Point-by-point extraction of parton distribution functions from SIDIS single transverse–spin asymmetries

Franco Bradamante
Trieste University and INFN
COMPASS results on Collins asymmetry

**Proton 2010**

![Graph showing Collins asymmetry for protons in 2010.](image)


**Deuteron 2002, 2003, 2004**

![Graph showing Collins asymmetry for deuterons from 2002 to 2004.](image)

COMPASS results on Collins asymmetry
COMPASS results on Collins asymmetry

**deuteron** 2003, 2004  

**proton** 2007, 2010  
Transversity from COMPASS p and d results

COMPASS results already used to extract the transversity PDFs

- Collins asymmetry (COMPASS p and d, HERMES p, Belle)
  Anselmino et al., Kang et al., …

- Dihadron asymmetry (COMPASS p and d, HERMES p, Belle)
  Bacchetta et al.

using parametrisations for transversity and spin-dependent FFs
Transversity from Collins asymmetries

COMPASS results already used to extract the transversity PDFs

- Collins asymmetry (COMPASS p and d, HERMES p, Belle)

Anselmino et al. 2013

Kang et al. 2015
Transversity from Collins asymmetries

\[ g_T = \delta u - \delta d \]

\[ \int_0^1 dx [h_1^q(x) - \bar{h}_1^q(x)] = \delta q. \]

T. Bhattacharya et al.,

Franco Bradamante
Transversity from COMPASS and Belle data

POINT - BY - POINT EXTRACTION:
results obtained using

• Belle results for pion and pion-pair asymmetries

• COMPASS results on
  • p and d dihadron asymmetries vs \( x \) (integrated over \( z, M \))
  • p and d Collins asymmetry vs \( x \) (integrated over \( z, p_T \))
    \( h^+ \) and \( h^- \) assuming that all hadrons are pions

• unpolarised PDFs and FFs parametrizations
  • PDFs: CTEQ5D
  • FFs: DSS LO
    • first shown @ QCD Evolution 2014, in Santa Fe, NM
    • then @ SPIN 2014 in Beijing, China
    • published as A. Martin, F. Bradamante and V. Barone,
      Phys.Rev. D91 (2015) no.1, 014034
Collins asymmetry – COMPASS data

- Charged hadrons
  - Proton 2010
  - Deuteron 2002-2004
Collins asymmetry – COMPASS data

\[ A_{Coll}^h(x, z) = \frac{\sum_{q\bar{q}} e_q^2 x h_1^q(x) \otimes H_{1q}^\pm}{\sum_{q\bar{q}} e_q^2 x f_1^q(x) \otimes D_{1q}^h} \]

“gaussian ansatz”:

\[ A_{Coll}^{\pm}(x, z) = C_G \cdot \frac{\sum_{q,\bar{q}} e_q^2 x h_1^q(x) H_{1q}^{\pm}(z)}{\sum_{q,\bar{q}} e_q^2 x f_1^q(x) D_{1q}^{\pm}(z)} \]

\[ H_{1q}^{\pm} = H_{1q}^{\perp(1/2)} h^\pm \]

\[ C_G = \frac{1}{\sqrt{1 + z^2 < p_{h_1}^2 > / < p_{H_1}^2 >}} \]

Efremov et al., PRD73 (2006)

we have assumed

• \( C_G = 1 \)
• the usual relations among the FFs
• \( H_{1s}^{\pm} = H_{1s}^{\perp} = 0 \) and c quark contributions to be negligible
the measured asymmetries as function of $x$ can be written as

\[
\begin{align*}
A_{Coll,p}^+ &= \frac{<H_1^{fav}>}{<D_1^{fav}>} \frac{4(x h_1^u + \alpha x h_1^\bar{u}) + (\alpha x h_1^d + x h_1^\bar{d})}{d_p^+} \\
A_{Coll,p}^- &= \frac{<H_1^{fav}>}{<D_1^{fav}>} \frac{4(\alpha x h_1^u + x h_1^\bar{u}) + (x h_1^d + \alpha x h_1^\bar{d})}{d_p^-} \\
A_{Coll,d}^+ &= \frac{<H_1^{fav}>}{<D_1^{fav}>} \frac{(x h_1^u + x h_1^d)(4 + \alpha) + (x h_1^\bar{u} + x h_1^\bar{d})(1 + 4\alpha)}{d_d^+} \\
A_{Coll,d}^- &= \frac{<H_1^{fav}>}{<D_1^{fav}>} \frac{(x h_1^u + x h_1^d)(4\alpha + 1) + (x h_1^\bar{u} + x h_1^\bar{d})(4 + \alpha)}{d_d^-}
\end{align*}
\]

\[\alpha = \frac{<H_1^{dis}>}{<H_1^{fav}>} = \begin{bmatrix} -1 \\ - \frac{<D_1^{dis}>}{<D_1^{fav}>} \end{bmatrix} \text{ (a1, a2)}\]

corresponding quantities with unpol PDFs and FFs

from Belle
Collins asymmetry – COMPASS data

The measured asymmetries as function of $x$ can be written as

$$A^{+}_{\text{Coll.p}} = \frac{\langle H_{1}^{f_{av}} \rangle}{\langle D_{1}^{f_{av}} \rangle} \frac{4(xh_{1}^{u} + \alpha xh_{1}^{\bar{u}}) + (\alpha xh_{1}^{d} + xh_{1}^{\bar{d}})}{d_{p}^{+}}$$

$$A^{-}_{\text{Coll.p}} = \frac{\langle H_{1}^{f_{av}} \rangle}{\langle D_{1}^{f_{av}} \rangle} \frac{4(\alpha xh_{1}^{u} + xh_{1}^{\bar{u}}) + (xh_{1}^{d} + \alpha xh_{1}^{\bar{d}})}{d_{p}^{-}}$$

$$A^{+}_{\text{Coll.d}} = \frac{\langle H_{1}^{f_{av}} \rangle}{\langle D_{1}^{f_{av}} \rangle} \frac{(xh_{1}^{u} + xh_{1}^{d})(4 + \alpha) + (xh_{1}^{\bar{u}} + xh_{1}^{\bar{d}})(1 + 4\alpha)}{d_{d}^{+}}$$

$$A^{-}_{\text{Coll.d}} = \frac{\langle H_{1}^{f_{av}} \rangle}{\langle D_{1}^{f_{av}} \rangle} \frac{(xh_{1}^{u} + xh_{1}^{d})(4\alpha + 1) + (xh_{1}^{\bar{u}} + xh_{1}^{\bar{d}})(4 + \alpha)}{d_{d}^{-}}$$

$$\alpha = \frac{\langle H_{1}^{dis} \rangle}{\langle H_{1}^{f_{av}} \rangle} = \begin{bmatrix} -1 & a1 \\ -\frac{\langle D_{1}^{dis} \rangle}{\langle D_{1}^{f_{av}} \rangle} & a2 \end{bmatrix}$$

In each $x$ bin, the only unknowns are the $u$, $u$-bar, $d$ and $d$-bar quark transversity PDFs.
Collins asymmetry – Belle data

We have used the asymmetry (corrected for charm contribution)

\[
A_{12}^{UL}(\chi_1, \chi_2) = \frac{<s^2>}{<1+c^2>} \left[ P_U(\chi_1, \chi_2) - P_L(\chi_1, \chi_2) \right]
\]

integrated over \(M_1, M_2\)

Where

\[
P_U(\chi_1, \chi_2) = \frac{\sum_q e_q^2[H_{1q}^+(\chi_1)H_{1q}^-(\chi_2) + H_{1q}^-(\chi_1)H_{1q}^+(\chi_2)]}{\sum_q e_q^2[D_{1q}^+(\chi_1)D_{1q}^-(\chi_2) + D_{1q}^-(\chi_1)D_{1q}^+(\chi_2)]}
\]

\[
P_L(\chi_1, \chi_2) = \frac{\sum_q e_q^2[H_{1q}^+(\chi_1)H_{1q}^+(\chi_2) + H_{1q}^-(\chi_1)H_{1q}^-(\chi_2)]}{\sum_q e_q^2[D_{1q}^+(\chi_1)D_{1q}^+(\chi_2) + D_{1q}^-(\chi_1)D_{1q}^-(\chi_2)]}
\]

\[
H_{1q}^\pm = H_1^{\perp(1/2)}(q \rightarrow \pi^\pm), \quad D_{1q}^\pm = D_1(q \rightarrow \pi^\pm)
\]

Efremov et al., PRD73 (2006)
Bacchetta et al., PLB659 (2008)
Anselmino et al., PRD75 (2007)
Seidl et al., PRD78 (2008)
Collins asymmetry – Belle data

for the FFs we have made the assumptions

\[ H_{1u}^{fav} = H_{1d}^{+} = H_{1u}^{-} = H_{1d}^{-} = H_{1}^{+} \]  
\[ H_{1u}^{dis} = H_{1d}^{-} = H_{1u}^{+} = H_{1d}^{+} = H_{1}^{-} \]  

(same for \( D \))

ignoring the c and s quark contributions, in the case \( z_1 = z_2 = z \) it is

\[ A_{12}^{UL}(z) = \frac{<s^2>}{<1+c^2>} \left[ \frac{H_{1}^{fav}(z)}{D_{1}^{fav}(z)} \right]^2 B(z) \]

where

\[ B(z) = \frac{b(z)[1+a^2(z)] - [1+b^2(z)]a(z)}{b(z)[1+b^2(z)]} \]
\[ a(z) = \frac{H_{1}^{dis}(z)}{H_{1}^{fav}(z)} \]
\[ b(z) = \frac{D_{1}^{dis}(z)}{D_{1}^{fav}(z)} \]
Collins asymmetry – Belle data

we have done 2 alternative assumptions

\[ H_{1}^{fav}(z) = - H_{1}^{dis}(z) \quad \text{i.e.} \quad a(z) = -1 \]

\[ \frac{H_{1}^{fav}(z)}{D_{1}^{fav}(z)} = - \frac{H_{1}^{dis}(z)}{D_{1}^{dis}(z)} \quad \text{i.e.} \quad a(z) = -b(z) \]

both in agreement with the considerations on the “interplay between the Collins and the dihadron FFs”

and already used / suggested / found as a result of global fits

• these assumptions allow to evaluate \( \frac{H_{1}^{fav}(z)}{D_{1}^{fav}(z)} \) in the four \( z \) bins

• the values are then fitted with a function of \( z \)
Collins asymmetry – Belle data

finally:

\[
\begin{align*}
\text{a1} & \quad \frac{\langle H_1^{fav} \rangle}{\langle D_1^{fav} \rangle} \sim 0.12 \\
\text{a2} & \quad \frac{\langle H_1^{fav} \rangle}{\langle D_1^{fav} \rangle} \sim 0.17
\end{align*}
\]

assumed to be constant in \( Q^2 \)

i.e. the evolution of \( H_1^{fav} \) is the same as that of \( D_1^{fav} \)

if the evolution of \( H_1^{fav} \) is negligible, the analysing powers at COMPASS \( Q^2 \) decrease by \( \sim 10\% \)
Collins asymmetry – transversity

$x h_{1u}^v$ and $x h_{1d}^v$

for a1 and a2
Collins asymmetry – transversity

$xh_{1}^{u_{v}}$ and $xh_{1}^{d_{v}}$
for a1 and a2

Anselmino et al., 2013
Collins asymmetry – transversity

\[ xh_{1u}^{a1} \text{ and } xh_{1d}^{d} \text{ for } a1 \text{ and } a2 \]

\[ xh_{1u}^{a1} \text{ and } xh_{1d}^{d} \]
Collins asymmetry – transversity

$x h^u_1$ and $x h^d_1$
from single-hadron
(closed points)
and
from di-hadron
(open points)
asymmetries
point-by-point extraction

The same method used for the Collins asymmetry has been used for the point-by-point extraction of the Sivers function from the pion and kaon Sivers asymmetries for proton and deuteron

A. Martin, F. Bradamante and V. Barone,
Phys.Rev. D95 (2017) no.9, 094024
Sivers asymmetries

\[ A_{Siv} \]

**Protons:**

- PLB 744 (2015) 250

\[ \pi^+ \]
\[ \pi^- \]

**Deuterons:**

- PLB 673 (2009) 127

\[ K^+ \]
\[ K^- \]
Sivers functions

results

A. Martin, F. Bradamante and V. Barone,
Phys.Rev. D95 (2017) no.9, 094024
**NEW**

The same method used for the extraction of the Collins asymmetry has been used to estimate the impact of one year of COMPASS SIDIS measurement with

- 160 GeV/c momentum muons
- $^6$LiD transversely polarized target
Transversity – valence

with one year of COMPASS SIDIS measurement with $^6$LiD transversely polarized target
SIDIS experiments, transversely polarised target

- JLab (n only, over)
- HERMES (p only, over)
- COMPASS (p and d)
- JLab12 (p and d/n, future)
- EIC (p and d, far future)
THANK YOU

for your attention