Monte Carlo simulation of polarized quark fragmentation with the string $+^{3}P_{0}$ model

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In collaboration with X. ARTRU and A. MARTIN
string + $^3P_0$


different approach w.r.t Matevosyan, Kotzinian and collaborators PRD95.1 (2017), p. 014021.

← remnant side (diquark or anti-quark)

string axis defined by the momentum of $q_A$

transversely polarized (along $\hat{y}$ axis)

$x_1 = q_A$
string + $^3P_0$


← remnant side (diquark or anti-quark)

string breaking via tunnelling of $q_2 \bar{q}_2$ pair in $^3P_0$ state

$L = 1, \quad S = 1$

$J = L + S = 0 \rightarrow \langle L \rangle = -\langle S \rangle$

the pair has vacuum quantum numbers $0^{++}$

pseudo-scalar meson ($\pi, K, \eta, \eta'$)

quark spin

quark transverse momentum

relative orbital angular momentum

rank 1

X

remnant side (diquark or anti-quark)
string + $^3P_0$


← remnant side (diquark or anti-quark)
string $+ {^3P_0}$


← remnant side (diquark or anti-quark)
string + $^3P_0$


left-right asymmetry with respect to the plane defined by the quark direction of motion and its spin

→ recursive model for the Collins effect
string + $^3P_0$

u-quark polarized along $\hat{y}$

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π$^+$ and π$^-$ are emitted on opposite sides
→ qualitative agreement with experimental data
→ predicts also a di-hadron asymmetry (local compensation of $k_T$)

- classical picture which gives only qualitative predictions
- MC implementation necessary: first prototype for $pp^\uparrow \rightarrow \pi X$ based on probabilities by X. Artru et al.[*], spin effects limited to the leading hadron
- to take into account the quark spin → quantum mechanical formulation of the string+ $^3P_0$ model

Recursivity: the elementary splitting

String decay can be viewed as the recursive repetition of the elementary splitting

\( \rho(q') \)

4-momentum \( k' \)

2x2 spin density matrix

flavour \( q' \)

\( \text{type } h = q\bar{q}' \) (ps = pseudo-scalar)

4-momentum \( p = k - k' \)

no spin information

\( \rho(q) = (I + \sigma \cdot S_q)/2 \)

polarization vector = \( S_q \)

flavour \( q \)

4-momentum \( k \)

2x2 spin density matrix

X. Artru, Z. Belghobsi DSPIN-2011
X. Artru, Z. Belghobsi DSPIN-2013
recursivity: the elementary splitting

string decay can be viewed as the recursive repetition of the elementary splitting

\[ \rho(q') \]

flavour \( q' \)
4-momentum \( k' \)
2x2 spin density matrix

hadron

\[ \rho(q) = \frac{(I + \sigma \cdot S_q)}{2} \]
polarization vector = \( S_q \)

\[ \text{type } h = q\bar{q}' \text{ (ps = pseudo-scalar)} \]
4-momentum \( p = k - k' \)
no spin information

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Transition amplitude \( T_{q',h,q} \)
Recursivity: the elementary splitting

String decay can be viewed as the recursive repetition of the elementary splitting:

$\rho(q') \propto T \rho(q) T^\dagger$

4-momentum $p = k - k'$

No spin information

Type $h = q\bar{q}'$ (ps = pseudo-scalar)

Flavour $q'$

4-momentum $k'$

2x2 spin density matrix

Transition amplitude $T_{q',h,q}$

Flavour $q$

4-momentum $k$

2x2 spin density matrix

$\rho(q) = (I + \sigma \cdot S_q)/2$

Polarization vector $= S_q$

Splitting function: probability of emitting $h$ with longitudinal momentum fraction $Z = p^+/k^+$ and transverse momentum $p_T = k_T - k'_T$ from a quark $q$ with polarization $S_q$
**Recursivity: The Elementary Splitting**

String decay can be viewed as the recursive repetition of the elementary splitting.

**Type** $h = q\bar{q}'$ (ps = pseudo-scalar)

4-momentum $p = k - k'$

No spin information

**Flavour** $q'$

4-momentum $k'$

2x2 spin density matrix

$\rho(q') \propto T \rho(q) T^\dagger$

**Hadron**

**Quark**

**Quark**

**Flavour** $q$

4-momentum $k$

2x2 spin density matrix

$\rho(q) = (I + \sigma \cdot S_q)/2$

Polarization vector $= S_q$

**Transition Amplitude** $T_{q',h,q}$

$$T_{q',h,q} = C_{q',h,q} \times \left(\frac{1 - Z}{\zeta_h^2}\right)^{a/2} e^{-\frac{b_L}{2Z} (\zeta_h^2)} \times e^{-\frac{b_T}{2} k_T'^2} \times \tilde{g}(\zeta_h^2) \times [\mu + \sigma_z \sigma \cdot k'_T] \times \sigma_z \times \hat{u}^{-\frac{1}{2}}(k_T)$$

$T = \text{flavour} \times \text{Lund string model} \times k'_T \text{ supp.} \times \text{input func.} \times ^3P_0 \text{ operator} \times \text{ps coupling} \times \text{single quark density}$
recursivity: the elementary splitting

the model has:
- 5 free parameters
  \( b_L \sim \) probability of having a string cutting point
  \( b_T \sim \) suppression of \( k_T \) at string breaking
  \( a \sim \) suppression of large \( Z \)
  \( \mu \rightarrow \) «complex mass» introduced by the \( ^3P_0 \) mechanism
  \( \text{Im}(\mu)/|\mu| \) responsible for transverse spin effects

- input function
  \( \bar{g}(\varepsilon_h^2) \rightarrow \) governs spin-independent \( k_T - k_T' \) correlations

\[
T_{q',\h,q} = C_{q',\h,q} \times \left( \frac{1 - Z}{\varepsilon_h^2} \right)^{a/2} \times e^{-\frac{b_L}{2Z}(\varepsilon_h^2)} \times e^{-\frac{b_T}{2}k_T'^2} \times \bar{g}(\varepsilon_h^2) \times [\mu + \sigma_z \sigma \cdot k_T'] \times \sigma_z \times \hat{u}^{-\frac{1}{2}}(k_T)
\]

\( T = \) flavour \( \times \) Lund string model \( \times k_T' \) supp. \( \times \) input func. \( \times ^3P_0 \text{ operator} \times \text{ps coupling} \)

- already in the Lund string Model (PYTHIA, LEPTO,..)
- encode string fragm. dynamics

single quark density in \( k_T \otimes \) spin space
possible choices for the input function

\[
\hat{\mathcal{g}}(\varepsilon_h^2) = (\varepsilon_h^2)^a
\]

more complete \quad \hat{\mathcal{g}} \quad \text{more simple}

Model **M18** published in 2018

*PRD97 (2018) no.7, 074010*

- spin independent \(k_T - k'_T\) correlations
- complicates simulations

Model **M19** published in 2019

*PRD100 (2019) no.1, 014003*

\[
\hat{\mathcal{g}}^2(\varepsilon_h^2) = \left[ \int_0^1 dZZ^{-1} \left( \frac{1-Z}{\varepsilon_h^2} \right)^a e^{-\frac{bL\varepsilon_h^2}{Z}} \right]^{-1}
\]

- no spin independent \(k_T - k'_T\) correlations
- analytically/numerically simpler
- unpolarized splitting function \(\sim\) in PYTHIA

both implemented in a stand alone Monte Carlo programs
both restricted to pseudo-scalar meson emission
General structure of the MC program

• For each event define initial quark $q_A \equiv q_1$ with flavour $u, d, s$, its energy and its spin density matrix $\rho(q_A)$

1. Generate a $q_2 \bar{q}_2$ pair and form the hadron $h_1(q_A \bar{q}_2)$
2. Construct the four-momentum of $h_1$ by drawing $Z_1$ and $p_{1T}$ using $F_{q_2 h_1 q_A}$
3. Calculate the spin density matrix of $q_2$

• Iterate points 1-3 until the exit condition is reached (enough renamining c.m. energy to produce at least one baryonic resonance)
The free parameters of the model

Values of the free parameters in M18 have been fixed in order to have a qualitative agreement between simulation results with

- parameterizations of unpolarized fragmentation functions and experimental data on unpolarized SIDIS
- $e^+e^-$ Collins asymmetries

\[ b_L = 0.5 \left( \frac{\text{GeV}}{c^2} \right)^{-2}, \quad b_T = 5.17 \left( \frac{\text{GeV}}{c} \right)^{-2}, \quad a = 0.9 \]

\[ \mu = (0.42 + i0.76) \text{ GeV}/c^2 \]
The initial conditions in simulations

• **Fragmentation chains of transversely and fully polarized u quarks** with initial spin density matrix $\rho(q_A) = (1 + \sigma_y)/2$

• Energy of fragmenting quark calculated from a $\{x_B, Q^2\}$ sample of SIDIS events
Collins analysing power as function of rank

- Azimuthal spectrum of hadrons

\[ N_h(\phi_C) = N_U [1 + a^{u \rightarrow h \rightarrow X} \sin \phi_C] \]

\[ \phi_C = \phi_h - \phi_{s_u} \]

- Analysing power calculated as

\[ a^{u \rightarrow h \rightarrow X} = 2\langle \sin \phi_C \rangle \]

- Classical picture reproduced
- The quark spin information decays along the fragmentation chain
Collins analysing power as function of $z_h$ and $p_T$

- mirror symmetry for opposite charges
- decreases with $z_h = E_h / E_{\text{frag. quark}}$
- change of sign of $a_{u^\uparrow \rightarrow h+X}^u(p_T)$ due to a second rank $\pi^+$ produced after a first rank $\pi^0$
Collins analysing power as function of $z_h$ and $p_T$

- mirror symmetry for opposite charges
- decreases with $z_h = E_h/E_{frag.\,quark}$
- change of sign of $a^{u\uparrow \to h+X}(p_T)$ due to a second rank $\pi^+$ produced after a first rank $\pi^0$

- MC scaled by $\lambda \sim \langle h_1^u/f_1^u \rangle = 0.055 \pm 0.010$ because in reality quarks are partially polarized
- $\lambda$ is estimated by comparison with the COMPASS (proton) asymmetries
Asymmetry of oppositely charged hadron pairs in the same jet

- MC points scaled by the same factor $\lambda \sim \langle h_1^u/f_1^u \rangle = 0.055 \pm 0.010$
possible choices for the input function

\[ \tilde{g}^2(\varepsilon_h^2) = (\varepsilon_h^2)^a \]

- more complete
- more simple

\[ \tilde{g}^2(\varepsilon_h^2) = \left[ \int_0^1 dZZ^{-1} \left( \frac{1 - Z}{\varepsilon_h^2} \right)^a e^{-\frac{bL\varepsilon_h^2}{Z}} \right]^{-1} \]

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both implemented in a stand alone Monte Carlo programs
both restricted to pseudo-scalar meson emission
Comparison between M19 and M18

re-tuning \((b_T)_{M19} = 1.63 \times (b_T)_{M20}\) to have same \(p_T^2\) distributions

Very similar:
1. kinematical distributions
2. Collins analysing power as function of rank, \(z_h\) and \(p_T\)
3. dihadron analysing power

Conclusion:

M19 gives the same results as M18

in spite of being by far simpler
vector meson production

a very recent development using M19 (*) → model M20

(*) A. Kerbizi, PhD thesis, XXXII cycle, Trieste University [2020]

necessary
- in fragmentation vm are produced roughly as many as pseudo-scalar mesons
- decay products contribute to the final hadrons

non - trivial task
- spin 1 → 3x3 spin density matrix → non isotropic decay distributions
- spin of vm correlated in an entangled way with spin of q’ → different spin transfer mechanism from q to q’

what exists
- a prototype of MC code for $pp^\uparrow \rightarrow \rho + X$ based on probabilities, spin effects limited to the first rank hadron -> Czyzewski (*)
- coupling $q' - vm - q$ → two new complex coupling constants -> Artru (**)
- recipe for propagation of spin correlations in simulations which take into account entanglement -> Collins, Knowles (***)

in the following slides
- systematic account of vector mesons in the simulation of a polarized quark fragmentation chain in a way which is consistent with quantum mechanics

Suppose this is a $\rho^0$ with prob. $f_{vm}/f_{vm}+f_{ps}$

$f_v/f_ps$ taken as in PYTHIA 8 = 0.62 (u,d), 0.725 (s)

First rank $\rightarrow \pi^+$

Calculate the spin density matrix of $q_2$
Introduction of vector mesons

generate mass and momentum of $\rho^0$

depends on $|G_L|/|G_T|$
- regulates the strength of the Collins effect for the vector meson

$1^{st}$ new free param.

relativistic Breit-Wigner

splitting function for vm production

$F_{q',h,q} = \sum_{\alpha=L,T} tr T^\alpha_{q',h,q} \rho(q) T^\alpha_{q',h,q}^+$

$T^\alpha = \text{mass distrib} \times \text{.. same as ps} \times \text{vm coupling} \times \text{single quark density in } k_T \otimes \text{spin space}$

(*) $\Gamma^\alpha_h = \begin{cases} G_T \times \sigma_z \sigma_T \\ G_L \times I_{2 \times 2} \end{cases}$
- $G_T$ coupling with transversely polarized vm
- $G_L$ coupling with longitudinally polarized vm

$G_L$ and $G_T$ complex free parameters

(*) X. Artru [arXiv:1001.1061, 2009]
Introduction of vector mesons

in the vector meson rest frame

\[ \hat{\rho}_{\alpha \alpha'}(vm) \propto \text{tr} \ T_{q',h,q}^{\alpha} \rho(q) T_{q',h,q}^{\alpha\dagger} \]

depends on \(|GL|/GT|\) and \(\theta = \arg(G_L/G_T)\)

\( \rightarrow 2^{nd} \) new free parameter

\( \rightarrow \) gives an oblique linear polarization which can enhance or reduce the Collins effect of the decay products
Introduction of vector mesons

come back with a «decay matrix» $\tilde{\rho}_{\alpha \alpha'}$

$\tilde{\rho}_{\alpha \alpha'} = \hat{R}_{\alpha} \hat{R}_{\alpha'}$

Simulate the decay of the $\rho^0$ in its rest frame and then boost to the string rest frame

angular distribution in the $\rho^0$ rest frame

$$\frac{dN}{d\Omega} = \frac{3}{4\pi} \rho_{\alpha \alpha'} \hat{R}_{\alpha} \hat{R}_{\alpha'}$$

Finally calculate the spin density matrix of $q_3$

$$\rho(q_3) = \tilde{\rho}_{\alpha \alpha'} tr \ T_{q',h,q}^\alpha \rho(q) T_{q',h,q}^{\alpha'}$$

- Collins-Knowles recipe

→ takes into account entangled spin-correlations between $\rho^0$ and $q_3$

- this mechanism is repeated until the end of the chain $\rho, K^*, \omega, \phi$ included
Simulations with M20

- Fragmentations of **fully transversely polarized** $u$ quarks
- No tuning of $|G_L|/|G_T|$ and $\theta_{LT}$
- Model by Czyzewski [*] based on the non relativistic quark model gives

$$\frac{a_p^{u \rightarrow \rho^+ + X}}{a_p^{u \rightarrow \pi^+ + X}} \bigg|_{rank \ 1} = -\frac{1}{3}$$

- M20 gives

$$\frac{a_p^{u \rightarrow \rho^+ + X}}{a_p^{u \rightarrow \pi^+ + X}} \bigg|_{rank \ 1} = -\frac{|G_L|^2}{2|G_T|^2 + |G_L|^2}$$

→ current choice of free parameters $\frac{|G_L|}{|G_T|} = 1$ and $\theta_{LT} = 0$

however $|G_L|/|G_T|=1$ and $\theta_{LT}=0$

need not be necessarily true!

Collins analysing power of $\rho^\pm, \rho^0$

- opposite sign of Collins analysing power for $\rho^+$ and $\pi^+$
  - $a^{u\uparrow}\rightarrow\rho^- + X \sim a^{u\uparrow}\rightarrow\rho^+ + X$ by chance, not true for $|G_L|/|G_T| \neq 1$
Collins analysing power of pions from $\rho^+$ decay

Pions with large $z_h$ come preferentially from decay of rank 1 $\rho^+$ linearly polarized along $\hat{z}$ for which $a^{u\rightarrow \rho^+_L + X} \sim 3a^{u\rightarrow \rho^+_{unpol} + X}$.

- Same analysing power for $\pi^+$ and $\pi^0$ -> decay process invariant under parity.

- Pions from
  - rank 1 $\rho^+$ linearly pol. along $p_T(\rho^+)$
  - $\rho^+$ linearly pol. along $\hat{z} \times p_T(\rho^+)$

in both cases

$$a^{u\rightarrow \rho^+_L + X} \sim 3a^{u\rightarrow \rho^+_{unpol} + X}$$
Collins analysing power for pions from M20

- analysing power of primary pions reduced by a factor of 2 as a consequence of $\rho$ meson decay!
- $K^*, \omega, \phi$ contribute less to the pion analysing power
- the trend as function of $z_h$ for $\pi^+$ is no more linear
Dihadron analysing power

- large effect on the dihadron analysing power, decreased by a factor of 2!

\[
|G_L|/|G_T| = 1 \\
\theta_{LT} = 0
\]

\[
\begin{align*}
\text{VM decay OFF} \\
\text{M19} \\
\text{VM decay ON}
\end{align*}
\]
Sensitivity to $|G_L|/|G_T|$
Sensitivity to $\theta_{LT}$

all vm decays switched ON

- $\theta_{LT}$ can enhance or reduce the Collins analysing power of final adrons $\rightarrow$ oblique polarization of the vm
- large effect for all $z_h$ and $p_T < 0.5$ (GeV/c)
- small effect on dihadron analysing power

open markers $\rightarrow \pi^-$
full markers $\rightarrow \pi^+$
$|G_L|/|G_T| = 1$

$\theta_{LT} = +\pi/2$
$\theta_{LT} = 0$
$\theta_{LT} = -\pi/2$
Comparison with COMPASS asymmetries

full points -> MC
with vector mesons $|G_L|/|G_T| = 1$

open points -> COMPASS PLB 717 (2012) 376
only pseudoscalar mesons

$\theta_{LT} = 0$

$\lambda' \sim 0.11 \pm 0.02$

$\lambda \sim 0.055 \pm 0.010$

- MC points scaled by the factors $\lambda'$ and $\lambda$
- M20 promising but tuning of parameters is necessary
- different trends as function of $z_h$ for $\pi^-$?
To exploit the true predictive power of the model, **M19 has been interfaced with PYTHIA 8 for SIDIS processes** introducing for the first time spin effects for pseudoscalar mesons in a complete event generator

a parameterization for the transversity PDF for valence u and d quarks is implemented

→ **allows to simulate the Collins and dihadron asymmetries**

for the description of the interface see


a write-up is in preparation [A. Kerbizi and L. Lönnblad]
Collins and dihadron asymmetries on proton from PYTHIA+\(^3\)P\(_0\)

**PYTHIA + \(^3\)P\(_0\) only pseudo-scalar mesons**

Collins asymmetry for charged pions

- similar trends as the Collins asymmetry seen in data as function of \(x_B\) and \(p_T\)
- different \(z_h\) trends -> no vector mesons

**very promising results!**

**PYTHIA + \(^3\)P\(_0\) only pseudo-scalar mesons**

dihadron asymmetry for h+h- pairs

similar trends as the measured asymmetry
conclusions

- The fragmentation process of polarized quarks with pseudoscalar and vector meson emission within the quantum mechanical formulation of the string+3P0 model has been simulated for the first time

- The results of the stand alone MC are in good qualitative agreement with the present data

- The interface with Pythia 8 for pseudo-scalar meson production has been developed and will be public soon

- The first step is done.. but this is not the end of the story
  - better tuning of parameters
  - study of e+e- annihilation
  - ...

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backup slides
Kinematic distributions: comparison between M18 and M19

$z_h > 0.2$

$p_T > 0.1 \text{ GeV/c}$

$\langle p_T \rangle (\text{GeV/c})^2$

Counts

$10^0$ $10^1$ $10^2$ $10^3$ $10^4$ $10^5$

$z_h$

$1$ $0.5$ $1$ $1.5$ $2$ $2.5$ $3$

$p_T^2 (\text{GeV/c})^2$

$0$ $0.2$ $0.4$ $0.6$

$z_h$

$0$ $0.2$ $0.4$ $0.6$ $0.8$ $1$

$h^+ \text{ M18}$

$h^+ \text{ M19}$

$h^{-} \text{ M18}$

$h^{-} \text{ M19}$

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M20: kinematic distributions for primary $\pi^+$ and $\rho^+$

fraction $\text{vm/ps}$ depends on the kinematics

- Prediction of the string+$^3P_0$ model with vector meson production:
  \[ \langle p_T^2 \rangle (\rho^+) < \langle p_T^2 \rangle (\text{prim. } \pi^+) \]

This is at variance with PYTHIA!
- already expected in X. Artru and Z. Belghobsi DSPIN2011