a direct extraction of the Sivers function from SIDIS data

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this work

point-by-point determination of the first k_T^2 moment of the Sivers distribution from SIDIS data

the extraction is based on

- the use of Sivers asymmetries measured with transversely polarised **proton and deuteron targets** at the **same x, Q^2 values**
- some **simple assumptions**

  a **Gaussian form** is assumed to allow an analytical computation of the convolutions, but
  no specific parametrization is required

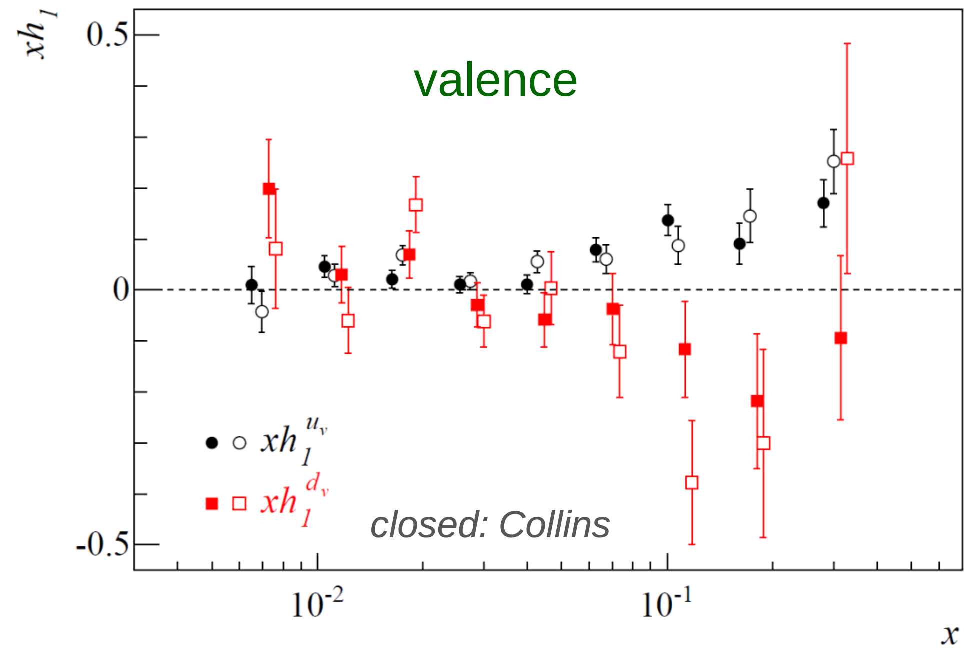
the method is essentially the same already used to extract the transversity PDF

transversity

we used the COMPASS p and d data for Collins and di-hadron asymmetries at the same x, Q_2 values and the Belle data to extract the transversity PDF

- simple assumptions on FFs
- intrinsic transverse momentum neglected
- no parametrization for PDF e FF

results



A.M., F. Bradamante, V , Barone
Phys.Rev. D91 (2015) no.1, 014034

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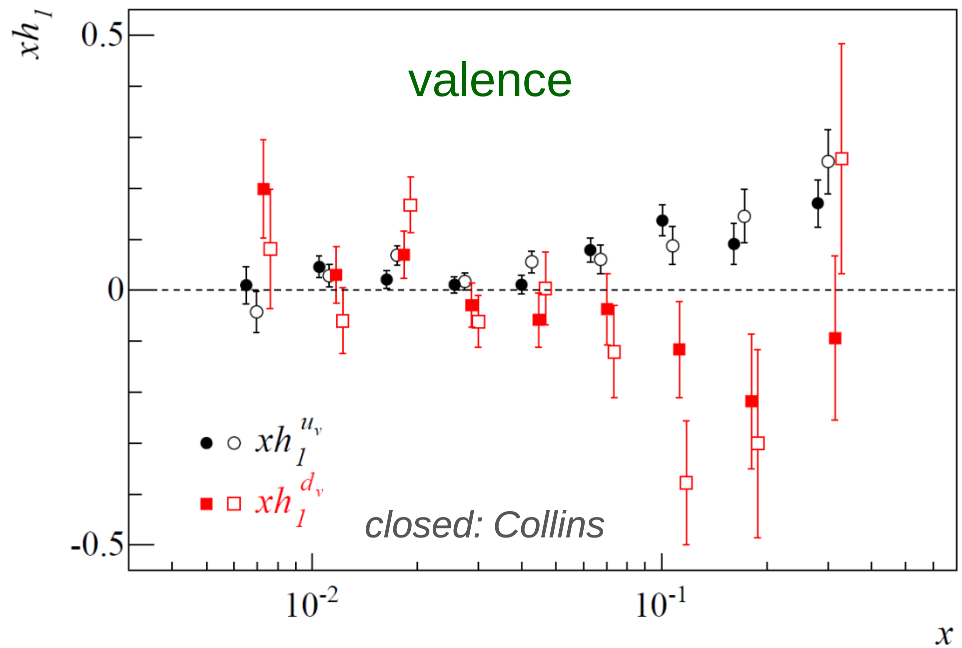

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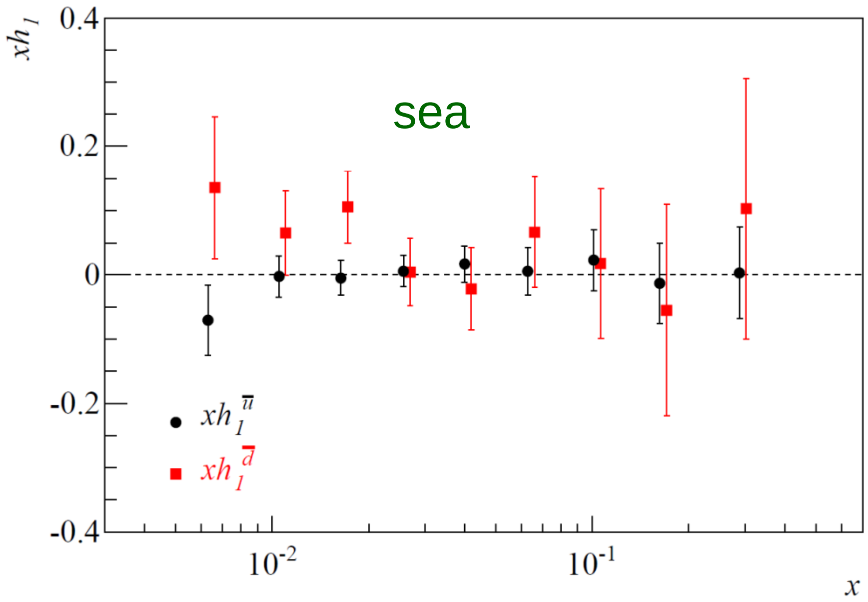

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Sivers asymmetry in SIDIS

$$A^h_{Siv} = \frac{\sum_q e_q^2 \cdot x f_{1T}^{\perp q} \otimes D_{1q}^h}{\sum_q e_q^2 \cdot x f_1^q \otimes D_{1q}^h}$$

convolutions over transverse momenta

$$f_1 \otimes D_1 = \int d^2\vec{P}_T \int d^2\vec{k}_T \int d^2\vec{p}_T \, \delta^2(z\vec{k}_T + \vec{p}_T - \vec{P}_T) \; f_1 D_1 = f_1 \cdot D_1$$

$$f_{1T}^{\perp} \otimes D_1 = \int d^2\vec{P}_T \int d^2\vec{k}_T \int d^2\vec{p}_T \, \delta^2(z\vec{k}_T + \vec{p}_T - \vec{P}_T) \frac{\vec{k}_T \cdot \vec{P}_T}{M P_T} f_{1T}^{\perp} D_1$$

we use the **Gaussian model** for the transverse-momentum dependent distributions and fragmentation functions:

$$f_{1T}^{\perp h}(x, k_T^2, Q^2) = f_{1T}^{\perp h}(x, Q^2) \frac{1}{\pi \langle k_T^2 \rangle_S} e^{-k_T^2/\langle k_T^2 \rangle_S}$$

$$D_1(z, p_T^2, Q^2) = D_1(z, Q^2) \frac{1}{\pi \langle p_T^2 \rangle} e^{-p_T^2/\langle p_T^2 \rangle}$$

↓

fragmentation function integrated over the transverse momentum.

Sivers asymmetry in SIDIS

the asymmetry becomes

$$A_{Siv}^{h}(x,z,Q^2) = G\,z\,\frac{\sum_q e_q^2 \cdot x f_{1T}^{\perp(1)q}(x,Q^2) \cdot D_{1q}^{h}(z,Q^2)}{\sum_q e_q^2 \cdot x f_1^{q}(x,Q^2) \cdot D_{1q}^{h}(z,Q^2)}$$

with $f_{1T}^{\perp(1)q} = \int d^2\vec{k}_T \frac{k_T^2}{2M^2} f_{1T}^{\perp q}(k_T^2)$ transverse moment of the Sivers function we would like to extract

and $G = \frac{\sqrt{\pi}M}{\sqrt{z^2\langle k_T^2\rangle_S + \langle p_T^2\rangle}}$

in the Gaussian model, writing the average transverse momentum of the produced hadrons as $\langle P_T\rangle = \frac{\sqrt{\pi}}{2}\sqrt{z^2\langle k_T^2\rangle + \langle p_T^2\rangle}$

$$\simeq \frac{\pi M}{2\langle P_T\rangle}$$ known

Sivers asymmetry in SIDIS

integrating over z

$$A_{Siv}^{h}(x, Q^2) = G \frac{\sum_q e_q^2 \cdot x f_{1T}^{\perp(1)q}(x, Q^2) \cdot \widetilde{D}_{1q}^{(1)h}(Q^2)}{\sum_q e_q^2 \cdot x f_1^q(x, Q^2) \cdot \widetilde{D}_{1q}^{h}(Q^2)}$$

where $$\widetilde{D}_{1q}^{h}(Q^2) = \int dz\, D_{1q}^{h}(z, Q^2)$$

known

$$\widetilde{D}_{1q}^{(1)h}(Q^2) = \int dz\, z\, D_{1q}^{h}(z, Q^2)$$

Sivers asymmetry in SIDIS

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known

$$\widetilde{D}_{1q}^{(1)h}(Q^2) = \int dz\, z\, D_{1q}^{h}(z,Q^2)$$

it is convenient to distinguish
favored and unfavored fragmentation functions
according to $h^{\pm} = \pi^{\pm}$ (or $K^{\pm}$)

Sivers function from pion asymmetries

pions

$$D^{\pi}_{1,\mathrm{fav}} \equiv D^{\pi^+}_{1u} = D^{\pi^-}_{1d} = D^{\pi^-}_{1\bar{u}} = D^{\pi^+}_{1\bar{d}}$$
$$D^{\pi}_{1,\mathrm{unf}} \equiv D^{\pi^-}_{1u} = D^{\pi^+}_{1d} = D^{\pi^+}_{1\bar{u}} = D^{\pi^-}_{1\bar{d}}$$
$$D^{\pi^\pm}_{1s} = D^{\pi^\pm}_{1\bar{s}} = N\, D^{\pi}_{1,\mathrm{unf}}$$

the denominators $\sum_q e_q^2 \cdot x f_1^q(x,Q^2) \cdot \widetilde{D}^h_{1q}(Q^2)$ become

$$p,\pi^+ : \quad x\,[4(f_1^u + \beta_\pi f_1^{\bar{u}}) + (\beta_\pi f_1^d + f_1^{\bar{d}}) + N\beta_\pi(f_1^s + f_1^{\bar{s}})]\,\widetilde{D}^{\pi}_{1,\mathrm{fav}} \equiv x f_p^{\pi^+}\,\widetilde{D}^{\pi}_{1,\mathrm{fav}},$$
$$d,\pi^+ : \quad x\,[(4+\beta_\pi)(f_1^u + f_1^d) + (1+4\beta_\pi)(f_1^{\bar{u}} + f_1^{\bar{d}}) + 2N\beta_\pi(f_1^s + f_1^{\bar{s}})]\,\widetilde{D}^{\pi}_{1,\mathrm{fav}} \equiv x f_d^{\pi^+}\,\widetilde{D}^{\pi}_{1,\mathrm{fav}},$$
$$p,\pi^- : \quad x\,[4(\beta_\pi f_1^u + f_1^{\bar{u}}) + (f_1^d + \beta_\pi f_1^{\bar{d}}) + N\beta_\pi(f_1^s + f_1^{\bar{s}})]\,\widetilde{D}^{\pi}_{1,\mathrm{fav}} \equiv x f_p^{\pi^-}\,\widetilde{D}^{\pi}_{1,\mathrm{fav}},$$
$$d,\pi^- : \quad x\,[(1+4\beta_\pi)(f_1^u + f_1^d) + (4+\beta_\pi)(f_1^{\bar{u}} + f_1^{\bar{d}}) + 2N\beta_\pi(f_1^s + f_1^{\bar{s}})]\,\widetilde{D}^{\pi}_{1,\mathrm{fav}} \equiv x f_d^{\pi^-}\,\widetilde{D}^{\pi}_{1,\mathrm{fav}},$$

$$\beta_\pi(Q^2) = \frac{\widetilde{D}^{\pi}_{1,\mathrm{unf}}(Q^2)}{\widetilde{D}^{\pi}_{1,\mathrm{fav}}(Q^2)}$$

and similar expressions can be written for the numerators, with

$$\widetilde{D}_1 \rightarrow \widetilde{D}_1^{(1)},\ f_1 \rightarrow f_{1T}^{\perp(1)}, \quad \beta_\pi \rightarrow \quad \beta_\pi^{(1)}(Q^2) = \frac{\widetilde{D}^{\pi(1)}_{1,\mathrm{unf}}(Q^2)}{\widetilde{D}^{\pi(1)}_{1,\mathrm{fav}}(Q^2)}$$

Sivers function from pion asymmetries

pions

finally one gets the **valence distributions for u and d quarks** separately

$$x f_{1T}^{\perp(1)u_v} = \frac{1}{5G\rho_\pi(1-\beta_\pi^{(1)})}\left[(x f_p^{\pi^+} A_p^{\pi^+} - x f_p^{\pi^-} A_p^{\pi^-}) + \frac{1}{3}(x f_d^{\pi^+} A_d^{\pi^+} - x f_d^{\pi^-} A_d^{\pi^-})\right]$$

$$x f_{1T}^{\perp(1)d_v} = \frac{1}{5G\rho_\pi(1-\beta_\pi^{(1)})}\left[\frac{4}{3}(x f_d^{\pi^+} A_d^{\pi^+} - x f_d^{\pi^-} A_d^{\pi^-}) - (x f_p^{\pi^+} A_p^{\pi^+} - x f_p^{\pi^-} A_p^{\pi^-})\right]$$

$$\rho_\pi(Q^2) = \frac{\widetilde{D}_{1,\mathrm{fav}}^{\pi(1)}(Q^2)}{\widetilde{D}_{1,\mathrm{fav}}^{\pi}(Q^2)}$$

directly **from the measured asymmetries**

Sivers function from pion asymmetries

pions

finally one gets the **valence distributions for u and d quarks** separately

$$xf_{1T}^{\perp(1)u_v} = \frac{1}{5G\rho_\pi(1-\beta_\pi^{(1)})}\left[(xf_p^{\pi^+}A_p^{\pi^+} - xf_p^{\pi^-}A_p^{\pi^-}) + \frac{1}{3}(xf_d^{\pi^+}A_d^{\pi^+} - xf_d^{\pi^-}A_d^{\pi^-})\right]$$

$$xf_{1T}^{\perp(1)d_v} = \frac{1}{5G\rho_\pi(1-\beta_\pi^{(1)})}\left[\frac{4}{3}(xf_d^{\pi^+}A_d^{\pi^+} - xf_d^{\pi^-}A_d^{\pi^-}) - (xf_p^{\pi^+}A_p^{\pi^+} - xf_p^{\pi^-}A_p^{\pi^-})\right]$$

$$\rho_\pi(Q^2) = \frac{\widetilde{D}_{1,\mathrm{fav}}^{\pi(1)}(Q^2)}{\widetilde{D}_{1,\mathrm{fav}}^{\pi}(Q^2)}$$

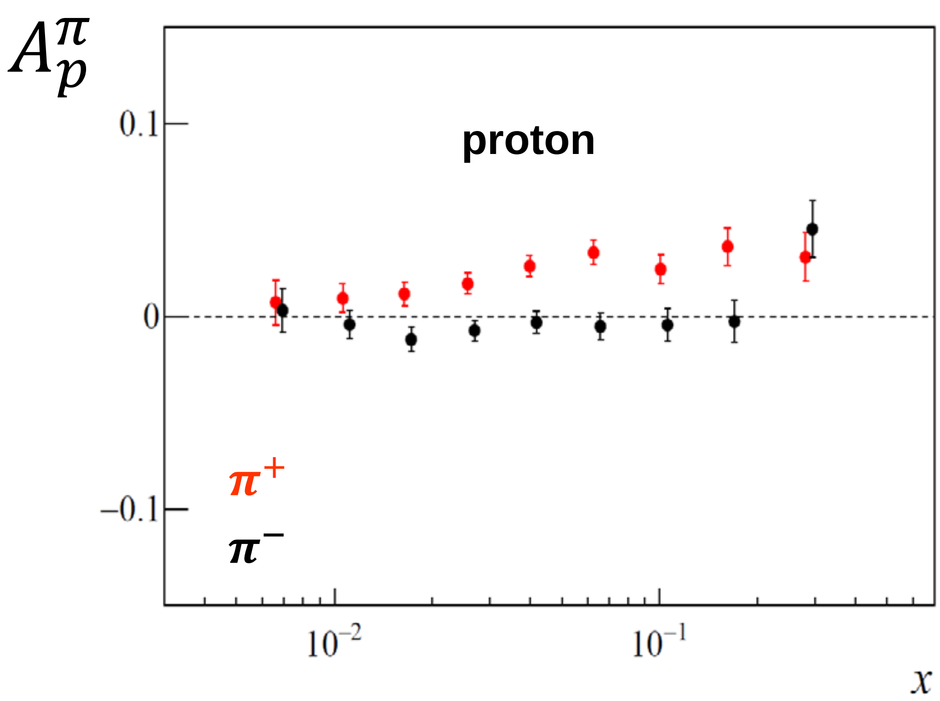
and the **sea** combination

$$xf_{1T}^{\perp(1)\bar{u}} - xf_{1T}^{\perp(1)\bar{d}} = \frac{1}{15G\rho_\pi\left(1-\beta_\pi^{(1)2}\right)}\left[2(1-4\beta_\pi^{(1)})xf_p^{\pi^+}A_p^{\pi^+} + 2(4-\beta_\pi^{(1)})xf_p^{\pi^-}A_p^{\pi^-}\right.$$
$$\left. - (1-4\beta_\pi^{(1)})xf_d^{\pi^+}A_d^{\pi^+} - (4-\beta_\pi^{(1)})xf_d^{\pi^-}A_d^{\pi^-}\right].$$

directly **from the measured asymmetries**

pion asymmetries

COMPASS results for Sivers asymmetries



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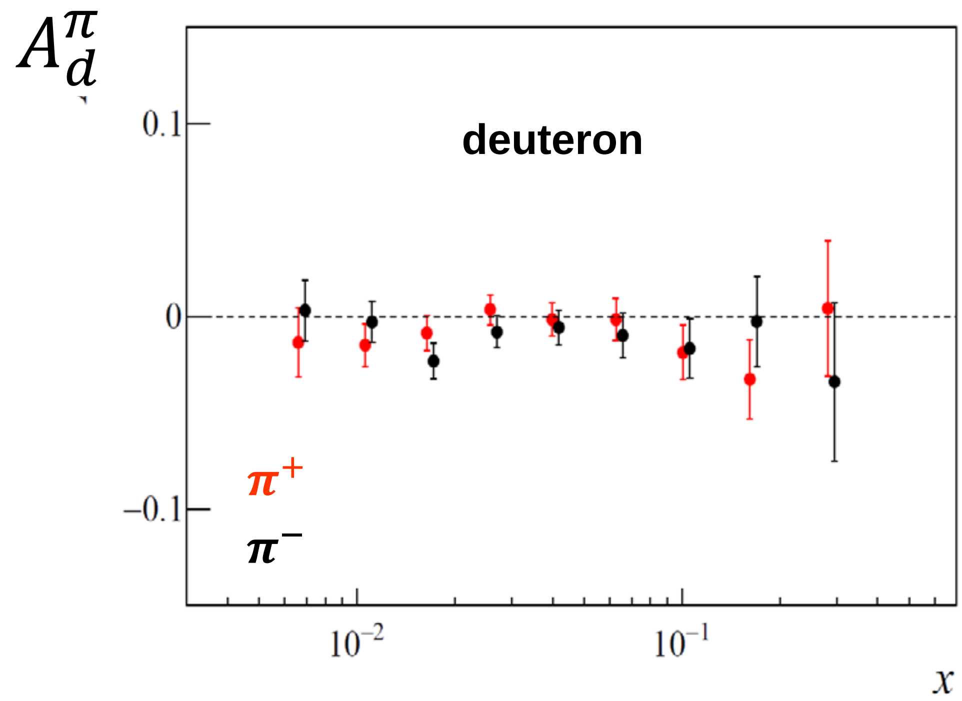

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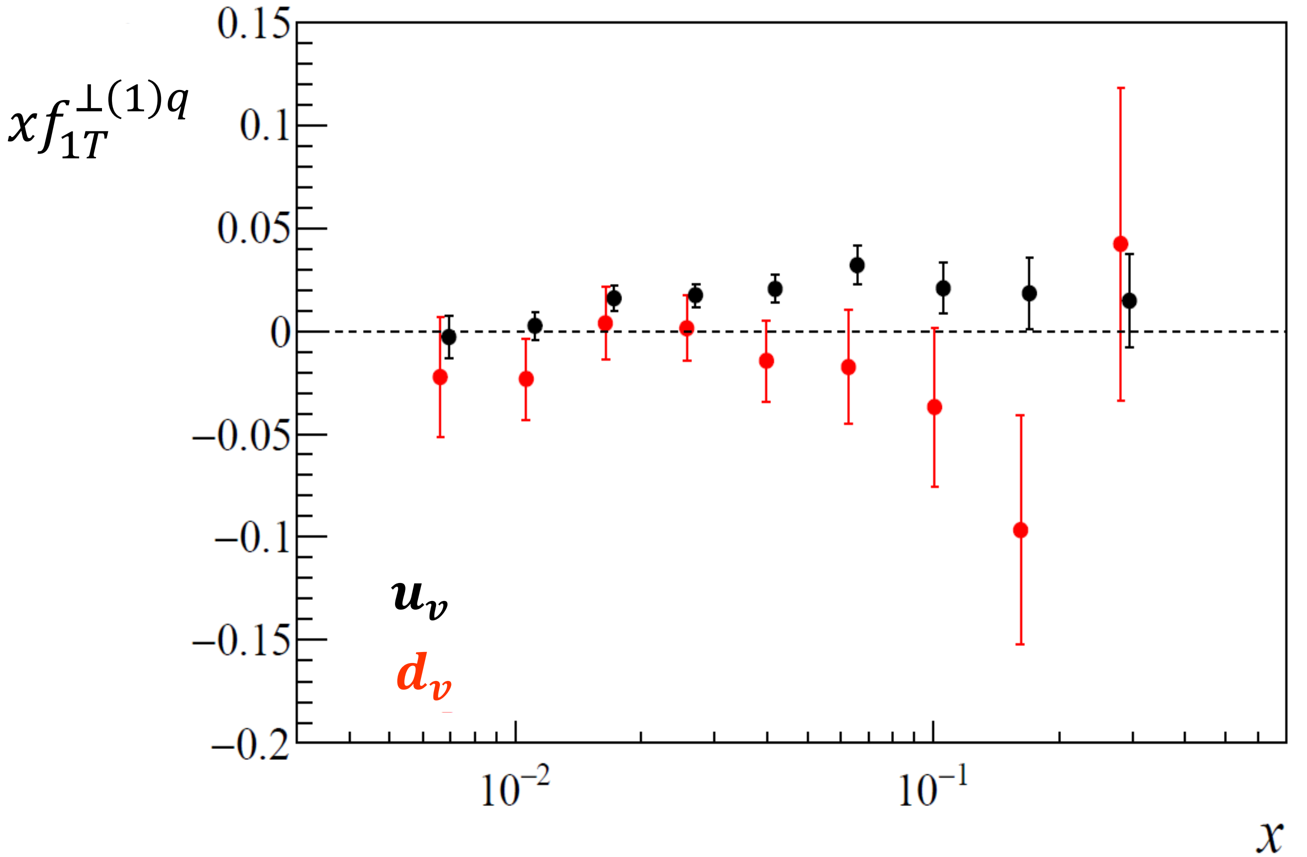
marginal statistics

Sivers function from pion asymmetries

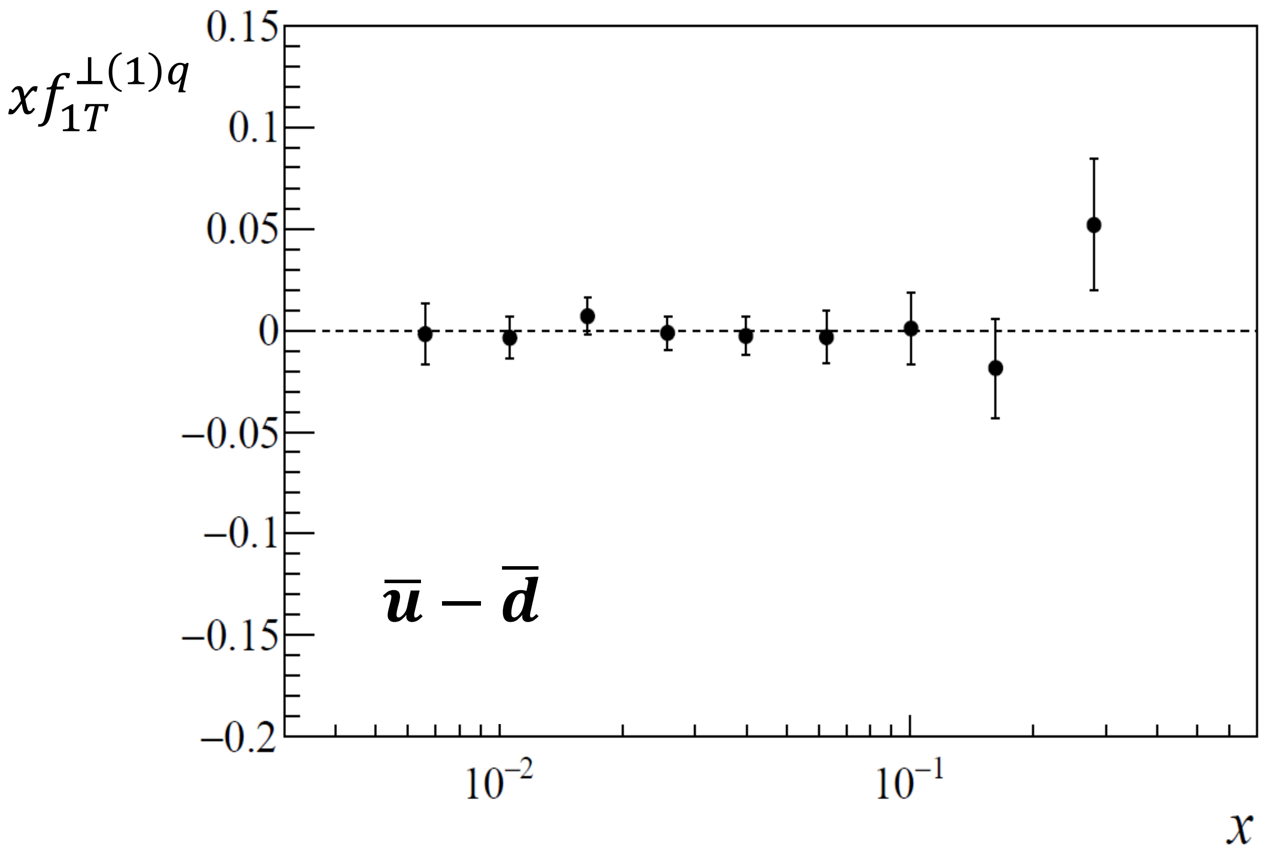
extracted Sivers functions $xf_{1T}^{\perp(1)u_v}$ $xf_{1T}^{\perp(1)d_v}$



no attempt to correct for the different values of Q^2 in the different x bins

Sivers function from pion asymmetries

extracted values of $xf_{1T}^{\perp(1)\bar{u}} - xf_{1T}^{\perp(1)\bar{d}}$



Sivers function from kaon asymmetries

kaons

$$D^K_{1,\mathrm{fav}} \equiv D^{K^+}_{1u} = D^{K^-}_{1\bar{u}}$$
$$D'^K_{1,\mathrm{fav}} \equiv D^{K^+}_{1\bar{s}} = D^{K^-}_{1s}$$
$$D^K_{1,\mathrm{unf}} \equiv D^{K^\pm}_{1d} = D^{K^\pm}_{1\bar{d}} = D^{K^+}_{1\bar{u}} = D^{K^-}_{1u} = D^{K^+}_{1s} = D^{K^-}_{1\bar{s}}$$

the denominators $\sum_q e_q^2 \cdot x f_1^q(x,Q^2) \cdot \widetilde{D}^h_{1q}(Q^2)$ become

$$p, K^+: \quad x\left[4(f_1^u + \beta_K f_1^{\bar{u}}) + \beta_K(f_1^d + f_1^{\bar{d}}) + (\beta_K f_1^s + \gamma_K f_1^{\bar{s}})\right] \widetilde{D}^K_{1,\mathrm{fav}} \equiv x f_p^{K^+}\, \widetilde{D}^K_{1,\mathrm{fav}},$$
$$d, K^+: \quad x\left[(4+\beta_K)(f_1^u + f_1^d) + 5\beta_K(f_1^{\bar{u}} + f_1^{\bar{d}}) + 2(\beta_K f_1^s + \gamma_K f_1^{\bar{s}})\right] \widetilde{D}^K_{1,\mathrm{fav}} \equiv x f_d^{K^+}\, \widetilde{D}^K_{1,\mathrm{fav}},$$
$$p, K^-: \quad x\left[4(\beta_K f_1^u + f_1^{\bar{u}}) + \beta_K(f_1^d + f_1^{\bar{d}}) + (\gamma_K f_1^s + \beta_K f_1^{\bar{s}})\right] \widetilde{D}^K_{1,\mathrm{fav}} \equiv x f_p^{K^-}\, \widetilde{D}^K_{1,\mathrm{fav}},$$
$$d, K^-: \quad x\left[5\beta_K(f_1^u + f_1^d) + (4+\beta_K)(f_1^{\bar{u}} + f_1^{\bar{d}}) + 2(\gamma_K f_1^s + \beta_K f_1^{\bar{s}})\right] \widetilde{D}^K_{1,\mathrm{fav}} \equiv x f_d^{K^-}\, \widetilde{D}^K_{1,\mathrm{fav}},$$

$$\beta_K(Q^2) = \frac{\widetilde{D}^K_{1,\mathrm{unf}}(Q^2)}{\widetilde{D}^K_{1,\mathrm{fav}}(Q^2)}, \qquad \gamma_K(Q^2) = \frac{\widetilde{D}'^K_{1,\mathrm{fav}}(Q^2)}{\widetilde{D}^K_{1,\mathrm{fav}}(Q^2)}$$

and similar expressions can be written for the numerators

Sivers function from kaon asymmetries

assuming $f_{1T}^{\perp(1)s} = f_{1T}^{\perp(1)\bar{s}}$

the u and quark Sivers functions can be obtained the kaon Sivers asymmetries alone

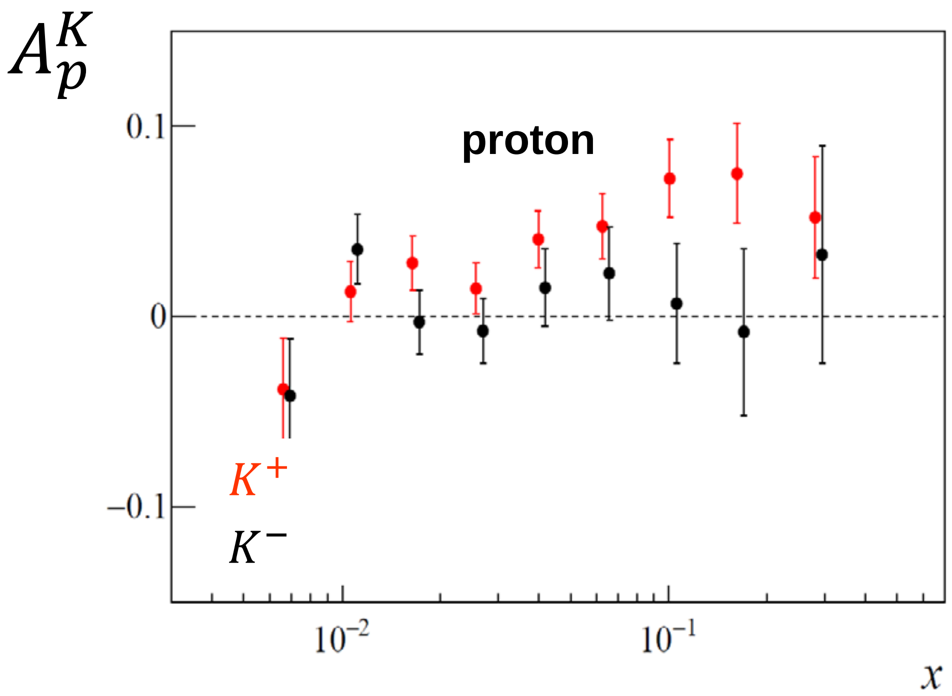
$$x f_{1T}^{\perp(1)u_v} = \frac{1}{4G\rho_K(1-\beta_K^{(1)})}\left[x f_p^{K^+} A_p^{K^+} - x f_p^{K^-} A_p^{K^-}\right]$$

kaons

$$x f_{1T}^{\perp(1)d_v} = -\frac{1}{4G\rho_K(1-\beta_K^{(1)})}\left[x f_p^{K^+} A_p^{K^+} - x f_p^{K^-} A_p^{K^-} - (x f_d^{K^+} A_d^{K^+} - x f_d^{K^-} A_d^{K^-})\right]$$

kaon asymmetries

COMPASS results for Sivers asymmetries



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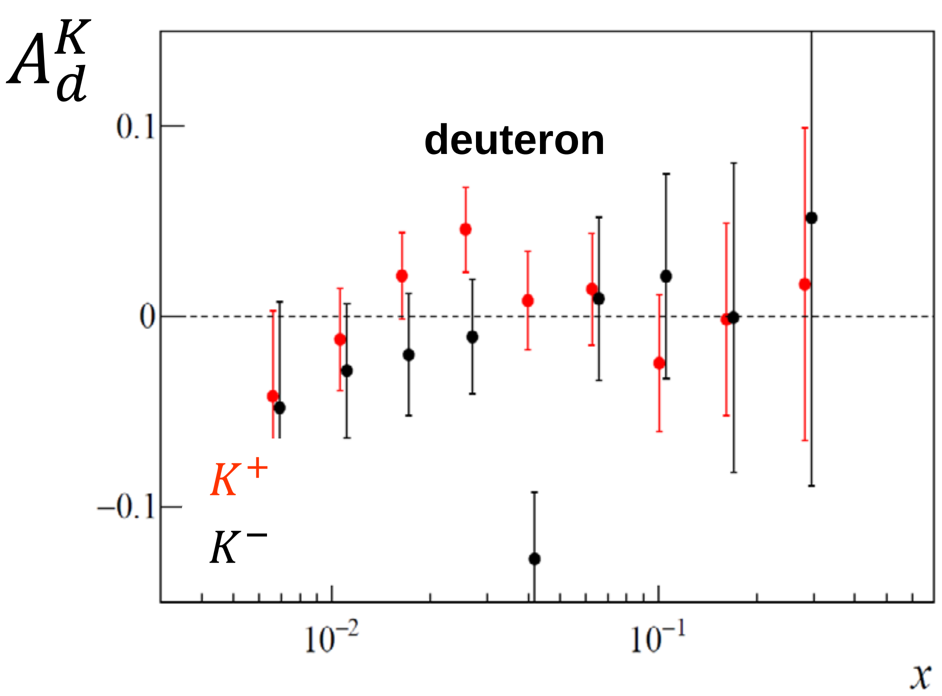

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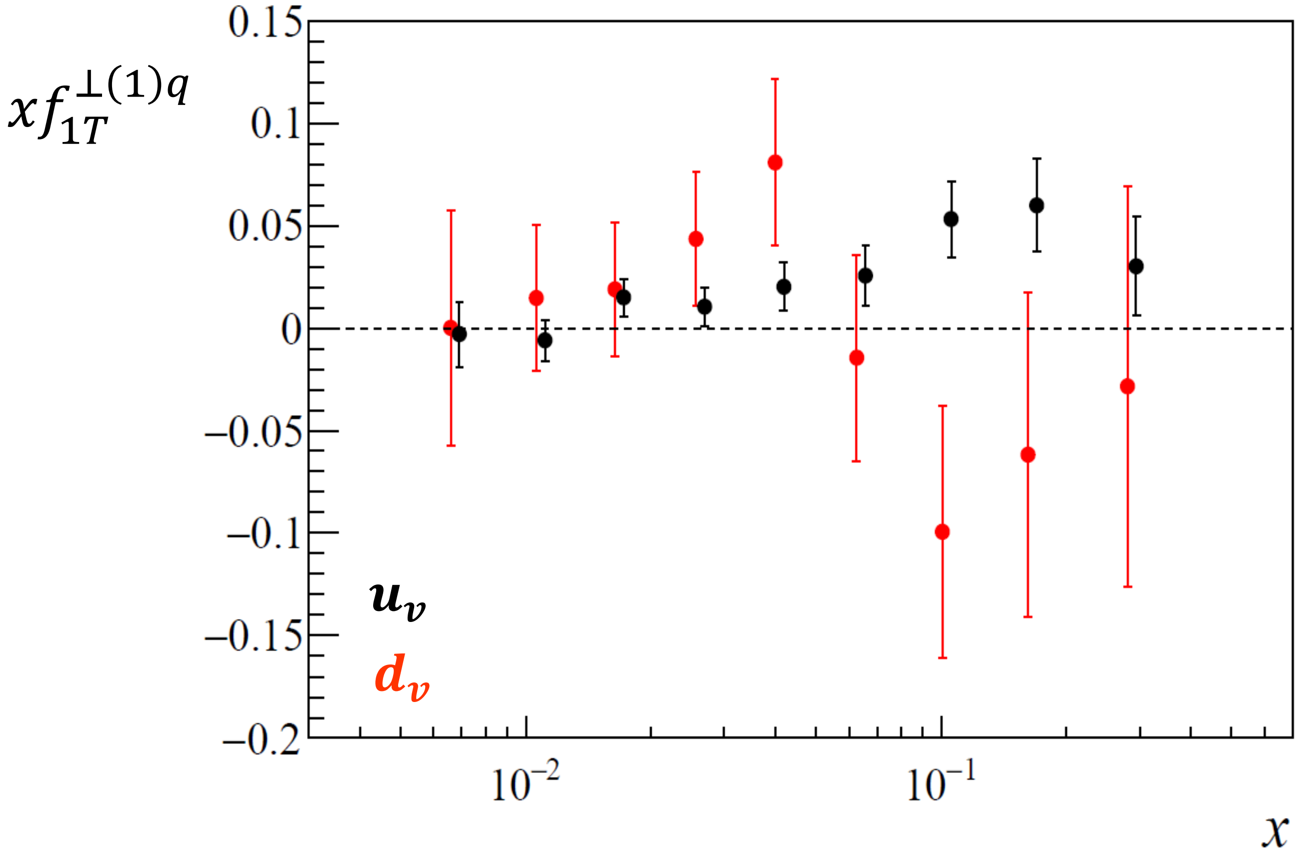
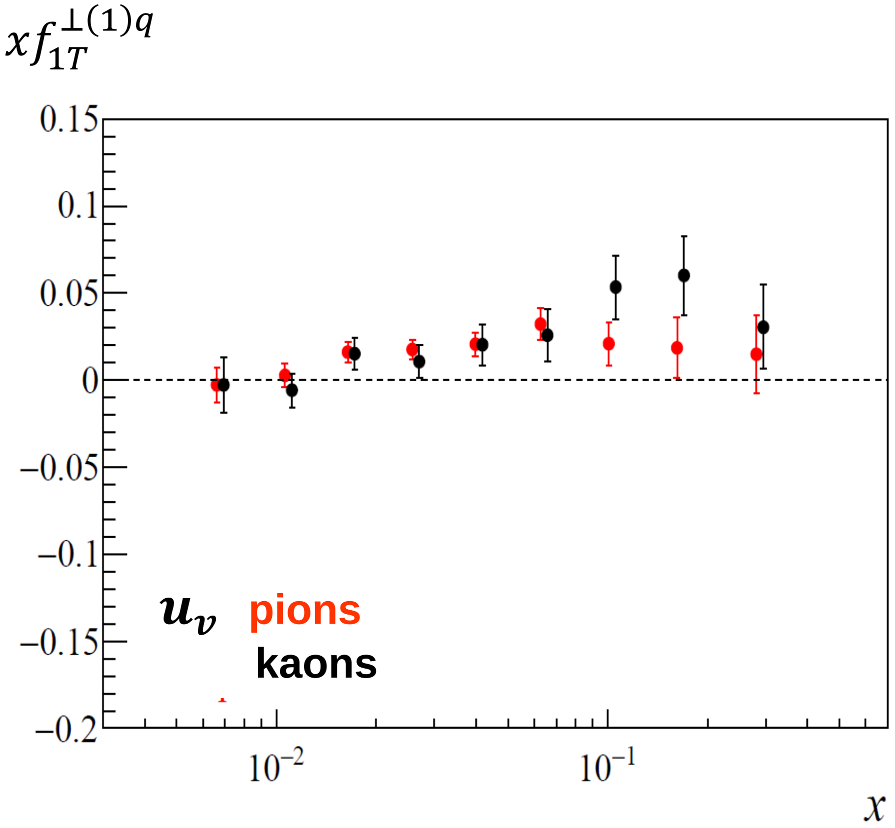
lower statistics wrt pions!

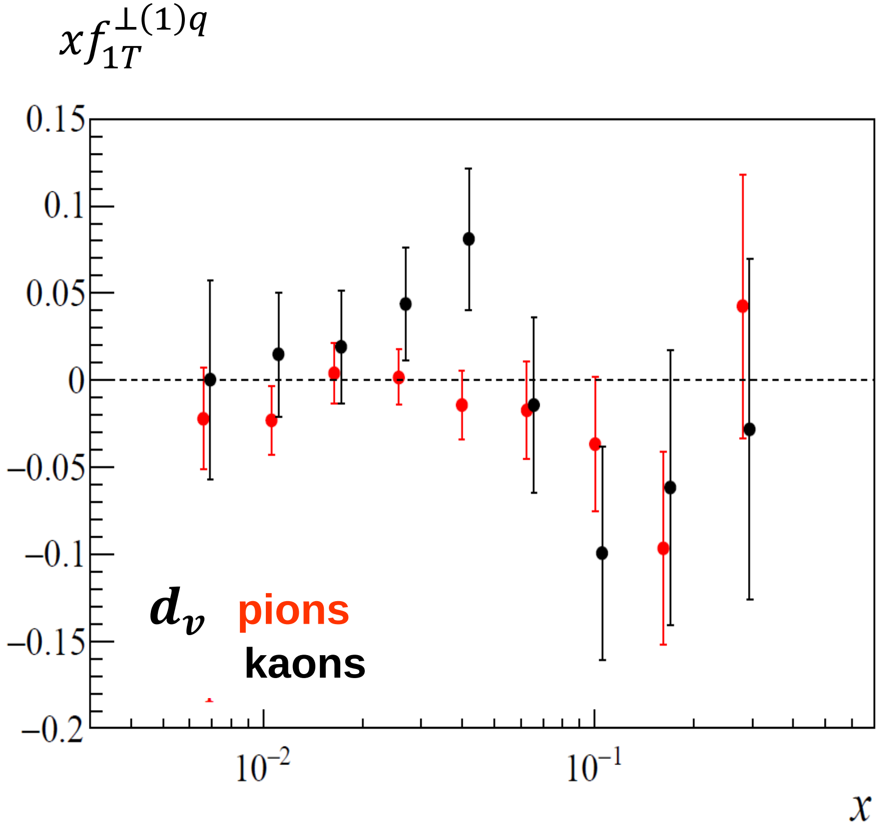
Sivers function from kaon asymmetries

extracted Sivers functions $xf_{1T}^{\perp(1)u_v}$ $xf_{1T}^{\perp(1)d_v}$

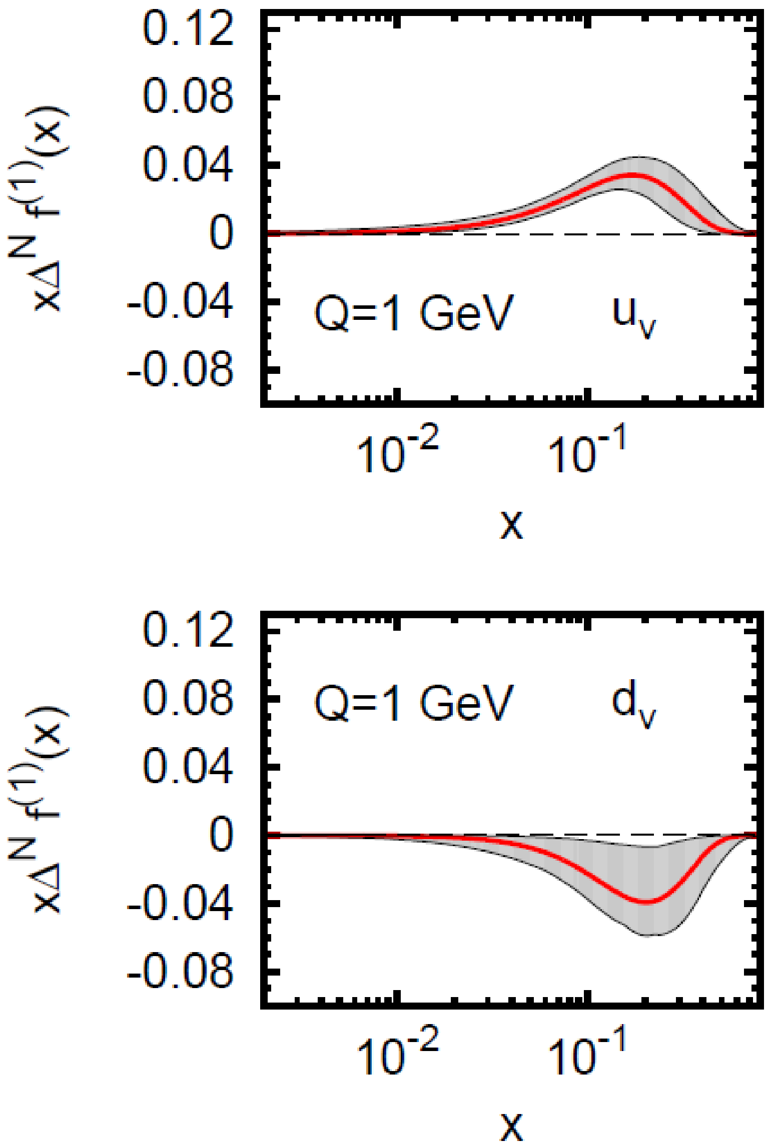


Sivers function from pions and kaons





Sivers function from pions and kaons



SIVERS FUNCTION - DGLAP

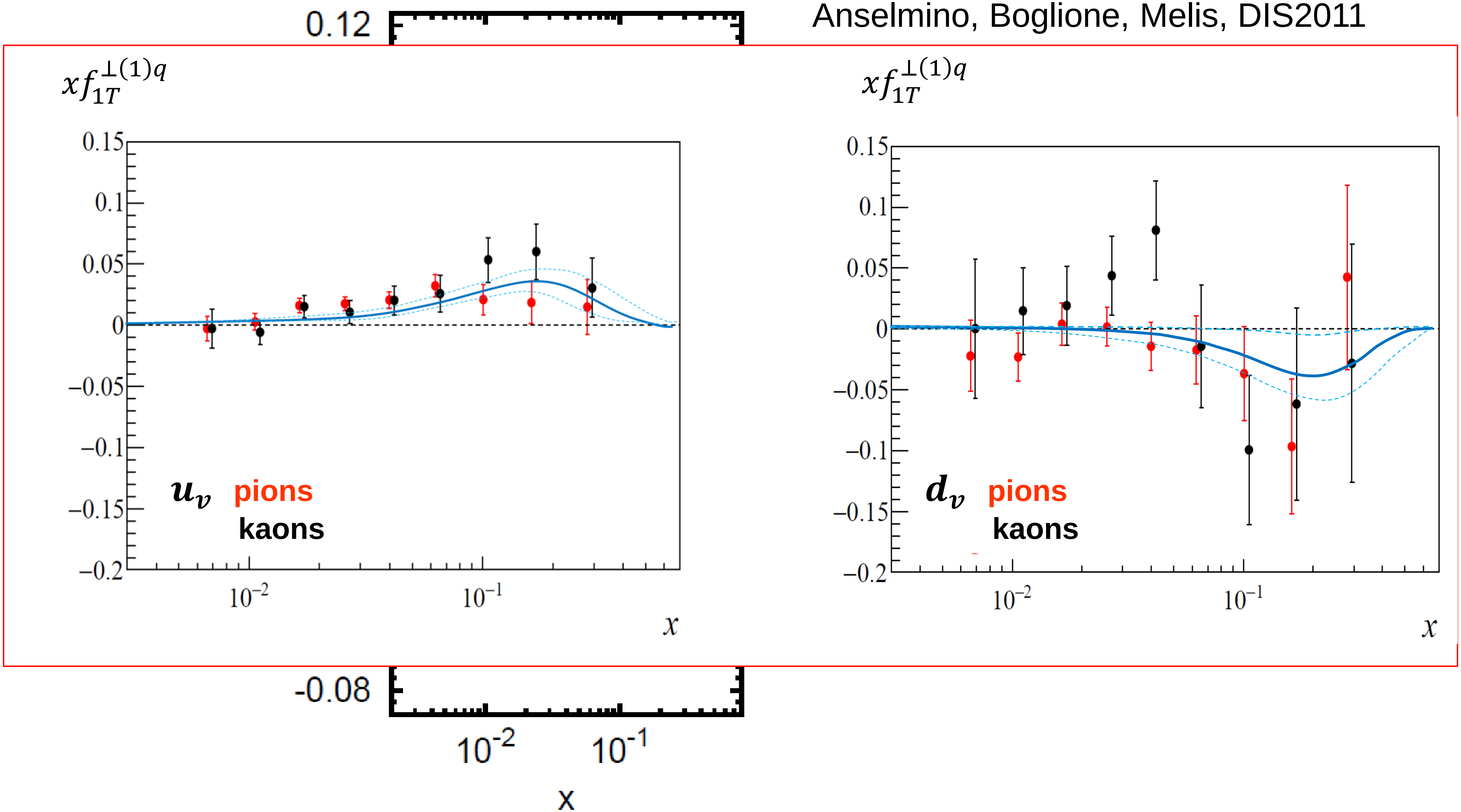
Anselmino, Boglione, Melis, DIS2011

fits to HERMES and COMPASS Sivers asymmetries

Sivers function from pions and kaons

SIVERS FUNCTION - DGLAP

Anselmino, Boglione, Melis, DIS2011



summary

the p and d SIDIS data at the same x, Q^2 allow for a point-by-point determination of the first k_T^2 moment of **the Sivers function**

the Sivers functions for the valence u and d quark are obtained independently from charged pion and charged kaon data

- the present statistics of the d data does not allow to constrain the d quark distribution
- the results from kaon and pions are compatible and have about the same statistical uncertainty

the pion data also allow to measure $f_{1T}^{\perp(1)\bar{u}} - f_{1T}^{\perp(1)\bar{d}}$

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the method is promising

and we plan to use it for the P_T weighted asymmetries, which do not require any assumption on the k_T dependence and will provide a test of the Gaussian model