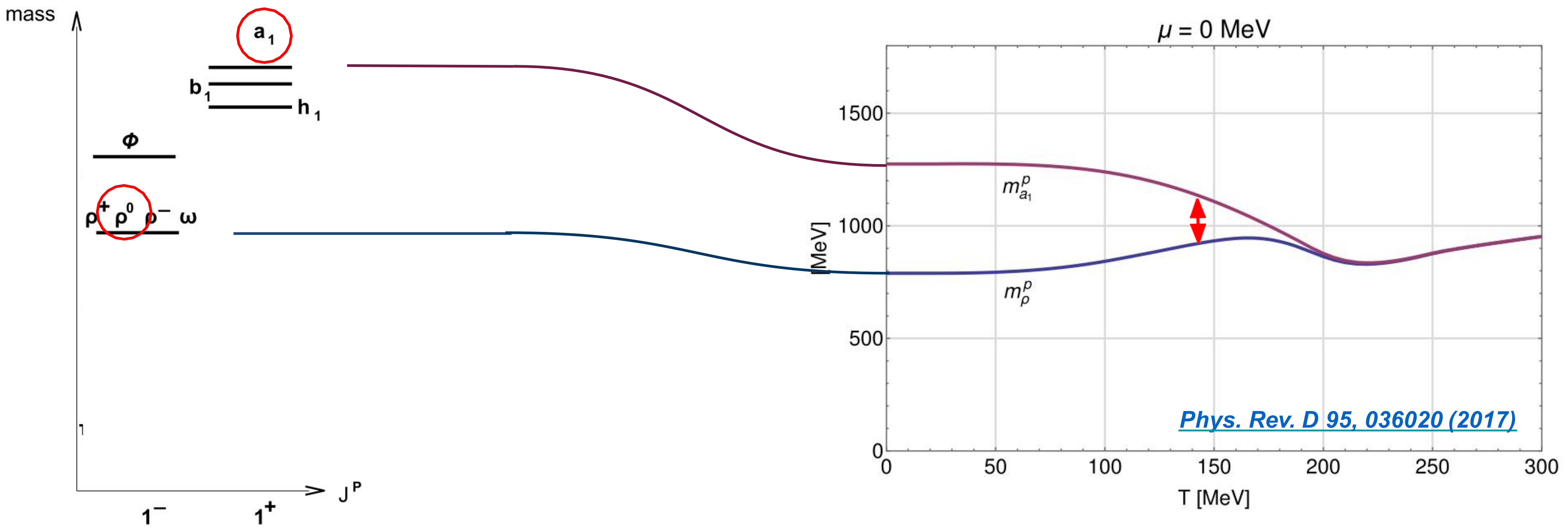


Intensity in P-wave



Chiral Symmetry Restauration (parity doubling)

Nucleon mass: 1% from quark mass (Higgs mechanism) → 99% from the strong interaction (QCD)



Spontaneously broken: chiral symmetry

- Hadrons with different parity do **not have same mass**

Unique test of fundamental QCD property:

- Change order parameter → change temperature
- Symmetry restoration at high temperatures
  - Measure hadron properties (spectral functions)

Conclusion

- Low energy meson dynamics described by χPT
- Experiments address key observables (also sensitive to quark masses)
- Precision experiments allow to determine low energy constants of effective theory → universal description of QCD at low energies
- Extension into strangeness sector ongoing (see AMBER proposal)
- PWA important tool: extract S-wave scattering and single out Primakoff processes from high backgrounds
- Important test: Parity doubling in hot medium with restored chiral symmetry

Kπ elastic scatttering

- LASS data and projected JLAB data
- Measure t-dependeence and extrapolate to $t = m_\pi^2$