Transverse-momentum-weighted transverse-spin asymmetries at COMPASS

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University and INFN of Trieste

On behalf of the COMPASS Collaboration

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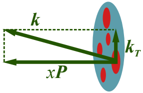
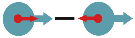
Outline

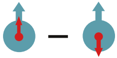
Hadron structure: Transverse momentum dependent PDFs



Number density.



Helicity.

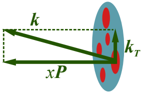


Transversity.

- Parton distribution functions (PDFs)
  - Structure in longitudinal momentum space.
  - $f(x, Q^2)$, the dependence on Q^2 calculable.
- Transverse Momentum Dependent (TMD) PDFs:
  - If parton intrinsic $\boldsymbol{k}_\mathrm{T}$ is not integrated over,
  - "three-dimensional" objects $f(x, k_\mathrm{T}^2, Q^2)$.
  - Accessible in
    - semi-inclusive deep-inelastic scattering (SIDIS),
    - Drell–Yan dilepton production,
    - proton–proton collisions…

		Parent hadron polarization		
		Unpolarised	Longitudinal	Transverse
Parton polarisation	U	$f_1(x, k_\mathrm{T}^2)$ (number density)		$f_{1T}^\perp(x, k_\mathrm{T}^2)$ (Sivers)
	L		$g_1(x, k_\mathrm{T}^2)$ (helicity)	$g_{1T}(x, k_\mathrm{T}^2)$
	T	$h_1^\perp(x, k_\mathrm{T}^2)$ (Boer–Mulders)	$h_{1L}^\perp(x, k_\mathrm{T}^2)$	$h_1(x, k_\mathrm{T}^2)$ (transversity) $h_{1T}^\perp(x, k_\mathrm{T}^2)$ (pretzelosity)

Hadron structure: Transverse momentum dependent PDFs



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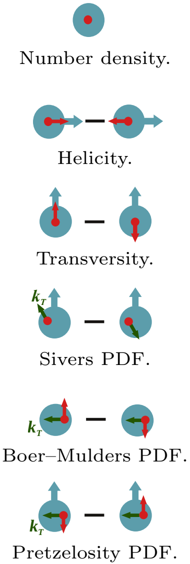
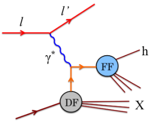
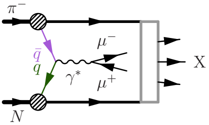


Hadron structure: SIDIS and Drell–Yan

COMPASS





SIDIS on transversely polarised nucleons

- Structure functions F:
  $F = \mathrm{PDF}_{q,\mathrm{p}} \otimes \mathrm{FF}_{q\to h}$.
- For example:
  - $F_{\mathrm{UU}}^{\cos\phi_h}$ and $F_{\mathrm{UU}}^{\cos 2\phi_h}$ linked to $h_{1,\mathrm{p}}^{\perp}$,
  - $F_{\mathrm{UT,T}}^{\sin(\phi_h-\phi_S)} = f_{1\mathrm{T,p}}^{\perp} \otimes D_1$.
  - $F_{\mathrm{UT}}^{\sin(\phi_h+\phi_S)} = h_{1,\mathrm{p}} \otimes H_1^{\perp}$,

Drell–Yan on transversely polarised nucleons

- Structure functions F:
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- For example:
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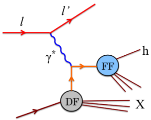
  - $F_{\mathrm{T}}^{\sin\phi_S} = f_{1,\pi} \otimes f_{1\mathrm{T,p}}^{\perp}$,
  - $F_{\mathrm{T}}^{\sin(2\phi-\phi_S)} = h_{1,\pi}^{\perp} \otimes h_{1,\mathrm{p}}$.

A sign change predicted for Sivers and Boer–Mulders functions:

$$f_{1\mathrm{T}}^{\perp q}\big|_{\mathrm{SIDIS}} = -f_{1\mathrm{T}}^{\perp q}\big|_{\mathrm{DY}}$$

$$h_{1}^{\perp q}\big|_{\mathrm{SIDIS}} = -h_{1}^{\perp q}\big|_{\mathrm{DY}}$$

[J. Collins, Phys.Lett. B536 (2002) 43]



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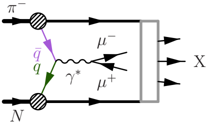

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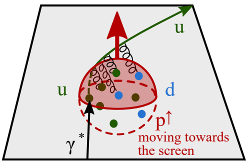

Sivers effect in SIDIS (as described by [M. Burkardt, Nucl.Phys. A735 (2004) 185].

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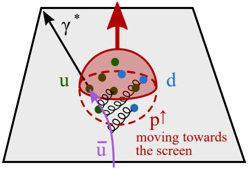
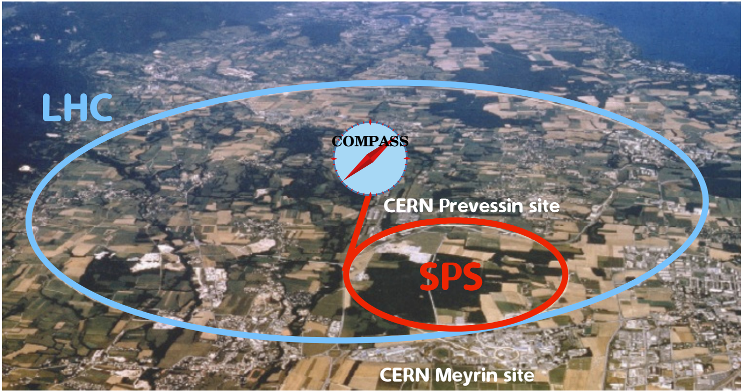


Sivers effect in Drell–Yan drawn in the same manner.

Outline

COMPASS experiment: Introduction

- COMPASS Collaboration: 24 institutions from 13 countries ($\approx$ 220 physicists).
- Experimental area: CERN Super Proton Synchrotron (SPS) North Area.
- Two-stage spectrometer, about 350 detector planes, μ identification.
- Multi-purpose apparatus with rich physics program since 2002 aimed at hadron structure and spectroscopy.



COMPASS experiment: SIDIS and Drell–Yan setups

Both programs

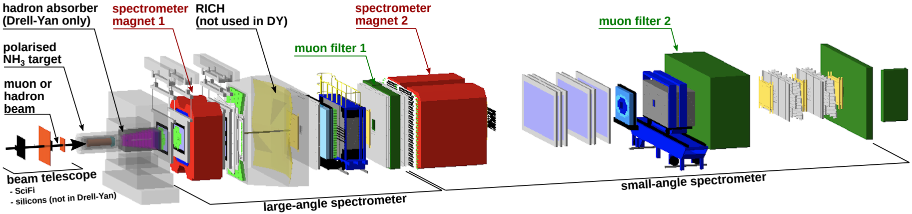
- Polarised p (NH_3) target polarisation 70–80 %, 2 or 3 oppositely-polarised cells.
- Two-stage spectrometer, about 350 detector planes, μ identification.

SIDIS with transversely-polarised target (2007, 2010)

- 160 GeV/c μ^+ beam (about 3.5×10^8 μ/spill of 10 s).
- Triggering on scattered μ.

Drell–Yan with transversely-polarised target (2015, 2018)

- 190 GeV/c π^- beam (about 10^9 π/spill of 10 s).
- Triggering on dimuons.



Outline

Weighted asymmetries: Motivation

- For example, SIDIS cross section as a function of $\Phi_{\mathrm{Siv}} = \phi_h - \phi_S$ is

$$\sigma(x, y, z, P_{h\mathrm{T}}, \phi_{\mathrm{Siv}}) = C(x, y, Q^2)\left[F_{\mathrm{UU,T}} + |\boldsymbol{S}_{\mathbf{T}}| F_{\mathrm{UT,T}}^{\sin\Phi_{\mathrm{Siv}}} \sin(\Phi_{\mathrm{Siv}})\right],$$

- Sivers asymmetry is the amplitude of the $\sin\Phi_{\mathrm{Siv}}$ modulation,

$$A_{\mathrm{UT,T}}^{\sin\Phi_{\mathrm{Siv}}}(x, z, P_{h\mathrm{T}}) = \frac{F_{\mathrm{UT,T}}^{\sin\Phi_{\mathrm{Siv}}}(x, z, P_{h\mathrm{T}})}{F_{\mathrm{UU,T}}(x, z, P_{h\mathrm{T}})} = \frac{\mathcal{C}\left[\frac{\boldsymbol{P}_{h\mathrm{T}}\cdot\boldsymbol{k}_{\mathrm{T}}}{P_{h\mathrm{T}}M} f_{1\mathrm{T}}^{\perp}(x, k_{\mathrm{T}}^2) D_1(z, p_{\perp}^2)\right]}{\mathcal{C}\left[f_1(x, k_{\mathrm{T}}^2) D_1(z, p_{\perp}^2)\right]}$$

- In TMD factorisation: F are flavour sums of convolutions of TMD PDFs and TMD FFs over intrinsic transverse momenta $\boldsymbol{k}_{\mathrm{T}}$ and $\boldsymbol{p}_{\perp}$.
- Full 3(4)D analysis: too demanding for the statistics yet.
- Integrating σ over $\boldsymbol{P}_{h\mathrm{T}}$:

$$\int \mathrm{d}^2\boldsymbol{P}_{h\mathrm{T}} F_{\mathrm{UU,T}} = \int \mathrm{d}^2\boldsymbol{P}_{h\mathrm{T}}\, x \sum_q e_q^2 \int \mathrm{d}^2\boldsymbol{p}_{\perp}\mathrm{d}^2\boldsymbol{k}_{\mathrm{T}}\, \delta(\boldsymbol{P}_{h\mathrm{T}} - \boldsymbol{p}_{\perp} - z\boldsymbol{k}_{\mathrm{T}}) f_1^q(x, k_{\mathrm{T}}^2) D_1^q(z, p_{\perp}^2)$$

$$= x \sum_q e_q^2 f_1^q(x) D_1^q(z) \qquad \text{(flavour sum of standard PDFs!)},$$

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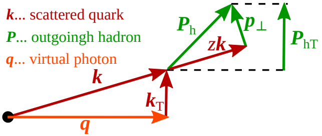

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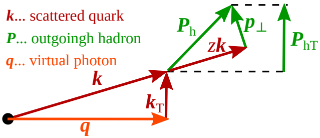

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Weighted asymmetries: Principle

- Another solution: weighting with powers of transverse momentum.
  ([A. Kotzinian, P. Mulders, Phys.Lett. B406 (1997) 373], [D. Boer, P. Mulders, Phys.Rev. D57 (1998) 5780])
- The integration of $F_{\mathrm{UT,T}}^{\sin\Phi_{\mathrm{Siv}}}$ over $\boldsymbol{P_{h\mathrm{T}}}$ with weight $P_{h\mathrm{T}}/(zM)$:

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$$= 2x\sum_q e_q^2 f_{1\mathrm{T}}^{\perp q(1)}D_1^q(z),$$

  where $f_{1\mathrm{T}}^{\perp q(1)}$ is the 1st k_{T}^2-moment of the Sivers function.
- In general, the n-th moments of a TMD PDF and FF are defined as

$$f^{(n)}(x) = \int \mathrm{d}^2\boldsymbol{k_{\mathrm{T}}}\left(\frac{k_{\mathrm{T}}^2}{2M^2}\right)^n f(x,k_{\mathrm{T}}^2) \qquad D^{(n)}(z) = \int \mathrm{d}^2\boldsymbol{p}_\perp\left(\frac{p_\perp^2}{2z^2M^2}\right)^n D(x,p_\perp^2)$$

- Utilising this, we define the $P_{h\mathrm{T}}/(zM)$-weighted Sivers asymmetry as

$$A_{\mathrm{UT,T}}^{\sin\Phi_{\mathrm{Siv}}\frac{P_{h\mathrm{T}}}{zM}} = \frac{\int \mathrm{d}^2\boldsymbol{P_{h\mathrm{T}}}\frac{P_{h\mathrm{T}}}{zM}F_{\mathrm{UT,T}}^{\sin\Phi_{\mathrm{Siv}}}}{\int \mathrm{d}^2\boldsymbol{P_{h\mathrm{T}}}F_{\mathrm{UU,T}}} = 2\frac{\sum_q e_q^2 f_{1\mathrm{T}}^{\perp q(1)}D_1^q(z)}{\sum_q e_q^2 f_1^q(x)D_1^q(z)}.$$

- Every structure function F in SIDIS has a weight $\propto (P_{h\mathrm{T}})^n$ that solves the convolution.
- Similar convolution exists in Drell–Yan (over the q and q̄ intrinsic transverse momenta).

$$\mathcal{C}[wf_\pi f_{\mathrm{p}}] = \frac{1}{N_{\mathrm{c}}}\sum_q e_q^2\int \mathrm{d}^2\boldsymbol{k_{\pi\mathrm{T}}}\mathrm{d}^2\boldsymbol{k_{\mathrm{pT}}}\delta(\boldsymbol{q_{\mathrm{T}}}-\boldsymbol{k_{\pi\mathrm{T}}}-\boldsymbol{k_{\mathrm{pT}}})\,w\left[f_\pi^{\bar{q}}(x_\pi,k_{\pi\mathrm{T}}^2)\,f_{\mathrm{p}}^q(x_{\mathrm{p}},k_{\mathrm{pT}}^2)+(q\leftrightarrow\bar{q})\right].$$

- Weighting with the γ^* momentum $(q_{\mathrm{T}})^n$ can be used to solve the convolutions.

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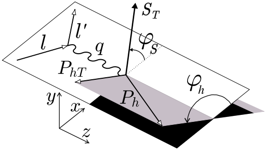
$$\mathcal{C}[wf_\pi f_\mathrm{p}] = \frac{1}{N_\mathrm{c}}\sum_q e_q^2 \int \mathrm{d}^2\boldsymbol{k_{\pi\mathrm{T}}}\mathrm{d}^2\boldsymbol{k_{\mathrm{pT}}}\delta(\boldsymbol{q_\mathrm{T}} - \boldsymbol{k_{\pi\mathrm{T}}} - \boldsymbol{k_{\mathrm{pT}}})\,w\left[f_\pi^{\bar q}(x_\pi,k_{\pi\mathrm{T}}^2)\,f_\mathrm{p}^q(x_\mathrm{p},k_{\mathrm{pT}}^2) + (q\leftrightarrow\bar q)\right].$$

- Weighting with the γ^* momentum $(q_\mathrm{T})^n$ can be used to solve the convolutions.

Outline

Cross section for transversely polarised target at leading twist
[A. Bacchetta *et al.*, JHEP 0702 (2007) 093]:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}\phi_S\mathrm{d}\phi_h\mathrm{d}P_{hT}^2} = \frac{\alpha^2}{xyQ^2}\left(1+\frac{\gamma^2}{2x}\right)\Big\{\frac{2-2y+y^2}{2}F_{UU,T} + (2-y)\sqrt{1-y}\,\cos\phi_h F_{UU}^{\cos\phi_h} + (1-y)\cos(2\phi_h)F_{UU}^{\cos 2\phi_h} + |\boldsymbol{S}_{\mathbf{T}}|\Big[\frac{2-2y+y^2}{2}\sin(\phi_h-\phi_S)F_{UT,T}^{\sin(\phi_h-\phi_S)} + (1-y)\sin(\phi_h+\phi_S)F_{UT}^{\sin(\phi_h+\phi_S)} + (1-y)\sin(3\phi_h-\phi_S)F_{UT}^{\sin(3\phi_h-\phi_S)}\Big]\Big\}.$$



SIDIS process in the $\gamma^* N$ frame.

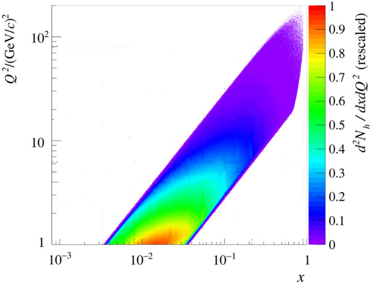
Weighted asymmetries

- Weighted Sivers asymmetry – published [COMPASS, Nucl.Phys. B940 (2019) 34]

$$A_{UT,T}^{\sin(\phi_h-\phi_S)\frac{P_{hT}}{zM}} = \frac{\int \mathrm{d}^2\boldsymbol{P}_{\boldsymbol{h}\mathbf{T}}\frac{P_{hT}}{zM}F_{UT,T}^{\sin(\phi_h-\phi_S)}}{\int \mathrm{d}^2\boldsymbol{P}_{\boldsymbol{h}\mathbf{T}}F_{UU,T}} = 2\frac{\sum_q e_q^2 f_{1T}^{\perp q(1)}D_1^q(z)}{\sum_q e_q^2 f_1^q(x)D_1^q(z)}.$$

- The weighting can be used also for other structure functions:
  - The other two UT-type modulations.
  - The UU-type modulations (polarisation-independent).
  - Also the LU- and LT-type (note that the COMPASS μ beam is longitudinally polarised).

Weighted TSAs in SIDIS: Data analysis

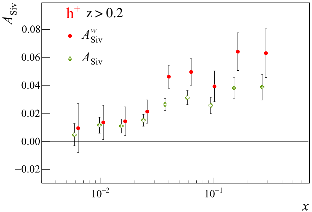


The kinematic range in x and Q^2.

- Data collected in 2010 in $\mu^+ + \mathrm{p}^\uparrow \rightarrow \mu^+ + h + X$.
- 160 GeV/c μ^+ beam and NH_3 target.
- Event selection like in standard Sivers asymmetry analysis [COMPASS, Phys.Lett. B717 (2012) 383].
- In particular, the same kinematic cuts were applied:
  - $Q^2 > 1\ (\mathrm{GeV}/c)^2$,
  - $0.1 < y < 0.9$,
  - $W > 5\ \mathrm{GeV}/c^2$,
  - $P_{h\mathrm{T}} > 0.1\ \mathrm{GeV}/c$,
  - $z > 0.2$, the region $0.1 < z < 0.2$ analysed separately.

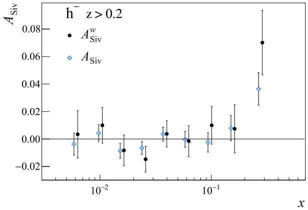
[COMPASS, Nucl.Phys. B940 (2019) 34]

The $P_{hT}/(zM)$-weighted Sivers asymmetries (here A^{w}_{Siv}) and the standard ones (here A_{Siv})



For h^+ the u quarks are dominant,

$$A^{w,h^+}_{\mathrm{Siv}}(x) \approx 2\frac{f_{1T}^{\perp u(1)}(x)}{f_1^{u}(x)}.$$



For h^- there is an approximate cancellation,

$$f_{1T}^{\perp u(1)} D_{\mathrm{unfav.}} - f_{1T}^{\perp d(1)} D_{\mathrm{fav.}} \approx 0.$$

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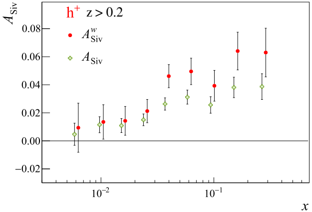

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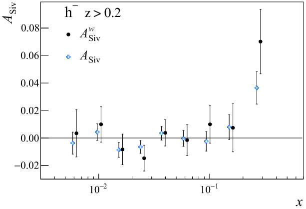

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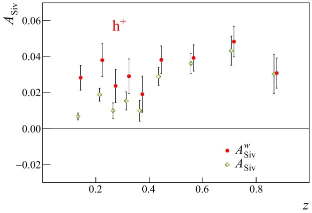
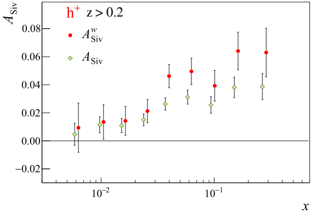


For h^+ A^{w}_{Siv} is almost constant in z.

Weighted TSAs in SIDIS: Results in x

[COMPASS, Nucl.Phys. B940 (2019) 34]

The $P_{hT}/(zM)$-weighted Sivers asymmetries (here A^w_{Siv}) and the standard ones (here A_{Siv})



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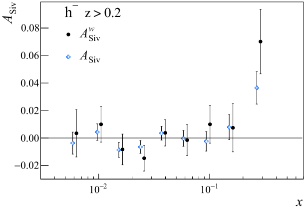

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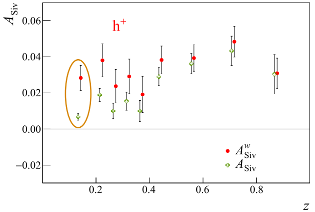

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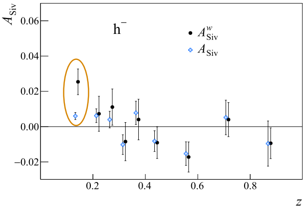


At small z, $D_{\mathrm{unfav.}} = D_{\mathrm{fav.}}$, so $A^{w,h^-}_{\mathrm{Siv}} = A^{w,h^+}_{\mathrm{Siv}}$.

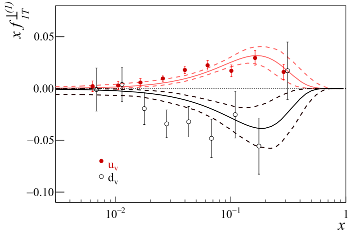
Weighted TSAs in SIDIS: Sivers function extraction

[COMPASS, Nucl.Phys. B940 (2019) 34]

- The Sivers function can be extracted from the weighted asymmetries point-by-point without any assumption on its shape neither in x nor in k_{T}^2.
- Assuming zero Sivers function of the sea quarks,

$$A_{\mathrm{UT,T}}^{\sin(\phi_h-\phi_S)\frac{P_{h\mathrm{T}}}{zM}}(x,Q^2) = 2\,\frac{\frac{4}{9}\,f_{1\mathrm{T}}^{\perp(1)\mathrm{u}}(x,Q^2)\,\tilde{D}_1^{\mathrm{u}}(Q^2) + \frac{1}{9}\,f_{1\mathrm{T}}^{\perp(1)\mathrm{d}}(x,Q^2)\,\tilde{D}_1^{\mathrm{d}}(Q^2)}{\sum_q e_q^2\,f_1^q(x,Q^2)\tilde{D}_1^q(Q^2)},$$

- where $\tilde{D}_1^q(Q^2) = \int_{z_{\min}}^{z_{\max}} \mathrm{d}z D_1^q(z,Q^2)$.
- $f_1^q(x,Q^2)$ taken from CTEQ5D and $D_1^q(z,Q^2)$ from DSS 2007.



Our results compared with [Anselmino *et al.*, Phys.Rev. D86 (2012) 014028].

- To learn about sea quarks and to improve on the d-quark, deuteron data are needed.
- COMPASS will take data with polarized deuteron target in 2021,
  - The statistics and acceptance at least as good as in 2010 proton data.
  - Both standard and weighted asymmetries will be measured.

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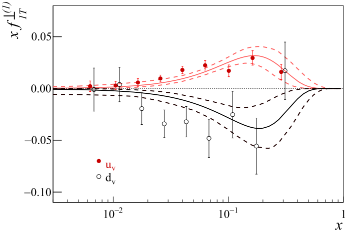

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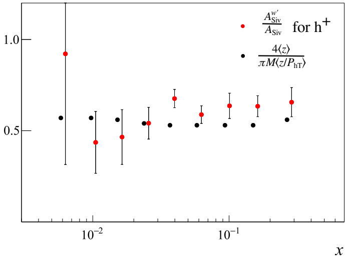
[COMPASS, Nucl.Phys. B940 (2019) 34]

- The $P_{h\mathrm{T}}/M$-weighted Sivers asymmetry is

$$A_{\mathrm{Siv}}^{w'} = A_{\mathrm{UT,T}}^{\sin(\phi_h-\phi_S)\frac{P_{h\mathrm{T}}}{M}} = 2\frac{\sum_q e_q^2 x f_{1\mathrm{T}}^{\perp(1)q}(x) z D_1^q(z)}{\sum_q e_q^2 x f_1^q(x) D_1^q(z)}.$$

- The standard asymmetry assuming Gaussian dependence of f and D on $\boldsymbol{k}_{\mathbf{T}}$ and $\boldsymbol{p}_{\perp}$ is

$$A_{\mathrm{Siv}} = A_{\mathrm{UT,T}}^{\sin(\phi_h-\phi_S)} = \frac{\pi M}{2\langle P_{h\mathrm{T}}\rangle}\frac{\sum_q e_q^2 x f_{1\mathrm{T}}^{\perp(1)q}(x) z D_1^q(z)}{\sum_q e_q^2 x f_1^q(x) D_1^q(z)} = \frac{\pi M}{4\langle P_{h\mathrm{T}}\rangle} A_{\mathrm{Siv}}^{w'}$$



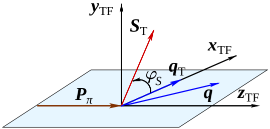
The ratio of the $P_{h\mathrm{T}}/M$-weighted Sivers asymmetry to the standard one.

Outline

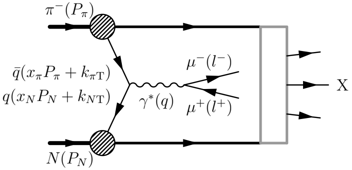
Weighted TSAs in Drell–Yan: Introduction

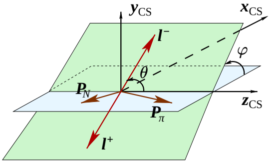
Cross section for transversely polarised target at leading twist
[S. Arnold, A. Metz, M. Schlegel, Phys.Rev. D79 (2009) 034005]:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_\pi \mathrm{d}x_N \mathrm{d}q_\mathrm{T}^2 \mathrm{d}\phi_S \mathrm{d}\cos\theta \mathrm{d}\phi} = C_0 \Big\{ (1+\cos^2\theta) F_\mathrm{U}^1 + \sin^2\theta \, \cos 2\phi \, F_\mathrm{U}^{\cos 2\phi} + |\boldsymbol{S}_\mathrm{T}| \Big[(1+\cos^2\theta)\sin\phi_S \, F_\mathrm{T}^{\sin\phi_S} + \sin^2\theta \, \sin(2\phi+\phi_S) F_\mathrm{T}^{\sin(2\phi+\phi_S)} + \sin^2\theta \, \sin(2\phi-\phi_S) F_\mathrm{T}^{\sin(2\phi-\phi_S)} \Big] \Big\},$$



Target frame.





Collins–Soper frame.

- The structure functions F are convolutions over the intrinsic transverse momenta of q and $\bar{\mathrm{q}}$,

$$\mathcal{C}[w f_\pi f_\mathrm{p}] = \frac{1}{N_\mathrm{c}} \sum_q e_q^2 \int \mathrm{d}^2\boldsymbol{k_{\pi\mathrm{T}}} \mathrm{d}^2\boldsymbol{k_{\mathrm{pT}}} \delta(\boldsymbol{q_\mathrm{T}} - \boldsymbol{k_{\pi\mathrm{T}}} - \boldsymbol{k_{\mathrm{pT}}}) \, w \big[f_\pi^{\bar{q}}(x_\pi, k_{\pi\mathrm{T}}^2) \, f_\mathrm{p}^q(x_\mathrm{p}, k_{\mathrm{pT}}^2) + (q \leftrightarrow \bar{q}) \big].$$

- The weighting in Drell–Yan is done with powers of the γ^* transverse momentum q_T.

- Weighted Sivers asymmetry

$$A_{\mathrm{T}}^{\sin\phi_S \frac{q_{\mathrm{T}}}{M_{\mathrm{p}}}}(x_\pi, x_N) = -2\frac{\sum_q e_q^2\left[f_{1,\pi}^{\bar{q}}(x_\pi)\, f_{1\mathrm{T,p}}^{\perp(1)q}(x_N) + (q \leftrightarrow \bar{q})\right]}{\sum_q e_q^2\left[f_1^{\bar{q}}(x_\pi)\, f_1^q(x_N) + (q \leftrightarrow \bar{q})\right]} \approx -2\frac{f_{1\mathrm{T,p}}^{\perp(1)\mathrm{u}}(x_N)}{f_{1,\mathrm{p}}^{\mathrm{u}}(x_N)}.$$

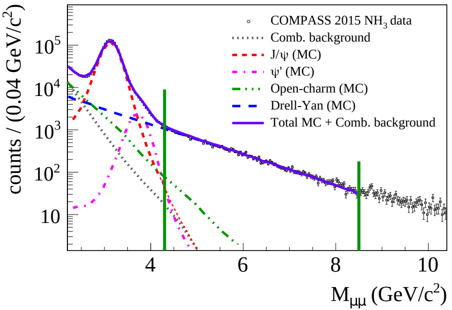
- Weighted asymmetry induced by proton transversity and pion Boer–Mulders function

$$A_{\mathrm{T}}^{\sin(2\phi-\phi_S)\frac{q_{\mathrm{T}}}{M_\pi}}(x_\pi, x_N) = -2\frac{\sum_q e_q^2\left[h_{1,\pi}^{\perp(1)\bar{q}}(x_\pi)\, h_{1,\mathrm{p}}^q(x_N) + (q \leftrightarrow \bar{q})\right]}{\sum_q e_q^2\left[f_1^{\bar{q}}(x_\pi)\, f_1^q(x_N) + (q \leftrightarrow \bar{q})\right]}$$

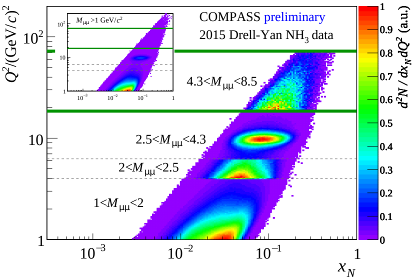
- Weighted asymmetry induced by proton pretzelosity and pion Boer–Mulders function

$$A_{\mathrm{T}}^{\sin(2\phi+\phi_S)\frac{q_{\mathrm{T}}^3}{2M_\pi M_{\mathrm{p}}^2}}(x_\pi, x_N) = -2\frac{\sum_q e_q^2\left[h_{1,\pi}^{\perp(1)\bar{q}}(x_\pi)\, h_{1\mathrm{T,p}}^{\perp(2)q}(x_N) + (q \leftrightarrow \bar{q})\right]}{\sum_q e_q^2\left[f_1^{\bar{q}}(x_\pi)\, f_1^q(x_N) + (q \leftrightarrow \bar{q})\right]}$$

- The weighting can be used also for $F_{\mathrm{U}}^{\cos 2\phi}$.



2015 data and reconstructed MC.

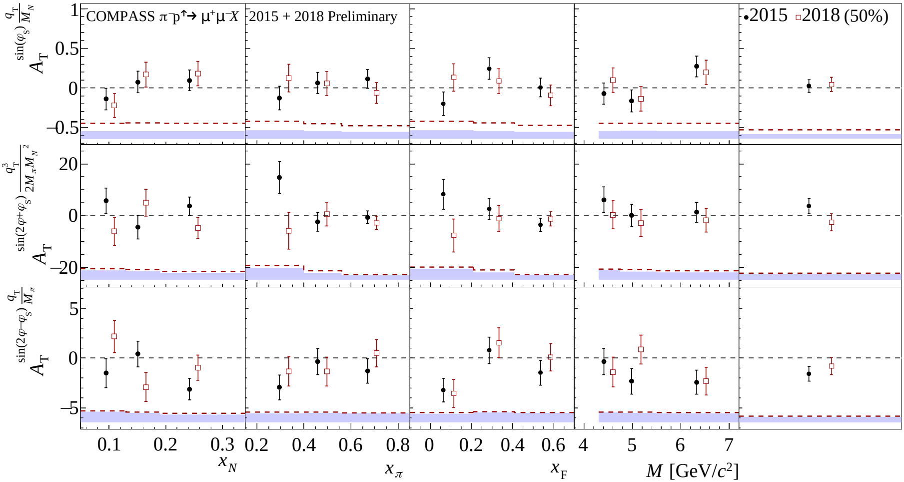


Kinematic coverage in x_N and Q^2

Event selection is almost the same for weighted and "standard" TSAs ([COMPASS, Phys.Rev.Lett. 119(11), 112002 (2017)], and a talk by R. Longo this afternoon).

- $\mu^+\mu^-$ pairs (μ candidates: $X/X_0 > 30$).
- Vertex reconstructed in the target.
- $M_{\mu\mu} \in [4.3, 8.5]$ GeV/c^2.
- But no cut on q_T.

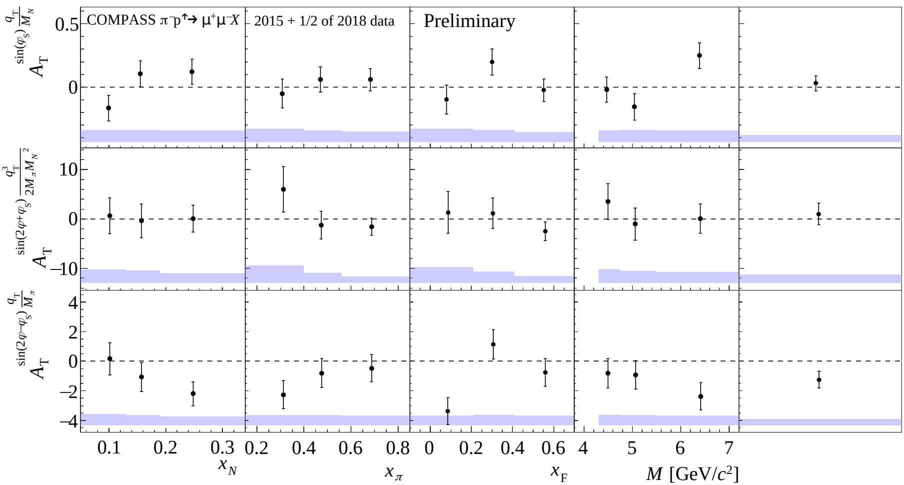
[R. Longo (COMPASS), PoS DIS2019 (2019) 186]



Comparison of the results from 2015 data and about 50 % of 2018 data.

Weighted TSAs in Drell–Yan: Results

[R. Longo (COMPASS), PoS DIS2019 (2019) 186]



The results combining the 2015 data and about 50 % of 2018 data.

Weighted TSAs in Drell–Yan: Sivers function sign change

COMPASS

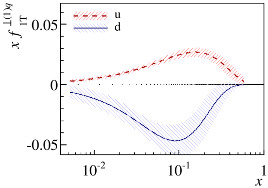
[R. Longo (COMPASS), PoS DIS2019 (2019) 186]

- How can we compare the Sivers function in SIDIS and Drell–Yan?
- A straightforward way utilising the weighted asymmetries:

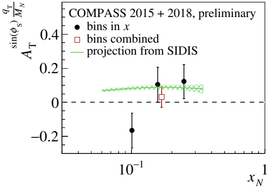
$$\text{DY}: \quad A_{\mathrm{T}}^{\sin\phi_S \frac{q_{\mathrm{T}}}{M_{\mathrm{p}}}}(x_N) \approx -2\frac{\left[f_{1\mathrm{T}}^{\perp(1)\mathrm{u}}(x_N)\right]_{\mathrm{DY}}}{f_{1,\mathrm{p}}^{\mathrm{u}}(x_N)} \overset{\text{sign change}}{=} 2\frac{f_{1\mathrm{T}}^{\perp(1)\mathrm{u}}(x_N)}{f_{1,\mathrm{p}}^{\mathrm{u}}(x_N)}$$

$$\text{SIDIS}: \quad A_{\mathrm{UT,T}}^{\sin(\phi_h-\phi_S)\frac{P_{h\mathrm{T}}}{zM}}(x) \approx 2\frac{\frac{4}{9}f_{1\mathrm{T}}^{\perp(1)\mathrm{u}}(x,Q^2)\,\tilde{D}_1^{\mathrm{u}}(Q^2) + \frac{1}{9}f_{1\mathrm{T}}^{\perp(1)\mathrm{d}}(x,Q^2)\,\tilde{D}_1^{\mathrm{d}}(Q^2)}{\sum_q e_q^2\, f_1^q(x,Q^2)\tilde{D}_1^q(Q^2)}.$$

- where $\tilde{D}_1^q(Q^2) = \int_{z_{\min}}^{z_{\max}} \mathrm{d}z\, D_1^q(z,Q^2)$ is an integrated FF.



The Sivers functions extracted from $A_{\mathrm{UT,T}}^{\sin(\phi_h-\phi_S)\frac{P_{h\mathrm{T}}}{zM}}(x)$ in SIDIS assuming $x f_{1\mathrm{T}}^{\perp(1)q}(x) = a_q\, x^{b_q}\,(1-x)^{c_q}$.



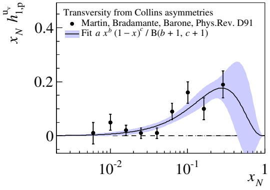
The comparison of the expectation from SIDIS with the measurement in Drell–Yan. Only statistical errors are shown.

The interpretation of the $\sin(2\phi - \phi_S)$ weighted asymmetry:

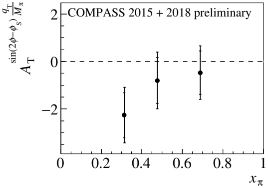
$$A_{\mathrm{T}}^{\sin(2\phi-\phi_S)\frac{q_{\mathrm{T}}}{M_\pi}}(x_\pi, x_N) = -2\frac{\sum_q e_q^2 \left[h_{1,\pi}^{\perp(1)\bar{q}}(x_\pi)\, h_{1,\mathrm{p}}^{q}(x_N) + (q \leftrightarrow \bar{q})\right]}{\sum_q e_q^2 \left[f_{1,\pi}^{\bar{q}}(x_\pi)\, f_{1,\mathrm{p}}^{q}(x_N) + (q \leftrightarrow \bar{q})\right]}$$

$$\approx -2\frac{e_{\mathrm{u}}^2\, h_{1,\pi}^{\perp(1)\bar{\mathrm{u}}}(x_\pi)\, h_{1,\mathrm{p}}^{\mathrm{u}}(x_N)}{\sum_{q=\mathrm{u,d,s}} e_q^2 \left[f_{1,\pi}^{\bar{q}}(x_\pi)\, f_{1,\mathrm{p}}^{q}(x_N) + (q \leftrightarrow \bar{q})\right]}$$

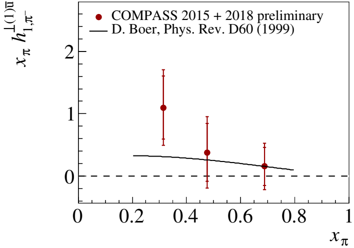
- $f_{1,\mathrm{p}}^{q}(x_\pi, Q^2)$ from CTEQ5D, $f_{1,\mathrm{p}}^{q}(x_N, Q^2)$ from GRV-PI.
- $h_{1,\mathrm{p}}^{\mathrm{u}}(x)$ from the point-by-point extraction [A. Martin *et al.*, Phys.Rev. D91 (2015) 014034].
- We obtained the 1st k_{T}^2-moment of valence Boer–Mulders function of the pion.
- We compared it with Boer's result based on older unpolarised Drell–Yan experiments. [D. Boer, Phys.Rev. D60 (1999) 014012]



The transversity, interpolated by a simple fit.



The weighted asymmetry in bins of x_π.



The first k_{T}^2-moment of the valence Boer–Mulders function of the pion.

[R. Longo (COMPASS), PoS DIS2019 (2019) 186]

Outline

Conclusion

- The transverse momentum weighted asymmetries in general:
  - A way to overcome the convolution over intrinsic $\boldsymbol{k}_{\mathrm{T}}$ without any ansatz.
  - Direct access to the relevant k_{T}^2-moments of TMD PDFs.
  - Price to pay: larger statistical uncertainty.
- In SIDIS:
  - Recently published weighted Sivers asymmetry.
  - Results consistent with the standard ones and with the Gaussian ansatz.
  - Straightforward extraction of Sivers function.
  - Other weighted asymmetries to be extracted, e.g. Collins- and pretzelosity-induced ones:

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Thank you for your attention!

Backup: Measurement method

- Recalling the SIDIS cross-section and the weighted Sivers asymmetry,

$$\sigma(x, y, z, P_{hT}, \phi_{\mathrm{Siv}}) = C(x, y, Q^2)\left[F_{\mathrm{UU,T}} + |\boldsymbol{S}_{\mathbf{T}}| F_{\mathrm{UT,T}}^{\sin\Phi_{\mathrm{Siv}}} \sin\Phi_{\mathrm{Siv}}\right],$$

$$A_{\mathrm{UT,T}}^{\sin\Phi_{\mathrm{Siv}}\frac{P_{hT}}{zM}} = \frac{\int \mathrm{d}^2\boldsymbol{P}_{\boldsymbol{h}\mathbf{T}} \frac{P_{hT}}{zM} F_{\mathrm{UT,T}}^{\sin\Phi_{\mathrm{Siv}}}}{\int \mathrm{d}^2\boldsymbol{P}_{\boldsymbol{h}\mathbf{T}} F_{\mathrm{UU,T}}}$$

  we can get the numerator by weighting each event i with $w_i = \sin\Phi_{\mathrm{Siv},i}\frac{P_{hT,i}}{z_i M}$,

$$A_{\mathrm{UT,T}}^{\sin\Phi_{\mathrm{Siv}}\frac{P_{hT}}{zM}}(x, z) = \frac{2}{|\boldsymbol{S}_{\mathbf{T}}|}\frac{\langle w_i\rangle_{\text{kinematic bin}}}{\langle 1\rangle_{\text{kinematic bin}}} = \frac{2}{|\boldsymbol{S}_{\mathbf{T}}|}\frac{\int \mathrm{d}^2\boldsymbol{P}_{\boldsymbol{h}\mathbf{T}}\mathrm{d}\Phi_{\mathrm{Siv}} w_i \sigma(x, z, P_{hT}, \phi_{\mathrm{Siv}})}{\int \mathrm{d}^2\boldsymbol{P}_{\boldsymbol{h}\mathbf{T}}\mathrm{d}\Phi_{\mathrm{Siv}}\sigma(x, z, P_{hT}, \phi_{\mathrm{Siv}})}.$$

- However, this ignores the experimental acceptance!

- COMPASS used target with 2 or 3 cells with opposite and alternating polarisation and measured the weighted transverse-spin-dependent asymmetries (TSAs) through the ratio

$$\frac{W_1^{\uparrow}W_2^{\uparrow} - W_1^{\downarrow}W_2^{\downarrow}}{\sqrt{(W_1^{\uparrow}W_2^{\uparrow} + W_1^{\downarrow}W_2^{\downarrow})(N_1^{\uparrow}N_2^{\uparrow} + N_1^{\downarrow}N_2^{\downarrow})}} \approx \tilde{D}_{\sin\Phi_{\mathrm{Siv}}}\overline{|\boldsymbol{S}_{\mathbf{T}}|}\sin\Phi_{\mathrm{Siv}} A_{\mathrm{UT,T}}^{\sin\Phi_{\mathrm{Siv}}\frac{P_{hT}}{zM}},$$

  where e.g. $N_i^{\uparrow}(\Phi_{\mathrm{Siv}})$ is the number of events in the cells polarised $\uparrow$ in period i and $W_i^{\uparrow}(\Phi_{\mathrm{Siv}})$ is the sum of the weights $\frac{P_{hT}}{zM}$ of the same events.
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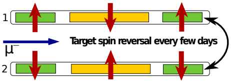
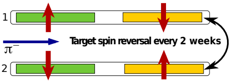


Target cells in 2007 + 2010 SIDIS.



Target cells in 2015 + 2018 Drell–Yan.

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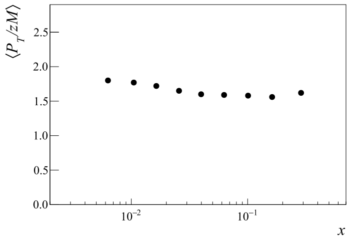
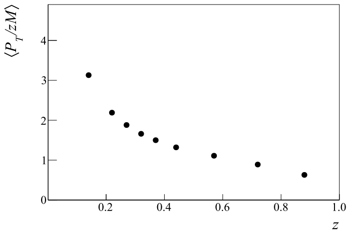
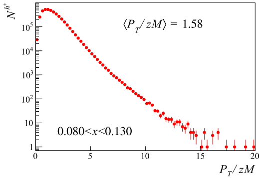


The average weights as a function of x.



The average weights as a function of z.



Example: the distribution of weights in the 7th x-bin.

Backup: Boer–Mulders func. sign change

To check that $h_{1,\mathrm{p}}^{\perp q}\big|_{\mathrm{SIDIS}} = -h_{1,\mathrm{p}}^{\perp q}\big|_{\mathrm{DY}}$ is more complicated...

The Boer–Mulders function in Drell–Yan at COMPASS:

$$A_{\mathrm{U}}^{\cos 2\phi} \propto h_{1,\pi}^{\perp q} \otimes h_{1,\mathrm{p}}^{\perp q} \quad \text{or} \quad A_{\mathrm{U}}^{\cos 2\phi \frac{q_{\mathrm{T}}^2}{4M_\pi M_{\mathrm{p}}}} \propto h_{1,\pi}^{\perp(1)q} \times h_{1,\mathrm{p}}^{\perp(1)q} \qquad \text{(ongoing analysis)}$$

$$A_{\mathrm{T}}^{\sin(2\phi-\phi_S)} \propto h_{1,\pi}^{\perp q} \otimes h_{1,\mathrm{p}}^{q} \quad \text{or} \quad A_{\mathrm{T}}^{\sin(2\phi-\phi_S)\frac{q_{\mathrm{T}}}{M_\pi}} \propto h_{1,\pi}^{\perp(1)q} \times h_{1,\mathrm{p}}^{q}. \qquad \text{(published/preliminary)}$$

- Two asymmetries (possibly weighted) have to be measured to get the sign of $h_{1,\mathrm{p}}^{\perp q}\big|_{\mathrm{DY}}$.
- The knowledge of $h_{1,\mathrm{p}}^{q}$ is needed,
  - Can be obtained from the Collins or dihadron asymmetry in SIDIS.
  - Measured by COMPASS, HERMES...
  - Global fits exist (however, with considerable uncertainties).

The Boer–Mulders function in SIDIS:

- $h_{1,\mathrm{p}}^{\perp q}\big|_{\mathrm{SIDIS}}$ can be obtained from $A_{\mathrm{UU}}^{\cos 2\phi_h}$ (published by both COMPASS and HERMES).
- However, the extractions have faced problems.
- COMPASS is analysing data taken on liquid H in 2016 + 2017
  (A. Moretti has presented the status on Monday)