

Nucleon longitudinal spin structure

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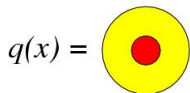


EINN2019 Workshop
The Road to the Electron-Ion Collider

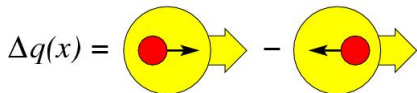
Paphos, 27 October – 02 November 2019

Partonic structure of the nucleon; distribution functions

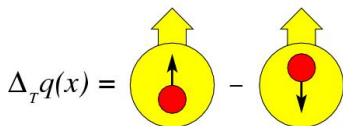
Three **twist-two** quark distributions in QCD and after integrating over the quark intrinsic k_t



Quark momentum DF;
well known (unpolarised DIS $\rightarrow \mathbf{F}_{1,2}(x, Q^2)$).



Difference in DF of quarks with spin parallel or antiparallel to the nucleon's spin in a longitudinally polarised nucleon;
less well known (polarised DIS $\rightarrow g_1(x, Q^2)$).

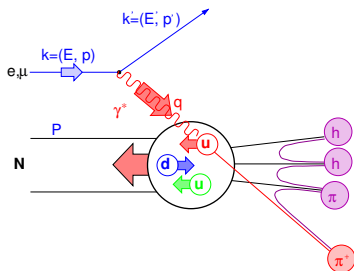


Difference in DF of quarks with spin parallel or antiparallel to the nucleon's spin in a transversely polarised nucleon;
poorly known (polarised DIS $\rightarrow h_1(x, Q^2)$).

Nonrelativistically: $\Delta_T q(x, Q^2) \equiv \Delta q(x, Q^2)$. **OBS.!** $\Delta_T q(x, Q^2)$ are C-odd and chiral-odd

Goal: measurement of $g_1(x, Q^2)$ and determination of $\Delta q(x, Q^2)$

Nucleon (spin) structure in DIS: $\vec{\mu} + \vec{N} \rightarrow \mu' + X$

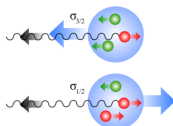


- $\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2}{2Mq^4} \frac{E'}{E} L_{\mu\nu} W^{\mu\nu}$
- Symmetric part of $W^{\mu\nu}$ – unpolarised DIS, antisymmetric – polarised DIS
- Nominally $F_{1,2}, q(x, Q^2) \rightarrow g_{1,2}, \Delta q(x, Q^2)$ where $q = q^+ + q^-, \Delta q = q^+ - q^-$, but...
 - ...anomalous gluon contribution to $g_1(x, Q^2)$
 - ... $g_2(x, Q^2)$ has no interpretation in terms of partons.

Definitions of DIS variables...

$Q^2 = -q^2$	γ^* virtuality
$x = Q^2/(2Pq)$	Bjorken variable
$y = Pq/(Pk)$	relative γ^* energy
$W = P + q$	γ^* -N cms energy

...and of the γ^* -N asymmetry (e.g. for γ^* -p):



$$A_1^p(x, Q^2) = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}}$$

$$A_2^p(x, Q^2) = \frac{2\sigma^{\text{TL}}}{\sigma_{1/2} + \sigma_{3/2}}$$

Full hadronic tensor for proton

$$\begin{aligned}
 W_{\mu\nu} = & (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}) F_1(x, Q^2) + \frac{\hat{P}_\mu \hat{P}_\nu}{P \cdot q} F_2(x, Q^2) - i\varepsilon_{\mu\nu\lambda\sigma} \frac{q^\lambda P^\sigma}{2P \cdot q} F_3(x, Q^2) \\
 & + i\varepsilon_{\mu\nu\lambda\sigma} \frac{q^\lambda S^\sigma}{P \cdot q} g_1(x, Q^2) + i\varepsilon_{\mu\nu\lambda\sigma} \frac{q^\lambda (P \cdot q S^\sigma - S \cdot q P^\sigma)}{(P \cdot q)^2} g_2(x, Q^2) \\
 & + \left[\frac{\hat{P}_\mu \hat{S}_\nu + \hat{S}_\mu \hat{P}_\nu}{2} - S \cdot q \frac{\hat{P}_\mu \hat{P}_\nu}{(P \cdot q)} \right] \frac{g_3(x, Q^2)}{P \cdot q} \\
 & + S \cdot q \frac{\hat{P}_\mu \hat{P}_\nu}{(P \cdot q)^2} g_4(x, Q^2) + (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}) \frac{(S \cdot q)}{P \cdot q} g_5(x, Q^2),
 \end{aligned}$$

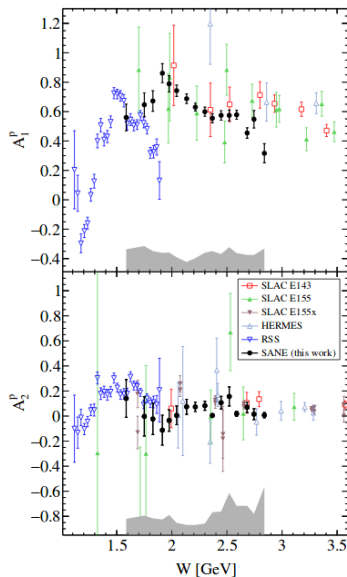
Blümlein, Kochelev, arXiv:hep-ph/9612318v2

- 5 spin-dependent structure functions: $g_1 \div g_5$; g_3, g_4, g_5 at CC
- 31 relations between them
- several sum rules (e.g. Burkhardt-Cottingham: $\int_0^1 dx g_2(x, Q^2) = 0$)
- g_2 and g_3 have both twist-2 and twist-3 contributions
- if no target mass corrections then the twist-3 matrix element

$$\tilde{d}_2 = \int_0^1 dx x^2 [2g_1(x, Q^2) + 3g_2(x, Q^2)]$$

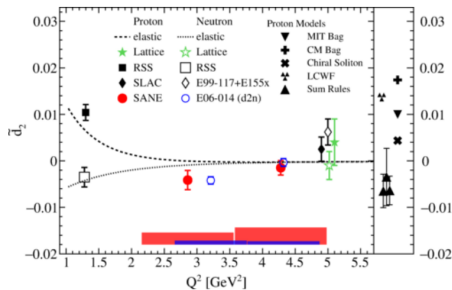
may be measured experimentally; also accessed in LQCD

Measurement of \tilde{d}_2 by SANE (JLab/Hall C)



Color Lorentz force in the proton??

SANE PRL 122 (2019) 022002

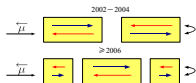


Method of extraction of g_1

- Inclusive asymmetry, $A_{meas}(x, Q^2)$, γ^* -N asymmetry, $A_1(x, Q^2)$, and $g_1(x, Q^2)$:

$$A_{meas} = \frac{1}{f P_T P_B} \left(\frac{N^{\rightarrow} - N^{\leftarrow}}{N^{\rightarrow} + N^{\leftarrow}} \right) \approx D A_1 = D \frac{g_1(x, Q^2)}{F_1(x, Q^2)} \stackrel{LO}{=} D \frac{\sum_q e_q^2 \Delta q(x, Q^2)}{\sum_q e_q^2 q(x, Q^2)}$$

f, D : dilution and depolarisation factors; P_T, P_B : target and beam polarisations;
 $N^{\rightarrow, \leftarrow}$: number of $\vec{\mu}$ interactions in each target cell:
 (upstream, downstream) or (outer, central)



- Then $g_1(x, Q^2)$:

$$g_1(x, Q^2) = A_1(x, Q^2) \cdot F_1(x, Q^2) = A_1(x, Q^2) \cdot \frac{F_2(x, Q^2)}{2x(1 + R(x, Q^2))}$$

- For the deuteron target:

$$(\text{per nucleon}) g_1^d = g_1^N \left(1 - \frac{3}{2} \omega_D\right) = \frac{g_1^p + g_1^n}{2} \left(1 - \frac{3}{2} \omega_D\right); \quad \omega_D = 0.05 \pm 0.01$$

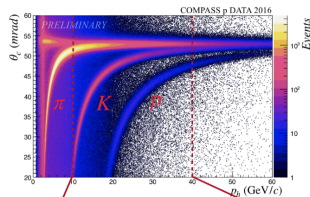
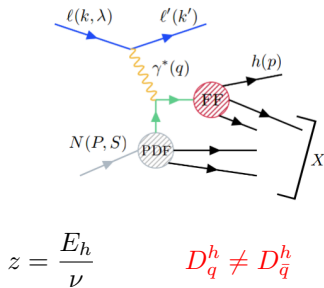
Method of extraction of g_1 in a $\bar{\mu}\vec{N}$ fixed-target experiment,... cont'd

- At LO, semi-inclusive (SIDIS) asymmetry, A_1^h :

$$A_1^h(x, z, Q^2) \approx \frac{\sum_q e_q^2 \Delta q(x, Q^2) D_q^h(z, Q^2)}{\sum_q e_q^2 q(x, Q^2) D_q^h(z, Q^2)}$$

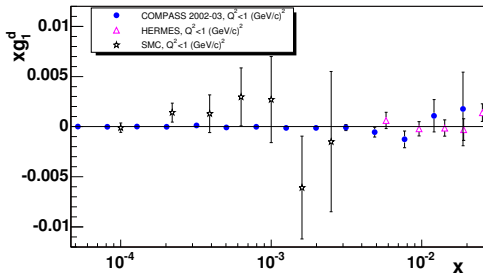
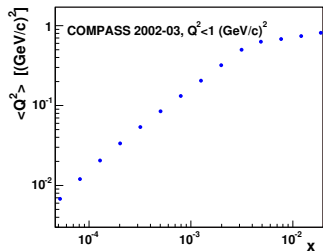
$$A^{\text{SIDIS}} \sim \text{pdf} \otimes \text{FF}$$

Nonperturbative fragmentation functions $D_q^h(z, Q^2)$ need to be determined from experiment!



g_1^d at low Q^2

g_1^d in the nonperturbative ($Q^2 < 1$ (GeV/c) 2 region)

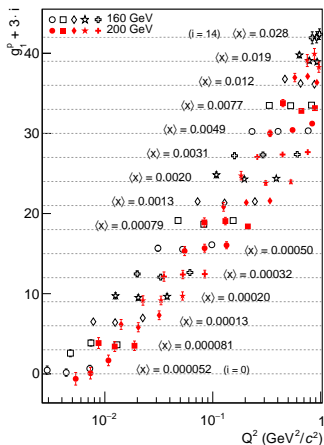
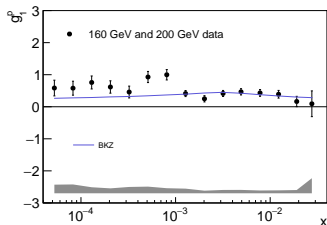
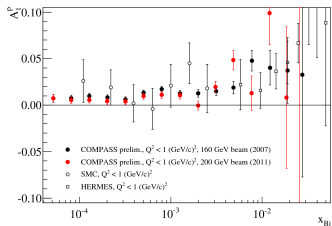


COMPASS PL B 647 (2007) 330

- More than ten-fold improvement over the statistical precision of SMC.
- Spin effects in g_1^d at low x and Q^2 absent ?

g_1^p at low Q^2

Low Q^2 : results on $A_1^P(x)$ and $g_1^P(x, Q^2)$



- More than ten-fold improvement over the statistical precision of SMC.
- Very clear spin effects in g_1^P at low x and Q^2

COMPASS PLB 781 (2018) 464



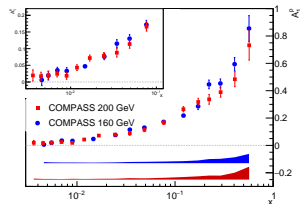
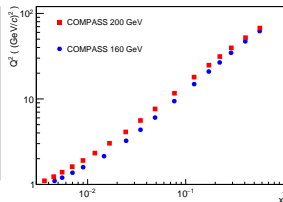
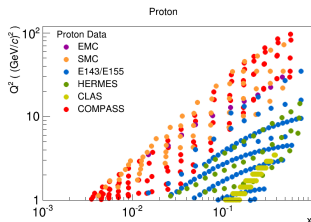
Importance of $g_1(x, Q^2)$ at low x, Q^2

- At low x : strong increase of parton density with decreasing x
⇒ parton recombination effects
- Low x effects particularly visible in $g_1^{\text{NS}} \Rightarrow \ln^2(1/x)$ terms
- BUT: in fixed-target experiments low x correlated with low Q^2 region
⇒ nonperturbative effects must be considered
and parton mechanisms have to be suitably extended to low Q^2 .
- Attempts to describe that region phenomenologically:
 - B. B., J. Kwiecinski, J. Kiriyluk, Phys. Rev. D61 (2000) 014009.
 - B. B., J. Kwiecinski, B. Ziaja, Eur. Phys. J. C26 (2002) 45.
 - B.I. Ermolaev, M. Greco, S.I. Troyan, Eur. Phys. J. C50 (2007) 823;
ibid. C51 (2007) 859.
 - W. Zhu, J. Ruan, Int. J. Mod. Phys. E24 (2015) 1550077.

g_1^p at $Q^2 > 1 \text{ (GeV}/c)^2$

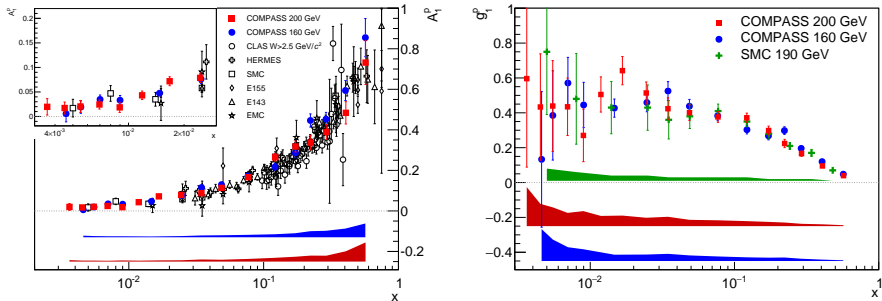
g_1^p : data samples

- Measurements at incident $E = 160$ GeV (from 2007): PLB 690 (2010) 466
- Measurements at incident $E = 200$ GeV (from 2011): PLB 753 (2016) 18
energy increased to reach lower x and higher Q^2
(and to balance the amount of data from the deuteron target)
- Final sample: 85 million events @ 160 GeV + 77 million events @ 200 GeV
- Results on A_1^p at both energies agree very well



Results on $A_1^P(x)$ and $g_1^P(x)$

- $A_1^P(x)$ and $g_1^P(x)$ shown at the measured values of Q^2
- Bands of systematic uncertainties for each energy separately



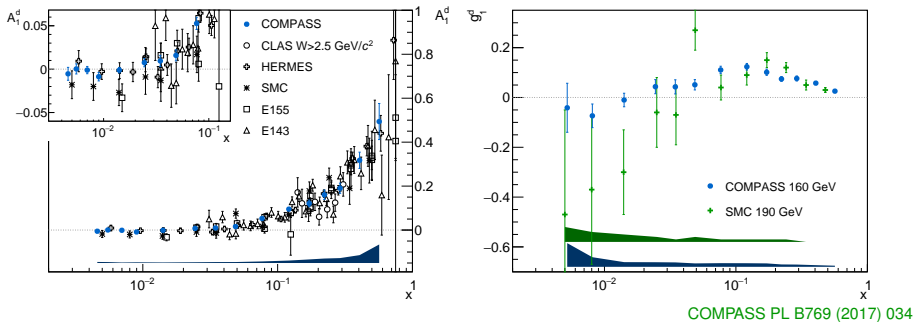
PLB 753 (2016) 18

- Good agreement of $A_1^P(x)$ and $g_1^P(x)$ with world data
- $g_1^P(x)$ clearly positive at lowest measured values of x

g_1^d at $Q^2 > 1 \text{ (GeV}/c)^2$

Results on $A_1^d(x)$ and $g_1^d(x)$

- Combined results from 2002–2004 and 2006
- $A_1^d(x)$ and $g_1^d(x)$ shown at the measured values of Q^2

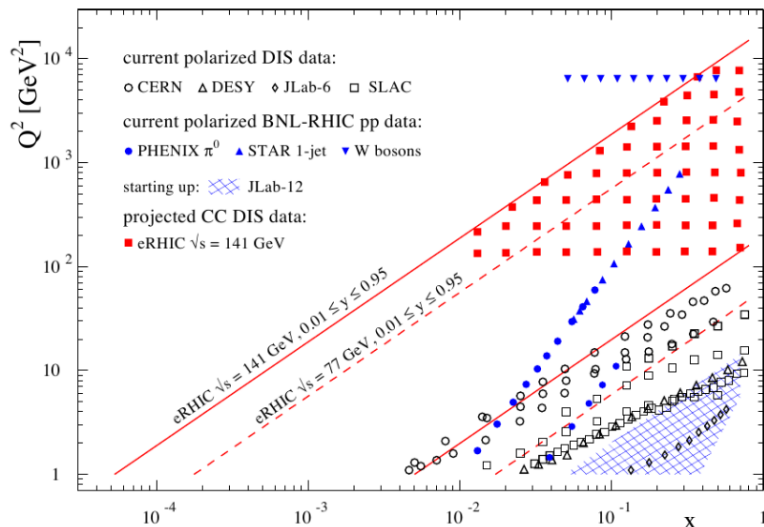


- Good agreement of $A_1^d(x)$ and $g_1^d(x)$ with world data
- $g_1^d(x)$ compatible with zero at lowest measured values of x , contrary to previous hints (SMC)

NLO QCD fit to p,d, ^3He world data

COMPASS, PLB 753 (2016) 18

Current polarised DIS data (including CC DIS)



arXiv: 1409.1633

NLO QCD fit: conditions

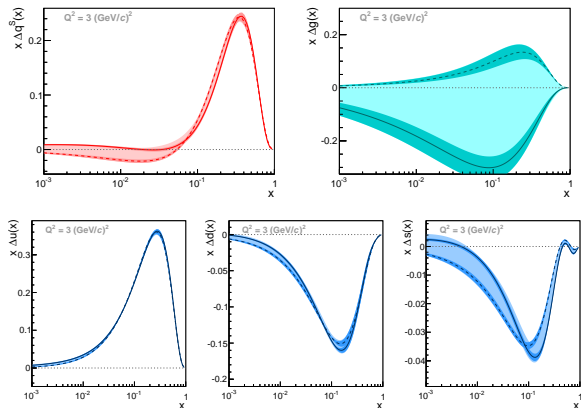
- A fit of the g_1^p , g_1^d , $g_1^{3\text{He}}$ inclusive data
- $\overline{\text{MS}}$ scheme
- $W^2 < 10 \text{ (GeV}/c^2)^2$ excluded
- Number of data points total/COMPASS: 495/138
- Fitted: Δg , $\Delta q^s = \Delta(u + \bar{u}) + \Delta(d + \bar{d}) + \Delta(s + \bar{s})$, gluons, singlet
 $\Delta q_3 = \Delta(u + \bar{u}) - \Delta(d + \bar{d})$, nonsinglet
 $\Delta q_8 = \Delta(u + \bar{u}) + 2\Delta(d + \bar{d}) - \Delta(s + \bar{s})$, nonsinglet
- Parameterisation (at $Q_0^2 = 1 \text{ (GeV}/c)^2$):

$$\Delta f_k(x) = \eta_k \frac{x^{\alpha_k} (1-x)^{\beta_k} (1 + \gamma_k x)}{\int_0^1 x^{\alpha_k} (1-x)^{\beta_k} (1 + \gamma_k x) dx}, \quad (k = s, 3, 8, g)$$

η_k = first moment of $\Delta f_k(x)$ at Q_0^2

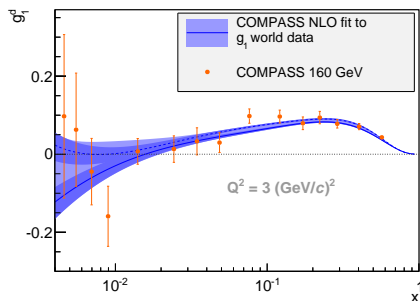
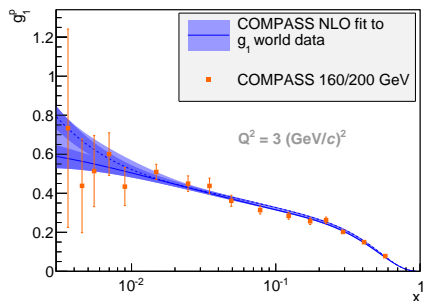
- Number of free parameters in the fitted pdfs: 11
- Only statistical errors taken, normalisations of each data set varied
- Required at every iteration:
 $|\Delta q(x) + \Delta \bar{q}(x)| \leq q(x) + \bar{q}(x)$, $|\Delta g(x)| \leq g(x)$ at $Q^2 = 1 \text{ (GeV}/c)^2$
- Studied: parameterisation $\Delta f_k(x)$ dependence and Q_0^2 dependence of the fit.

NLO QCD fit: results



- Statistical uncertainties (dark bands) \ll systematic (light bands)
- **Gluon polarisation poorly constrained \implies “direct” methods**
- Quark spin contribution to the nucleon spin: $0.26 < \Delta\Sigma < 0.36$ (due to poor Δg)

NLO QCD fit: results...cont'd



- g_1^p clearly positive at low x and raising with decreasing x
- g_1^d consistent with zero at low x ?

First moments of g_1 and singlet axial charge a_0

- First moments $\Gamma_1^p, \Gamma_1^d, \Gamma_1^N$
where $\Gamma_1^i = \int_0^1 g_1^i(x, Q^2) dx$

- In particular:

$$\begin{aligned}\Gamma_1^N(Q^2) &= \frac{1}{36} [4a_0 C_S(Q^2) + a_8 C_{NS}(Q^2)] \\ &= \int_0^1 \frac{g_1^d(x, Q^2)}{1 - 1.5\omega_D} dx\end{aligned}$$

- In the \overline{MS} : $a_0 = \Delta\Sigma = (\Delta u + \Delta\bar{u}) + (\Delta d + \Delta\bar{d}) + (\Delta s + \Delta\bar{s})$

- Γ_1^N approaches asymptotic value already at $Q^2 = 3 \text{ (GeV/c)}^2$

- From COMPASS data alone:

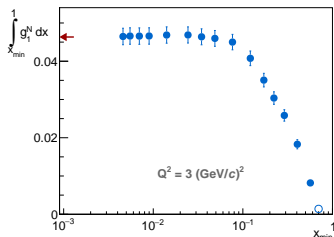
$$\Gamma_1^N(Q^2 = 3 \text{ (GeV/c)}^2) = 0.046 \pm 0.002_{\text{stat.}} \pm 0.004_{\text{syst.}} \pm 0.005_{\text{evol.}}$$

- From COMPASS data alone (and a_8 from PRD 82 (2010) 114018):

$$a_0(Q^2 = 3 \text{ (GeV/c)}^2) = 0.32 \pm 0.02_{\text{stat.}} \pm 0.04_{\text{syst.}} \pm 0.05_{\text{evol.}}$$

(consistent with value from the COMPASS NLO QCD fit of world data).

COMPASS PL B769 (2017) 034



Non-singlet structure function, $g_1^{\text{NS}}(x, Q^2)$

- Non-singlet structure function:

$$g_1^{\text{NS}} = g_1^{\text{P}}(x, Q^2) - g_1^{\text{n}}(x, Q^2)$$

$$= 2 \left[g_1^{\text{P}}(x, Q^2) - g_1^{\text{N}}(x, Q^2) \right]$$

- Its moment connected to the Bjorken sum rule:

$$\Gamma_1^{\text{NS}}(Q^2) = \int_0^1 g_1^{\text{NS}}(x, Q^2) dx = \frac{1}{6} \left| \frac{g_A}{g_V} \right| C_1^{\text{NS}}(Q^2)$$

- g_1^{NS} calculated, NLO QCD fitted (only Δq_3), evolved to $Q^2 = 3 \text{ (GeV}/c)^2$ and fit-extrapolated $x \rightarrow 0, 1$:

$$\Gamma_1^{\text{NS}} = 0.192 \pm 0.007_{\text{stat.}} \pm 0.015_{\text{sys.}}$$

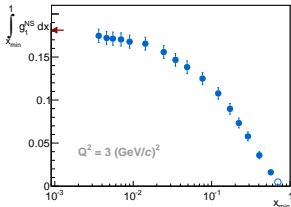
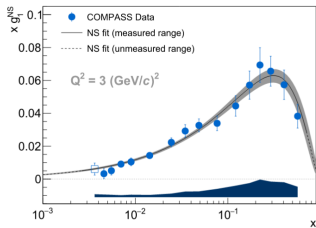
$$\left| \frac{g_A}{g_V} \right| = 1.29 \pm 0.05_{\text{stat.}} \pm 0.10_{\text{sys.}}$$

- Neutron β decay gives: $|g_A/g_V| = 1.2701 \pm 0.002$

PDG, PRD86 (2012) 010001

- This validates the Bjorken sum rule with an accuracy of 9%

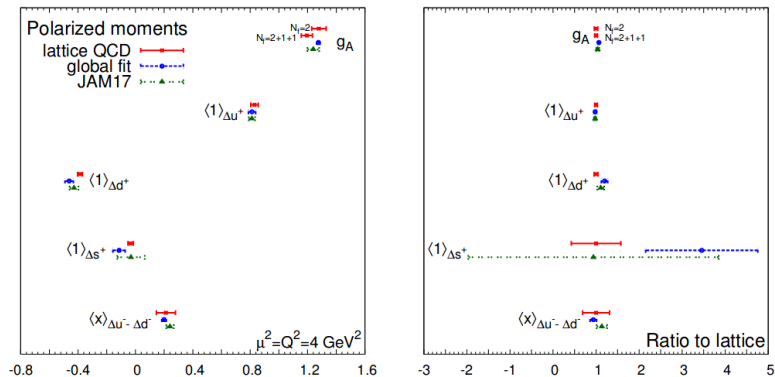
COMPASS PL B753 (2016) 18



Comparison of spin-dependent moments to lattice QCD

$$\langle 1 \rangle_{\Delta q}(Q^2) = \int_0^1 dx [\Delta q(x, Q^2) - \Delta \bar{q}(x, Q^2)] = 0\text{-th moment}$$

$$\langle x \rangle_{\Delta q}(Q^2) = \int_0^1 dx x \Delta q(x, Q^2) = 1\text{-st moment}$$



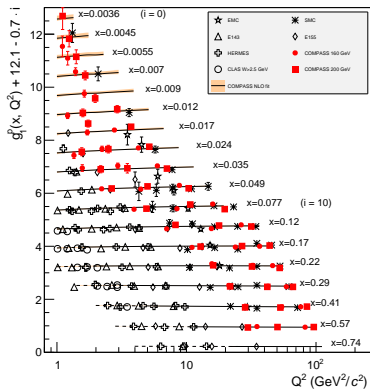
Surprising agreement!

arXiv: 1711.07916

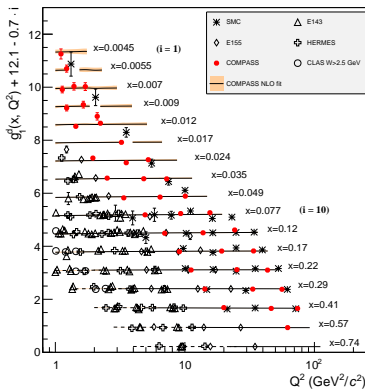
g_1^p and g_1^d , $Q^2 > 1$ (GeV/c)², COMPASS full statistics

COMPASS NLO QCD fit to the world data at $W^2 > 10$ (GeV/c)²
dashed line: extrapolation to $W^2 < 10$ (GeV/c)²

proton



deuteron



Phys.Lett.B753(2016)18

COMPASS PL B769 (2017) 034

COMPASS measurements at high Q^2 important for the QCD analysis! but little sensitive to Δg

SIDIS measurements

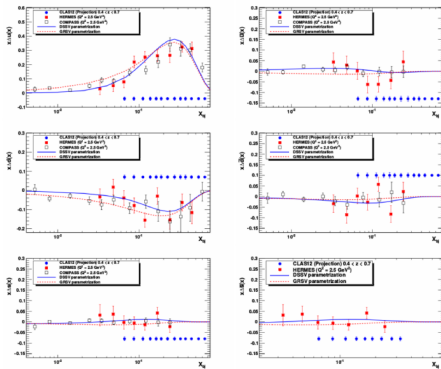
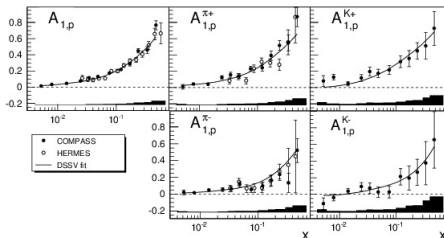
Semi-inclusive asymmetries, A_1^h and parton distributions

- COMPASS: measured on both proton and deuteron targets for identified, positive and negative pions and (for the first time) kaons

COMPASS, Phys. Lett. B **693** (2010) 227

DSSV, Phys. Rev. D **80** (2009) 034030

CLAS12, Update to E12-09-007



- COMPASS: LO DSS fragm. functions and LO unpolarised MRST assumed here.
- NLO parameterisation of DSSV describes the data well.

Polarisation of quark sea

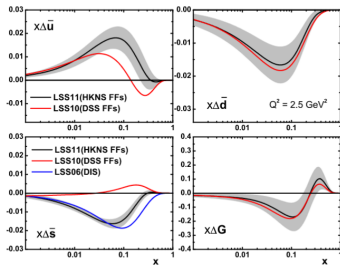
- Δs puzzle. Strange quark polarisation:

$2\Delta S = \int_0^1 (\Delta s(x) + \Delta \bar{s}(x)) dx = -0.09 \pm 0.01 \pm 0.02$ from incl. asymmetries + SU_3 ,
 while from SIDIS it is compatible with zero
 but depends upon chosen FFs.

Most critical: $R_{SF} = \frac{\int D_{\bar{s}}^{K^+}(z) dz}{\int D_u^{K^+}(z) dz}$

\implies COMPASS extracts it from multiplicities.

- Example of sensitivity to FFs at $Q^2=2.5$ (GeV/c)²



- The sea is not unsymmetric:

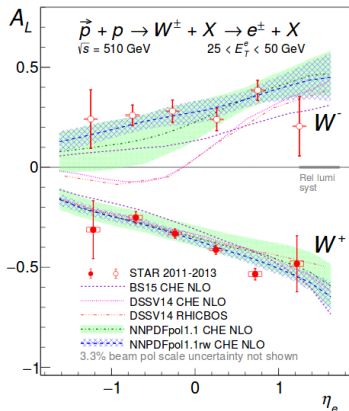
$$\int_{0.004}^{0.3} [\Delta \bar{u}(x, Q^2) - \Delta \bar{d}(x, Q^2)] dx = 0.06 \pm 0.04 \pm 0.02 \text{ @ } Q^2 = 3 \text{ (GeV/c)}^2$$

Thus the data disfavour models predicting $\Delta \bar{u} - \Delta \bar{d} \gg \bar{d} - \bar{u}$

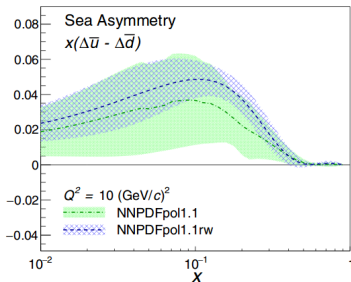
LSS, PRD D84 (2011) 014002

COMPASS, PL B693 (2010) 227

Recent results from STAR on sea polarisation



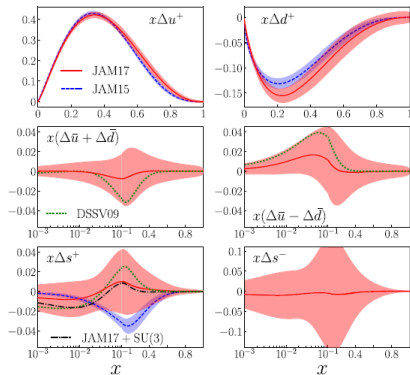
STAR, PR D99 (2019) 051102



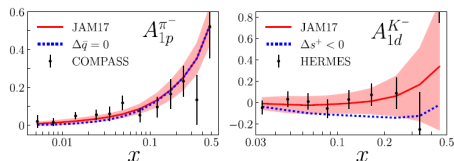
- The polarised sea is probably unsymmetric (STAR) contrary to COMPASS result
- This is opposite to spin-averaged quark sea asymmetry

Global QCD fits: JAM17

A global NLO $\overline{\text{MS}}$ QCD analysis of DIS, SIDIS, e^+e^- data and of FFs



- Inclusion of SIDIS changes Δs^+
- Sensitivity to imposing SU_3 !



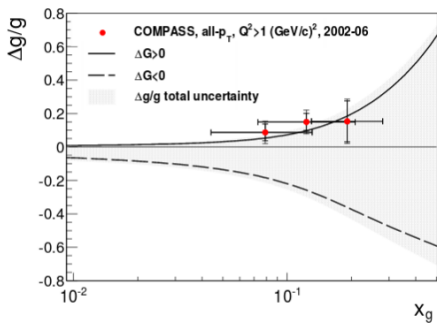
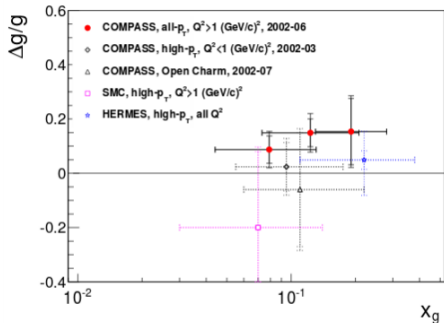
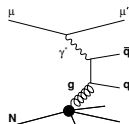
JAM PRL 119 (2017) 132001

“Direct” measurements of Δg

Direct measurements of $\Delta g(x)$ by COMPASS

Direct measurements – *via* the cross section asymmetry for the photon–gluon fusion (PGF) with subsequent fragmentation into

$c\bar{c}$ (LO, NLO) or $q\bar{q}$ (high p_T hadron pair (LO)): $A_{\gamma N}^{\text{PGF}} \approx \langle a_{LL}^{\text{PGF}} \rangle \frac{\Delta g}{g}$



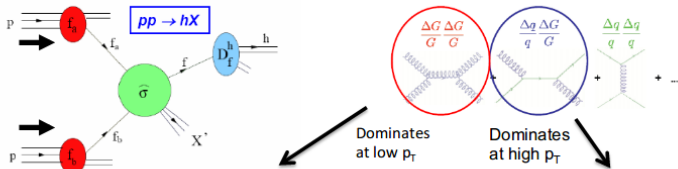
COMPASS from SIDIS on d for any $(p_T)_h$ and at LO:

$\Delta g/g = 0.113 \pm 0.038(\text{stat.}) \pm 0.036(\text{syst.})$ at $\langle Q^2 \rangle \approx 3$ (GeV/c) 2 , $\langle x_g \rangle \approx 0.10$
 clearly positive gluon polarisation!

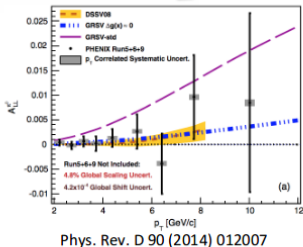
COMPASS, EPJC 77(2017) 209



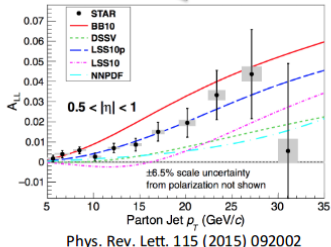
Measurement of the gluon polarization Δg at RHIC



D. de Florian *et al*,
PRL 113 (2014) 012001



E. Nocera *et al*,
NPB 887 (2014) 276



Surrow *et al* on sea quark spin
from W production at RHIC

$$\int_{0.01}^1 dx \Delta g(x, Q^2=10 \text{ GeV}^2) = 0.20^{+0.06}_{-0.07} \quad \text{DSSV++}$$

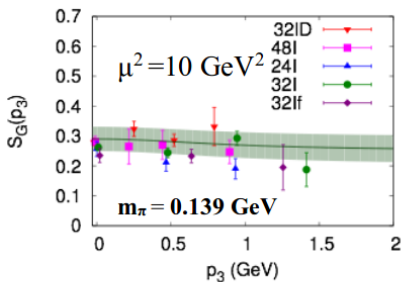
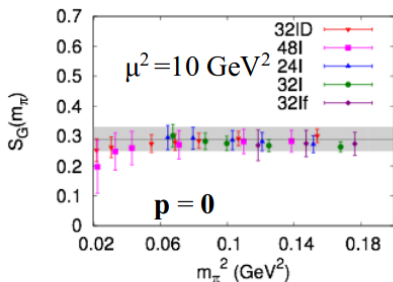
$$\int_{0.05}^{0.2} dx \Delta g(x, Q^2=10 \text{ GeV}^2) = 0.17 \pm 0.06 \quad \text{NNPDFpol1.1}$$

$$\int_{0.05}^{0.8} dx \Delta g(x, Q^2=1 \text{ GeV}^2) = 0.5 \pm 0.4 \quad \text{JAM15}$$

H.Gao, DIS2018

Gluon Spin From Lattice QCD

First lattice QCD calculation of the gluon spin in the nucleon



$$\Delta G \approx S_g(|\mathbf{p}| \rightarrow \infty) : \quad 0.251(47)(16) \quad (\mu^2 = 10 \text{ GeV}^2)$$

50(9)(3)% of the total proton spin

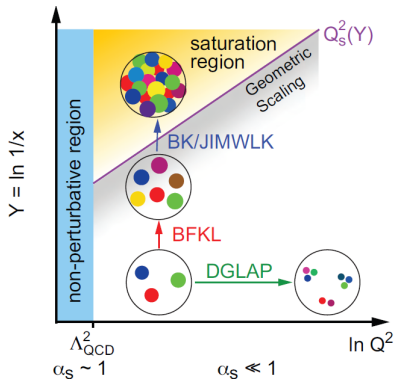
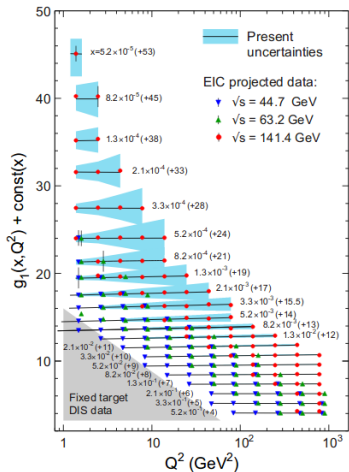
Y.-B. Yang *et al.* (χ QCD Collaboration), *Phys. Rev. Lett.* **118**, 102001 (2017).

8

H.Gao, DIS2018



Inclusive $g_1(x, Q^2)$ at EIC (pseudo-data)



Errors statistical (EIC: expected, modest parameters); bands: from gluon helicity uncertainty

arXiv:1708.01527v3

"White paper", arXiv:1212.1701

“Low x physics”: gluon saturation

BFKL equation has $[\alpha_s \ln(1/x)]^n$ (DGLAP: $[\alpha_s \ln(Q^2)]^n$)

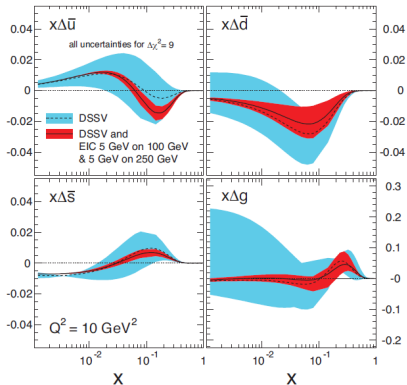
Even more, a new approach needed: non-linear QCD evolution

- Gluon splitting “infinite” \rightarrow recombination needed! (BFKL \rightarrow BK)
- Saturation at certain scale Q_s^2
- New evolution equations to describe low x
- Colour Glass Condensate as an effective theory

Easy signature of BFKL in g_1 : at low x it is sensitive to $\ln^2(1/x)$!!!!

Parton separation at EIC pseudo-data (inclusive and semi-inclusive)

DIS + SIDIS



EW DIS

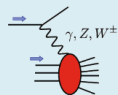
- $\Delta g(x)$ from scaling violation
- $\Delta\bar{u}, \Delta\bar{d}, \Delta s$ from SIDIS
- Flavor separation at high Q^2 via CC DIS:

$$g_1^{W^+} = \Delta\bar{u} + \Delta d + \Delta\bar{c} + \Delta s$$

$$g_1^{W^-} = \Delta u + \Delta\bar{d} + \Delta c + \Delta\bar{s}$$

$$g_5^{W^+} = \Delta\bar{u} - \Delta d + \Delta\bar{c} - \Delta s$$

$$g_5^{W^-} = -\Delta u + \Delta\bar{d} - \Delta c + \Delta\bar{s}$$

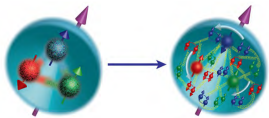


From "White paper", arXiv:1212.1701

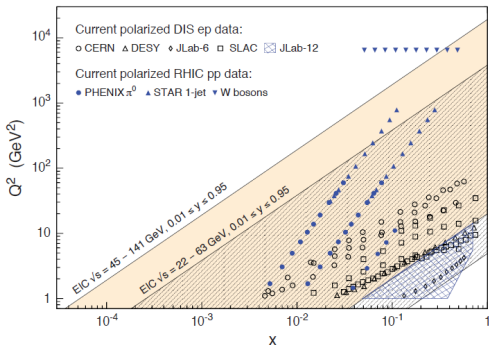
E. Aschenauer, SPIN2016

Spin “puzzle” (> 30 years old!)

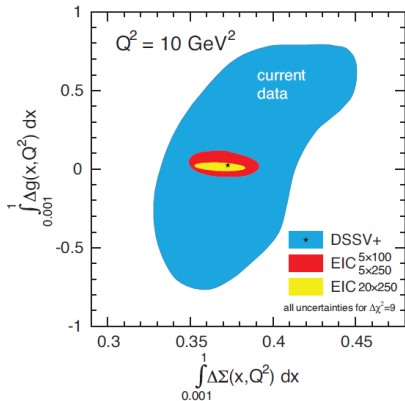
Nucleon spin “puzzle” at EIC



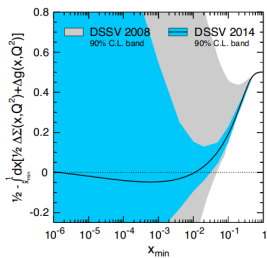
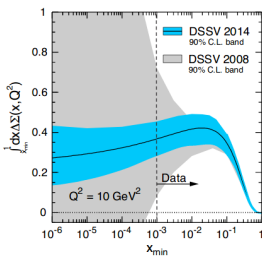
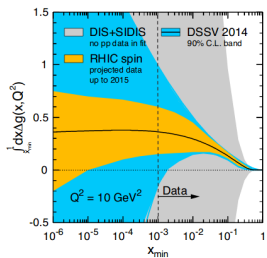
$$\frac{1}{2} = \frac{1}{2} \overset{\text{quark spins}}{\Delta\Sigma} + \overset{\text{gluon spins}}{\Delta G} + \overset{\text{quark\&gluon orbital motion}}{L_z}$$



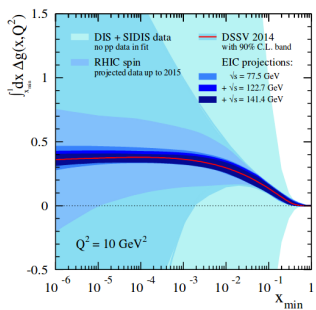
From “White paper2017”, arXiv:1708.01527v3



From “White paper”, arXiv:1212.1701



arXiv: 1708.01527



arXiv: 1509.06489

Accuracy of “spin puzzle” contributions improves with time

Hadron multiplicities and quark fragmentation functions

Nonperturbative component: Fragmentation Functions (FF)

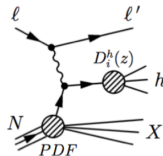
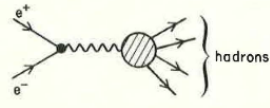
FF for quarks

- e^+e^- : gives the cleanest access but sensitive only to fragmentation of $q + \bar{q}$; limited flavour separation
- **SIDIS**: a convolution of q distribution and FF

$$\sigma^{lN \rightarrow l'NX} \propto \sum_q f(x) \otimes \sigma^{lq \rightarrow l'q} \otimes D_q^h(z)$$

but separation to q and \bar{q} and to flavours possible
(in LO QCD, D has a sense of a probability density)

SIDIS are crucial!



FF for gluons

- through high p_T hadron production in pp

Fragmentation Functions...cont'd

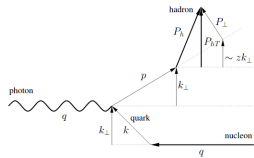
- Unpolarised SIDIS cross-section, σ^h , depends on 5 kinematic variables:
 x, Q^2, z, P_{hT}, ϕ

$$\frac{d^4\sigma^h(x, Q^2, z)}{dx dQ^2 dz dP_{hT}^2} \sim \sum_q e_q^2 \int d^2\vec{k}_T d^2\vec{p}_{h\perp} \delta^{(2)}(\vec{P}_{hT} - z\vec{k}_T - \vec{p}_{h\perp}) q(x, Q^2, k_T) D_q^h(z, Q^2, p_{h\perp})$$

(neglecting ϕ dependence); $\vec{p}_{h\perp}$ - w.r.t. fragmenting q ; \vec{P}_{hT} - w.r.t. γ^*

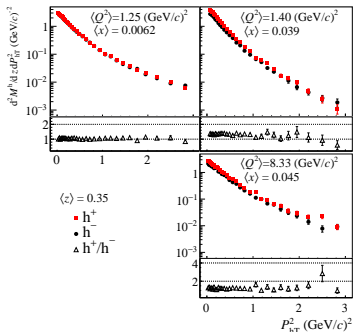
- This σ^h is accessed via multiplicities, $M^h(x, Q^2, z, P_{hT}^2)$

$$\frac{d^2 M^h(x, Q^2, z, P_{hT}^2)}{dz dP_{hT}^2} = \left(\frac{d^4 \sigma^h}{dx dQ^2 dz dP_{hT}^2} \right) / \left(\frac{d^2 \sigma^{\text{DIS}}}{dx dQ^2} \right)$$

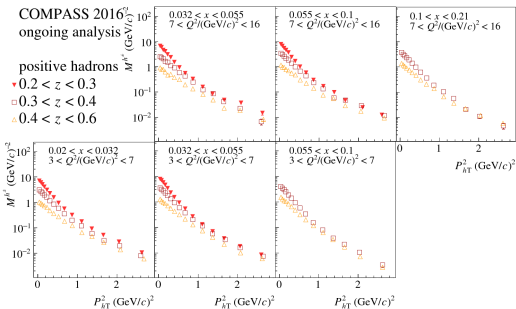


A. Bacchetta et al., JHEP06 (2017) 081

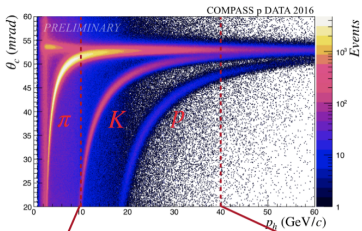
Multiplicities measurements at COMPASS in (x, Q^2, z, P_{HT}^2)



COMPASS, PR D97 (2018) 0322006

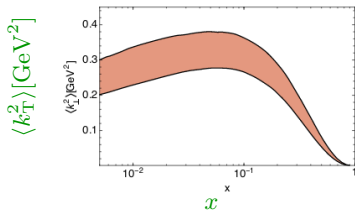


COMPASS, A. Moretti, DIS2019



Multiplicities in SIDIS \implies TMD QCD

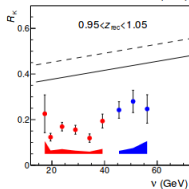
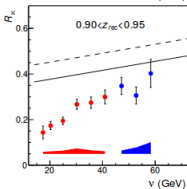
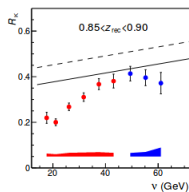
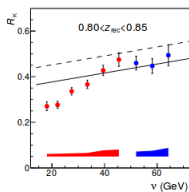
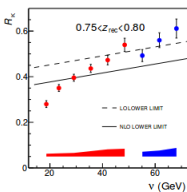
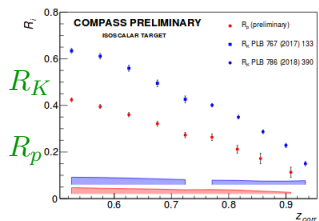
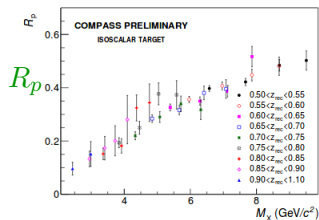
- Transverse Momentum Dependent QCD has **factorisation and evolution eqs different than collinear QCD!**
- **A process sensitive to TMD:** if some transverse momentum component, p_{hT} , is much less than the hard scale of the process, $p_{hT} \ll Q^2$.
- **Possible processes:** SIDIS, Drell-Yan, Z boson production
- First global analysis attempt



A. Bacchetta et al., JHEP 06 (2017) 081

Simultaneous fit of unpolarised
TMD PDF (f_1^q) and of TMD FF (D_q^h);
NLL accuracy.

Multiplicity measurements: problems



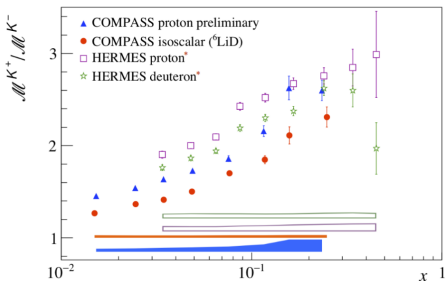
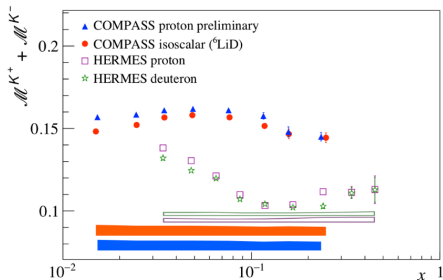
R_K vs ν in bins of z

$$R_h = \frac{M_{h^-}}{M_{h^+}} \text{ vs } M_X, z$$

Sign of saturation at lower z ?

M. Stolarski, DIS2019

Fragmentation Functions: problems...cont'd



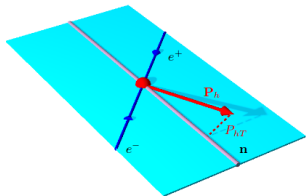
N.Pierre, DIS2019

M.Stolarski, DIS2019:

- In COMPASS the pQCD prediction fails in the corner of the phase space
- However, taking into account the observed ν and/or M_X dependence in experiments at lower energy much larger region of phase-space is affected
 - Note that selection of large Q^2 is not enough!
- This gives interesting insights:
 - It can be a reason for discrepancy between COMPASS and HERMES kaon multiplicity ratio and sum

SHOULD QCD in SIDIS BE REVISITED ?

Fragmentation Functions...cont'd; long time awaited

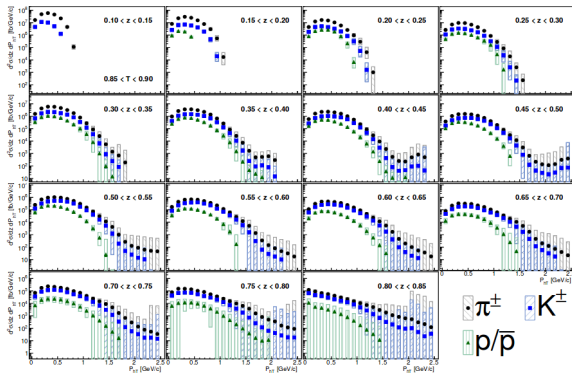


$$T \equiv \frac{\sum_h |\mathbf{P}_h^{\text{CMS}} \cdot \hat{\mathbf{n}}|}{\sum_h |\mathbf{P}_h^{\text{CMS}}|}$$

data of Belle [Phys.Rev. D99 \(2019\) 112006](#)

$$\frac{d^2\sigma}{dzdP_{hT}} \text{ vs } P_{hT}$$

(bins of z, T)



Take away menu

- Measurements of $g_1^p(x, Q^2)$ and $g_1^d(x, Q^2)$ for DIS ($Q^2 > 1$ (GeV/c)²) and nonperturbative ($Q^2 < 1$ (GeV/c)²) regions cannot be improved \implies EIC !
- The $g_1^p(x)$ at low x and low Q^2 is clearly positive ($g_1^d(x)$ is consistent with zero) \implies first observation of the spin effect at such low x
- The $g_1^p(x)$ at low x is positive (and $g_1^d(x) \approx 0$) also at $Q^2 > 1$ (GeV/c)²
- NLO QCD fit of g_1 world data gave well constrained quark distributions; gluons poorly determined. Quark helicity contribution to nucleon spin:
 $0.26 < \Delta\Sigma < 0.36$ but hope is in global fits (PDFs and FFs)
- Need to understand hadron multiplicities \implies FFs !
- From the COMPASS data alone:
 - first moments determined and Bjorken sum rule verified to 9% accuracy
 - flavour-singlet axial charge a_0 extracted:
 $a_0(Q^2 = 3 \text{ (GeV/c)}^2) = 0.32 \pm 0.02_{\text{stat.}} \pm 0.04_{\text{syst.}} \pm 0.05_{\text{evol.}}$
(in $\overline{\text{MS}}$ identified with total contribution of quark helicities to the nucleon spin)