Polarized Quark Hadronization

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Outlook

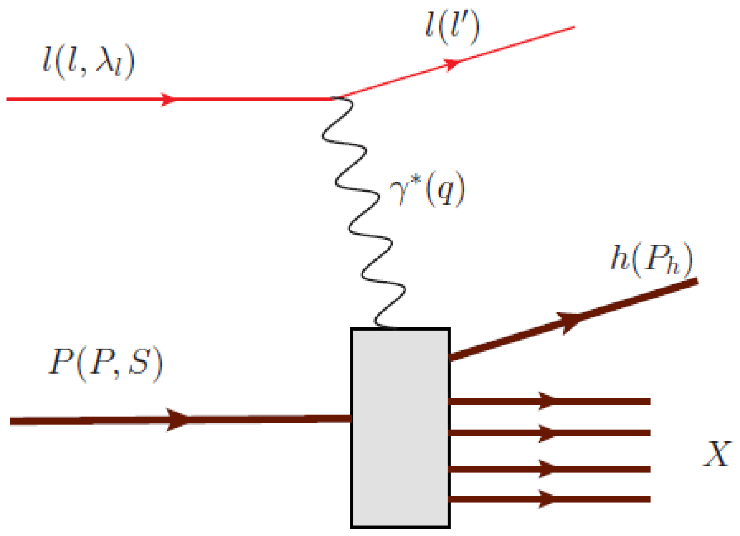
- Introduction
- Recursive model for quark hadronization
  - Collinear Field Feynman model
  - Generalization to spin and transverse momentum (STMD) case
- Monte Carlo implementation
  - Validation
  - Results for one and two hadron FFs
- Conclusions

Nucleon 3D partonic structure: Twist-2 STMD qDFs

Nucleon Polarization \ Quark polarization	U	L	T
U	$f_1^q(x,k_T^2)$		$\frac{\epsilon_T^{ij} k_T^{\ j}}{M} h_1^{\perp q}(x,k_T^2)$
L		$S_L g_{1L}^q(x,k_T^2)$	$S_L \frac{\mathbf{k}_T}{M} h_{1L}^{\perp q}(x,k_T^2)$
T	$\frac{\mathbf{k}_T \times \mathbf{S}_T}{M} f_{1T}^{\perp q}(x,k^2)$	$\frac{\mathbf{k}_T \cdot \mathbf{S}_T}{M} g_{1T}^{\perp q}(x,k_T^2)$	$\mathbf{S}_T h_{1T}^q(x,k_T^2) + \frac{\mathbf{k}_T}{M}\frac{(\mathbf{k}_T \cdot \mathbf{S}_T)}{M} h_{1T}^{\perp q}(x,k_T^2)$

All azimuthal dependences are in prefactors. TMDs do not depend on them

QCD TMD factorization: SIDIS CFR



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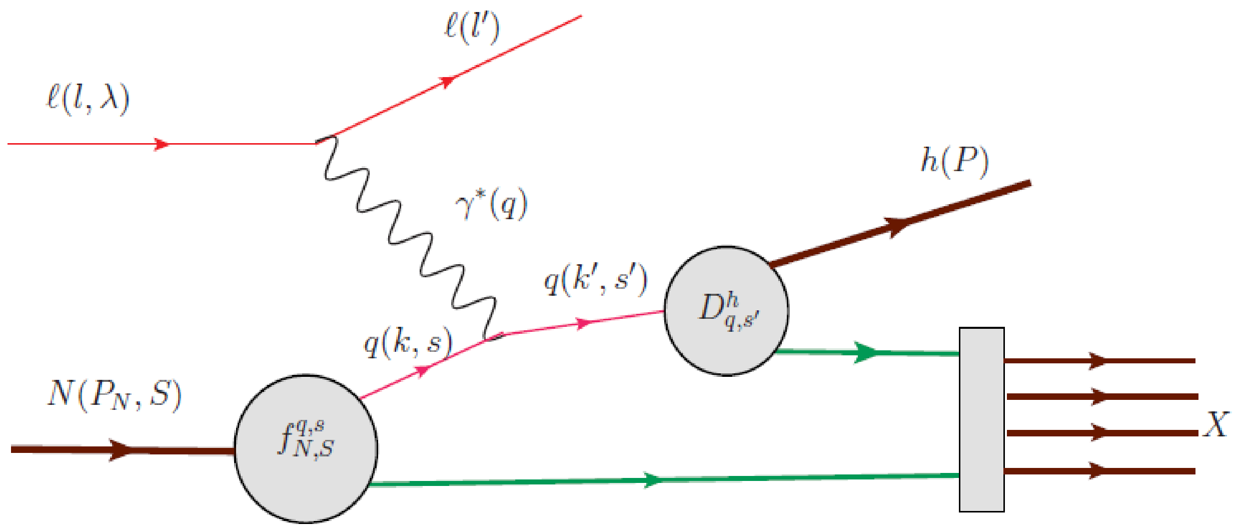
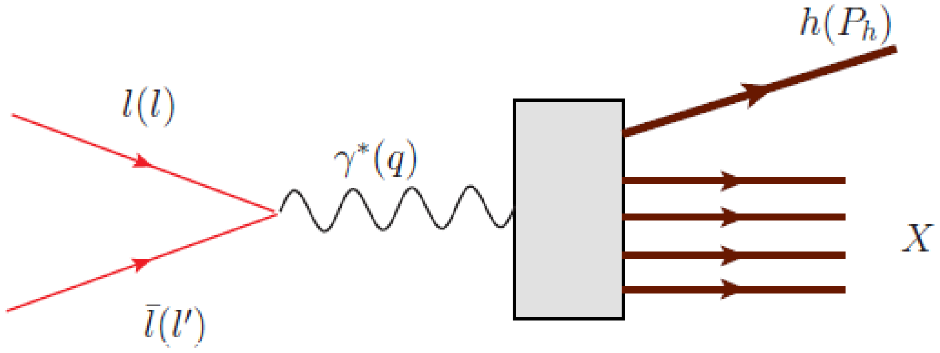


Access to nucleon PDFs and quark FFs

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_T} = f_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2} \otimes D^{h_1}_{q,s'}$$

$$D^{h_1}_{q,s'}(z,\mathbf{p}_T) = D_1(z,p_T^2) + \frac{\mathbf{p}_T \times \mathbf{s}'_T}{m_h} H_1(z,p_T^2)$$

QCD TMD factorization: SIA



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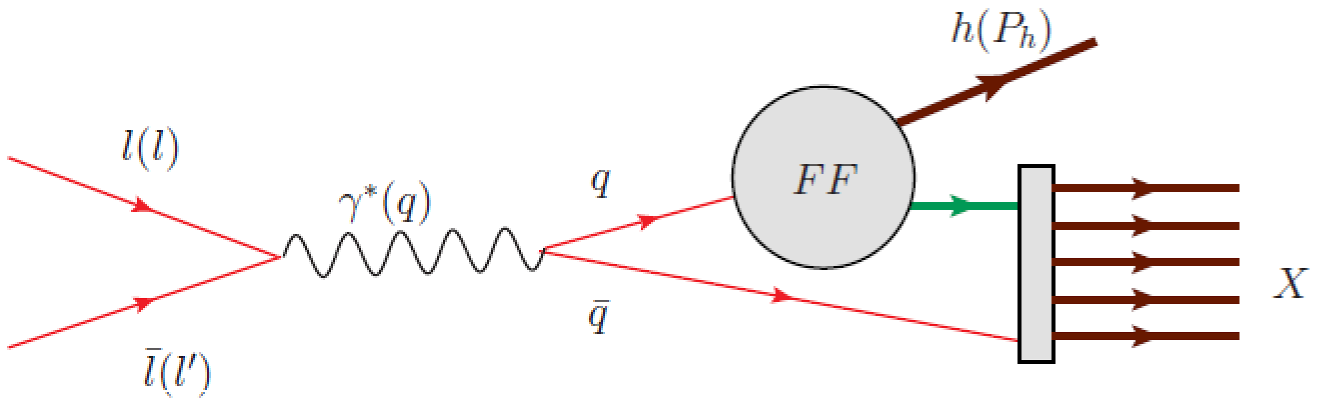


Access to $q+\bar{q}$ fragmentation finctions $D^h_{q+\bar{q}}(\mathrm{z},\mathrm{p}^2_\perp)$

Two hadron production in opposite hemispheres: acces to Collins FF $H^h_{1q}(\mathrm{z},\mathrm{p}^2_\perp)$

Two di-hadron production in opposite hemispheres:

acces to $H^{\sphericalangle}_q(\mathrm{z})$, $H^\perp_q(\mathrm{z})$ and $\mathrm{G}^\perp_q(\mathrm{z})$

Event generators PYTHIA, LEPTO…

- String fragmentation. No polarization effects in hadronization
- Modify generators to include Sivers effect
  - AK, Matevosyan,Thomas, **PRL 113, 062003 (2014), PR D 90, 074006 (2014)**
  - Matevosyan, AK, Aschenauer, Thomas, **PR D 92, 054028 (2015)** (predictions for EIC and CLAS12)
    - Validation: Good description of COMPASS data
    - Predictions for 1h and 2h production in CFR and TFR of SIDIS

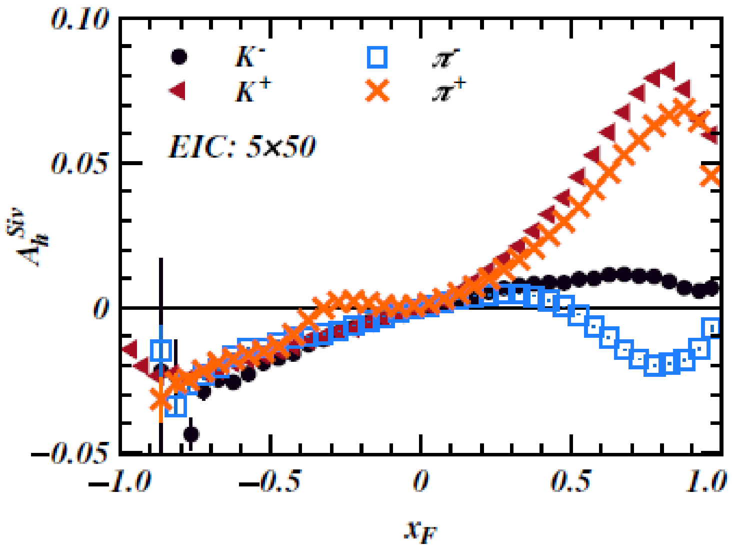


FIG. 13 (color online). EIC model SSAs for 5×50 SIDIS kinematics for charged pions and kaons versus x_F. The Sivers asymmetry is present both in the current and target fragmentation regions.

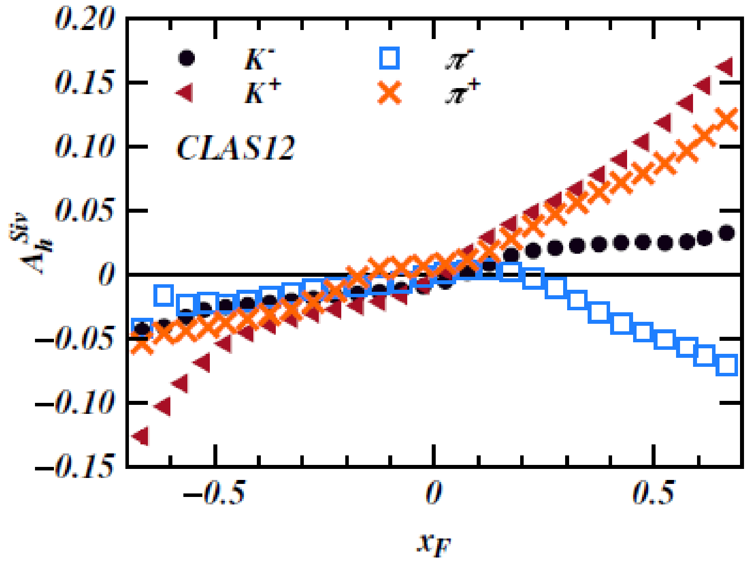


FIG. 17 (color online). Predictions for SSAs for charged pions and kaons versus x_F at CLAS12. The Sivers asymmetry is present both in the current and target fragmentation regions.

Modeling FFs: Recursive FF model

Field, Feynman PRD 15(1977)2590, NPB 136(1078)1 (**A PARAMETRIZATION OF THE PROPERTIES OF QUARK JETS**)

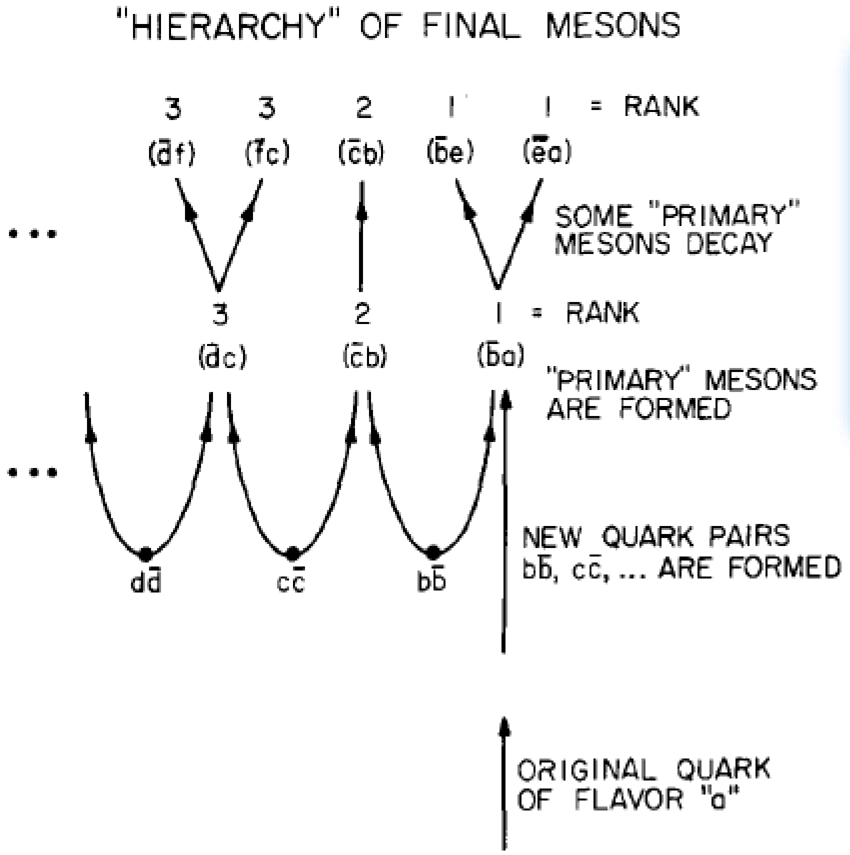


Fig. 1. Illustration of the "hierarchy" structure of the final mesons produced when a quark of type "a" fragments into hadrons. New quark pairs $b\bar{b}$, $c\bar{c}$, etc., are produced and "primary" mesons are formed. The "primary" meson $\bar{b}a$ that contains the original quark is said to have "rank" one and primary meson $\bar{c}b$ rank two, etc. Finally, some of the primary mesons decay and we assign all the decay products to have the rank of the parent. The order in "hierarchy" is *not* the same as order in momentum or rapidity.

assumed that for very high momenta, all distributions scale so that they depend only on ratios of the hadron momenta to the quark momenta. Given these assumptions, complete knowledge of the structure of a quark jet is determined by one unknown function $f(\eta)$ and three parameters describing flavor, primary meson spin, and transverse momentum to be discussed later. The function $f(\eta)$ is defined by

$f(\eta)\,d\eta$ = the probability that the first hierarchy (rank-1) primary meson *leaves* the fraction of momentum η to the remaining cascade, (2.1)

f(η) – elementary q→q' fragmentation or splitting function

Recursive FF: integral equation

$$S = 1 + x + x^2 + x^3 + \cdots = 1 + x\left(1 + x + x^2 + \cdots\right) = 1 + xS, \quad S = 1/(1-x)$$

2.2. Single-particle decay distribution $F(z)$

The above ansatz leads to an obvious and simple Monte Carlo calculation of a jet as well as to a straightforward recursive integral equation. For example, if we define a single-particle distribution in the quark jet as

$F(z)\,\mathrm{d}z$ = the probability of finding any primary meson (independent of hierarchy) with fractional momentum z within $\mathrm{d}z$ in a quark jet, (2.4)

then $F(z)$ must satisfy the following integral equation (take $W_0 = 1$)

$$F(z) = f(1-z) + \int_z^1 f(\eta) F(z/\eta)\,\mathrm{d}\eta/\eta\,, \qquad (2.5)$$

where the limits are automatic since we define $f(1-z) = 0$ and $F(z) = 0$ for $z > 1$ or $z < 0$. Eq. (2.5) arises because the primary meson might be the first in rank (with probability $f(1-z)\,\mathrm{d}z$) or if not, then the first-rank primary meson has left a momentum fraction η with probability $f(\eta)\,\mathrm{d}\eta$, and in this remaining cascade the probability to find z in $\mathrm{d}z$ is $F(z/\eta)\,\mathrm{d}z/\eta$ by the scaling principle. Dividing out the $\mathrm{d}z$ leaves eq. (2.5).

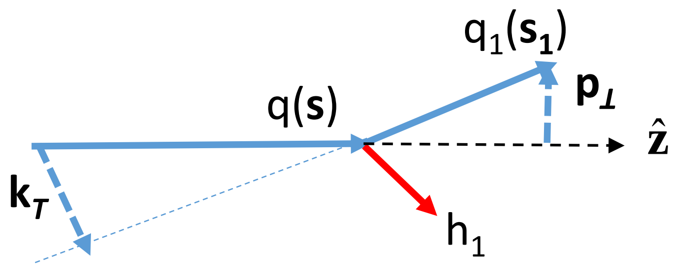
Only longitudinal scaled momentum flow is taken into account

Generalization of recursive mechanism to STMD FFs

- X. Artru & collaborators: string fragmentation
  - See talk by Artru
- H. Matevosyan, A.W. Thomas & collaborators
  - First MC study with constant spin transfer
    - Matevosyan, AK, Thomas **PLB 731(2014)208**
  - Theory framework: include polarization and transverse momentum flow in recursive FF approach
    - Bentz, AK, Matevosyan, Ninomiya, Thomas, Yazaki, **PR D94 (2016)034004**
  - MC implementation and results: validation and examples
    - Matevosyan, AK, Thomas **PR D95 (2017) 014021,** one hadron production
    - Matevosyan, AK, Thomas (2017), **arXiv:1707.04999,** two hadron production

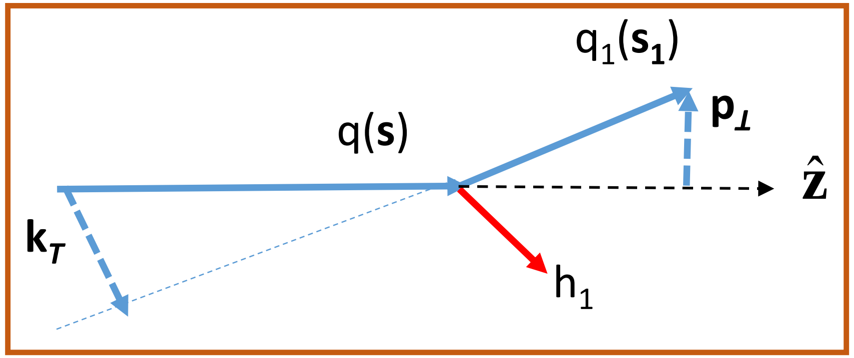
Twist-2 quark to quark STMD FFs

		Final quark polarization		
		U	L	T
Initial quark polarization	U	$D(z, p_\perp^2)$		$-\frac{\mathbf{k}_T \times \hat{\mathbf{z}}}{\mathcal{M}} D_T^\perp(z, p_\perp^2)$
	L		$s_L G_L(z, p_\perp^2)$	$s_L \frac{\mathbf{k}_T}{\mathcal{M}} G_T(z, p_\perp^2)$
	T	$-\frac{(\mathbf{k}_T \times \mathbf{s}_T) \cdot \hat{\mathbf{z}}}{\mathcal{M}} H^\perp(z, p_\perp^2)$	$\frac{\mathbf{k}_T \cdot \mathbf{s}_T}{\mathcal{M}} H_L^\perp(z, p_\perp^2)$	$\mathbf{s}_T H_T(z, p_\perp^2) + \frac{\mathbf{k}_T}{\mathcal{M}} \frac{(\mathbf{k}_T \cdot \mathbf{s}_T)}{\mathcal{M}} H_T^\perp(z, p_\perp^2)$



$$\mathbf{k}_T = -\mathbf{p}_T / z$$

STMD splitting function (SF) probability distribution



Polarized quark to polarized quark SF

$$F^{q\to q_1}(z,\mathbf{p}_\perp;\mathbf{s}_1,\mathbf{s}) = D(z,\mathbf{p}_\perp^2) - \frac{1}{M}(\mathbf{k}_T\times\mathbf{s}_{1T})\cdot\hat{\mathbf{z}}D_T^\perp(z,\mathbf{p}_\perp^2)$$
$$+(\mathbf{s}_T\cdot\mathbf{s}_{1T})H_T(z,\mathbf{p}_\perp^2) + \frac{1}{M}s_{1L}(\mathbf{k}_T\cdot\mathbf{s}_T)H_L^\perp(z,\mathbf{p}_\perp^2)$$
$$+\frac{1}{M^2}(\mathbf{s}_{1T}\cdot\mathbf{k}_T)(\mathbf{s}_T\cdot\mathbf{k}_T)H_T^\perp(z,\mathbf{p}_\perp^2) - \frac{1}{M}(\mathbf{k}_T\times\mathbf{s}_T)\cdot\hat{\mathbf{z}}H^\perp(z,\mathbf{p}_\perp^2)$$
$$+(s_{1L}s_L)G_L(z,\mathbf{p}_\perp^2) + \frac{1}{M}s_L(\mathbf{s}_{1T}\cdot\mathbf{k}_T)G_T(z,\mathbf{p}_\perp^2)$$

Polarized quark to unpolarized hadron SF

$$F^{q\to h_1}(z,\mathbf{p}_\perp;\mathbf{s}) = F^{q\to q_1}(1-z,-\mathbf{p}_\perp;\mathbf{s}_1=0,\mathbf{s}) = D(1-z,\mathbf{p}_\perp^2) + \frac{1}{M}(\mathbf{k}_T\times\mathbf{s}_T)\cdot\hat{\mathbf{z}}H^\perp(1-z,\mathbf{p}_\perp^2)$$

Quark polarization after hadron emission

$$F^{q\to q_1}\left(z,\mathbf{p}_\perp;\mathbf{s}_1,\mathbf{s}\right)=\alpha\left(z,\mathbf{p}_\perp;\mathbf{s}\right)+\boldsymbol{\beta}\left(z,\mathbf{p}_\perp;\mathbf{s}\right)\cdot\mathbf{s}_1,\quad \mathbf{s}=\left(\mathbf{s}_T,s_L\right)$$

$$\alpha\left(z,\mathbf{p}_\perp;\mathbf{s}\right)=D(z,\mathbf{p}_\perp^2)-\frac{1}{M}\left(\mathbf{k}_T\times\mathbf{s}_T\right)\cdot\hat{\mathbf{z}}H^\perp(z,\mathbf{p}_\perp^2)$$

$$\beta_L\left(z,\mathbf{p}_\perp;\mathbf{s}\right)=s_L G_L(z,\mathbf{p}_\perp^2)-\frac{1}{M}\left(\mathbf{k}_T\cdot\mathbf{s}_T\right)H_L^\perp(z,\mathbf{p}_\perp^2)$$

$$\boldsymbol{\beta}_\perp\left(z,\mathbf{p}_\perp;\mathbf{s}\right)=-\frac{\mathbf{k}_T^{'}}{M}D_T^\perp(z,\mathbf{p}_\perp^2)+s_L\frac{\mathbf{k}_T}{M}G_T(z,\mathbf{p}_\perp^2)$$

$$+\mathbf{s}_T H_T(z,\mathbf{p}_\perp^2)+\frac{\mathbf{k}_T}{M^2}\left(\mathbf{s}_T\cdot\mathbf{k}_T\right)H_T^\perp(z,\mathbf{p}_\perp^2)$$

α and $\boldsymbol{\beta}$ are linear functions of $\mathbf{s}$

$\mathbf{k}_T^{'}=(-k_y,k_x)$

The final quark spin is completely determined by elementary splitting functions and depends on z, $\mathbf{p}_\perp$ and initial quark polarization **s**

$$\left\langle\mathbf{s}_1\right\rangle=\frac{\boldsymbol{\beta}\left(z,\mathbf{p}_\perp;\mathbf{s}\right)}{\alpha\left(z,\mathbf{p}_\perp;\mathbf{s}\right)}$$

Integral equations for hadron production

Inserting everything into (III.29) we obtain the following two coupled integral equations [10]:

$$D^{(q\to\pi)}(z,\mathbf{p}_\perp^2) = \hat{d}^{(q\to\pi)}(z,\mathbf{p}_\perp^2)$$
$$+2\int \mathcal{D}^2\eta \int \mathcal{D}^4 p_\perp\, \delta(z-\eta_1\eta_2)\delta^{(2)}(\mathbf{p}_\perp - \mathbf{p}_{2\perp} - \eta_2 \mathbf{p}_{1\perp})$$
$$\times \Big[\hat{d}^{(q\to Q)}(\eta_1,\mathbf{p}_{1\perp}^2)\, D^{(Q\to\pi)}(\eta_2,\mathbf{p}_{2\perp}^2) + \frac{1}{M m_\pi z}$$
$$\times (\mathbf{p}_{1\perp}\cdot\mathbf{p}_{2\perp})\, \hat{d}_T^{\perp(q\to Q)}(\eta_1,\mathbf{p}_{1\perp}^2)\, H^{\perp(Q\to\pi)}(\eta_2,\mathbf{p}_{2\perp}^2)\Big], \qquad \text{(III.39)}$$

$$(\mathbf{p}_\perp\times\mathbf{s}_T)^3\, H^{\perp(q\to\pi)}(z,\mathbf{p}_\perp^2) = (\mathbf{p}_\perp\times\mathbf{s}_T)^3\, \hat{h}^{\perp(q\to\pi)}(z,\mathbf{p}_\perp^2)$$
$$+2\int \mathcal{D}^2\eta \int \mathcal{D}^4 p_\perp\, \delta(z-\eta_1\eta_2)\delta^{(2)}(\mathbf{p}_\perp - \mathbf{p}_{2\perp} - \eta_2 \mathbf{p}_{1\perp})$$
$$\times \Big[\frac{m_\pi}{M}\eta_2\, (\mathbf{p}_{1\perp}\times\mathbf{s}_T)^3\, \hat{h}^{\perp(q\to Q)}(\eta_1,\mathbf{p}_{1\perp}^2) D^{(Q\to\pi)}(\eta_2,\mathbf{p}_{2\perp}^2)$$
$$+\Big(\eta_1\, (\mathbf{p}_{2\perp}\times\mathbf{s}_T)^3\, \hat{h}_T^{(q\to Q)}(\eta_1,\mathbf{p}_{1\perp}^2) - \frac{1}{M^2\eta_1}(\mathbf{s}_T\cdot\mathbf{p}_{1\perp})$$
$$\times (\mathbf{p}_{1\perp}\times\mathbf{p}_{2\perp})^3\, \hat{h}_T^{\perp(q\to Q)}(\eta_1,\mathbf{p}_{1\perp}^2)\Big)\, H^{\perp(Q\to\pi)}(\eta_2,\mathbf{p}_{2\perp}^2)\Big].$$

$$\int \mathcal{D}^N\eta \equiv \int_0^1 \mathrm{d}\eta_1 \int_0^1 \mathrm{d}\eta_2 \cdots \int_0^1 \mathrm{d}\eta_N\,,$$
$$\int \mathcal{D}^{2N}p_\perp \equiv \int \mathrm{d}^2p_{1\perp} \int \mathrm{d}^2p_{2\perp} \cdots \int \mathrm{d}^2p_{N\perp}$$

Longitudinal and transverse momentum (Schäfer-Teryaev) sum rules

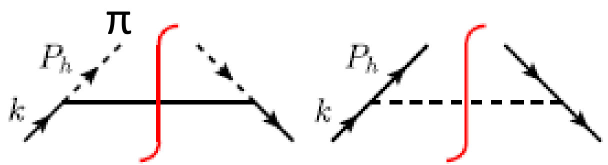
$$\sum_h \gamma_h \int_0^1 \mathrm{d}z\, z \int \mathrm{d}^2p_\perp\, D^{(q\to h)}(z,\mathbf{p}_\perp^2) = 1\,, \qquad \text{(II.22)}$$

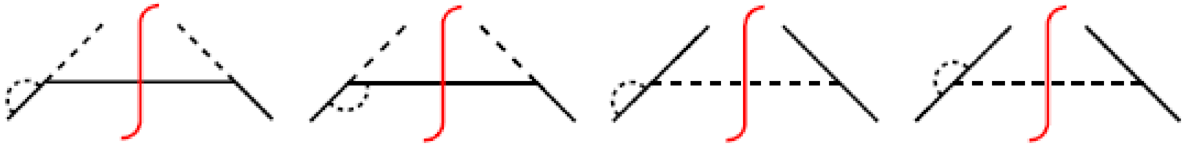
$$\sum_h \gamma_h \int_0^1 \frac{\mathrm{d}z}{2z\, M_h} \int \mathrm{d}^2p_\perp \cdot \mathbf{p}_\perp^2\, H^{\perp(q\to h)}(z,\mathbf{p}_\perp^2) = 0\,, \qquad \text{(II.23)}$$

where γ_h is the spin degeneracy factor of the hadron and M_h its mass. A similar derivation can be given for the

Lowercase $\hat{d}^{\,q\to\pi}$ *etc* - elementary SF

Spectator model for elementary SFs





T-even

$$\hat{D}(z, p_\perp^2) = C[p_\perp^2 + (1-z)^2 M^2],$$

$$\hat{G}_L(z, p_\perp^2) = C[-p_\perp^2 + (1-z)^2 M^2],$$

$$\hat{G}_T(z, p_\perp^2) = C[2z(1-z)M^2],$$

$$\hat{H}_T(z, p_\perp^2) = -\hat{D}(z, p_\perp^2),$$

$$\hat{H}_L^\perp(z, p_\perp^2) = \hat{G}_T(z, p_\perp^2),$$

$$\hat{H}_T^\perp(z, p_\perp^2) = C[2z^2 M^2],$$

$$C(z, p_\perp^2) = \frac{1-z}{12}\frac{g_\pi^2}{(2\pi)^3}\frac{1}{(p_\perp^2 + M^2(1-z)^2 + zm_\pi^2)^2}.$$

T-even SFs saturate positivity bounds

T-odd: $\hat{h}^\perp$ and $\hat{d}_T^\perp$

Positivity bound violation

For MC implementation we use :

$$\hat{D}(z) = 1.1\hat{D}_{\text{tree}}(z)$$

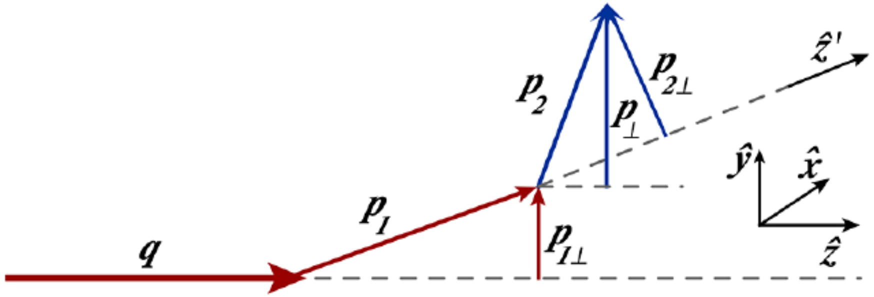
$$\frac{p_\perp}{zM}\frac{\hat{H}^{\perp(q\to h)}(z, p_\perp^2)}{\hat{D}^{(q\to h)}(z, p_\perp^2)} = 0.4\frac{2p_\perp M_Q}{p_\perp^2 + M_Q^2},$$

$$\hat{D}^{(q\to h)}(z, p_\perp^2) = \hat{D}^{(q\to q_1)}(1-z, p_\perp^2),$$

$$\hat{H}^{\perp(q\to h)}(z, p_\perp^2) = -\hat{H}^{\perp(q\to q_1)}(1-z, p_\perp^2),$$

$$\hat{H}^{\perp(q\to q_1)}(z, p_\perp^2) = -\hat{D}_T^{\perp(q\to q_1)}(z, p_\perp^2),$$

Monte Carlo implementation: Validation, two-step process, 1



$$z = z_1 z_2,$$

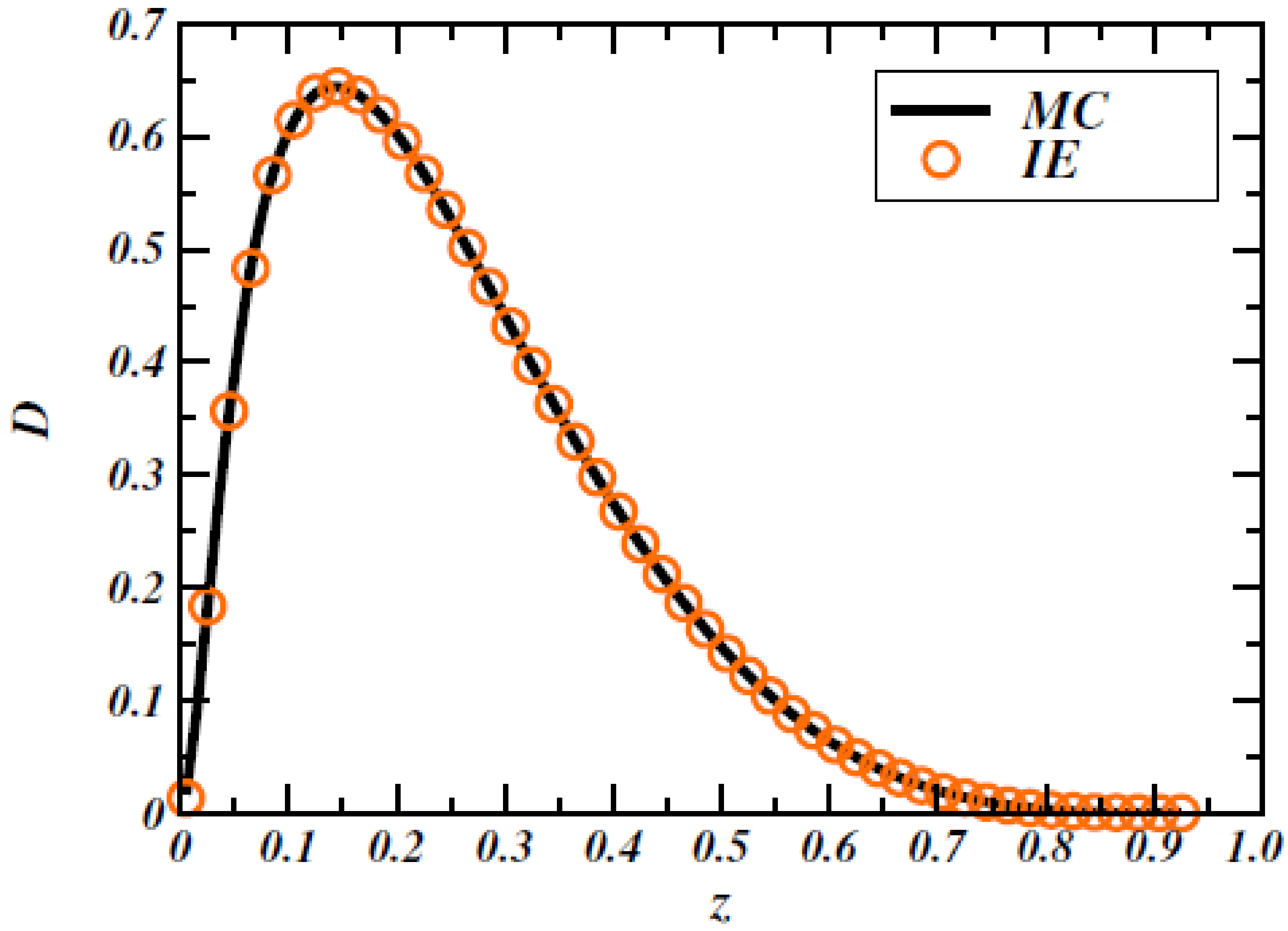
$$\boldsymbol{p}_\perp = z_2 \boldsymbol{p}_{1\perp} + \boldsymbol{p}_{2\perp}$$

$$D^{(2)}_{q\to h}(z, p_\perp^2) = 2\sum_{q_1}\int_0^1 dz_1 \int_0^1 dz_2 \int d^2\boldsymbol{p}_{1\perp}\int d^2\boldsymbol{p}_{2\perp} \times \delta(z - z_1 z_2)\delta^2(\boldsymbol{p}_\perp - z_2\boldsymbol{p}_{1\perp} - \boldsymbol{p}_{2\perp})$$
$$\times \left[\hat{D}^{q\to q_1}(z, p_{1\perp}^2)\hat{D}^{q_1\to h}(z_2, p_{2\perp}^2) + \frac{1}{z\mathcal{M}m_h}(\boldsymbol{p}_{1\perp}\cdot\boldsymbol{p}_{2\perp})\hat{D}_T^{\perp(q\to q_1)}(z_1, p_{1\perp}^2)\hat{H}^{\perp(q_1\to h)}(z_2, p_{2\perp}^2)\right],$$

0

$$H^{\perp(2)}_{q\to h}(z, p_\perp^2) = 2\frac{z m_h}{(\boldsymbol{p}_\perp \times \boldsymbol{s}_T)\cdot\hat{z}}\sum_{q_1}\int_0^1 dz_1\int_0^1 dz_2\int d^2\boldsymbol{p}_{1\perp}\int d^2\boldsymbol{p}_{2\perp}\times\delta(z - z_1 z_2)\delta^2(\boldsymbol{p}_\perp - z_2\boldsymbol{p}_{1\perp} - \boldsymbol{p}_{2\perp})$$
$$\times\left[\frac{1}{z_1\mathcal{M}}(\boldsymbol{p}_{1\perp}\times\boldsymbol{s}_T)\cdot\hat{\boldsymbol{z}}\hat{H}^{\perp(q\to q_1)}(z_1, p_{1\perp}^2)\hat{D}^{(q_1\to h)}(z_2, p_{2\perp}^2)\right.$$
$$\left.+\frac{1}{z_2 m_h}\left(\boldsymbol{p}_{2\perp}\times\left\{\boldsymbol{s}_T\hat{H}_T^{(q\to q_1)}(z_1, p_{1\perp}^2) + \boldsymbol{p}_{1\perp}(\boldsymbol{p}_{1\perp}\cdot\boldsymbol{s}_T)\frac{1}{z_1^2\mathcal{M}^2}\hat{H}_T^{\perp(q\to q_1)}(z_1, p_{1\perp}^2)\right\}\right)\cdot\hat{\boldsymbol{z}}\hat{H}^{\perp(q_1\to h)}(z_2, p_{2\perp}^2)\right]$$

Monte Carlo implementation: Validation, two-step u→π⁺, 2.1



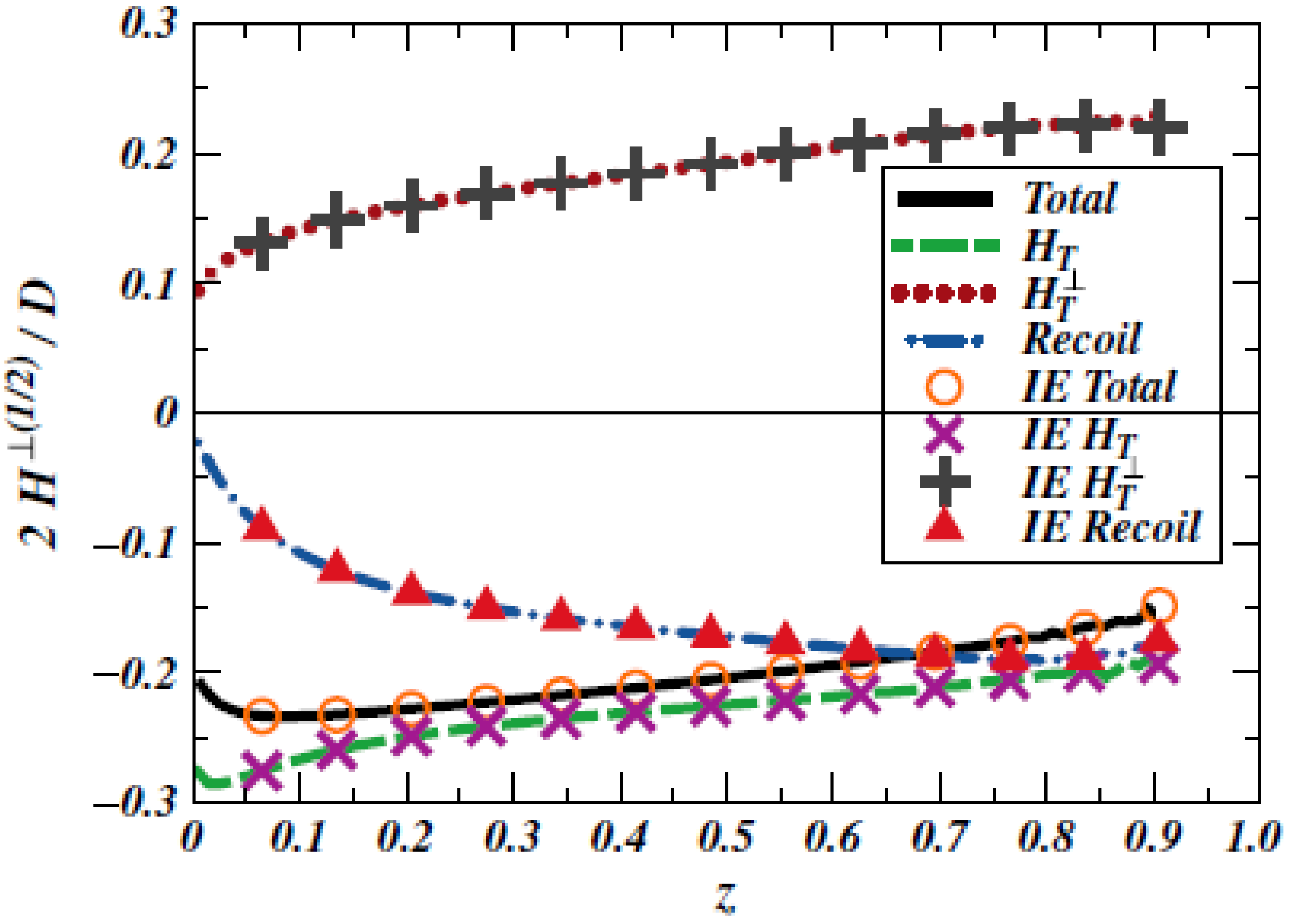
Labels:

Recoil: $\hat{H}^{\perp(q\to q_1)} \otimes \hat{D}^{(q_1\to \mathrm{h})}$

$\hat{H}_T$: $\hat{H}_T^{(q\to q_1)} \otimes \hat{H}^{\perp(q\to h)}$

$\hat{H}_T^{\perp}$: $\hat{H}_T^{\perp(q\to q_1)} \otimes \hat{H}^{\perp(q\to h)}$

Monte Carlo implementation: Validation, two-step u→π^+, 2.2



Labels:

Recoil: $\hat{H}^{\perp(q\to q_1)} \otimes \hat{D}^{(q_1\to \mathrm{h})}$

$\hat{H}_T : \hat{H}_T^{(q\to q_1)} \otimes \hat{H}^{\perp(q\to h)}$

$\hat{H}_T^{\perp} : \hat{H}_T^{\perp(q\to q_1)} \otimes \hat{H}^{\perp(q\to h)}$

Monte Carlo implementation: Validation, two-step process, 2

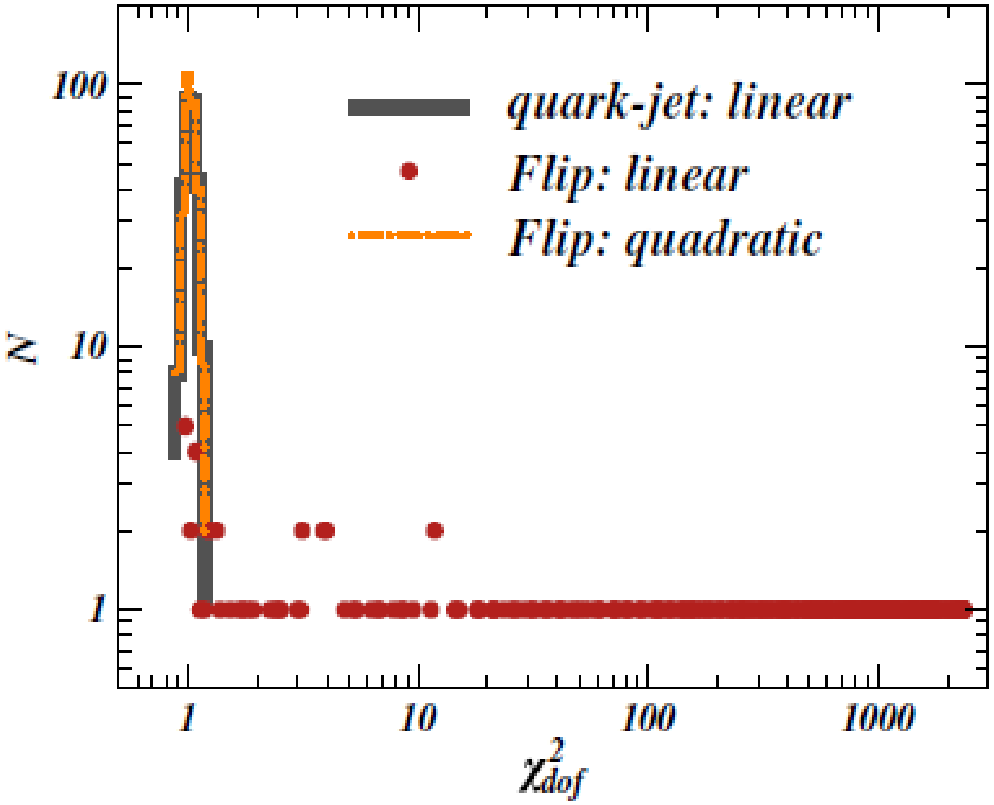


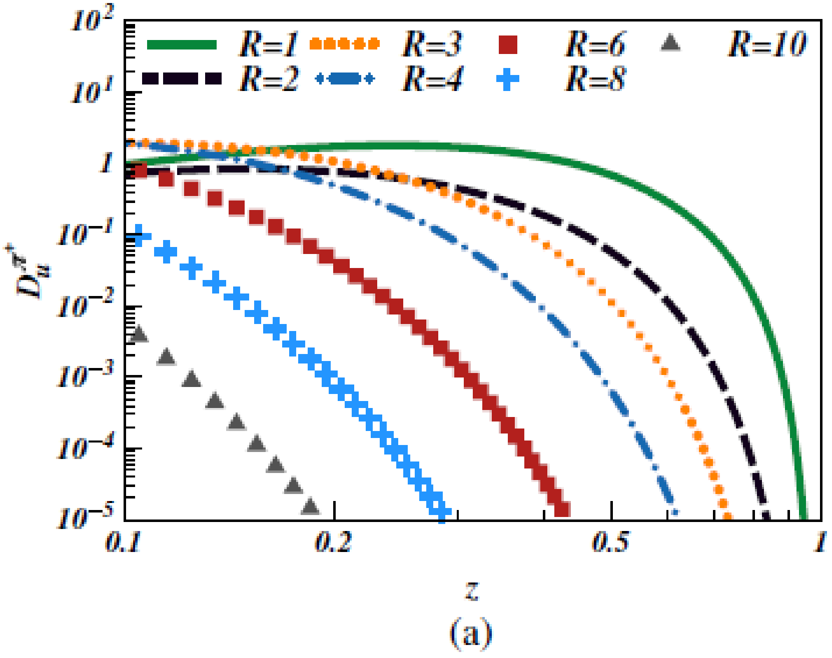
FIG. 4. Histogram of the values of χ^2_{dof} for fits of all polarized fragmentation functions of the u quark to rank-2 pions, fitted with linear and quadratic polynomials in $\sin(\varphi_C)$ of Eqs. (42) and (43) for MC simulations of rank-2 hadrons. The label "Flip" denotes the simulations where the transverse polarization of the quark is simply flipped after each hadron emission step.

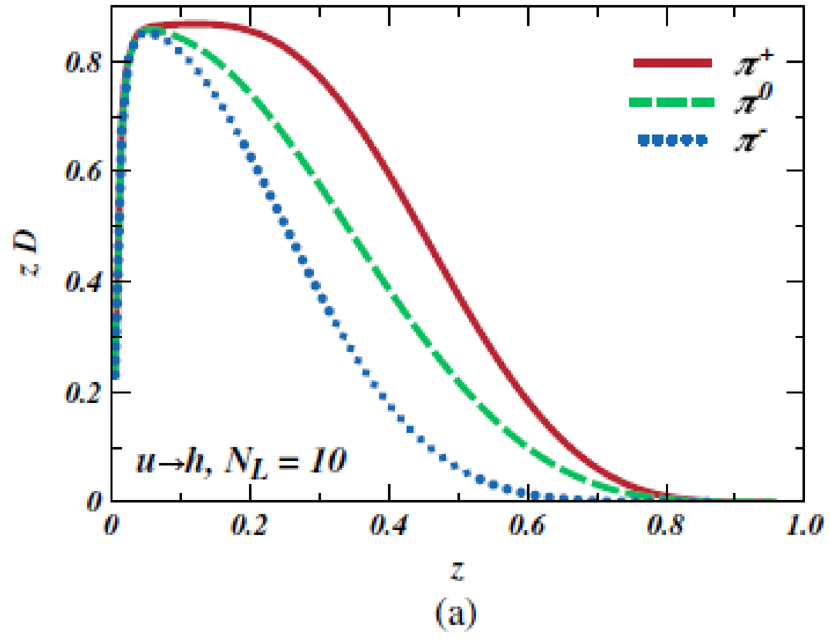
Fit functions

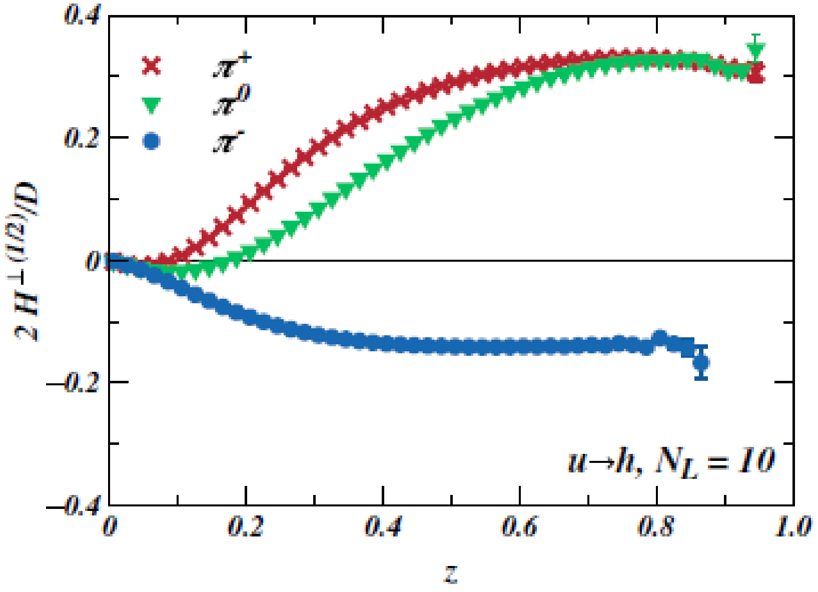
$$P_1(\phi_C) = c_0 + c_1 \sin(\phi_C)$$

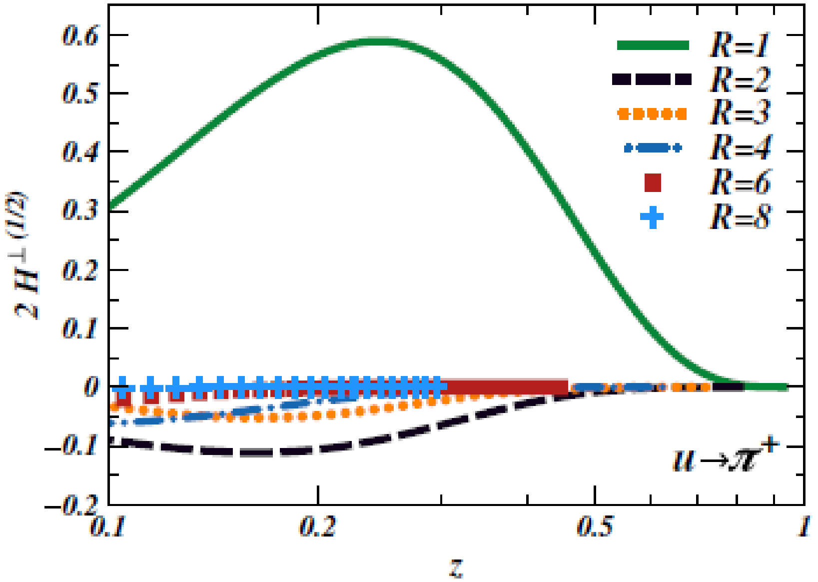
$$P_2(\phi_C) = c_0 + c_1 \sin(\phi_C) + c_2 \sin^2(\phi_C)$$

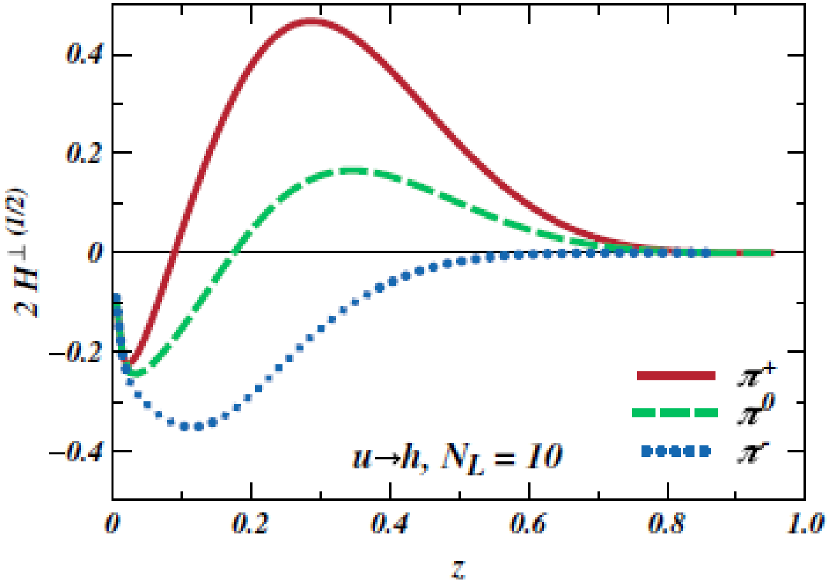
Monte Carlo implementation: Collins effect



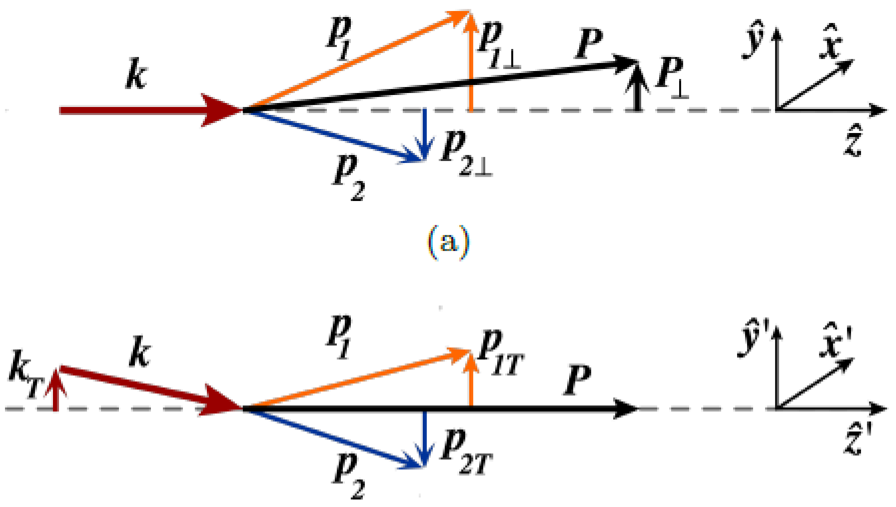








Dihadron FFs: definition



$$P \equiv P_h = P_1 + P_2,$$
$$R = \frac{1}{2}(P_1 - P_2),$$

$$z = z_1 + z_2,$$
$$\xi = \frac{z_1}{z} = 1 - \frac{z_2}{z}$$
$$z_i = P_i^- / k^-$$

$$\boldsymbol{P}_{1T} = \boldsymbol{P}_{1\perp} + z_1 \boldsymbol{k}_T,$$
$$\boldsymbol{P}_{2T} = \boldsymbol{P}_{2\perp} + z_2 \boldsymbol{k}_T\,.$$

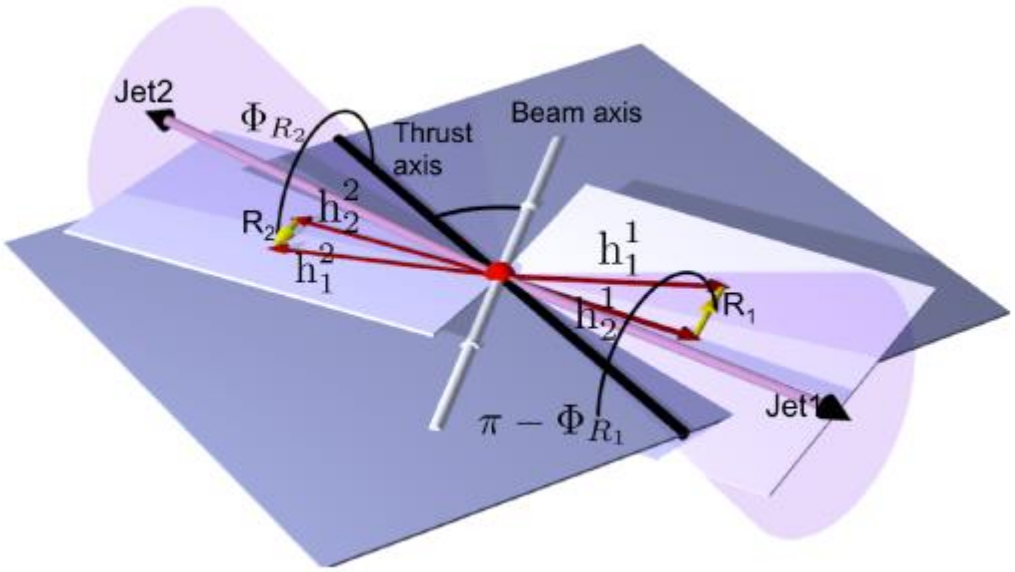
$$\boldsymbol{k}_T = -\frac{\boldsymbol{P}_\perp}{z},$$
$$\boldsymbol{R}_T = \frac{z_2 \boldsymbol{P}_{1\perp} - z_1 \boldsymbol{P}_{2\perp}}{z} = (1-\xi)\boldsymbol{P}_{1\perp} - \xi \boldsymbol{P}_{2\perp}.$$

$$\Delta_{ij}(k; P_1, P_2) = \sum_X \int d^4\zeta e^{ik\cdot\zeta} \langle 0|\psi_i(\zeta)|P_1 P_2, X\rangle\langle P_1 P_2, X|\bar{\psi}_j(0)|0\rangle.$$

$$\Delta^\Gamma(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T = \frac{1}{4z}\int dk^+ \mathrm{Tr}[\Gamma\Delta(k, P_1, P_2)]\big|_{k^- = P_h^-/z}.$$

$$\Delta^{[\gamma^-]} = D_1(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T),$$
$$\Delta^{[\gamma^-\gamma_5]} = \frac{\epsilon_T^{ij} R_{Ti} k_{Tj}}{M_1 M_2} G_1^\perp(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T),$$
$$\Delta^{[i\sigma^{i-}\gamma_5]} = \frac{\epsilon_T^{ij} R_{Tj}}{M_1 + M_2} H_1^\sphericalangle(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)$$
$$+ \frac{\epsilon_T^{ij} k_{Tj}}{M_1 + M_2} H_1^\perp(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)$$

Belle results on $G_1^\perp$ induced asymmetry



Theory: Boer, Jakob, Radici, PR D **67**, 094003 (2003)

BELLE: arXiv:1505.08020v1 [hep-ex] 29 May 2015

$$\langle \cos(2(\Phi_{R_1'} - \Phi_{R_2'})) \rangle \propto \sum_{q,\bar{q}} e_q^2 G_1^{\perp,q}(z, M^2) G_1^{\perp,\bar{q}}(\bar{z}, \bar{M}^2)$$

$$D_1(z, M_h^2) = \int d\xi \int d\varphi_R \int d^2\boldsymbol{k}_T\, D_1(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T),$$

$$G_1^\perp(z, M_h^2) = \int d\xi \int d\varphi_R \int d^2\boldsymbol{k}_T (\boldsymbol{k}_T \cdot \boldsymbol{R}_T)\, G_1^\perp(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)$$

$$G_1^\perp\left(z, \xi, k_T^2, R_T^2 \cos\left(\phi_R - \phi_k\right)\right) = c_0 + c_1 \cos\left(\phi_R - \phi_k\right) + c_2 \cos 2\left(\phi_R - \phi_k\right) + \cdots$$

Fourier series (FS) c_1

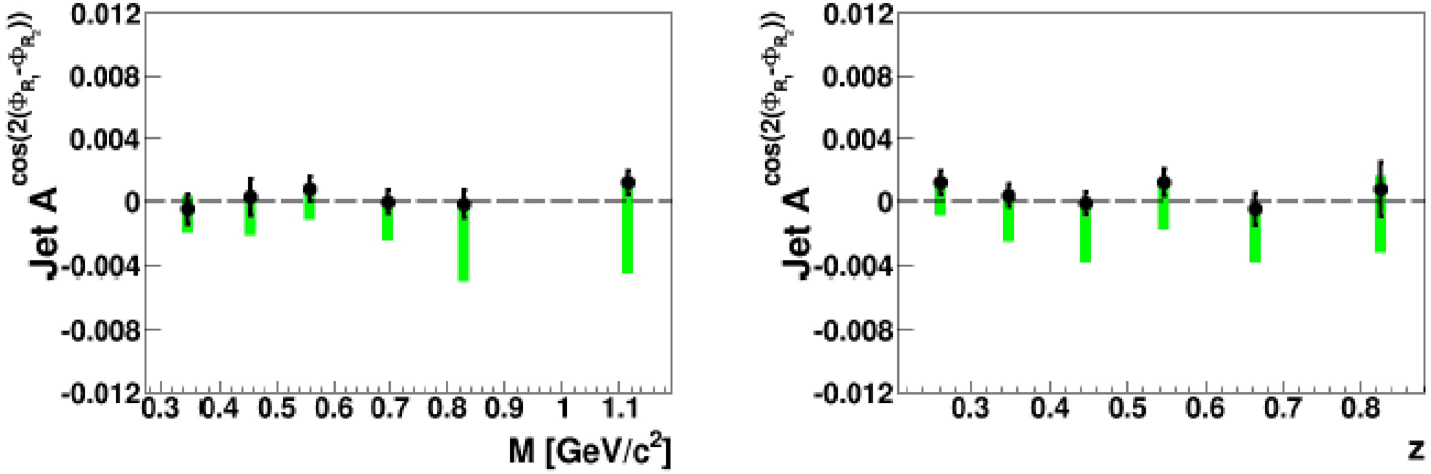
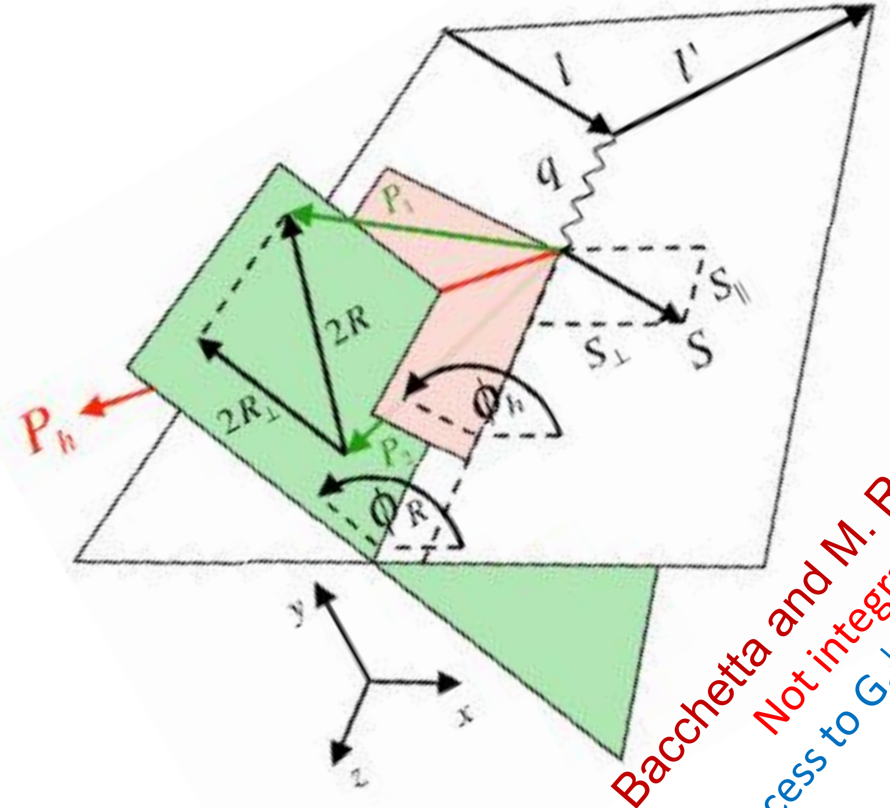


FIG. 2: Results for $A^{\cos(2(\Phi_{R_1} - \Phi_{R_2}))}$ binned in M and z. The black error bars are statistical and the green bands show the systematic uncertainty.

COMPASS Dihadron results: longitudinal polarized target

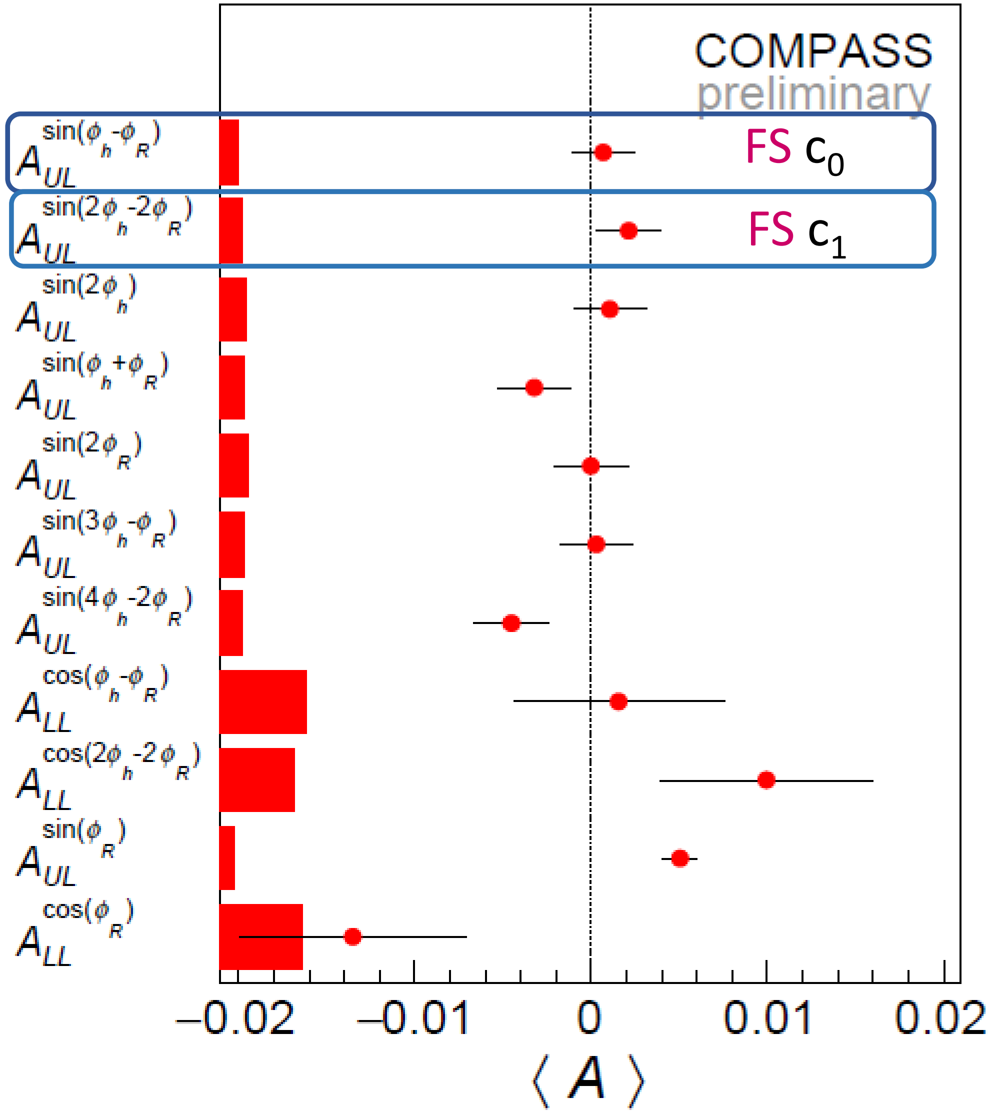
COMPASS (preliminary): arXiv:1702.07317v1 [hep-ex] 23 Feb 2017



S. Gliske, A. Bacchetta and M. Radici: PR D 90, 114027 (2014)

Not integrated over ϕ_h

Access to $G_1^\perp$ spherical harmonics



Extracting D_1 and $G_1^\perp$ from number densities

Number density: $F(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T;s_L) = D_1(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\cos(\varphi_{RK})) - s_L \frac{R_T k_T \sin(\varphi_{RK})}{M_1 M_2} G_1^\perp(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\cos(\varphi_{RK}))$

$$\varphi_{RK} = \varphi_R - \varphi_k$$

As an example we consider M^2 integrated Dihadron FFs

$$D_1(z) = \int d\xi \int d^2\boldsymbol{R}_T \int d^2\boldsymbol{k}_T \times D_1(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T),$$

$$G_1^\perp(z) = \int d\xi \int d^2\boldsymbol{R}_T \int d^2\boldsymbol{k}_T \times (\boldsymbol{k}_T\cdot\boldsymbol{R}_T)\, G_1^\perp(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

In terms of number density

$$D_1(z) = \int d\xi \int d^2\boldsymbol{R}_T \int d^2\boldsymbol{k}_T \times F(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T;s_L),$$

$$G^\perp(z) = -\frac{M_1 M_2}{s_L} \int d\xi \int d^2\boldsymbol{R}_T \int d^2\boldsymbol{k}_T \times \cot(\varphi_{RK}) F(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T;s_L)$$

Calculate number density F from 10^{12} events generated using extended quark jet model

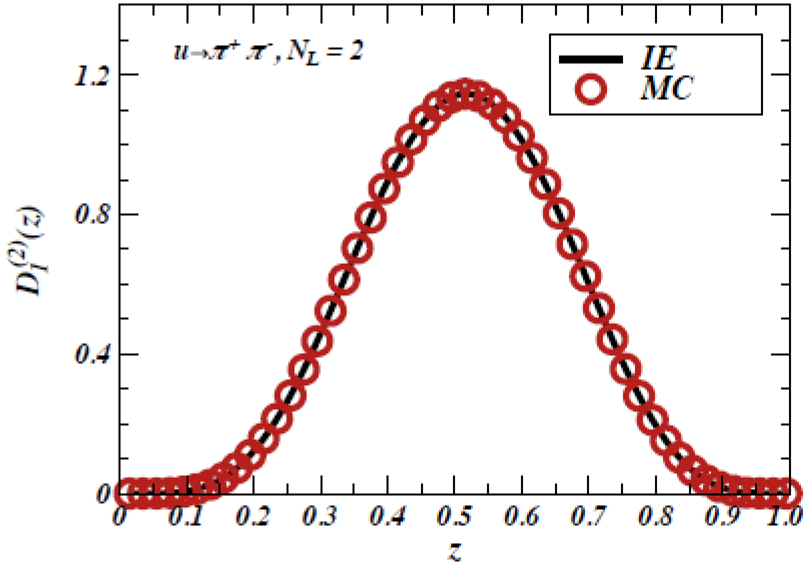
Validation: two step (N_L=2) u→$\pi^+ \pi^-$ case

$$D_1^{(2)}(z) = \int_0^1 d\eta_1 \int_0^1 d\eta_2 \; \delta\Big(z - 1 + \eta_1(1-\eta_2)\Big) \times \; \hat{D}^{q\to q_1}(\eta_1) \; \hat{D}^{q_1\to h_2}(\eta_2),$$

$$\tilde{G}_1^{\perp(2)}(z) = -\pi^2 \int_0^1 d\eta_1 \int_0^1 d\eta_2 \; \delta\Big(z - 1 + \eta_1(1-\eta_2)\Big) \times \int dp_{1\perp}^2 \int dp_{2\perp}^2 \, \frac{\eta_2(1-\eta_2)p_{1\perp}^2 - (1-\eta_1)p_{2\perp}^2}{z} \times \frac{1}{\eta_1 M_q}\hat{G}_T^{q\to q_1}(\eta_1, p_{1\perp}^2) \; \frac{1}{\eta_2 M_2} \; \hat{H}^{\perp(q_1\to h_2)}(\eta_2, p_{2\perp}^2)$$

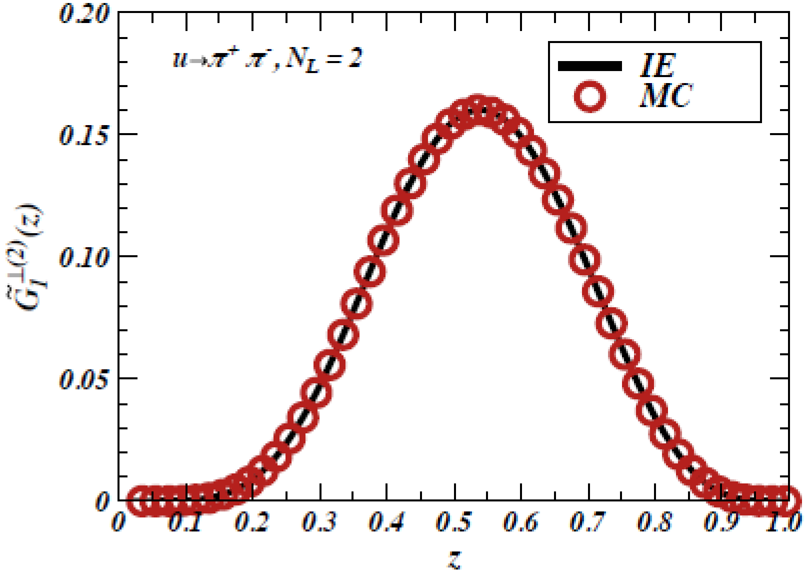
Transverse polarization of q_1 in $q\to q_1$ splitting

Collins effect in $q_1\to h$ splitting



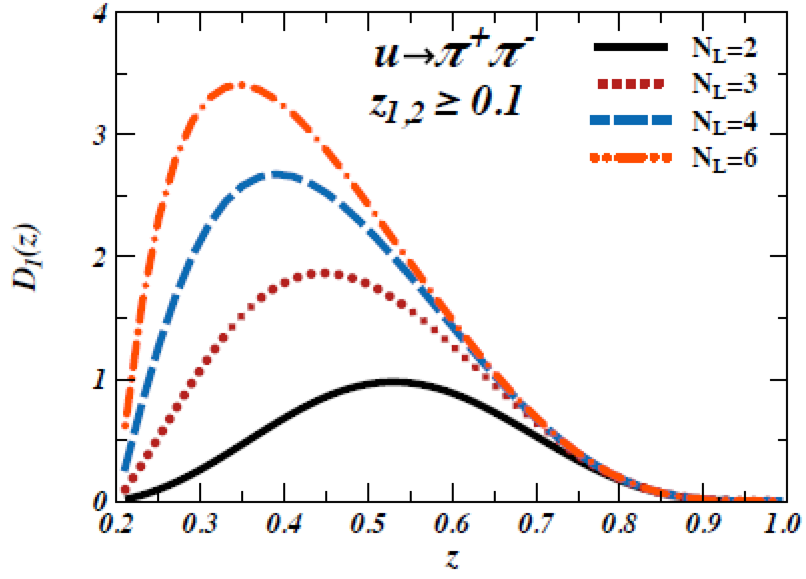
(a)

$$\tilde{G}_1^{\perp}(z) \equiv \frac{1}{M_1 M_2} G_1^{\perp}(z)$$

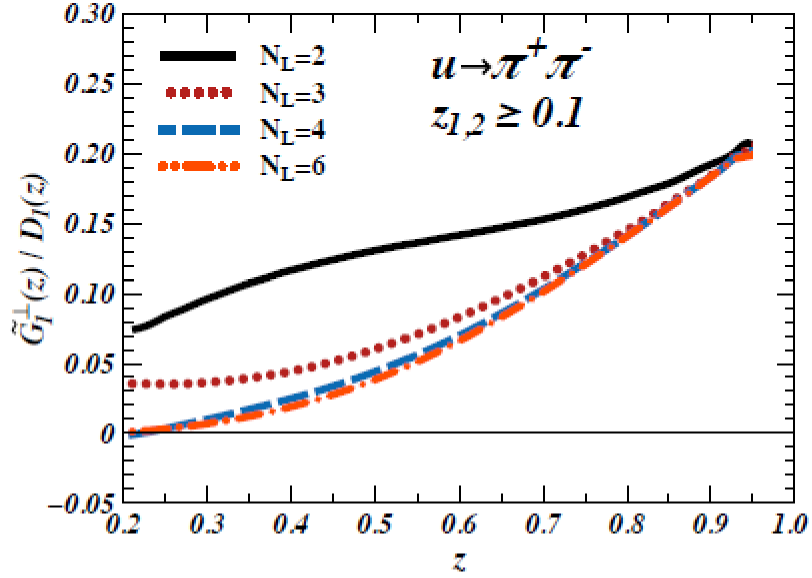


Twist-2 STMD one-hadron splitting functions generates nonzero quark longitudinal polarization dependent FF without interference effects

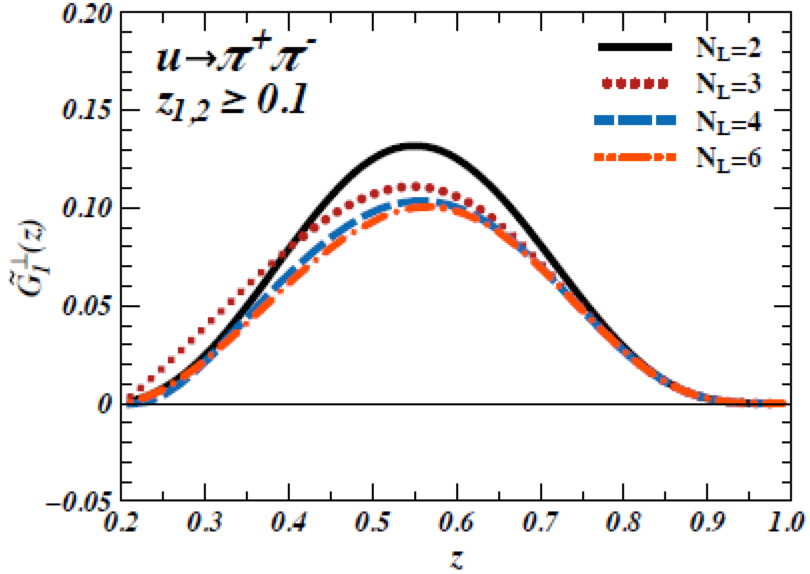
Results for $G_1^\perp$ FFs

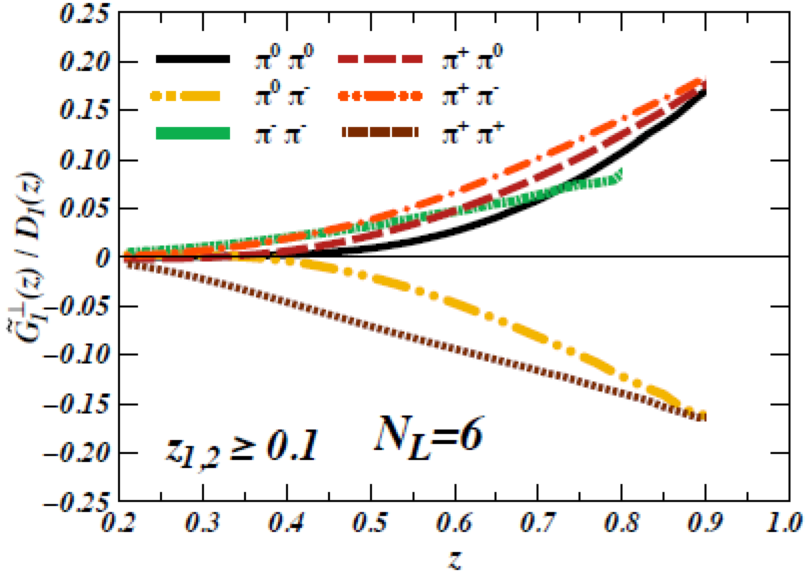


(a)



Fast saturation when increasing number of produced pions





Small (~2–3%) analyzing power for z about 0.4–0.5.

Analyzing power of Collins effect is about 20% using the same model for SFs

Conclusions

- Complete framework for polarized quark hadronization has been developed
- MC implementation and extraction of STMD FFs:
  - Validation
  - Results for one and two hadron production
    - Collins effect in one-hadron production
    - Longitudinally polarized quark fragmentation for two-hadron production
  - Transversely polarized quark fragmentation for two-hadron production coming soon
- For the moment only light quarks and pions are included in MC
  - Generalization for s quark fragmentation and kaons production is straightforward
    - Was already done for unpolarized TMD FFs also taking into account vector meson production, Matevosyan *et al*
- To do list
  - Develop MC framework for STMD string hadronization
  - Include vector meson production and strong decay for polarized case

Additional slides