2^{-+} resonance poles from COMPASS data on 3π

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for the JPAC&COMPASS collaborations

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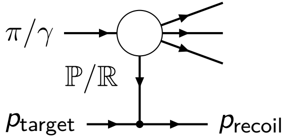
Content

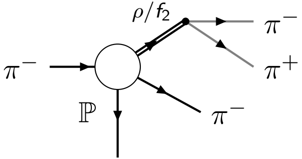
Motivation and opportunities

Study meson spectrum through peripheral resonance production

- High-energy beam,
- Pomeron/Reggeon t-channel exchange dominates,
- Recoil particle is kinematically decoupled



- Analysis at COMPASS
  - Large data sample with high purity
  - Breit-Wigner fit of major waves is done (see F.Krinner, B9 15:30)
  - JPAC&COMPASS collaboration to perform theoretically advanced analysis on the complete data set
- Opportunities at GlueX



Constraints from fundamental principles

Unitarity

$$\hat{S} = \hat{\mathbb{I}} + i\hat{T}, \quad \hat{S}\hat{S}^{\dagger} = \hat{\mathbb{I}} \quad \Rightarrow \quad \hat{T} - \hat{T}^{\dagger} = i\hat{T}\hat{T}^{\dagger},$$

- probability conservation,
- hermitian analyticity allows continuation to the second sheet.

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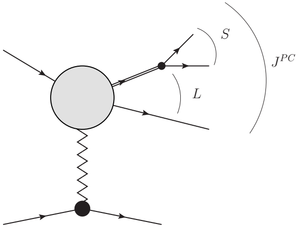

Partial-wave decomposition

- $J^{PC}M^\epsilon$ quantum numbers of system
- in case of three-body final state ξ is isobar state with spin S

$$A = \langle \text{final} | \hat{T} | \text{initial} \rangle = \sum_{JMLS} F^{JM}_{LS} \, \text{PW}^{JM}_{LS}(\tau)$$

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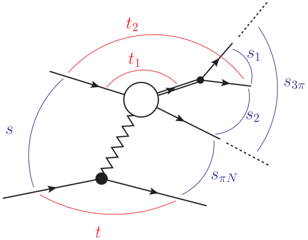

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$$A = \langle \text{final}|\,\hat{T}\,|\text{initial}\rangle = \sum_{JMLS} F_{LS}^{JM}\,\mathrm{PW}_{LS}^{JM}(\tau)$$

Constraints on the full amplitude

$$A = \langle 3\pi | \, \hat{T} \, | \pi \mathbb{P} \rangle = \sum_{JMLS} F_{LS}^{JM} \, \mathrm{PW}_{LS}^{JM}(\tau)$$

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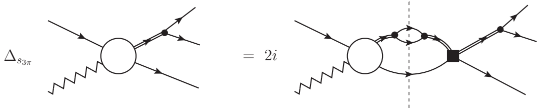
Unitarity condition for the full amplitude $\Delta A = i \int A \, \mathrm{d}\Phi \, T^\dagger$



$$\Delta_{w^2} F_{LS}^{JM}(s_1, w^2) = 2i \sum_{L'S'} \int_{(2m_\pi)^2}^{(w-m_\pi)^2} ds_1' \, \rho_3(s_1', w^2) \, T_{LSL'S'}^{JM*}(s_1, w^2, s_1') F_{L'S'}^{JM}(s_1', w^2)$$

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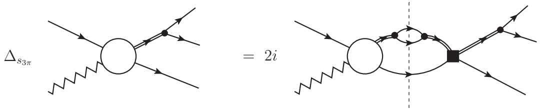

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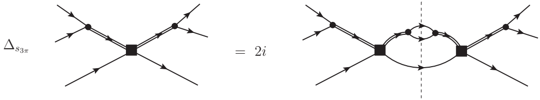
- We use elastic approximation for 3π, i.e. we neglect inelasticity.
- Interaction can be considered for pairs (subsystems, isobars), i.e. three-body forces are negligible.

Scattering amplitude

$3\pi \to 3\pi$ matrix form

$$T = \langle 3\pi | \hat{T} | 3\pi \rangle = \sum_{JMLSL'S'} T^{JM}_{LSL'S'} \, \mathrm{PW}^{JM}_{LS}(\tau) \, \mathrm{PW}^{JM}_{L'S'}(\tau')$$

Unitarity condition for rescattering amplitude $\Delta T = i \int T \, \mathrm{d}\Phi \, T^\dagger$



$$\Delta_{w^2} T^{JM}_{LSL'S'}(s_1, w^2, s_1') = 2i \sum_{L''S''} \int_{(2m_\pi)^2}^{(w-m_\pi)^2} \mathrm{d}s_1'' \rho_3(s_1'', w^2) \, T^{JM*}_{LSL''S''}(s_1, w^2, s_1'') \, T^{JM}_{L''S''L'S'}(s_1'', w^2, s_1')$$

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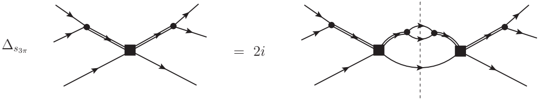

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- The $(\pi\pi)$-subsystems freely scatter to each other.

$$T = \langle \xi\pi | \, \hat{T} \, | \xi'\pi \rangle, \quad \text{e.g. } T = \begin{pmatrix} T_{\rho\pi\to\rho\pi} & T_{\rho\pi\to f_2\pi} \\ T_{f_2\pi\to\rho\pi} & T_{f_2\pi\to f_2\pi} \end{pmatrix}.$$

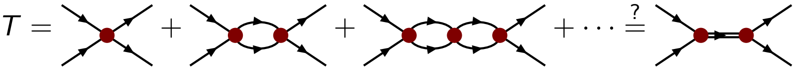
- Unitarity equations are written in matrix form.

K-matrix approach

$$T = \frac{K}{1 - i\tilde{\rho}K} = K + K\,[i\tilde{\rho}]\,K + K\,[i\tilde{\rho}]\,K\,[i\tilde{\rho}]\,K\,[i\tilde{\rho}]\,K + \dots$$

where $i\tilde{\rho}$ is a diagonal matrix of loop integrals (Chew-Mandelstam).

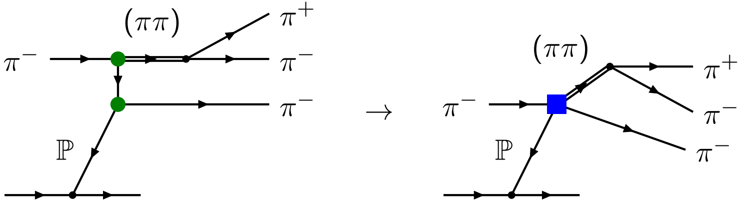
- K-matrix parametrization of T satisfies unitarity by construction.
- The approach takes into account rescattering through K-potential (resonances).



- Fit K-matrix parameters to data and extract resonance information

$$K_{ij}(s) = \sum_r \frac{g_i^r g_j^r}{m_r^2 - s} + \sum_n \gamma_{ij}^n s^n$$

Production process

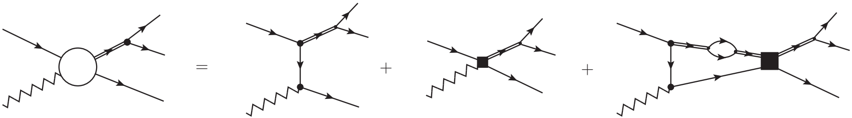


Long-range(only LHC) and Short-range production amplitudes.

- Consider $\pi + \mathbb{P} \to (\pi\pi)\,\pi$ scattering via t-exchanges.
- Interaction range is determined by the mass of the exchange particle
- Pion is lightest exchange particle with range $\sim 1\,\mathrm{fm}$.

Unitarized model [Basdevant, Berger, 1967]

Everything which is produced is supposed to scatter



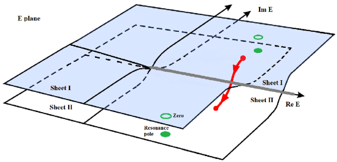
- **Production process via an exchange does not satisfy probability conservation.**
- Rescattering (Unitarisation) term has to be added.
- In the limit of short range the production amplitude is approximated by a constant c_{LS}.
- Amplitude has correct threshold behavior due to factors C_{LS}.

$$F_{LS}(w^2) = b_{LS}(w^2) + C_{LS}(w^2)\hat{T}_{LSL'S'}(w^2)c_{L'S'} + \\ + C_{LS}(w^2)\frac{\hat{T}_{LSL'S'}(w^2)}{2\pi}\int \frac{\rho_{L'S'}(s')b_{L'S'}(s')C_{L'S'}(s')}{s'-w^2}\mathrm{d}s'$$

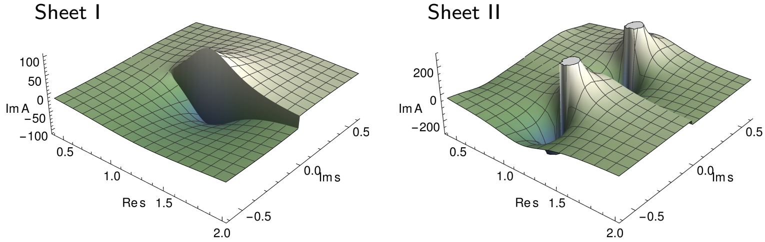
Analytic structure

General note

- We consider the amplitude as complex function of invariant mass squared w^2 and explore the structure.
- The physical region is $A(s + i\epsilon)$

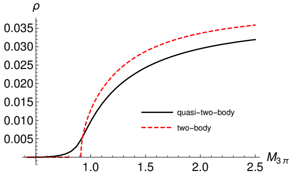


Imaginary part of Breit-Wigner amplitude on the complex plane



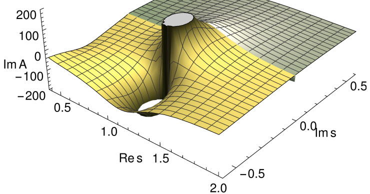
Quasi-two-body approximation

- In stable-isobar limit, phase space is 2-body: $\rho_i \sim \sqrt{(s-s_i)/s}$
- Decaying isobar introduces $\pi^+\pi^-$ scattering amplitude $f(s)$
- Phase-space factor changes to quasi-two-body phase-space factor
- Affects how we continue to unphysical sheets, isobar cut ("Woolly" cut) introduced in the complex plane.

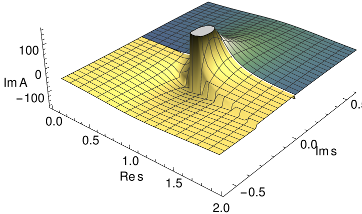


$$\rho_{\text{Quasi}}(s) \sim \int_{4m_\pi^2}^{(\sqrt{s}-m_\pi)^2} ds' \, \rho_{Isobar-\pi}(s') \text{Im} \, f(s')$$

Sheet I&II
two-body system



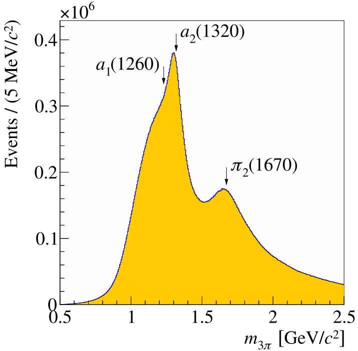
Sheet I&II
quasi-two-body system



3π at COMPASS

Step 1: mass-independent analysis

- The largest data set (50×10^6 events) on diffractively produced 3π systems.
- High-energy beam guaranties peripheral reaction $\sqrt{s} \approx 19\,\text{GeV}$.
- many resonances are seen in the raw spectrum.



[C. Adolph et al. [COMPASS Collaboration], arXiv:1509.00992]

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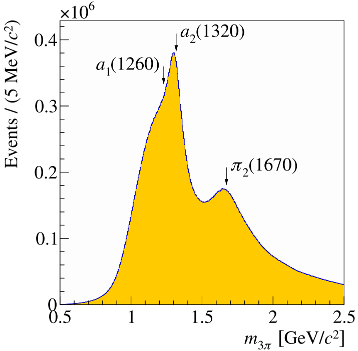

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COMPASS 3π PWA:

- $\pi^-\pi^+\pi^-$ final state, $m_{3\pi} < 2.5\,\mathrm{GeV}$, $0.1 < t' < 1\,\mathrm{GeV}^2$,
- Independent PWA in $M_{3\pi} \times t'$ bins (100×11 bins),
- $\pi^+\pi^-$-resonances: $f_0(500)$, ρ, $f_0(980)$, f_2, $\rho_3(1670)$.
- PWA model consists of 88 waves $J^{PC} = 0^{-+}, 1^{++}, 1^{-+}, 2^{++}, 2^{-+}, \dots$

Partial waves in the 2^{-+} sector

Partial wave
$2^{-+}0^{+}$
$f_2\pi$ S
$f_2\pi$ D
$\rho\pi$ P
$\rho\pi$ F
$(\pi\pi)_S$ D
$f_0\pi$ D
$\rho_3\pi$ P
$f_2\pi$ G
$2^{-+}1^{+}$
$\rho\pi$ P
$f_2\pi$ S
$\rho\pi$ F
$(\pi\pi)_S$ D
$\rho_3\pi$ P
$f_2\pi$ D
$2^{-+}2^{+}$
$\rho\pi$ P
$f_2\pi$ S
$f_2\pi$ D

Partial waves in the 2^{-+} sector

Partial wave
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$f_2\pi$ S
$f_2\pi$ D
$\rho\pi$ P
$\rho\pi$ F
$(\pi\pi)_S$ D
$f_0\pi$ D
$\rho_3\pi$ P
$f_2\pi$ G
$2^{-+}1^{+}$
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$f_2\pi$ D
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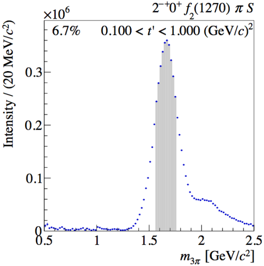
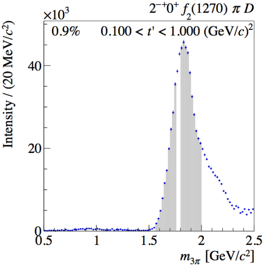
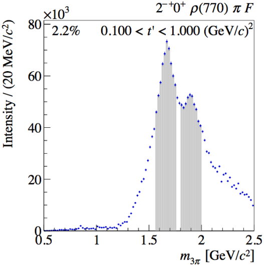
$f_2\pi$ S
$f_2\pi$ D





- intensity peak for $f_2\pi$ S- and $f_2\pi$ D-waves appear at different places
- $\rho\pi$ F-wave shows two separated peaks

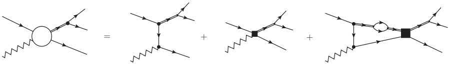


Fit of all t' slices

Description

Simultaneous fit of 5 intensities & 4 phases in 11 t'-bins

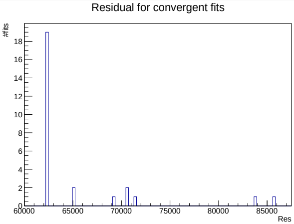
Model



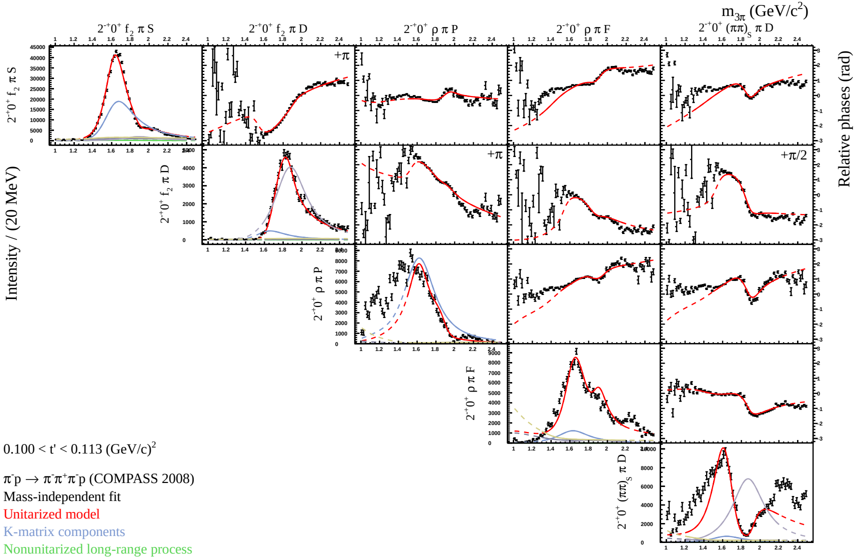
1. T matrix has 5 channels, 4 poles. It does not depend on t'.
2. Production includes short- and long-range processes.
3. A new set of parameters for every t' is used.

Fit

1. 145 independent parameters.
2. 12 steps fit. MC sampling of starting values.



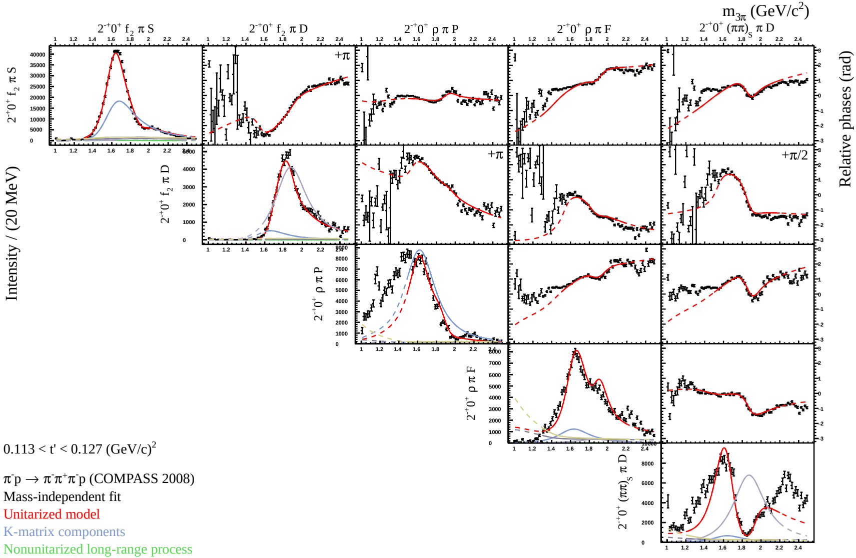
Fit over all t' slices



0.100 < t' < 0.113 (GeV/c)^2

$\pi^- p \to \pi^-\pi^+\pi^- p$ (COMPASS 2008)
Mass-independent fit
Unitarized model
K-matrix components
Nonunitarized long-range process

Fit over all t' slices



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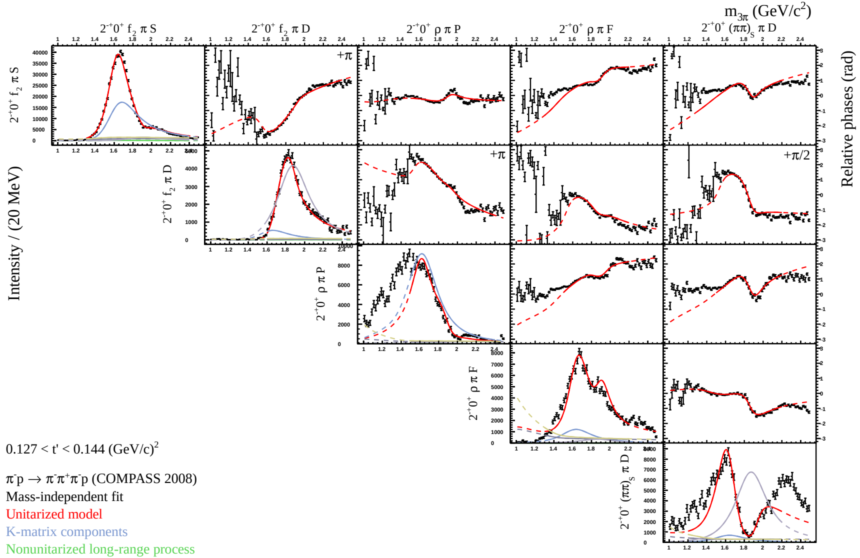

$0.127 < t' < 0.144$ (GeV/c)^2

$\pi^- p \to \pi^-\pi^+\pi^- p$ (COMPASS 2008)
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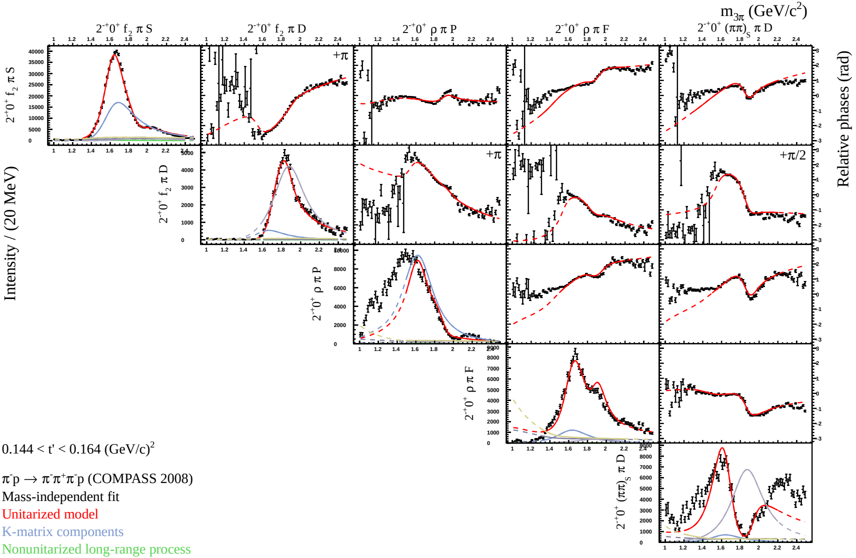
Fit over all t' slices



$0.144 < t' < 0.164$ (GeV/c)^2

$\pi^- p \to \pi^-\pi^+\pi^- p$ (COMPASS 2008)

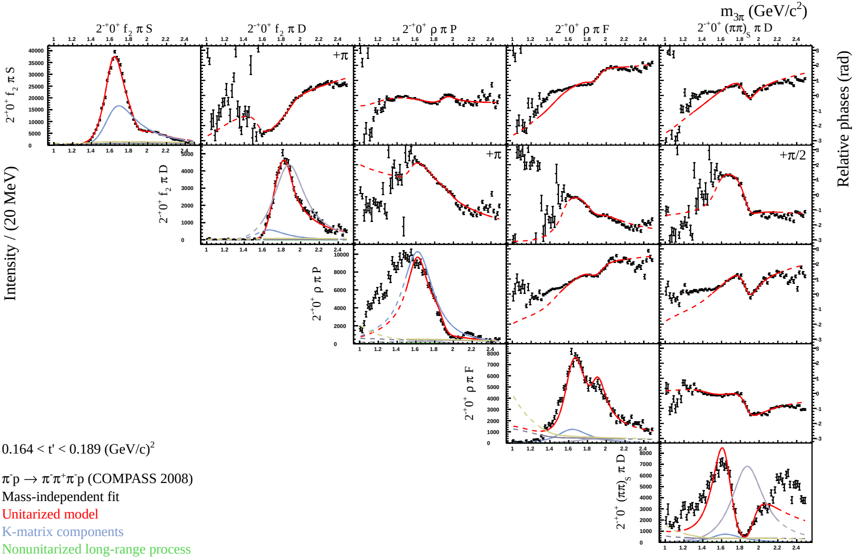
Mass-independent fit

Unitarized model

K-matrix components

Nonunitarized long-range process

Fit over all t' slices



$0.164 < t' < 0.189$ (GeV/c)^2

$\pi^- p \to \pi^-\pi^+\pi^- p$ (COMPASS 2008)

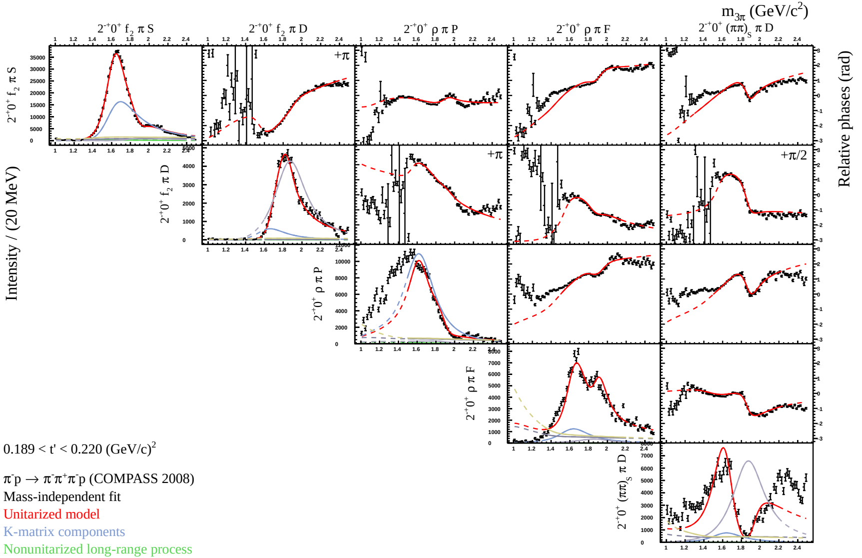
Mass-independent fit

Unitarized model

K-matrix components

Nonunitarized long-range process

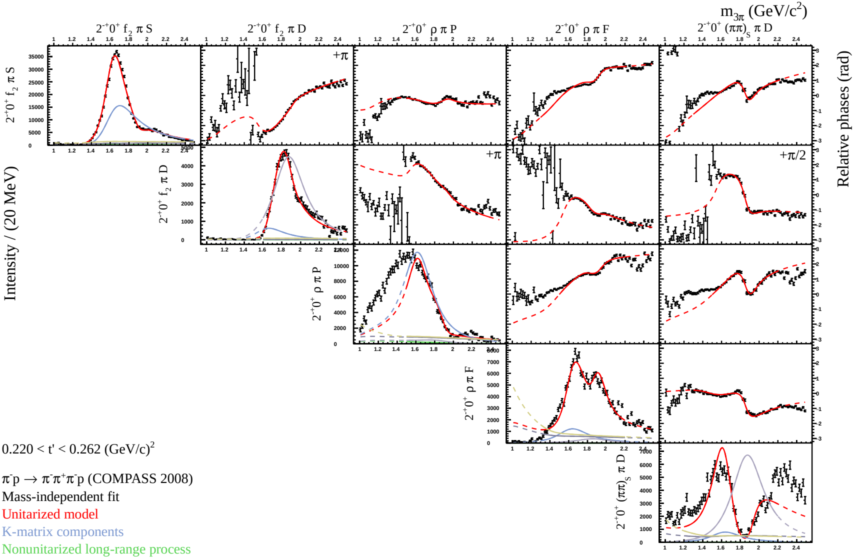
Fit over all t' slices



$0.189 < t' < 0.220$ (GeV/c)^2

$\pi^- p \to \pi^-\pi^+\pi^- p$ (COMPASS 2008)
Mass-independent fit
Unitarized model
K-matrix components
Nonunitarized long-range process

Fit over all t' slices



$0.220 < t' < 0.262$ (GeV/c)^2

$\pi^- p \to \pi^-\pi^+\pi^- p$ (COMPASS 2008)
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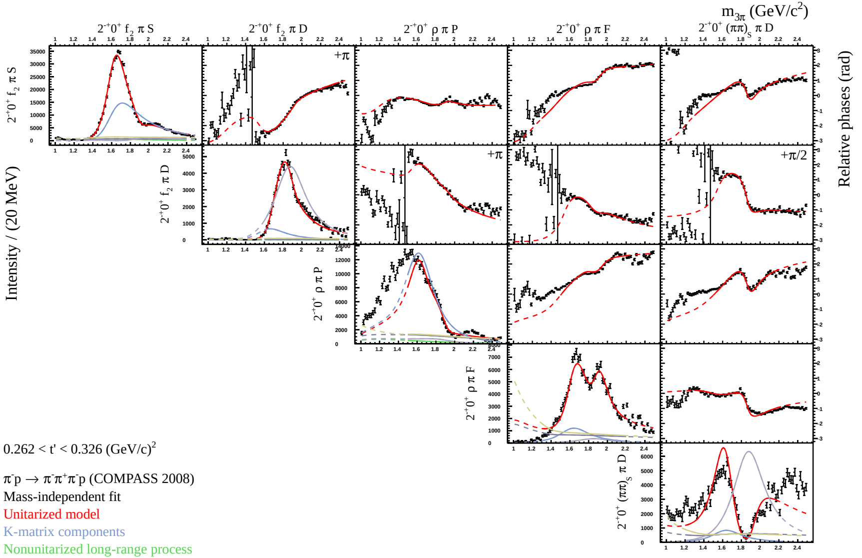
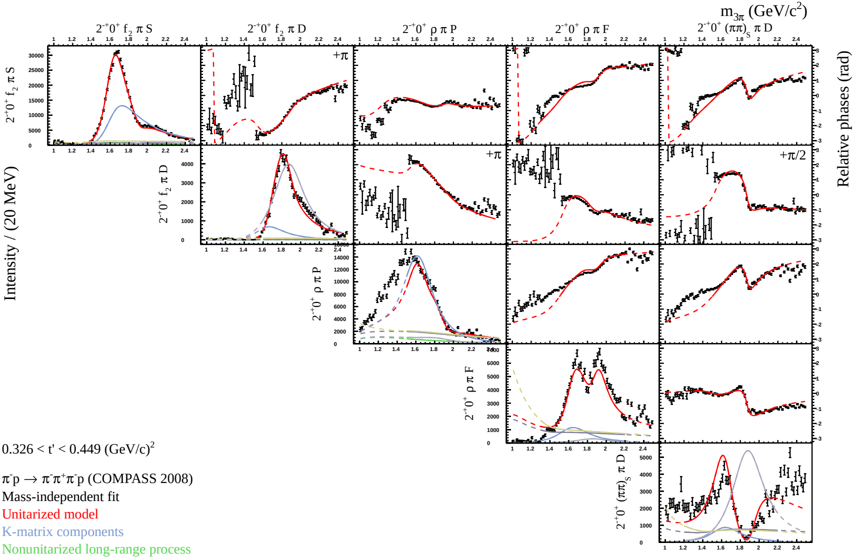
Fit over all t' slices



Fit over all t' slices



$0.326 < t' < 0.449$ (GeV/c)^2

$\pi^- p \to \pi^- \pi^+ \pi^- p$ (COMPASS 2008)

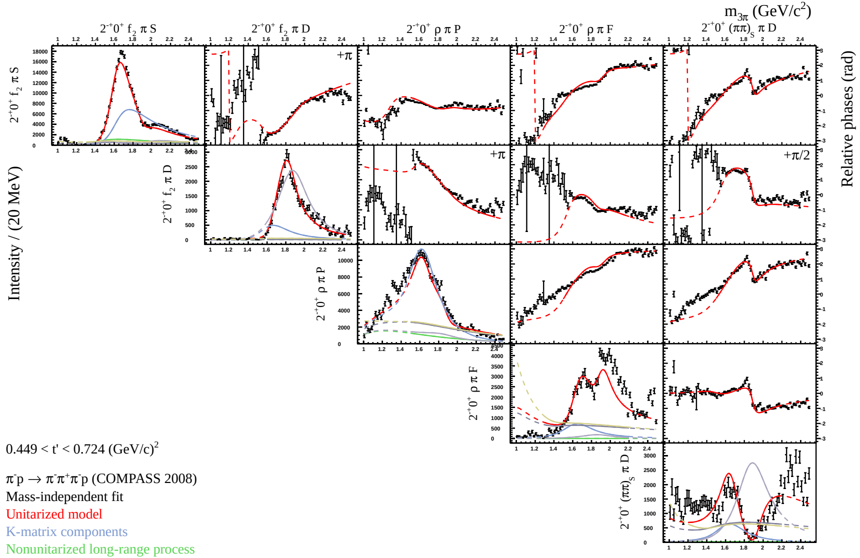
Mass-independent fit

Unitarized model

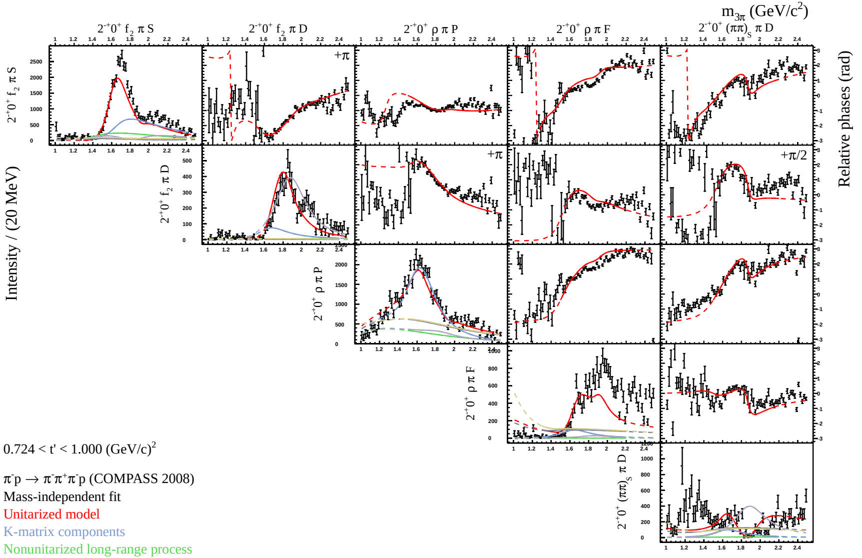
K-matrix components

Nonunitarized long-range process

Fit over all t' slices



Fit over all t' slices

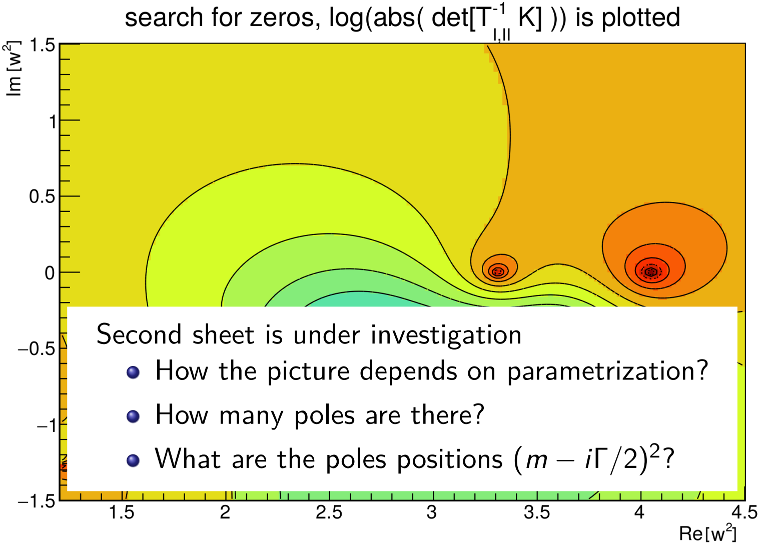


$0.724 < t' < 1.000$ (GeV/c)^2

$\pi^- p \to \pi^-\pi^+\pi^- p$ (COMPASS 2008)
Mass-independent fit
Unitarized model
K-matrix components
Nonunitarized long-range process

2^{-+} resonances

poles on the second sheet



Second sheet is under investigation

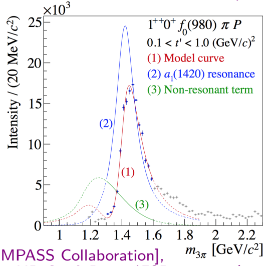
- How the picture depends on parametrization?
- How many poles are there?
- What are the poles positions $(m - i\Gamma/2)^2$?

Future developments for COMPASS Analysis

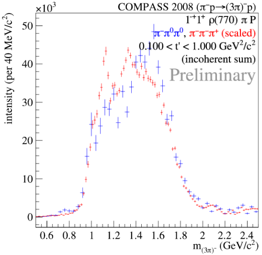
Many ideas to continue:

- Extend 5-waves-fit to available data for 2^{-+} sector, extract pole positions
- Apply the formalism to other $J^{PC}M^{\epsilon}$ sectors of 3π data. Several interesting cases along the way:
  - 1^{++} sector:
    - $a_1(1260)$ and $a_1(1420)$,
    - $K^{\star}K$ inelasticity and triangle singularity
      [MM et al., Phys. Rev. D **91**, no. 9, 094015 (2015)]
  - 1^{-+} sector: exotics.
- make 3π scattering amplitudes available for use in other experiments, MC generators
- Analysis of peripheral $\eta\pi/\eta'\pi$ production on the COMPASS data is in progress

[COMPASS Collaboration], Phys. Rev. Lett. 115, 082001 (2015)



[COMPASS Collaboration], AIP Conf. Proc. **1735**, 020007 (2016)



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- A new approach to study peripheral production and scattering dynamics has been developed.
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  - the amplitude satisfies the principle of unitarity,
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- Excellent 3π data from COMPASS experiment allow us to shine new light to many open questions in spectroscopy.
- Fit to $J^{PC} M^{\epsilon} = 2^{-+}0^{+}$ COMPASS 3π data has been performed.
  - Main features of the data are reproduced by the fit.
  - Continuation to the pole region is done, studies on stability and systematics are in progress.