Polarized semi-inclusive DY process: fracture functions formalism

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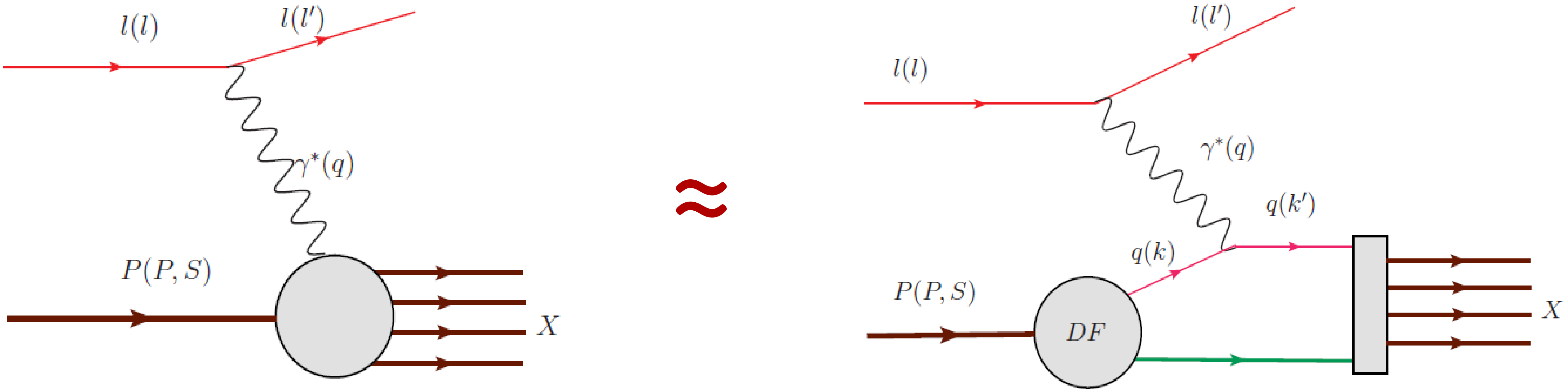
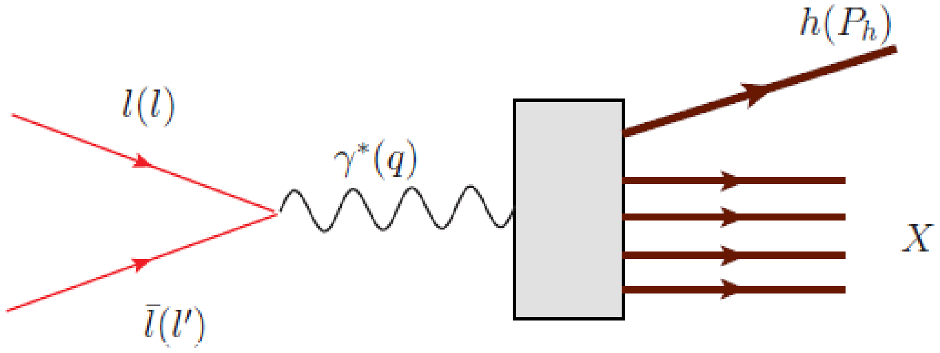
- **Introduction**
  - Only leading twist STMDs
  - Processes with electromagnetic hard probe
  - Parton Distribution Functions: STMD PDF
    - DIS, DY, SIDIS
  - Parton Fragmentation Functions: STMD FF
    - Hadron production in e^+e^- annihilation: SIA, SIDIS
- STMD Fracture Functions
  - SIDIS
  - SIDY
- Conclusions

QCD factorization: DIS

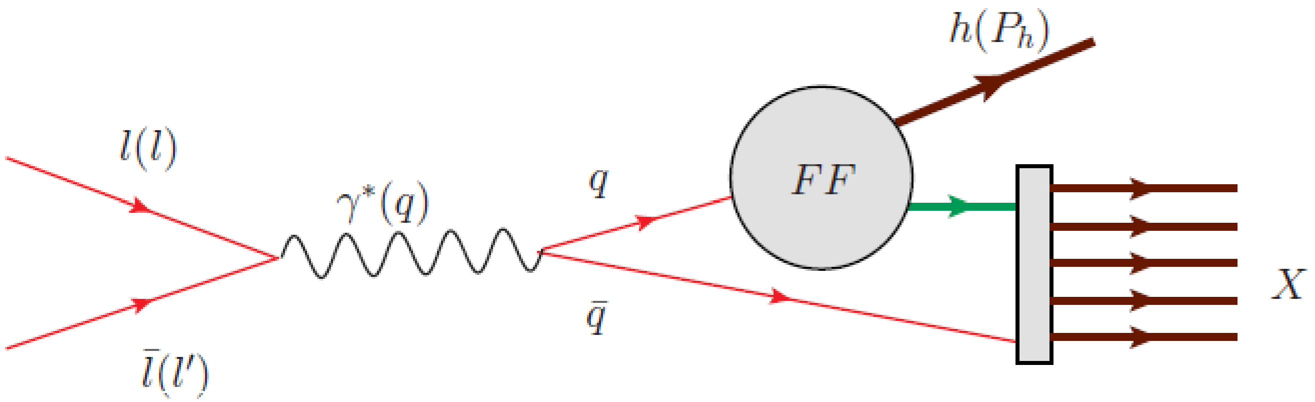


Access to nucleon $f_1^{q+\bar{q}}(x)$ and $g_1^{q+\bar{q}}(x)$ leading twist PDFs

QCD TMD factorization: SIA



$\approx$



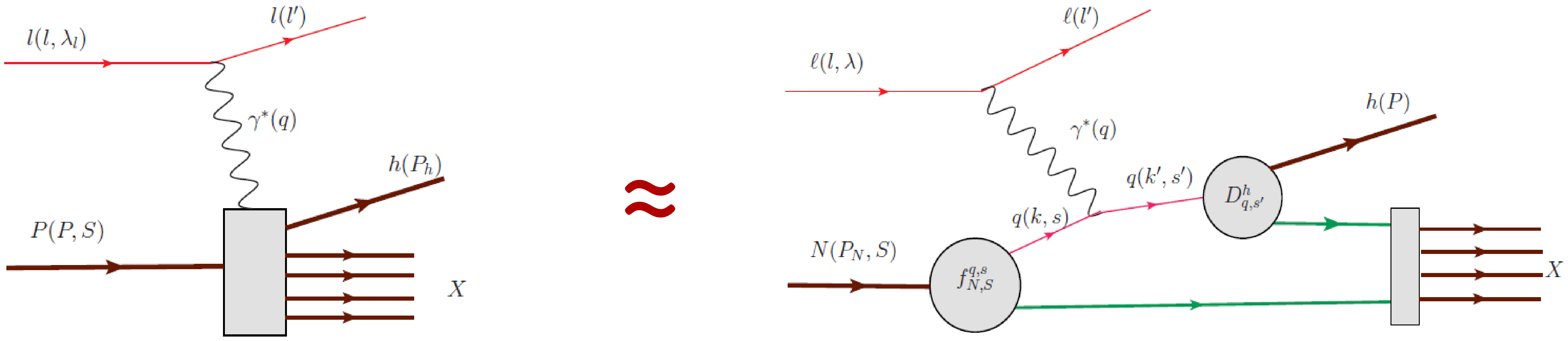
Access to $q+\bar{q}$ fragmentation finctions $D^h_{q+\bar{q}}(\mathrm{z},\mathrm{p}^2_\perp)$

Two hadron production in opposite hemispheres: acces to Collins FF $H^h_{1q}(\mathrm{z},\mathrm{p}^2_\perp)$

Two di-hadron production in opposite hemispheres:

acces to $H^{\sphericalangle}_q(\mathrm{z}),\ H^\perp_q(\mathrm{z})$ and $\mathrm{G}^\perp_q(\mathrm{z})$

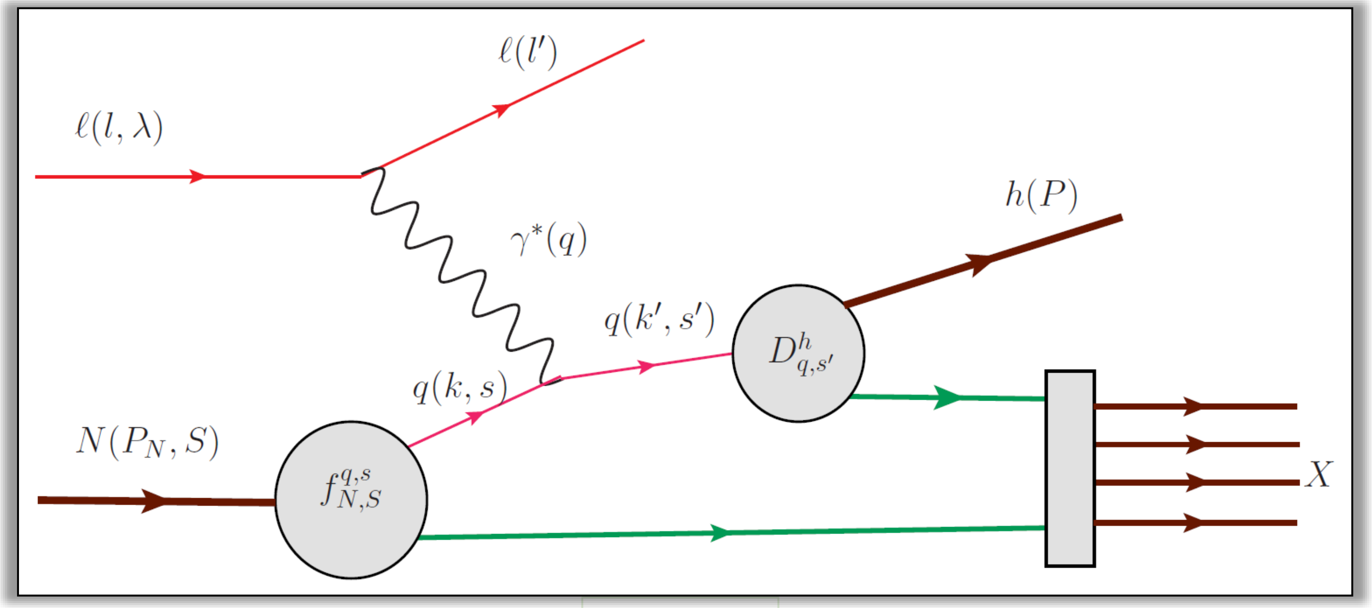
QCD TMD factorization: SIDIS



Access to leading twist nucleon TMD PDFs $f_1^q(x,k_T^2)$, $g_1^q(x,k_T^2)$, $h_1^q(x,k_T^2)$

and Collins FF $H_1(z,p_T^2)$

SIDIS: CFR



$$x_F > 0$$

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_T} = f_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2} \otimes D^{h_1}_{q,s'}$$

Two FFs:

$$D^{h_1}_{q,s'}(z,\mathbf{p}_T) = D_1(z,p_T^2) + \frac{\mathbf{p}_T \times \mathbf{s}'_T}{m_h} H_1(z,p_T^2)$$

← Measured in semi inclusive $e^+e^- \to h_1h_2X$ annihilation (SIA)

Twist-2 STMD qDFs

Nucleon Polarization \ Quark polarization	U	L	T
U	$f_1^q(x,k_T^2)$		$\frac{\epsilon_T^{ij}k_T{}^j}{M}h_1^{\perp q}(x,k_T^2)$
L		$S_L g_{1L}^q(x,k_T^2)$	$S_L\frac{\mathbf{k}_T}{M}h_{1L}^{\perp q}(x,k_T^2)$
T	$\frac{\mathbf{k}_T\times\mathbf{S}_T}{M}f_{1T}^{\perp q}(x,k^2)$	$\frac{\mathbf{k}_T\cdot\mathbf{S}_T}{M}g_{1T}^{\perp q}(x,k_T^2)$	$\mathbf{S}_T h_{1T}^q(x,k_T^2)+\frac{\mathbf{k}_T}{M}\frac{(\mathbf{k}_T\cdot\mathbf{S}_T)}{M}h_{1T}^{\perp q}(x,k_T^2)$

All azimuthal dependences are in prefactors. TMDs do not depend on them

LO cross section in SIDIS CFR

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}(x_F>0)}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_T}=\frac{\alpha^2 x}{yQ^2}\left(1+(1-y)^2\right)\times$$

$$\times\left[\begin{array}{l} F_{UU,T}+D_{nn}(y)F_{UU}^{\cos 2\phi_h}\cos(2\phi_h)+ \\ S_L D_{nn}(y)F_{UL}^{\sin 2\phi_h}\sin(2\phi_h)+\lambda S_L D_{ll}(y)F_{LL}+ \\ S_T\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}\sin(\phi_h-\phi_S)+D_{nn}(y)\left(\begin{array}{l}F_{UT}^{\sin(\phi_h+\phi_S)}\sin(\phi_h+\phi_S)+ \\ F_{UT}^{\sin(3\phi_h-\phi_S)}\sin(3\phi_h-\phi_S)\end{array}\right)\right)+ \\ \lambda S_T D_{ll}(y)F_{LT}^{\cos(\phi_h-\phi_S)}\cos(\phi_h-\phi_S)\end{array}\right]$$

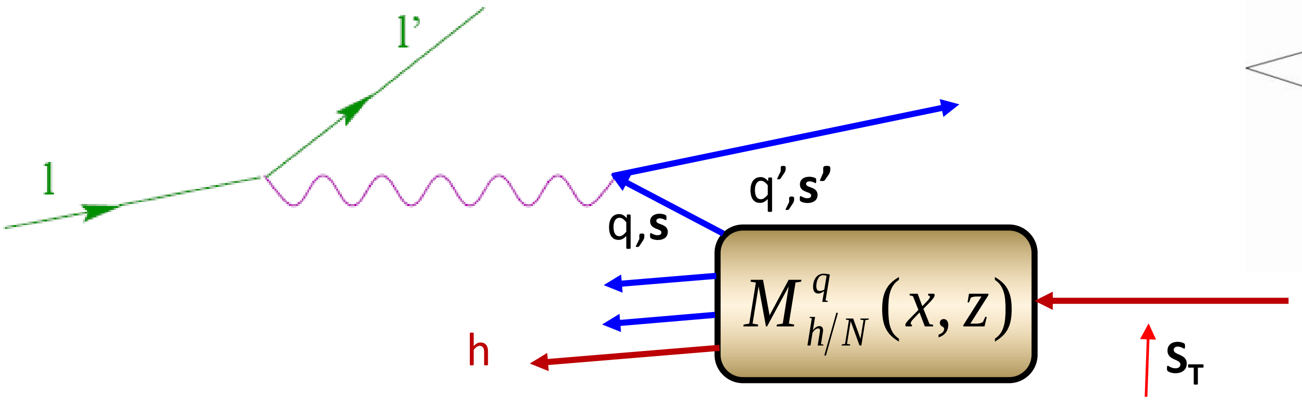
$$D_{ll}(y)=\frac{y(2-y)}{1+(1-y)^2},\qquad D_{nn}(y)=\frac{2(1-y)}{1+(1-y)^2}$$

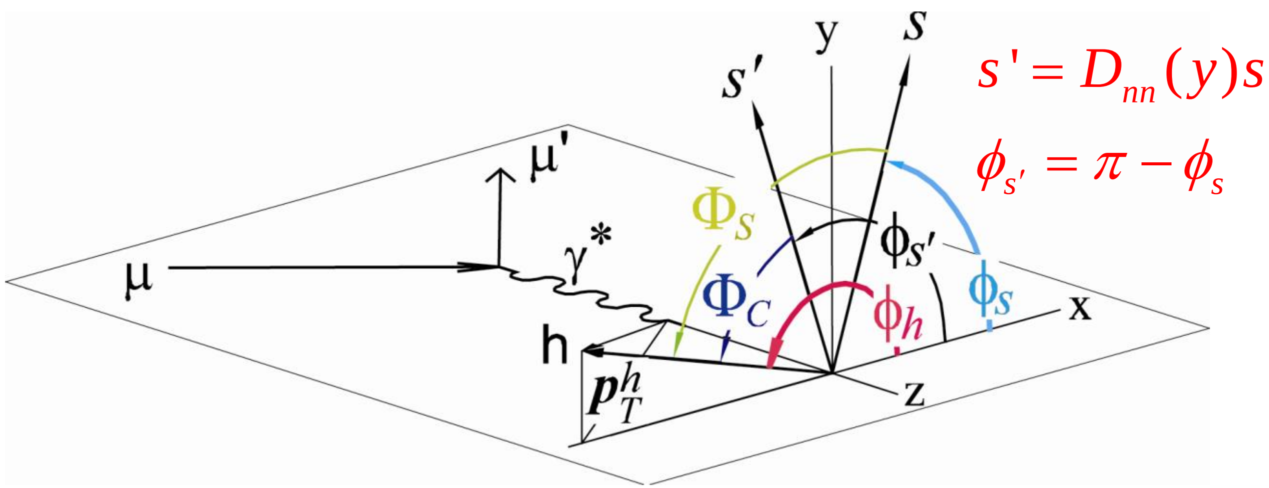
8 terms out of 18 Structure Functions, 6 azimuthal modulations

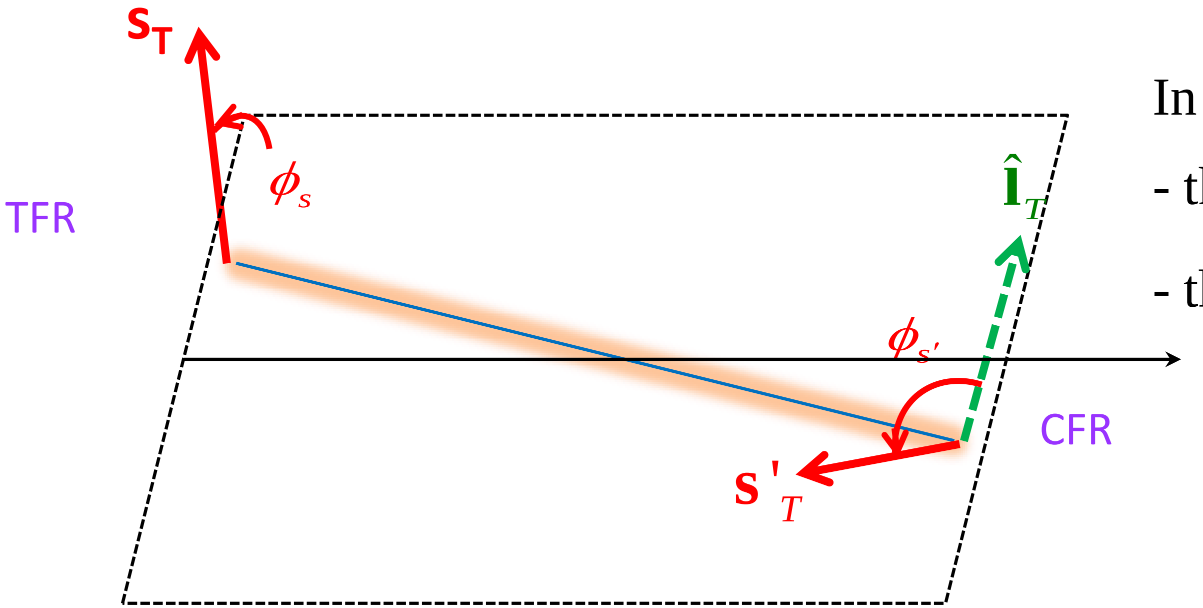
4 terms are generated by Collins effect in fragmentation

Are there Collins-like correlation in TFR of SIDIS?

AK, Yerevan Transversity Workshop, 2009



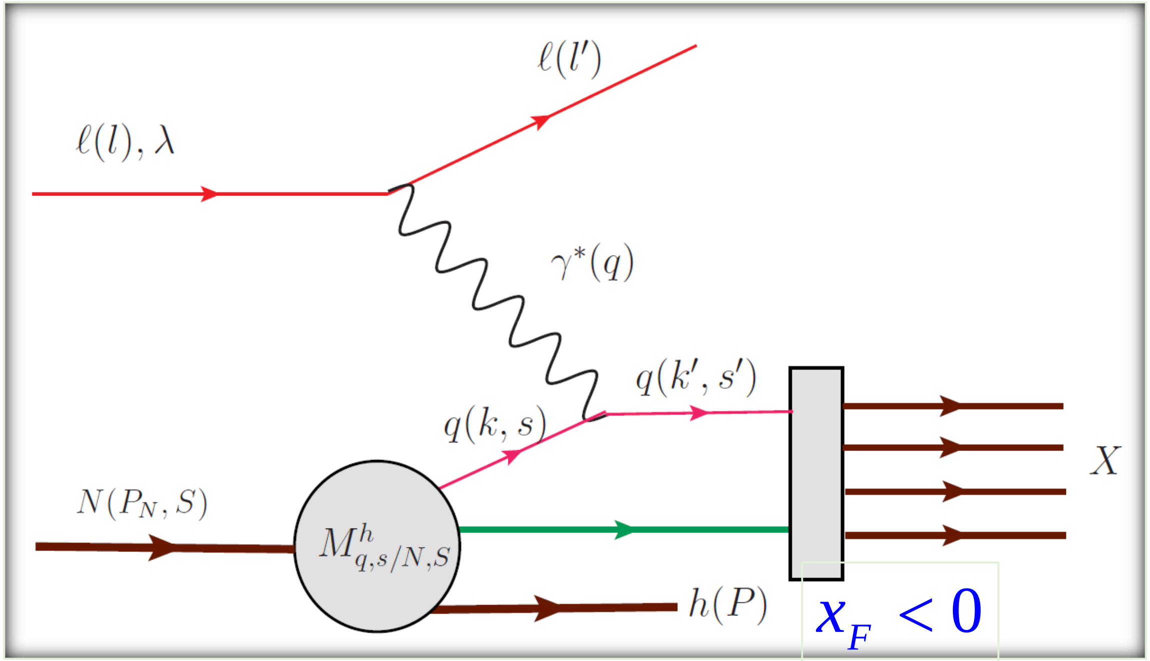




In contrast with CFR for hadrons produced in TFR of SIDIS

- the factor $D_{nn}(y) = \dfrac{2(1-y)}{1+(1-y)^2}$ will be absent in asymmetry
- the modulation induced by combined effect of transversity and Collins-like correlation will be different: $\sin(\phi_h + \phi_s) \rightarrow \sin(\phi_h - \phi_s)$

SIDIS: Fracture Functions formalism for TFR



Trentadue, Veneziano 1994
Graudenz 1994
Collins 1998, 2000, 2002
de Florian, Sassot 1997, 1998
Grazzini, Trentadue, Veneziano 1998
Ceccopieri, Trentadue 2006, 2007, 2008
Sivers 2009
Ceccopieri , Mancusi 2013
Ceccopieri 2013
……..

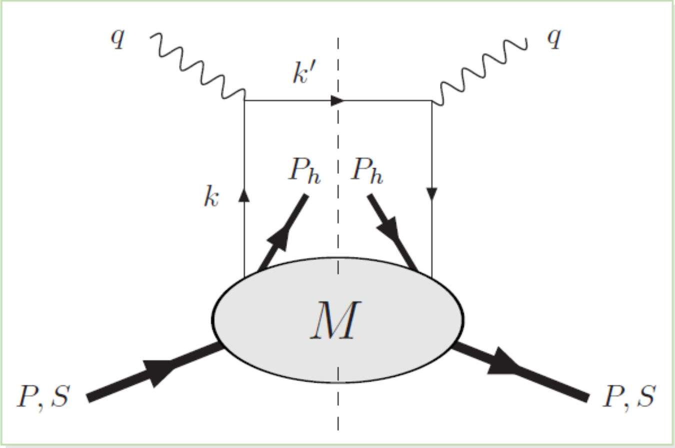
STMD fracture functions, Anselmino, Barone and AK, PL B 699 (2011)108**; 706** (2011)46**; 713** (2012)317

Both the nucleon and quark polarization are taken into account as well as produced hadron and quark transverse momenta

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}}{dx\,dQ^2\,d\phi_S\,d\zeta\,d^2P_T} = M^h_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2}$$

$$\zeta = \frac{P^-}{P_N^-} \approx |x_F|(1-x)$$

Quark correlator for fracture functions



$$\mathcal{M}^{[\Gamma]}(x_B,\vec{k}_\perp,\zeta,\vec{P}_{h\perp}) = \frac{1}{4\zeta}\int\frac{d\xi^+ d^2\xi_\perp}{(2\pi)^6} e^{i(x_B P^- \xi^+ - \vec{k}_\perp\cdot\vec{\xi}_\perp)}\sum_X\int\frac{d^3P_X}{(2\pi)^3 2E_X}\times$$

$$\times\langle P,S|\bar{\psi}(0)\Gamma|P_h,S_h;X\rangle\langle P_h,S_h;X|\psi(\xi^+,0,\vec{\xi}_\perp)|P,S\rangle$$

$$\Gamma = \gamma^-,\quad \gamma^-\gamma_5,\quad i\sigma^{i-}\gamma_5 \;\leftrightarrow\; \text{leading twist}$$

At LO 16 STMD fracture functions. Probabilistic interpretation at LO:
Conditional probability of finding a quark $q(x,k_\perp)$ in the fast moving proton fragmenting to $h(\zeta,P_{h\perp})$ moving in same direction ⇒ STMD CPDFs

Leading twist decomposition of quark correlator

$$\mathcal{M}^{[\gamma^-]} = \hat{u}_1 + \frac{\mathbf{P}_T \times \mathbf{S}_T}{m_2}\hat{u}_{1T}^{h} + \frac{\mathbf{k}_T \times \mathbf{S}_T}{m_N}\hat{u}_{1T}^{\perp} + \frac{S_L(\mathbf{k}_T \times \mathbf{P}_T)}{m_N m_2}\hat{u}_{1L}^{\perp h}$$

$$\mathcal{M}^{[\gamma^-\gamma_5]} = S_L \hat{l}_{1L} + \frac{\mathbf{P}_T \cdot \mathbf{S}_T}{m_2}\hat{l}_{1T}^{h} + \frac{\mathbf{k}_T \cdot \mathbf{S}_T}{m_N}\hat{l}_{1T}^{\perp} + \frac{\mathbf{k}_T \times \mathbf{P}_T}{m_N m_2}\hat{l}_{1}^{\perp h}$$

$$\mathcal{M}^{[i\sigma^{i-}\gamma_5]} = S_T^i \hat{t}_{1T} + \frac{S_L P_T^i}{m_2}\hat{t}_{1L}^{h} + \frac{S_L k_T^i}{m_N}\hat{t}_{1L}^{\perp}$$
$$+ \frac{(\mathbf{P}_T \cdot \mathbf{S}_T) P_T^i}{m_2^2}\hat{t}_{1T}^{hh} + \frac{(\mathbf{k}_T \cdot \mathbf{S}_T) k_T^i}{m_N^2}\hat{t}_{1T}^{\perp\perp}$$
$$+ \frac{(\mathbf{k}_T \cdot \mathbf{S}_T) P_T^i - (\mathbf{P}_T \cdot \mathbf{S}_T) k_T^i}{m_N m_2}\hat{t}_{1T}^{\perp h} + \frac{\epsilon_\perp^{ij} P_{Tj}}{m_2}\hat{t}_1^{h} + \frac{\epsilon_\perp^{ij} k_{Tj}}{m_N}\hat{t}_1^{\perp}$$

STMD fracture functions depend on $x, k_T^2, \zeta, P_T^2, \mathbf{k}_T \cdot \mathbf{P}_T$

$\mathbf{k}_T \cdot \mathbf{P}_T = k_T P_T \cos(\phi_h - \phi_q)$ – azimuthal dependence in fracture functions

STMD Fracture Functions for spinless hadron production

		Quark polarization		
		U	**L**	**T**
Nucleon Polarization	**U**	$\hat{u}_1$	$\frac{\mathbf{k}_T \times \mathbf{P}_T}{m_N m_h} \hat{l}_1^{\perp h}$	$\frac{\epsilon_T^{ij} P_T^j}{m_h} \hat{t}_1^h + \frac{\epsilon_T^{ij} k_T^j}{m_N} \hat{t}_1^\perp$
	L	$\frac{S_L (\mathbf{k}_T \times \mathbf{P}_T)}{m_N m_h} \hat{u}_{1L}^{\perp h}$	$S_L \hat{l}_{1L}$	$\frac{S_L \mathbf{P}_T}{m_h} \hat{t}_{1L}^h + \frac{S_L \mathbf{k}_T}{m_N} \hat{t}_{1L}^\perp$
	T	$\frac{\mathbf{P}_T \times \mathbf{S}_T}{m_h} \hat{u}_{1T}^h + \frac{\mathbf{k}_T \times \mathbf{S}_T}{m_N} \hat{u}_{1T}^\perp$	$\frac{\mathbf{P}_T \cdot \mathbf{S}_T}{m_h} \hat{l}_{1T}^h + \frac{\mathbf{k}_T \cdot \mathbf{S}_T}{m_N} \hat{l}_{1T}^\perp$	$\mathbf{S}_T \hat{t}_{1T} + \frac{\mathbf{P}_T (\mathbf{P}_T \cdot \mathbf{S}_T)}{m_h^2} \hat{t}_{1T}^{hh} + \frac{\mathbf{k}_T (\mathbf{k}_T \cdot \mathbf{S}_T)}{m_N^2} \hat{t}_{1T}^{\perp\perp} + \frac{\mathbf{P}_T (\mathbf{k}_T \cdot \mathbf{S}_T) - \mathbf{k}_T \cdot (\mathbf{P}_T \cdot \mathbf{S}_T)}{m_N m_h} \hat{t}_{1T}^{\perp h}$

Nomenclature:

- subscript 1 $\leftrightarrow$ twist 2
- subscript L,T $\leftrightarrow$ long.(tran.) target polarization
- $\hat{u}_1 \leftrightarrow$ unpolarized quark
- $\hat{l}_1 \leftrightarrow$ long. pol. quark

$\hat{t}_1 \leftrightarrow$ tran. pol. quark

$\perp \leftrightarrow k_T$

$h \leftrightarrow P_T$

Sum Rules

$$\sum_h \int \zeta d\zeta \int d^2 P_T \, \hat{u}_1 = (1-x)\, f_1(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\hat{u}_{1T}^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}}{k_T^2} \hat{u}_{1T}^{h} \right) = -(1-x)\, f_{1T}^{\perp}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \, \hat{l}_{1L} = (1-x) g_{1L}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\hat{l}_{1T}^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}}{k_T^2} \hat{l}_{1T}^{h} \right) = (1-x) g_{1T}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\hat{t}_{1L}^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}}{k_T^2} \hat{t}_{1L}^{h} \right) = (1-x) h_{1L}^{\perp}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\hat{t}_{1}^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}}{k_T^2} \hat{t}_{1}^{h} \right) = -(1-x) h_{1}^{\perp}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\hat{t}_{1T}^{\perp\perp} + \frac{m_N^2}{m_h^2} \frac{2\left(\mathbf{k}_T \cdot \mathbf{P}\right)^2 - k_T^2 P_T^2}{k_T^4} \hat{t}_{1T}^{hh} \right) = (1-x) h_{1T}^{\perp}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\hat{t}_{1T} + \frac{k_T^2}{2m_N^2} \hat{t}_{1T}^{\perp\perp} + \frac{P_T^2}{2m_h^2} \hat{t}_{1T}^{hh} \right) = (1-x) h_1(x, k_T^2)$$

Nonzero fracture functions u,l,t. Useful for modeling.

LO cross-section in TFR

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}(x_F<0)}{dx\,dQ^2\,d\phi_S\,d\zeta\,d^2P_T} = \frac{\alpha^2 x}{yQ^4}\left(1+(1-y)^2\right)\sum_q e_q^2 \times$$

$$\times\left[\begin{array}{l} \tilde{u}_1\left(x,\zeta,P_T^2\right) - S_T\frac{P_T}{m_h}\tilde{u}_{1T}^h\left(x,\zeta,P_T^2\right)\sin(\phi_h-\phi_S)+ \\ \lambda y(2-y)\left(S_L\tilde{l}_{1L}\left(x,\zeta,P_T^2\right)+S_T\frac{P_T}{m_h}\tilde{l}_{1T}^h\left(x,\zeta,P_T^2\right)\cos(\phi_h-\phi_S)\right) \end{array}\right]$$

$$\tilde{u}_1(x_B,\zeta_2,P_{T2}^2) = \int d^2k_T\,\hat{u}_1$$

$$\tilde{u}_{1T}^h(x_B,\zeta_2,P_{T2}^2) = \int d^2k_T\left\{\hat{u}_{1T}^h + \frac{m_h}{m_N}\frac{\mathbf{k}_T\cdot\mathbf{P}_{T2}}{P_{T2}^2}\hat{u}_{1T}^\perp\right\}$$

$$\tilde{l}_{1L}(x_B,\zeta_2,P_{T2}^2) = \int d^2k_T\,\hat{l}_{1L}$$

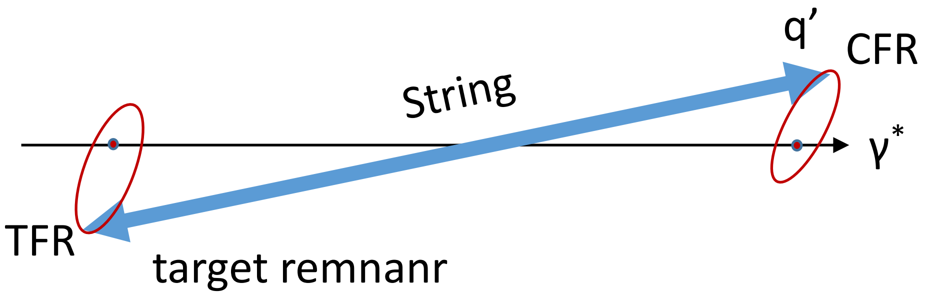
$$\tilde{l}_{1T}^h(x_B,\zeta_2,P_{T2}^2) = \int d^2k_T\left\{\hat{l}_{1T}^h + \frac{m_h}{m_N}\frac{\mathbf{k}_T\cdot\mathbf{P}_{T2}}{P_{T2}^2}\hat{l}_{1T}^\perp\right\}$$

At LO only 4 terms out of 18 Structure Functions,
Only 2 azimuthal modulations

No Collins-like $\sin(\phi_h-\phi_S)$ modulation

No access to quark transverse polarization

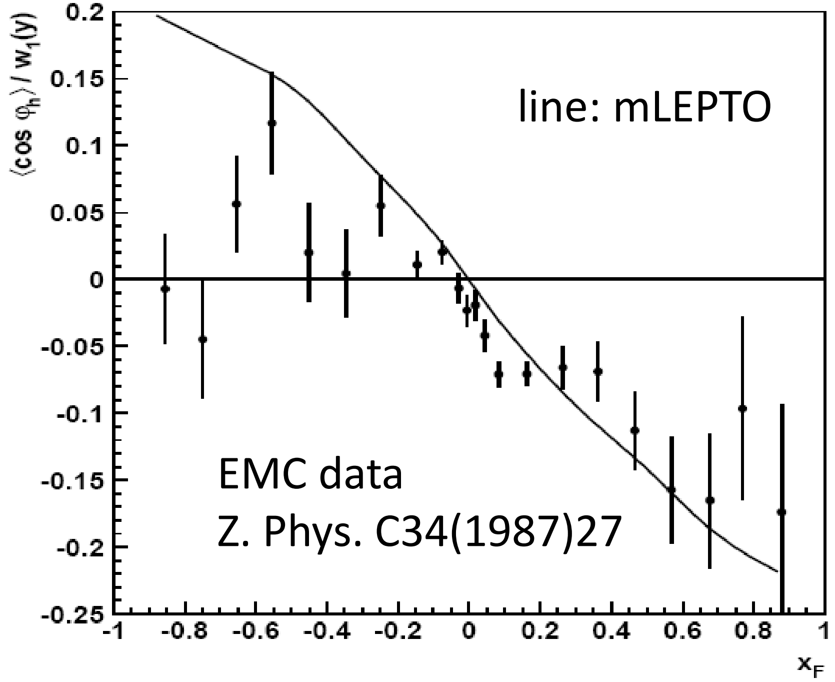
Including Cahn and Sivers effects in the event generators



Cahn: $1 - \frac{(2+y)\sqrt{1-y}}{1+(1-y)^2}\frac{k_\perp}{Q}\cos\varphi$

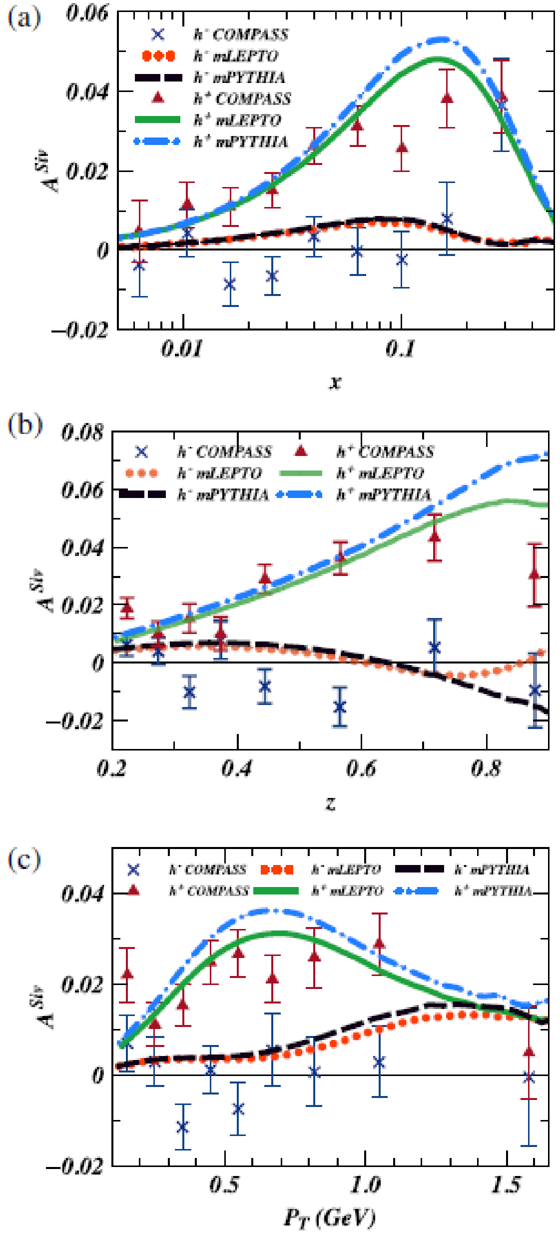
Sivers: $1 + |\boldsymbol{S}_T| \mathcal{N}_q(x) h(k_\perp)\, \sin\phi_{Siv}$

Generate the final quark azimuth
according to above distributions



Sivers effect in the event generators

Matevosyan, AK, Aschenauer, Avakian, Thomas, PRD 92, 054028 (2015)



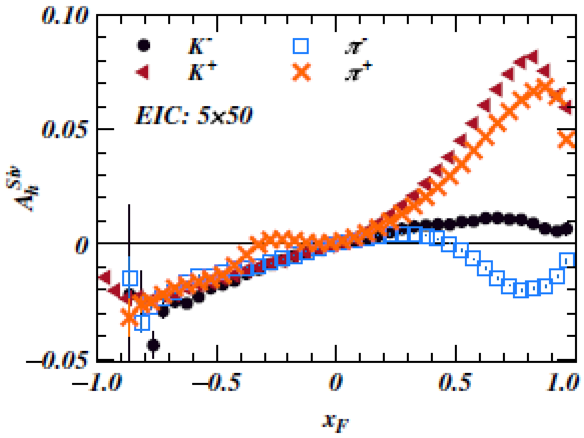


FIG. 13 (color online). EIC model SSAs for 5×50 SIDIS kinematics for charged pions and kaons versus x_F. The Sivers asymmetry is present both in the current and target fragmentation regions.

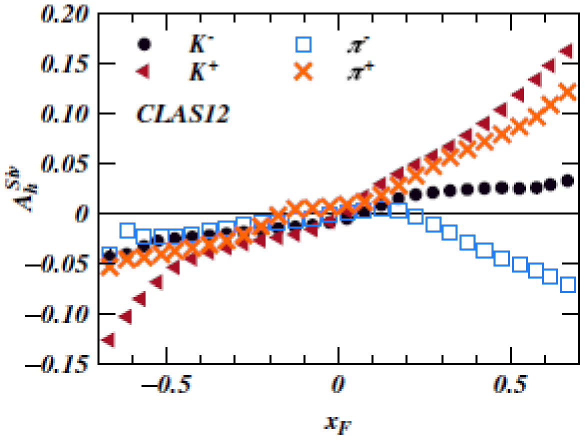
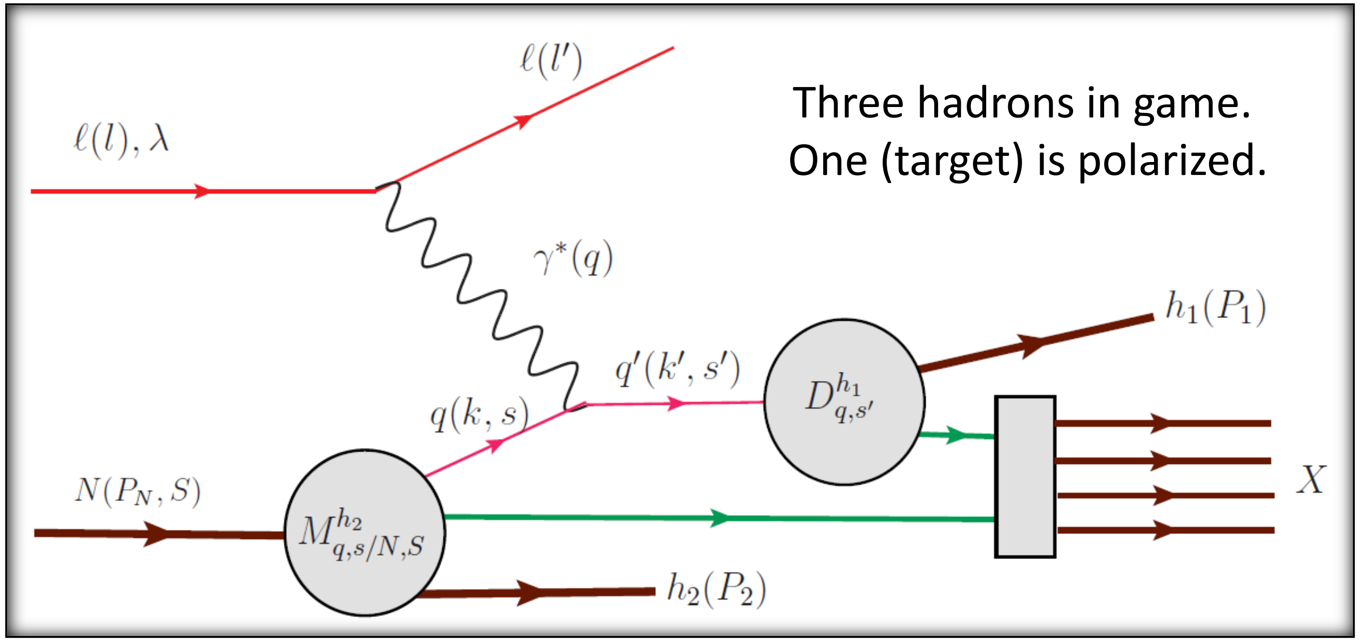


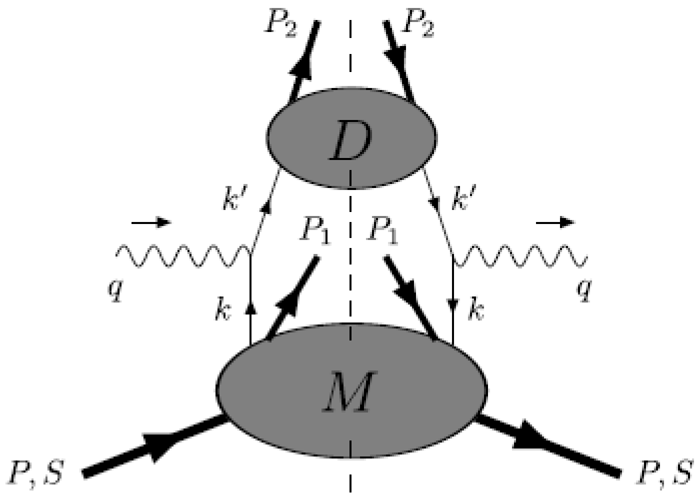
FIG. 17 (color online). Predictions for SSAs for charged pions and kaons versus x_F at CLAS12. The Sivers asymmetry is present both in the current and target fragmentation regions.

Only correlation of target $\mathbf{S_T}$ and struck quark $\mathbf{k_T}$ is explicitly parametrized using Sivers PDFs. Then this correlation is transferred to produced hadrons via unpolarized string fragmentation .

$$\tilde{u}_{1T}^{h}(x_B,\zeta_2,P_{T2}^2)=\int d^2k_T\left\{\hat{u}_{1T}^{h}+\frac{m_2}{m_N}\frac{\mathbf{k}_T\cdot\mathbf{P}_{T2}}{P_{T2}^2}\hat{u}_{1T}^{\perp}\right\}$$

Double hadron production in DIS (DSIDIS): TFR & CFR





$$x_{F2} < 0, \quad x_{F1} > 0$$

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_{T1}\,d\zeta\,d^2P_{T2}} = M^{h_2}_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2} \otimes D^{h_1}_{q,s'}$$

$$D^{h_1}_{q,s'}(z,\mathbf{p}_T) = D_1(z, p_T^2) + \frac{\mathbf{p}_T \times \mathbf{s}'_T}{m_h} H_1(z, p_T^2)$$

Unintegrated DSIDIS cross-section

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_{T1}\,d\zeta\,d^2P_{T2}} =$$

$$= \frac{\alpha^2 x}{Q^4 y}\left(1+(1-y)^2\right)\left(\hat{u}^{h_2} \otimes D_1^{h_1} + \lambda D_{ll}(y)\hat{l}^{h_2} \otimes D_1^{h_1} + \hat{t}^{h_2} \otimes \frac{\mathbf{p}_T \times \mathbf{s}'_T}{m_{h_1}} H_1^{h_1} \right)$$

$$= \frac{\alpha^2 x}{Q^4 y}\left(1+(1-y)^2\right)\left(\sigma_{UU} + S_L\sigma_{UL} + S_T\sigma_{UT} + \lambda D_{ll}\left(\sigma_{LU} + S_L\sigma_{LL} + S_T\sigma_{LT}\right) \right)$$

DSIDIS cross section is a sum of polarization independent, single and double spin dependent terms, similarly to 1h SIDIS cross section.

DSIDIS azimuthal modulations

AK @ DIS2011

$$\sigma_{UU} = F_0^{\hat{u}\cdot D_1} - D_{nn}\left(\frac{P_{T1}^2}{m_1 m_N} F_{kp1}^{\hat{t}_1^\perp \cdot H_1} \cos(2\phi_1) + \frac{P_{T1}P_{T2}}{m_1 m_2} F_{p1}^{\hat{t}_1^h \cdot H_1} \cos(\phi_1+\phi_2) + \left(\frac{P_{T2}^2}{m_1 m_N} F_{kp2}^{\hat{t}_1^\perp \cdot H_1} + \frac{P_{T2}^2}{m_1 m_2} F_{p2}^{\hat{t}_1^h \cdot H_1} \right) \cos(2\phi_2) \right)$$

$$D_{nn}(y) = \frac{2(1-y)}{1+(1-y)^2}$$

$$F_{k1}^{\hat{M}\cdot D} = C\left[\hat{M}\cdot D \frac{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})(\mathbf{P}_{T2}\cdot\mathbf{k}) - (\mathbf{P}_{T1}\cdot\mathbf{k})\mathbf{P}_{T2}^2}{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2 - \mathbf{P}_{T1}^2\mathbf{P}_{T2}^2} \right]$$

Structure functions $F_{...}^{\hat{u}\cdot D}$ depend on $x, z, \zeta, P_{T1}^2, P_{T2}^2$ and $(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})$

$$\mathbf{P}_{T1}\cdot\mathbf{P}_{T2} = P_{T1}P_{T2}\cos(\Delta\phi), \text{ with } \Delta\phi = \phi_1 - \phi_2$$

σ_{UL}

$$\sigma_{UL} = -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{u}_{1L}^{\perp h}\cdot D_1} \sin(\phi_1 - \phi_2)$$

$$+D_{nn}\left(\frac{P_{T1}^2}{m_1 m_N} F_{kp1}^{\hat{t}_{1L}^{\perp}\cdot H_1} \sin(2\phi_1) + \frac{P_{T1}P_{T2}}{m_1 m_2} F_{p1}^{\hat{t}_{1L}^{h}\cdot H_1} \sin(\phi_1 + \phi_2) + \left(\frac{P_{T2}^2}{m_1 m_N} F_{kp2}^{\hat{t}_{1L}^{\perp}\cdot H_1} + \frac{P_{T2}^2}{m_1 m_2} F_{p2}^{\hat{t}_{1L}^{h}\cdot H_1} \right) \sin(2\phi_2) \right)$$

σ_{UT}

$$
\begin{aligned}
\sigma_{UT} = & -\frac{P_{T1}}{m_N} F_{k1}^{\hat{u}_{1T}^{\perp}\cdot D_1} \sin(\phi_1-\phi_S) \\
& -\left(\frac{P_{T2}}{m_2} F_0^{\hat{u}_{1T}^{h}\cdot D_1} + \frac{P_{T2}}{m_N} F_{k2}^{\hat{u}_{1T}^{\perp}\cdot D_1} \right) \sin(\phi_2-\phi_S) \\
& + D_{nn}(y) \Bigg[\left(\frac{P_{T1}}{m_1} F_{p1}^{\hat{t}_{1T}\cdot H_1} + \frac{P_{T1}P_{T2}^2}{2m_1m_2^2} F_{p1}^{\hat{t}_{1T}^{hh}\cdot H_1} - \frac{P_{T1}P_{T2}^2}{2m_1m_2m_N} F_{kp3}^{\hat{t}_{1T}^{\perp h}\cdot H_1} \right. \\
& \qquad \left. + \frac{P_{T1}^3}{2m_1m_N^2} F_{kkp1}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T1}P_{T2}^2}{2m_1m_N^2} F_{kkp4}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T1}}{m_1m_N^2} F_{kkp5}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(\phi_1+\phi_S) \\
& \qquad + \left(\frac{P_{T2}}{m_1} F_{p2}^{\hat{t}_{1T}\cdot H_1} + \frac{P_{T2}^3}{2m_1m_2^2} F_{p2}^{\hat{t}_{1T}^{hh}\cdot H_1} + \frac{P_{T1}^2P_{T2}}{2m_1m_2m_N} F_{kp1}^{\hat{t}_{1T}^{\perp h}\cdot H_1} + \frac{P_{T2}}{m_1m_2m_N} F_{kp4}^{\hat{t}_{1T}^{\perp h}\cdot H_1} \right. \\
& \qquad \left. + \frac{P_{T1}^2P_{T2}}{2m_1m_N^2} F_{kkp2}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T2}^3}{2m_1m_N^2} F_{kkp3}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T2}}{m_1m_N^2} F_{kkp6}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(\phi_2+\phi_S) \\
& \qquad + \frac{P_{T1}^3}{2m_1m_N^2} F_{kkp1}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \sin(3\phi_1-\phi_S) \\
& \qquad + \left(\frac{P_{T2}^3}{2m_1m_2^2} F_{p2}^{\hat{t}_{1T}^{hh}\cdot H_1} + \frac{P_{T2}^3}{2m_1m_N^2} F_{kkp3}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(3\phi_2-\phi_S) \\
& \qquad + \left(\frac{P_{T1}P_{T2}^2}{2m_1m_2^2} F_{p1}^{\hat{t}_{1T}^{hh}\cdot H_1} + \frac{P_{T1}P_{T2}^2}{2m_1m_N^2} F_{kkp4}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(\phi_1+2\phi_2-\phi_S) \\
& \qquad - \frac{P_{T1}^2P_{T2}}{2m_1m_2m_N} F_{kp1}^{\hat{t}_{1T}^{\perp h}\cdot H_1} \sin(2\phi_1-\phi_2+\phi_S) \\
& \qquad - \frac{P_{T1}P_{T2}^2}{2m_1m_2m_N} F_{kp3}^{\hat{t}_{1T}^{\perp h}\cdot H_1} \sin(\phi_1-2\phi_2-\phi_S) \\
& \qquad + \frac{P_{T1}^2P_{T2}}{2m_1m_N^2} F_{kkp2}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \sin(2\phi_1+\phi_2-\phi_S) \Bigg]
\end{aligned}
$$

σ_{LU}, σ_{LL}, σ_{LT}

$$\sigma_{LU} = -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{l}_1^{\perp h}\cdot D_1} \sin(\phi_1 - \phi_2)$$

Twist two!

$$\sigma_{LL} = F_0^{\hat{l}_1\cdot D_1}$$

$$\sigma_{LT} = \frac{P_{T1}}{m_N} F_{k1}^{\hat{l}_{1T}^{\perp}\cdot D_1} \cos(\phi_1 - \phi_S) + \left(\frac{P_{T2}}{m_2} F_0^{\hat{l}_{1T}^{h}\cdot D_1} + \frac{P_{T2}}{m_N} F_{k2}^{\hat{l}_{1T}^{\perp}\cdot D_1} \right) \cos(\phi_2 - \phi_S)$$

A_{LU} asymmetry

Anselmino, Barone and AK, **713** (2012)317

$F_{\dots}^{\hat{u}\cdot D}$ depend on $x, z, \zeta, P_{T1}^2, P_{T2}^2$ and $(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})$

$\mathbf{P}_{T1}\cdot\mathbf{P}_{T2} = P_{T1}P_{T2}\cos(\Delta\phi)$, with $\Delta\phi = \phi_1 - \phi_2$

One can choose as independent angles $\Delta\phi$ and ϕ_2 $(\phi_1 = \Delta\phi + \phi_2)$

Integrating σ_{UU} and σ_{lU} over ϕ_2 we obtain

$$A_{LU} = \frac{\int d\phi_2 \sigma_{LU}}{\int d\phi_2 \sigma_{UU}} = $$

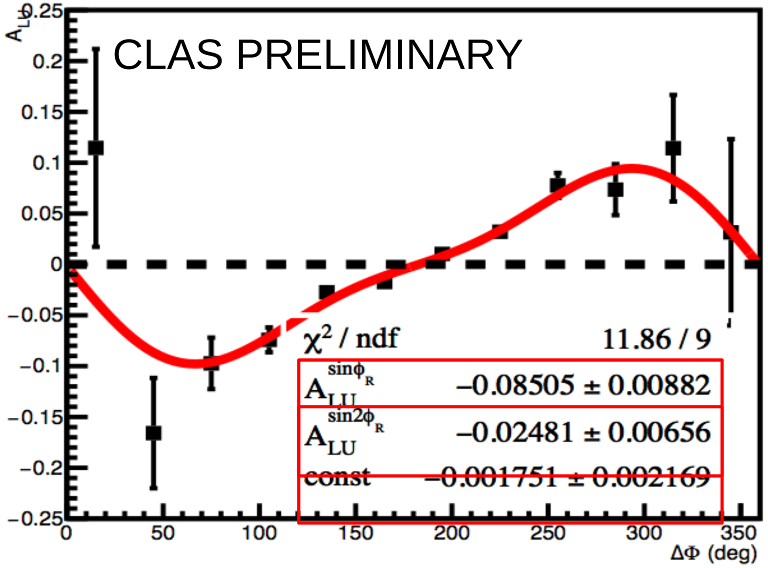
$$= \frac{-\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{l}_1^{\perp h}\cdot D_1}\left(x, z, \zeta, P_{T1}^2, P_{T2}^2, \cos(\Delta\phi)\right)\sin(\Delta\phi)}{F_0^{\hat{u}\cdot D_1}\left(x, z, \zeta, P_{T1}^2, P_{T2}^2, \cos(\Delta\phi)\right)}$$

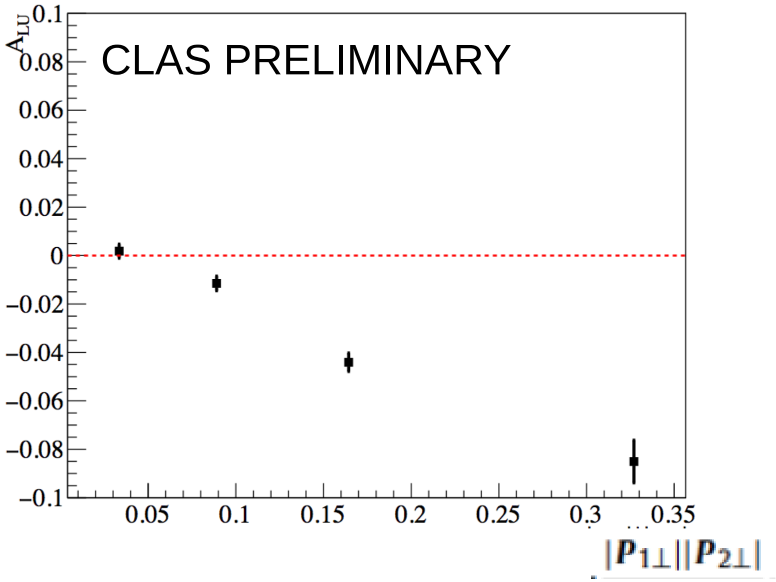
A_{LU} @ CLAS

$$A_{LU} = \frac{\sigma_{LU}(x,z,\zeta,P_{T1}^2,P_{T2}^2)\left(1+a_{LU1}\cos(\Delta\phi)+a_{LU2}\cos(2\Delta\phi)+\cdots\right)\sin(\Delta\phi)}{\sigma_{UU}(x,z,\zeta,P_{T1}^2,P_{T2}^2)\left(1+a_{UU1}\cos(\Delta\phi)+a_{UU2}\cos(2\Delta\phi)+\cdots\right)} \approx$$

$$\approx p_1\sin(\Delta\phi)+p_2\sin(2\Delta\phi)+\cdots$$

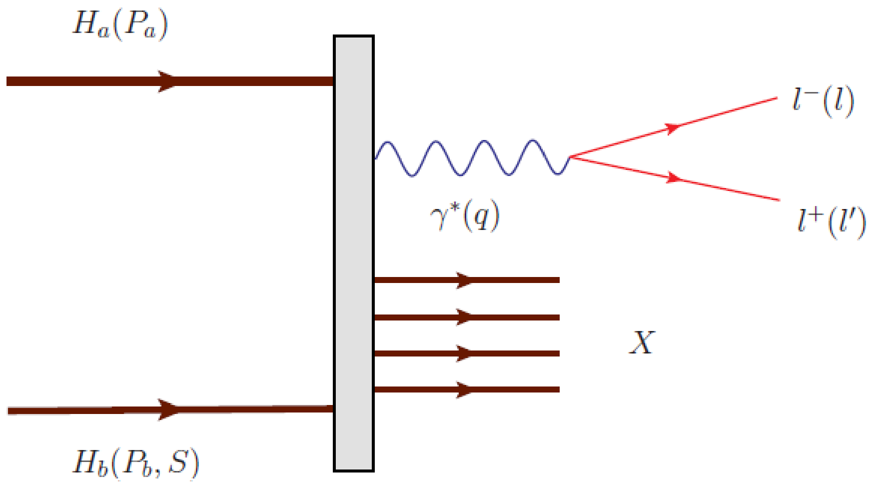
Courtesy of S.Pisano & H.Avakian





Presence of higher harmonics indicate that $\sigma_{LU}(\Delta\Phi) \neq \sigma_{UU}(\Delta\Phi)$

DY processes: QCD TMD factorization OK



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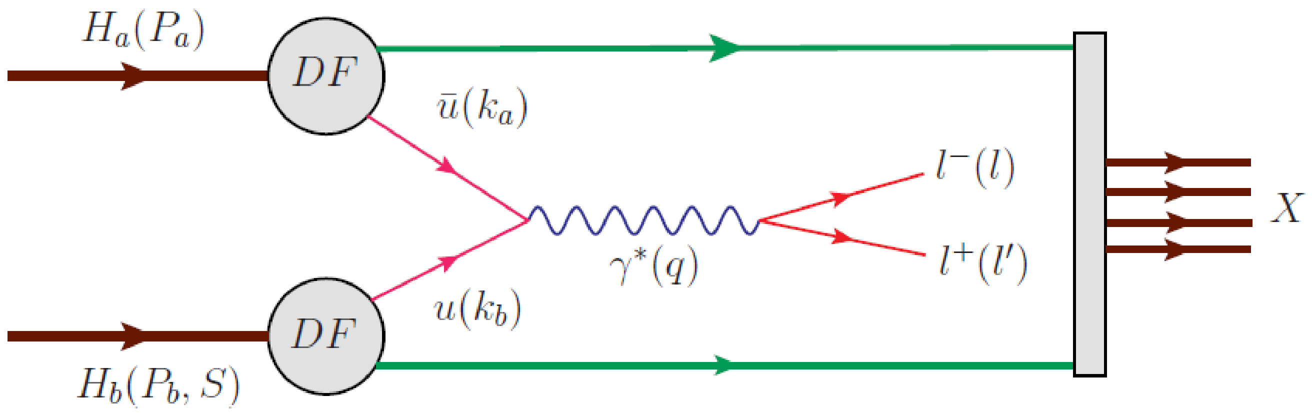
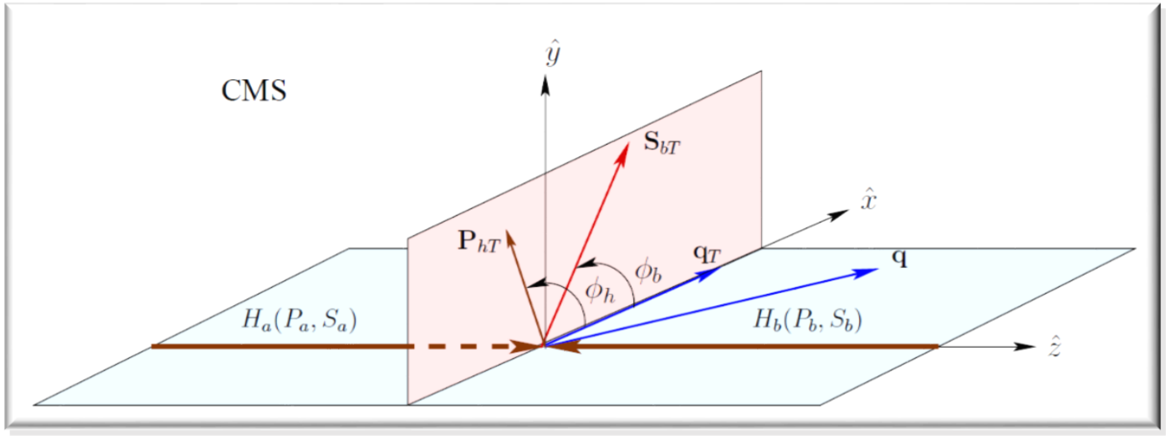
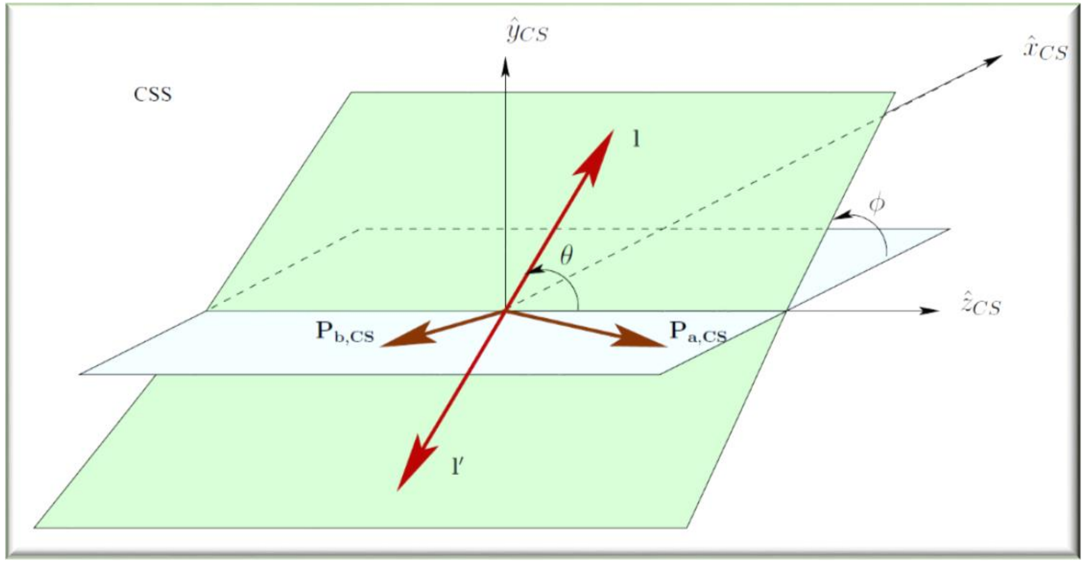






DY cross section: general expression

Arnold.Metz.Schlegel, PhysRevD.79.034005

$$
\begin{aligned}
\frac{d\sigma}{d^4q\,d\Omega} = \frac{\alpha_{\rm em}^2}{Fq^2}\{ & ((1+\cos^2\theta)F_{UU}^1 + (1-\cos^2\theta)F_{UU}^2 + \sin2\theta\cos\phi F_{UU}^{\cos\phi} + \sin^2\theta\cos2\phi F_{UU}^{\cos2\phi}) \\
& + S_{aL}(\sin2\theta\sin\phi F_{LU}^{\sin\phi} + \sin^2\theta\sin2\phi F_{LU}^{\sin2\phi}) + S_{bL}(\sin2\theta\sin\phi F_{UL}^{\sin\phi} + \sin^2\theta\sin2\phi F_{UL}^{\sin2\phi}) \\
& + |\vec{S}_{aT}|[\sin\phi_a((1+\cos^2\theta)F_{TU}^1 + (1-\cos^2\theta)F_{TU}^2 + \sin2\theta\cos\phi F_{TU}^{\cos\phi} + \sin^2\theta\cos2\phi F_{TU}^{\cos2\phi}) \\
& + \cos\phi_a(\sin2\theta\sin\phi F_{TU}^{\sin\phi} + \sin^2\theta\sin2\phi F_{TU}^{\sin2\phi})] + |\vec{S}_{bT}|[\sin\phi_b((1+\cos^2\theta)F_{UT}^1 + (1-\cos^2\theta)F_{UT}^2 \\
& + \sin2\theta\cos\phi F_{UT}^{\cos\phi} + \sin^2\theta\cos2\phi F_{UT}^{\cos2\phi}) + \cos\phi_b(\sin2\theta\sin\phi F_{UT}^{\sin\phi} + \sin^2\theta\sin2\phi F_{UT}^{\sin2\phi})] \\
& + S_{aL}S_{bL}((1+\cos^2\theta)F_{LL}^1 + (1-\cos^2\theta)F_{LL}^2 + \sin2\theta\cos\phi F_{LL}^{\cos\phi} + \sin^2\theta\cos2\phi F_{LL}^{\cos2\phi}) \\
& + S_{aL}|\vec{S}_{bT}|[\cos\phi_b((1+\cos^2\theta)F_{LT}^1 + (1-\cos^2\theta)F_{LT}^2 + \sin2\theta\cos\phi F_{LT}^{\cos\phi} + \sin^2\theta\cos2\phi F_{LT}^{\cos2\phi}) \\
& + \sin\phi_b(\sin2\theta\sin\phi F_{LT}^{\sin\phi} + \sin^2\theta\sin2\phi F_{LT}^{\sin2\phi})] + |\vec{S}_{aT}|S_{bL}[\cos\phi_a((1+\cos^2\theta)F_{TL}^1 + (1-\cos^2\theta)F_{TL}^2 \\
& + \sin2\theta\cos\phi F_{TL}^{\cos\phi} + \sin^2\theta\cos2\phi F_{TL}^{\cos2\phi}) + \sin\phi_a(\sin2\theta\sin\phi F_{TL}^{\sin\phi} + \sin^2\theta\sin2\phi F_{TL}^{\sin2\phi})] \\
& + |\vec{S}_{aT}||\vec{S}_{bT}|[\cos(\phi_a+\phi_b)((1+\cos^2\theta)F_{TT}^1 + (1-\cos^2\theta)F_{TT}^2 + \sin2\theta\cos\phi F_{TT}^{\cos\phi} + \sin^2\theta\cos2\phi F_{TT}^{\cos2\phi}) \\
& + \cos(\phi_a-\phi_b)((1+\cos^2\theta)\bar{F}_{TT}^1 + (1-\cos^2\theta)\bar{F}_{TT}^2 + \sin2\theta\cos\phi \bar{F}_{TT}^{\cos\phi} + \sin^2\theta\cos2\phi \bar{F}_{TT}^{\cos2\phi}) \\
& + \sin(\phi_a+\phi_b)(\sin2\theta\sin\phi F_{TT}^{\sin\phi} + \sin^2\theta\sin2\phi F_{TT}^{\sin2\phi}) \\
& + \sin(\phi_a-\phi_b)(\sin2\theta\sin\phi \bar{F}_{TT}^{\sin\phi} + \sin^2\theta\sin2\phi \bar{F}_{TT}^{\sin2\phi})]\}.
\end{aligned}
$$

$\theta,\ \phi:$ CS angles

$\phi_a,\ \phi_b:$ azimuths of $\mathbf{S}_{aT},\ \mathbf{S}_{bT}$

48 structure functions

DY cross section: parton model approximation

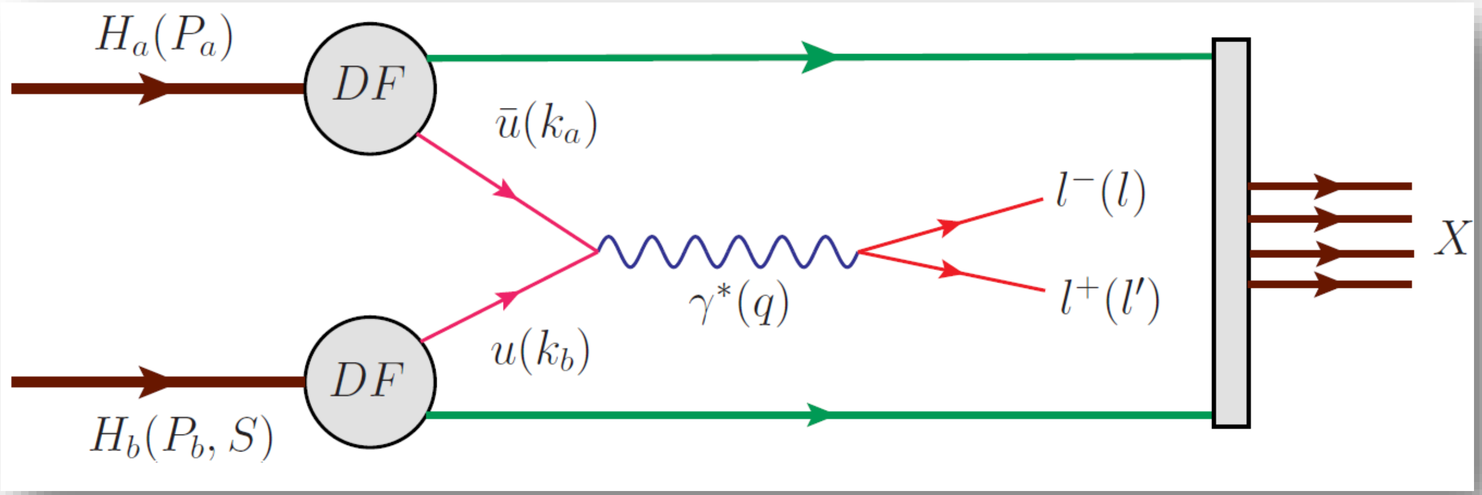
$$\frac{d\sigma}{d^4q\,d\Omega} = \frac{\alpha_{\rm em}^2 x_a x_b}{2q^4}\frac{1}{N_c}\sum_q e_q^2 \int d^2\vec{k}_{aT} d^2\vec{k}_{bT}$$
$$\times\, \delta^{(2)}(\vec{q}_T - \vec{k}_{aT} - \vec{k}_{bT})[(1+\cos^2\theta)(\Phi^{q[\gamma^+]}\bar{\Phi}^{q[\gamma^-]}$$
$$+\, \Phi^{q[\gamma^+\gamma_5]}\bar{\Phi}^{q[\gamma^-\gamma_5]})$$
$$+\, \sin^2\theta(\cos 2\phi(\delta^{i1}\delta^{j1} - \delta^{i2}\delta^{j2})$$
$$+\, \sin 2\phi(\delta^{i1}\delta^{j2} + \delta^{i2}\delta^{j1}))\Phi^{q[i\sigma^{i+}\gamma_5]}\bar{\Phi}^{q[i\sigma^{j-}\gamma_5]}]$$
$$+\, \{\Phi \leftrightarrow \bar{\Phi}\} + \mathcal{O}(1/q). \qquad (88)$$

$$\Phi^{q[\gamma^+]} = f_1^q(x_a, \vec{k}_{aT}^2) - \frac{\varepsilon_T^{ij} k_{aT}^i S_{aT}^j}{M_a} f_{1T}^{\perp q}(x_a, \vec{k}_{aT}^2), \qquad (81)$$

$$\Phi^{q[\gamma^+\gamma_5]} = S_{aL}\, g_{1L}^q(x_a, \vec{k}_{aT}^2) + \frac{\vec{k}_{aT}\cdot\vec{S}_{aT}}{M_a} g_{1T}^q(x_a, \vec{k}_{aT}^2), \qquad (82)$$

$$\Phi^{q[i\sigma^{i+}\gamma_5]} = S_{aT}^i h_1^q(x_a, \vec{k}_{aT}^2)$$
$$+\, \frac{k_{aT}^i(\vec{k}_{aT}\cdot\vec{S}_{aT}) - \frac{1}{2}\vec{k}_{aT}^2 S_{aT}^i}{M_a^2} h_{1T}^{\perp q}(x_a, \vec{k}_{aT}^2)$$
$$+\, S_{aL}\frac{k_{aT}^i}{M_a} h_{1L}^{\perp q}(x_a, \vec{k}_{aT}^2) + \frac{\varepsilon_T^{ij} k_{aT}^j}{M_a} h_1^{\perp q}(x_a, \vec{k}_{aT}^2).$$

DY-LO, transversely polarized target



$$\frac{d\sigma}{d^4q\,d\Omega} \overset{LO}{=} \frac{\alpha_{em}^2}{F q^2}\hat{\sigma}_U \Big\{1 + D_{[\sin^2\theta]} A_U^{\cos 2\phi} \cos 2\phi$$

$$+ S_T\Big[A_T^{\sin\phi_S}\sin\phi_S + D_{[\sin^2\theta]}\Big(A_T^{\sin(2\phi+\phi_S)}\sin(2\phi+\phi_S) + A_T^{\sin(2\phi-\phi_S)}\sin(2\phi-\phi_S)\Big)\Big]\Big\}$$

$$\hat{\sigma}_U \overset{LO}{=} F_U^1(1+\cos^2\theta),\ \ D_{[f(\theta)]} \overset{LO}{=} \frac{f(\theta)}{1+\cos^2\theta}$$

Only 5 structure functions $F_A^{f(\phi,\phi_S)}$

DY-LO asymmetries, transversely polarized target

$$A_U^{\cos 2\phi} \stackrel{LO}{=} \mathcal{C}\left[\left(2(\mathbf{h}\cdot\mathbf{k}_{aT})(\mathbf{h}\cdot\mathbf{k}_{bT}) - \mathbf{k}_{aT}\cdot\mathbf{k}_{bT}\right) h_1^{\perp} \bar{h}_1^{\perp}\right] \Big/ M_a M_b F_U^1$$

$$A_T^{\sin\phi_S} \stackrel{LO}{=} \tilde{A}_T^{\sin\phi_S} \stackrel{LO}{=} \mathcal{C}\left[\mathbf{h}\cdot\mathbf{k}_{bT}\; f_1 \bar{f}_{1T}^{\perp}\right] \Big/ M_b F_U^1$$

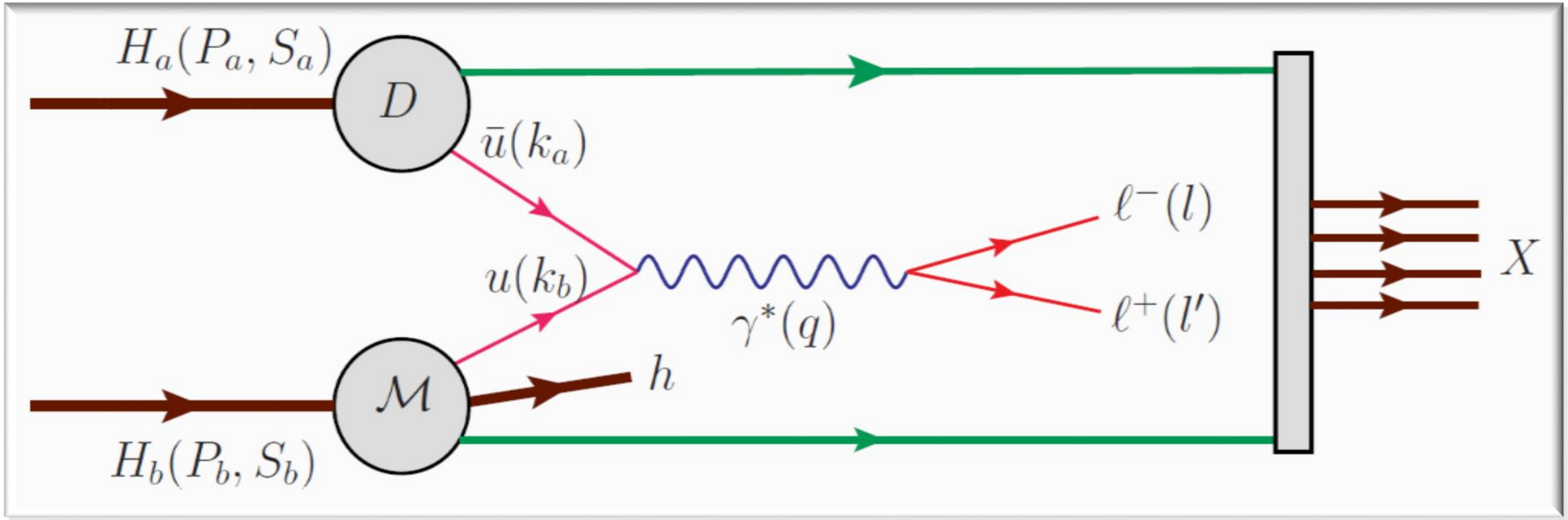
$$A_T^{\sin(2\phi+\phi_S)} \stackrel{LO}{=} -\mathcal{C}\left[\left(2(\mathbf{h}\cdot\mathbf{k}_{bT})[2(\mathbf{h}\cdot\mathbf{k}_{aT})(\mathbf{h}\cdot\mathbf{k}_{bT}) - \mathbf{k}_{aT}\cdot\mathbf{k}_{bT}] - \mathbf{k}_{bT}^2(\mathbf{h}\cdot\mathbf{k}_{aT})\right) h_1^{\perp} \bar{h}_{1T}^{\perp}\right] \Big/ 4 M_a M_b^2 F_U^1$$

$$A_T^{\sin(2\phi-\phi_S)} \stackrel{LO}{=} -\mathcal{C}\left[\mathbf{h}\cdot\mathbf{k}_{aT} h_1^{\perp} \bar{h}_1\right] \Big/ 2 M_a F_U^1$$

$$F_U^1 \stackrel{LO}{=} \mathcal{C}\left[f_a \bar{f}_a\right] \qquad \mathbf{h} \doteq \frac{\mathbf{q}_T}{q_T}$$

$$\mathcal{C}\left[w(\mathbf{k}_{aT},\mathbf{k}_{bT}) f_1 \bar{f}_2\right] \doteq \frac{1}{N_c}\sum_q e_q^2 \int d^2\mathbf{k}_{aT} d^2\mathbf{k}_{bT}\, \delta^{(2)}(\mathbf{q}_T - \mathbf{k}_{aT} - \mathbf{k}_{bT}) w(\mathbf{k}_{aT},\mathbf{k}_{bT})$$
$$\times \left[f_1^q(x_a,\mathbf{k}_{aT}^2) f_2^{\bar{q}}(x_b,\mathbf{k}_{bT}^2) + f_1^{\bar{q}}(x_a,\mathbf{k}_{aT}^2) f_2^q(x_b,\mathbf{k}_{bT}^2)\right]$$

SIDY processes



Generalisation of Arnold.Metz.Schlegel formalism to the case when
one hadron with low transverse momentum is detected in coincidence with dimuon.
We will consider only LO contributions

Disclaimer: factorization analyzer

For low p_T hadron production arguments against factorization have been given by
Collins, Frankfurt, Strikman, PLB 307 (1993) 161, Berera, Soper, PRD 50 (1994) 4328, Collins, PRD 57 (1998) 3051.

Ceccopieri, Trentadue, PLB 668 (2008) 319,
Semi-inclusive Drell-Yan Process as a Factorization Analyzer
'It is therefore highly desirable to have a standard perturbative framework which can be used as a "*factorization analyzer*"...'

Example: Ceccopieri, https://arxiv.org/abs/1606.06134
Predictions for **Single-diffractive Drell-Yan pair production at the LHC**

$$p(P_1) + p(P_2) \to p(P) + \gamma^*(\to l^+(p_3) + l^-(p_4)) + X$$

obtained using the fits **to HERA leading proton data**

$$l(k) + p(P) \to l(k') + p(P') + X(p_X)$$

WA11 Collaboration, Pietrzik *et al, PLB113(1982)105,* I

Thanks to Jen-Chieh Peng for pointing to this article

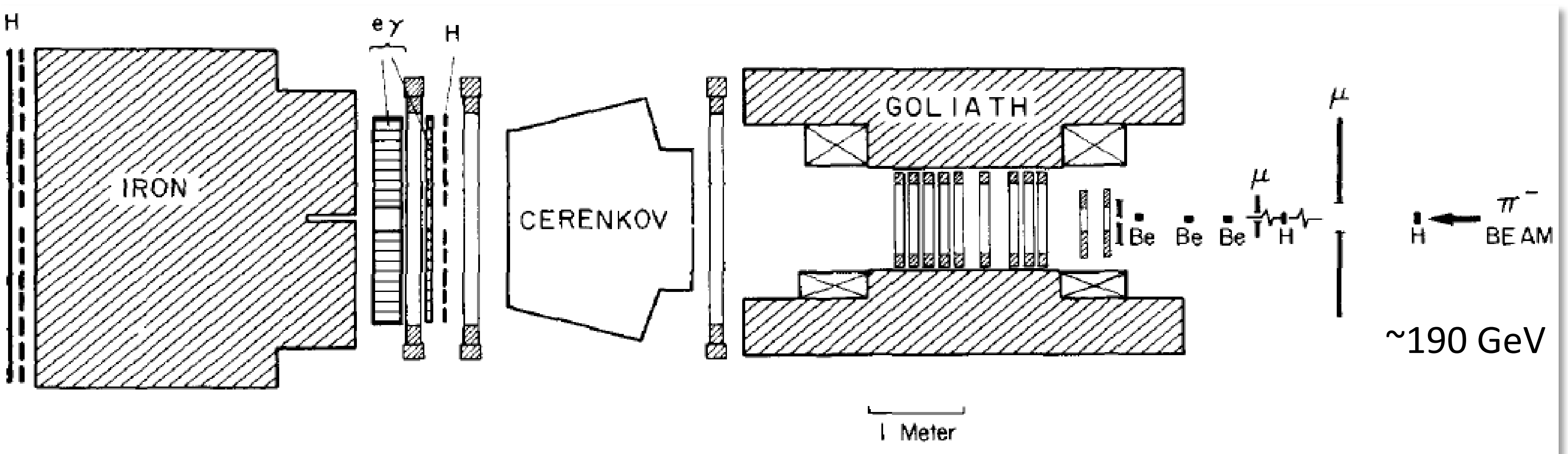
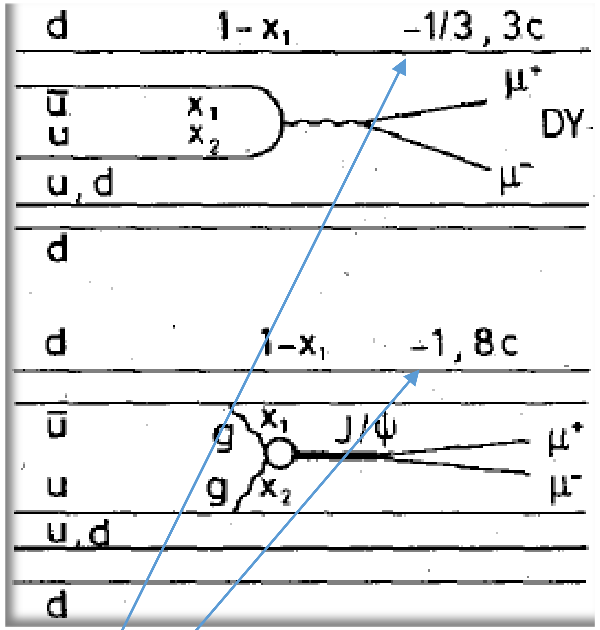


Fig. 1. Experimental layout. H represents a scintillator hodoscope; μ is a muon beam halo detector.



Charge asymmetry: $\frac{1}{N_{ev}}\left(\frac{dN^{+}}{dy}-\frac{dN^{-}}{dy}\right)$

Naïve expectations:
-1/3 for y>0 in DY production
-1 for y>0 in J/ψ production by gg fusion

WA11 Collaboration, Pietrzik *et al*, *PLB113(1982)105*, II

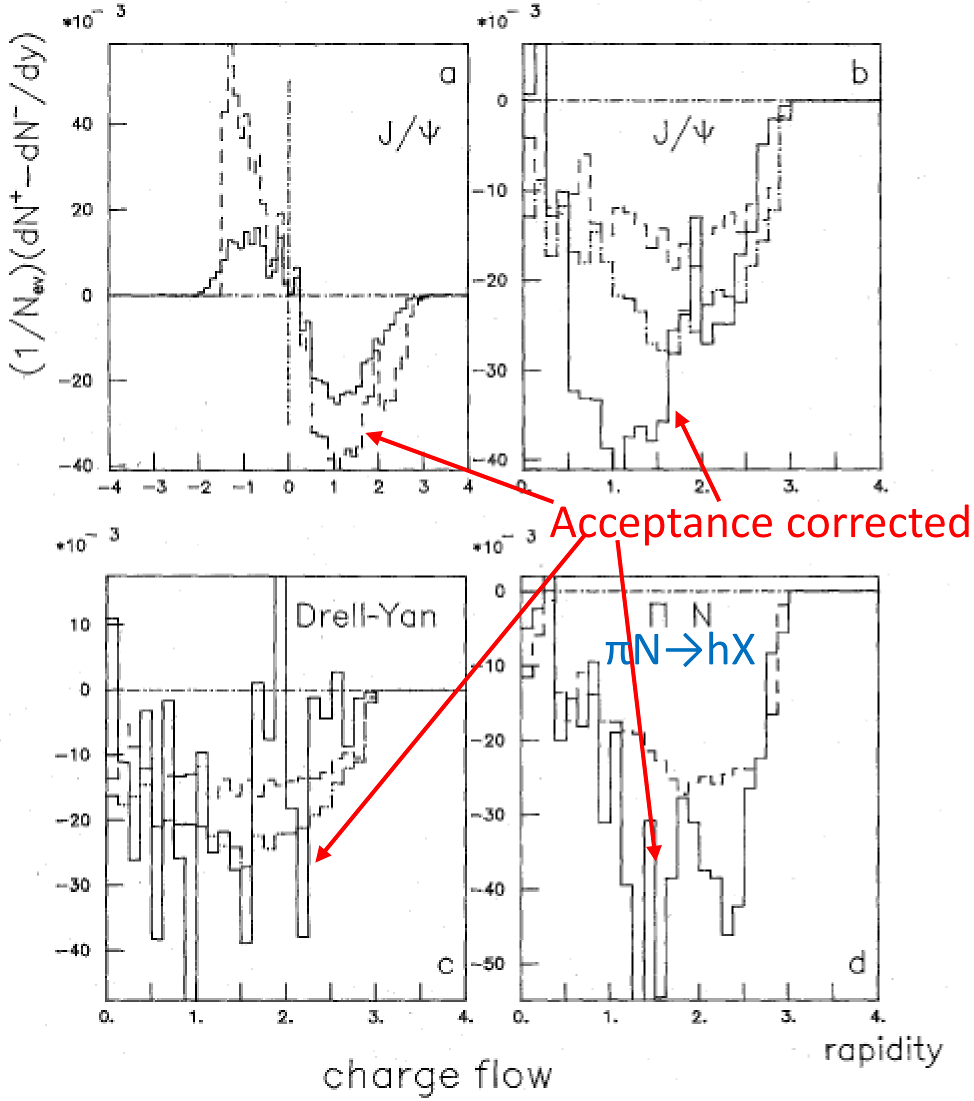


Fig. 3. Distribution of charge versus the rapidity of every particle. (a) Distribution corrected for acceptance (dashed line) and not corrected for acceptance (solid line). (b), (c) and (d) Data corrected for acceptance (solid line), d quark MC (dashed line), and $\bar{u}$ quark MC (dashed–dotted line).

Integrated charge of produced at y>0 hadrons

DY production: -0.36 ± 0.05

Close to expected $-1/3$

J/ψ production : -0.60 ± 0.08

J/ψ produced mainly by gg-fusion

$\pi^- N \to hX$: -0.76 ± 0.1

Charge "leak" at y=0

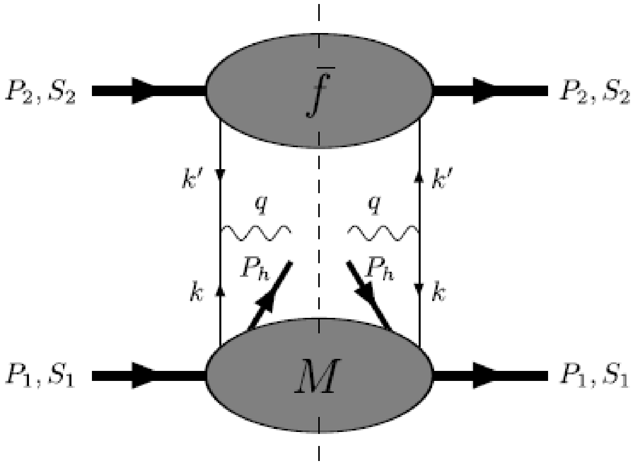
Polarized SIDY cross section in parton model approximation

$$\frac{d\sigma}{d^4q\,d\Omega\,d\zeta\,d^2P_T} = \frac{\alpha_{em}^2 x_a x_b}{2q^4}\frac{1}{N_c}\sum_q e_q^2 \int d^2\vec{k}_{aT}\,d^2\vec{k}_{bT}\,\delta^{(2)}\left(\vec{q}_T - \vec{k}_{aT} - \vec{k}_{bT}\right)\times$$

$$\times\left(\begin{array}{l}\left(1+\cos^2\theta\right)\left(\Phi^{q[\gamma^+]}\overline{\mathcal{M}}^{q[\gamma^-]} + \Phi^{q[\gamma^+\gamma_5]}\overline{\mathcal{M}}^{q[\gamma^-\gamma_5]}\right) \\ +\sin^2\theta\left(\begin{array}{l}\cos 2\phi\left(\delta^{i1}\delta^{j1} - \delta^{i2}\delta^{j2}\right) \\ +\sin 2\phi\left(\delta^{i1}\delta^{j2} + \delta^{i2}\delta^{j1}\right)\end{array}\right)\Phi^{q[i\sigma^{i+}\gamma_5]}\overline{\mathcal{M}}^{q[i\sigma^{j-}\gamma_5]} \\ +\{\Phi \leftrightarrow \overline{\Phi}, \overline{\mathcal{M}} \leftrightarrow \mathcal{M}\} + \mathcal{O}(1/q)\end{array}\right)$$

$$= \frac{\alpha_{em}^2 x_a x_b}{2q^4}\left(\begin{array}{l}\sigma_{UU} + S_{bL}\sigma_{UL} + S_{bT}\sigma_{UT} \\ +S_{aL}\sigma_{LU} + S_{aL}S_{bL}\sigma_{LL} + S_{aL}S_{bT}\sigma_{LT} \\ +S_{aT}\sigma_{TU} + S_{aT}S_{bL}\sigma_{TL} + S_{aT}S_{bT}\sigma_{TT}\end{array}\right)$$

One of the simple quark correlator is replaced with fracture function correlator



σ$_{UU}$

$$\sigma_{UU} = \left(1+\cos^2\theta\right) F_{UU} - \sin^2\theta \left[F_{UU}^{\cos(2\phi)} \cos(2\phi) + F_{UU}^{\cos(2\phi-\phi_h)} \cos(2\phi-\phi_h) + F_{UU}^{\cos(2\phi-2\phi_h)} \cos(2\phi-2\phi_h) \right]$$

$$F_{UU} = F_0^{f_1 \cdot \hat{u}_1}$$

$$F_{UU}^{\cos(2\phi)} = \frac{q_T^2}{M_a M_b} F_{ab2}^{h_1^\perp \cdot \hat{t}_1^\perp}$$

$$F_{UU}^{\cos(2\phi-\phi_h)} = \frac{P_T q_T}{m M_a} F_{a2}^{h_1^\perp \cdot \hat{t}_1^h}$$

$$F_{UU}^{\cos(2\phi-2\phi_h)} = \frac{P_T^2}{m M_a} F_{a1}^{h_1^\perp \cdot \hat{t}_1^h} + \frac{P_T^2}{M_a M_b} F_{ab1}^{h_1^\perp \cdot \hat{t}_1^\perp}$$

Takahiro Sawada talk:
open dpectrometer
@J-PARC?

Fracture functions		Quark polarization		
		U	L	T
Nucleon Polarization	U	*		*
	L			
	T			

σ_{UL}

$$\sigma_{UL} = \left(1+\cos^2\theta\right) F_{UL}^{\sin(\phi_h)} \sin\left(\phi_h\right) - \sin^2\theta \left[F_{UL}^{\sin(2\phi)} \sin\left(2\phi\right) + F_{UL}^{\sin(2\phi-\phi_h)} \sin\left(2\phi-\phi_h\right) + F_{UL}^{\sin(2\phi-2\phi_h)} \sin\left(2\phi-2\phi_h\right) \right]$$

Fracture functions		Quark polarization		
		U	L	T
Nucleon Polarization	U			
	L	*		*
	T			

σ_{UT}

$$\sigma_{UT} = \left(1+\cos^2\theta\right)\left[\begin{array}{l} F_{UT}^{\sin(\phi_b-\phi_h)}\sin(\phi_b-\phi_h) \\ +F_{UT}^{\sin(\phi_b)}\sin(\phi_b) \end{array}\right]$$

$$-\sin^2\theta\left[\begin{array}{l} F_{UT}^{\sin(2\phi+\phi_b)}\sin(2\phi+\phi_b) \\ +F_{UT}^{\sin(2\phi-\phi_b+\phi_h)}\sin(2\phi-\phi_b+\phi_h) \\ +F_{UT}^{\sin(2\phi+\phi_b-\phi_h)}\sin(2\phi+\phi_b-\phi_h) \\ +F_{UT}^{\sin(2\phi+\phi_b-3\phi_h)}\sin(2\phi+\phi_b-3\phi_h) \\ +F_{UT}^{\sin(2\phi-\phi_b-2\phi_h)}\sin(2\phi-\phi_b-2\phi_h) \\ +F_{UT}^{\sin(2\phi+\phi_b-2\phi_h)}\sin(2\phi+\phi_b-2\phi_h) \\ +F_{UT}^{\sin(2\phi-\phi_b-\phi_h)}\sin(2\phi-\phi_b-\phi_h) \\ +F_{UT}^{\sin(2\phi-\phi_b)}\sin(2\phi-\phi_b) \end{array}\right]$$

Fracture functions		Quark polarization		
		U	L	T
Nucleon Polarization	U			
	L			
	T	*		*

σ_{LU}

$$\sigma_{LU} = \left(1+\cos^2\theta\right) F_{LU}^{\sin(\phi_h)} \sin(\phi_h) - \sin^2\theta \left[\begin{array}{l} F_{LU}^{\sin(2\phi-2\phi_h)} \sin(2\phi-2\phi_h) \\ +F_{LU}^{\sin(2\phi-\phi_h)} \sin(2\phi-\phi_h) \\ +F_{LU}^{\sin(2\phi)} \sin(2\phi) \end{array} \right]$$

Fracture functions		Quark polarization		
		U	L	T
Nucleon Polarization	U		*	*
	L			
	T			

σ_{LL}

$$
\sigma_{LL} = \left(1+\cos^2\theta\right) F_{LL}
$$

$$
+\sin^2\theta \left[F_{LL}^{\cos(2\phi-2\phi_h)} \cos\left(2\phi - 2\phi_h\right) + F_{LL}^{\cos(2\phi-\phi_h)} \cos\left(2\phi - \phi_h\right) + F_{LL}^{\cos(2\phi)} \cos(2\phi) \right]
$$

Fracture functions		Quark polarization		
		U	L	T
Nucleon Polarization	U			
	L		*	*
	T			

σ_{LT}

$$\sigma_{LT} = \left(1+\cos^2\theta\right)\left[\begin{array}{l} F_{LT}^{\cos(\phi_b-\phi_h)}\cos\left(\phi_b-\phi_h\right) \\ +F_{LT}^{\cos(\phi_b)}\cos\left(\phi_b\right) \end{array}\right]$$

$$+\sin^2\theta\left[\begin{array}{l} F_{LT}^{\cos(2\phi-\phi_b+\phi_h)}\cos\left(2\phi-\phi_b+\phi_h\right) \\ +F_{LT}^{\cos(2\phi+\phi_b)}\cos\left(2\phi+\phi_b\right)+ \\ F_{LT}^{\cos(2\phi+\phi_b-\phi_h)}\cos\left(2\phi+\phi_b-\phi_h\right) \\ +F_{LT}^{\cos(2\phi+\phi_b-3\phi_h)}\cos\left(2\phi+\phi_b-3\phi_h\right) \\ +F_{LT}^{\cos(2\phi-\phi_b-2\phi_h)}\cos\left(2\phi-\phi_b-2\phi_h\right) \\ +F_{LT}^{\cos(2\phi+\phi_b-2\phi_h)}\cos\left(2\phi+\phi_b-2\phi_h\right) \\ +F_{LT}^{\cos(2\phi-\phi_b-\phi_h)}\cos\left(2\phi-\phi_b-\phi_h\right) \\ +F_{LT}^{\cos(2\phi-\phi_b)}\cos\left(2\phi-\phi_b\right) \end{array}\right]$$

Fracture functions		Quark polarization		
		U	L	T
Nucleon Polarization	U			
	L			
	T		*	*

σ_{TU}

$$\sigma_{TU} = \left(1+\cos^2\theta\right)\begin{bmatrix} F_{TU}^{\sin(\phi_a-\phi_h)}\sin(\phi_a-\phi_h) \\ +F_{TU}^{\sin(\phi_a)}\sin(\phi_a) \\ +F_{TU}^{\sin(\phi_a+\phi_h)}\sin(\phi_a+\phi_h) \\ +F_{TU}^{\sin(\phi_a-2\phi_h)}\sin(\phi_a-2\phi_h) \end{bmatrix}$$

$$+\sin^2\theta\begin{bmatrix} F_{TU}^{\sin(2\phi-\phi_a-\phi_h)}\sin(2\phi-\phi_a-\phi_h) \\ +F_{TU}^{\sin(2\phi-\phi_a)}\sin(2\phi-\phi_a) \\ +F_{TU}^{\sin(2\phi+\phi_a-3\phi_h)}\sin(2\phi+\phi_a-3\phi_h)+ \\ F_{TU}^{\sin(2\phi+\phi_a-\phi_h)}\sin(2\phi+\phi_a-\phi_h) \\ +F_{TU}^{\sin(2\phi+\phi_a-2\phi_h)}\sin(2\phi+\phi_a-2\phi_h) \\ +F_{TU}^{\sin(2\phi+\phi_a)}\sin(2\phi+\phi_a) \end{bmatrix}$$

Fracture functions		Quark polarization		
		U	L	T
Nucleon Polarization	U	*	*	*
	L			
	T			

σ_{TL}

$$\sigma_{TL} = \left(1+\cos^2\theta\right)\left[\begin{array}{l} F_{TL}^{\cos(\phi_a-\phi_h)}\cos(\phi_a-\phi_h) \\ +F_{TL}^{\cos(\phi_a)}\cos(\phi_a) \\ +F_{TL}^{\cos(\phi_a+\phi_h)}\cos(\phi_a+\phi_h) \\ +F_{TL}^{\cos(\phi_a-2\phi_h)}\cos(\phi_a-2\phi_h) \end{array}\right]$$

$$+\sin^2\theta\left[\begin{array}{l} F_{TL}^{\cos(2\phi+\phi_a-3\phi_h)}\cos(2\phi+\phi_a-3\phi_h) \\ +F_{TL}^{\cos(2\phi+\phi_a-2\phi_h)}\cos(2\phi+\phi_a-2\phi_h) \\ +F_{TL}^{\cos(2\phi+\phi_a)}\cos(2\phi+\phi_a) \\ +F_{TL}^{\cos(2\phi-\phi_a-\phi_h)}\cos(2\phi-\phi_a-\phi_h) \\ +F_{TL}^{\cos(2\phi-\phi_a)}\cos(2\phi-\phi_a) \end{array}\right]$$

Fracture functions		Quark polarization		
		U	L	T
Nucleon Polarization	U			
	L		*	*
	T			

σ_{TT}

$$\sigma_{TT} = \left(1+\cos^2\theta\right)\begin{bmatrix} F_{TT}^{\cos(\phi_a-\phi_b)}\cos(\phi_a-\phi_b) \\ +F_{TT}^{\cos(\phi_a+\phi_b-2\phi_h)}\cos(\phi_a+\phi_b-2\phi_h) \\ +F_{TT}^{\cos(\phi_a-\phi_b+\phi_h)}\cos(\phi_a-\phi_b+\phi_h) \\ +F_{TT}^{\cos(\phi_a+\phi_b-\phi_h)}\cos(\phi_a+\phi_b-\phi_h) \\ +F_{TT}^{\cos(\phi_a+\phi_b)}\cos(\phi_a+\phi_b) \\ +F_{TT}^{\cos(\phi_a-\phi_b-\phi_h)}\cos(\phi_a-\phi_b-\phi_h) \end{bmatrix}$$

$$+\sin^2\theta\begin{bmatrix} F_{TT}^{\cos(2\phi-\phi_a-\phi_b-\phi_h)}\cos(2\phi-\phi_a-\phi_b-\phi_h) \\ +F_{TT}^{\cos(2\phi-\phi_a-\phi_b+\phi_h)}\cos(2\phi-\phi_a-\phi_b+\phi_h) \\ +F_{TT}^{\cos(2\phi+\phi_a+\phi_b)}\cos(2\phi+\phi_a+\phi_b) \\ +F_{TT}^{\cos(2\phi+\phi_a-\phi_b+\phi_h)}\cos(2\phi+\phi_a-\phi_b+\phi_h)+ \\ F_{TT}^{\cos(2\phi-\phi_a+\phi_b)}\cos(2\phi-\phi_a+\phi_b) \\ +F_{TT}^{\cos(2\phi-\phi_a+\phi_b-2\phi_h)}\cos(2\phi-\phi_a+\phi_b-2\phi_h) \\ +F_{TT}^{\cos(2\phi+\phi_a+\phi_b-4\phi_h)}\cos(2\phi+\phi_a+\phi_b-4\phi_h) \\ +F_{TT}^{\cos(2\phi+\phi_a-\phi_b-3\phi_h)}\cos(2\phi+\phi_a-\phi_b-3\phi_h) \\ +F_{TT}^{\cos(2\phi-\phi_a-\phi_b)}\cos(2\phi-\phi_a-\phi_b) \\ +F_{TT}^{\cos(2\phi+\phi_a-\phi_b-2\phi_h)}\cos(2\phi+\phi_a-\phi_b-2\phi_h) \\ +F_{TT}^{\cos(2\phi+\phi_a+\phi_b-2\phi_h)}\cos(2\phi+\phi_a+\phi_b-2\phi_h) \\ +F_{TT}^{\cos(2\phi+\phi_a-\phi_b-\phi_h)}\cos(2\phi+\phi_a-\phi_b-\phi_h) \\ +F_{TT}^{\cos(2\phi+\phi_a-\phi_b)}\cos(2\phi+\phi_a-\phi_b) \end{bmatrix}$$

Fracture functions		Quark polarization: U	L	T
Nucleon Polarization	U			
	L			
	T	*	*	*

CONCLUSIONS

- New members of the polarized TMDs family -- 16 LO STMD fracture functions
- For hadron produced in the TFR of SIDIS, only 4 k_T-integrated fracture functions of unpolarized and longitudinally polarized quarks are probed.
  - SSA contains only a Sivers-type modulation $\sin(\phi_h-\phi_S)$ but no Collins-type $\sin(\phi_h+\phi_S)$ or $\sin(3\phi_h-\phi_S)$. The eventual observation of Collins-type asymmetry will indicate that LO factorized approach fails and long range correlations between the struck quark polarization and P_T of produced in TFR hadron might be important.
- DSIDIS cross section at LO contains 2 azimuthal independent and 20 azimuthally modulated terms.
- SIDY cross section at LO contains 2 azimuthal independent, 20 lepton azimuth independent and 52 lepton azimuth dependent terms
- The ideal place to test the fracture functions factorization and measure these new nonperturbative objects are JLab12 and EIC facilities with full coverage of phase space and polarized SIDY
- PQCD factorization issues (SIDIS, DSIDIS, SIDY).
  - Structure of Wilson lines. SIDIS ←→ DY universality: sign changes of some fracture functions? Higher twist. Polarized hadron production. Phenomenology: parameterizations , simple models.
  - Application to other processes:

  $P\uparrow + P \rightarrow \pi + X$, $P\uparrow + P \rightarrow \pi + \text{jet} + X$, ...

Additional slides

Convolutions & tensorial decomposition

$$C[\hat{M}\cdot Dw] = \sum_a e_a^2 \int d^2k_T\, d^2p_T\, \delta^{(2)}(z\mathbf{k}_T + \mathbf{p}_T - \mathbf{P}_{T1})\hat{M}_a(x,\zeta,k_T^2,P_{T2}^2,\mathbf{k}_T\cdot\mathbf{P}_{T2})D_a(z,p_T^2)w$$

$$C[\hat{M}\cdot D] = F_0^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot Dk^i] = P_{T1}^i F_{k1}^{\hat{M}\cdot D} + P_{T2}^i F_{k2}^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot D\,p^i] = P_{T1}^i F_{p1}^{\hat{M}\cdot D} + P_{T2}^i F_{p2}^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot Dk^ik^j] = P_{T1}^iP_{T1}^j F_{kk1}^{\hat{M}\cdot D} + P_{T2}^iP_{T2}^j F_{kk2}^{\hat{M}\cdot D} + \delta^{ij} F_{kk3}^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot Dk^ip^j] = P_{T1}^iP_{T1}^j F_{kp1}^{\hat{M}\cdot D} + P_{T2}^iP_{T2}^j F_{kp2}^{\hat{M}\cdot D} + \left(P_{T1}^iP_{T2}^j - P_{T1}^jP_{T2}^i\right)F_{kp3}^{\hat{M}\cdot D} + \delta^{ij} F_{kp4}^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot Dk^ik^jp^k] = P_{T1}^iP_{T1}^jP_{T1}^k F_{kkp1}^{\hat{M}\cdot D} + P_{T1}^iP_{T1}^jP_{T2}^k F_{kkp2}^{\hat{M}\cdot D} + P_{T2}^iP_{T2}^jP_{T2}^k F_{kkp3}^{\hat{M}\cdot D}$$
$$+P_{T2}^iP_{T2}^jP_{T1}^k F_{kkp4}^{\hat{M}\cdot D} + P_{T1}^k\delta^{ij} F_{kkp5}^{\hat{M}\cdot D} + P_{T2}^k\delta^{ij} F_{kkp6}^{\hat{M}\cdot D}$$

where $F_{\dots}^{\hat{M}\cdot D}$ depend on $x, z, \zeta, P_{T1}^2, P_{T2}^2, \left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)$

Structure functions

$$F_{k1}^{\hat{M}\cdot D} = C\left[\hat{M}\cdot D\,\frac{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})(\mathbf{P}_{T2}\cdot\mathbf{k})-(\mathbf{P}_{T1}\cdot\mathbf{k})\mathbf{P}_{T2}^2}{\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2}\right]$$

$$F_{k2}^{\hat{M}\cdot D} = C\left[\hat{M}\cdot D\,\frac{(\mathbf{P}_{T1}\cdot\mathbf{k})(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})-(\mathbf{P}_{T2}\cdot\mathbf{k})\mathbf{P}_{T1}^2}{\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2}\right]$$

$$F_{kk1}^{\hat{M}\cdot D} = C\left[\hat{M}\cdot D\,\frac{\left(-2\left(\mathbf{P}_{T1}\cdot\mathbf{k}\right)^2+\mathbf{k}^2\mathbf{P}_{T1}^2\right)\mathbf{P}_{T2}^4+\left(2\left(\mathbf{P}_{T2}\cdot\mathbf{k}\right)^2-\mathbf{k}^2\mathbf{P}_{T2}^2\right)\left(2\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)}{4\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2\left(\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)}\right]$$

$$F_{kk2}^{\hat{M}\cdot D} = C\left[\hat{M}\cdot D\,\frac{\left(2\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)\left(\mathbf{P}_{T1}\cdot\mathbf{k}\right)^2+\mathbf{P}_{T1}^2\left(\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2-\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2\right)\mathbf{k}^2-\left(\mathbf{P}_{T2}\cdot\mathbf{k}\right)^2\mathbf{P}_{T1}^4}{2\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2\left(\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)}\right]$$

$$F_{kk3}^{\hat{M}\cdot D} = C\left[\hat{M}\cdot D\,\frac{\left(\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2+\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)\mathbf{k}^2-\left(\mathbf{P}_{T2}\cdot\mathbf{k}\right)^2\mathbf{P}_{T1}^2-\left(\mathbf{P}_{T1}\cdot\mathbf{k}\right)^2\mathbf{P}_{T2}^2}{2\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2}\right]$$

Structure functions of σ_{UU}, σ_{UL}, σ_{UT}

$$F_{UU} = F_0^{f_1 \cdot \hat{M}}$$

$$F_{UU}^{\cos(2\phi)} = \frac{q_T^2}{M_a M_b} F_{ab2}^{h_1^\perp \cdot \Delta_T \hat{M}^\perp}$$

$$F_{UU}^{\cos(2\phi-\phi_h)} = \frac{P_T q_T}{m M_a} F_{a2}^{h_1^\perp \cdot \Delta_T \hat{M}^h}$$

$$F_{UU}^{\cos(2\phi-2\phi_h)} = \frac{P_T^2}{m M_a} F_{a1}^{h_1^\perp \cdot \Delta_T \hat{M}^h} + \frac{P_T^2}{M_a M_b} F_{ab1}^{h_1^\perp \cdot \Delta_T \hat{M}^\perp}$$

$$F_{UL}^{\sin(\phi_h)} = \frac{P_T q_T}{m M_b} F_{b2}^{f_1 \cdot \hat{M}_L^{\perp h}}$$

$$F_{UL}^{\sin(2\phi)} = \frac{q_T^2}{M_a M_b} F_{ab2}^{h_1^\perp \cdot \Delta_T \hat{M}_L^\perp}$$

$$F_{UL}^{\sin(2\phi-\phi_h)} = \frac{P_T q_T}{m M_a} F_{a2}^{h_1^\perp \cdot \Delta_T \hat{M}_L^h}$$

$$F_{UL}^{\sin(2\phi-2\phi_h)} = \frac{P_T^2}{m M_a} F_{a1}^{h_1^\perp \cdot \Delta_T \hat{M}_L^h} + \frac{P_T^2}{M_a M_b} F_{ab1}^{h_1^\perp \cdot \Delta_T \hat{M}_L^\perp}$$

$$F_{UT}^{\sin(\phi_b-\phi_h)} = \frac{P_T}{m} F_0^{f_1 \cdot \hat{M}_T^h} + \frac{P_T}{M_b} F_{b1}^{f_1 \cdot \hat{M}_T^\perp}, \qquad F_{UT}^{\sin(\phi_b)} = \frac{q_T}{M_b} F_{b2}^{f_1 \cdot \hat{M}_T^\perp}$$

$$F_{UT}^{\sin(2\phi-\phi_b+\phi_h)} = -\frac{P_T q_T^2}{2m M_a M_b} F_{ab2}^{h_1^\perp \cdot \Delta_T \hat{M}_T^{\perp h}}, \qquad F_{UT}^{\sin(2\phi+\phi_b)} = \frac{q_T^3}{2 M_a M_b^2} F_{abb3}^{h_1^\perp \cdot \Delta_T \hat{M}_T^{\perp\perp}}, \qquad F_{UT}^{\sin(2\phi+\phi_b-\phi_h)} = \frac{P_T q_T^2}{2 M_a M_b^2} F_{abb4}^{h_1^\perp \cdot \Delta_T \hat{M}_T^{\perp\perp}}$$

$$F_{UT}^{\sin(2\phi+\phi_b-3\phi_h)} = \frac{P_T^3}{2m^2 M_a} F_{a1}^{h_1^\perp \cdot \Delta_T \hat{M}_T^{hh}} + \frac{P_T^3}{2 M_a M_b^2} F_{abb1}^{h_1^\perp \cdot \Delta_T \hat{M}_T^{\perp\perp}}, \qquad F_{UT}^{\sin(2\phi-\phi_b-2\phi_h)} = \frac{P_T^2 q_T}{2m M_a M_b} F_{ab3}^{h_1^\perp \cdot \Delta_T \hat{M}_T^{\perp h}}$$

$$F_{UT}^{\sin(2\phi+\phi_b-2\phi_h)} = \frac{P_T^2 q_T}{2m^2 M_a} F_{a2}^{h_1^\perp \cdot \Delta_T \hat{M}_T^{hh}} + \frac{P_T^2 q_T}{2 M_a M_b^2} F_{abb2}^{h_1^\perp \cdot \Delta_T \hat{M}_T^{\perp\perp}}$$

$$F_{UT}^{\sin(2\phi-\phi_b-\phi_h)} = \left(\begin{array}{l} \frac{P_T}{M_a} F_{a1}^{h_1^\perp \cdot \Delta_T \hat{M}_T} + \frac{P_T^3}{2m^2 M_a} F_{a1}^{h_1^\perp \cdot \Delta_T \hat{M}_T^{hh}} + \frac{P_T^3}{2 M_a M_b^2} F_{abb1}^{h_1^\perp \cdot \Delta_T \hat{M}_T^{\perp\perp}} + \frac{P_T q_T^2}{2m M_a M_b} F_{ab2}^{h_1^\perp \cdot \Delta_T \hat{M}_T^{\perp h}} \\ + \frac{P_T q_T^2}{2 M_a M_b^2} F_{abb4}^{h_1^\perp \cdot \Delta_T \hat{M}_T^{\perp\perp}} + \frac{P_T}{m M_a M_b} F_{ab4}^{h_1^\perp \cdot \Delta_T \hat{M}_T^{\perp h}} + \frac{P_T}{M_a M_b^2} F_{abb5}^{h_1^\perp \cdot \Delta_T \hat{M}_T^{\perp\perp}} \end{array} \right)$$

$$F_{UT}^{\sin(2\phi-\phi_b)} = \left(\begin{array}{l} \frac{q_T}{M_a} F_{a2}^{h_1^\perp \cdot \Delta_T \hat{M}_T} + \frac{P_T^2 q_T}{2m^2 M_a} F_{a2}^{h_1^\perp \cdot \Delta_T \hat{M}_T^{hh}} + \frac{q_T^3}{2 M_a M_b^2} F_{abb3}^{h_1^\perp \cdot \Delta_T \hat{M}_T^{\perp\perp}} \\ - \frac{P_T^2 q_T}{2m M_a M_b} F_{ab3}^{h_1^\perp \cdot \Delta_T \hat{M}_T^{\perp h}} + \frac{P_T^2 q_T}{2 M_a M_b^2} F_{abb2}^{h_1^\perp \cdot \Delta_T \hat{M}_T^{\perp\perp}} + \frac{q_T}{M_a M_b^2} F_{abb6}^{h_1^\perp \cdot \Delta_T \hat{M}_T^{\perp\perp}} \end{array} \right)$$

Structure functions of σ_{LU}, σ_{LL}

$$F_{LU}^{\sin(\phi_h)} = \frac{P_T q_T}{mM_b} F_{b2}^{g_{1L}\cdot\Delta\hat{M}^{\perp h}}$$

$$F_{LU}^{\sin(2\phi-2\phi_h)} = \frac{P_T^2}{mM_a} F_{a1}^{h_{1L}^{\perp}\cdot\Delta_T\hat{M}^{h}} + \frac{P_T^2}{M_a M_b} F_{ab1}^{h_{1L}^{\perp}\cdot\Delta_T\hat{M}^{\perp}}$$

$$F_{LU}^{\sin(2\phi-\phi_h)} = \frac{P_T q_T}{mM_a} F_{a2}^{h_{1L}^{\perp}\cdot\Delta_T\hat{M}^{h}}$$

$$F_{LU}^{\sin(2\phi)} = \frac{q_T^2}{M_a M_b} F_{ab2}^{h_{1L}^{\perp}\cdot\Delta_T\hat{M}^{\perp}}$$

$$F_{LL} = F_0^{g_{1L}\cdot\Delta\hat{M}_L}$$

$$F_{LL}^{\cos(2\phi-2\phi_h)} = \frac{P_T^2}{mM_a} F_{a1}^{h_{1L}^{\perp}\cdot\Delta_T\hat{M}_L^{h}} + \frac{P_T^2}{M_a M_b} F_{ab1}^{h_{1L}^{\perp}\cdot\Delta_T\hat{M}_L^{\perp}}$$

$$F_{LL}^{\cos(2\phi-\phi_h)} = \frac{P_T q_T}{mM_a} F_{a2}^{h_{1L}^{\perp}\cdot\Delta_T\hat{M}_L^{h}}$$

$$F_{LL}^{\cos(2\phi)} = \frac{q_T^2}{M_a M_b} F_{ab2}^{h_{1L}^{\perp}\cdot\Delta_T\hat{M}_L^{\perp}}$$

Structure functions of σ_{LT}

$$F_{LT}^{\cos(\phi_b-\phi_h)} = \frac{P_T}{m} F_0^{g_{1L}\cdot\Delta\hat{M}_T^h} + \frac{P_T}{M_b} F_{b1}^{g_{1L}\cdot\Delta\hat{M}_T^\perp}$$

$$F_{LT}^{\cos(\phi_b)} = \frac{q_T}{M_b} F_{b2}^{g_{1L}\cdot\Delta\hat{M}_T^\perp}$$

$$F_{LT}^{\cos(2\phi-\phi_b+\phi_h)} = -\frac{P_T q_T^2}{2mM_aM_b} F_{ab2}^{h_{1L}^\perp\cdot\Delta_T\hat{M}_T^{\perp h}}$$

$$F_{LT}^{\cos(2\phi+\phi_b)} = \frac{q_T^3}{2M_aM_b^2} F_{abb3}^{h_{1L}^\perp\cdot\Delta_T\hat{M}_T^{\perp\perp}}$$

$$F_{LT}^{\cos(2\phi+\phi_b-\phi_h)} = \frac{P_T q_T^2}{2M_aM_b^2} F_{abb4}^{h_{1L}^\perp\cdot\Delta_T\hat{M}_T^{\perp\perp}}$$

$$F_{LT}^{\cos(2\phi+\phi_b-3\phi_h)} = \frac{P_T^3}{2m^2M_a} F_{a1}^{h_{1L}^\perp\cdot\Delta_T\hat{M}_T^{hh}} + \frac{P_T^3}{2M_aM_b^2} F_{abb1}^{h_{1L}^\perp\cdot\Delta_T\hat{M}_T^{\perp\perp}}$$

$$F_{LT}^{\cos(2\phi-\phi_b-2\phi_h)} = \frac{P_T^2 q_T}{2mM_aM_b} F_{ab3}^{h_{1L}^\perp\cdot\Delta_T\hat{M}_T^{\perp h}}$$

$$F_{LT}^{\cos(2\phi+\phi_b-2\phi_h)} = \frac{P_T^2 q_T}{2m^2M_a} F_{a2}^{h_{1L}^\perp\cdot\Delta_T\hat{M}_T^{hh}} + \frac{P_T^2 q_T}{2M_aM_b^2} F_{abb2}^{h_{1L}^\perp\cdot\Delta_T\hat{M}_T^{\perp\perp}}$$

$$F_{LT}^{\cos(2\phi-\phi_b-\phi_h)} = \left(\frac{P_T}{M_a} F_{a1}^{h_{1L}^\perp\cdot\Delta_T\hat{M}_T} + \frac{P_T^3}{2m^2M_a} F_{a1}^{h_{1L}^\perp\cdot\Delta_T\hat{M}_T^{hh}} + \frac{P_T^3}{2M_aM_b^2} F_{abb1}^{h_{1L}^\perp\cdot\Delta_T\hat{M}_T^{\perp\perp}} + \frac{P_T q_T^2}{2mM_aM_b} F_{ab2}^{h_{1L}^\perp\cdot\Delta_T\hat{M}_T^{\perp h}} + \frac{P_T q_T^2}{2M_aM_b^2} F_{abb4}^{h_{1L}^\perp\cdot\Delta_T\hat{M}_T^{\perp\perp}} + \frac{P_T}{mM_aM_b} F_{ab4}^{h_{1L}^\perp\cdot\Delta_T\hat{M}_T^{\perp h}} + \frac{P_T}{M_aM_b^2} F_{abb5}^{h_{1L}^\perp\cdot\Delta_T\hat{M}_T^{\perp\perp}} \right)$$

$$F_{LT}^{\cos(2\phi-\phi_b)} = \left(\frac{q_T}{M_a} F_{a2}^{h_{1L}^\perp\cdot\Delta_T\hat{M}_T} + \frac{P_T^2 q_T}{2m^2M_a} F_{a2}^{h_{1L}^\perp\cdot\Delta_T\hat{M}_T^{hh}} + \frac{q_T^3}{2M_aM_b^2} F_{abb3}^{h_{1L}^\perp\cdot\Delta_T\hat{M}_T^{\perp\perp}} - \frac{P_T^2 q_T}{2mM_aM_b} F_{ab3}^{h_{1L}^\perp\cdot\Delta_T\hat{M}_T^{\perp h}} + \frac{P_T^2 q_T}{2M_aM_b^2} F_{abb2}^{h_{1L}^\perp\cdot\Delta_T\hat{M}_T^{\perp\perp}} + \frac{q_T}{M_aM_b^2} F_{abb6}^{h_{1L}^\perp\cdot\Delta_T\hat{M}_T^{\perp\perp}} \right)$$

Structure functions of σ_{TU}

$$F_{TU}^{\sin(\phi_a-\phi_h)} = -\frac{P_T}{M_a}F_{a1}^{f_{1T}^{\perp}\cdot M} - \frac{P_T q_T^2}{2mM_aM_b}F_{ab2}^{g_{1T}\cdot\Delta M^{\perp h}} - \frac{P_T}{mM_aM_b}F_{ab4}^{g_{1T}\cdot\Delta M^{\perp h}}$$

$$F_{TU}^{\sin(\phi_a)} = -\frac{q_T}{M_a}F_{a2}^{f_{1T}^{\perp}\cdot M} + \frac{P_T^2 q_T}{2mM_aM_b}F_{ab3}^{g_{1T}\cdot\Delta M^{\perp h}}$$

$$F_{TU}^{\sin(\phi_a+\phi_h)} = \frac{P_T q_T^2}{2mM_aM_b}F_{ab2}^{g_{1T}\cdot\Delta M^{\perp h}}$$

$$F_{TU}^{\sin(\phi_a-2\phi_h)} = -\frac{P_T^2 q_T}{2mM_aM_b}F_{ab3}^{g_{1T}\cdot\Delta M^{\perp h}}$$

$$F_{TU}^{\sin(2\phi-\phi_a-\phi_h)} = -\frac{P_T}{M_b}F_{b1}^{h_1\cdot\Delta_T M^{\perp}} - \frac{P_T}{m}F_0^{h_1\cdot\Delta_T M^{h}}$$

$$F_{TU}^{\sin(2\phi-\phi_a)} = -\frac{q_T F_{b2}^{h_1\cdot\Delta_T M^{\perp}}}{M_b}$$

$$F_{TU}^{\sin(2\phi+\phi_a-3\phi_h)} = -\frac{P_T^3}{2mM_a^2}F_{aa1}^{h_{1T}^{\perp}\cdot\Delta_T M^{h}} - \frac{P_T^3}{2M_a^2M_b}F_{aab1}^{h_{1T}^{\perp}\cdot\Delta_T M^{\perp}}$$

$$F_{TU}^{\sin(2\phi+\phi_a-\phi_h)} = -\frac{P_T q_T^2}{2mM_a^2}F_{aa2}^{h_{1T}^{\perp}\cdot\Delta_T M^{h}} - \frac{P_T q_T^2}{2M_a^2M_b}F_{aab4}^{h_{1T}^{\perp}\cdot\Delta_T M^{\perp}}$$

$$F_{TU}^{\sin(2\phi+\phi_a-2\phi_h)} = -\frac{P_T^2 q_T}{2M_a^2M_b}F_{aab2}^{h_{1T}^{\perp}\cdot\Delta_T M^{\perp}}$$

$$F_{TU}^{\sin(2\phi+\phi_a)} = -\frac{q_T^3}{2M_a^2M_b}F_{aab3}^{h_{1T}^{\perp}\cdot\Delta_T M^{\perp}}$$

Structure functions of σ_{TL}

$$F_{TL}^{\cos(\phi_a-\phi_h)} = \frac{P_T}{M_a} F_{a1}^{g_{1T}\cdot\Delta M_L} - \frac{P_T q_T^2}{2mM_aM_b} F_{ab2}^{f_{1T}^{\perp}\cdot\Delta M_L^{\perp h}} - \frac{P_T}{mM_aM_b} F_{ab4}^{f_{1T}^{\perp}\cdot\Delta M_L^{\perp h}}$$

$$F_{TL}^{\cos(\phi_a)} = \frac{q_T}{M_a} F_{a2}^{g_{1T}\cdot\Delta M_L} + \frac{P_T^2 q_T}{2mM_aM_b} F_{ab3}^{f_{1T}^{\perp}\cdot\Delta M_L^{\perp h}}$$

$$F_{TL}^{\cos(\phi_a+\phi_h)} = \frac{P_T q_T^2}{2mM_aM_b} F_{ab2}^{f_{1T}^{\perp}\cdot\Delta M_L^{\perp h}}$$

$$F_{TL}^{\cos(\phi_a-2\phi_h)} = -\frac{P_T^2 q_T}{2mM_aM_b} F_{ab3}^{f_{1T}^{\perp}\cdot\Delta M_L^{\perp h}}$$

$$F_{TL}^{\cos(2\phi+\phi_a-3\phi_h)} = \frac{P_T^3}{2mM_a^2} F_{aa1}^{h_{1T}^{p}\cdot\Delta_T M_L^{h}} + \frac{P_T^3}{2M_a^2M_b} F_{aab1}^{h_{1T}^{p}\cdot\Delta_T M_L^{\perp}}$$

$$F_{TL}^{\cos(2\phi+\phi_a-2\phi_h)} = \frac{P_T^2 q_T}{2M_a^2M_b} F_{aab2}^{h_{1T}^{p}\cdot\Delta_T M_L^{\perp}}$$

$$F_{TL}^{\cos(2\phi+\phi_a)} = \frac{q_T^3}{2M_a^2M_b} F_{aab3}^{h_{1T}^{p}\cdot\Delta_T M_L^{\perp}}$$

$$F_{TL}^{\cos(2\phi-\phi_a-\phi_h)} = \frac{P_T}{M_b} F_{b1}^{h_1\cdot\Delta_T M_L^{\perp}} + \frac{P_T}{m} F_0^{h_1\cdot\Delta_T M_L^{h}}$$

$$F_{TL}^{\cos(2\phi-\phi_a)} = \frac{q_T}{M_b} F_{b2}^{h_1\cdot\Delta_T M_L^{\perp}}$$

Structure functions σ_{TT}

$$F_{TT}^{\cos(\phi_a-\phi_b)} = \begin{pmatrix} -\frac{P_T^2}{2mM_a}F_{a1}^{f_{1T}^{\perp}\cdot M_T^h} + \frac{P_T^2}{2mM_a}F_{a1}^{g_{1T}\cdot\Delta M_T^h} - \frac{P_T^2}{2M_aM_b}F_{ab1}^{f_{1T}^{\perp}\cdot M_T^{\perp}} + \frac{P_T^2}{2M_aM_b}F_{ab1}^{g_{1T}\cdot\Delta M_T^p} \\ -\frac{q_T^2}{2M_aM_b}F_{ab2}^{f_{1T}^{\perp}\cdot M_T^{\perp}} + \frac{q_T^2}{2M_aM_b}F_{ab2}^{g_{1T}\cdot\Delta M_T^p} - \frac{1}{M_aM_b}F_{ab4}^{f_{1T}^{\perp}\cdot M_T^{\perp}} + \frac{1}{M_aM_b}F_{ab4}^{g_{1T}\cdot\Delta M_T^p} \end{pmatrix}$$

$$F_{TT}^{\cos(\phi_a+\phi_b-2\phi_h)} = \frac{P_T^2}{2mM_a}F_{a1}^{f_{1T}^{\perp}\cdot M_T^h} + \frac{P_T^2}{2mM_a}F_{a1}^{g_{1T}\cdot\Delta M_T^h} + \frac{P_T^2}{2M_aM_b}F_{ab1}^{f_{1T}^{\perp}\cdot M_T^{\perp}} + \frac{P_T^2}{2M_aM_b}F_{ab1}^{g_{1T}\cdot\Delta M_T^p}$$

$$F_{TT}^{\cos(\phi_a-\phi_b+\phi_h)} = -\frac{P_Tq_T}{2mM_a}F_{a2}^{f_{1T}^{\perp}\cdot M_T^h} + \frac{P_Tq_T}{2mM_a}F_{a2}^{g_{1T}\cdot\Delta M_T^h} + \frac{P_Tq_T}{2M_aM_b}F_{ab3}^{f_{1T}^{\perp}\cdot M_T^{\perp}} - \frac{P_Tq_T}{2M_aM_b}F_{ab3}^{g_{1T}\cdot\Delta M_T^p}$$

$$F_{TT}^{\cos(\phi_a+\phi_b-\phi_h)} = \frac{P_Tq_T}{2mM_a}F_{a2}^{f_{1T}^{\perp}\cdot M_T^h} + \frac{P_Tq_T}{2mM_a}F_{a2}^{g_{1T}\cdot\Delta M_T^h}, \quad F_{TT}^{\cos(\phi_a+\phi_b)} = \frac{q_T^2}{2M_aM_b}F_{ab2}^{f_{1T}^{\perp}\cdot M_T^{\perp}} + \frac{q_T^2}{2M_aM_b}F_{ab2}^{g_{1T}\cdot\Delta M_T^p}$$

$$F_{TT}^{\cos(\phi_a-\phi_b-\phi_h)} = \frac{P_Tq_T}{2M_aM_b}F_{ab3}^{g_{1T}\cdot\Delta M_T^p} - \frac{P_Tq_T}{2M_aM_b}F_{ab3}^{f_{1T}^{\perp}\cdot M_T^{\perp}}, \quad F_{TT}^{\cos(2\phi-\phi_a-\phi_b-\phi_h)} = \frac{P_Tq_T}{2mM_b}F_{b2}^{h_1\cdot\Delta_T\hat{M}_T^{\perp h}}$$

$$F_{TT}^{\cos(2\phi-\phi_a-\phi_b+\phi_h)} = -\frac{P_Tq_T}{2mM_b}F_{b2}^{h_1\cdot\Delta_T\hat{M}_T^{\perp h}}, \quad F_{TT}^{\cos(2\phi+\phi_a+\phi_b)} = \frac{q_T^4}{4M_a^2M_b^2}F_{aabb3}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp\perp}}, \quad F_{TT}^{\cos(2\phi+\phi_a-\phi_b+\phi_h)} = -\frac{P_Tq_T^3}{4mM_a^2M_b}F_{aab3}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp h}}$$

$$F_{TT}^{\cos(2\phi-\phi_a+\phi_b)} = \frac{q_T^2}{2M_b^2}F_{bb2}^{h_1\cdot\Delta_T\hat{M}_T^{\perp\perp}}, \quad F_{TT}^{\cos(2\phi-\phi_a+\phi_b-2\phi_h)} = \frac{P_T^2}{2m^2}F_0^{h_1\cdot\Delta_T\hat{M}_T^{hh}} + \frac{P_T^2}{2M_b^2}F_{bb1}^{h_1\cdot\Delta_T\hat{M}_T^{\perp\perp}}$$

$$F_{TT}^{\cos(2\phi+\phi_a+\phi_b-4\phi_h)} = \frac{P_T^4}{4m^2M_a^2}F_{aa1}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{hh}} + \frac{P_T^4}{4M_a^2M_b^2}F_{aabb1}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp\perp}}, \quad F_{TT}^{\cos(2\phi+\phi_a-\phi_b-3\phi_h)} = \frac{P_T^3q_T}{4mM_a^2M_b}F_{aab2}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp h}}$$

$$F_{TT}^{\cos(2\phi-\phi_a-\phi_b)} = F_0^{h_1\cdot\Delta_T\hat{M}_T} + \frac{P_T^2}{2m^2}F_0^{h_1\cdot\Delta_T\hat{M}_T^{hh}} + \frac{P_T^2}{2M_b^2}F_{bb1}^{h_1\cdot\Delta_T\hat{M}_T^{\perp\perp}} + \frac{q_T^2}{2M_b^2}F_{bb2}^{h_1\cdot\Delta_T\hat{M}_T^{\perp\perp}} + \frac{1}{M_b^2}F_{bb3}^{h_1\cdot\Delta_T\hat{M}_T^{\perp\perp}}$$

$$F_{TT}^{\cos(2\phi+\phi_a-\phi_b-2\phi_h)} = \frac{P_T^2}{2M_a^2}F_{aa1}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T} + \frac{P_T^4}{4m^2M_a^2}F_{aa1}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{hh}} + \frac{P_T^4}{4M_a^2M_b^2}F_{aabb1}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp\perp}} + \frac{P_T^2}{2M_a^2M_b^2}F_{aabb5}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp\perp}} + \frac{P_T^2q_T^2}{4M_a^2M_b^2}F_{aabb2}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp\perp}}$$

$$F_{TT}^{\cos(2\phi+\phi_a+\phi_b-2\phi_h)} = \frac{P_T^2q_T^2}{4m^2M_a^2}F_{aa2}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{hh}} + \frac{P_T^2q_T^2}{4M_a^2M_b^2}F_{aabb2}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp\perp}} + \frac{P_T^2q_T^2}{4M_a^2M_b^2}F_{aabb4}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp\perp}}$$

$$F_{TT}^{\cos(2\phi+\phi_a-\phi_b-\phi_h)} = \frac{P_Tq_T^3}{4mM_a^2M_b}F_{aab3}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp h}} - \frac{P_T^3q_T}{4mM_a^2M_b}F_{aab2}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp h}}$$

$$F_{TT}^{\cos(2\phi+\phi_a-\phi_b)} = \frac{q_T^2}{2M_a^2}F_{aa2}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T} + \frac{P_T^2q_T^2}{4m^2M_a^2}F_{aa2}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{hh}} + \frac{q_T^4}{4M_a^2M_b^2}F_{aabb3}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp\perp}} + \frac{P_T^2q_T^2}{4M_a^2M_b^2}F_{aabb4}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp\perp}} + \frac{q_T^2}{2M_a^2M_b^2}F_{aabb6}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp\perp}}$$

Old ↔ New Notations

Old

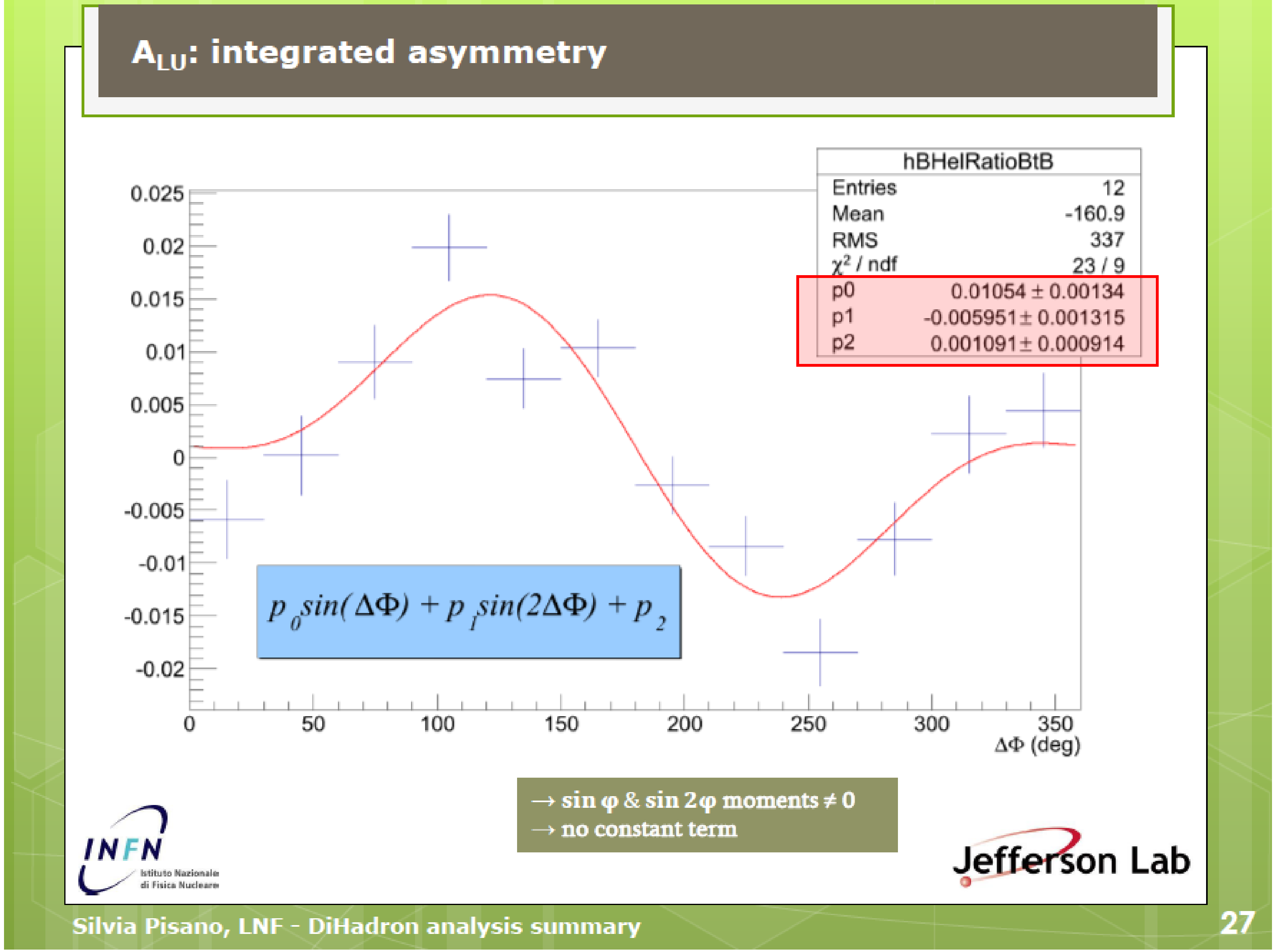
Nucleon Polarization	Quark polarization: U	L	T
U	$\hat{M}$	$\frac{\mathbf{k}_T\times\mathbf{P}_T}{m_N m_h}\Delta\hat{M}^{\perp h}$	$\frac{\epsilon_T^{ij}P_T^j}{m_h}\Delta_T\hat{M}^h+\frac{\epsilon_T^{ij}k_T^j}{m_N}\Delta_T\hat{M}^\perp$
L	$\frac{S_L(\mathbf{k}_T\times\mathbf{P}_T)}{m_N m_h}\hat{M}_L^{\perp h}$	$S_L\Delta\hat{M}_L$	$\frac{S_L\mathbf{P}_T}{m_h}\Delta_T\hat{M}_L^h+\frac{S_L\mathbf{k}_T}{m_N}\Delta_T\hat{M}_L^\perp$
T	$\frac{\mathbf{P}_T\times\mathbf{S}_T}{m_h}\hat{M}_T^h+\frac{\mathbf{k}_T\times\mathbf{S}_T}{m_N}\hat{M}_T^\perp$	$\frac{\mathbf{P}_T\cdot\mathbf{S}_T}{m_h}\Delta\hat{M}_T^h+\frac{\mathbf{k}_T\cdot\mathbf{S}_T}{m_N}\Delta\hat{M}_T^\perp$	$\mathbf{S}_T\Delta_T\hat{M}_T+\frac{\mathbf{P}_T(\mathbf{P}_T\cdot\mathbf{S}_T)}{m_h^2}\Delta_T\hat{M}_T^{hh}+\frac{\mathbf{k}_T(\mathbf{k}_T\cdot\mathbf{S}_T)}{m_N^2}\Delta_T\hat{M}_T^{\perp\perp}+\frac{\mathbf{P}_T(\mathbf{k}_T\cdot\mathbf{S}_T)-\mathbf{k}_T\cdot(\mathbf{P}_T\cdot\mathbf{S}_T)}{m_N m_h}\Delta_T\hat{M}_T^{\perp h}$

New

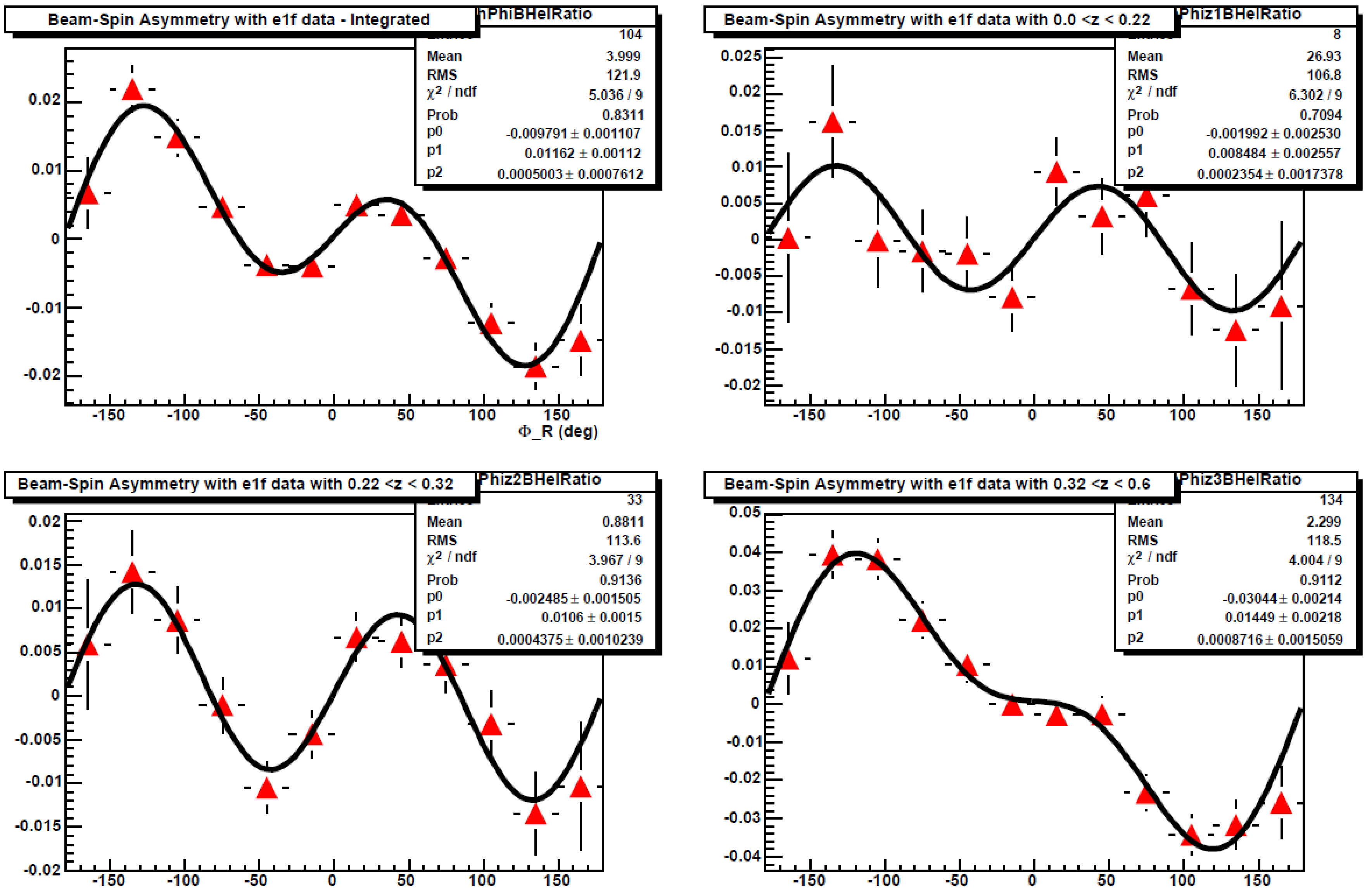
Nucleon Polarization	Quark polarization: U	L	T
U	$\hat{u}_1$	$\frac{\mathbf{k}_T\times\mathbf{P}_T}{m_N m_h}\hat{l}_1^{\perp h}$	$\frac{\epsilon_T^{ij}P_T^j}{m_h}\hat{t}_1^h+\frac{\epsilon_T^{ij}k_T^j}{m_N}\hat{t}_1^\perp$
L	$\frac{S_L(\mathbf{k}_T\times\mathbf{P}_T)}{m_N m_h}\hat{u}_{1L}^{\perp h}$	$S_L\hat{l}_{1L}$	$\frac{S_L\mathbf{P}_T}{m_h}\hat{t}_{1L}^h+\frac{S_L\mathbf{k}_T}{m_N}\hat{t}_{1L}^\perp$
T	$\frac{\mathbf{P}_T\times\mathbf{S}_T}{m_h}\hat{u}_{1T}^h+\frac{\mathbf{k}_T\times\mathbf{S}_T}{m_N}\hat{u}_{1T}^\perp$	$\frac{\mathbf{P}_T\cdot\mathbf{S}_T}{m_h}\hat{l}_{1T}^h+\frac{\mathbf{k}_T\cdot\mathbf{S}_T}{m_N}\hat{l}_{1T}^\perp$	$\mathbf{S}_T\hat{t}_{1T}+\frac{\mathbf{P}_T(\mathbf{P}_T\cdot\mathbf{S}_T)}{m_h^2}\hat{t}_{1T}^{hh}+\frac{\mathbf{k}_T(\mathbf{k}_T\cdot\mathbf{S}_T)}{m_N^2}\hat{t}_{1T}^{\perp\perp}+\frac{\mathbf{P}_T(\mathbf{k}_T\cdot\mathbf{S}_T)-\mathbf{k}_T\cdot(\mathbf{P}_T\cdot\mathbf{S}_T)}{m_N m_h}\hat{t}_{1T}^{\perp h}$

JLab CLAS, preliminary (**courtesy of Silvia Pisano**)

π^+ in CFR, π^- in TFR,



JLab Very Preliminary



Presence of higher harmonics indicate that $\sigma_{LU}(\Delta\Phi) \neq \sigma_{UU}(\Delta\Phi)$