

New way to access the quark fragmentation functions in e^+e^- annihilation reactions

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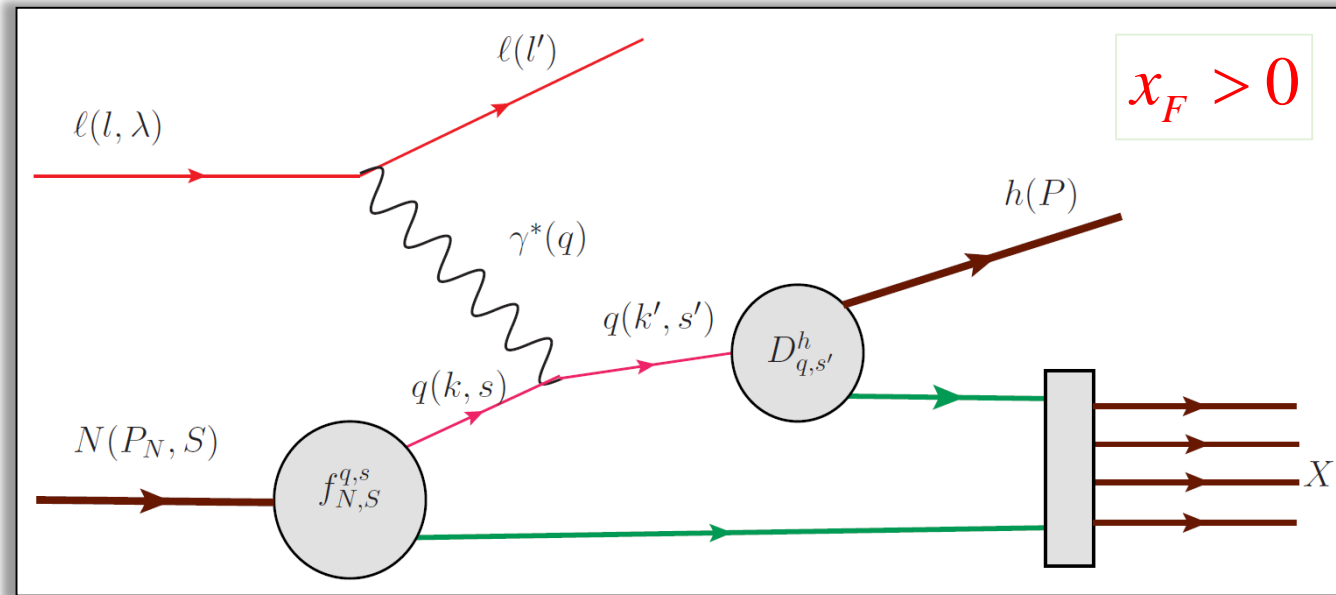
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• Introduction

- accessing TMD PDFs and FFs with electromagnetic probe
- Examples of weighted asymmetries to access DiFFs

SIDIS Current Fragmentation Region (CFR)

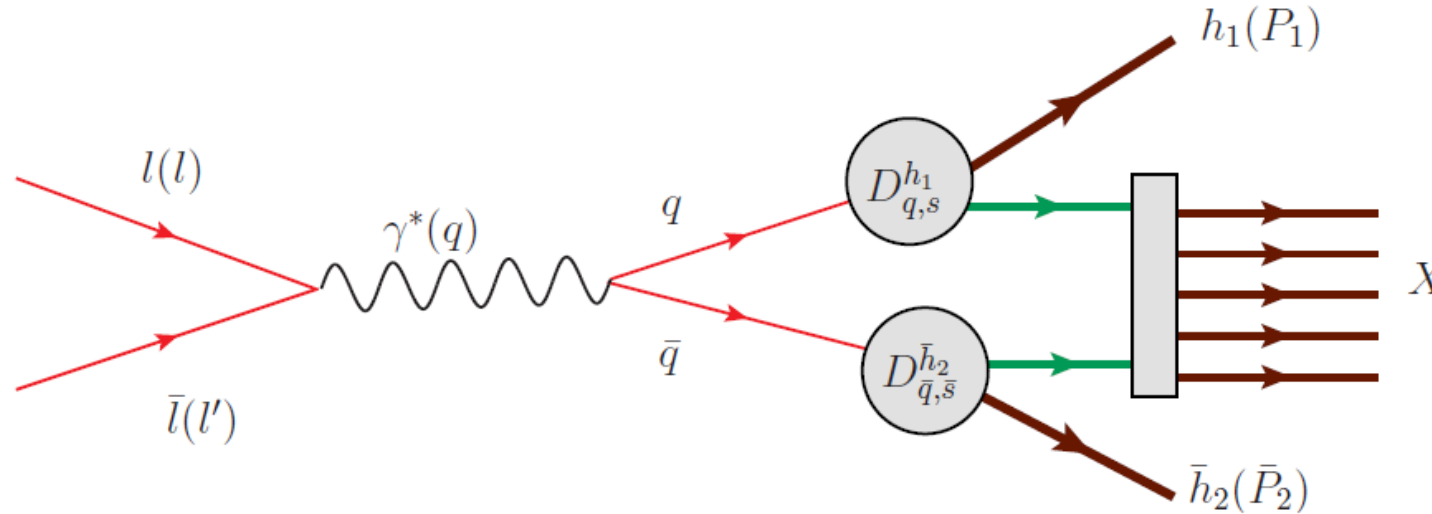


$$\frac{d\sigma^{\ell(l, \lambda) + N(P_N, S) \rightarrow \ell(l') + h(P) + X}}{dx dQ^2 d\phi_S dz d^2 P_T} = f_{q,s/N,S}^{q,s} \otimes \frac{d\sigma^{\ell(l, \lambda) + q(k, s) \rightarrow \ell(l') + q(k', s')}}{dQ^2} \otimes D_{q,s'}^h$$

$$D_{q,s'}^h(z, \mathbf{p}_T) = D_{1q}^h(z, p_T^2) + \frac{\mathbf{p}_T \times \mathbf{s}'_T}{m_h} H_{1q}^h(z, p_T^2)$$

H_1 was measured by BABAR and BELLE to 2 back-to-back jets $e^+e^- \rightarrow h_1 h_2 + X$

h_1+h_2 Semi-Inclusive Annihilation (SIA)



Two hadron production in opposite hemispheres: access to Collins FF $H_{1q}^h(z, p_{\perp}^2)$.

Quarks are unpolarized, but their transverse polarization are correlated, inducing an azimuthal correlation of produced hadrons in opposite jets.

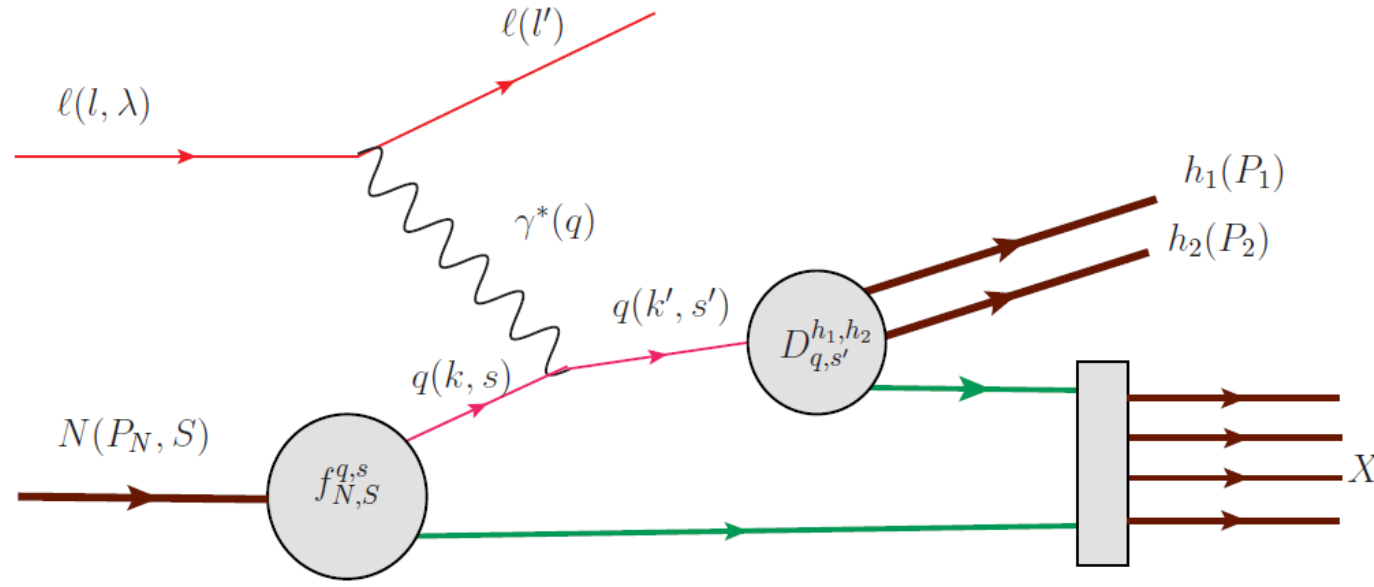
Obtained $H_{1q}^h(z, p_{\perp}^2)$ FFs are used for transversity $h_1(x, k_T^2)$ extraction from SIDIS data.

Twist-2 TMD qDFs

		Quark polarization		
		U	L	T
Nucleon Polarization	U	$f_1^q(x, k_T^2)$		$\frac{\epsilon_T^{ij} k_T^j}{M} h_1^{\perp q}(x, k_T^2)$
	L		$S_L g_{1L}^q(x, k_T^2)$	$S_L \frac{\mathbf{k}_T}{M} h_{1L}^{\perp q}(x, k_T^2)$
	T	$\frac{\mathbf{k}_T \times \mathbf{S}_T}{M} f_{1T}^{\perp q}(x, k_T^2)$	$\frac{\mathbf{k}_T \cdot \mathbf{S}_T}{M} g_{1T}^{\perp q}(x, k_T^2)$	$\mathbf{S}_T h_{1T}^q(x, k_T^2) + \frac{\mathbf{k}_T (\mathbf{k}_T \cdot \mathbf{S}_T)}{M} h_{1T}^{\perp q}(x, k_T^2)$

All azimuthal dependences are in prefactors. TMDs do not depend on them

2h SIDIS: CFR

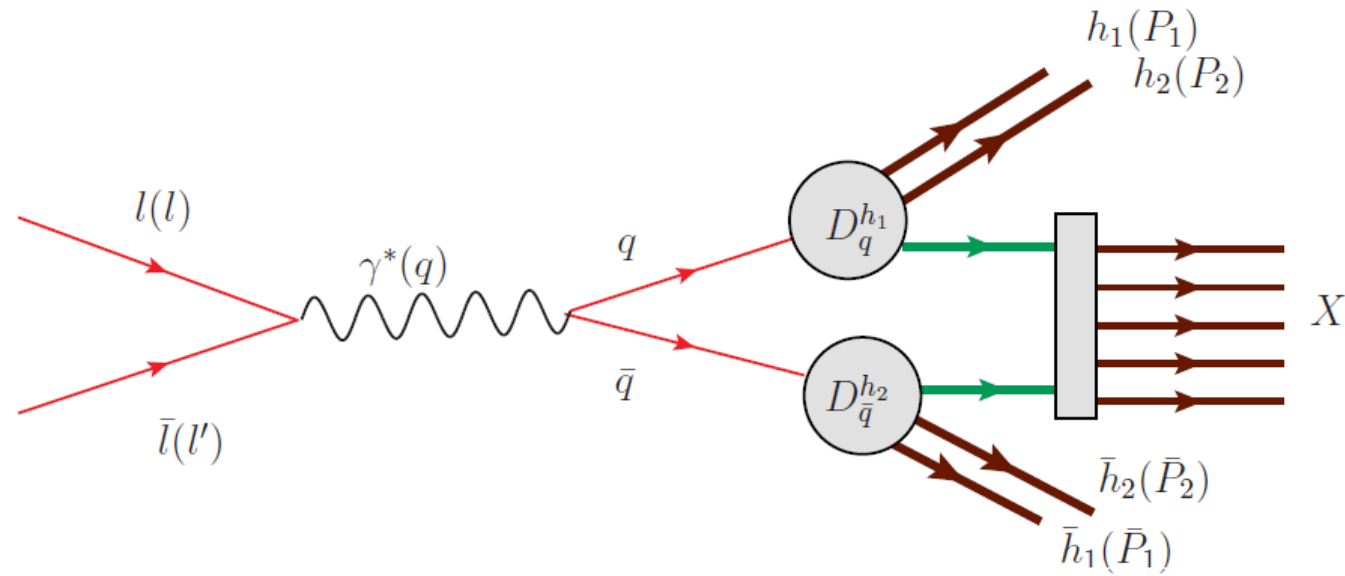


$$x_{F,1} > 0, x_{F,2} > 0$$

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\rightarrow\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx dQ^2 d\phi_S dz_1 d^2 P_{1T} dz_2 d^2 P_{2T}} = f_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\rightarrow\ell(l')+q(k',s')}}{dQ^2} \otimes D_{q,s'}^{h_1,h_2}$$

New objects: DiFFs $D_{q,s'}^{h_1,h_2}$

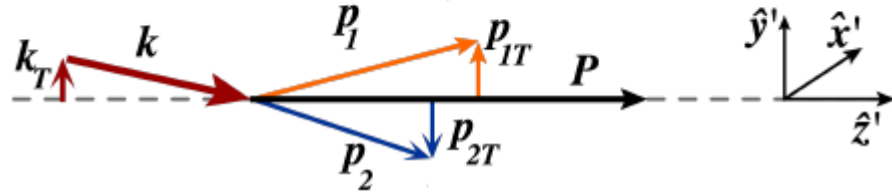
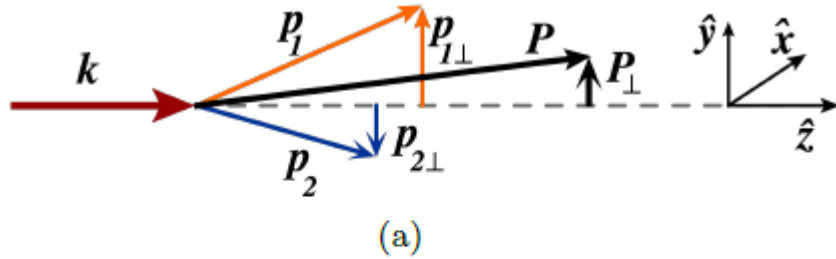
2h+2h SIA



Measured by BELLE: dihadrons production in back-to-back jets in SIA

Access to spin dependent DiFFs $D_{q,s'}^{h_1,h_2}$

Dihadron FFs: pQCD definition



$$P \equiv P_h = P_1 + P_2,$$

$$R = \frac{1}{2}(P_1 - P_2),$$

$$z = z_1 + z_2,$$

$$\xi = \frac{z_1}{z} = 1 - \frac{z_2}{z}$$

$$z_i = P_i^- / k^-$$

$$P_{1T} = P_{1\perp} + z_1 k_T,$$

$$P_{2T} = P_{2\perp} + z_2 k_T.$$

$$k_T = -\frac{P_{\perp}}{z},$$

$$R_T = \frac{z_2 P_{1\perp} - z_1 P_{2\perp}}{z} = (1 - \xi)P_{1\perp} - \xi P_{2\perp}.$$

$$R_T^2 = \xi(1 - \xi)M_h^2 - M_1^2(1 - \xi) - M_2^2\xi.$$

$$\Delta_{ij}(k; P_1, P_2) = \sum_X \int d^4\zeta e^{ik \cdot \zeta} \langle 0 | \psi_i(\zeta) | P_1 P_2, X \rangle \langle P_1 P_2, X | \bar{\psi}_j(0) | 0 \rangle.$$

$$\Delta^\Gamma(z, \xi, k_T^2, R_T^2, k_T \cdot R_T) = \frac{1}{4z} \int dk^+ \text{Tr}[\Gamma \Delta(k, P_1, P_2)]|_{k^- = P_h^- / z}.$$

$$\Delta^{[\gamma^-]} = D_1(z, \xi, k_T^2, R_T^2, k_T \cdot R_T),$$

$$\Delta^{[\gamma^- \gamma_5]} = \frac{\epsilon_T^{ij} R_{Ti} k_{Tj}}{M_1 M_2} G_1^{\perp}(z, \xi, k_T^2, R_T^2, k_T \cdot R_T),$$

$$\Delta^{[i\sigma^{i-} \gamma_5]} = \frac{\epsilon_T^{ij} R_{Tj}}{M_1 + M_2} H_1^{\triangleleft}(z, \xi, k_T^2, R_T^2, k_T \cdot R_T)$$

$$+ \frac{\epsilon_T^{ij} k_{Tj}}{M_1 + M_2} H_1^{\perp}(z, \xi, k_T^2, R_T^2, k_T \cdot R_T)$$

Number density distribution in quark to 2h fragmentation

q pol.	U	L	T
DiFF	D_1	G_1^\perp	H_1^\times, H_1^\perp

Unpolarized DiFF

Longitudinal handedness

Interference DiFF (IFF)

Collins-like DiFF

$$\begin{aligned}
 F(z, \xi, \mathbf{k}_T, \mathbf{R}_T; s) = & D_1(z, \xi, \mathbf{k}_T^2, R_T^2, \cos(\varphi_{RK})) \\
 & - s_L \frac{R_T k_T \sin(\varphi_{RK})}{M_1 M_2} G_1^\perp(z, \xi, \mathbf{k}_T^2, R_T^2, \cos(\varphi_{RK})) \\
 & + s_T \frac{R_T \sin(\varphi_R - \varphi_S)}{M_1 + M_2} H_1^\times(z, \xi, \mathbf{k}_T^2, R_T^2, \cos(\varphi_{RK})) \\
 & + s_T \frac{k_T \sin(\varphi_k - \varphi_S)}{M_1 + M_2} H_1^\perp(z, \xi, \mathbf{k}_T^2, R_T^2, \cos(\varphi_{RK}))
 \end{aligned}$$

$$\cos(\varphi_{RK}) \doteq \cos(\varphi_R - \varphi_k)$$

$$\mathbf{k}_T = -\frac{\mathbf{P}_{h\perp}}{z}$$

Transverse momentum weighted asymmetries

AK, Mulders: PRD 54 (1996) 1229; PLB 406 (1997) 373-380; Boer, Mulders: PRD 57 (1998) 5780

$$\frac{d\sigma_{\uparrow\downarrow}^{\ell(l,\lambda)+N(P_N,S)\rightarrow\ell(l')+h(P)+X}}{dx dQ^2 d\phi_S dz d^2 P_T} = C(x, Q^2) (\sigma_U \pm S_T \sigma_{Siv} + \dots),$$

$$\sigma_U(x, Q^2, z, \mathbf{P}_T) = \sum_q \int d^2 k_T d^2 p_T \delta^2(\mathbf{P}_T - \mathbf{p}_T - z\mathbf{k}_T) f_1^q(x, k_T^2) D_{1q}^h(z, p_T^2)$$

$$\sigma_{Siv}(x, Q^2, z, \mathbf{P}_T, \hat{\mathbf{s}}) = \sum_q \int d^2 k_T d^2 p_T \delta^2(\mathbf{P}_T - \mathbf{p}_T - z\mathbf{k}_T) \frac{k_T}{M} \sin(\phi_k - \phi_S) f_{1T}^{\perp q}(x, k_T^2) D_{1q}^h(z, p_T^2)$$

Integrate over \mathbf{P}_T with weight $W_{Siv}(\mathbf{P}_T, \hat{\mathbf{s}}) = \frac{P_T}{Mz} \sin(\phi_h - \phi_S)$

$$\sigma_{Siv}^{W_{Siv}}(x, Q^2, z, \hat{\mathbf{s}}) = \int d^2 P_T W_{Siv}(\mathbf{P}_T, \hat{\mathbf{s}}) \sigma_{Siv}(x, Q^2, z, \mathbf{P}_T, \hat{\mathbf{s}})$$

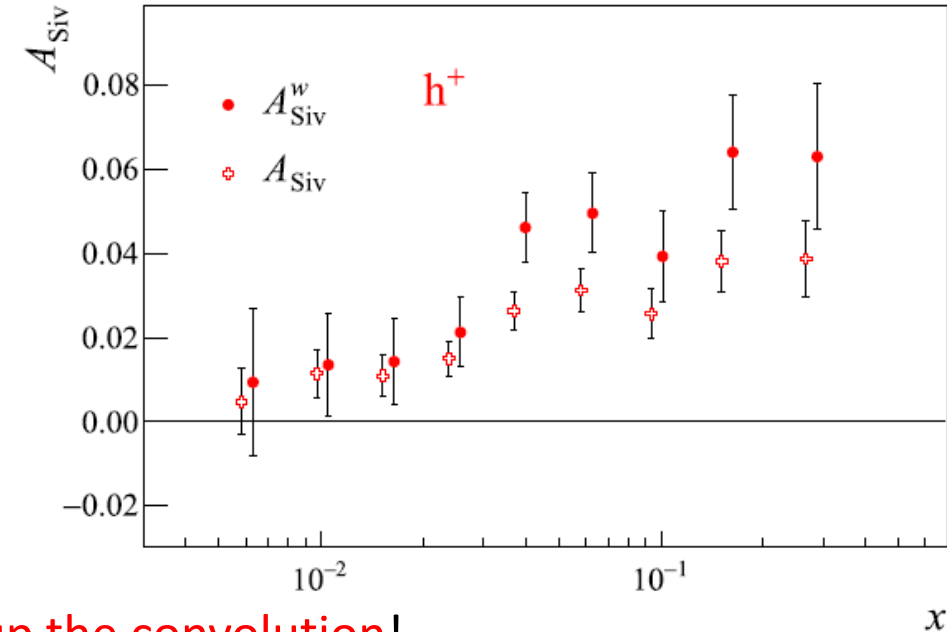
$$A_{Siv}^{W_{Siv}}(x, Q^2, z) = \frac{\sigma_{Siv}^{W_{Siv}}}{\sigma_U}$$

$$\sigma_{Siv}^{W_{Siv}}(x, Q^2, z, \hat{\mathbf{s}}) = \sum_q f_{1T}^{\perp q(1)}(x) D_{1q}^h(z),$$

Weighting break-up the convolution!

$$f_{1T}^{\perp q(1)}(x) = \int d^2 k_T \frac{k_T^2}{2M^2} f_{1T}^{\perp q}(x, k_T^2) \text{ Enters in Burkard sum rule: } \sum_{i=q,\bar{q},g} \int_0^1 dx f_{1T}^{\perp i(1)}(x) = 0$$

COMPASS: NPB 940 (2019) 34

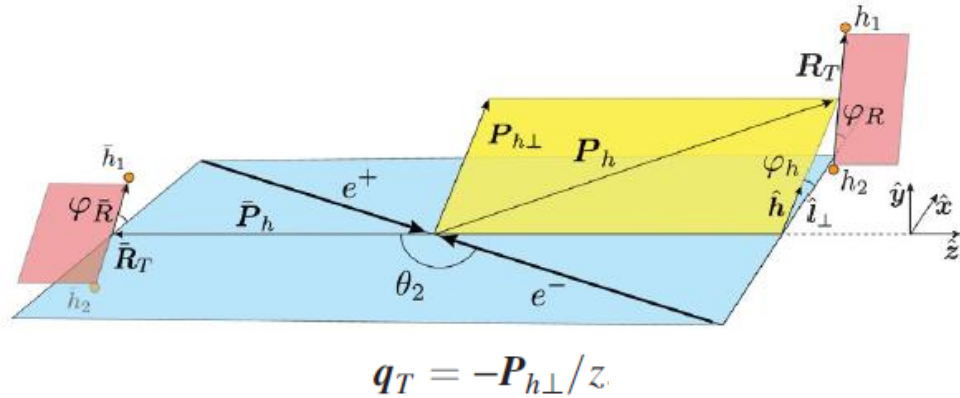


H. Xing, S. Yoshida: [arXiv:1904.00416](https://arxiv.org/abs/1904.00416) [nucl-th]

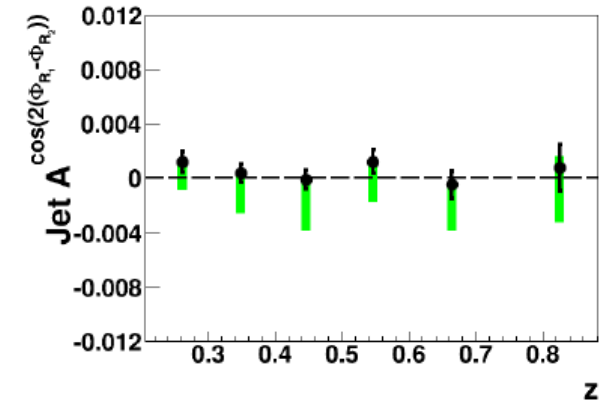
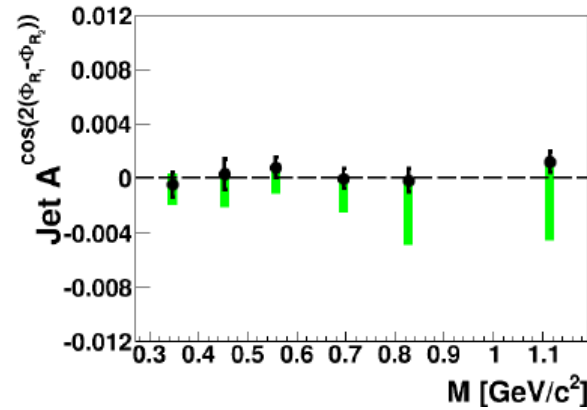
“the transverse-momentum-weighted technique as a useful tool to derive the scale evolution equation for the twist-3 collinear function which is expressed by the first moment of the TMD function.”

Handedness DiFF in e^+e^- and SIDIS

D. Boer, R. Jakob, and M. Radici, Phys. Rev. D 67, 094003 (2003): $W = \cos 2(\varphi_R - \varphi_{\bar{R}})$



BELLE: [arXiv:1505.08020](https://arxiv.org/abs/1505.08020)



Matevosyan, AK, Thomas: PRL 120, 252, 001 (2018): $A^f(\varphi_R, \varphi_{\bar{R}}) \equiv 0$ for $\forall f$

Matevosyan, Bacchetta, Boer, Courtoy, AK, Radici, Thomas: Phys. Rev. D 97, 074019 (2018)

New weight: $W_{New} = q_T^2 [3 \sin(\varphi_q - \varphi_R) \sin(\varphi_q - \varphi_{\bar{R}}) + \cos(\varphi_q - \varphi_R) \cos(\varphi_q - \varphi_{\bar{R}})]$

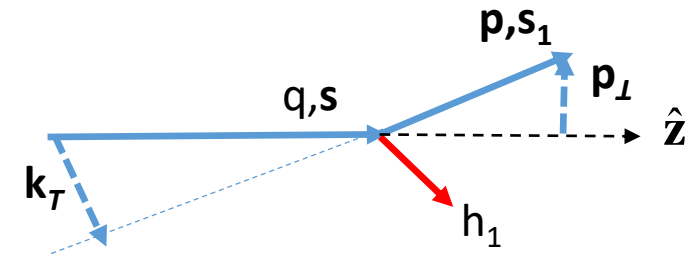
$$A_{e^+e^-}^{W_{New}}(z, \bar{z}, M_h^2, \bar{M}_h^2) = 4 \frac{\sum_a G_1^{\perp a}(z, M_h^2) G_1^{\perp \bar{a}}(\bar{z}, \bar{M}_h^2)}{\sum_a D_1^a(z, M_h^2) D_1^{\bar{a}}(\bar{z}, \bar{M}_h^2)}$$

$$A_{SIDIS}^{m_h \frac{P_h^\perp \sin(\varphi_h - \varphi_R)}{M_h^2}}(x, z, M_h^2) = S_L \frac{\sum_a g_1^a(x) z G_1^{\perp \bar{a}}(z, M_h^2)}{\sum_a f_1^a(z, M_h^2) D_1^a(z, M_h^2)}$$

$$G_1^{\perp a}(z, M_h^2) \equiv G_1^{\perp a, [0]}(z, M_h^2) - G_1^{\perp a, [2]}(z, M_h^2)$$

Twist-2 quark to spin ½ hadron STMD FFs

		Final hadron polarization		
		U	L	T
Initial quark polarization	U	$D_1(z, p_\perp^2)$		$-\frac{\mathbf{k}_T \times \hat{\mathbf{z}}}{\mathcal{M}} D_{1T}^\perp(z, p_\perp^2)$
	L		$s_L G_{1L}(z, p_\perp^2)$	$s_L \frac{\mathbf{k}_T}{\mathcal{M}} G_{1T}(z, p_\perp^2)$
	T	$-\frac{(\mathbf{k}_T \times \mathbf{s}_T) \cdot \hat{\mathbf{z}}}{\mathcal{M}} H_1^\perp(z, p_\perp^2)$	$\frac{\mathbf{k}_T \cdot \mathbf{s}_T}{\mathcal{M}} H_{1L}^\perp(z, p_\perp^2)$	$\frac{\mathbf{s}_T H_{1T}(z, p_\perp^2) + \mathbf{k}_T (\mathbf{k}_T \cdot \mathbf{s}_T)}{\mathcal{M} \mathcal{M}} H_{1T}^\perp(z, p_\perp^2)$



$$\mathbf{k}_T = -\mathbf{p}_\perp / z$$

$$z = \frac{p^0 + p^3}{q^0 + q^3}$$

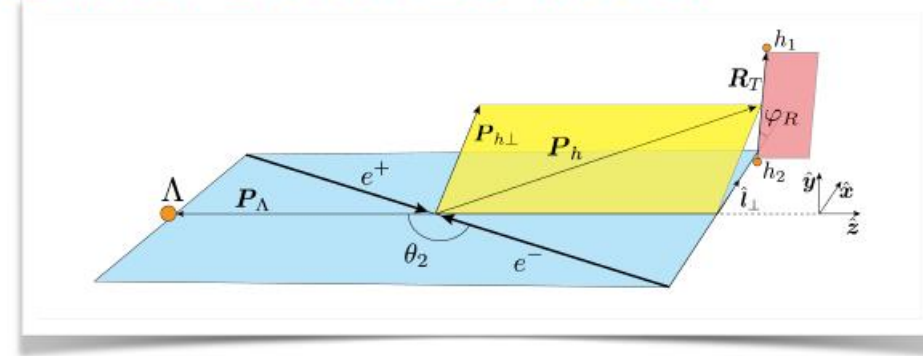
Single hadron and dihadron production in e^+e^-

Matevosyan , AK, Thomas: JHEP 1810 (2018) 008 .

- Use the standard kinematics to derive LO x-sec.

$$\frac{d\sigma(e^+e^- \rightarrow (h_1 h_2) + \Lambda + X)}{d^2q_T dz d\varphi_R dM_h^2 d\xi dz dy} = \frac{3\alpha_{em}^2}{(2\pi)^2 Q^2} z^2 \bar{z}^2 \sum_a e_a^2$$

$$\times \left\{ \begin{aligned} & A(y) \mathcal{F} \left[D_1^{a \rightarrow h_1 h_2} D_1^{\bar{a} \rightarrow \Lambda} \right] \\ & - S_T A(y) \mathcal{F} \left[\frac{\bar{k}_T}{M_\Lambda} \sin(\varphi_{\bar{k}} - \varphi_S) D_1^{a \rightarrow h_1 h_2} D_{1T}^{\perp, \bar{a} \rightarrow \Lambda} \right] \\ & + \lambda_\Lambda A(y) \mathcal{F} \left[\frac{k_T R_T}{M_h^2} \sin(\varphi_k - \varphi_R) G_1^{\perp, a \rightarrow h_1 h_2} G_{1L}^{\bar{a} \rightarrow \Lambda} \right] \\ & + S_T A(y) \mathcal{F} \left[\frac{k_T R_T}{M_h^2} \sin(\varphi_k - \varphi_R) \frac{\bar{k}_T}{M_\Lambda} \cos(\varphi_{\bar{k}} - \varphi_S) G_1^{\perp, a \rightarrow h_1 h_2} G_{1T}^{\bar{a} \rightarrow \Lambda} \right] \\ & + S_T B(y) \mathcal{F} \left[\left(\frac{k_T}{M_h} \sin(\varphi_k + \varphi_S) H_1^{\perp, a \rightarrow h_1 h_2} \right. \right. \\ & \quad \left. \left. + \frac{R_T}{M_h} \sin(\varphi_R + \varphi_S) H_1^{\langle, a \rightarrow h_1 h_2} \right) H_{1T}^{\bar{a} \rightarrow \Lambda} \right] \\ & + \lambda_\Lambda B(y) \mathcal{F} \left[\left(\frac{k_T}{M_h} \sin(\varphi_k + \varphi_{\bar{k}}) H_1^{\perp, a \rightarrow h_1 h_2} \right. \right. \\ & \quad \left. \left. + \frac{R_T}{M_h} \sin(\varphi_R + \varphi_{\bar{k}}) H_1^{\langle, a \rightarrow h_1 h_2} \right) \frac{\bar{k}_T}{M_\Lambda} H_{1L}^{\perp, \bar{a} \rightarrow \Lambda} \right] \\ & + S_T B(y) \mathcal{F} \left[\left(\frac{k_T}{M_h} \sin(\varphi_k + \varphi_{\bar{k}}) H_1^{\perp, a \rightarrow h_1 h_2} \right. \right. \\ & \quad \left. \left. + \frac{R_T}{M_h} \sin(\varphi_R + \varphi_{\bar{k}}) H_1^{\langle, a \rightarrow h_1 h_2} \right) \frac{\bar{k}_T^2}{M_\Lambda^2} \cos(\varphi_{\bar{k}} - \varphi_S) H_{1T}^{\perp, \bar{a} \rightarrow \Lambda} \right] \\ & + B(y) \mathcal{F} \left[\left(\frac{k_T}{M_h} \cos(\varphi_k + \varphi_{\bar{k}}) H_1^{\perp, a \rightarrow h_1 h_2} \right. \right. \\ & \quad \left. \left. + \frac{R_T}{M_h} \cos(\varphi_R + \varphi_{\bar{k}}) H_1^{\langle, a \rightarrow h_1 h_2} \right) \frac{\bar{k}_T}{M_\Lambda} H_{1T}^{\perp, \bar{a} \rightarrow \Lambda} \right] \end{aligned} \right\},$$



$$q_T = -P_{h\perp}/z.$$

$$A(y) = \frac{1}{2} y - y + y^2$$

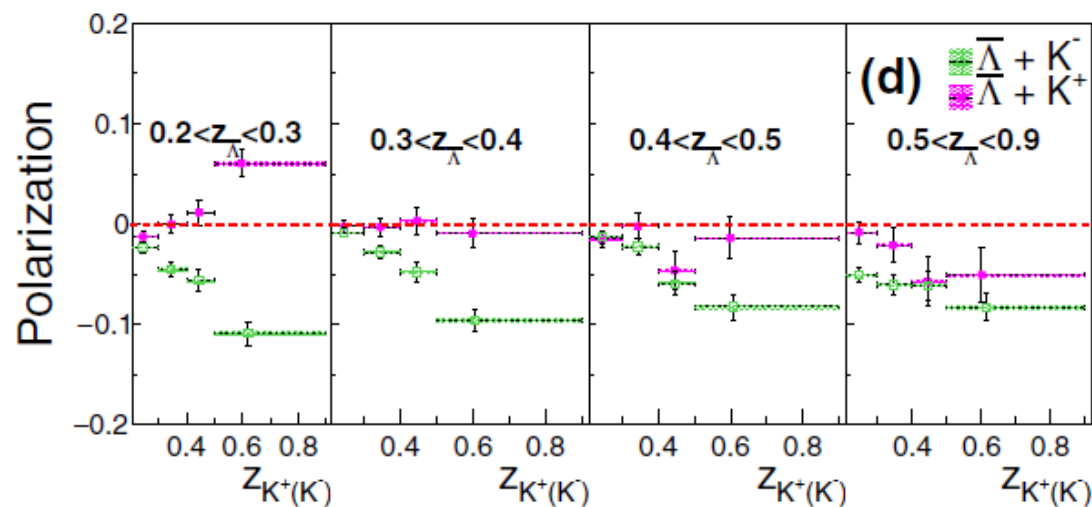
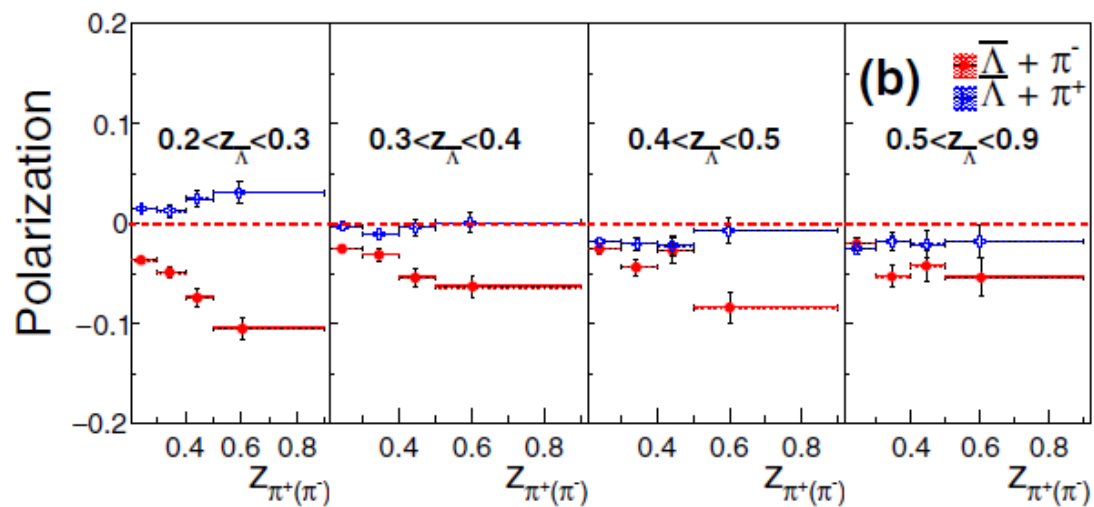
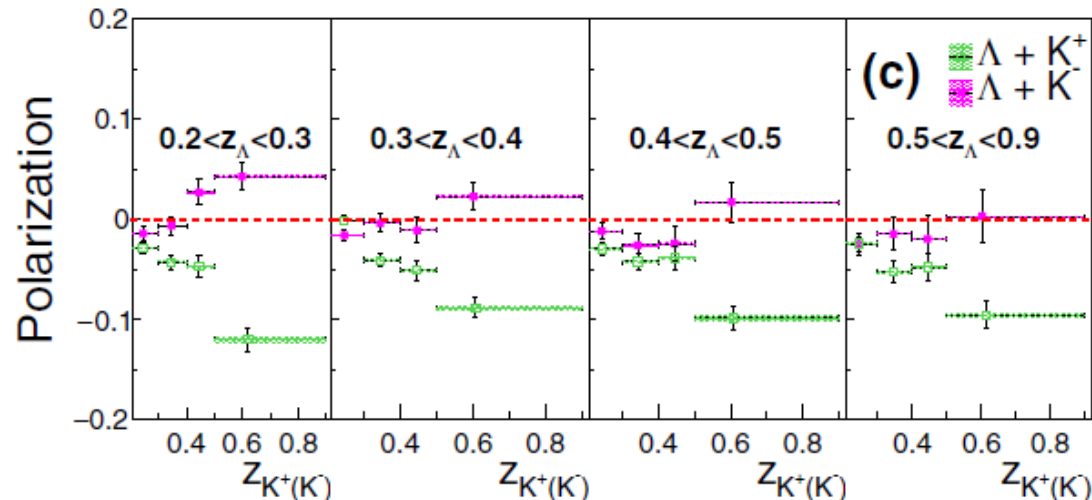
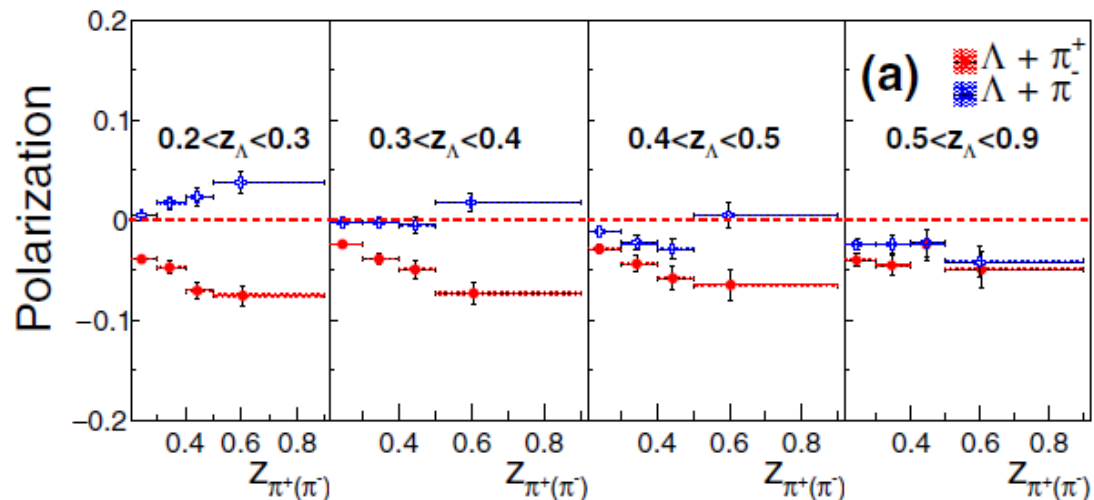
$$B(y) = y(1 - y)$$

$$y = \left(1 + \frac{1 + \cos^2 \theta_2}{2} \right)$$

BELLE: single hadron + Λ transverse polarization

Phys.Rev.Lett. 122 (2019) no.4, 042001

access to polarizing FF D_{1T}^\perp



Flavor decomposition of DiFFs

❖ Integrated cross section

$$\frac{d\sigma(e^+e^- \rightarrow (h_1 h_2) + \Lambda + X)}{dz dM_h^2 d\bar{z} dy} = \frac{3\alpha_{em}^2}{(2\pi)^2 Q^2} A(y) \sum_a e_a^2 D_1^{a \rightarrow h_1 h_2}(z, M_h^2) \bar{D}_1^{\bar{a} \rightarrow \Lambda}(\bar{z}),$$

❖ Isospin symmetry

$$D_1^{u \rightarrow \pi^+ \pi^-} = D_1^{\bar{u} \rightarrow \pi^+ \pi^-} \approx D_1^{d \rightarrow \pi^+ \pi^-} = D_1^{\bar{d} \rightarrow \pi^+ \pi^-},$$
$$D_1^{s \rightarrow \pi^+ \pi^-} = D_1^{\bar{s} \rightarrow \pi^+ \pi^-}.$$

❖ One pair inclusive: cannot disentangle the flavor dependence

$$d\sigma(e^+e^- \rightarrow (h_1 h_2) + X) \sim \sum_q e_q^2 D_1^{q \rightarrow \pi^+ \pi^-} \approx \frac{5}{9} D_1^{u \rightarrow \pi^+ \pi^-}(z) + \frac{1}{9} D_1^{s \rightarrow \pi^+ \pi^-}(z)$$

❖ New process: use the knowledge of single hadron FFs!

$$d\sigma(e^+e^- \rightarrow (h_1 h_2) + \pi^+ + X) \sim \frac{5}{9} D_1^{u \rightarrow \pi^+ \pi^-}(z) D_1^{u^+ \rightarrow \pi^+}(\bar{z}) + \frac{1}{9} D_1^{s \rightarrow \pi^+ \pi^-}(z) D_1^{s^+ \rightarrow \pi^+}(\bar{z})$$

$$D_1^{q^+ \rightarrow h}(\bar{z}) \equiv D_1^{q \rightarrow h}(\bar{z}) + D_1^{\bar{q} \rightarrow h}(\bar{z})$$

Weighted asymmetries in e^+e^- annihilation: unpolarized hadron

- ❖ Unpolarized hadrons: Accessing Collins x IFF.

$$\left\langle \frac{q_T}{M_\Lambda} \cos(\varphi_q + \varphi_R) \right\rangle = \frac{3\alpha_{em}^2}{(2\pi)^2 Q^2} \frac{B(y)}{M_\Lambda^2 M_h} \times \sum_a e_a^2 \int d\xi \int d\varphi_R \int d^2 \mathbf{q}_T \int d^2 \mathbf{k}_T \int d^2 \bar{\mathbf{k}}_T \delta^2(\mathbf{k}_T + \bar{\mathbf{k}}_T - \mathbf{q}_T) q_T \cos(\varphi_q + \varphi_R) \times \left[\left(k_T \bar{k}_T \cos(\varphi_k + \varphi_{\bar{k}}) H_1^{\perp, a \rightarrow h_1 h_2} + R_T \bar{k}_T \cos(\varphi_R + \varphi_{\bar{k}}) H_1^{\triangleleft, a \rightarrow h_1 h_2} \right) H_1^{\perp, \bar{a} \rightarrow \Lambda} \right],$$

any unpolarized hadron

- ❖ Momentum weighing helps to disentangle TM convolutions.

$$\int d^2 \mathbf{q}_T \delta^2(\mathbf{k}_T + \bar{\mathbf{k}}_T - \mathbf{q}_T) q_T \cos(\varphi_q + \varphi_R) = (k_T \cos(\varphi_k + \varphi_R) + \bar{k}_T \cos(\varphi_{\bar{k}} + \varphi_R)).$$

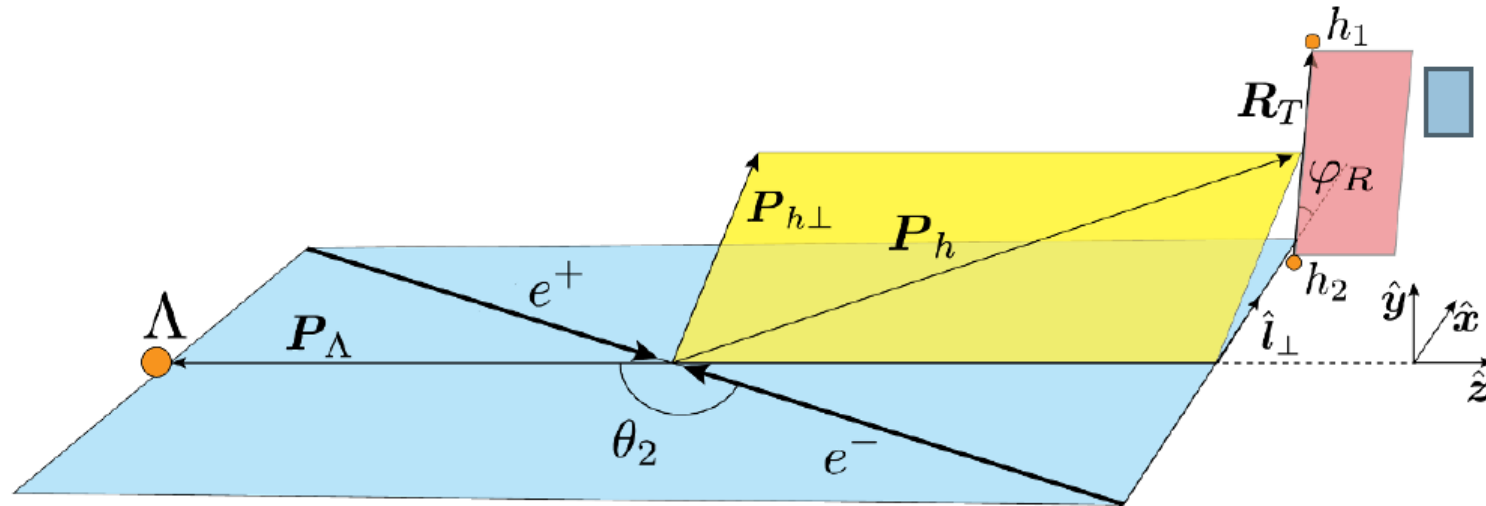
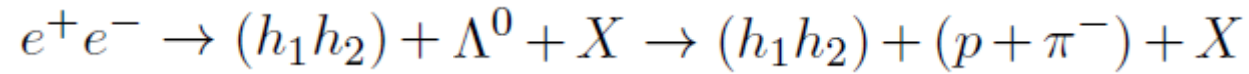
- ❖ Resulting moment and the asymmetry.

$$\left\langle \frac{q_T}{M_\Lambda} \cos(\varphi_q + \varphi_R) \right\rangle = \frac{3\alpha_{em}^2}{(2\pi)^2 Q^2} B(y) \sum_a e_a^2 H_1^{\triangleleft, a \rightarrow h_1 h_2}(z, M_h^2) H_1^{\perp, \bar{a}, [1]}(\bar{z}),$$

$$A^{Coll} = \frac{B(y) \sum_a e_a^2 H_1^{\triangleleft, a \rightarrow h_1 h_2}(z, M_h^2) H_1^{\perp, \bar{a}, [1]}(\bar{z})}{A(y) \sum_a e_a^2 D_1^{a \rightarrow h_1 h_2}(z, M_h^2) \bar{D}_1^{\bar{a} \rightarrow \Lambda}(\bar{z})}.$$

First moment of
single hadron
Collins FF

Λ polarization treatment



$$\frac{dN}{Nd \cos \theta} \sim 1 + \alpha_\Lambda S_\Lambda \cos(\theta),$$

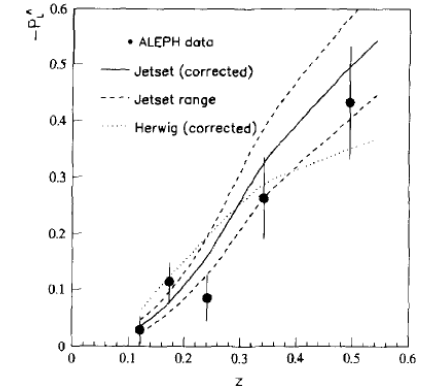
$$S_L \sim \left\langle \cos(\theta_p) \frac{qT}{M_h} \sin(\varphi_q - \varphi_R) \right\rangle \sim \alpha_\Lambda G_1^{\perp, a \rightarrow h_1 h_2} G_{1L}^{\bar{a} \rightarrow \Lambda},$$

Weighted “polarization” asymmetries: Λ longitudinal polarization

The correlations between the longitudinal polarizations of the fragmenting quark and antiquark \rightarrow longitudinal polarization s_L of Λ

$$\langle s_L \rangle^{\sin(\varphi_q - \varphi_R)}(z, M_h^2, \bar{z}, y) = \frac{\sum_a e_a^2 G_1^{\perp, a \rightarrow h_1 h_2}(z, M_h^2) G_{1L}^{\bar{a} \rightarrow \Lambda}(\bar{z})}{\sum_a e_a^2 D_1^{a \rightarrow h_1 h_2}(z, M_h^2) \bar{D}_1^{\bar{a} \rightarrow \Lambda}(\bar{z})}$$

ALEPH: PLB 374 (1996) 319



The correlations between the transverse polarizations of the fragmenting quark and antiquark \rightarrow longitudinal polarization s_L of Λ

$$\langle s_L \rangle^{\sin(\varphi_q + \varphi_R)}(z, M_h^2, \bar{z}, y) = \frac{B(y)}{A(y)} \frac{\sum_a e_a^2 H_1^{\leftarrow, a \rightarrow h_1 h_2}(z, M_h^2) H_{1L}^{\perp \bar{a}, [1]}(\bar{z})}{\sum_a e_a^2 D_1^{a \rightarrow h_1 h_2}(z, M_h^2) \bar{D}_1^{\bar{a} \rightarrow \Lambda}(\bar{z})}$$

Weighted “polarizations” asymmetries: Λ transverse polarization

The correlations between the transverse polarizations of the fragmenting quark and antiquark \rightarrow longitudinal polarization s_L of Λ

$$\langle s_T \rangle_x^{\sin(\varphi_R)}(z, M_h^2, \bar{z}, y) = \langle s_T \rangle_y^{\cos(\varphi_R)}(z, M_h^2, \bar{z}, y) = \frac{1}{2} \frac{B(y)}{A(y)} \frac{\sum_a e_a^2 H_1^{\leftarrow, a \rightarrow h_1 h_2}(z, M_h^2) H_1^{\bar{a}}(\bar{z})}{\sum_a e_a^2 D_1^{a \rightarrow h_1 h_2}(z, M_h^2) \bar{D}_1^{\bar{a} \rightarrow \Lambda}(\bar{z})}$$

$$\frac{\langle s_y \rangle^{\cos(\varphi_q)}(z, M_h^2, \bar{z}, y) - \langle s_x \rangle^{\sin(\varphi_q)}(z, M_h^2, \bar{z}, y)}{M_\Lambda} = 2 \frac{\sum_a e_a^2 D_1^{a \rightarrow h_1 h_2}(z, M_h^2) D_{1T}^{\perp \bar{a}, [1]}(\bar{z})}{\sum_a e_a^2 D_1^{a \rightarrow h_1 h_2}(z, M_h^2) \bar{D}_1^{\bar{a} \rightarrow \Lambda}(\bar{z})}$$

$$\frac{\langle s_y \rangle^{\cos(\varphi_q)}(z, M_h^2, \bar{z}, y) + \langle s_x \rangle^{\sin(\varphi_q)}(z, M_h^2, \bar{z}, y)}{M_h} = \frac{2B(y)}{A(y)} \frac{\sum_a e_a^2 H_1^{\perp, a \rightarrow h_1 h_2}(z, M_h^2) H_1^{\bar{a} \rightarrow \Lambda}(\bar{z})}{\sum_a e_a^2 D_1^{a \rightarrow h_1 h_2}(z, M_h^2) \bar{D}_1^{\bar{a} \rightarrow \Lambda}(\bar{z})}$$

Disentangling chiral-even and chiral-odd contributions

Weighted polarized asymmetries gives access to all twist-2 STMD FFs

		Final hadron polarization		
		U	L	T
Initial quark polarization	U	$D_1(z, p_\perp^2)$		$-\frac{\mathbf{k}_T \times \hat{\mathbf{z}}}{\mathcal{M}} D_{1T}^\perp(z, p_\perp^2)$
	L		$s_L G_{1L}(z, p_\perp^2)$	$s_L \frac{\mathbf{k}_T}{\mathcal{M}} G_{1T}(z, p_\perp^2)$
	T	$-\frac{(\mathbf{k}_T \times \mathbf{s}_T) \cdot \hat{\mathbf{z}}}{\mathcal{M}} H_1^\perp(z, p_\perp^2)$	$\frac{\mathbf{k}_T \cdot \mathbf{s}_T}{\mathcal{M}} H_{1L}^\perp(z, p_\perp^2)$	$s_T H_1(z, p_\perp^2)$

Too long and complicated expression to single out contribution from this “worm-gear” function

Summary

- In our recent works we proposed new measurements in e^+e^- annihilation to probe different combinations of FFs \otimes DiFFs
 - The BELLE zero result in quark handedness TMD DiFF study was explained
 - New weighted asymmetries are proposed for measurement of this DiFFs both in SIDIS and SIA
 - New weighted “polarized” asymmetries to access different STMD FFs for spin $\frac{1}{2}$ hadron production in coincidence with dihadron from opposite jet are derived
- Study of these asymmetries will help for flavor decomposition of FFs and DiFFs and for test of their universality
- Can be done at BELLE II, Jlab 12 and future EIC

Fourier moments of DiFFs

$$D_1(z, \xi, \mathbf{k}_T^2, \mathbf{R}_T^2, \cos(\varphi_{KR})) = \frac{1}{\pi} \sum_{n=0}^{\infty} \frac{\cos(n \cdot \varphi_{KR})}{1 + \delta_{0,n}} D_1^{[n]}(z, \xi, |\mathbf{k}_T|, |\mathbf{R}_T|),$$
$$F^{[n]} = \int d\varphi_{KR} \cos(n\varphi_{KR}) F(\cos(\varphi_{KR}))$$

We define Fourier moments of integrated over pair total momentum weighted DiFFs and

$$D_1^a(z, M_h^2) = z^2 \int d^2\mathbf{k}_T \int d\xi D_1^{a,[0]}(z, \xi, \mathbf{k}_T^2, \mathbf{R}_T^2)$$

$$G_1^{\perp a,[n]}(z, M_h^2) = z^2 \int d^2\mathbf{k}_T \int d\xi \left(\frac{\mathbf{k}_T^2}{2M_h^2} \right) \frac{|\mathbf{R}_T|}{M_h} G_1^{\perp a,[n]}(z, \xi, \mathbf{k}_T^2, \mathbf{R}_T^2)$$

$$H_1^{\triangleleft,[n]}(z, M_h^2) = z^2 \int d^2\mathbf{k}_T \int d\xi \frac{|\mathbf{R}_T|}{M_h} H_1^{\triangleleft,[n]}(z, \xi, |\mathbf{k}_T|, |\mathbf{R}_T|)$$

$$H_1^{\perp,[n]}(z, M_h^2) = z^2 \int d^2\mathbf{k}_T \int d\xi \frac{|\mathbf{k}_T|}{M_h} H_1^{\perp,[n]}(z, \xi, |\mathbf{k}_T|, |\mathbf{R}_T|)$$

Rederiving dihadron production cross-sections in e^+e^- and SIDIS

Matevosyan , AK, Thomas: PRL 120, 252, 001 (2018), : [arXiv:1712.06384](https://arxiv.org/abs/1712.06384).

Matevosyan, Bacchetta, Boer, Courtoy, AK, Radici, Thomas: Phys. Rev. D 97, 074019 (2018), [arXiv:1802.01578](https://arxiv.org/abs/1802.01578)

Fully differential cross section

$$\begin{aligned}
 & \frac{d\sigma(e^+e^- \rightarrow (h_1 h_2)(\bar{h}_1 \bar{h}_2)X)}{d^2\mathbf{q}_T dz d\xi d\varphi_R dM_h^2 d\bar{z} d\bar{\xi} d\varphi_{\bar{R}} d\bar{M}_h^2 dy} \\
 &= \frac{3\alpha^2}{\pi Q^2} z^2 \bar{z}^2 \sum_{a,\bar{a}} e_a^2 \left\{ A(y) \mathcal{F}[D_1^a \bar{D}_1^{\bar{a}}] + B(y) \mathcal{F}\left[\frac{|\mathbf{k}_T| |\bar{\mathbf{k}}_T|}{M_h \bar{M}_h} \cos(\varphi_k + \varphi_{\bar{k}}) H_1^{\perp a} \bar{H}_1^{\perp \bar{a}}\right] \right. \\
 &+ B(y) \mathcal{F}\left[\frac{|\mathbf{R}_T| |\bar{\mathbf{R}}_T|}{M_h \bar{M}_h} \cos(\varphi_R + \varphi_{\bar{R}}) H_1^{\leftarrow a} \bar{H}_1^{\leftarrow \bar{a}}\right] + B(y) \mathcal{F}\left[\frac{|\mathbf{k}_T| |\bar{\mathbf{R}}_T|}{M_h \bar{M}_h} \cos(\varphi_k + \varphi_{\bar{R}}) H_1^{\perp a} \bar{H}_1^{\leftarrow \bar{a}}\right] \\
 &\left. + B(y) \mathcal{F}\left[\frac{|\mathbf{R}_T| |\bar{\mathbf{k}}_T|}{M_h \bar{M}_h} \cos(\varphi_R + \varphi_{\bar{k}}) H_1^{\leftarrow a} \bar{H}_1^{\perp \bar{a}}\right] - A(y) \mathcal{F}\left[\frac{|\mathbf{R}_T| |\mathbf{k}_T| |\bar{\mathbf{R}}_T| |\bar{\mathbf{k}}_T|}{M_h^2 \bar{M}_h^2} \sin(\varphi_k - \varphi_R) \sin(\varphi_{\bar{k}} - \varphi_{\bar{R}}) G_1^{\perp a} \bar{G}_1^{\perp \bar{a}}\right] \right\}
 \end{aligned}$$

$$\mathcal{F}[w D^a \bar{D}^{\bar{a}}] = \int d^2\mathbf{k}_T d^2\bar{\mathbf{k}}_T \delta^2(\mathbf{k}_T + \bar{\mathbf{k}}_T - \mathbf{q}_T) w(\mathbf{k}_T, \bar{\mathbf{k}}_T, \mathbf{R}_T, \bar{\mathbf{R}}_T) D^a(z, \xi, \mathbf{k}_T^2, \mathbf{R}_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) D^{\bar{a}}(\bar{z}, \bar{\xi}, \bar{\mathbf{k}}_T^2, \bar{\mathbf{R}}_T^2, \bar{\mathbf{k}}_T \cdot \bar{\mathbf{R}}_T)$$

IFFs in e^+e^- and SIDIS

- *The asymmetry now involves exactly the same integrated IFF as in SIDIS!*

$$A^{\cos(\varphi_R + \varphi_{\bar{R}})} = \frac{1}{2} \frac{B(y)}{A(y)} \frac{\sum_{a, \bar{a}} e_a^2 H_1^{\triangleleft a}(z, M_h^2) \bar{H}_1^{\triangleleft \bar{a}}(\bar{z}, \bar{M}_h^2)}{\sum_{a, \bar{a}} e_a^2 D_1^a(z, M_h^2) \bar{D}_1^{\bar{a}}(\bar{z}, \bar{M}_h^2)}$$

$$D_1(z, M_h^2) \equiv z^2 \int d^2 \mathbf{k}_T \int d\xi D_1^{[0]}(z, \xi, |\mathbf{k}_T|, |\mathbf{R}_T|)$$

$$H_{1, e^+e^-}^{\triangleleft}(z, M_h^2) = H_1^{\triangleleft, [0]} + H_1^{\perp, [1]} \equiv H_{1, SIDIS}^{\triangleleft}(z, M_h^2)$$

- *All the previous extractions of the transversity are valid !*