Recent Results on Light-Meson Spectroscopy from COMPASS

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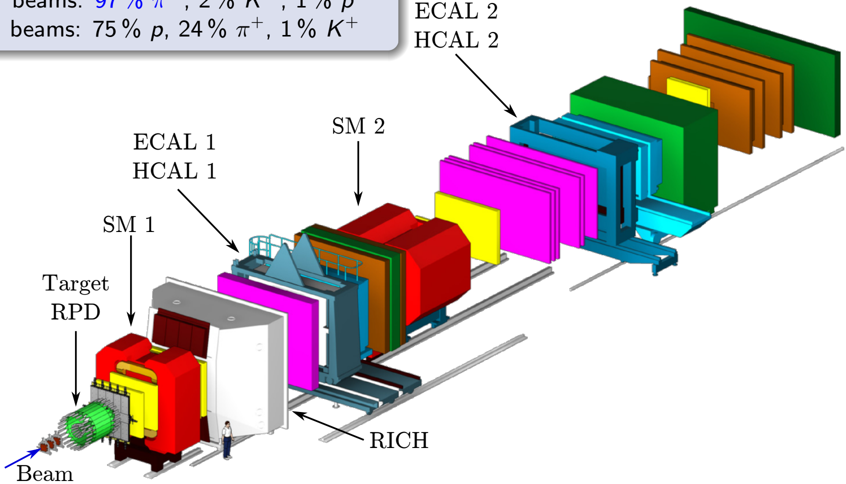
Hadron 2017

[NIMA 779 (2015) 69]

M2 beam line

- Located at CERN (SPS)
- 190 GeV/c secondary hadron beams
  - h^- beams: 97 % π^-, 2 % K^-, 1 % $\bar{p}$
  - h^+ beams: 75 % p, 24 % π^+, 1 % K^+



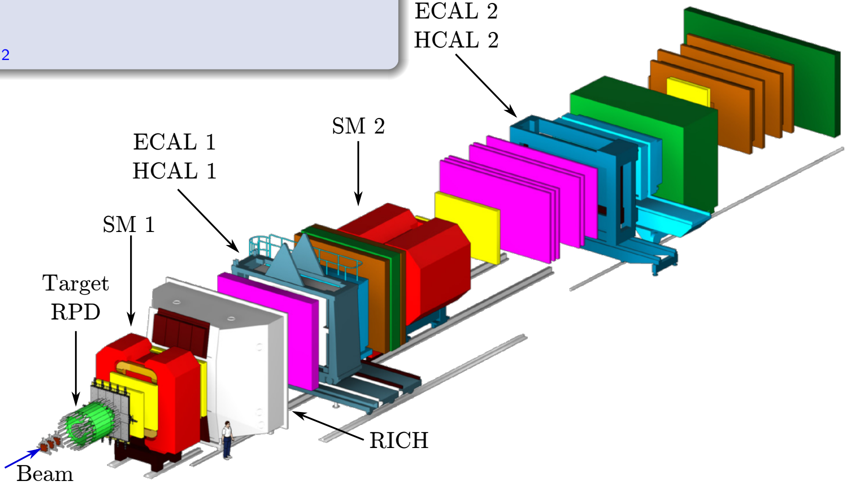
Introduction

COMPASS Setup for Hadron beams

[NIMA 779 (2015) 69]

Target

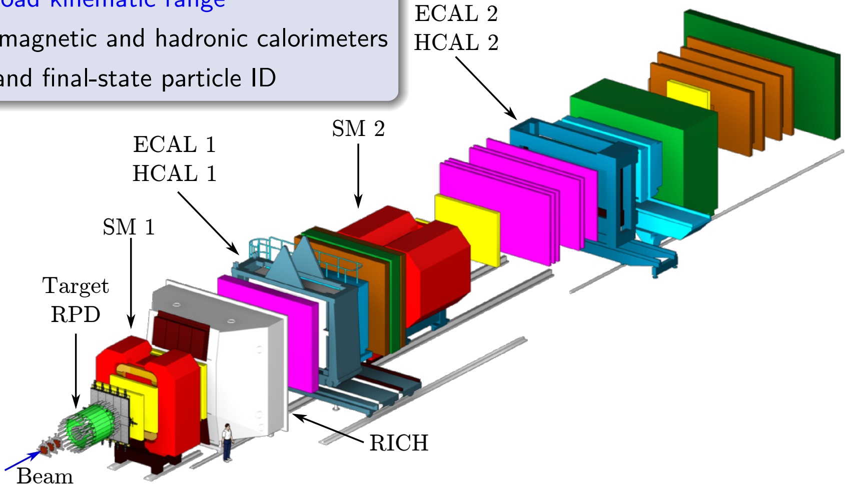
- Various targets:
  - Ni
  - Pb
  - W
  - ℓH_2



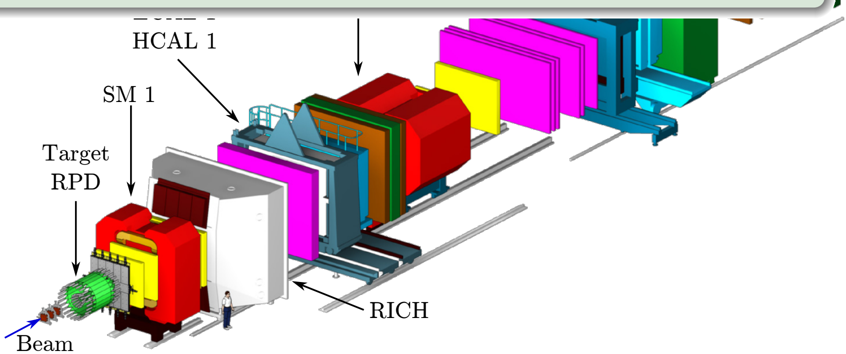
[NIMA 779 (2015) 69]

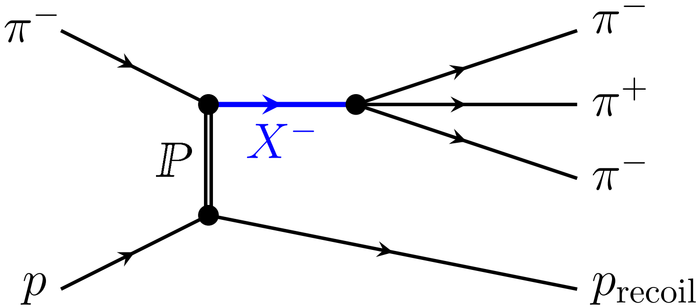
COMPASS spectrometer

- Two-stage magnetic spectrometer
  - ➥ Large acceptance
  - ➥ Broad kinematic range
- Electromagnetic and hadronic calorimeters
- Beam and final-state particle ID



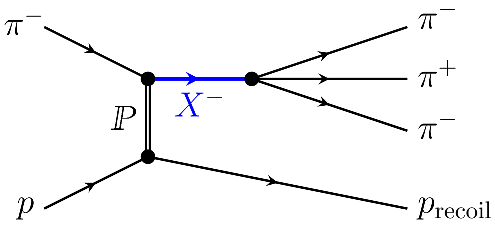
- Explore light-meson spectrum for $m \lesssim 2\,\mathrm{GeV}/c^2$
- High-precision measurement of known states
- Search for new forms of matter:
  - Multi-quark states
  - Hybrids
  - Glueballs
  - …

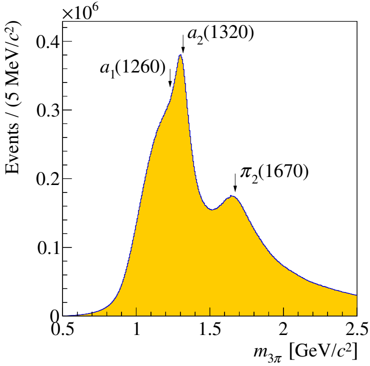




- Various production processes: diffractive production in πp scattering
- Light mesons appear as intermediate states
- Observed in decays into quasi-stable particles: $\pi^-\pi^-\pi^+$ final state

[Adolph et al., PRD 95, 032004 (2017)]

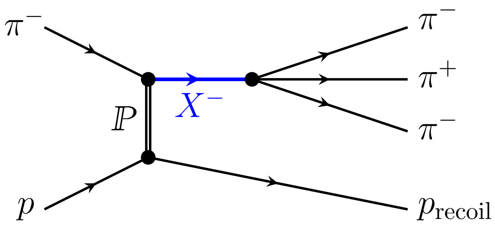


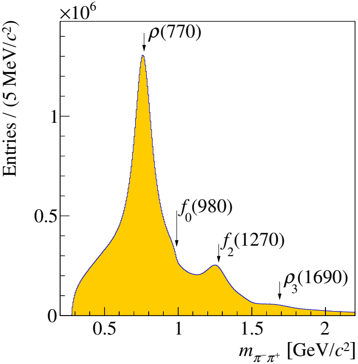


- Rich spectrum of overlapping and interfering X^-
  - Dominant well known states
  - States with lower intensity are “hidden”
- Also structure in $\pi^-\pi^+$ subsystem
  - ➡ Successive 2-body decay via $\pi^-\pi^+$ resonance called isobar

Partial-Wave Decomposition
Motivation

[Adolph et al., PRD 95, 032004 (2017)]





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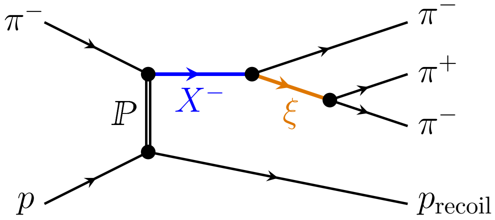

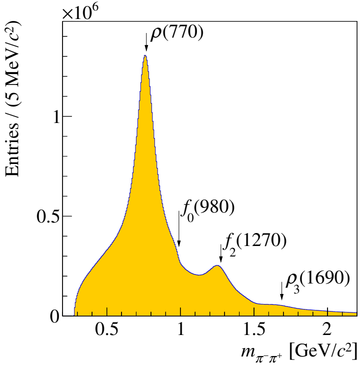

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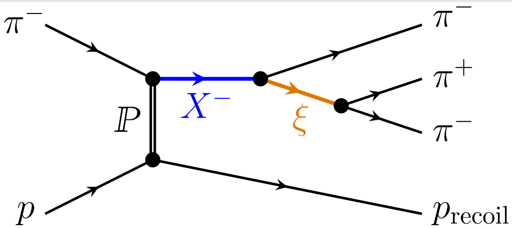
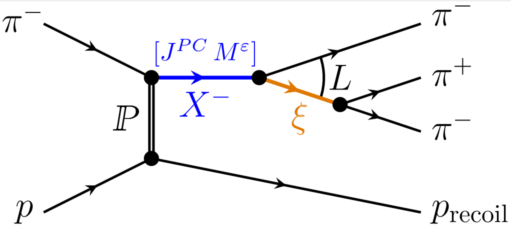


- Given partial wave $i = J^{PC}\, M^{\varepsilon}\, \xi\, \pi\, L$ at a fixed invariant mass of 3π system ($m_{3\pi}$)
  - ➥ Calculate 5D decay phase-space distribution of final state
- $\psi_i(\tau)$ describes distribution of wave i in decay phase-space variables τ
  - ➥ Total intensity distribution contains contribution of various partial waves
  - ➥ Perform maximum-likelihood fit in bins of $m_{3\pi}$
    - ➥ Decompose data into partial waves
    - ➥ Extract $m_{3\pi}$ dependence of partial-wave amplitudes



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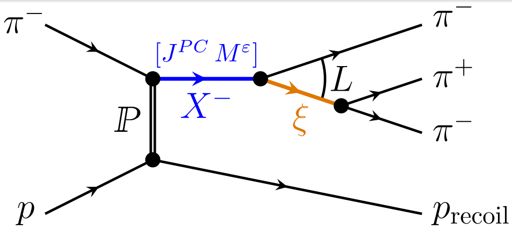
Partial-Wave Decomposition
Isobar Model



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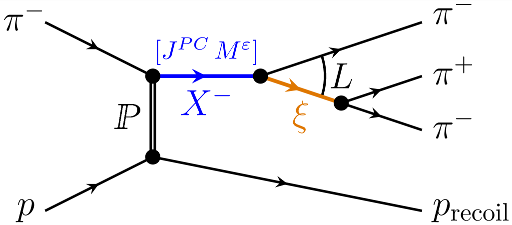

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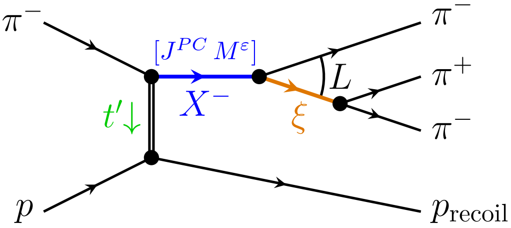
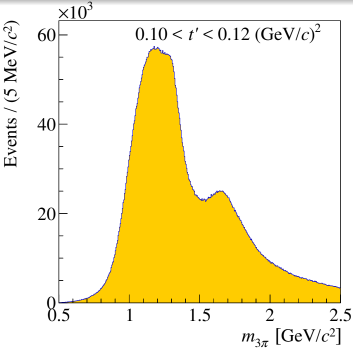
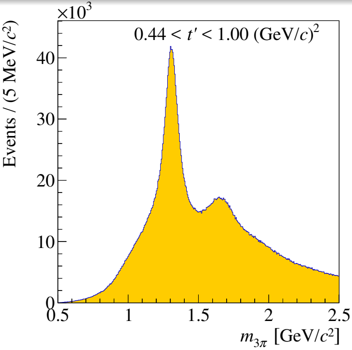


Challenge

- Production also depends on t'

- Large data set (≈ 50 M exclusive events)
  - ➡ Perform PWA also in narrow bins of t' (t'-resolved analysis)
  - ➡ Extract $m_{3\pi}$ AND t' dependence of partial-wave amplitudes





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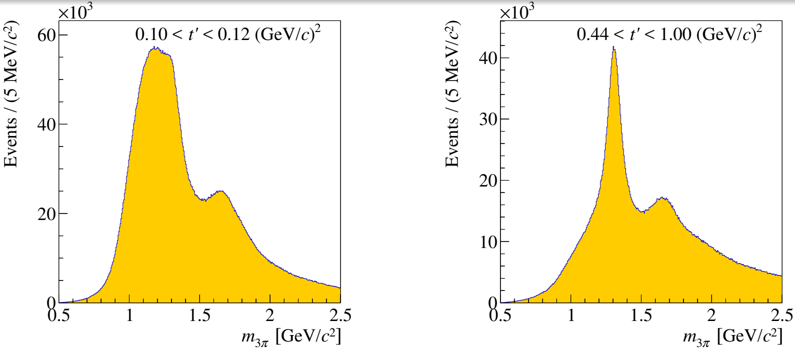

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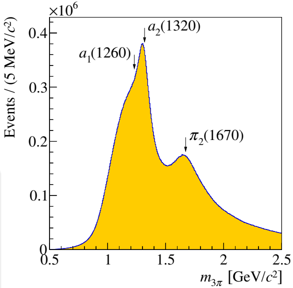
- Decompose into 88 partial waves
- $1^{++}\,0^{+}\,\rho(770)\,\pi\,S$
  - $a_1(1260)$
- $2^{++}\,1^{+}\,\rho(770)\,\pi\,D$
  - $a_2(1320)$
- $2^{-+}\,0^{+}\,f_2(1270)\,\pi\,S$
  - $\pi_2(1670)$



Partial-Wave Decomposition
Results

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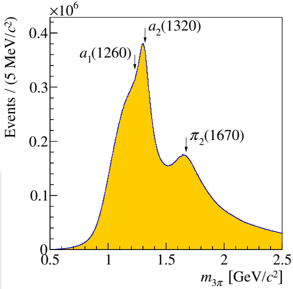

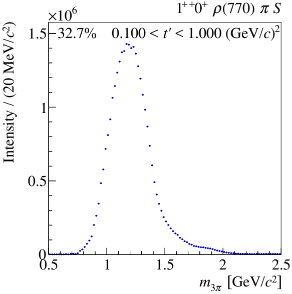

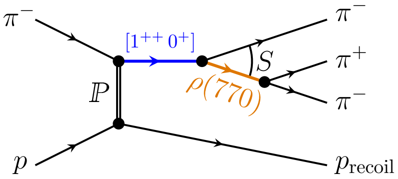

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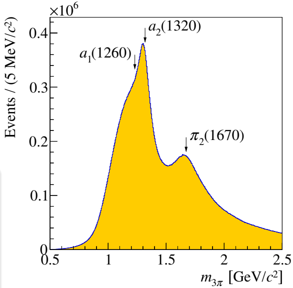

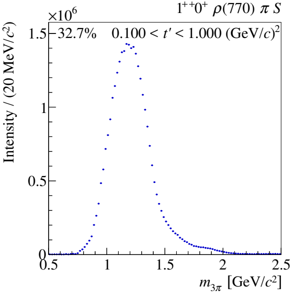

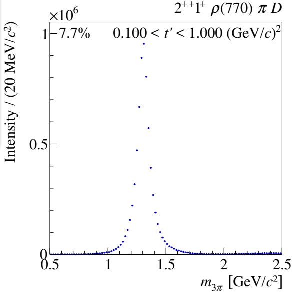

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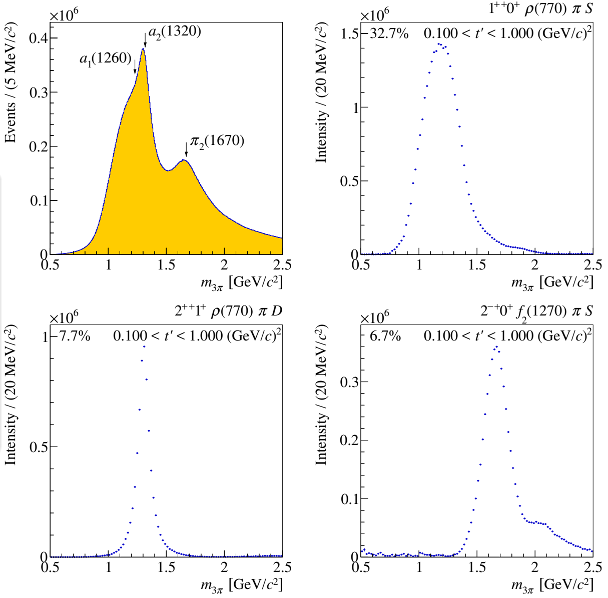


Data

Resonance Parameters

Masses and widths of the meson resonances

Data

(I) Partial-Wave Decomposition

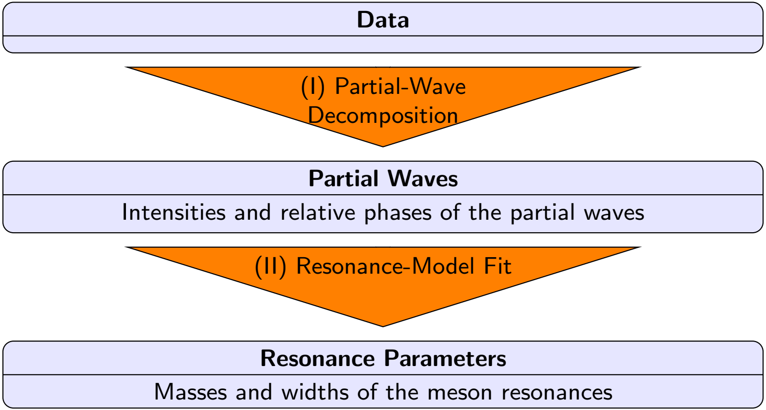
Partial Waves

Intensities and relative phases of the partial waves

Resonance Parameters

Masses and widths of the meson resonances

Resonance-Model Fit
Method



Modeling $m_{3\pi}$ dependence

- Parameterize $m_{3\pi}$ dependence of partial-wave amplitude (intensity & phase)

$$\mathcal{T}_\alpha(m_{3\pi}, t') = \sum_{k \in \mathrm{Comp}_\alpha} \mathcal{C}_\alpha^k(t') \cdot \mathcal{D}^k(m_{3\pi}, t'; \zeta_k)$$

- Dynamic amplitudes $\mathcal{D}^k(m_{3\pi}, t'; \zeta_k)$
  - For resonances: Breit-Wigner amplitude
  - For non-resonant term: Phenomenological parameterization
- "Coupling amplitudes" $\mathcal{C}_\alpha^k(t')$
  - Determine strength and phase of components
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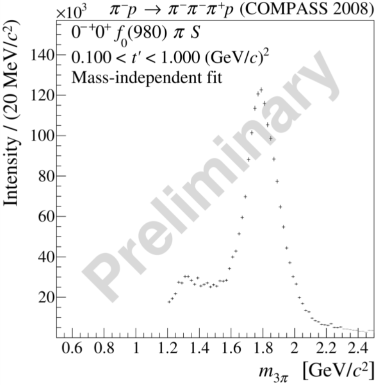

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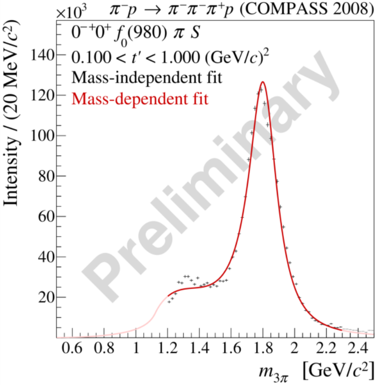

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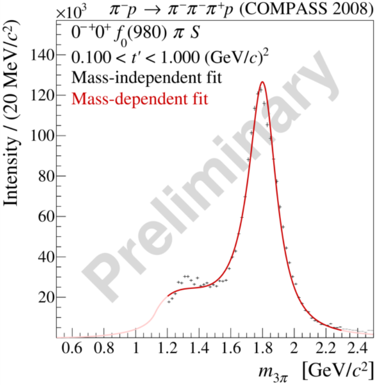

Resonance-Model Fit
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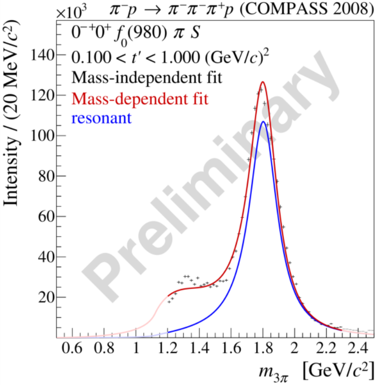

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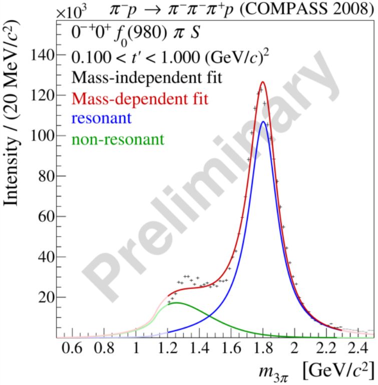

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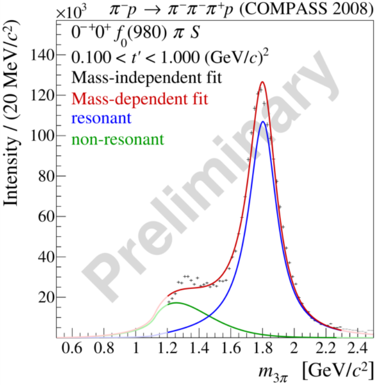
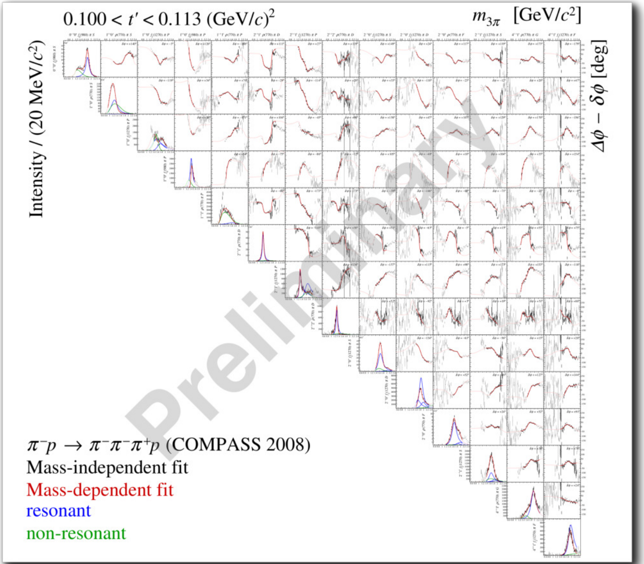


The fit

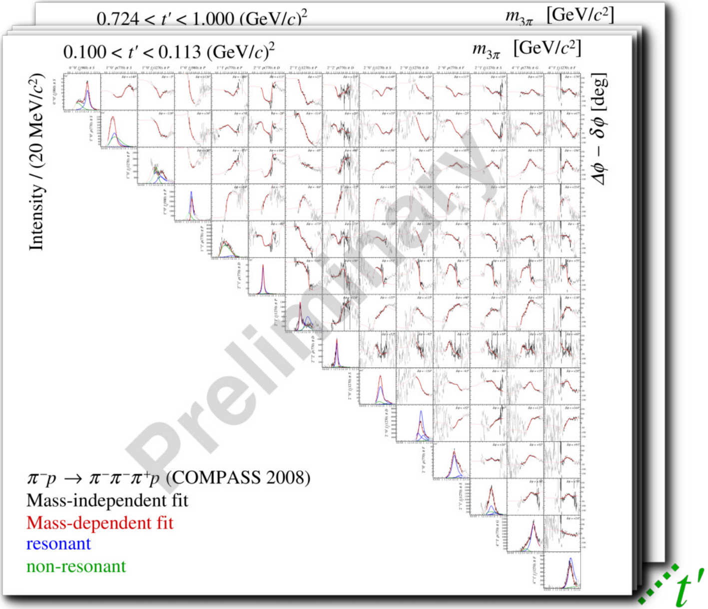
- Describe large fraction of data consistently
  - Simultaneously fit 14 waves ($\approx 60\,\%$ of total intensity)
  - Including 11 resonance components (a_1, a_2, a_4, π, π_1, π_2)
- Extract t' dependence of model components
- Computationally very expensive
  - 14×14 spin-density matrix $\times 11$ t' bins
  - 76 505 data points
  - 722 real fit parameters (51 shape parameters)
- Many systematic effects may influence the fit result
  - Parameterization of resonances and non-resonant terms
  - Selected subset of waves
  - ...

➡ We performed extensive systematic studies



$0.100 < t' < 0.113$ (GeV/c)^2
$m_{3\pi}$ [GeV/c^2]
Intensity / (20 MeV/c^2)
$\Delta\phi - \delta\phi$ [deg]
Preliminary
$\pi^- p \to \pi^-\pi^-\pi^+ p$ (COMPASS 2008)
Mass-independent fit
Mass-dependent fit
resonant
non-resonant

Resonance-Model Fit
Method



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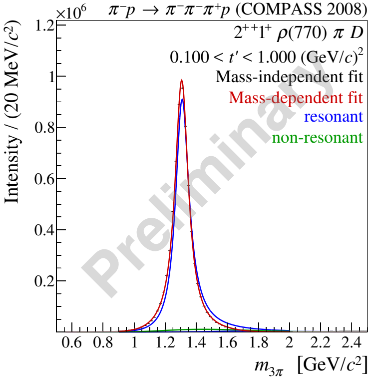
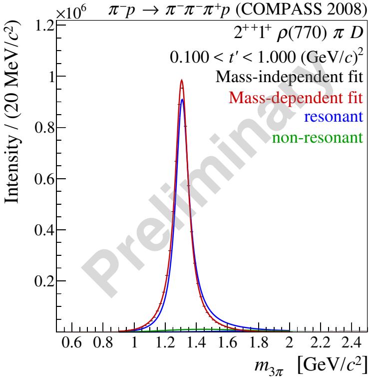
Resonance-Model Fit
2^{++} States

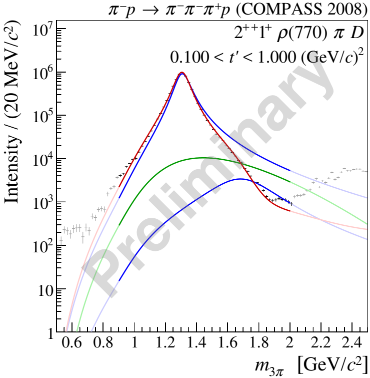


$J^{PC} = 2^{++}$: $a_2(1320)$

- Good description of $a_2(1320)$ peak and phase motion
  - $a_2(1320)$ parameters very stable with respect to variations of the fit model
- Potential $a_2(1700)$ appearing
  - Strongest evidence in $f_2(1270)\ \pi\ P$ decay

Resonance-Model Fit
2^{++} States





$J^{PC} = 2^{++}$: $a_2(1320)$, $a_2(1700)$

- Good description of $a_2(1320)$ peak and phase motion
  - $a_2(1320)$ parameters very stable with respect to variations of the fit model
- Potential $a_2(1700)$ appearing
  - Strongest evidence in $f_2(1270)\,\pi\,P$ decay

Resonance-Model Fit
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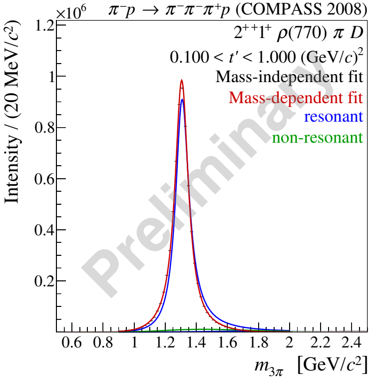

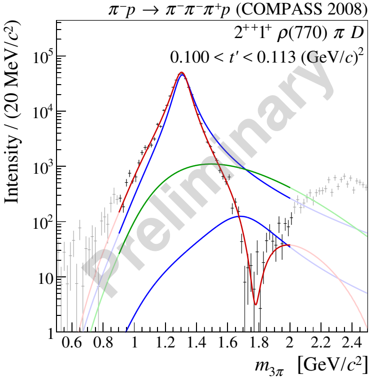

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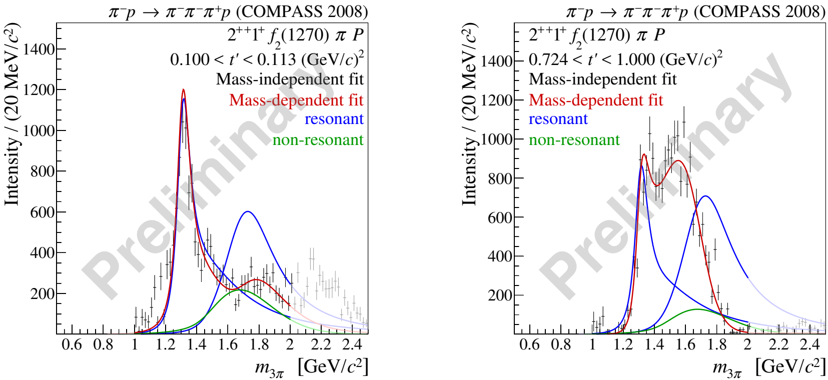

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Resonance-Model Fit

2^{++} States

$a_2(1320)$ resonance parameters — Preliminary

$$m_0 = 1314.2\,^{+1.0}_{-3.1}\,\mathrm{MeV}/c^2 \qquad \Gamma_0 = 106.7\,^{+3.5}_{-2.4}\,\mathrm{MeV}/c^2$$

- Measured width in agreement with PDG world average
- Measured mass is 4 MeV/c^2 lower compared to PDG average, but in agreement with some previous measurements

$a_2(1700)$ resonance parameters — Preliminary

$$m_0 = 1674\,^{+140}_{-32}\,\mathrm{MeV}/c^2 \qquad \Gamma_0 = 435\,^{+50}_{-15}\,\mathrm{MeV}/c^2$$

- Larger systematic uncertainties
- PDG lists $a_2(1700)$ as "omitted form summary table"
- Measured mass in agreement with PDG world average
- Measured width is 241 MeV/c^2 larger
  - Largest discrepancy with Belle measurement: $\Gamma_0 = 151 \pm 22_{\mathrm{stat}} \pm 24_{\mathrm{sys}}\,\mathrm{MeV}/c^2$
  - PDG excludes e.g. E835 measurement from average: $\Gamma_0 = 417 \pm 19\,\mathrm{MeV}/c^2$
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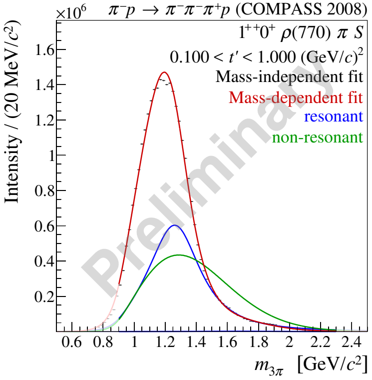
Resonance-Model Fit
Results: 1^{++} Waves



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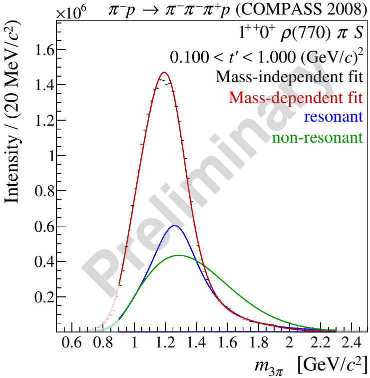

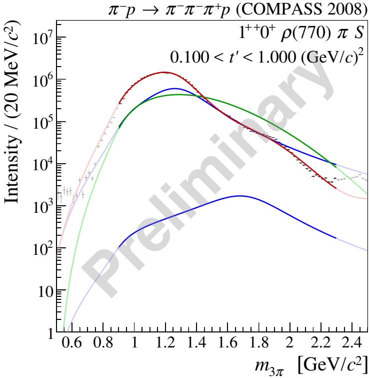

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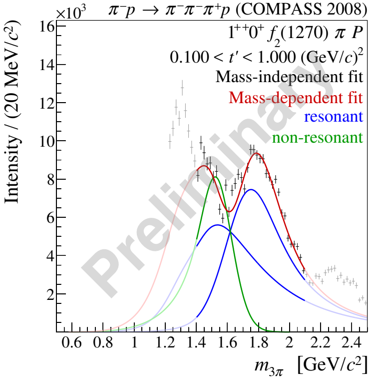

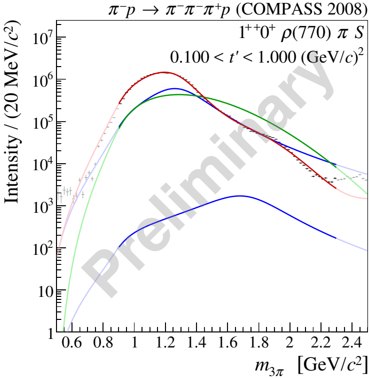

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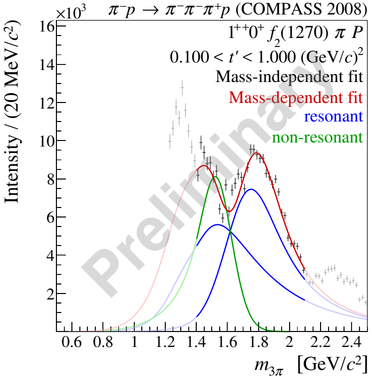

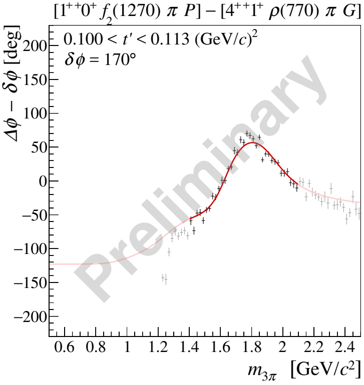

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$$m_0 = 1298\,^{+13}_{-22}\,\mathrm{MeV}/c^2 \qquad \Gamma_0 = 400\,^{+0}_{-100}\,\mathrm{MeV}/c^2$$

- Measured mass is compatible with the PDG world average
- Previous measurements cover a wide range of width values: 230 to 814 MeV/c^2

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$$m_0 = 1690\,^{+40}_{-70}\,\mathrm{MeV}/c^2 \qquad \Gamma_0 = 534\,^{+124}_{-20}\,\mathrm{MeV}/c^2$$

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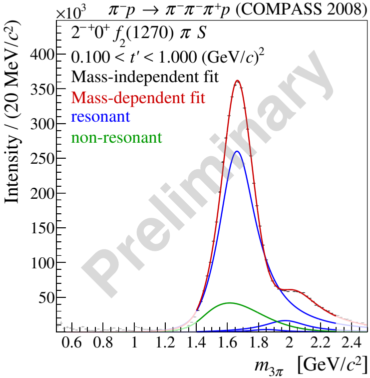

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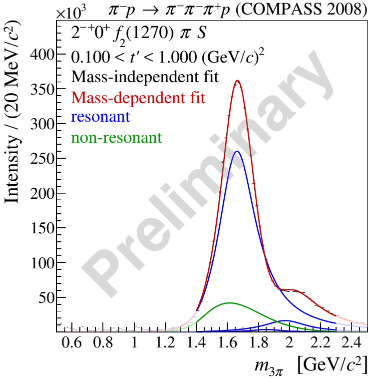
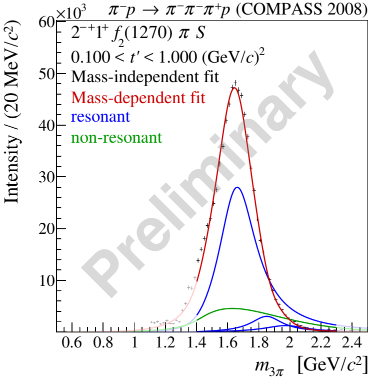
Resonance-Model Fit

Results: 2^{-+} Waves





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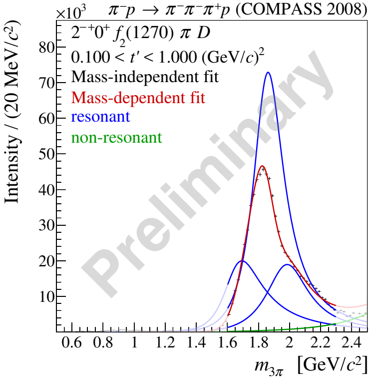

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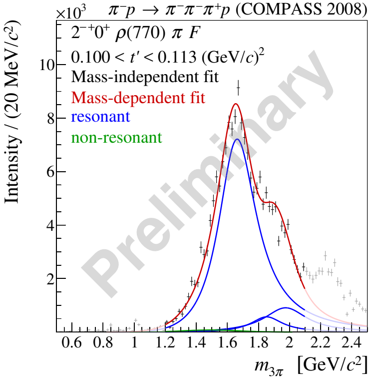

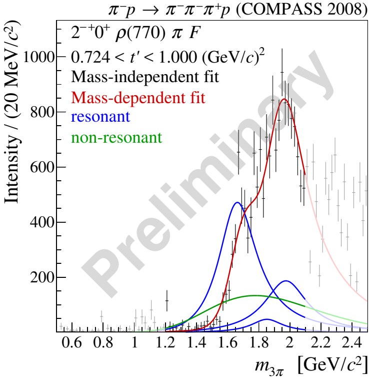

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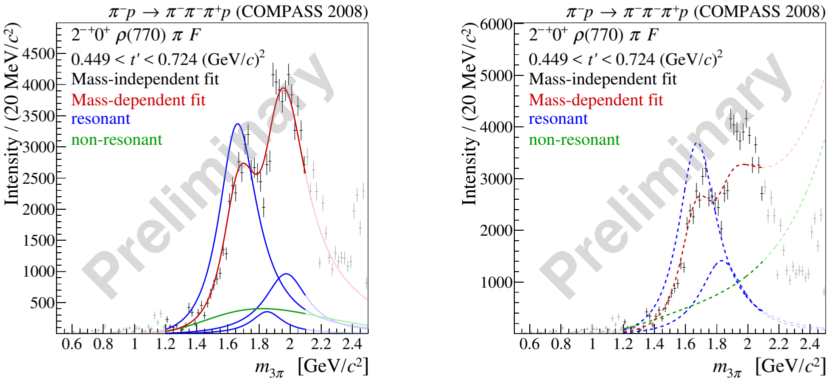

Evidence for $\pi_2(2005)$

- Worse description of intensity spectra and phase motions without $\pi_2(2005)$

Summary

3π final state

- Largest resonance-model used so far
  - Simultaneously fitting 14 partial waves including 4 $J^{PC} = 2^{-+}$ waves
- t' resolved resonance-model fit
- High precision measurement of known states, e.g. $a_1(1260)$
- Investigate excited states, e.g. $a_2(1700)$, $a_1(1640)$, $\pi_2(1880)$, $\pi_2(2005)$
- Investigate potential spin-exotic signals (talk by B. Ketzer, Fri. 11:40)
- Uncertainties limited by systematic effects: Large model dependences
  - ➥ Collaboration between JPAC and COMPASS

Further analysis projects

- Semi-automatized PWA model selection from data (talk by B. Grube, Tue.)
- Freed-isobar method (talk by F. Krinner, Wed. 9:45)
- $\eta\pi^-$: S-matrix amplitude analysis (talk by A. Jackura, Wed. 10:05)
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Backup

Outline

5. Model Selection
6. Freed-Isobar Method
7. Crosscheck with $\pi^-\pi^0\pi^0$
8. Resonance Parameters
9. 2^{-+} t' Dependencies
10. Evidence for $\pi_2(2005)$ in Phase Motion

Model Selection

Simplest Idea: Fit "all" Waves

Systematically construct wave set

- In principle, infinitely many waves possible $\Rightarrow$ have to cut somewhere
  - 6 isobars: broad $[\pi\pi]_S$, $\rho(770)$, $f_0(980)$, $f_2(1270)$, $f_0(1500)$, and $\rho_3(1690)$
  - Spin J up to 6, orbital angular momentum L up to 6
  - Positive and negative naturality of exchange particle
- 432 candidate waves $\Rightarrow$ "wave pool"

Test on Monte Carlo data with known partial-wave content

- Generate data according to fit result from real data with 88-wave model
- Focus on 2 mass bins
  - "Low mass": $m_{3\pi} = 1.0\,\text{GeV}/c^2$
  - "High mass": $m_{3\pi} = 1.8\,\text{GeV}/c^2$

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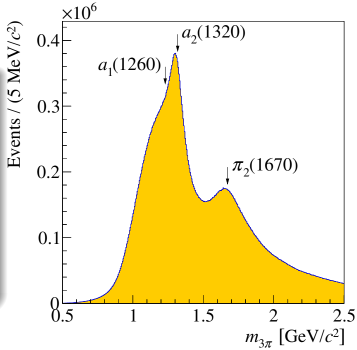

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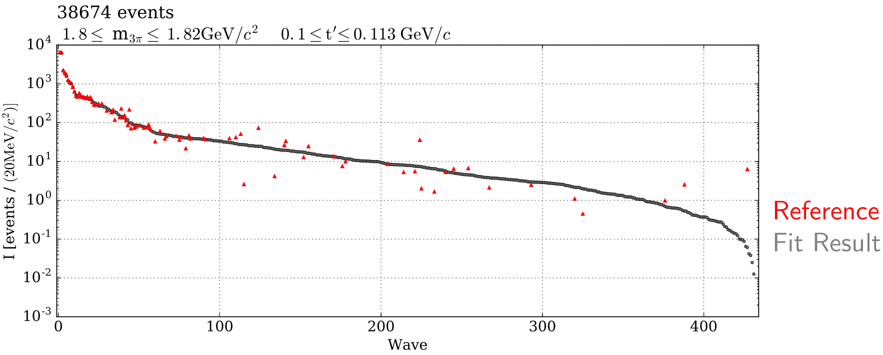
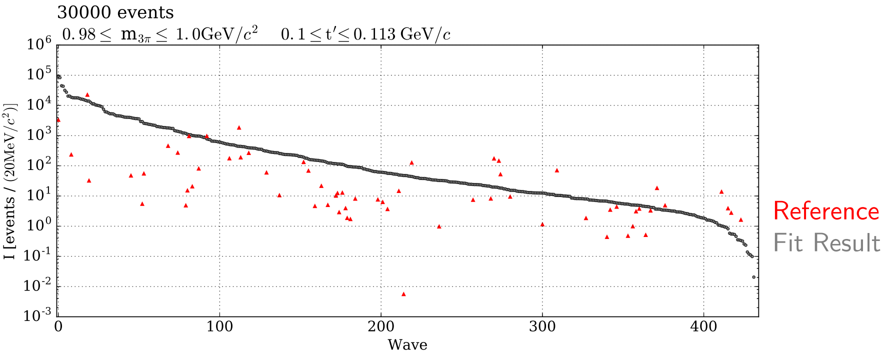


Mass bin at $m_{3\pi} = 1.8\,\mathrm{GeV}/c^2$ (MC Data)

- Waves in fit ordered by intensity
- Corresponding waves in 88-wave model in red
  - Fair agreement
  - But all waves pick up intensity

Model Selection
Fit with Systematically constructed Set of 432 Waves (MC Data)



Mass bin at $m_{3\pi} = 1.0\,\mathrm{GeV}/c^2$ (MC Data)

- All waves pick up intensity
- No agreement with input model
- Signs of overfitting

Model Selection
Maximum A-Posteriori Probability Estimate

Bayes' theorem

$$\underbrace{P(\text{theory}|\text{data})}_{\text{posterior}} = \frac{\overbrace{P(\text{data}|\text{theory})}^{\text{likelihood}}\ \overbrace{P(\text{theory})}^{\text{prior}}}{\underbrace{P(\text{data})}_{\text{evidence} = \text{const}}}$$

$$\propto P(\text{data}|\text{theory})\, P(\text{theory})$$

- Standard maximum likelihood: constant prior

Exploit model properties to construct useful prior

- Model components are waves $\Rightarrow$ want binary decision: include wave or not
- Partial-wave intensity $|\mathcal{T}_i|^2$ or magnitude $|\mathcal{T}_i|$ are measure for importance of wave
  - Normalization chosen such that $|\mathcal{T}_i|^2$ is given in terms of acceptance-corrected number of events
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Idea: prior-based model selection

Impose prior on $|\mathcal{T}_i|$ or $|\mathcal{T}_i|^2$ to suppress small waves

- Model components are waves $\Rightarrow$ want binary decision: include wave or not
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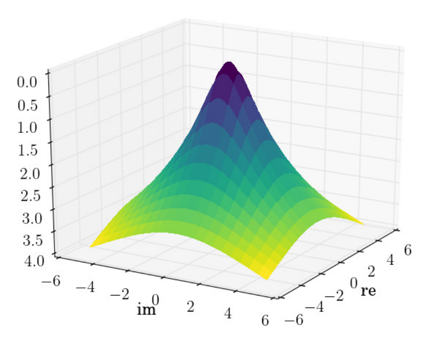
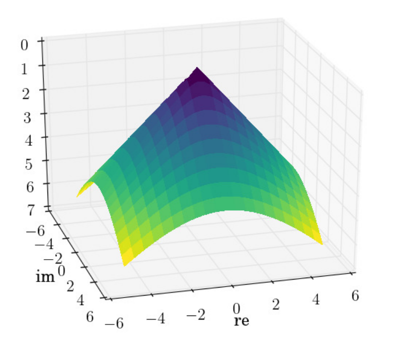
"Cauchy" prior

$$\mathcal{L}_{\text{Cauchy}} = \mathcal{L} \prod_{i}^{\text{waves}} \frac{1}{1 + \frac{|\mathcal{T}_i|^2}{\Gamma^2}}$$

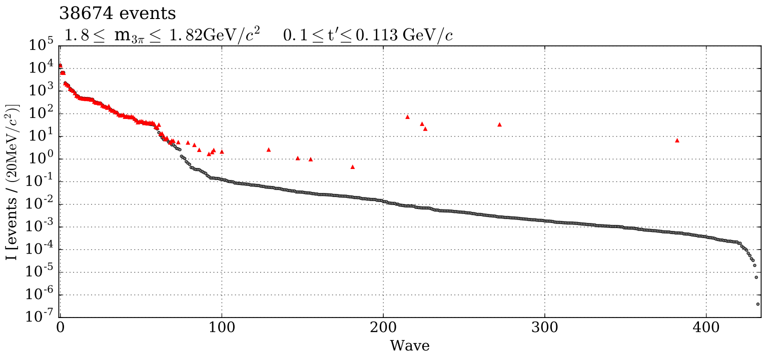


LASSO method

$$\mathcal{L}_{\text{LASSO}} = \mathcal{L} \prod_{i}^{\text{waves}} e^{-\lambda\,|\mathcal{T}_i|}$$



Model Selection
"Cauchy" Prior



Reference

$\Gamma = 0.5$

Mass bin at $m_{3\pi} = 1.8\,\mathrm{GeV}/c^2$ (MC Data)

- Small penalty $\Rightarrow$ no clear point to cut
- Clear drops in intensities $\Rightarrow$ clear place to cut
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- Height of drop increases with lower Γ

Model Selection
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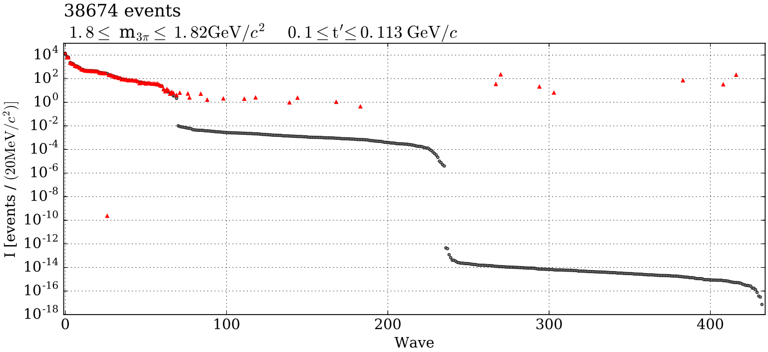

Reference
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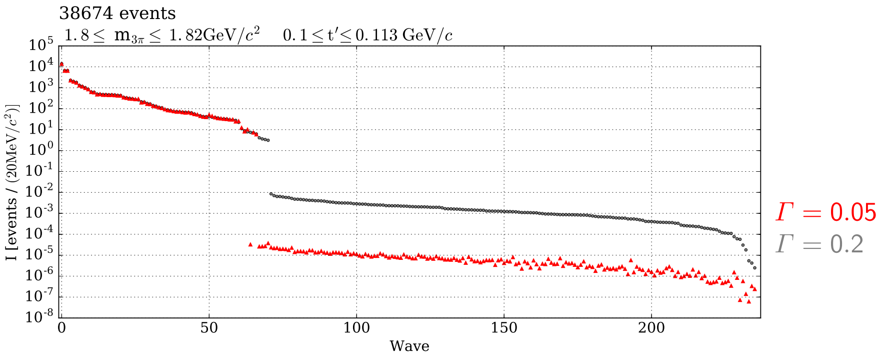

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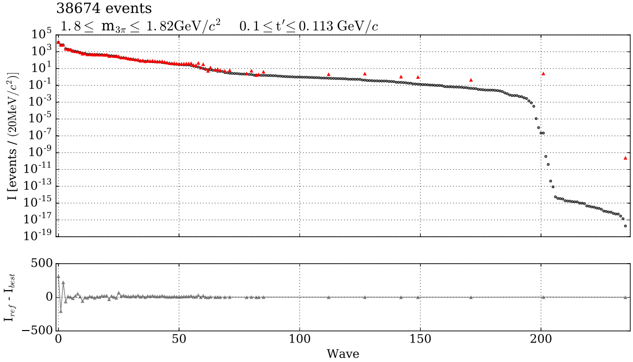
Model Selection
LASSO Method



Reference
$\lambda = 1.0$

Mass bin at $m_{3\pi} = 1.8\,\text{GeV}/c^2$ (MC Data)

- Lasso produces clear drop in intensity distribution
  - B. Guegan *et al.* used cut on relative intensity $> 10^{-3}$ folgt unnecessary
- Position depends strongly on value of λ $\Rightarrow$ dials wave-set size
- Increased bias in larger waves with increased λ
- Need criterion to tune λ

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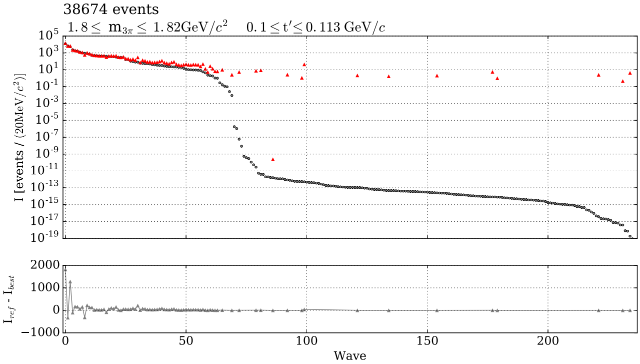

Reference
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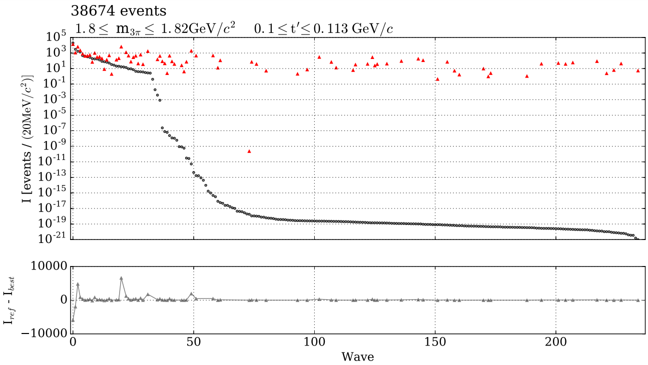

Reference
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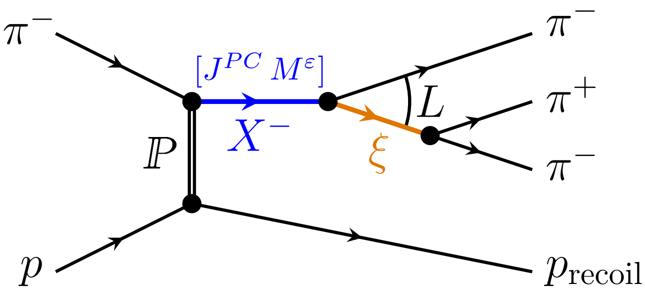
Freed-Isobar Method



Challenge

Need knowledge of isobar amplitude to calculate decay amplitudes $\psi_i(\tau)$

- How good are the parameterizations?
  - Single isobar may not be approximated well by a Breit-Wigner amplitude
  - Effects of rescattering may distort the isobar shape

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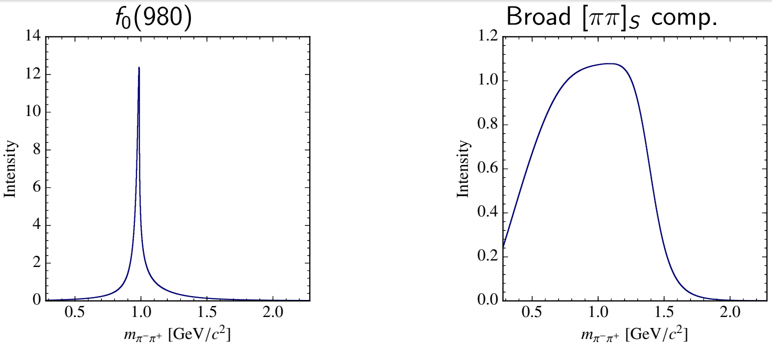

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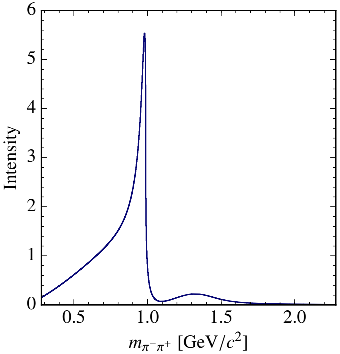
Freed-Isobar Method

Extract isobar amplitudes from data

- Replace model for isobar amplitude with step-like amplitude
- Extract binned shape from data
- Computationally more expensive
  - Up to 100 additional parameters per wave with freed isobar
- Needs large data sets

Total $[\pi\pi]_S$ isobar amplitude



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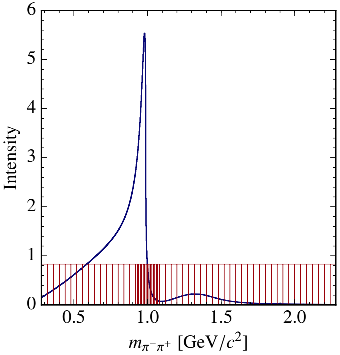

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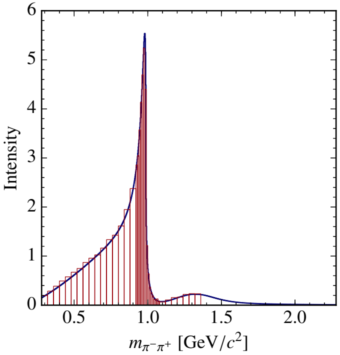

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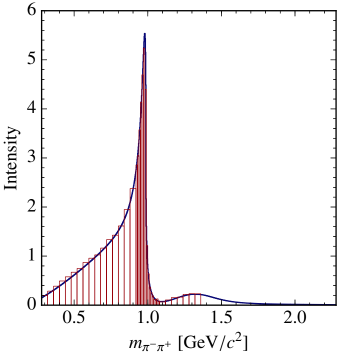

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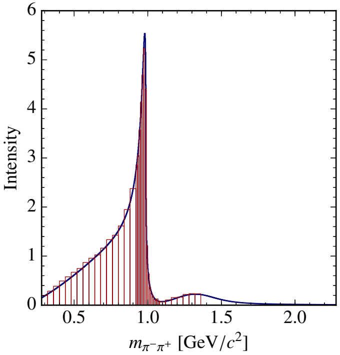

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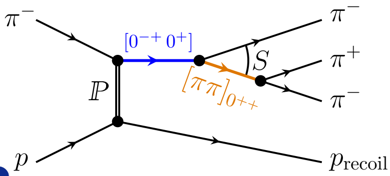
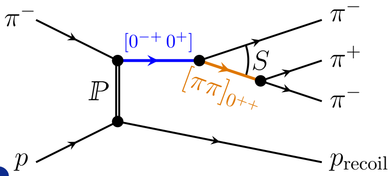


Example: $0^{-+}\,0^+\,[\pi\pi\ S\text{-wave}]\,\pi\,S$ wave

- Comparison of $0^{-+}\,0^+\,[\pi\pi\ S\text{-wave}]\,\pi\,S$ wave intensity between
  - sum of all conventional isobar waves
  - freed-isobar method
- Compatible shapes
- $\pi(1800)$ peak prominent

Freed-Isobar Method



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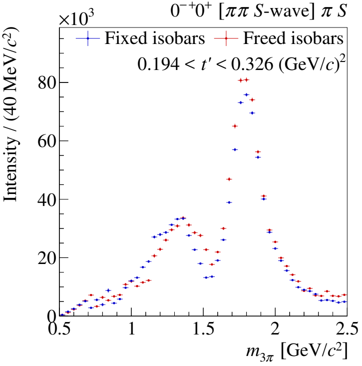
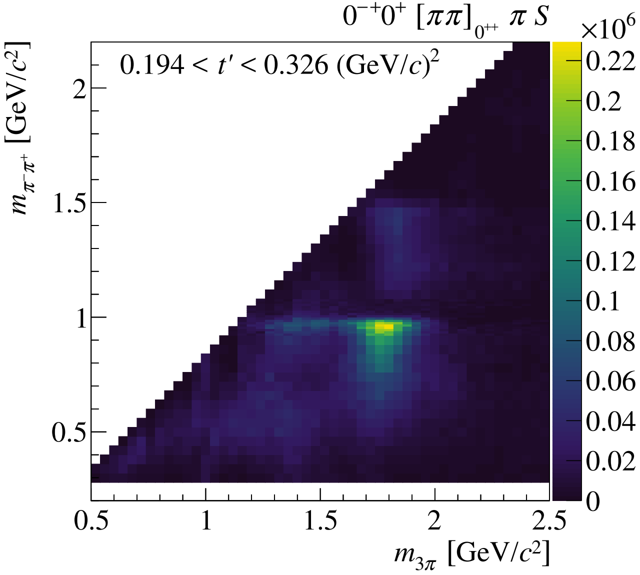


Freed-Isobar Method

[Adolph et al., PRD 95, 032004 (2017)]



This is not a Dalitz-plot

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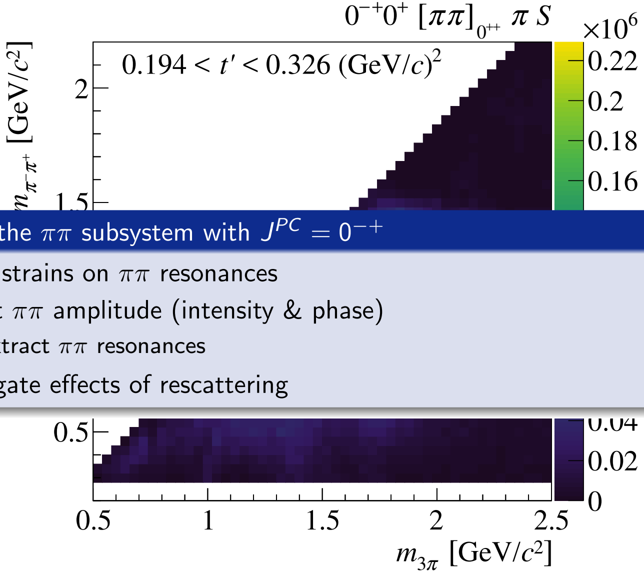


Investigate the $\pi\pi$ subsystem with $J^{PC} = 0^{-+}$

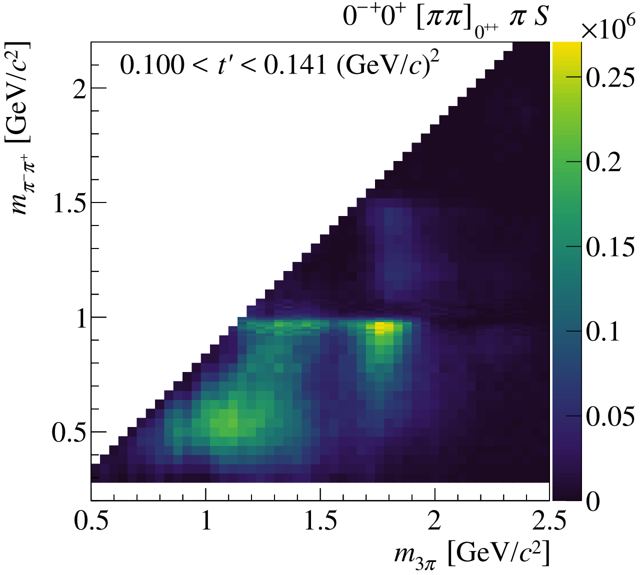
- No constrains on $\pi\pi$ resonances
- Extract $\pi\pi$ amplitude (intensity & phase)
  - Extract $\pi\pi$ resonances
- Investigate effects of rescattering

This is not a Dalitz-plot

Freed-Isobar Method

Freed-Isobar Method: $0^{-+}\,0^{+}\,[\pi\pi]_{0^{++}}\,\pi\,S$

[Adolph et al., PRD 95, 032004 (2017)]



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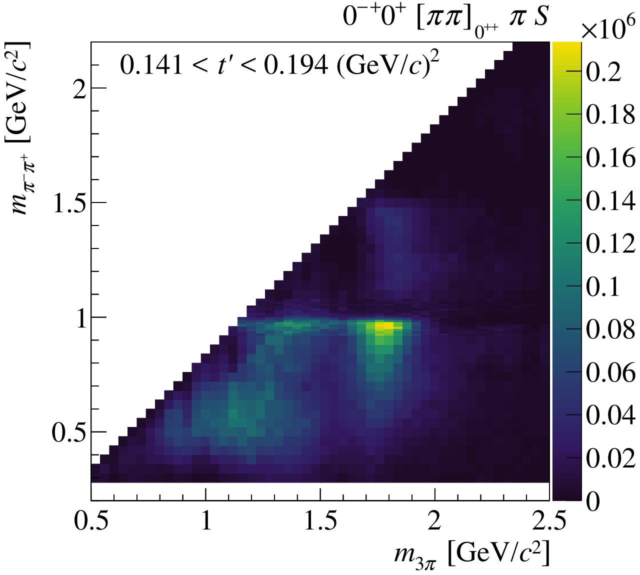

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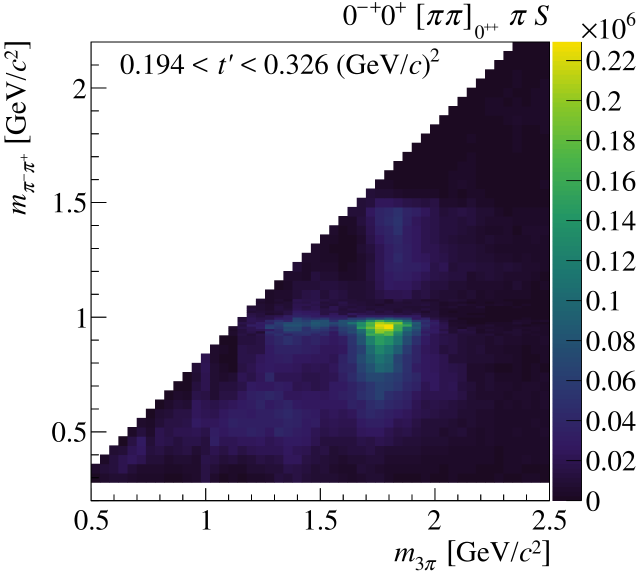

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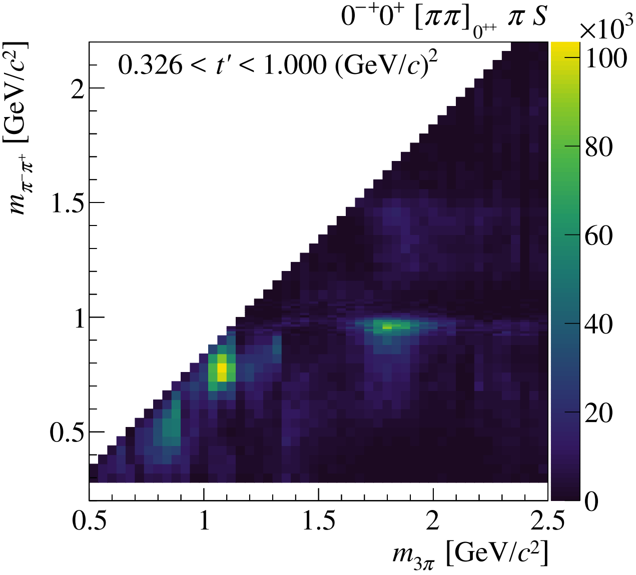

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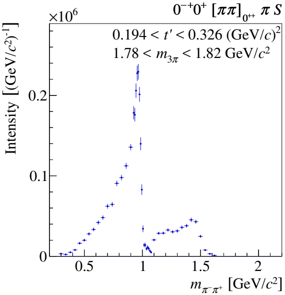

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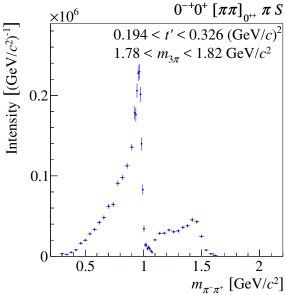

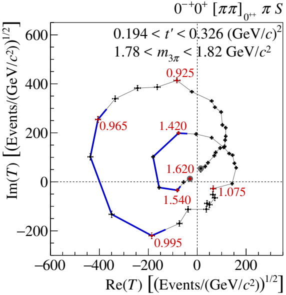

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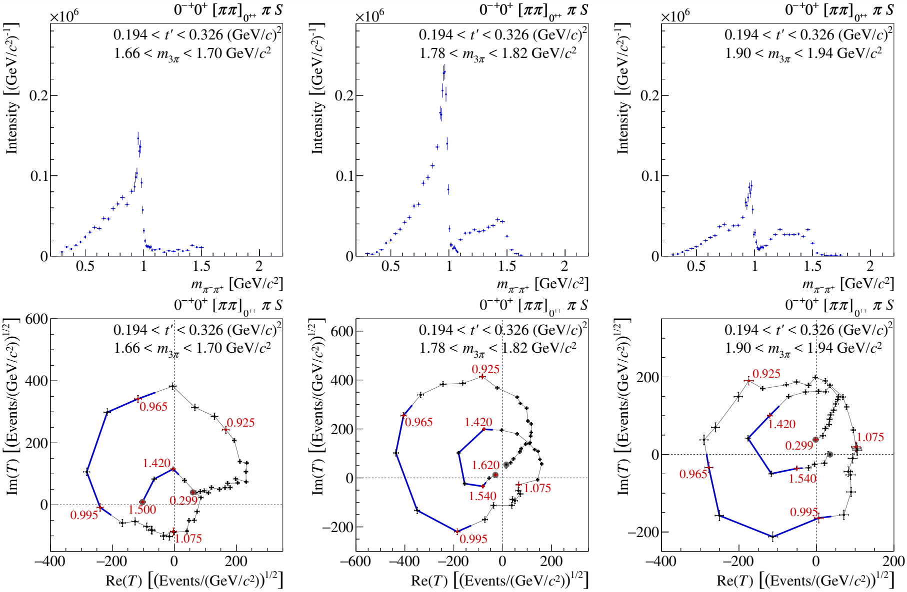
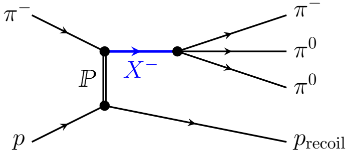


Crosscheck with $\pi^-\pi^0\pi^0$



Two data sets

- Large data set of $\pi^-\pi^0\pi^0$ final state (3.5×10^6 events)
- Access to the same resonances X^-
- Very different acceptance
- Neutral and charged isobars
  - $I = 1$ isobars: $\pi^-\pi^0$
  - $I = 0$ isobars: $\pi^0\pi^0$

Comparison

- Similar signals in both data sets
- Also for weaker signals

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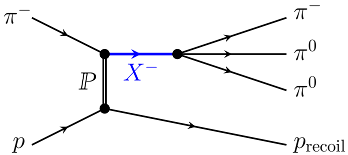

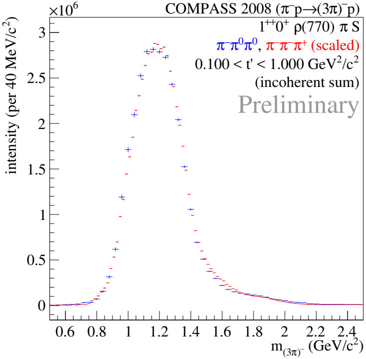

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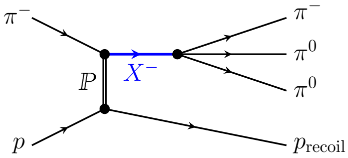

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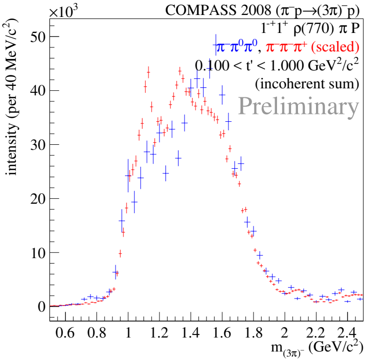

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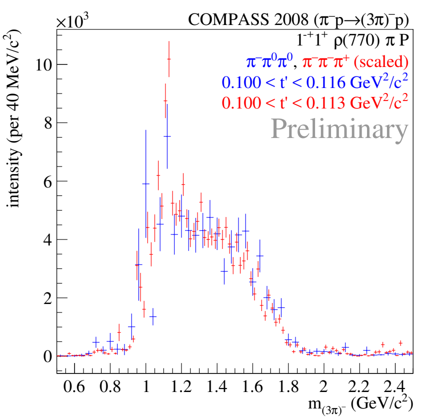
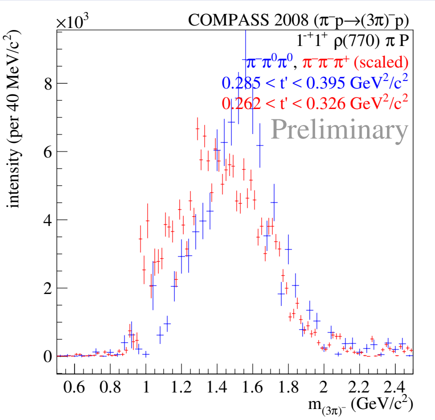
Low t'



High t'



Resonance Parameters

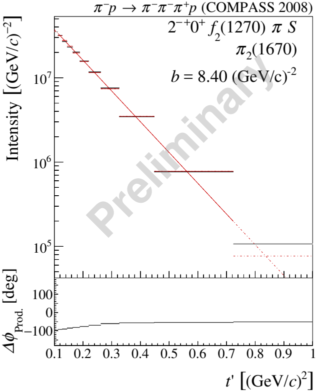
a_J resonances (Preliminary)

		$a_1(1260)$	$a_1(1640)$	$a_2(1320)$	$a_2(1700)$
COMPASS	Mass [MeV/c^2]	1298^{+13}_{-22}	1690^{+40}_{-70}	$1314.2^{+1.0}_{-3.1}$	1674^{+140}_{-32}
	Width [MeV/c^2]	400^{+0}_{-100}	534^{+124}_{-20}	$106.7^{+3.5}_{-2.4}$	435^{+50}_{-15}

π_J resonances (Preliminary)

		$\pi_2(1670)$	$\pi_2(1880)$	$\pi_2(2005)$
COMPASS	Mass [MeV/c^2]	1644^{+12}_{-3}	1847^{+14}_{-6}	1968^{+21}_{-21}
	Width [MeV/c^2]	306^{+14}_{-19}	247^{+41}_{-18}	340^{+50}_{-80}

2^{-+} t' Dependencies



t' dependence

- Intensity of resonances follows exponential model: $I \propto e^{-bt'}$
- For all three π_2 resonances
- Flattening of the t' slope of higher-lying states

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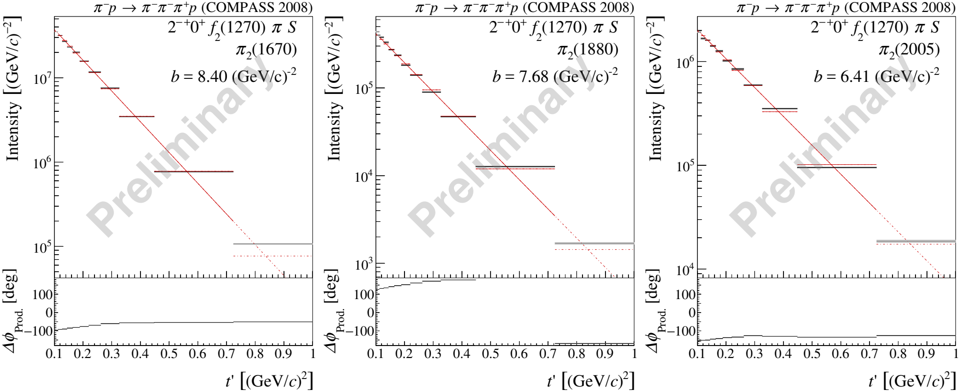

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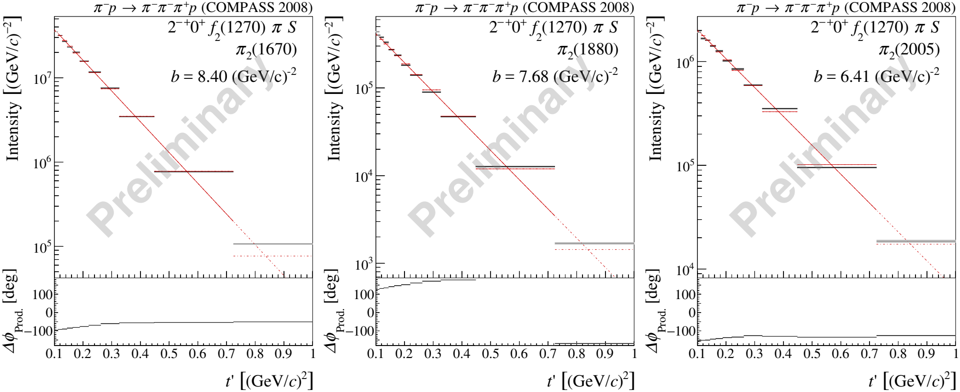

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Evidence for $\pi_2(2005)$ in Phase Motion