Fit of the $a_1(1420)$ as a Triangle Singularity

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supported by BMBF
The COMPASS Experiment
The COMPASS Experiment

- Secondary hadron beam, mostly $\pi^-$ ($\sim 97\%$)
- $E_{\text{beam}} = 190\ \text{GeV}$
- Liquid hydrogen target (40 cm)
- $\pi^- + p \rightarrow \pi^- + \pi^- + \pi^+ + p$

[COMPASS, NIM A779, 69-115 (2015)]

Fit of the $a_1(1420)$ as a Triangle Singularity
Isobar model: $X^- \rightarrow \pi^- + \xi \rightarrow \pi^- + \pi^+ + \pi^-$

Data binned in 100 $m_{3\pi}$ and 11 $t' = |t| - |t|_{\text{min}}$ slices

PWA with 88 waves [COMPASS, PRD 95, 032004 (2017)]
3π PWA

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\[ \begin{array}{c}
\pi^- \\
\downarrow t' \\
p \\
\uparrow \pi^+
\end{array} \quad \begin{array}{c}
X(J^{PC}) \\
L
\end{array} \quad \begin{array}{c}
\pi^- \\
\downarrow \pi^+
\end{array} \quad \begin{array}{c}
p \\
\uparrow \pi^-
\end{array} \]

\( \text{P}: \) Pomeron  
\( X: \) Resonance with \( J^{PC} \)  
\( \xi: \) Isobar


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\[ \begin{align*}
\text{P: Pomeran} \\
\text{X: Resonance with } J^{PC} \\
\xi: \text{Isobar}
\end{align*} \]

The $a_1(1420)$ signal
What makes this signal exotic? Only seen in the $J^{PC} = 1^{++}$ $f_0(980)$ $\pi P$ channel. Very close to the ground state $a_1(1260)$. Too narrow: 150 MeV (ground state has 250-600 MeV).

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[COMPASS, PRL 115, 082001 (2015)]
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Possible scenarios

- 4-quark state
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  [T. Gutsche et al. (2017)]

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- **$K^*\bar{K}$ molecule (similar to XYZ)**
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- **Dynamic effect of interference with Deck-amplitude**
  [Basdevant & Berger, PRL 114, 192001 (2015)]
Possible scenarios

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- **$K^*\bar{K}$ molecule (similar to XYZ)**
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- **Dynamic effect of interference with Deck-amplitude**
  - [Basdevant & Berger, PRL 114, 192001 (2015)]

- **Triangle singularity**
  - [Mikhasenko et al., PRD 91, 094015 (2015)],
  - [Aceti et al., PRD 94, 096015 (2016)]

Fit of the $a_1(1420)$ as a Triangle Singularity

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The $a_1(1420)$ signal

- The Triangle Diagram -
Include spin via partial-wave projection:

1. Look at the partial wave for $a_1(1260) \rightarrow K\bar{K}\pi$ with isobar $K^*$
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2. Project it onto the $3\pi$ final state with isobar $f_0(980)$
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1. Look at the partial wave for $a_1(1260) \to K\bar{K}\pi$ with isobar $K^*$
2. Project it onto the $3\pi$ final state with isobar $f_0(980)$
3. Obtain the first order approximation of the Khuri-Treiman approach
The Triangle Diagram

The $a_1(1420)$ signal

Kaons in the loop produce peak and phase motion at 1.4 GeV
The Triangle Diagram

The $a_1(1420)$ signal

- Ground state rescatters through other intermediate isobar $\xi$
- Using Feynman calculation treating everything as scalars

Fit of the $a_1(1420)$ as a Triangle Singularity

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The Triangle Diagram

The $a_1(1420)$ signal

- Other triangles similar to direct decay
- Phenomenological background is able to describe them

Fit of the $a_1(1420)$ as a Triangle Singularity

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The Fit

- Model -
Select interesting components from the pool of 88 partial waves:

1. $J^{PC} = 1^{++} \rho\pi S$-wave
   - Contains the ground state $a_1(1260)$
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   - Contains the signal of interest, the $a_1(1420)$
Select interesting components from the pool of 88 partial waves:

1. \( J^{PC} = 1^{++} \rho \pi \) S-wave
   - Contains the ground state \( a_1(1260) \)

2. \( J^{PC} = 1^{++} f_0(980) \pi \) P-wave
   - Contains the signal of interest, the \( a_1(1420) \)

3. \( J^{PC} = 2^{++} \rho \pi \) D-wave
   - Contains the \( a_2(1320) \)
   - Clean signal, small background
   - Interferometer
From PWA: #events per $m_{3\pi}$ and $t'$ slices for a given partial wave

Intensity: $\frac{d^2 N}{dm_{3\pi} dt'} \propto \frac{d^2 \sigma}{dm_{3\pi} dt'} \propto m_{3\pi} |M_{tot}|^2 \tilde{\Phi}_2,$

$\tilde{\Phi}_2$: quasi-2-body PS
From PWA: \#events per $m_{3\pi}$ and $t'$ slices for a given partial wave

Intensity: \[
\frac{d^2N}{dm_{3\pi} dt'} \propto \frac{d^2\sigma}{dm_{3\pi} dt'} \propto m_{3\pi} |M_{\text{tot}}|^2 \tilde{\Phi}_2,
\]

$\tilde{\Phi}_2$: quasi-2-body PS

Interferences between waves $\sim m_{3\pi} M_{\text{tot}}^{(1)*} M_{\text{tot}}^{(2)} \sqrt{\tilde{\Phi}_2^{(1)} \tilde{\Phi}_2^{(2)}}$

$M_{\text{tot}} = M_{\text{signal}} + M_{\text{bgd}}$
\[ \mathcal{M}_{\text{signal}} \propto \frac{1}{M_X^2 - m_{3\pi}^2 - i M_X \Gamma_X(m_{3\pi})} \]

- Propagator with energy-dependent width
- Multiplied by triangle amplitude for \( f_0 \pi \) \( P \)-wave
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- Propagator with energy-dependent width
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\[ M_{\text{bgd}} \propto \left( \frac{m_{3\pi} - m_{\text{thr}}}{m_{\text{thr}}} \right)^b e^{-c(t')\tilde{p}^2} \]

- Phenomenological parametrization for Deck background
- \( \tilde{p} = 4\pi m_{3\pi} \tilde{\Phi}_2 \)
Fit model
Signal
Background

\[ \rho \pi S \rightarrow a_1(1260) \]

\[ f_0 \pi P \rightarrow a_1(1260) \cdot \text{Triangle} \]

\[ \rho \pi D \rightarrow a_2(1320) \]

Intensities

Interferences (fitted via real and imaginary part)

\( 0.100 < t' / (\text{GeV/c})^2 < 0.113 \)

\( \pi p \rightarrow \pi \pi' p \) (COMPASS 2008)

Mass-independent fit

Fit model

Signal

Background

Fit of the \( a_1(1420) \) as a Triangle Singularity (15/19)

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$0.100 < t' / (\text{GeV}/c)^2 < 0.113$

$\pi^+ p \rightarrow \pi^- \pi^+ \pi^- p$ (COMPASS 2008)

Mass-independent fit

Fit model

Signal

Background

Fit of the $a_1 (1420)$ as a Triangle Singularity
Comparison with Breit-Wigner Fit

Fit of the $a_1 (1420)$ as a Triangle Singularity

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$J^{PC} M^\epsilon = 1^{++} 0^+ f_0 \pi P$ - Intensity

$J^{PC} M^\epsilon = 1^{++} 0^+ (\rho \pi S - f_0 \pi P)$ - Interference

Relative phases (deg)

Intensity / (20 MeV)

$0.100 < t'/(GeV/c)^2 < 0.113$

mass-independent fit

$0.100 < t'/(GeV/c)^2 < 0.113$

0.100 < $t'/(GeV/c)^2$ BW-model - model

$\Delta$ signal

background
Conclusion

- It is possible to fit $a_1(1420)$ with the Triangle-Model.

- The inclusion of spins is done via the partial-wave-projection method.

- Spin only affects the shape, not the position of peak.

- The Scalar-Triangle-Model is sufficient for first studies.

- Many systematic studies have been performed.
  - Very stable w.r.t. manipulations of the data and the fit model.

- Comparison of Triangle-Model and BW-Model:
  - Competitive fit quality between both models.
  - In the Triangle-Model: No free fit parameters are present to describe the peak position and width of the signal.
  - Rescattering has to be present!
Thank you for your attention!
Back-Up
Fit of the $a_1 (1420)$ as a Triangle Singularity

Structure of Complex Plane

$\text{Im} (\sqrt{s_1})$

$\text{Re} (\sqrt{s_1})$

- $K^* K \bar{K}$
- $\sigma \pi \pi$
- $\rho \pi \pi$
- $f_2 \pi \pi$
- $f_0 \pi \pi$

- branch point
- log-sing. on 2nd sheet
- log-sing. on ≥3rd sheet
- unitarity cut on 1st sheet
- branch cut on 2nd sheet
- branch cut on ≥3rd sheet
All Scalar Amplitudes

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Fit of the $a_1 (1420)$ as a Triangle Singularity

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Comparison with Scalar-Triangle

$J^P C^M = 1^{++} 0^+ f_0 \pi^0 P - $ Intensity

$J^P C^M = 1^{++} 0^+ (\rho \pi S - f_0 \pi^0 P) - $ Interference

Fit of the $a_1 (1420)$ as a Triangle Singularity

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Fit of the $a_1$ (1420) as a Triangle Singularity
Comparison with Direct Decay

$J^{PC}M^ε = 1^{++}0^+ f_0 π P$ - Intensity

$J^{PC}M^ε = 1^{++}0^+ (ρπ S - f_0 π P)$ - Interference

Fit of the $a_1 (1420)$ as a Triangle Singularity (7/10)

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Systematic Studies

Fit of the $a_1 (1420)$ as a Triangle Singularity

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\[ A(\tau) = \sum_{w=(JMLS)} \left[ F_w(s_{12})Z_w^*(\Omega_{3,12}) + F_w(s_{23})Z_w^*(\Omega_{1,23}) \right] \]

Simple model: \[ F_w(s_{12}) = C_{a_1} \cdot t_{K^*}(s_{12}) \]
\[ A(\tau) = \sum_{w=(JMLS)} \left[ F_w(s_{12})Z_w^*(\Omega_{3,12}) + F_w(s_{23})Z_w^*(\Omega_{1,23}) \right] \]

Projection to channel (23):

\[ A_w(s_{23}) = \int d\Omega_{1,23} Z_w(\Omega_{1,23})A(\tau) \]

\[ = F_w(s_{23}) + \hat{F}_w(s_{23}) \]

with \( \hat{F}_w(s_{23}) := \int dZ_w(s_{23}) \sum_{w'} F_{w'}(s_{12})Z_{w'}^*(\Omega_{3,12}) \)
PWP - Calculations

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with \( \hat{F}_w(s_{23}) := \int dZ_w(s_{23}) \sum_{w'} F_{w'}(s_{12}) Z_{w'}^*(\Omega_{3,12}) \)

unitarity for PW amplitude \( A_w \):

\[ \Rightarrow F_w(s_{23}) = t_\xi(s_{23}) \left[ C_w + \frac{1}{2\pi} \int_{s_{th}}^{\infty} \frac{\rho(\tilde{s}_{23}) \hat{F}_w(\tilde{s}_{23})}{\tilde{s}_{23} - s_{23}} d\tilde{s}_{23} \right] \]
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**Problem:** \( \hat{F} \) depends on \( F \) as well! \( \Rightarrow \) solve iteratively
\[ F(s_{23}) = t_{f_0}(s_{23}) \frac{1}{2\pi} \int_{4m_K^2}^{\infty} \rho(\tilde{s}_{23}) \int d\tilde{s}_{23} \frac{C_{a_1} t_{K^*}(s_{12}) Z_{K^*}^*(\Omega_{3,12})}{\tilde{s}_{23} - s_{23}} \]

Fit of the $a_1$ (1420) as a Triangle Singularity
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Fit of the \( a_1 (1420) \) as a Triangle Singularity

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Fit of the \( a_1 (1420) \) as a Triangle Singularity
PWP - Iterative Procedure

\[ F(s_{23}) = t_{f_0}(s_{23}) \frac{1}{2\pi} \int_{4m_K^2}^{\infty} d\tilde{s}_{23} \rho(\tilde{s}_{23}) \int dZ_{f_0}(\tilde{s}_{23}) C_{a_1} t_{K^*}(s_{12}) Z_{K^*}^*(\Omega_{3,12}) \]

\[ \tilde{s}_{23} - s_{23} \]

Fit of the \( a_1 (1420) \) as a Triangle Singularity (10/10)

Mathias Wagner (Uni Bonn, HISKP)
PWP - Iterative Procedure

\[ F(s_{23}) = t_{f_0}(s_{23}) \frac{1}{2\pi} \int_{4m_K^2}^{\infty} d\tilde{s}_{23} \rho(\tilde{s}_{23}) \int dZ_{f_0}(\tilde{s}_{23}) \ C_{a_1} \ t_{K^*}(s_{12}) \ Z_{K^*}(\Omega_{3,12}) \]

\[ \tilde{s}_{23} - s_{23} \]

Fit of the \( a_1 \) (1420) as a Triangle Singularity
PWP - Iterative Procedure

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\[ \tilde{s}_{23} - s_{23} = a_1 f_{0} \]

Fit of the \( a_1 (1420) \) as a Triangle Singularity