

**ECT\***

EUROPEAN CENTRE FOR THEORETICAL STUDIES  
IN NUCLEAR PHYSICS AND RELATED AREAS

ECT\* workshop

Dilepton Productions with Meson and Antiproton Beams

November 6-10, 2017

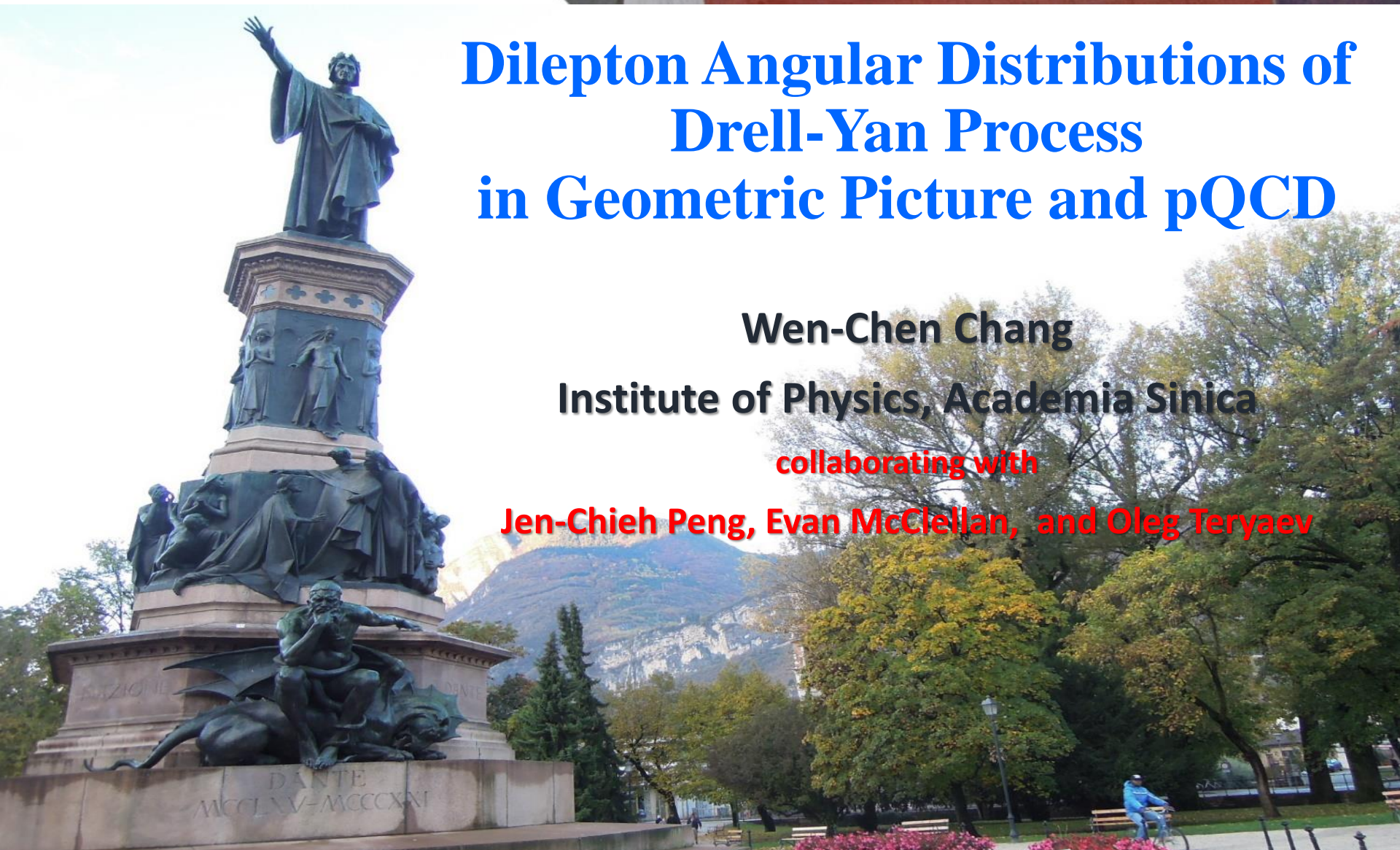
# Dilepton Angular Distributions of Drell-Yan Process in Geometric Picture and pQCD

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# The Drell-Yan Process

S.D. Drell and T.M. Yan, PRL 25 (1970) 316

MASSIVE LEPTON-PAIR PRODUCTION IN HADRON-HADRON COLLISIONS AT HIGH ENERGIES\*

Sidney D. Drell and Tung-Mow Yan

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

(Received 25 May 1970)

On the basis of a parton model studied earlier we consider the production process of large-mass lepton pairs from hadron-hadron inelastic collisions in the limiting region,  $s \rightarrow \infty$ ,  $Q^2/s$  finite,  $Q^2$  and  $s$  being the squared invariant masses of the lepton pair and the two initial hadrons, respectively. General scaling properties and connections with deep inelastic electron scattering are discussed. In particular, a rapidly decreasing cross section as  $Q^2/s \rightarrow 1$  is predicted as a consequence of the observed rapid falloff of the inelastic scattering structure function  $\nu W_2$  near threshold.

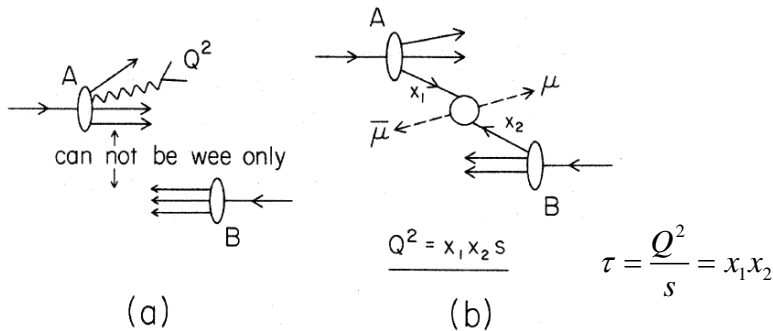


FIG. 1. (a) Production of a massive pair  $Q^2$  from one of the hadrons in a high-energy collision. In this case it is kinematically impossible to exchange "wee" partons only. (b) Production of a massive pair by parton-antiparton annihilation.

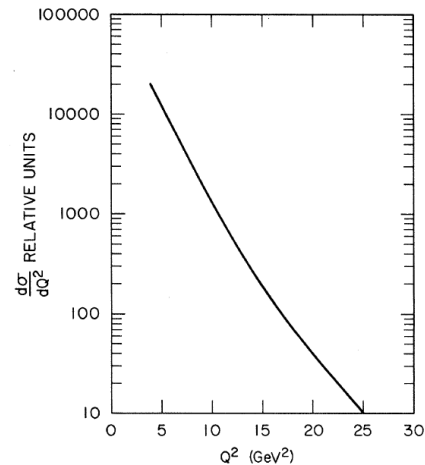
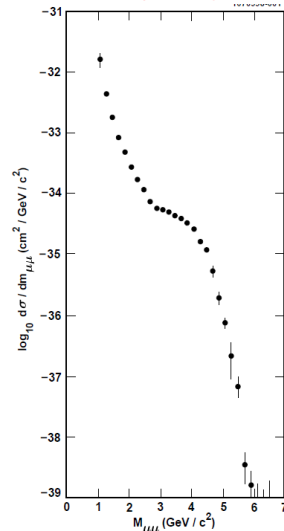


FIG. 2.  $d\sigma/dQ^2$  computed from Eq. (10) assuming identical parton and antiparton momentum distributions and with relative normalization.

PRL 25 (1970) 1523



$$\frac{d\sigma}{dQ^2} = \left( \frac{4\pi\alpha^2}{3Q^2} \right) \left( \frac{1}{Q^2} \right) \mathcal{F}(\tau) = \left( \frac{4\pi\alpha^2}{3Q^2} \right) \left( \frac{1}{Q^2} \right) \int_0^1 dx_1 \int_0^1 dx_2 \delta(x_1 x_2 - \tau) \sum_a \lambda_a^{-2} F_{2a}(x_1) F_{2\bar{a}}'(x_2),$$

# Angular Distribution in the “Naïve” Drell-Yan

VOLUME 25, NUMBER 5

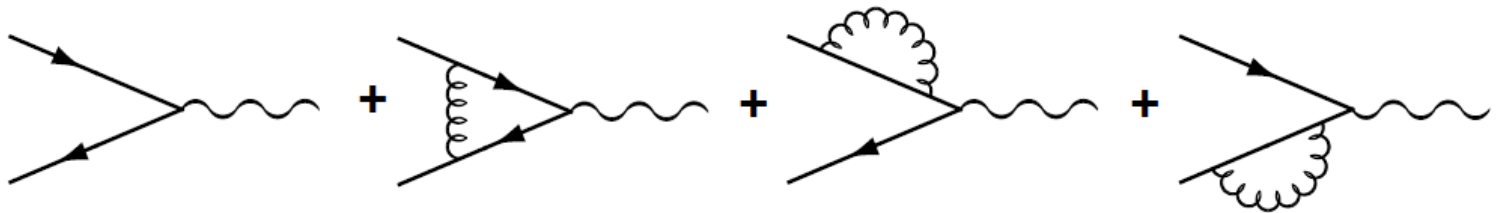
PHYSICAL REVIEW LETTERS

3 AUGUST 1970

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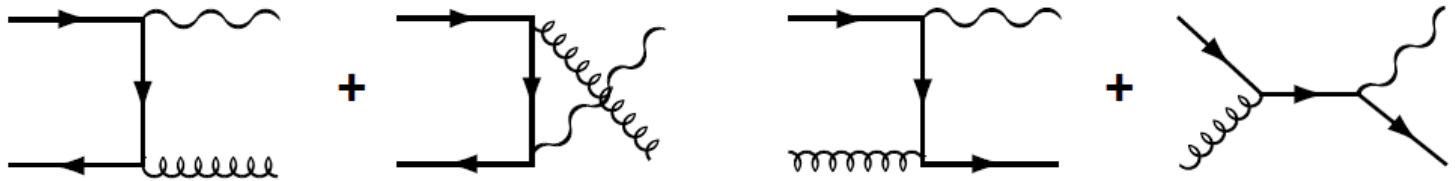
(3) The virtual photon will be predominantly transversely polarized if it is formed by annihilation of spin- $\frac{1}{2}$  parton-antiparton pairs. This means a distribution in the di-muon rest system varying as  $(1 + \cos^2\theta)$  rather than  $\sin^2\theta$  as found in Sakurai's<sup>10</sup> vector-dominance model, where  $\theta$  is the angle of the muon with respect to the time-like photon momentum. The model used in Fig.

# Drell-Yan Process with QCD Effect



( a )

Quark-antiquark annihilation



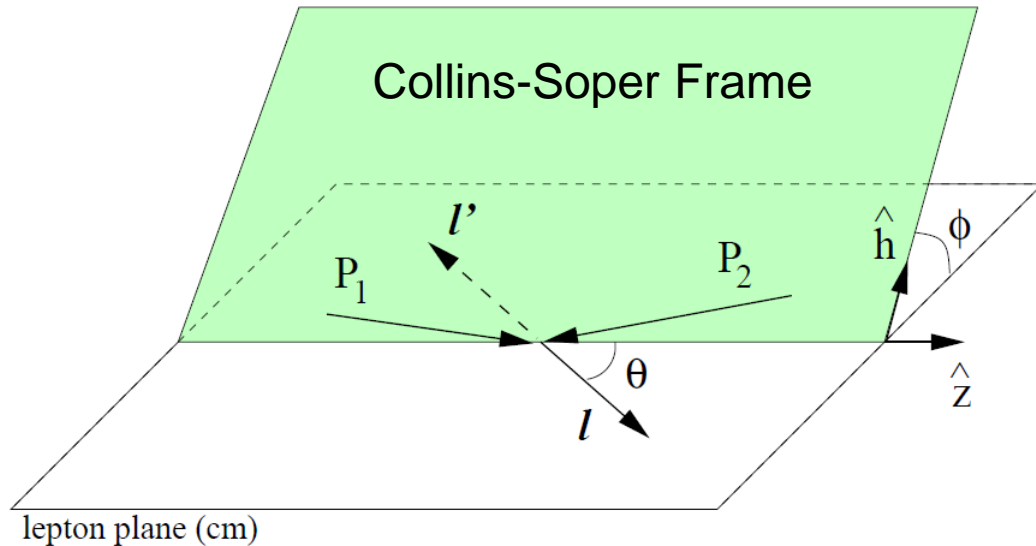
( b )

Quark-antiquark annihilation

( c )

Quark-gluon Compton scattering

# Angular Distributions of Lepton Pairs



$$\lambda = \frac{2 - 3A_0}{2 + A_0}$$

$$\mu = \frac{2A_1}{2 + A_0}$$

$$\nu = \frac{2A_2}{2 + A_0}$$

$$\frac{d\sigma}{d\Omega} \propto (1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi)$$

$$\propto [(1 + \cos^2 \theta) + \frac{A_0}{2} (1 - 3\cos^2 \theta) + A_1 \sin 2\theta \cos \phi + \frac{A_2}{2} \sin^2 \theta \cos 2\phi]$$

$q\bar{q}$  annihilation parton model:  $O(\alpha_s^0)$   $\lambda=1, \mu=\nu=0; A_0 = A_2 = 0$

pQCD:  $O(\alpha_s^1)$ , ;  $1 - \lambda - 2\nu = \frac{4(A_2 - A_0)}{2 + A_0} = 0$  ;  $A_0 = A_2$

# Angular Distributions of Lepton Pairs from $Z/\gamma^*$

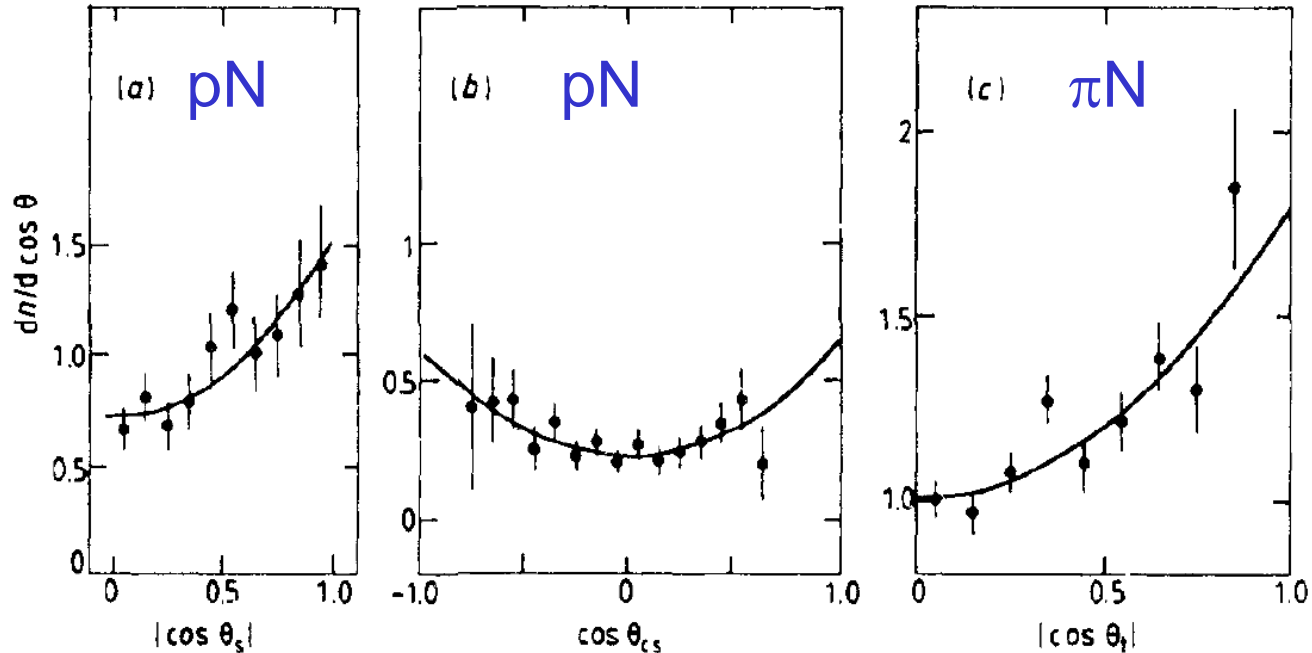
$$\begin{aligned} \frac{d\sigma}{d\Omega} \propto & [(1 + \cos^2 \theta) + \frac{A_0}{2}(1 - 3\cos^2 \theta) + A_1 \sin 2\theta \cos \phi + \frac{A_2}{2} \sin^2 \theta \cos 2\phi \\ & + A_3 \sin \theta \cos \phi + A_4 \cos \theta \\ & + A_5 \sin^2 \theta \sin 2\phi + A_6 \sin 2\theta \sin \phi + A_7 \sin \theta \sin \phi] \end{aligned}$$

$A_3, A_4$  :  $\gamma^* / Z$  interference, sensitive to  $\sin^2 \theta_W$

$A_5, A_6, A_7 := 0$ , up to  $O(\alpha_s^1)$

# Angular Distribution

I.R. Kenyon, Rep. Prog. Phys. 45 (1982) 1261

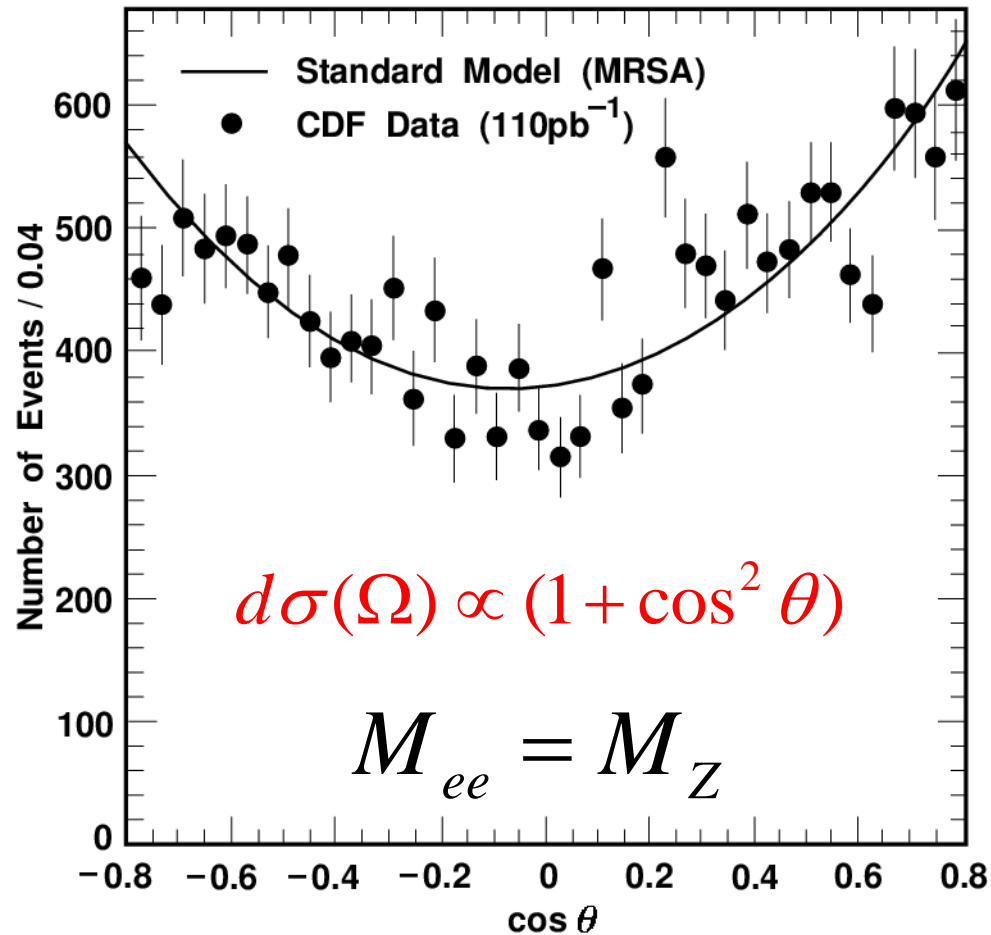


**Figure 17.** Measurements of the decay angular distribution of lepton pairs by Kourkouvelis *et al* (1980), Antreasyan *et al* (1980) and Badier *et al* (1980a). Fits to the form  $1 + \alpha \cos^2 \theta$  are shown as full curves and are discussed in the text. (a) ISR ABCS,  $4.5 < M < 8.7$  GeV, (b) ISR CHFMP,  $6 < M < 8$  GeV, (c) NA3,  $\pi^-$  200 GeV,  $4 < M < 6$  GeV,  $p_l < 1$  GeV.

$$d\sigma(\Omega) \propto (1 + \cos^2 \theta)$$

# Angular Distribution

CDF, PRL 77 (1996) 2616





# NA10 @ CERN: Violation of LT Relation

*Z. Phys.* 37 (1988) 545

Lam-Tung relation:  $1 - \lambda - 2\nu = 0$

$\pi^- + W$

140 GeV/c

194 GeV/c

286 GeV/c

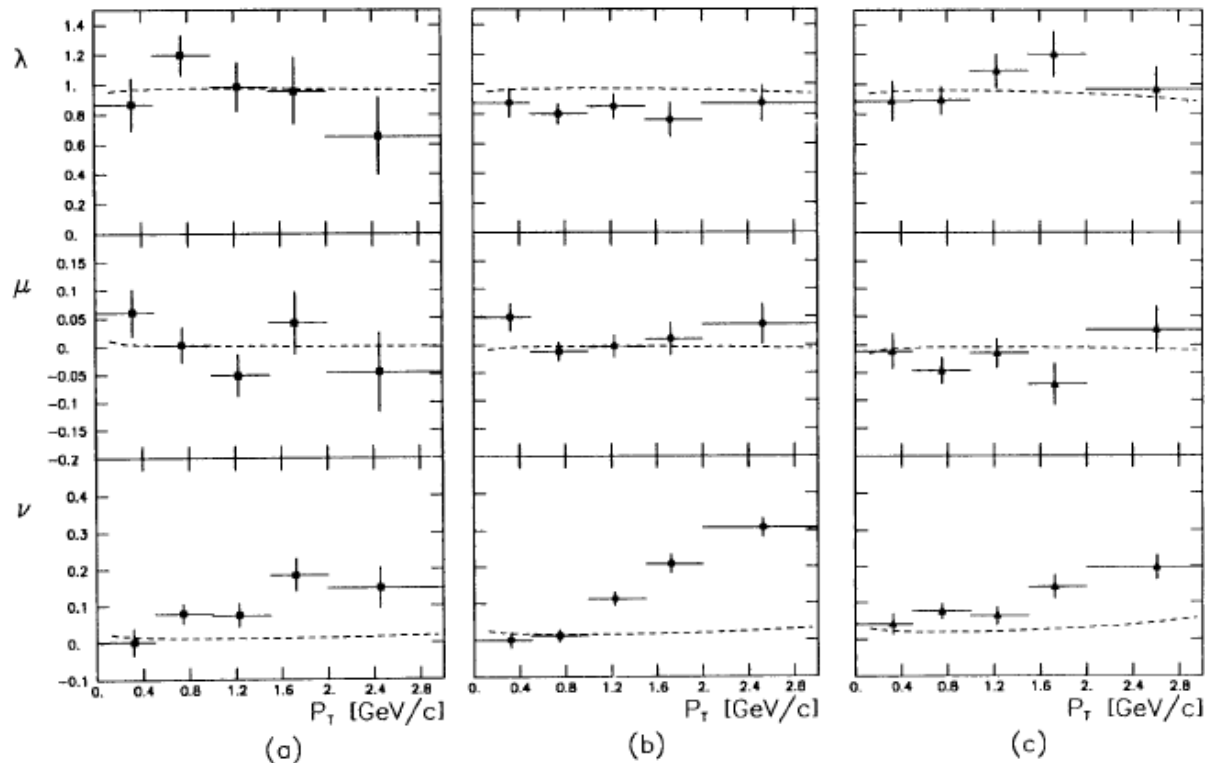


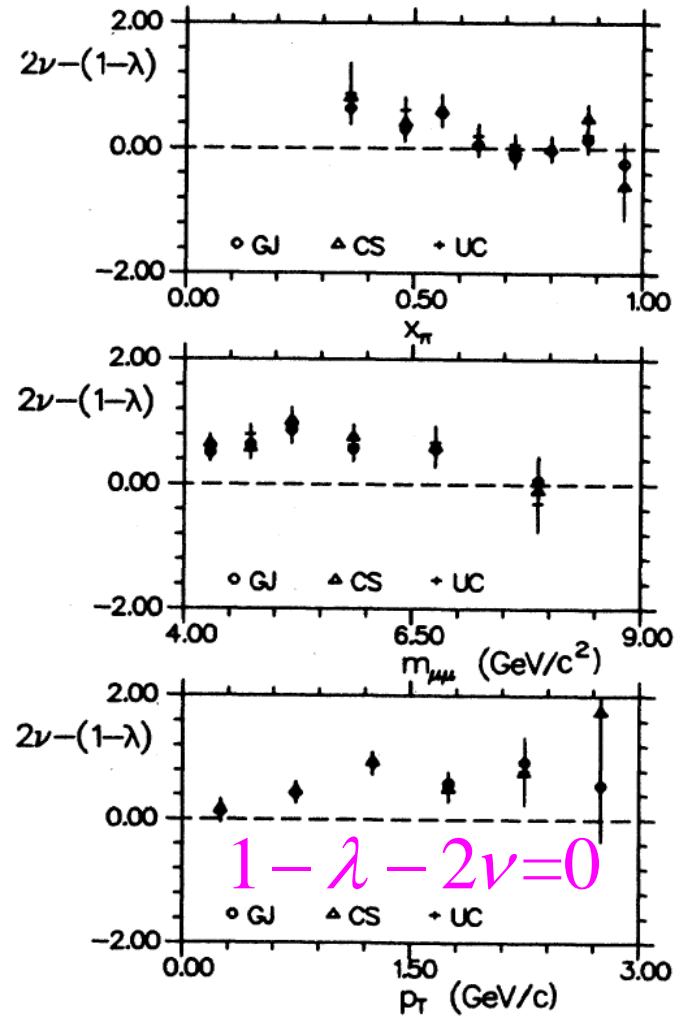
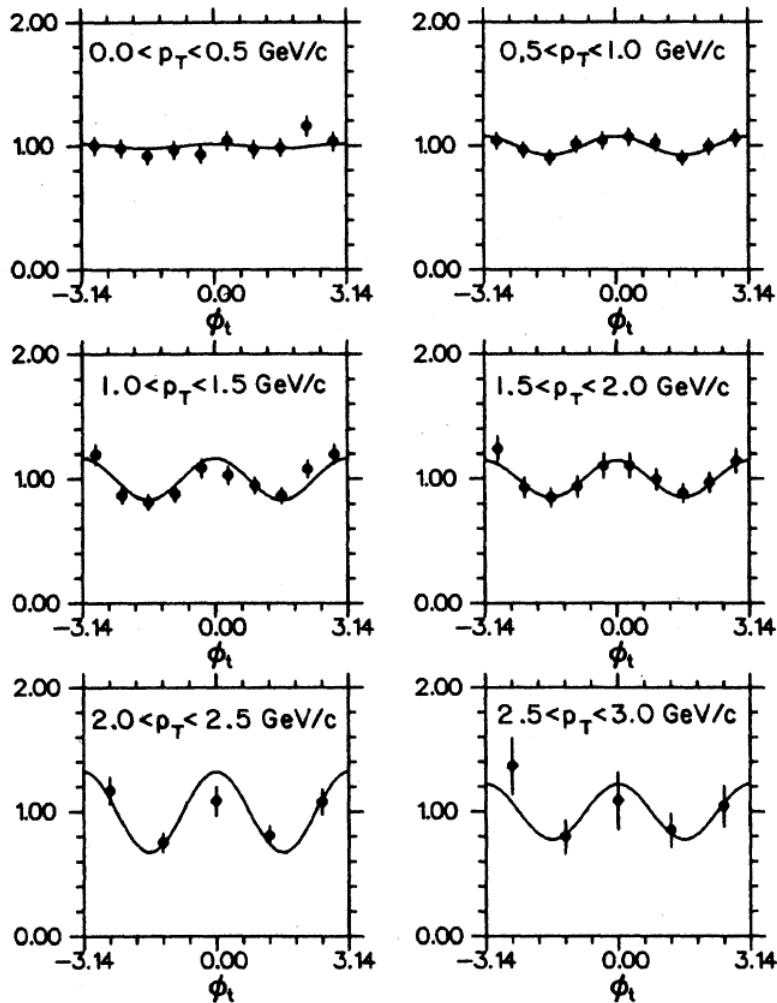
Fig. 3a-c. Parameters  $\lambda$ ,  $\mu$ , and  $\nu$  as a function of  $p_T$  in the CS frame. a 140 GeV/c; b 194 GeV/c; c 286 GeV/c. The error bars correspond to the statistical uncertainties only. The horizontal bars give the size of each interval. The dashed curves are the predictions of perturbative QCD [3]

$\nu \neq 0$  and  $\nu$  increases with  $p_T$

# E615 @ FNAL: Violation of LT Relation

*PRD 39, 92 (1989)*

252-GeV  $\pi^-+W$

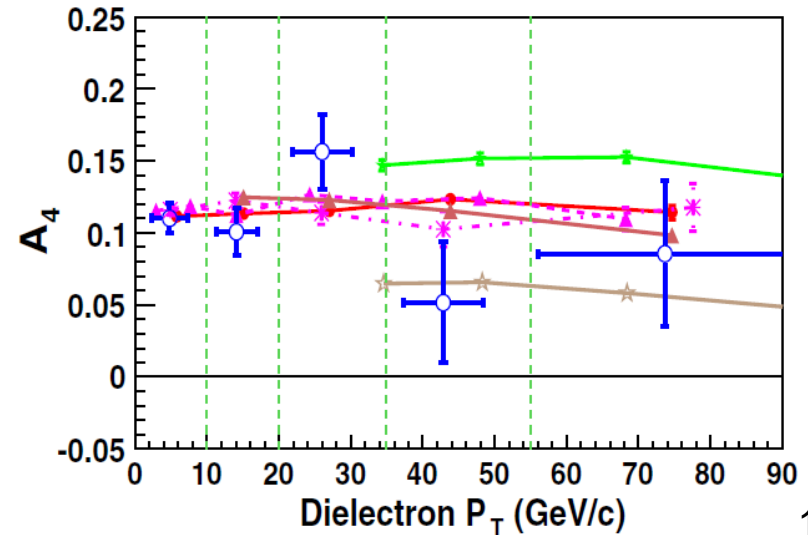
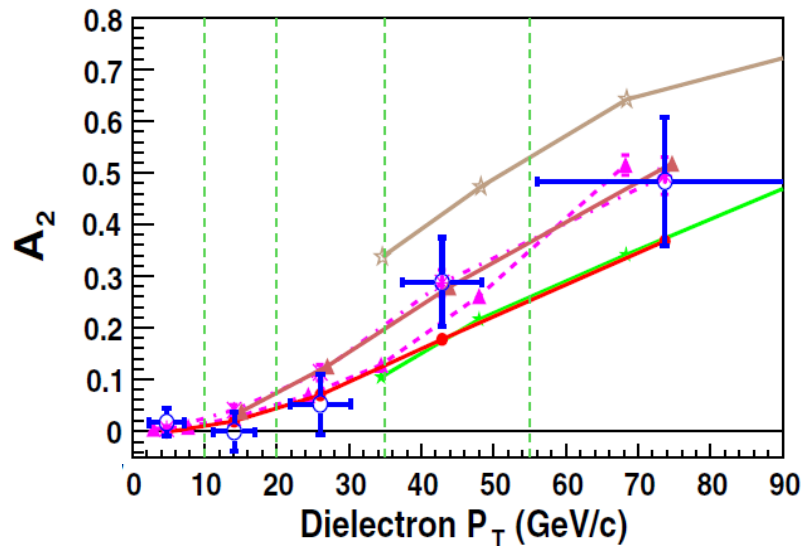
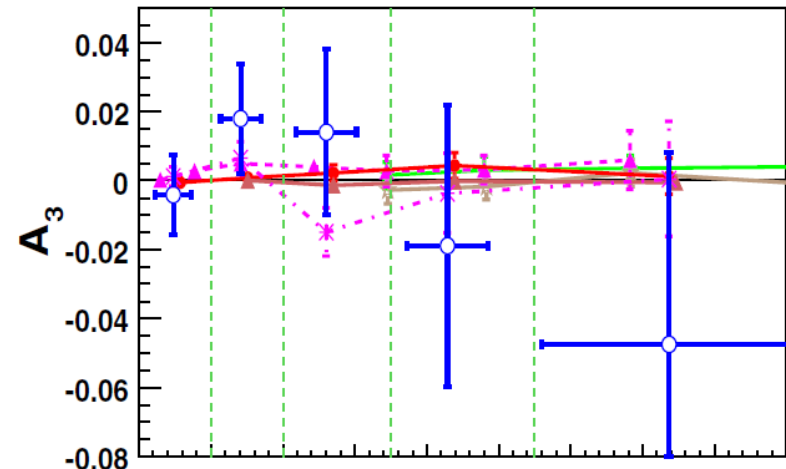
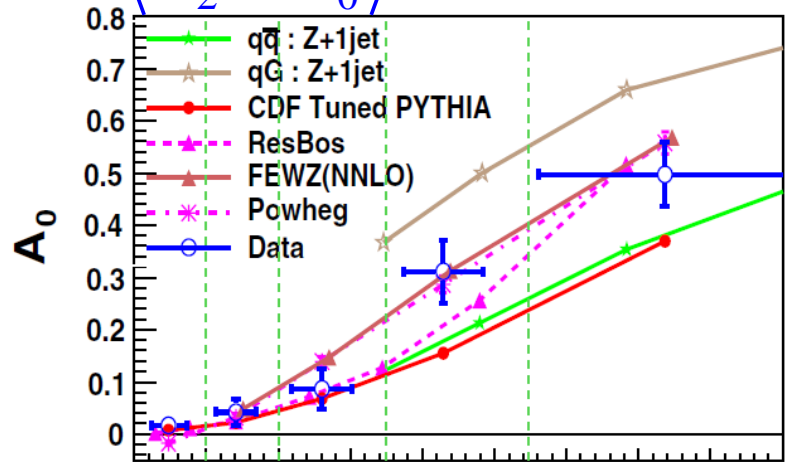


*cos 2 $\phi$  modulation at large  $p_T$*

# Angular Distributions of Z Production

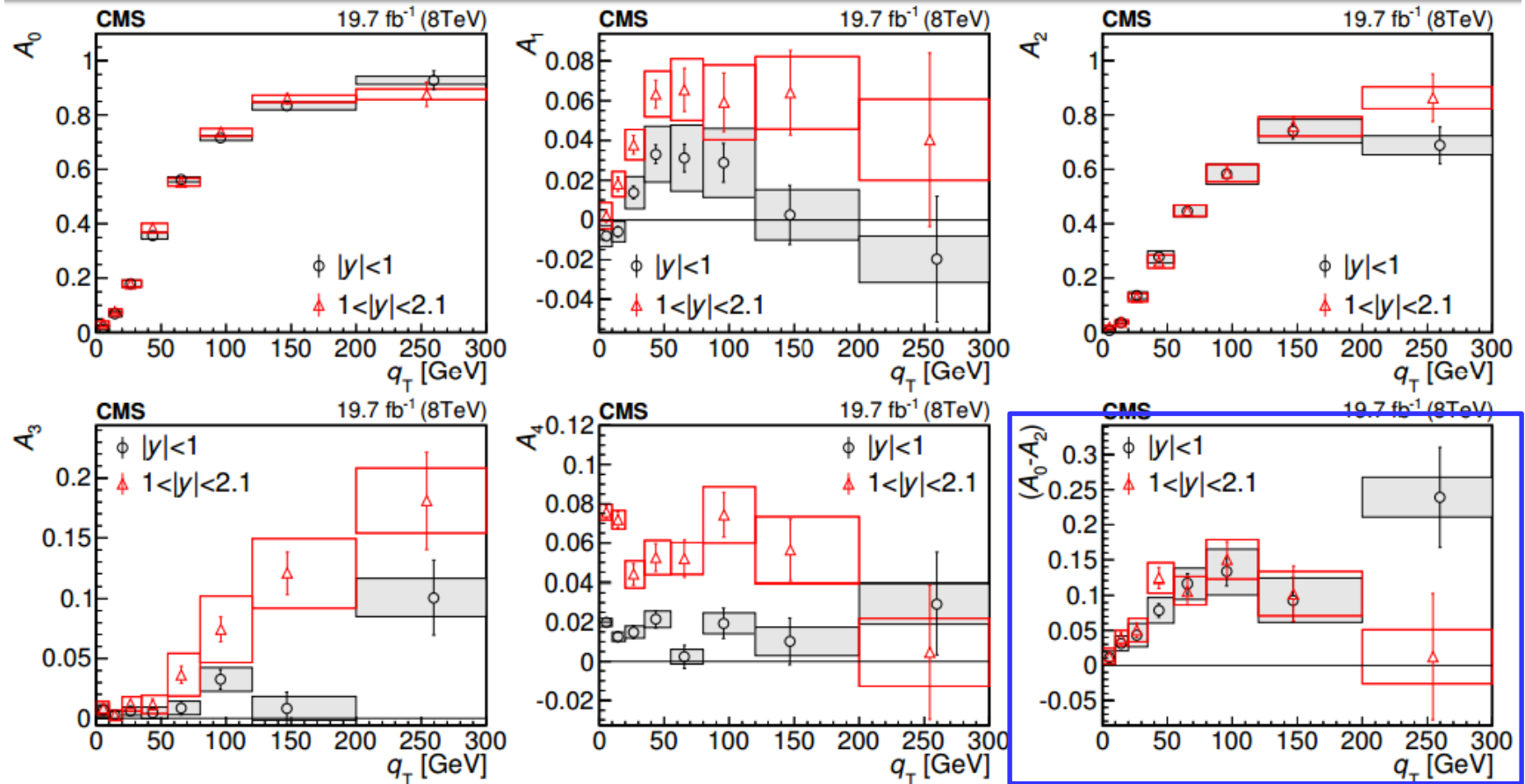
CDF, PRL 106, 241801 (2011)

$$\langle A_2 - A_0 \rangle = 0.02 \pm 0.02$$



# Angular Distributions of Z Production

CMS, PLB750, 154 (2015)



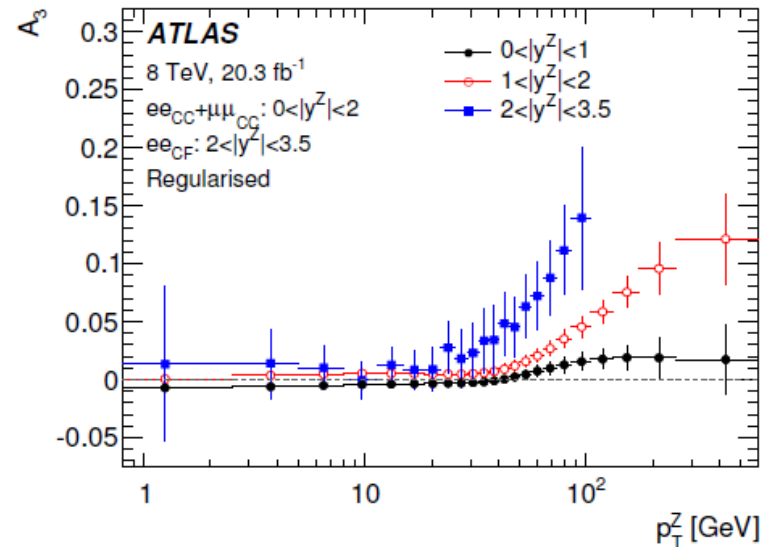
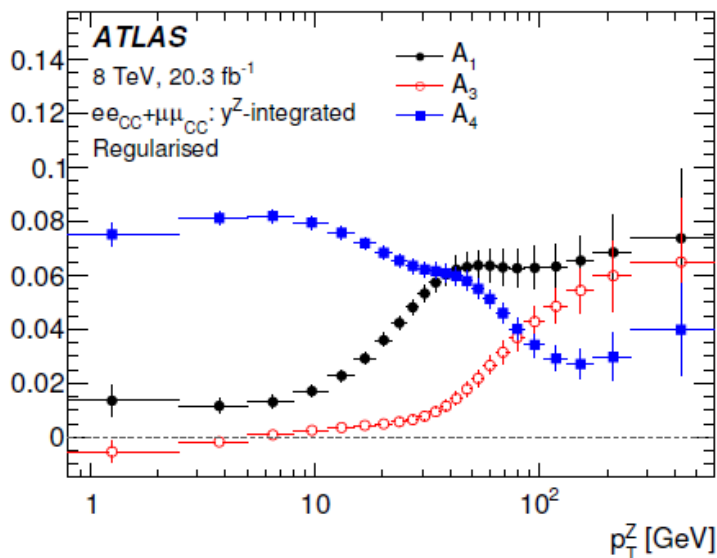
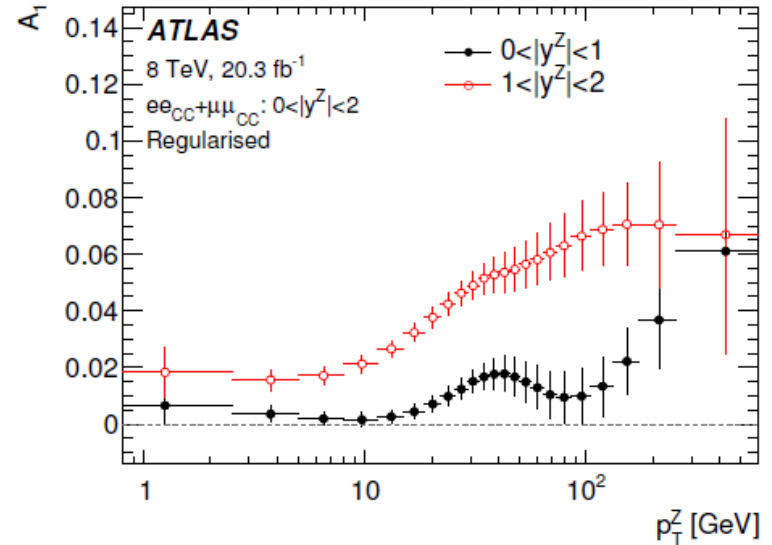
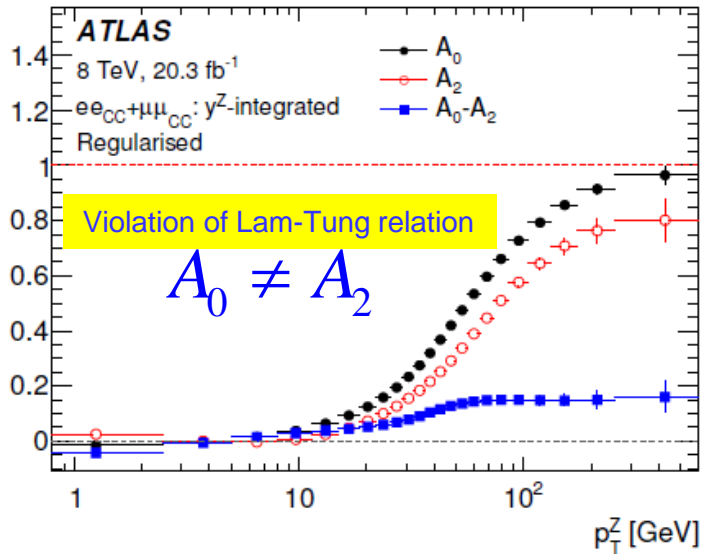
$$\frac{d^2\sigma}{d\cos\theta^* d\phi^*} \propto \left[ (1 + \cos^2\theta^*) + A_0 \frac{1}{2} (1 - 3\cos^2\theta^*) + A_1 \sin(2\theta^*) \cos\phi^* + A_2 \frac{1}{2} \sin^2\theta^* \cos(2\phi^*) \right. \\ \left. + A_3 \sin\theta^* \cos\phi^* + A_4 \cos\theta^* + A_5 \sin^2\theta^* \sin(2\phi^*) + A_6 \sin(2\theta^*) \sin\phi^* + A_7 \sin\theta^* \sin\phi^* \right].$$

Violation of Lam-Tung relation

$$A_0 \neq A_2^{12}$$

# Angular Distributions of Z Production

ATLAS, JHEP08, 159 (2016)



# Observations and Interpretations of DY Angular Distributions

- Strong  $q_T$  dependence and certain rapidity dependence of  $A_i$  ( $\lambda, \mu, \nu$ ).
- Lam-Tung Violation: ( $A_0 \neq A_2$ )
  - Low  $q_T$ :
    - Intrinsic partonic transverse momentum  $k_T$
    - Boer-Mulders functions (Boer 1999)
  - Large  $q_T$ :
    - Hard multi-gluon radiation ( $O(\alpha_s^2)$  or higher)
- Questions:
  - Is there a simple and intuitive understanding of the observed angular distributions?

# DY ANGULAR DISTRIBUTIONS WITHIN THE GEOMETRIC PICTURE

- **J.C. Peng, W.C. Chang, R.E. McClellan, O. Teryaev**  
Phys. Lett. B 758, 394 (2016)
- **W.C. Chang, R.E. McClellan, J.C. Peng, O. Teryaev**  
Phys. Rev. D 96, 054020 (2017)

# Interpretation of the CMS Z-production results

$$\begin{aligned}\frac{d\sigma}{d\Omega} \propto & (1 + \cos^2 \theta) + \frac{A_0}{2} (1 - 3 \cos^2 \theta) + A_1 \sin 2\theta \cos \phi \\ & + \frac{A_2}{2} \sin^2 \theta \cos 2\phi + A_3 \sin \theta \cos \phi + A_4 \cos \theta \\ & + A_5 \sin^2 \theta \sin 2\phi + A_6 \sin 2\theta \sin \phi + A_7 \sin \theta \sin \phi\end{aligned}$$

## Questions:

- How is the above expression derived?
- Can one express  $A_0 - A_7$  in terms of some quantities?
- Can one understand the  $q_T$  dependence of  $A_0, A_1, A_2$ , etc?
- Can one understand the origin of the violation of Lam-Tung relation?

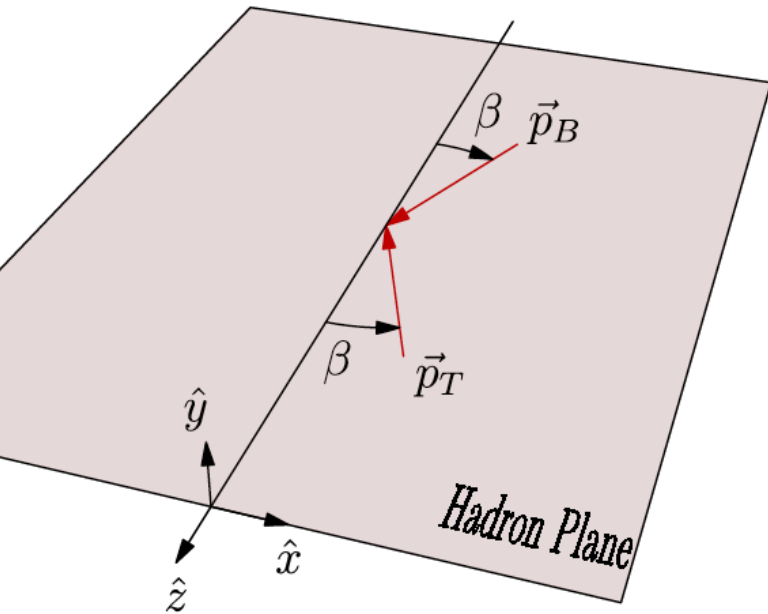


# How is the angular distribution expression derived?

## Define three planes in the Collins-Soper frame

1) Hadron Plane ( $\vec{P}_B \times \vec{P}_T$ )

- Contains the beam  $\vec{P}_B$  and target  $\vec{P}_T$  momenta
- Angle  $\beta$  satisfies the relation  $\tan \beta = q_T / Q$



Collins-Soper ( $\gamma^*$  /Z rest) Frame

# How is the angular distribution expression derived?

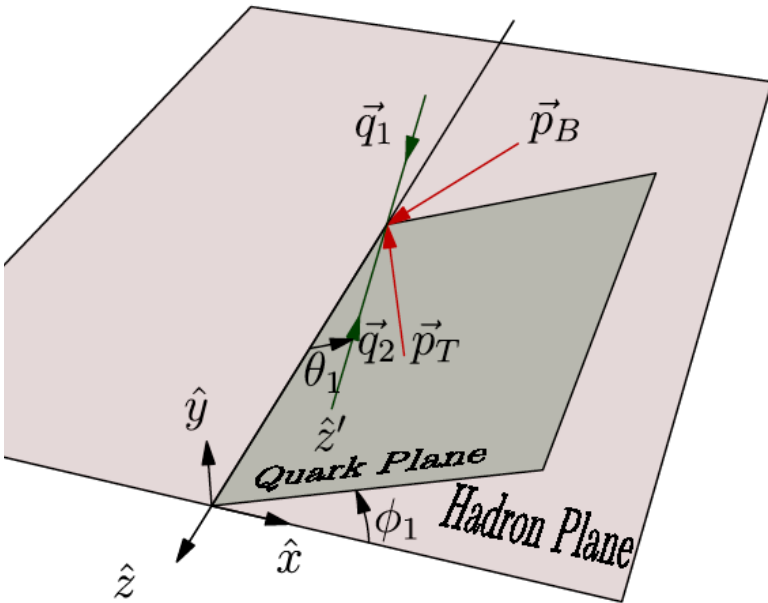
## Define three planes in the Collins-Soper frame

1) Hadron Plane ( $\vec{P}_B \times \vec{P}_T$ )

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2) Quark Plane ( $\hat{z}' \times \hat{z}$ )

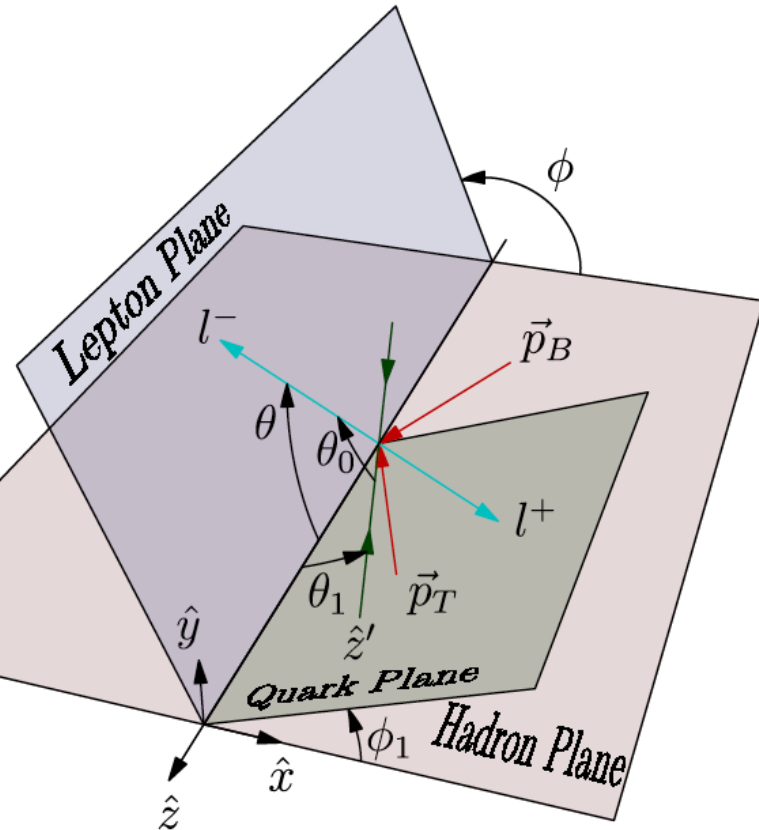
- $q$  and  $\bar{q}$  have head-on collision along the  $\hat{z}'$  axis
- $\hat{z}'$  axis has angles  $\theta_1$  and  $\varphi_1$  in the C-S frame



Collins-Soper ( $\gamma^*$  / Z rest) Frame

# How is the angular distribution expression derived?

## Define three planes in the Collins-Soper frame



1) Hadron Plane ( $\vec{P}_B \times \vec{P}_T$ )

- Contains the beam  $\vec{P}_B$  and target  $\vec{P}_T$  momenta
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2) Quark Plane ( $\hat{z}' \times \hat{z}$ )

- $q$  and  $\bar{q}$  have head-on collision along the  $\hat{z}'$  axis
- $\hat{z}'$  axis has angles  $\theta_1$  and  $\phi_1$  in the C-S frame

3) Lepton Plane ( $\vec{l}^- \times \hat{z}$ )

- $l^-$  and  $l^+$  are emitted back-to-back with equal  $|\vec{P}|$
- $l^-$  is emitted at angle  $\theta$  and  $\phi$  in the C-S frame

Collins-Soper ( $\gamma^*$  /Z rest) Frame

# How is the angular distribution expression derived?

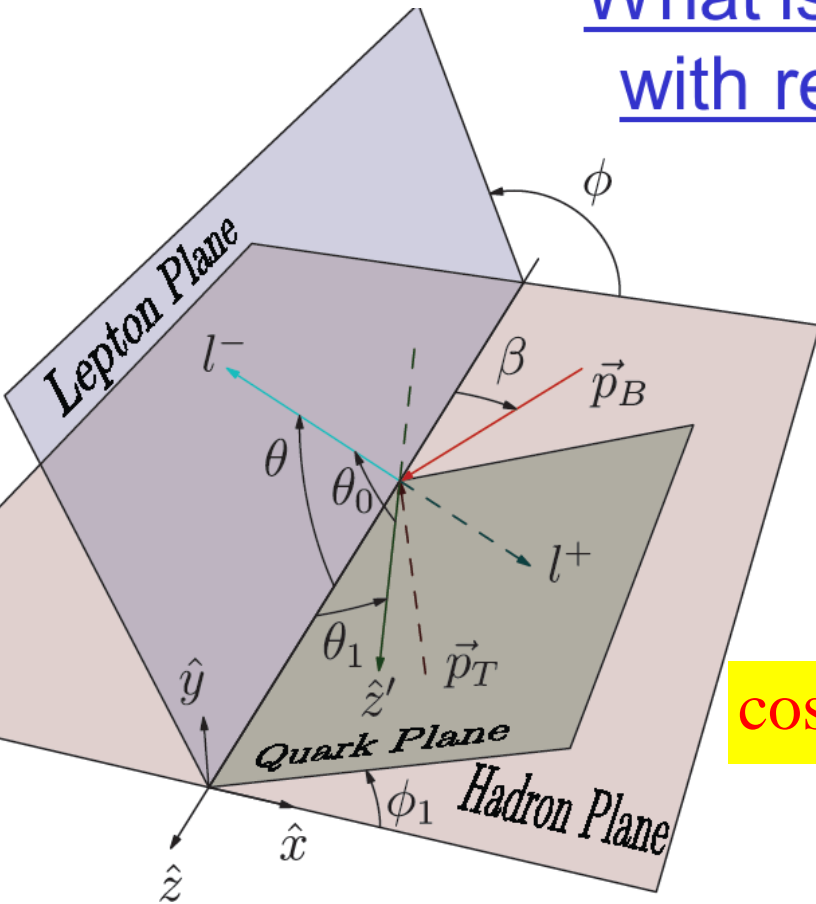
What is the lepton angular distribution with respect to the  $\hat{z}'$  (natural) axis?

$$\frac{d\sigma}{d\Omega} \propto 1 + a \cos \theta_0 + \cos^2 \theta_0$$

$a$ : forward-backward asymmetry coefficient

How to express the angular distribution in terms of  $\theta$  and  $\phi$ ?

$$\cos \theta_0 = \cos \theta \cos \theta_1 + \sin \theta \sin \theta_1 \cos(\phi - \phi_1)$$



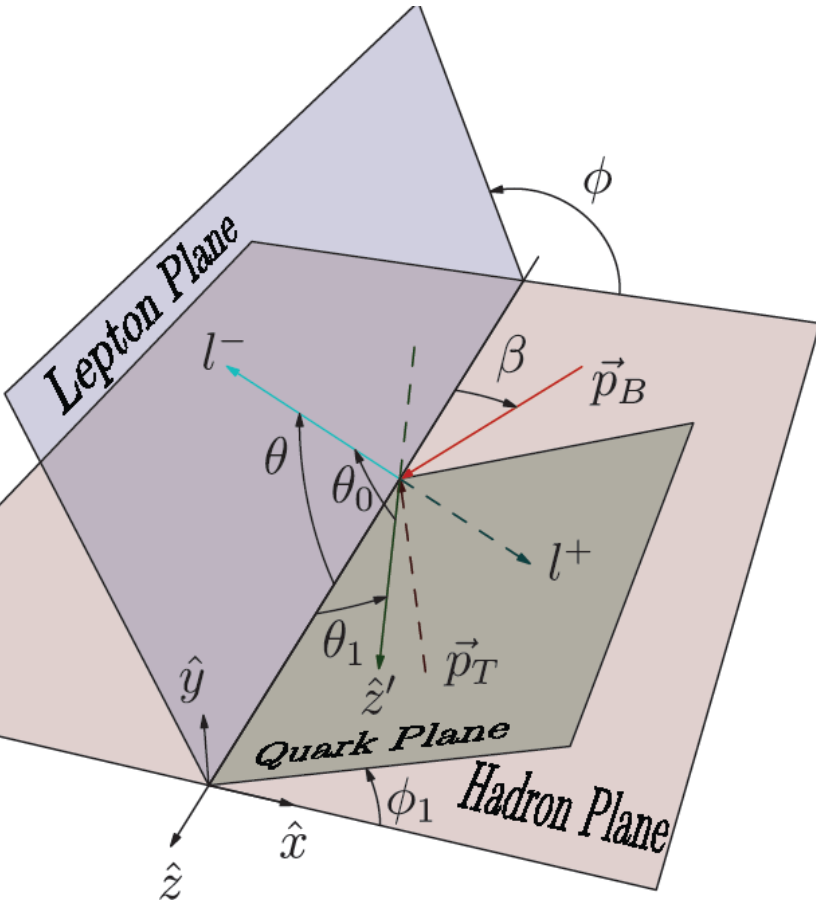
# How is the angular distribution expression derived?

$$\begin{aligned}\frac{d\sigma}{d\Omega} \propto & (1 + \cos^2 \theta) + \frac{\sin^2 \theta_1}{2} (1 - 3 \cos^2 \theta) \\ & + \left(\frac{1}{2} \sin 2\theta_1 \cos \phi_1\right) \sin 2\theta \cos \phi \\ & + \left(\frac{1}{2} \sin^2 \theta_1 \cos 2\phi_1\right) \sin^2 \theta \cos 2\phi \\ & + (a \sin \theta_1 \cos \phi_1) \sin \theta \cos \phi + (a \cos \theta_1) \cos \theta \\ & + \left(\frac{1}{2} \sin^2 \theta_1 \sin 2\phi_1\right) \sin^2 \theta \sin 2\phi \\ & + \left(\frac{1}{2} \sin 2\theta_1 \sin \phi_1\right) \sin 2\theta \sin \phi \\ & + (a \sin \theta_1 \sin \phi_1) \sin \theta \sin \phi.\end{aligned}$$

$$\begin{aligned}\frac{d\sigma}{d\Omega} \propto & (1 + \cos^2 \theta) + \frac{A_0}{2} (1 - 3 \cos^2 \theta) \\ & + A_1 \sin 2\theta \cos \phi \\ & + \frac{A_2}{2} \sin^2 \theta \cos 2\phi \\ & + A_3 \sin \theta \cos \phi + A_4 \cos \theta \\ & + A_5 \sin^2 \theta \sin 2\phi \\ & + A_6 \sin 2\theta \sin \phi \\ & + A_7 \sin \theta \sin \phi\end{aligned}$$

$A_0 - A_7$  are entirely described by  $\theta_1$ ,  $\phi_1$  and  $a$ .

# Angular distribution coefficients $A_0 - A_7$



$$A_0 = \langle \sin^2 \theta_1 \rangle$$

$$A_1 = \frac{1}{2} \langle \sin 2\theta_1 \cos \phi_1 \rangle$$

$$A_2 = \langle \sin^2 \theta_1 \cos 2\phi_1 \rangle$$

$$A_3 = a \langle \sin \theta_1 \cos \phi_1 \rangle$$

$$A_4 = a \langle \cos \theta_1 \rangle$$

$$A_5 = \frac{1}{2} \langle \sin^2 \theta_1 \sin 2\phi_1 \rangle$$

$$A_6 = \frac{1}{2} \langle \sin 2\theta_1 \sin \phi_1 \rangle$$

$$A_7 = a \langle \sin \theta_1 \sin \phi_1 \rangle$$

# Some implications of the angular distribution coefficients $A_0 - A_7$

$$A_0 = \langle \sin^2 \theta_1 \rangle$$

$$A_1 = \frac{1}{2} \langle \sin 2\theta_1 \cos \phi_1 \rangle$$

$$A_2 = \langle \sin^2 \theta_1 \cos 2\phi_1 \rangle$$

$$A_3 = a \langle \sin \theta_1 \cos \phi_1 \rangle$$

$$A_4 = a \langle \cos \theta_1 \rangle$$

$$A_5 = \frac{1}{2} \langle \sin^2 \theta_1 \sin 2\phi_1 \rangle$$

$$A_6 = \frac{1}{2} \langle \sin 2\theta_1 \sin \phi_1 \rangle$$

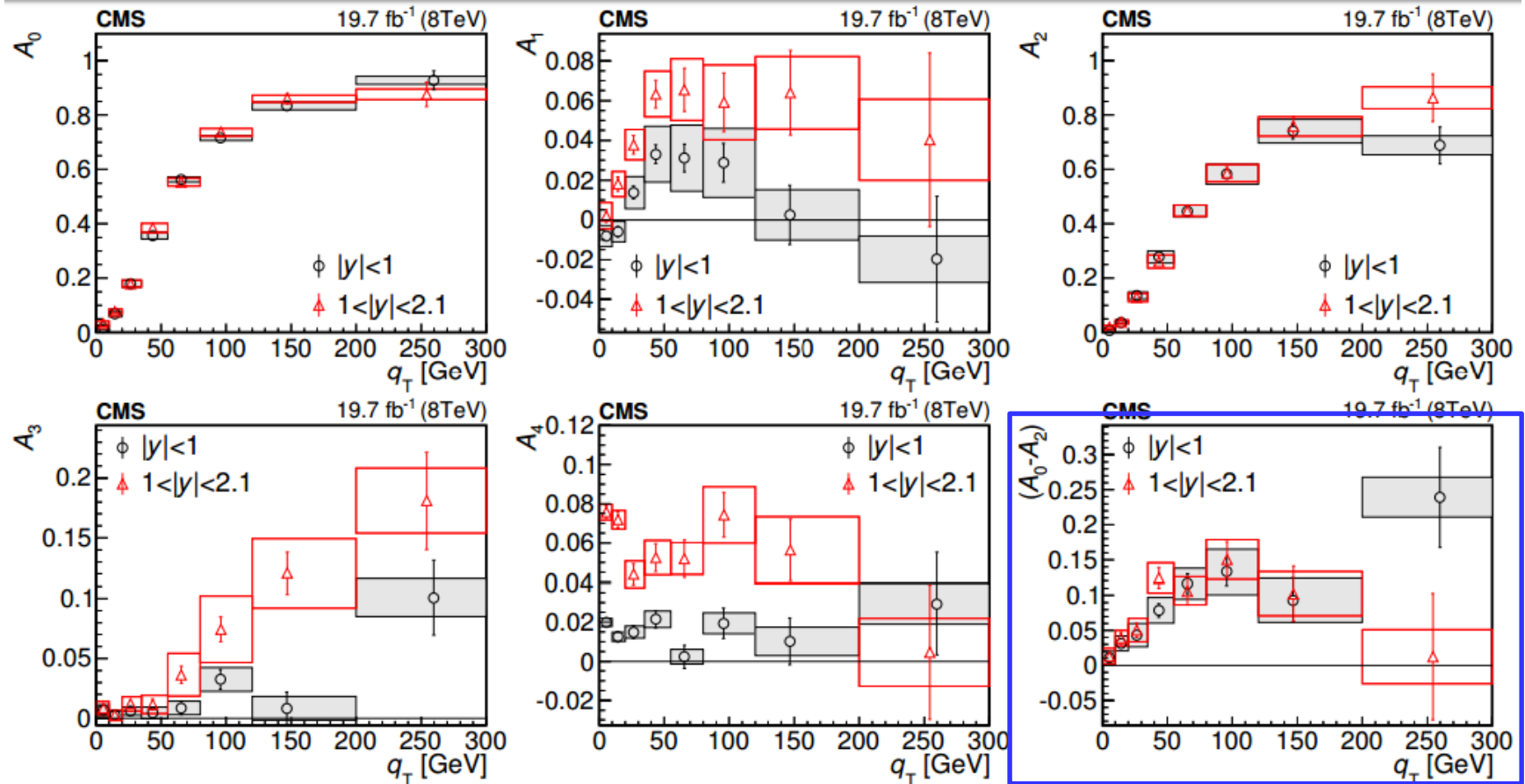
$$A_7 = a \langle \sin \theta_1 \sin \phi_1 \rangle$$

$$\tan \beta = q_T / Q$$

- $A_0 \geq A_2$  (or  $1 - \lambda - 2\nu \geq 0$ )
- Lam-Tung relation ( $A_0 = A_2$ ) is satisfied when  $\phi_1 = 0$
- Forward-backward asymmetry,  $a$ , is reduced by a factor of  $\langle \cos \theta_1 \rangle$  for  $A_4$
- $A_5, A_6, A_7$  are odd function of  $\phi_1$  and must vanish from symmetry consideration.
- $A_0, A_2$  and  $A_3$  increase with  $q_T$  monotonically while  $A_4$  decreases with  $q_T$  monotonically.
- $A_1$  ( $\sim \langle \sin 2\theta \rangle$ ) first increase with  $q_T$ , reaching a maximum, and then decrease.
- Some equality and inequality relations among  $A_0 - A_7$  can be obtained

# Angular Distributions of Z Production

CMS, PLB750, 154 (2015)



$$\frac{d^2\sigma}{d\cos\theta^* d\phi^*} \propto \left[ (1 + \cos^2\theta^*) + A_0 \frac{1}{2} (1 - 3\cos^2\theta^*) + A_1 \sin(2\theta^*) \cos\phi^* + A_2 \frac{1}{2} \sin^2\theta^* \cos(2\phi^*) \right. \\ \left. + A_3 \sin\theta^* \cos\phi^* + A_4 \cos\theta^* + A_5 \sin^2\theta^* \sin(2\phi^*) + A_6 \sin(2\theta^*) \sin\phi^* + A_7 \sin\theta^* \sin\phi^* \right].$$

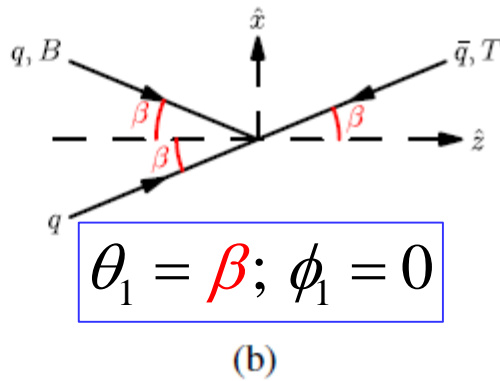
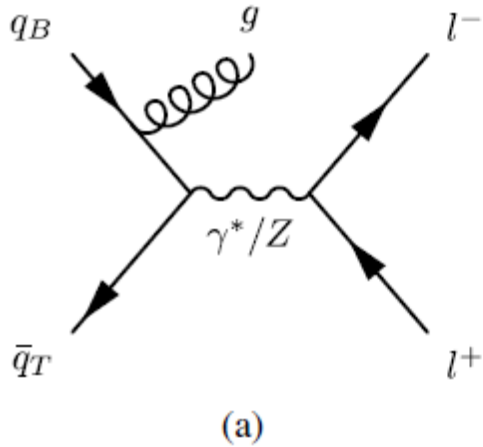
Violation of Lam-Tung relation

$$A_0 \neq A_2^{24}$$



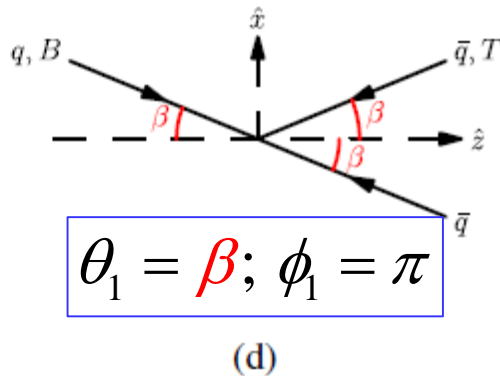
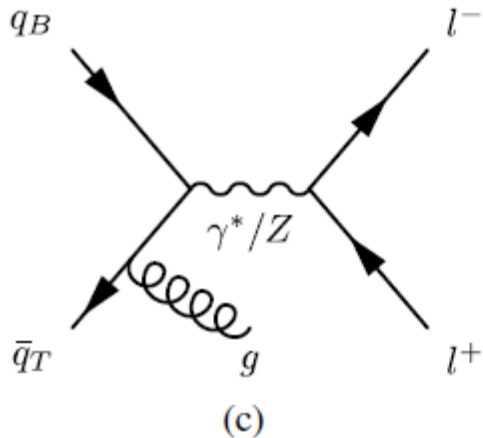
# $\theta_1$ and $\phi_1$ at $O(\alpha_s^1)$ : $q\bar{q} \rightarrow \gamma^*/Zg$

Collins-Soper ( $\gamma^*/Z$  rest) Frame



$$\theta_1 = \beta; \phi_1 = 0$$

$$A_0^{q\bar{q}} = \frac{q_T^2}{Q^2 + q_T^2} \text{ (Collins 1979)}$$



$$\theta_1 = \beta; \phi_1 = \pi$$

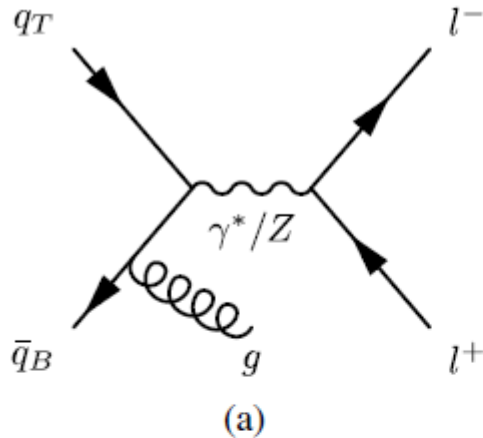
$$A_0 = A_2 = \langle \sin^2 \theta_1 \rangle = \frac{q_T^2}{Q^2 + q_T^2} > 0$$

$$\lambda = \frac{2 - 3A_0}{2 + A_0} = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2}$$

$$\nu = \frac{2A_2}{2 + A_0} = \frac{2q_T^2}{2Q^2 + 3q_T^2}$$

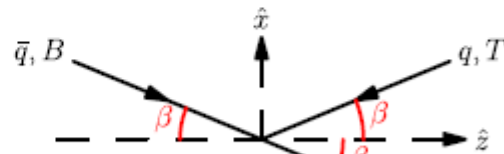
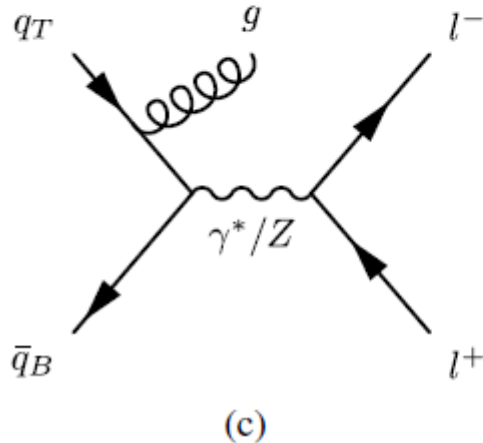
# $\theta_1$ and $\phi_1$ at $O(\alpha_s^1)$ : $q\bar{q} \rightarrow \gamma^*/Zg$

Collins-Soper ( $\gamma^*/Z$  rest) Frame



$$\theta_1 = \pi - \beta; \phi_1 = 0$$

$$A_0^{q\bar{q}} = \frac{q_T^2}{Q^2 + q_T^2} \text{ (Collins 1979)}$$



$$\theta_1 = \pi - \beta; \phi_1 = \pi$$

$$A_0 = A_2 = \langle \sin^2 \theta_1 \rangle = \frac{q_T^2}{Q^2 + q_T^2} > 0$$

$$\lambda = \frac{2 - 3A_0}{2 + A_0} = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2}$$

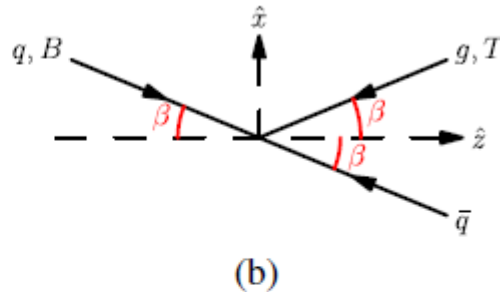
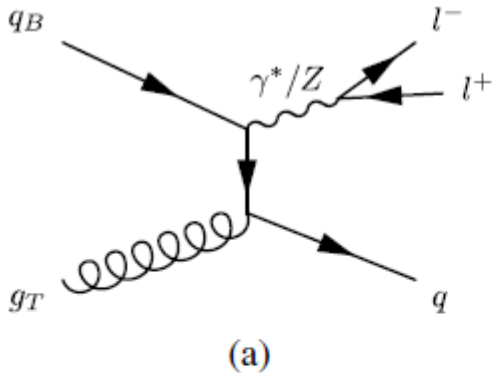
$$\nu = \frac{2A_2}{2 + A_0} = \frac{2q_T^2}{2Q^2 + 3q_T^2}$$

# $\theta_1$ and $\phi_1$ at $O(\alpha_s^1)$ : $qg \rightarrow \gamma^*/Zq$

Collins-Soper ( $\gamma^*/Z$  rest) Frame

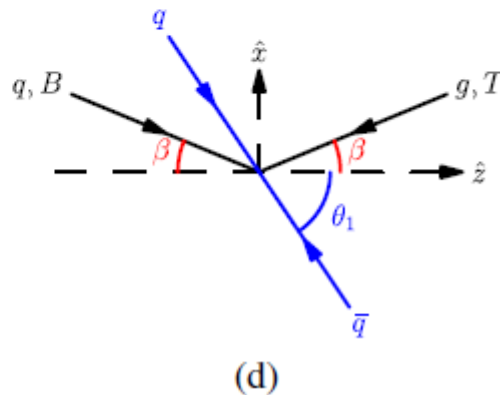
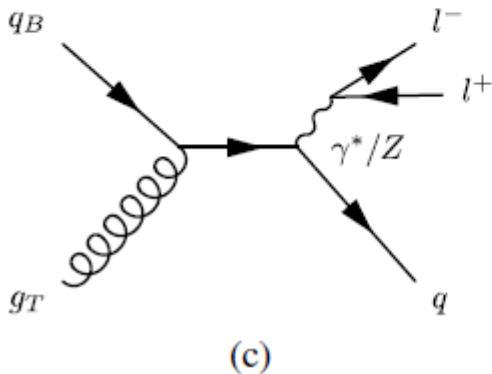
$$\theta_1 = \beta; \phi_1 = 0$$

$$A_0^{qg} = \frac{5q_T^2}{Q^2 + 5q_T^2} \text{ (Thews 1979)}$$



$$A_0 = A_2 = \langle \sin^2 \theta_1 \rangle \approx \frac{5q_T^2}{Q^2 + 5q_T^2} > 0$$

$$\lambda = \frac{2 - 3A_0}{2 + A_0} = \frac{2Q^2 - 5q_T^2}{2Q^2 + 15q_T^2}$$

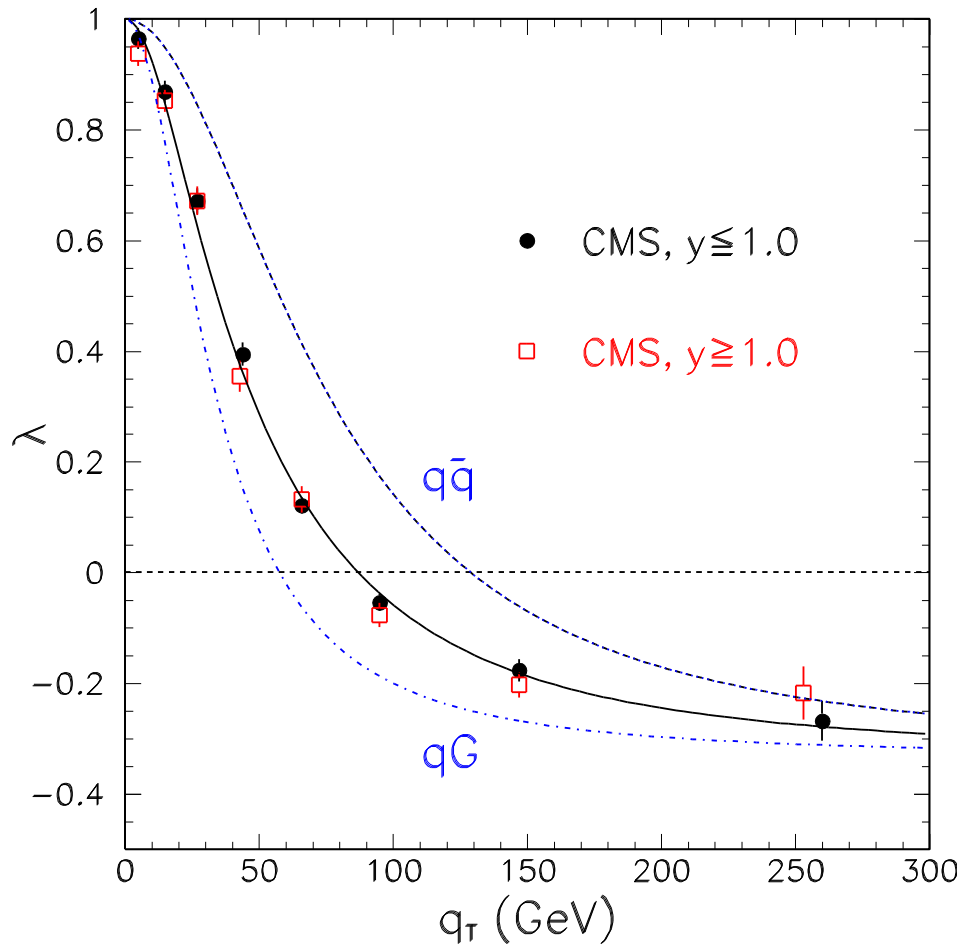


$$\theta_1 > \beta; \phi_1 = 0$$

$$v = \frac{2A_2}{2 + A_0} = \frac{10q_T^2}{2Q^2 + 15q_T^2}$$

# Compare with CMS data on $\lambda$

(Z production in  $p+p$  collision at 8 TeV)



$$\lambda = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2} \quad \text{for } q\bar{q} \rightarrow ZG$$

$$\lambda = \frac{2Q^2 - 5q_T^2}{2Q^2 + 15q_T^2} \quad \text{for } qG \rightarrow Zq$$

For both processes

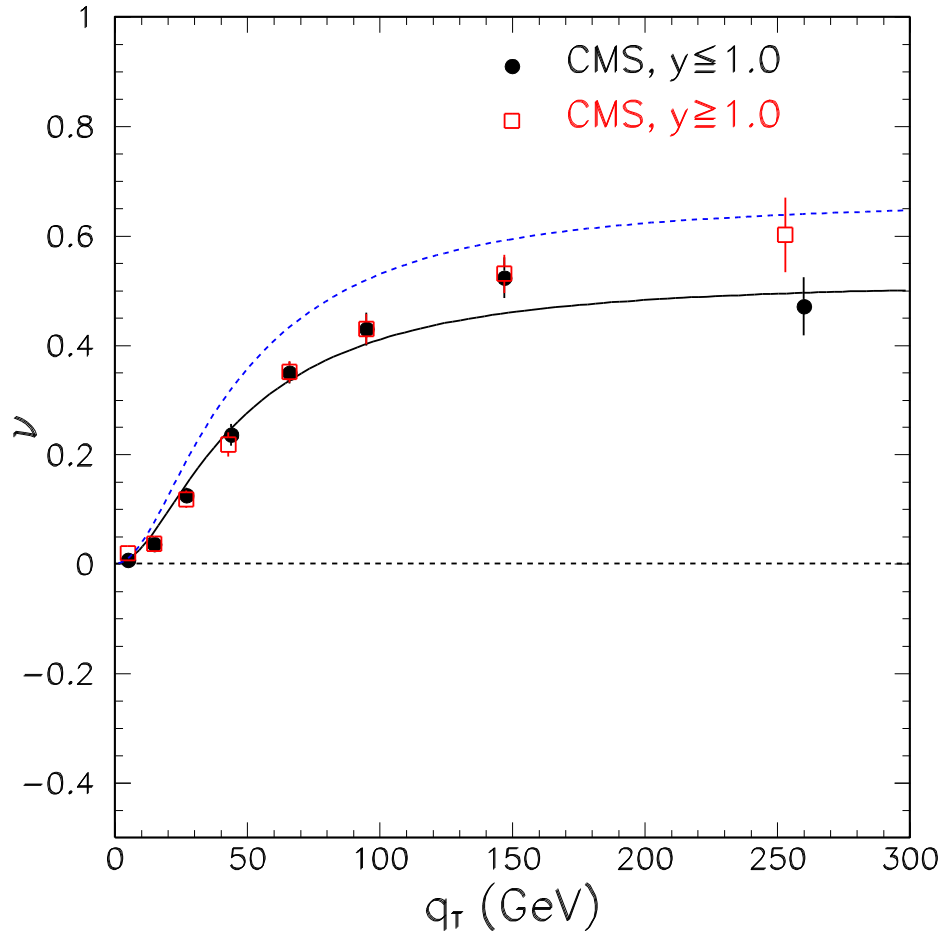
$$\lambda = 1 \text{ at } q_T = 0 \quad (\theta_1 = 0^\circ)$$

$$\lambda = -1/3 \text{ at } q_T = \infty \quad (\theta_1 = 90^\circ)$$

Data can be well described with a mixture of 58.5%  $qG$  and 41.5%  $q\bar{q}$  processes

# Compare with CMS data on $\nu$

( $Z$  production in  $p+p$  collision at 8 TeV)



$$\nu = \frac{2q_T^2}{2Q^2 + 3q_T^2} \quad \text{for } q\bar{q} \rightarrow ZG$$

$$\nu = \frac{10q_T^2}{2Q^2 + 15q_T^2} \quad \text{for } qG \rightarrow Zq$$

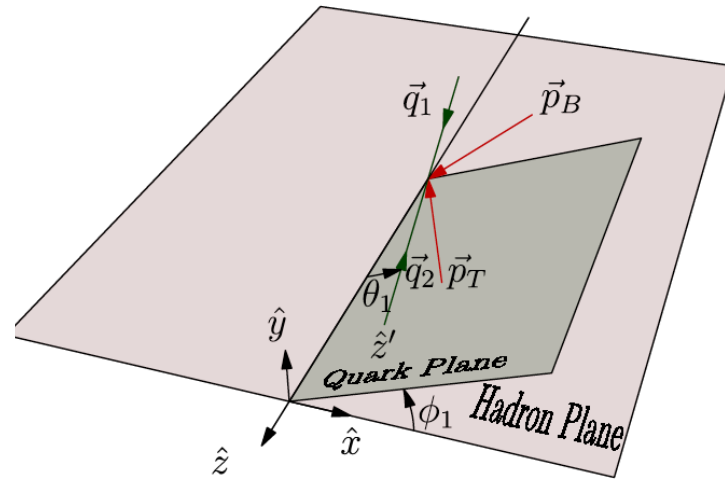
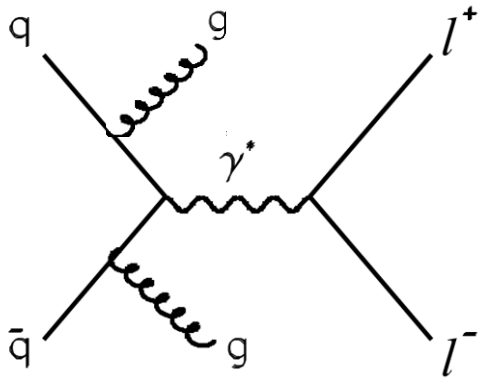
Dashed curve corresponds to a mixture of 58.5%  $qG$  and 41.5%  $q\bar{q}$  processes

Solid curve corresponds to  $\langle \sin^2 \theta_1 \cos 2\phi_1 \rangle / \langle \sin^2 \theta_1 \rangle = 0.77$

$q - \bar{q}$  axis is non-coplanar relative to the hadron plane

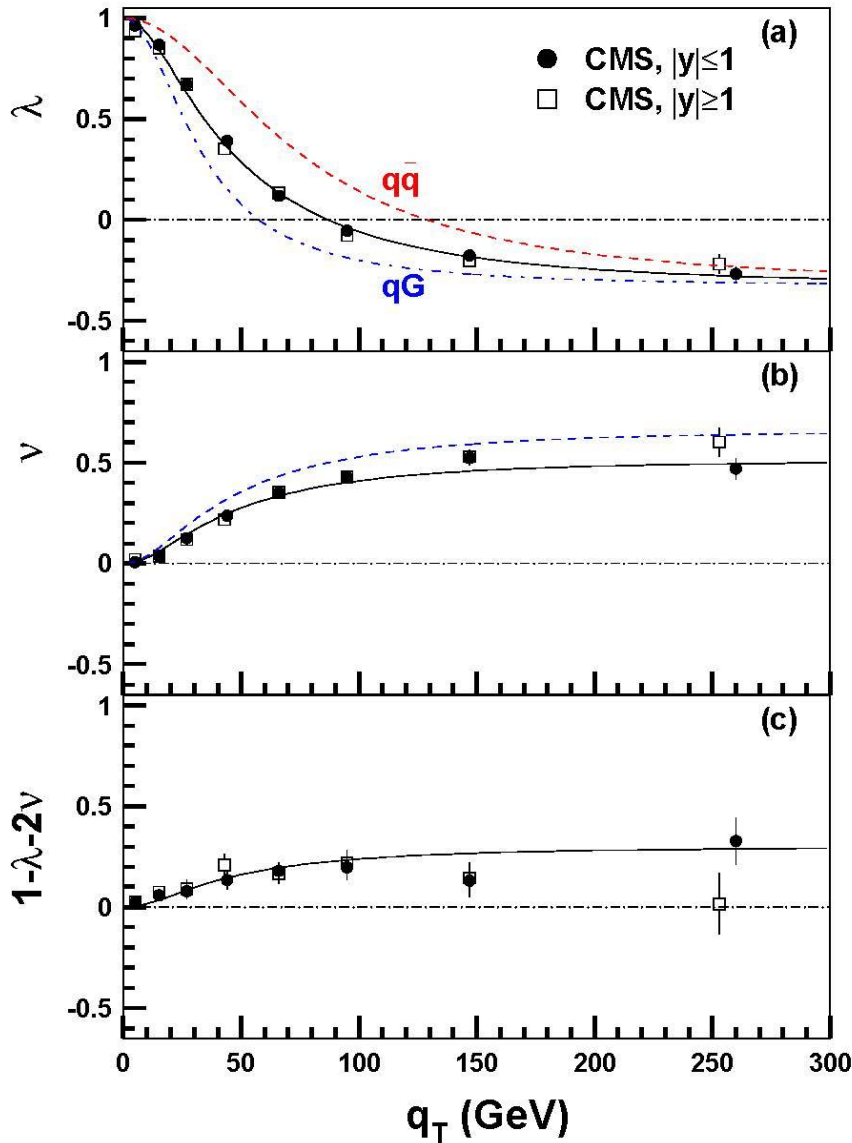
# Origins of the non-coplanarity

1) Processes at order  $\alpha_s^2$  or higher



2) Intrinsic  $k_T$  from interacting partons

# Compare with CMS data on Lam-Tung relation



Solid curves correspond to a mixture of 58.5%  $qG$  and 41.5%  $q\bar{q}$  processes, and

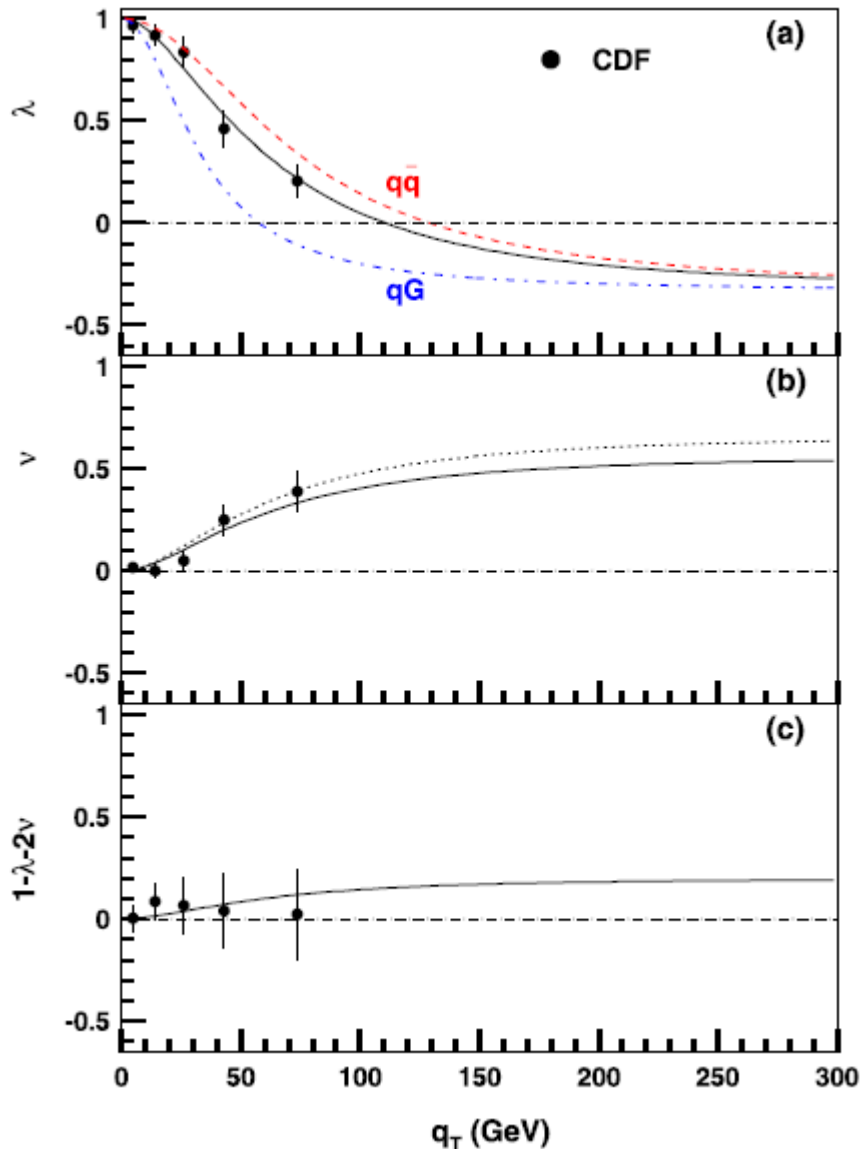
$$\langle \sin^2 \theta_1 \cos 2\phi_1 \rangle / \langle \sin^2 \theta_1 \rangle = 0.77$$

Violation of Lam-Tung relation is well described

J.C. Peng, W.C. Chang, R.E. McClellan, O. Teryaev  
 Phys. Lett. B758, 394 (2016)

# Compare with CDF data

(Z production in  $p + \bar{p}$  collision at 1.96 TeV)



Solid curves correspond to a mixture of 27.5%  $qG$  and 72.5%  $qq$  processes, and  $\langle \sin^2 \theta_1 \cos 2\phi_1 \rangle / \langle \sin^2 \theta_1 \rangle = 0.85$

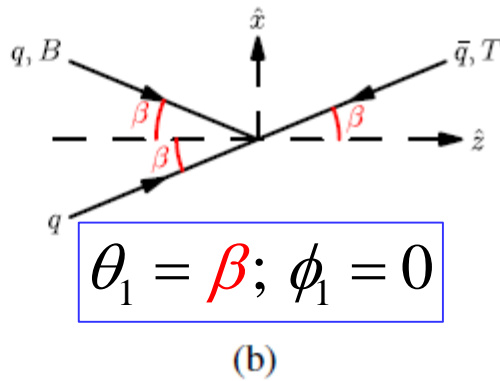
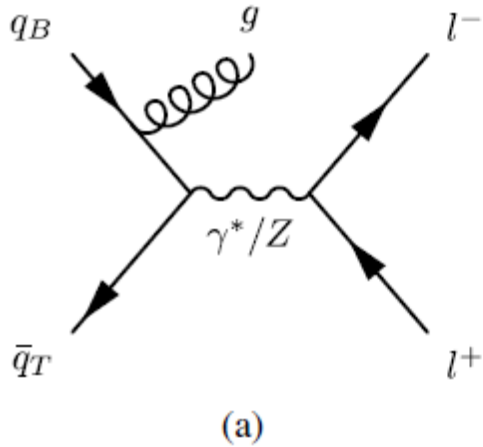
Violation of Lam-Tung relation is not ruled out

J.C. Peng, W.C. Chang, R.E. McClellan, O. Teryaev  
Phys. Lett. B758, 394 (2016)



# $\theta_1$ and $\phi_1$ at $O(\alpha_s^1)$ : $q\bar{q} \rightarrow \gamma^*/Zg$

Collins-Soper ( $\gamma^*/Z$  rest) Frame

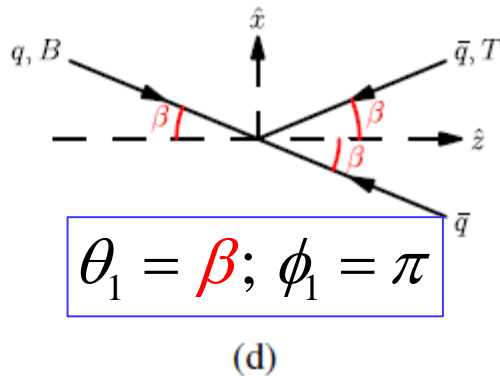
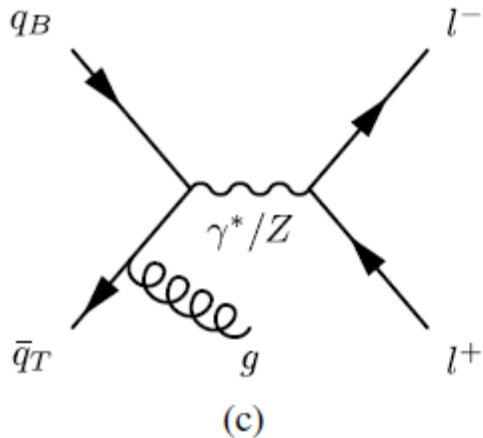


$$\theta_1 = \beta; \phi_1 = 0$$

$$A_1 = \frac{1}{2} \langle \sin 2\theta_1 \cos \phi_1 \rangle = \frac{q_T Q}{Q^2 + q_T^2} > 0$$

$$A_3 = \langle a \sin \theta_1 \cos \phi_1 \rangle = a \frac{q_T}{\sqrt{Q^2 + q_T^2}} > 0$$

$$A_4 = \langle a \cos \theta_1 \rangle = a \frac{Q}{\sqrt{Q^2 + q_T^2}} > 0$$



$$\theta_1 = \beta; \phi_1 = \pi$$

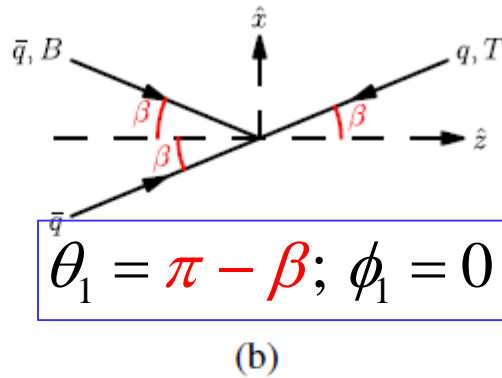
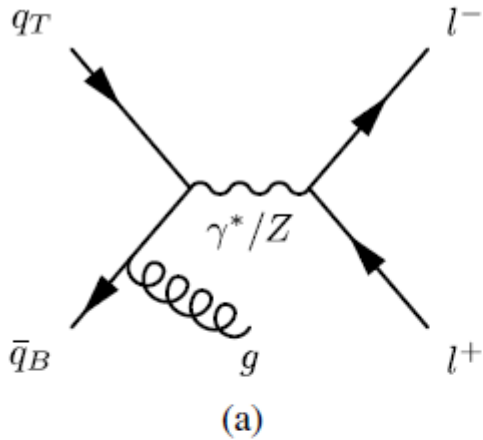
$$A_1 = \frac{1}{2} \langle \sin 2\theta_1 \cos \phi_1 \rangle = \frac{q_T Q}{Q^2 + q_T^2} < 0$$

$$A_3 = \langle a \sin \theta_1 \cos \phi_1 \rangle = a \frac{q_T}{\sqrt{Q^2 + q_T^2}} < 0$$

$$A_4 = \langle a \cos \theta_1 \rangle = a \frac{Q}{\sqrt{Q^2 + q_T^2}} > 0$$

# $\theta_1$ and $\phi_1$ at $O(\alpha_s^1)$ : $q\bar{q} \rightarrow \gamma^*/Zg$

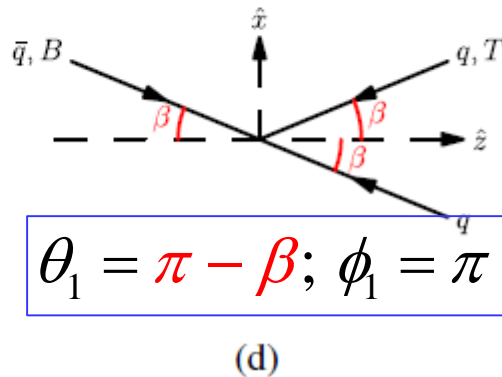
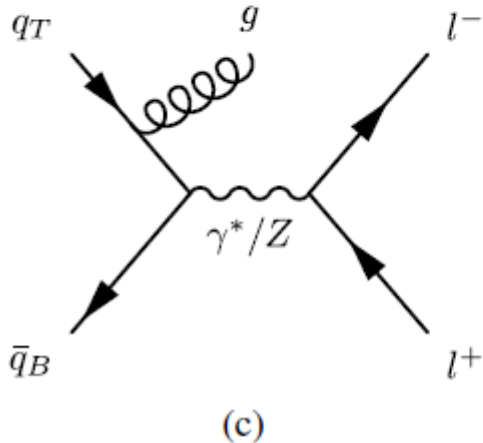
Collins-Soper ( $\gamma^*/Z$  rest) Frame



$$A_1 = \frac{1}{2} \langle \sin 2\theta_1 \cos \phi_1 \rangle = \frac{q_T Q}{Q^2 + q_T^2} < 0$$

$$A_3 = \langle a \sin \theta_1 \cos \phi_1 \rangle = a \frac{q_T}{\sqrt{Q^2 + q_T^2}} > 0$$

$$A_4 = \langle a \cos \theta_1 \rangle = a \frac{Q}{\sqrt{Q^2 + q_T^2}} < 0$$



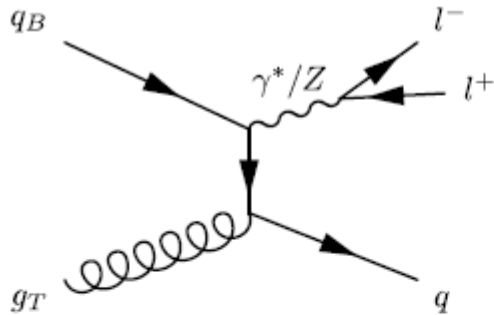
$$A_1 = \frac{1}{2} \langle \sin 2\theta_1 \cos \phi_1 \rangle = \frac{q_T Q}{Q^2 + q_T^2} > 0$$

$$A_3 = \langle a \sin \theta_1 \cos \phi_1 \rangle = a \frac{q_T}{\sqrt{Q^2 + q_T^2}} < 0$$

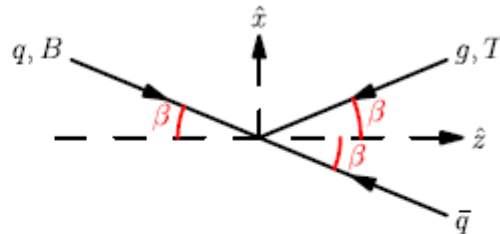
$$A_4 = \langle a \cos \theta_1 \rangle = a \frac{Q}{\sqrt{Q^2 + q_T^2}} < 0$$

# $\theta_1$ and $\phi_1$ at $O(\alpha_s^1)$ : $qg \rightarrow \gamma^*/Zq$

Collins-Soper ( $\gamma^*/Z$  rest) Frame

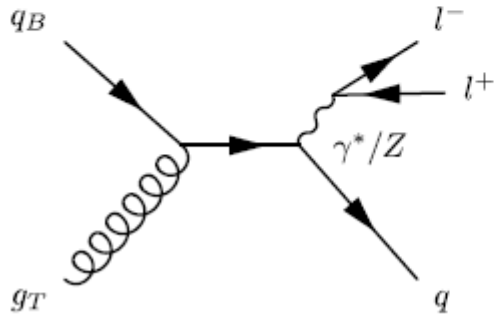


(a)

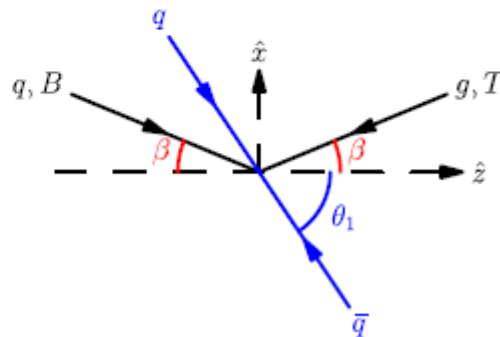


(b)

$$\theta_1 = \beta; \phi_1 = 0$$



(c)



(d)

$$\theta_1 > \beta; \phi_1 = 0$$

$$A_1 = \frac{1}{2} \langle \sin 2\theta_1 \cos \phi_1 \rangle = \frac{\sqrt{5}q_T Q}{Q^2 + 5q_T^2} > 0$$

$$A_3 = \langle a \sin \theta_1 \cos \phi_1 \rangle = a \frac{\sqrt{5}q_T}{\sqrt{Q^2 + 5q_T^2}} > 0$$

$$A_4 = \langle a \cos \theta_1 \rangle = a \frac{Q}{\sqrt{Q^2 + 5q_T^2}} > 0$$

$$A_1 = \frac{1}{2} \langle \sin 2\theta_1 \cos \phi_1 \rangle = \frac{\sqrt{5}q_T Q}{Q^2 + 5q_T^2} > 0$$

$$A_3 = \langle a \sin \theta_1 \cos \phi_1 \rangle = a \frac{\sqrt{5}q_T}{\sqrt{Q^2 + 5q_T^2}} > 0$$

$$A_4 = \langle a \cos \theta_1 \rangle = a \frac{Q}{\sqrt{Q^2 + 5q_T^2}} > 0$$

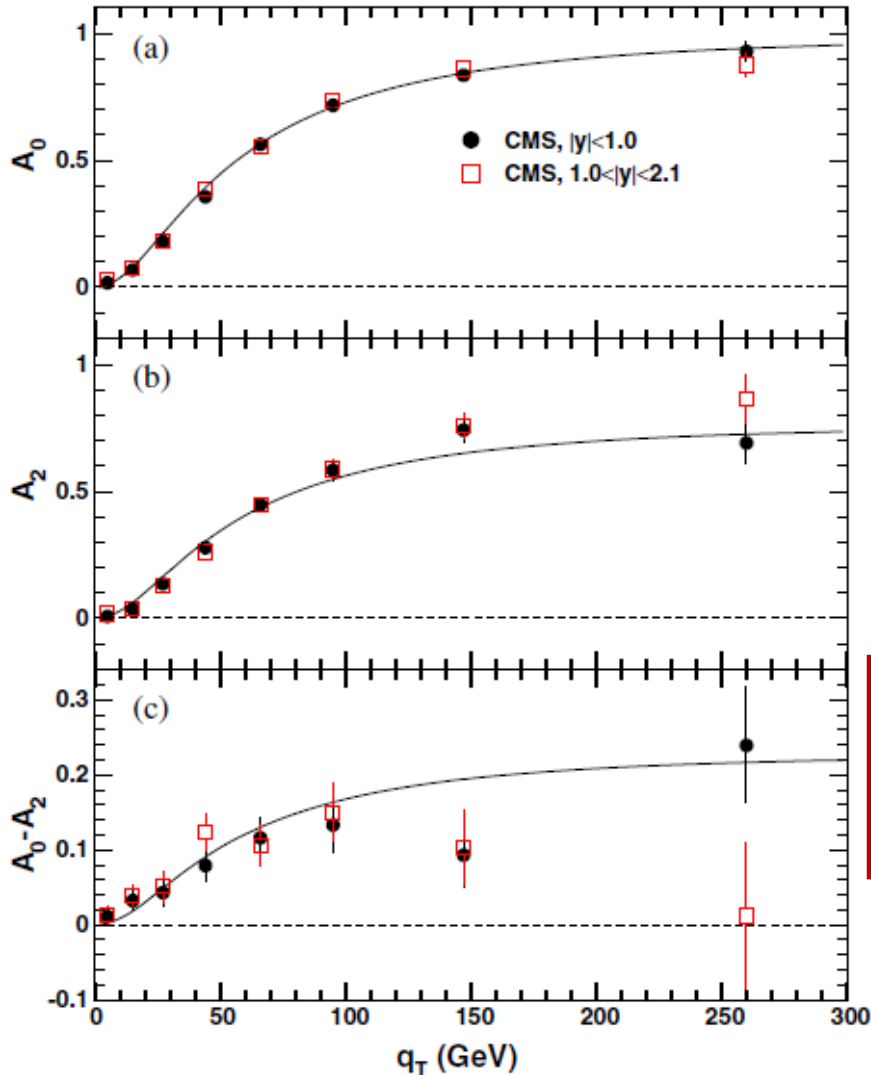
# Rapidity Dependence of $A_i$

TABLE I. Angles  $\theta_1$  and  $\phi_1$  for four cases of gluon emission in the  $q - \bar{q}$  annihilation process at order- $\alpha_s$ . The signs of  $A_0$  to  $A_4$  for the four cases are also listed.

Case	Gluon emitted from	$\theta_1$	$\phi_1$	$A_0$	$A_1$	$A_2$	$A_3$	$A_4$
1	Beam quark	$\beta$	0	+	+	+	+	+
2	Target antiquark	$\beta$	$\pi$	+	-	+	-	+
3	Beam antiquark	$\pi - \beta$	0	+	-	+	+	-
4	Target quark	$\pi - \beta$	$\pi$	+	+	+	-	-

A cancelation effect leads to a strong rapidity ( $y$ ) dependence of  $A_1$ ,  $A_3$  and  $A_4$ .

# Compare with CMS data on Lam-Tung relation

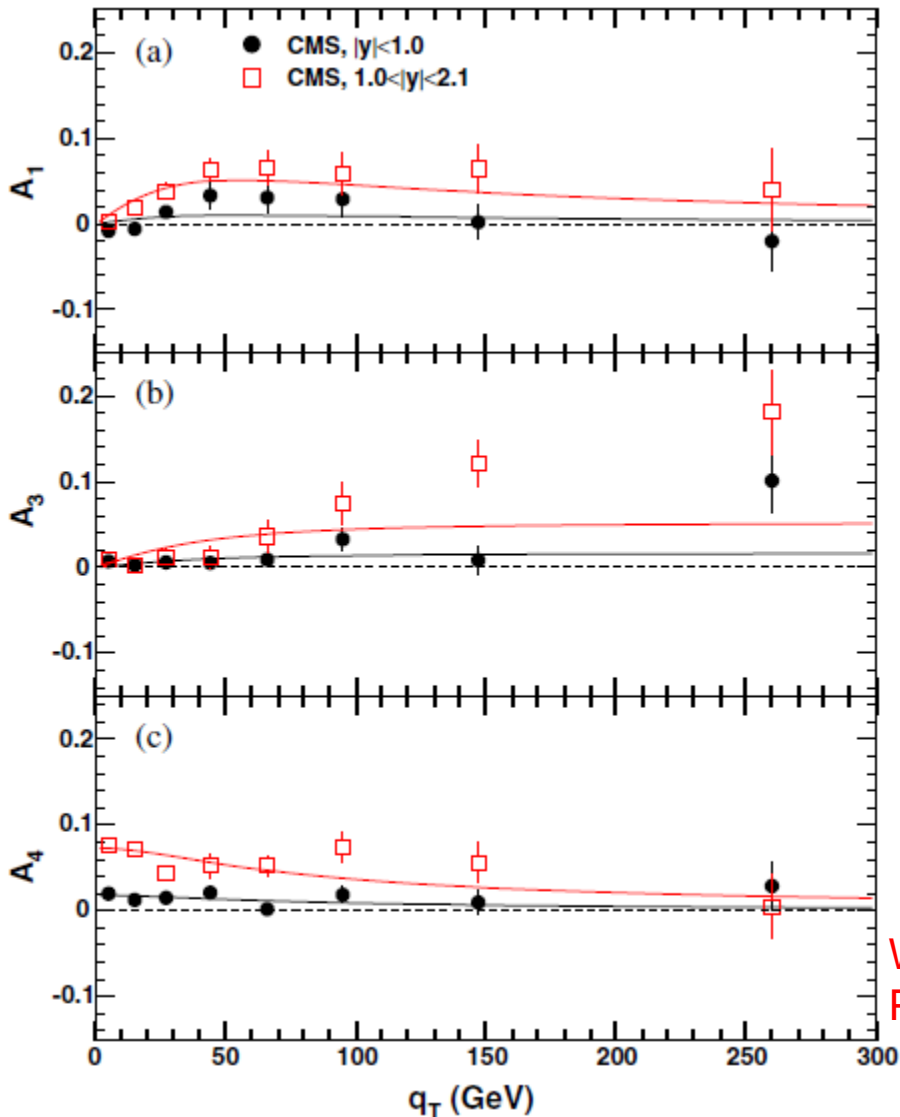


$$A_{0,2} = f \frac{q_T^2}{Q^2 + q_T^2} + (1-f) \frac{5q_T^2}{Q^2 + 5q_T^2}$$
$$f = 0.415$$

Weak Rapidity  
dependence of  $A_0$  and  $A_2$ .

W.C. Chang, R.E. McClellan, J.C. Peng, O. Teryaev  
Phys. Rev. D 96, 054020 (2017)

# Compare with CMS data on Lam-Tung relation



$$A_1 = r_1 \left[ f \frac{q_T Q}{Q^2 + q_T^2} + (1-f) \frac{\sqrt{5} q_T Q}{Q^2 + 5q_T^2} \right]$$

$$A_3 = r_3 \left[ f \frac{q_T}{\sqrt{Q^2 + q_T^2}} + (1-f) \frac{\sqrt{5} q_T}{\sqrt{Q^2 + 5q_T^2}} \right]$$

$$A_4 = r_4 \left[ f \frac{Q}{\sqrt{Q^2 + q_T^2}} + (1-f) \frac{Q}{\sqrt{Q^2 + 5q_T^2}} \right]$$

Rapidity of  $A_1$ ,  $A_3$  and  $A_4$   
are well described

W.C. Chang, R.E. McClellan, J.C. Peng, O. Teryaev  
Phys. Rev. D 96, 054020 (2017)

# Observations and Interpretations of DY Angular Distributions

- Strong  $q_T$  dependence and certain rapidity dependence of  $A_i$  ( $\lambda, \mu, \nu$ ).
- Lam-Tung Violation: ( $A_0 \neq A_2$ )
  - Low  $q_T$ :
    - Intrinsic partonic transverse momentum  $k_T$
    - Boer-Mulders functions (D. Boer, PRD 60 (1999) 014012)
  - Large  $q_T$ :
    - Hard multi-gluon radiation ( $O(\alpha_s^2)$  or higher)
- Questions:
  - How well does pQCD describe the data? Are they consistent with the geometric picture?

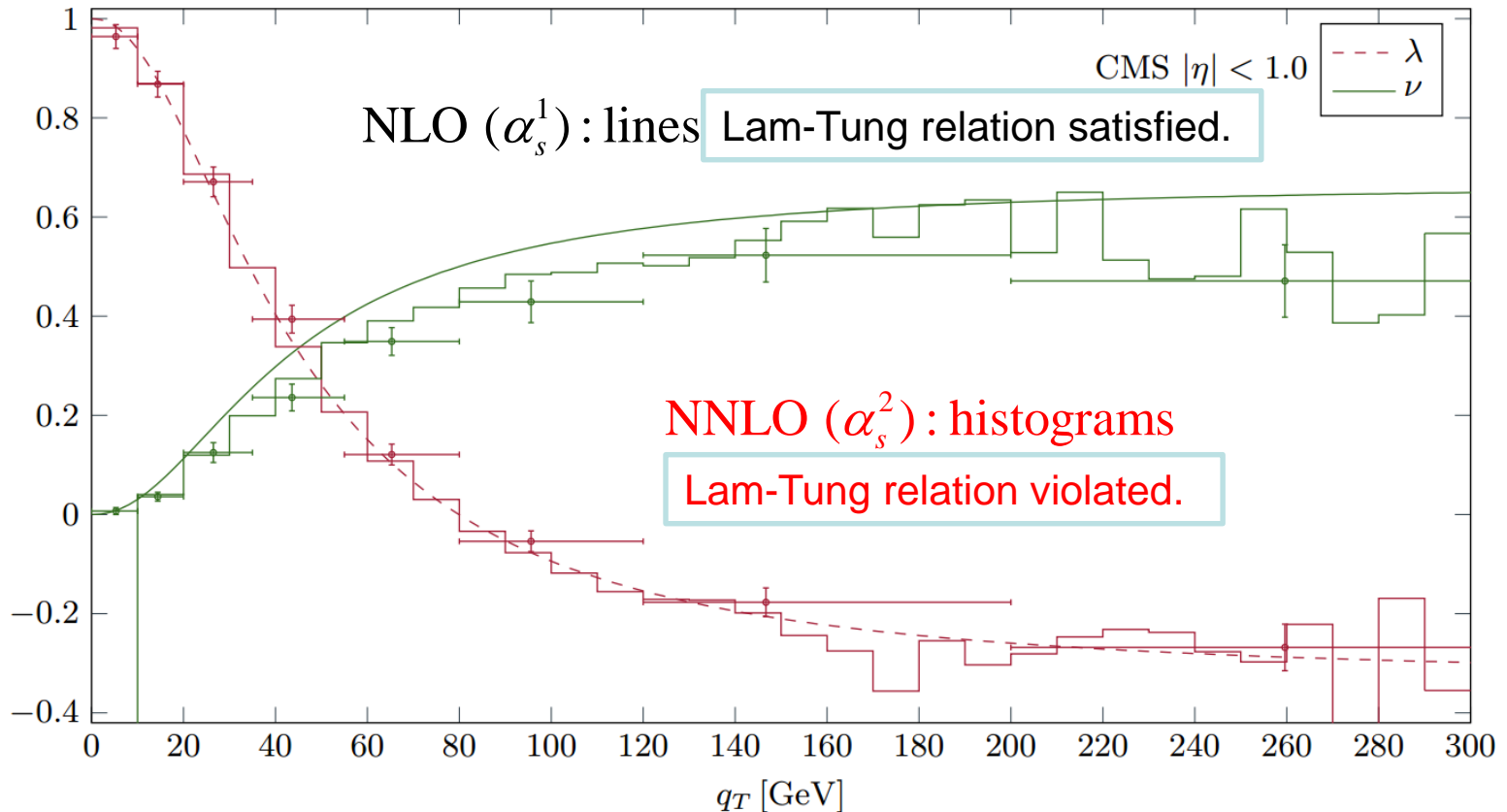
Priliminary

# **FIXED-ORDER pQCD (NLO, NNLO FROM DYNNLO)**



# pQCD NLO and NNLO Calculations

(M. Lambertsen and W. Vogelsang, Phys. Rev. D 93, 114013 (2016))



- NNLO pQCD calculation can describe the violation of Lam-Tung relation.

# DYNNLO

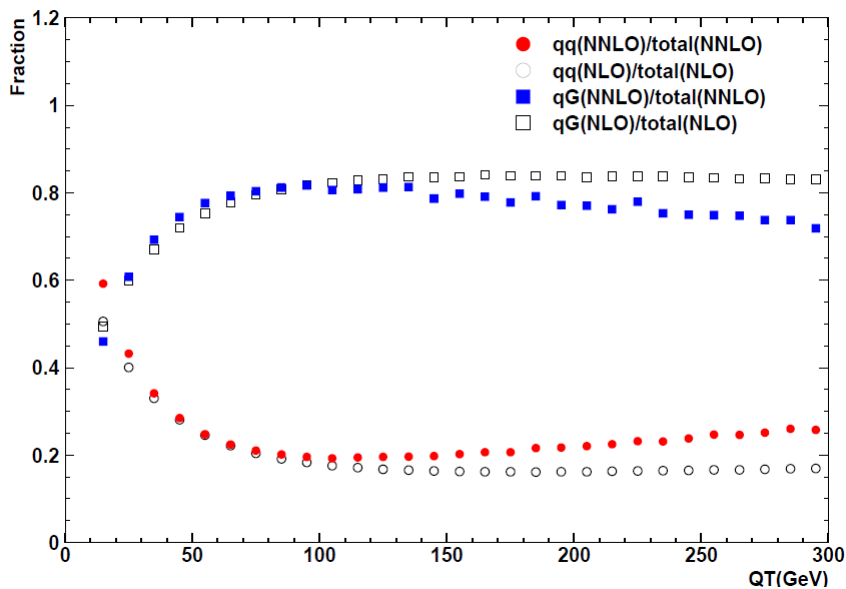
<http://theory.fi.infn.it/grazzini/dy.html>

- Parton level Monte Carlo program that computes the cross sections for vector boson production in  $pp$  and  $p\bar{p}$  collisions up to NNLO in QCD perturbation theory.
- The relative contribution and  $A_i$  for each sub-process like  $q\bar{q}$ ,  $qg$ , or  $gg$  can be calculated at specific  $q_T$  and  $y$  bins.

# Component Fractions

Preliminary

## LHC

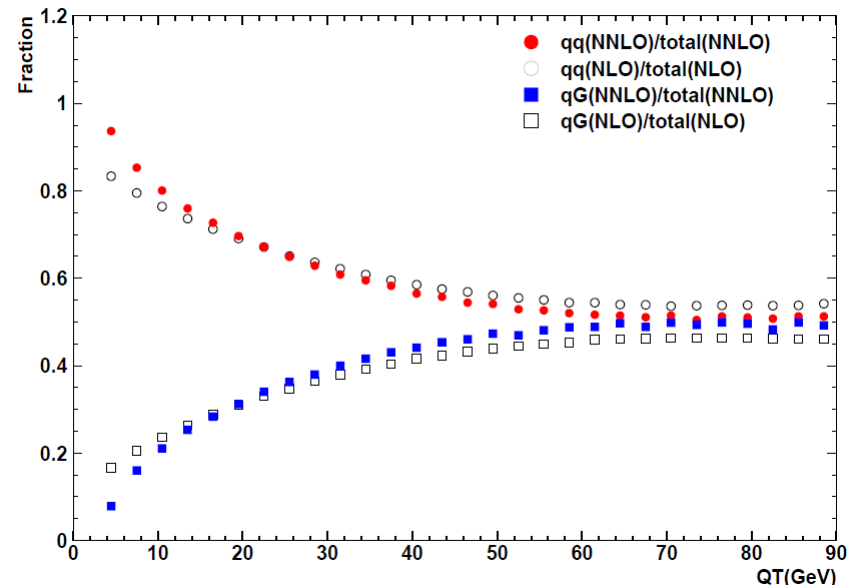


qG strongly dominates except at low QT.

qG~50%, qq~50% for  $q_T < 50$  GeV

qG~80%, qq~20% for  $q_T > 50$  GeV

## CDF



qG is comparable to qqbar at large QT.

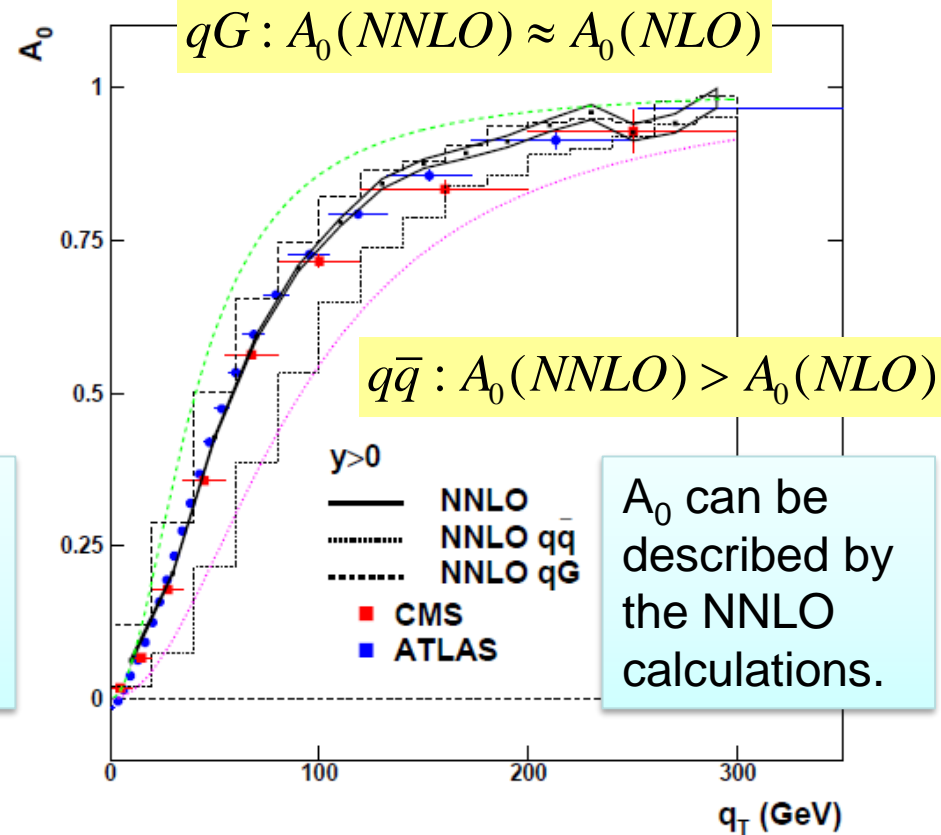
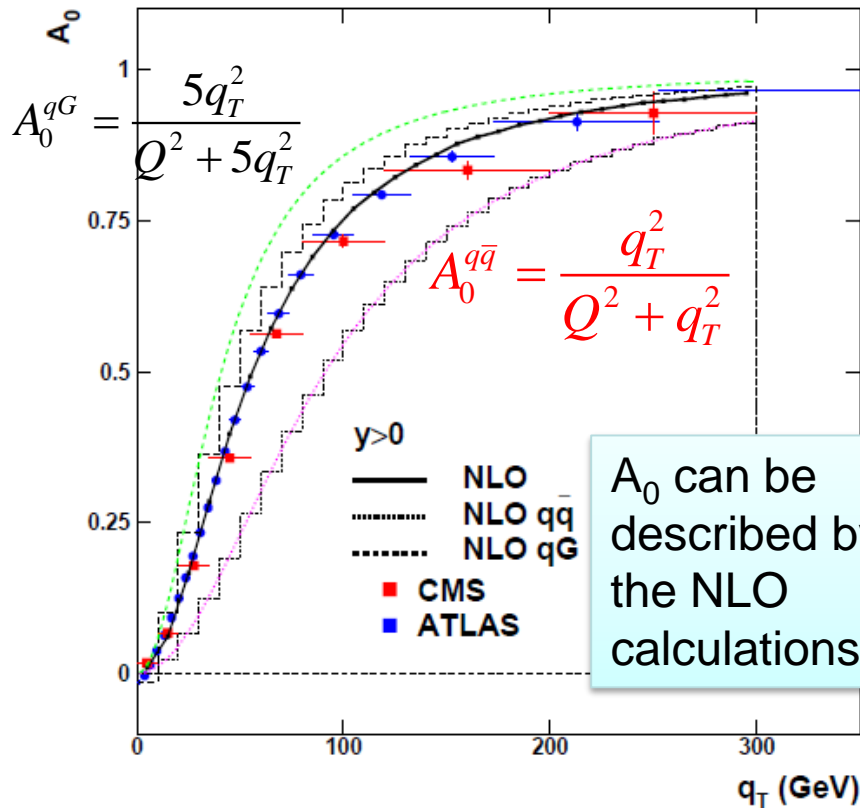
qG~20%, qq~80% for  $q_T < 50$  GeV

qG~50%, qq~50% for  $q_T > 50$  GeV

# LHC: $A_0$ , NLO vs. NNLO

Preliminary

$$A_0 = \sin^2 \theta_1$$

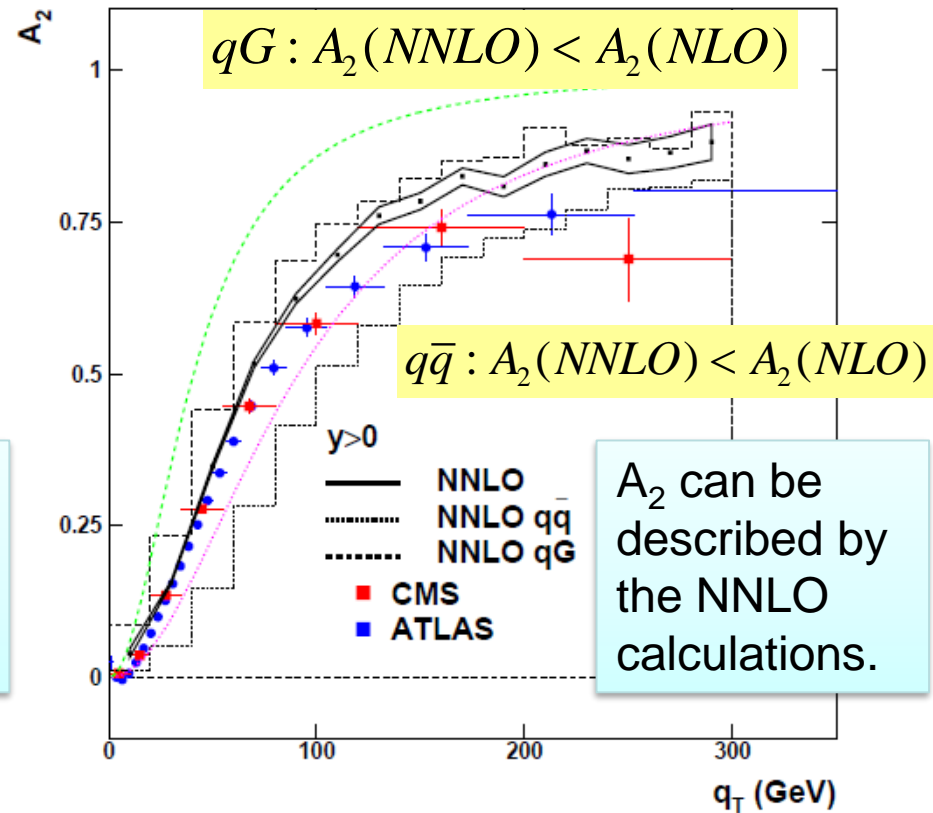
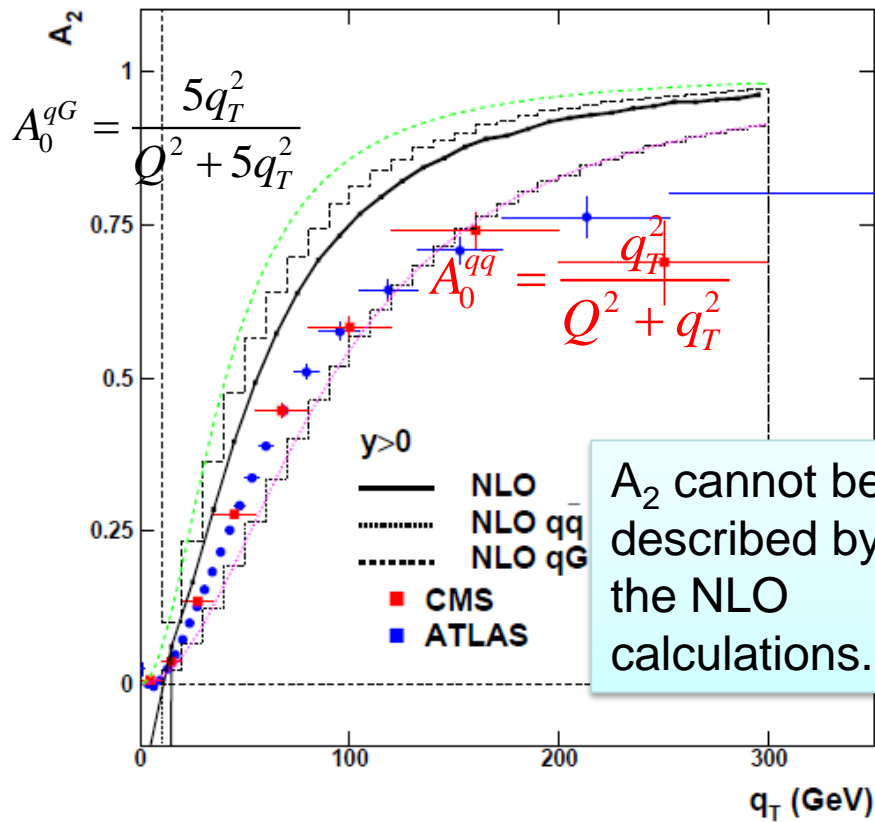


$$\langle \sin^2 \theta_1 \rangle_{NLO}^{qG} \approx \langle \sin^2 \theta_1 \rangle_{NNLO}^{qG} > \langle \sin^2 \theta_1 \rangle_{NNLO}^{q\bar{q}} > \langle \sin^2 \theta_1 \rangle_{NLO}^{q\bar{q}}$$

# LHC: $A_2$ , NLO vs. NNLO

Preliminary

$$A_2 = \sin^2 \theta_1 \cos 2\phi_1$$



$$\langle \cos 2\phi_1 \rangle_{NLO} = 1$$

$$\langle \cos 2\phi_1 \rangle_{NNLO} < 1$$

# LHC: $A_0 - A_2$ , NLO vs. NNLO

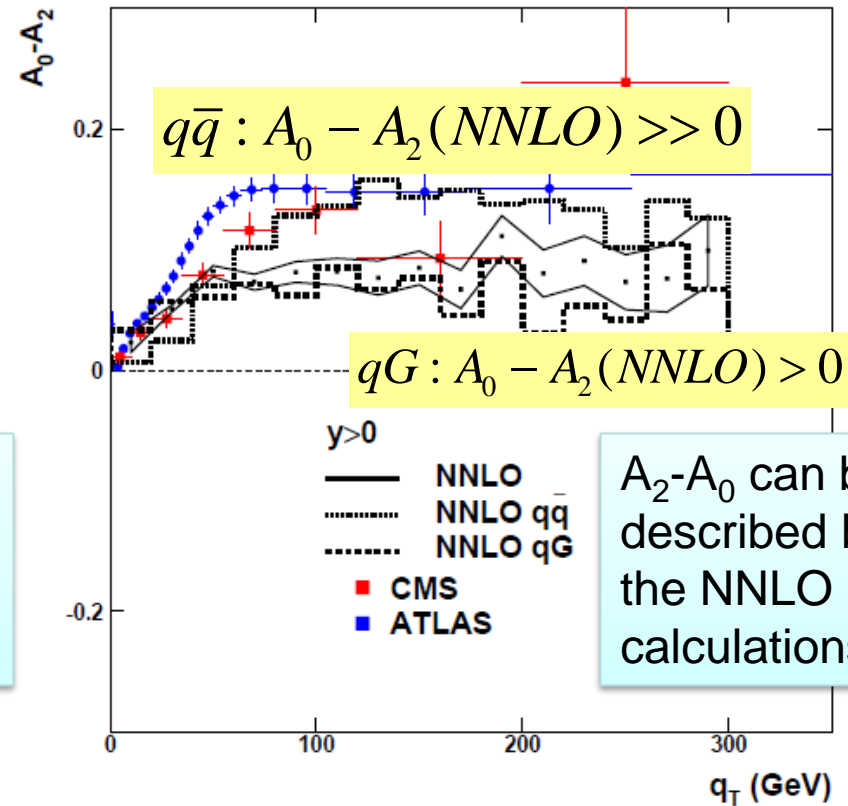
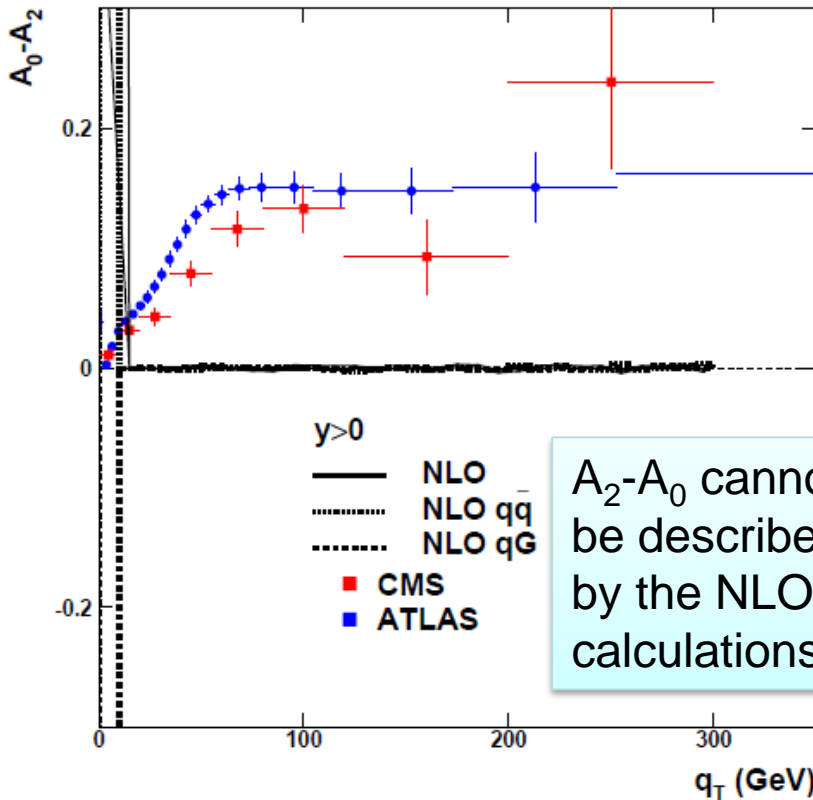
Preliminary

$$A_0 = \sin^2 \theta_1$$

$$A_2 = \sin^2 \theta_1 \cos 2\phi_1$$

$$\langle \cos 2\phi_1 \rangle_{NLO} = 1$$

$$\langle \cos 2\phi_1 \rangle_{NNLO} < 1$$



$q\bar{q} : A_0 - A_2(NNLO) \gg 0$

$qG : A_0 - A_2(NNLO) > 0$

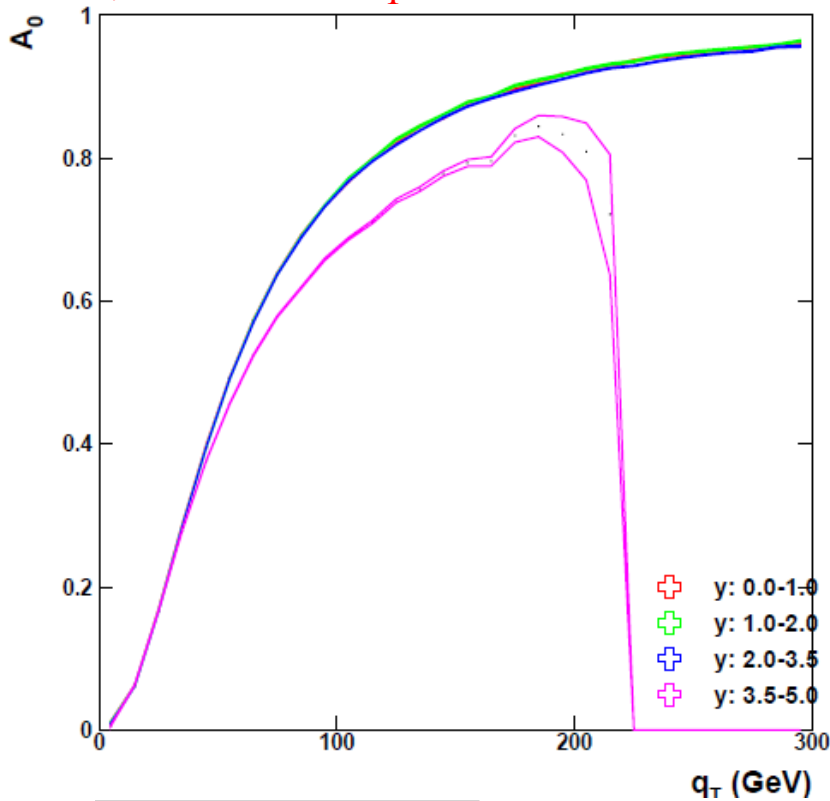
The degree of L-T violation  $A_2 - A_0$  is sub-process dependent!

# LHC: $A_0$ (NLO)

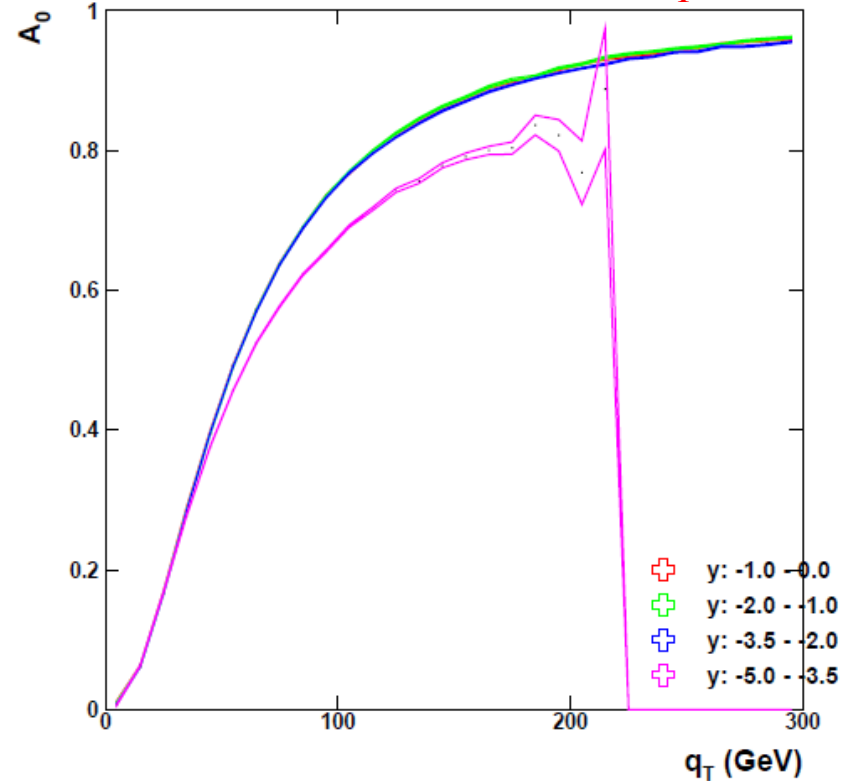
Preliminary

$$A_0 = \sin^2 \theta_1$$

$y > 0, 0 < \theta_1 < \pi/2$



$y > 0, \pi/2 < \theta_1 < \pi$



$\theta_1$	$\phi_1$	$A_0$	$A_1$	$A_2$	$A_3$	$A_4$
$\beta$	0	+	+	+	+	+
$\beta$	$\pi$	+	-	+	-	+
$\pi - \beta$	0	+	-	+	+	-
$\pi - \beta$	$\pi$	+	+	+	-	-

Small  $y$ -dependence True for both  $q\bar{q}$  and  $qg$  processes.

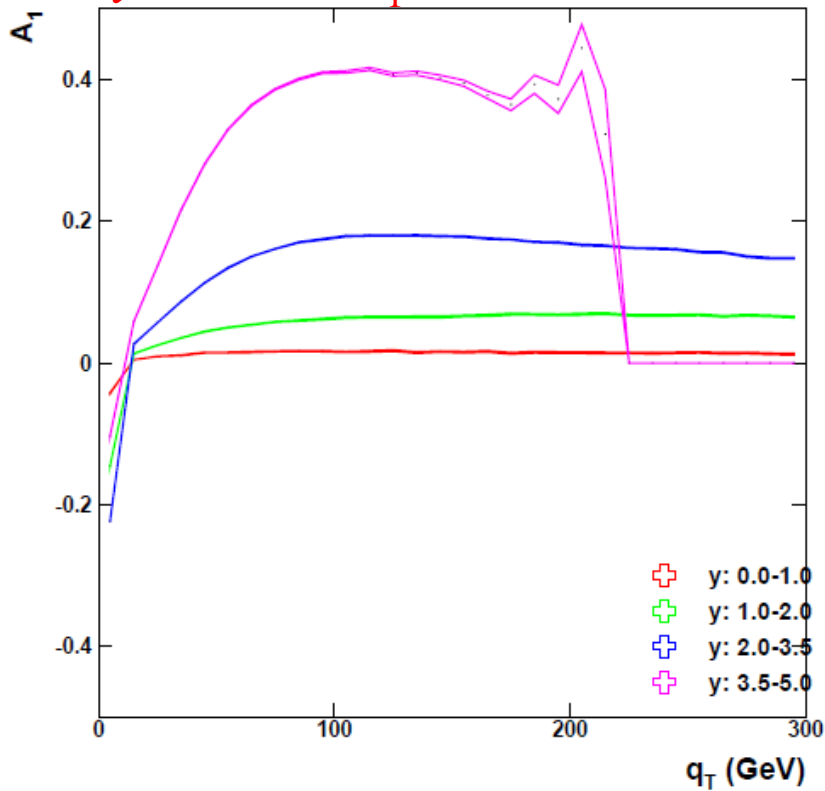
$$A_0(y) = A_0(-y)$$

# LHC: $A_1$ (NLO)

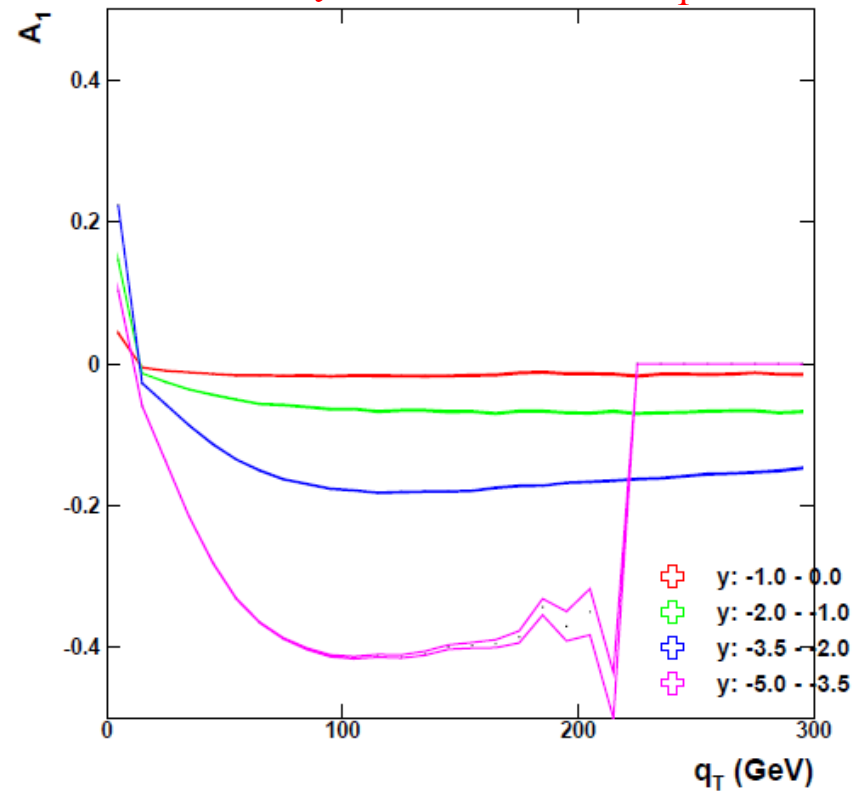
$$A_1 = \sin \theta_1 \cos \theta_1 \cos \phi_1$$

Preliminary

$y > 0, 0 < \theta_1 < \pi/2$



$y > 0, \pi/2 < \theta_1 < \pi$



$\theta_1$	$\phi_1$	$A_0$	$A_1$	$A_2$	$A_3$	$A_4$
$\beta$	0	+	+	+	+	+
$\beta$	$\pi$	+	-	+	-	+
$\pi - \beta$	0	+	-	+	+	-
$\pi - \beta$	$\pi$	+	+	+	-	-

Large  $y$ -dependence

True for both  $q\bar{q}$  and  $qg$  processes.

$$A_1(y) = -A_1(-y)$$

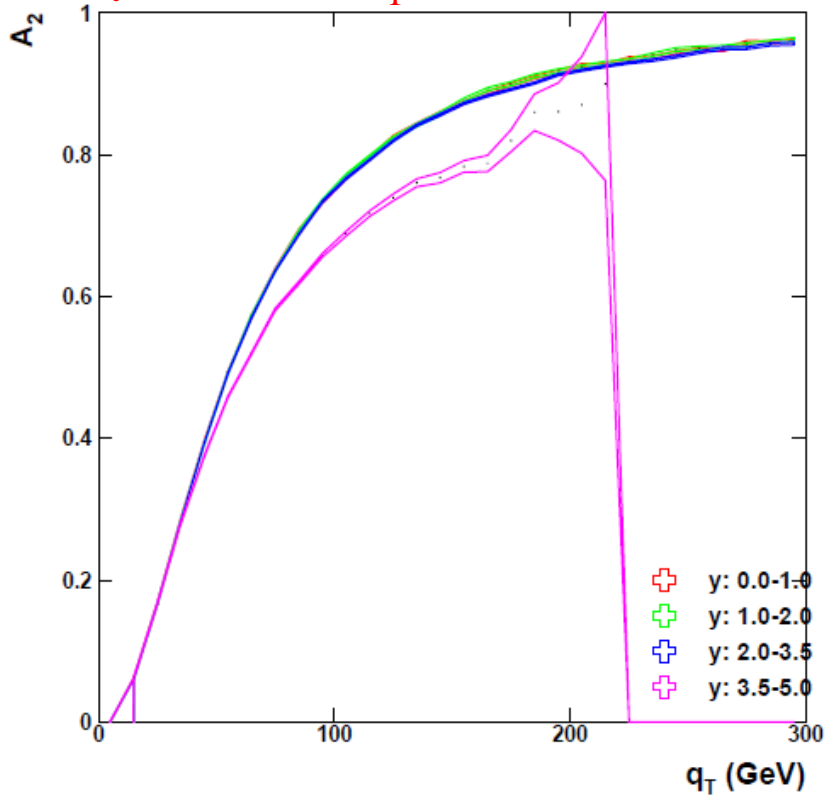


# LHC: $A_2$ (NLO)

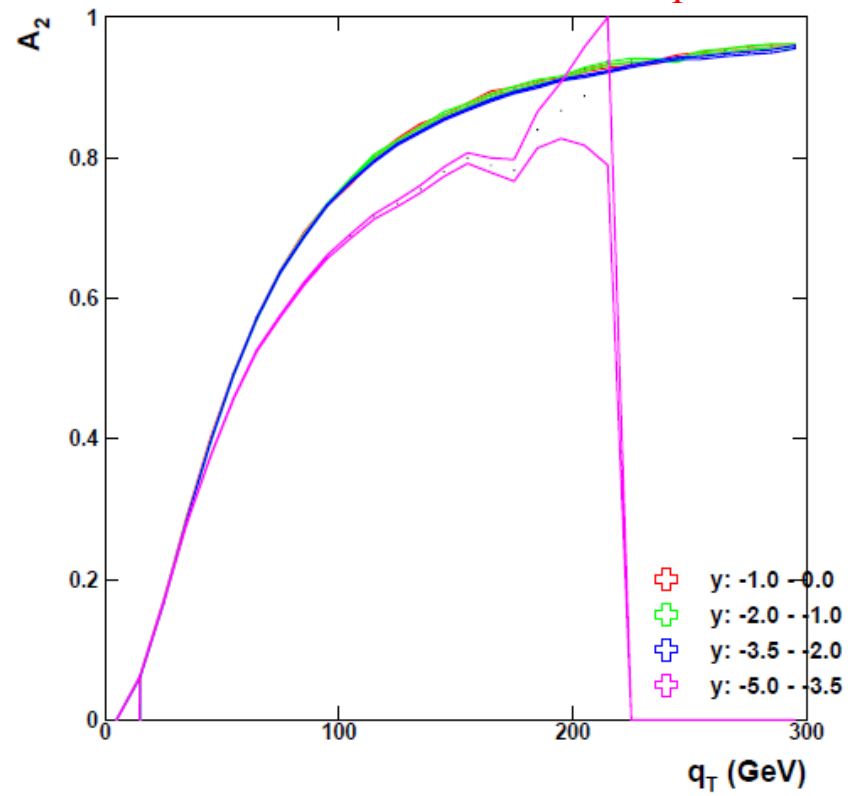
Preliminary

$$A_2 = \sin^2 \theta_1 \cos 2\phi_1$$

$y > 0, 0 < \theta_1 < \pi/2$



$y > 0, \pi/2 < \theta_1 < \pi$



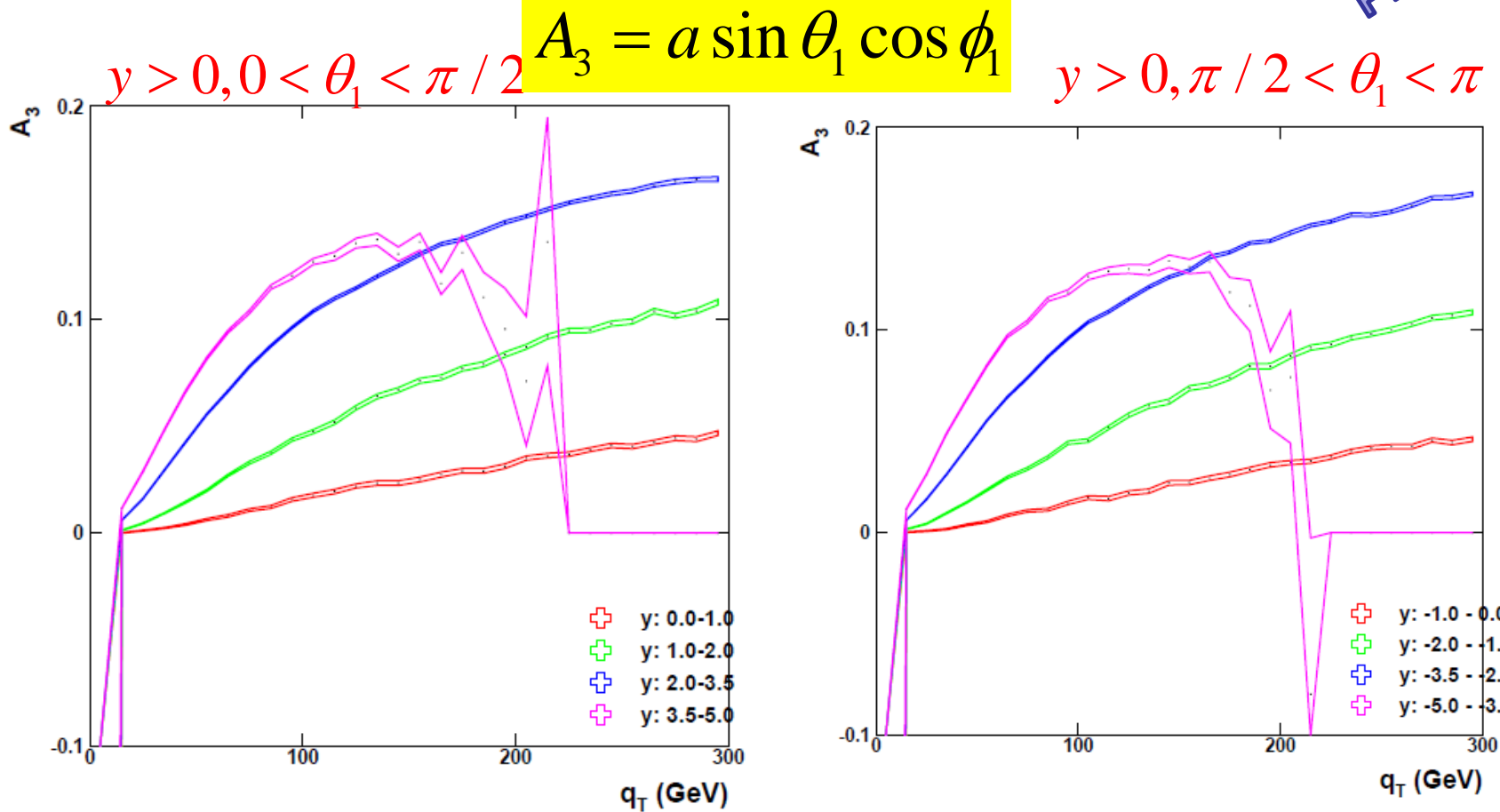
$\theta_1$	$\phi_1$	$A_0$	$A_1$	$A_2$	$A_3$	$A_4$
$\beta$	0	+	+	+	+	+
$\beta$	$\pi$	+	-	+	-	+
$\pi - \beta$	0	+	-	+	+	-
$\pi - \beta$	$\pi$	+	+	+	-	-

Small  $y$ -dependence True for both  $q\bar{q}$  and  $qg$  processes.

$$A_2(y) = A_2(-y)$$

# LHC: $A_3$ (NLO)

Preliminary



$\theta_1$	$\phi_1$	$A_0$	$A_1$	$A_2$	$A_3$	$A_4$
$\beta$	0	+	+	+	+	+
$\beta$	$\pi$	+	-	+	-	+
$\pi - \beta$	0	+	-	+	+	-
$\pi - \beta$	$\pi$	+	+	+	-	-

Large  $y$ -dependence True for both  $q\bar{q}$  and  $qg$  processes.

$A_3(y) = A_3(-y)$

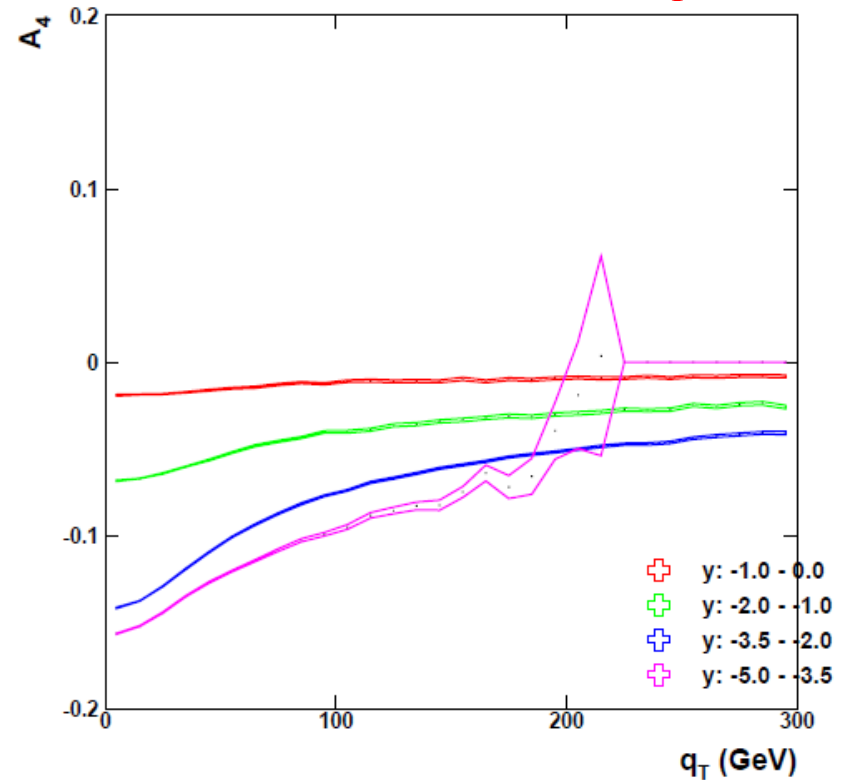
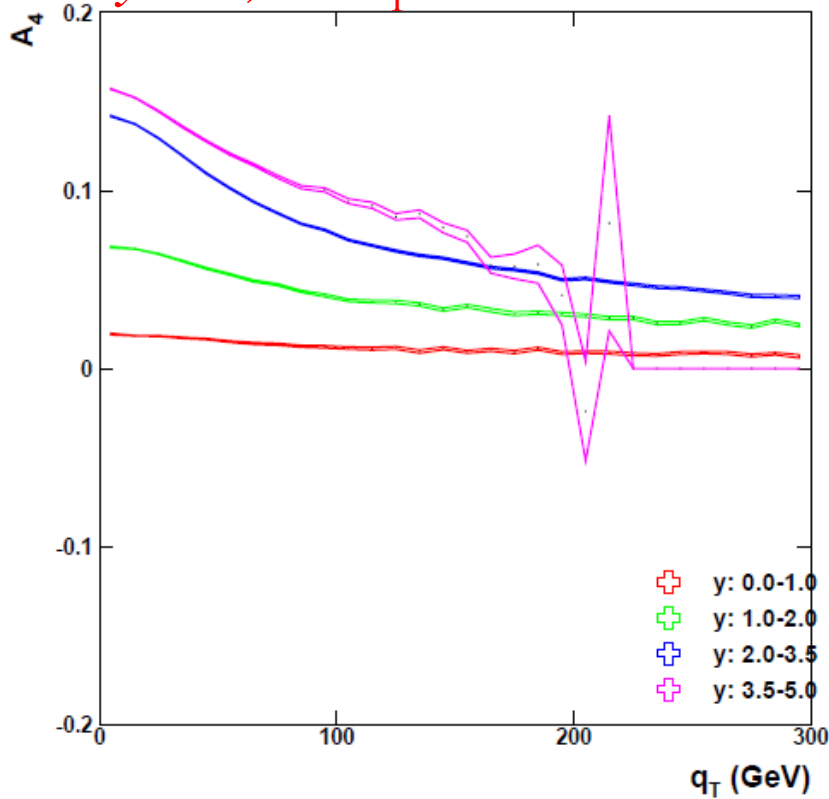
# LHC: $A_4$ (NLO)

Preliminary

$$A_4 = a \cos \theta_1$$

$y > 0, 0 < \theta_1 < \pi/2$

$y > 0, \pi/2 < \theta_1 < \pi$



$\theta_1$	$\phi_1$	$A_0$	$A_1$	$A_2$	$A_3$	$A_4$
$\beta$	0	+	+	+	+	+
$\beta$	$\pi$	+	-	+	-	+
$\pi - \beta$	0	+	-	+	+	-
$\pi - \beta$	$\pi$	+	+	+	-	-

Large  $y$ -dependence

True for both  $q\bar{q}$  and  $qg$  processes.

$$A_4(y) = -A_4(-y)$$

# Summary

- The lepton angular coefficients  $A_0$ - $A_7$  are described in terms of the polar and azimuthal angles of the  $q - \bar{q}$  axis.
- The striking  $q_T$  dependence of  $A_0$  (or  $\lambda$ ) can be well described by the mis-alignment of the  $q - \bar{q}$  axis and the CS z-axis, i.e. **finite  $\theta_1$** .
- Violation of the Lam-Tung relation ( $A_0 \neq A_2$ ) is described by the non-coplanarity of the  $q - \bar{q}$  axis and the hadron plane, i.e. **finite  $\phi_1$** .

# Summary

- Fixed-order pQCD calculations could describe quantitatively the data from colliders at large  $q_T$ .
- Many salient features of the data and fixed-order pQCD calculations could be nicely understood within the framework of geometric picture.