Extraction of the $\pi^+\pi^-$ Subsystem in Diffractively Produced $\pi^-\pi^+\pi^-$ at COMPASS

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for the COMPASS collaboration

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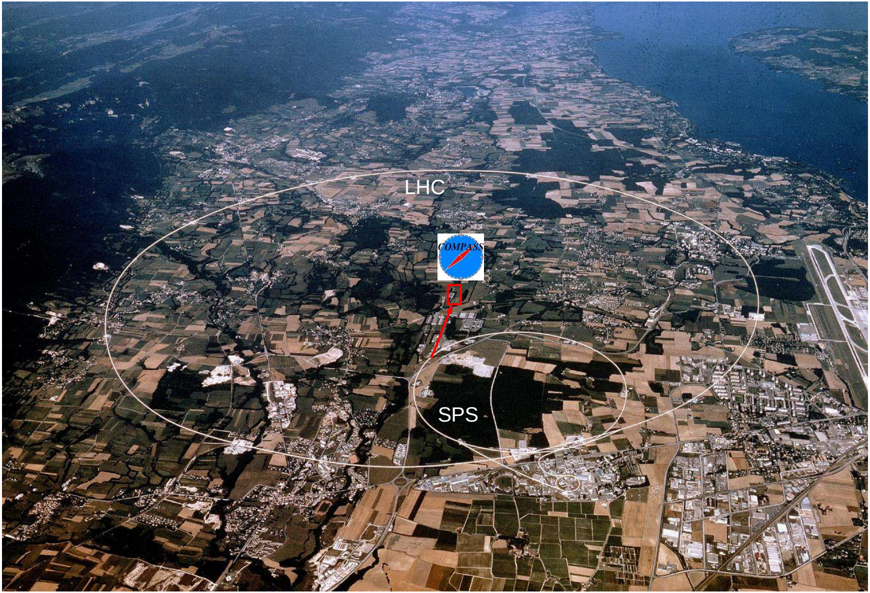
Technische Universität München

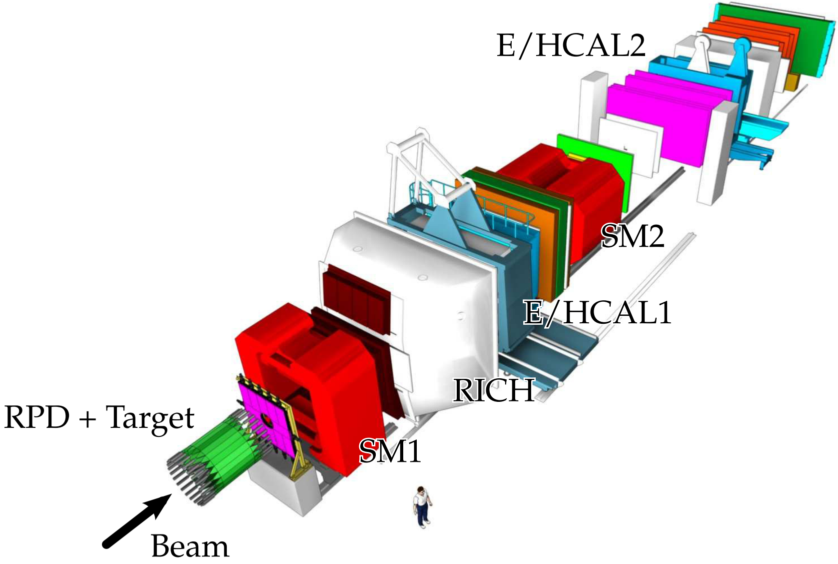
MENU 2016 KYOTO

Jul 29th 2016

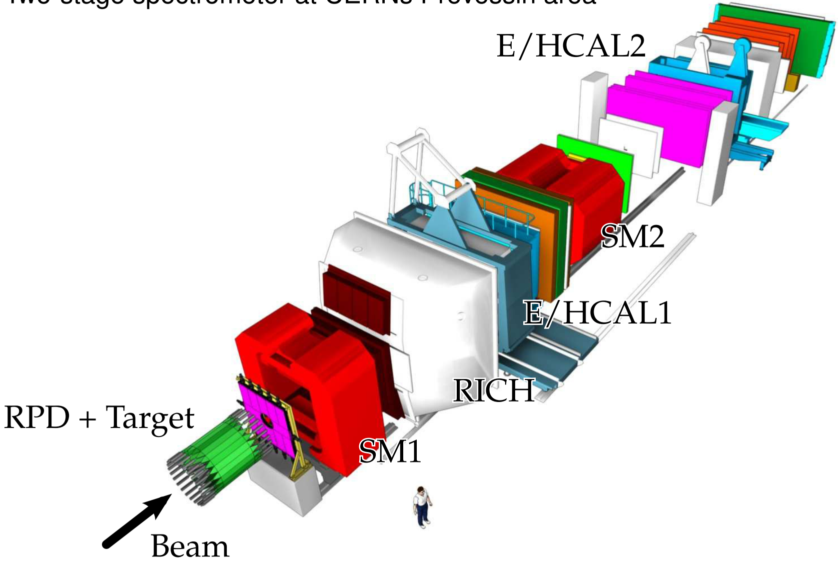
The COMPASS experiment
Located at CERN



The COMPASS experiment
Commom **M**uon **P**roton **A**pparatus for **S**tructure and **S**pectroscopy



Two-stage spectrometer at CERNs Prévessin area



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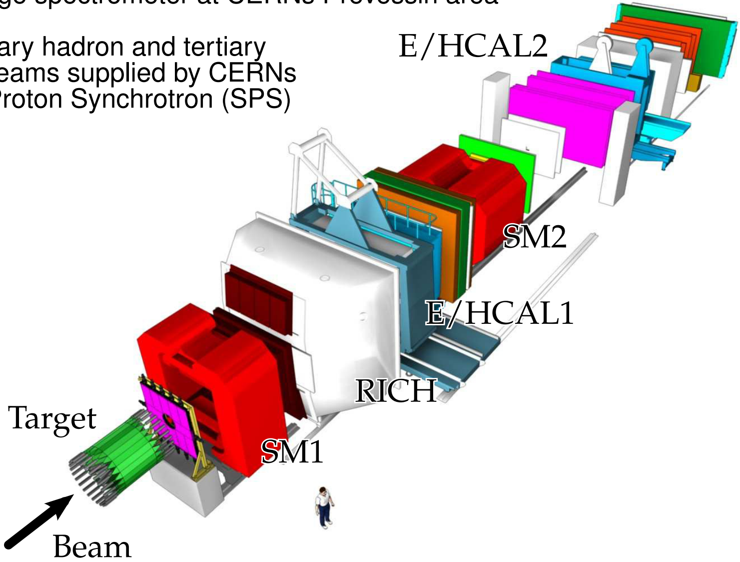
Secondary hadron and tertiary muon beams supplied by CERNs Super Proton Synchrotron (SPS)



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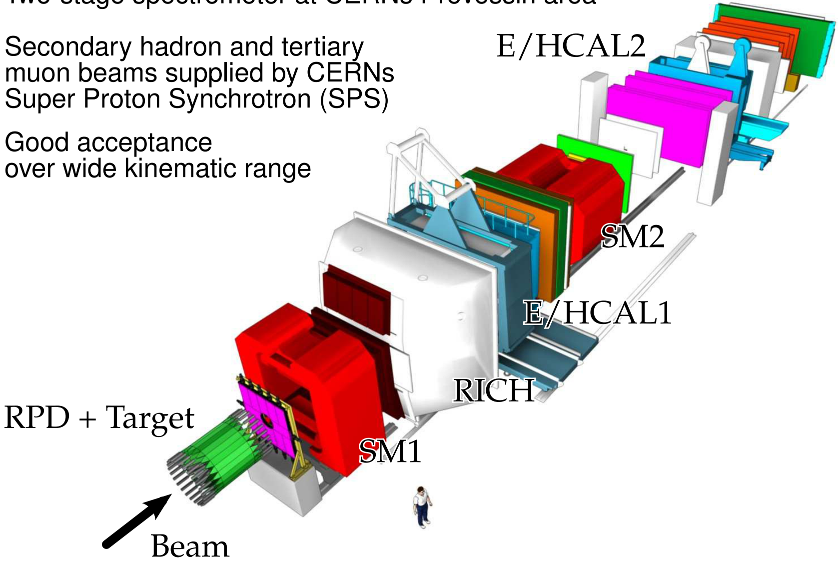
Good acceptance over wide kinematic range



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Broad physics program
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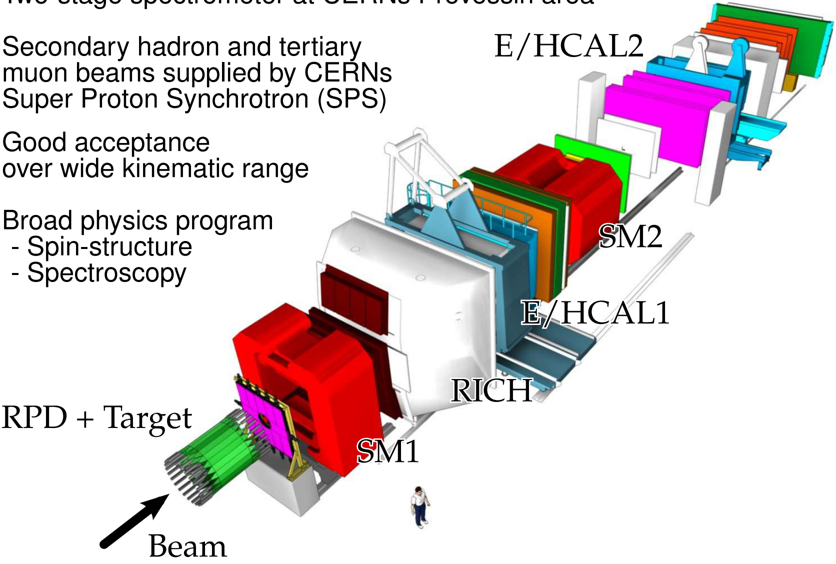
- Spectroscopy



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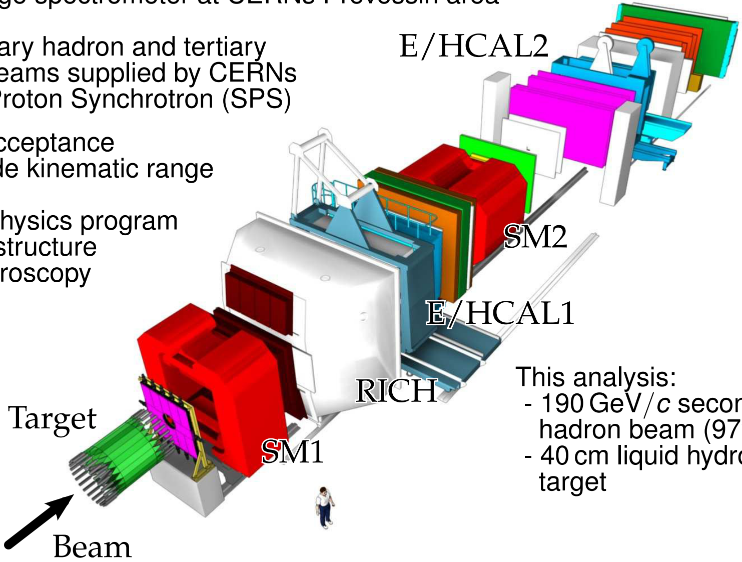
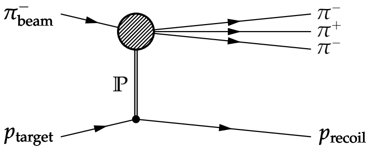
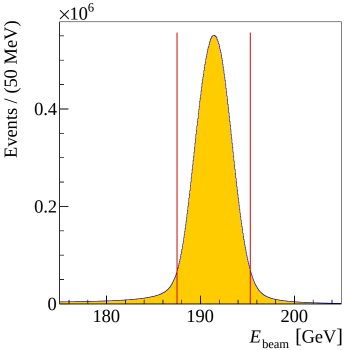


This analysis:
- 190 GeV/c secondary hadron beam (97% π^-)
- 40 cm liquid hydrogen target

Diffractive 3π production

- COMPASS: Currently world's largest data set for diffractive process
  $p + \pi^-_{\text{beam}} \rightarrow p + \pi^-\pi^+\pi^-$
  taken in 2008
  ($\sim 50 \cdot 10^6$ Events)
- Exclusive measurement





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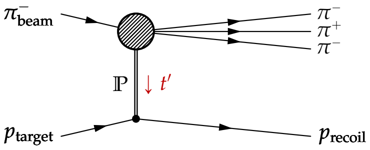
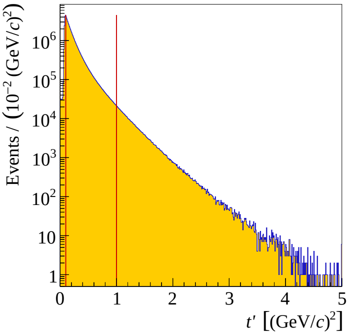
- Squared four-momentum transfer t' of Pomeron $\mathbb{P}$





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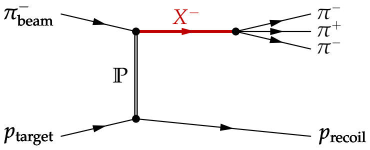
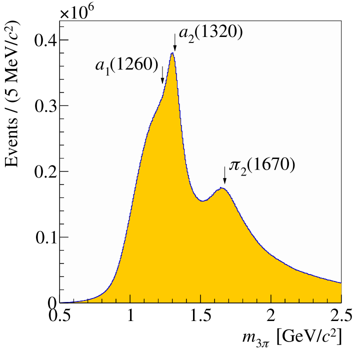
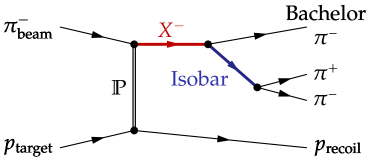
- Rich structure in $\pi^-\pi^+\pi^-$ mass spectrum

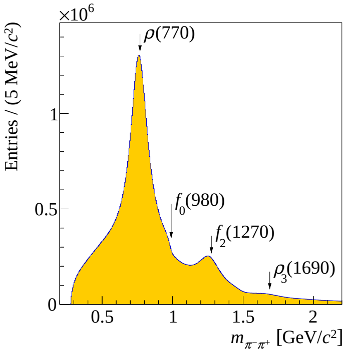




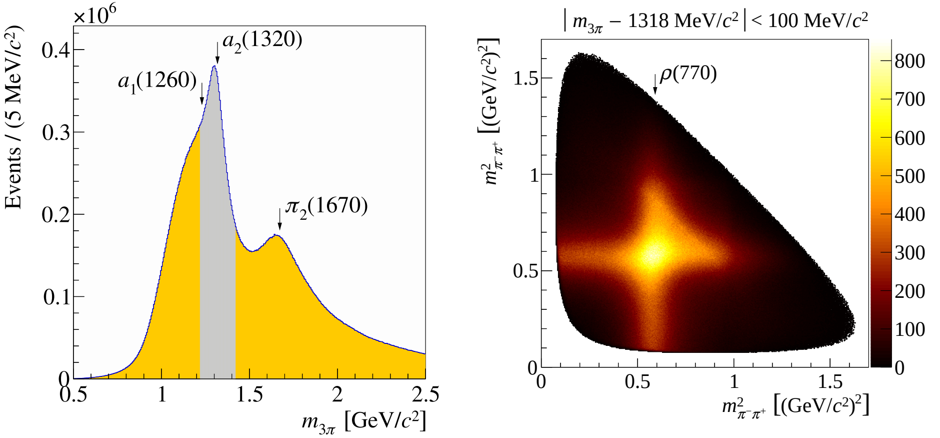
Diffractive 3π production

- COMPASS: Currently world's largest data set for diffractive process
  $p + \pi^-_{\text{beam}} \rightarrow p + \pi^-\pi^+\pi^-_{\text{bachelor}}$
  taken in 2008
  ($\sim 50 \cdot 10^6$ Events)
- Exclusive measurement
- Squared four-momentum transfer t' of Pomeron $\mathbb{P}$
- Rich structure in $\pi^-\pi^+\pi^-$ mass spectrum
- Also structure in $\pi^+\pi^-$ subsystem





Closer look at $\pi^+\pi^-$ substructures



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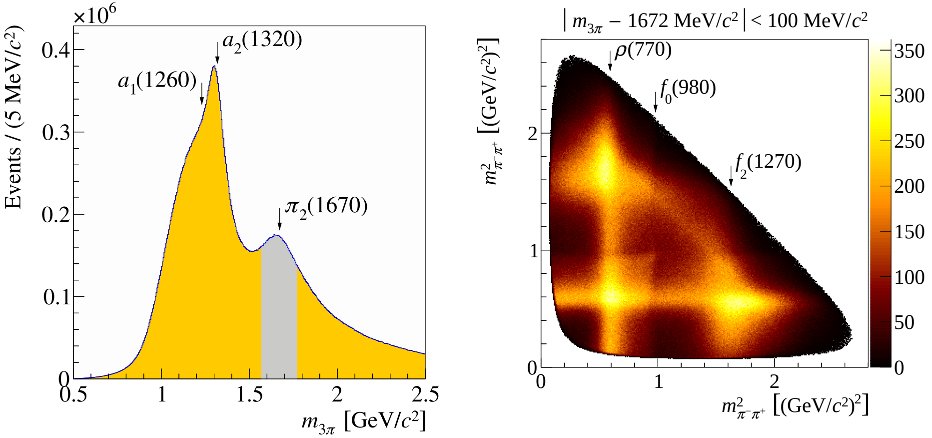

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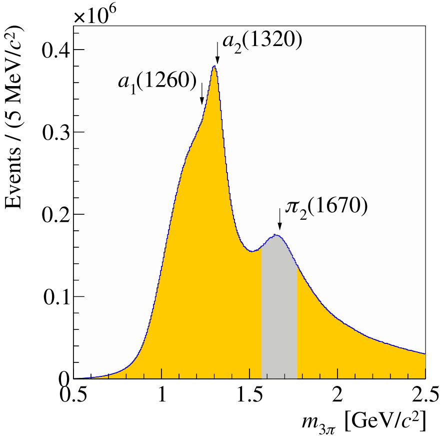
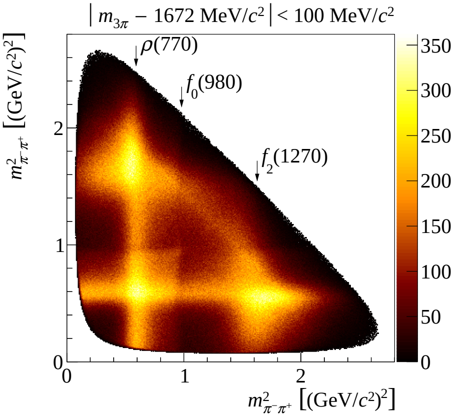
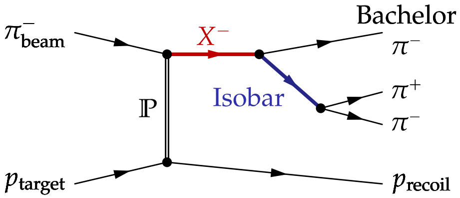




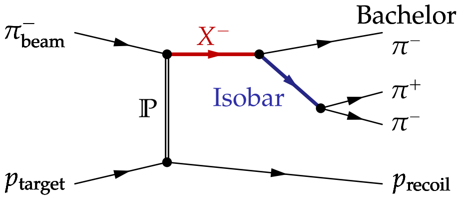
2π and 3π structures correlated

Use isobar model

- Beam pion excited to intermediate state X^-



- Beam pion excited to intermediate state X^-
- Subsequent two-particle decays: $X^- \to \xi\pi^- \to \pi^-\pi^+\pi^-$



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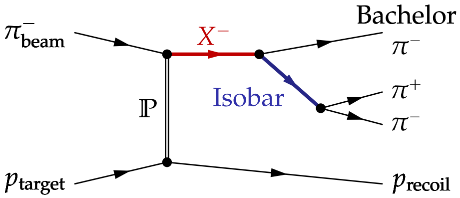

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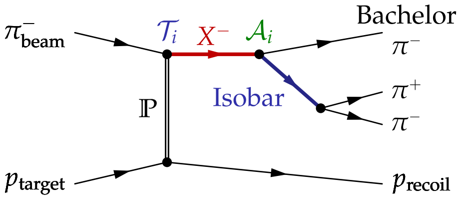
- Intensity modeled as $\mathcal{I} = |\mathcal{A}|^2$

$$\mathcal{A}(m_{3\pi}, \tau) = \sum_i^{\text{waves}} \mathcal{T}_i(m_{3\pi})\mathcal{A}_i(\tau)$$

phase-space variables τ



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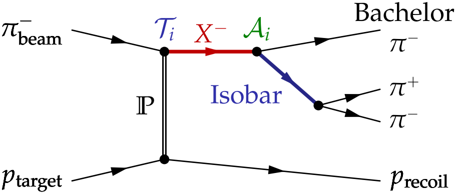
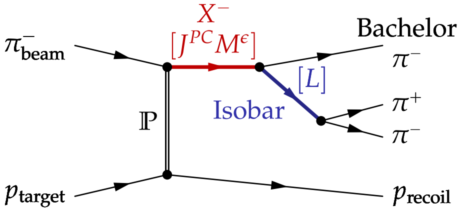
  phase-space variables τ
- Narrow bins in $m_{X^-} = m_{3\pi}$: No assumptions on shape of X^-



Naming scheme for partial waves

$$J^{PC} M^{\varepsilon} \xi \pi L$$



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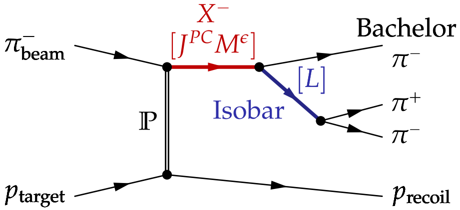

- J^{PC}: spin and eigenvalues under parity and charge conjugation of X^-

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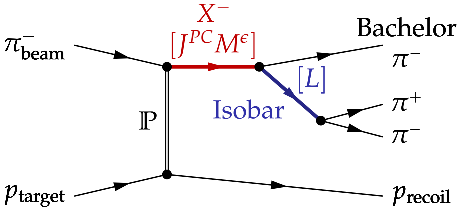

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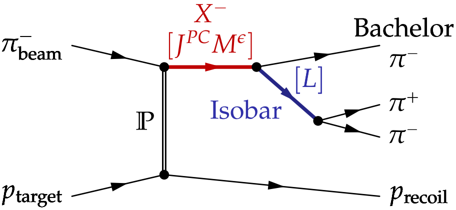

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- ξ: Appearing isobar, e.g. $\rho(770)$

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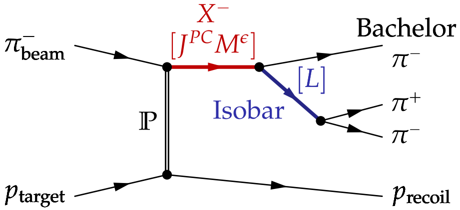

- J^{PC}: spin and eigenvalues under parity and charge conjugation of X^-
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- π: Indicating the bachelor π^-. Always the same

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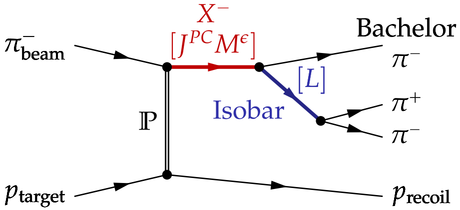

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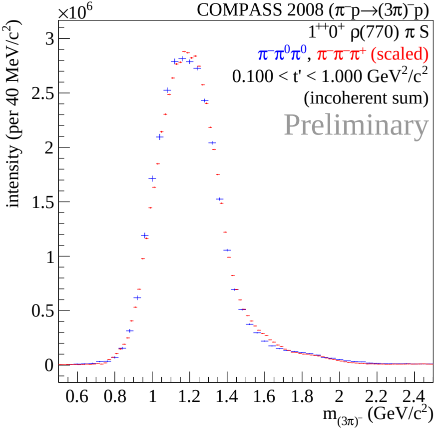
- π: Indicating the bachelor π^-. Always the same
- L: Orbital angular momentum between isobar and bachelor pion

Selected waves of established PWA

$1^{++}0^{+}\rho(770)\pi S$
$a_1(1260)$



- Intensity $|\mathcal{T}|^2$ plotted
- Each point is an independent fit
- Two 3π channels agree nicely

Selected waves of established PWA

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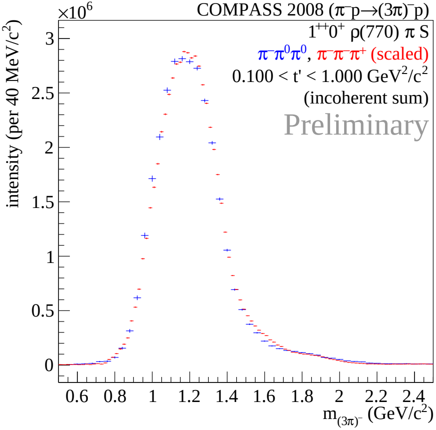
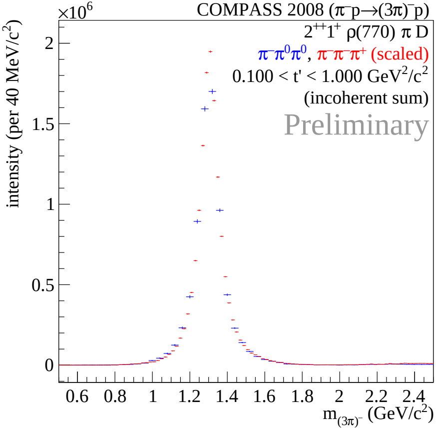
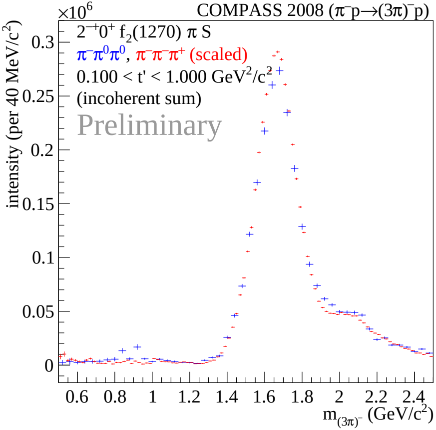


$2^{++}1^{+}\rho(770)\pi D$
$a_2(1320)$



$2^{-+}0^{+} f_2(1270)\pi S$

$\pi_2(1670)$



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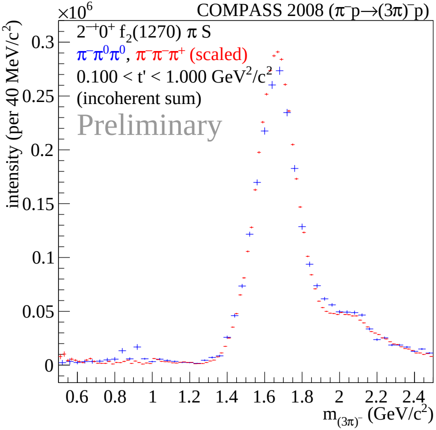
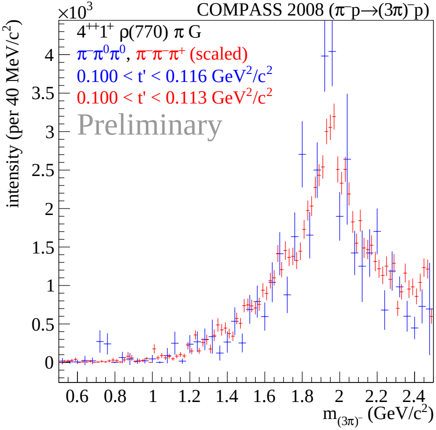
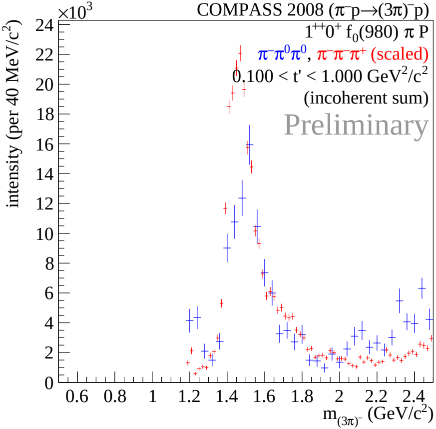


$4^{++}1^{+} \rho(770)\pi G$
$a_4(2040)$



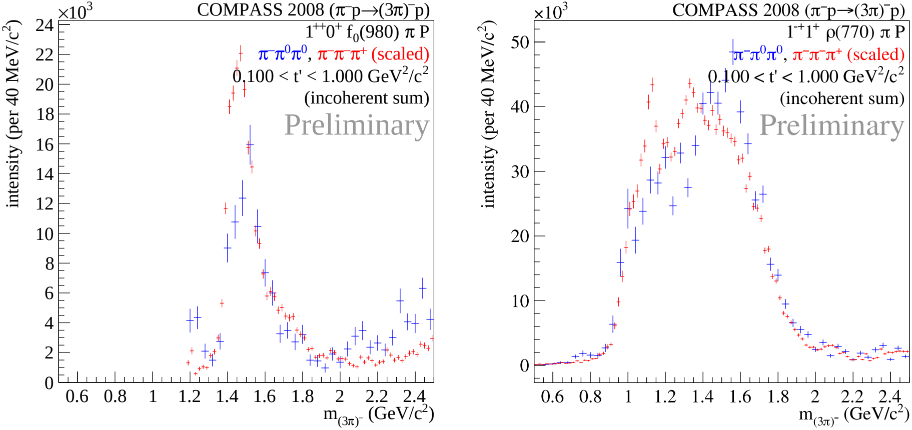
$1^{++}0^{+} f_0(980)\pi P$
New $a_1(1420)$



Selected waves of established PWA

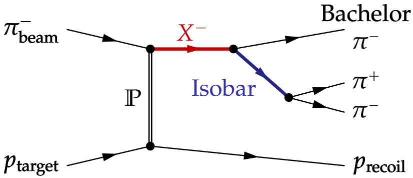
$1^{++}0^{+}f_0(980)\pi P$
New $a_1(1420)$

$1^{-+}1^{+}\rho(770)\pi P$
Possible $\pi_1(\ldots)$?



Novel approach

- Isobar amplitudes in established PWA:
  - J_{ξ}^{PC}: Isobar



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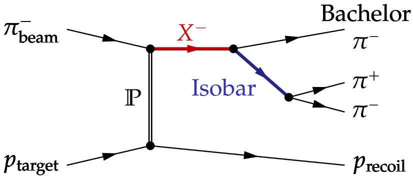
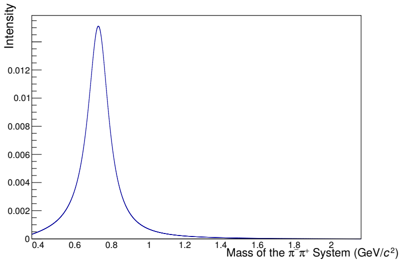
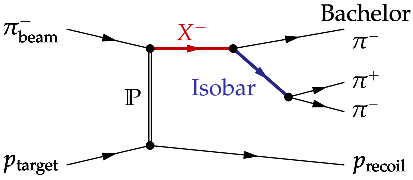
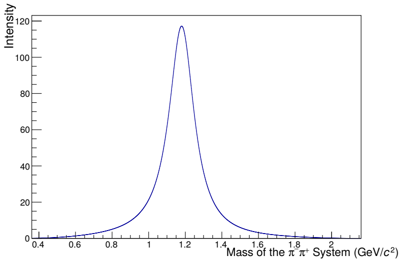




- Isobar amplitudes in established PWA:
  - J_{ξ}^{PC}: Isobar
  - 1^{--}: $\rho(770)$
  - 2^{++}: $f_2(1270)$





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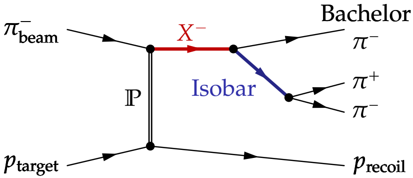

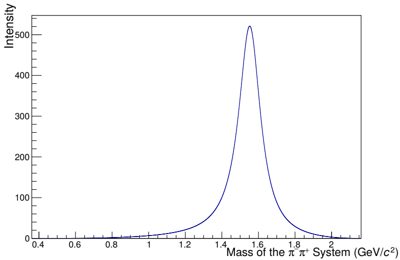

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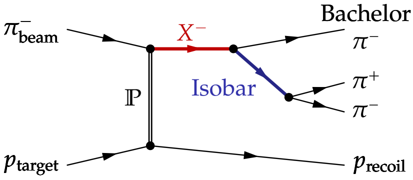
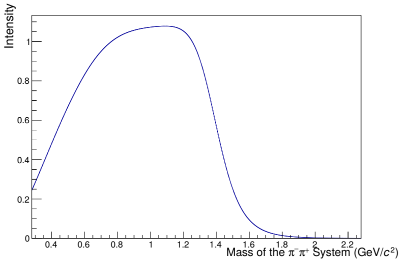
  - 0^{++}: $f_0(500)$





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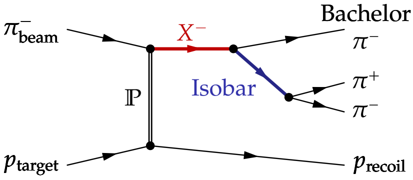
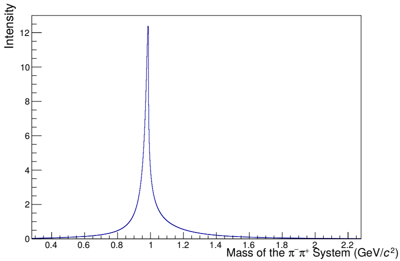
  - 0^{++}: $f_0(980)$





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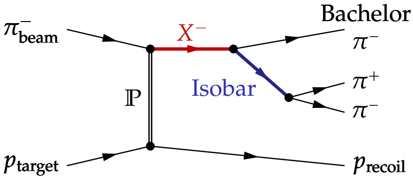
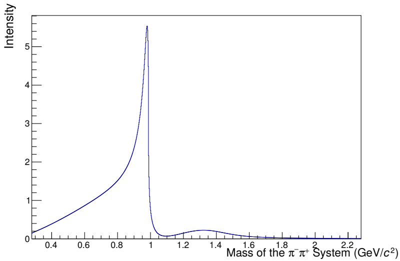
- Real shape may be complicated





Example: Shape of 0^{++} intensity resulting from interference of $f_0(500)$ and $f_0(980)$

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  - 0^{++}: $f_0(500)$
  - 0^{++}: $f_0(980)$

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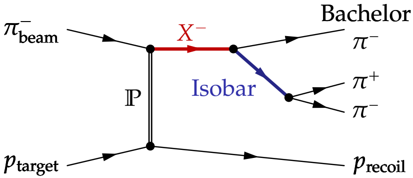
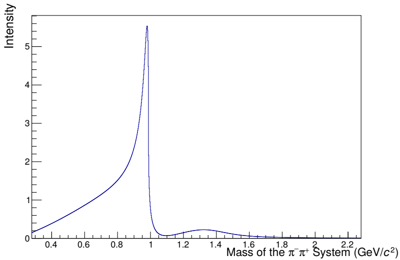
How good are the parametrizations used?

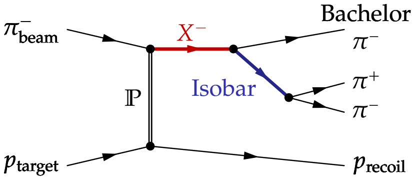
How good is the isobar model?

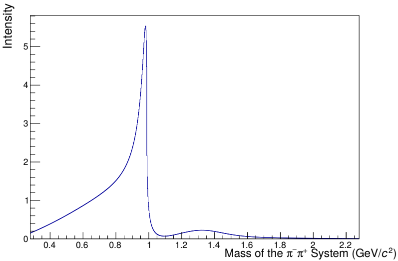




Example: Shape of 0^{++} intensity resulting from interference of $f_0(500)$ and $f_0(980)$

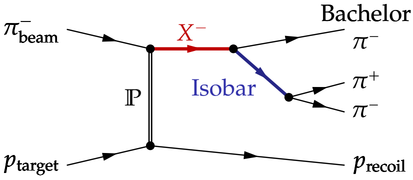
- Direct fit of isobar shapes computationally not feasible

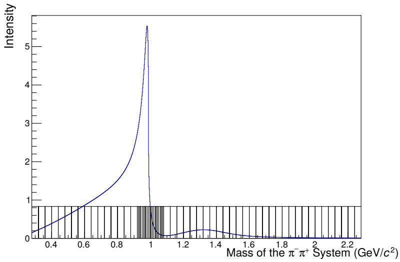




Example: Shape of 0^{++} intensity resulting from interference of $f_0(500)$ and $f_0(980)$

- Direct fit of isobar shapes computationally not feasible
- Replace with sets of step-like isobars

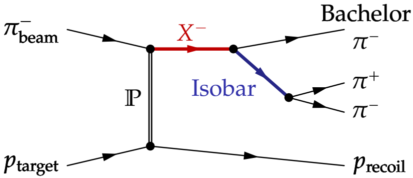


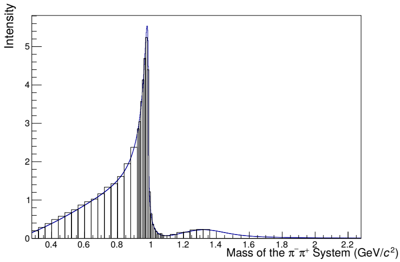


Example: Shape of 0^{++} intensity resulting from interference of $f_0(500)$ and $f_0(980)$

Novel approach
Step-like isobar amplitudes

- Direct fit of isobar shapes computationally not feasible
- Replace with sets of step-like isobars
- Extract binned shape





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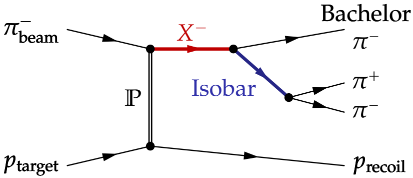
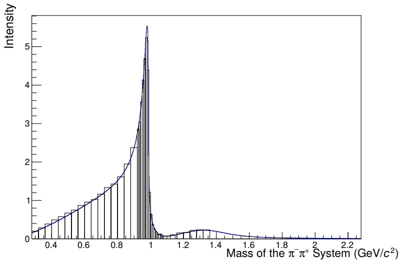
Novel approach

Step-like isobar amplitudes

- Direct fit of isobar shapes computationally not feasible
- Replace with sets of step-like isobars
- Extract binned shape
- Obtain isobar amplitudes directly from the data





Example: Shape of 0^{++} intensity resulting from interference of $f_0(500)$ and $f_0(980)$

- Total intensity in conventional PWA

$$\mathcal{I}(m_{3\pi}, m_{\pi^+\pi^-}, \tau) = \left|\sum_i^{\text{waves}} \mathcal{T}_i(m_{3\pi})\psi_i(\tau)\Delta_i(m_{\pi^+\pi^-})\right|^2$$

Fit parameters: Production amplitudes $\mathcal{T}_i(m_{3\pi})$
Fixed: Angular distributions $\psi(\tau)$ and isobar amplitudes $\Delta_i(m_{\pi^+\pi^-})$,
$\mathcal{A}_i = \psi(\tau)\Delta_i(m_{\pi^+\pi^-})$

Freed isobar fit

Formulation

- Total intensity in conventional PWA

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- Fixed isobar amplitudes $\rightarrow$ Sets of bins:

$$\Delta_i(m_{\pi^+\pi^-}) \rightarrow \sum_{\text{bins}} \Delta_i^{\text{bin}}(m_{\pi^+\pi^-}) \equiv [\pi\pi]_{J^{PC}}$$

$$\Delta_i^{\text{bin}}(m_{\pi^+\pi^-}) = \begin{cases} 1, & \text{if } m_{\pi^+\pi^-} \text{ in the bin.} \\ 0, & \text{otherwise.} \end{cases}$$

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- Each $m_{\pi^+\pi^-}$ bin behaves like an independent Partial Wave:

$$\mathcal{I} = \left| \sum_i^{\text{waves}} \sum_{\text{bin}}^{\text{bins}} \mathcal{T}_i^{\text{bin}}(m_{3\pi}) \psi_i(\tau) \Delta_i^{\text{bin}}(m_{\pi^+\pi^-}) \right|^2$$

Two dimensional results

- Conventional PWA: One-dimensional result $\mathcal{T}_i(m_{3\pi})$
- Freed isobar PWA: Two-dimensional result: $\mathcal{T}_i(m_{3\pi}, m_{\pi^+\pi^-})$
- First analysis: 3 waves with freed isobars:
  - $0^{-+}0^+[\pi\pi]_{0^{++}}\,\pi S$
  - $1^{++}0^+[\pi\pi]_{0^{++}}\,\pi P$
  - $2^{-+}0^+[\pi\pi]_{0^{++}}\,\pi D$

  arXiv:1509.00992 [hep-ex]
- Other waves still with fixed isobar amplitudes: $\rho(770)$, $f_2(1270)$, $\rho_3(1690)$
  - In principle also possible for 1^{--}, 2^{++}, … isobars

Two-dimensional intensity for waves with freed isobars

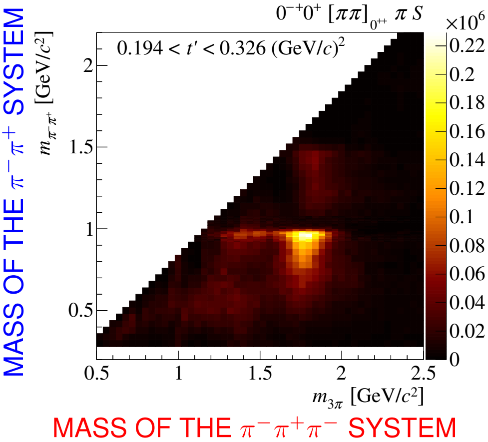
MASS OF THE $\pi^-\pi^+$ SYSTEM

MASS OF THE $\pi^-\pi^+\pi^-$ SYSTEM

This is not a Dalitz plot

Two-dimensional intensity for waves with freed isobars

$0^{-+}0^{+}[\pi\pi]_{0^{++}}\ \pi\ S$

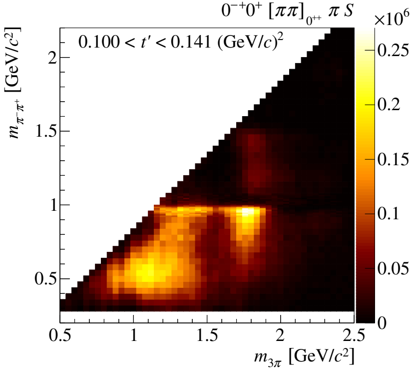


This is not a Dalitz plot

$0^{-+}0^{+}[\pi\pi]_{0^{++}}\,\pi S$

Different t' regions

$0.10 < t' < 0.14\,(\text{GeV}/c)^2$ $\qquad$ $0.19 < t' < 0.33\,(\text{GeV}/c)^2$



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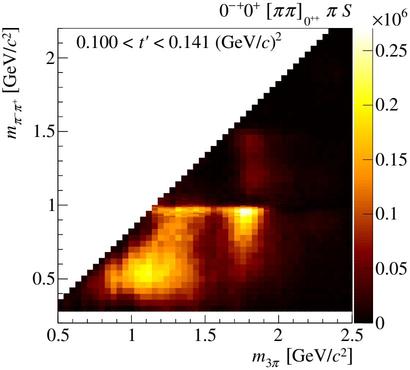

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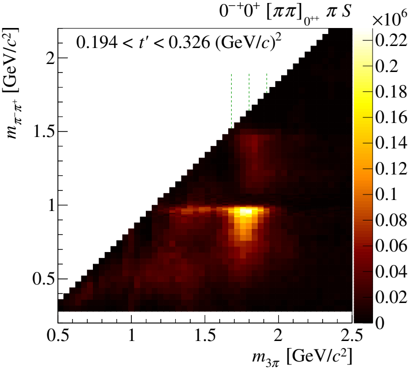
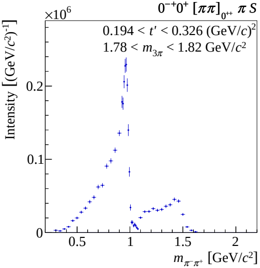
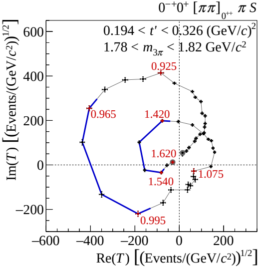


$0.19 < t' < 0.33 (\mathrm{GeV}/c)^2$

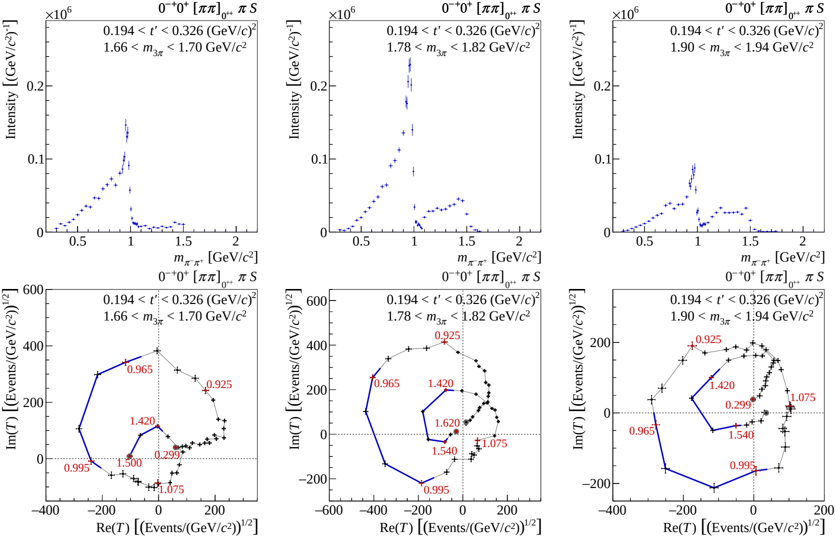
$1.78 < m_{3\pi} < 1.82$ GeV/c^2



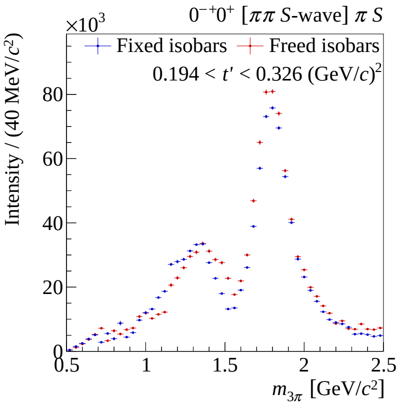


$1.66 < m_{3\pi} < 1.70$ GeV/c^2 | $1.78 < m_{3\pi} < 1.82$ GeV/c^2 | $1.90 < m_{3\pi} < 1.94$ GeV/c^2

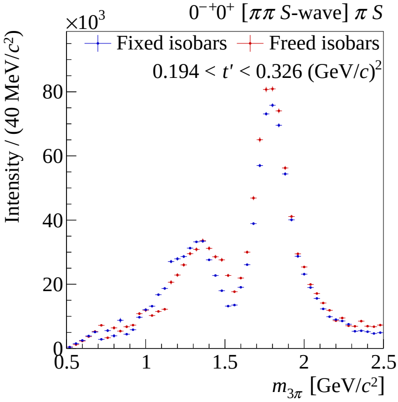
$0.19 < t' < 0.33(\text{GeV}/c)^2$



- Sum up all amplitudes in $m_{\pi^+\pi^-}$
- Compare with sum of conventional $f_0(\ldots)\pi^-$ amplitudes



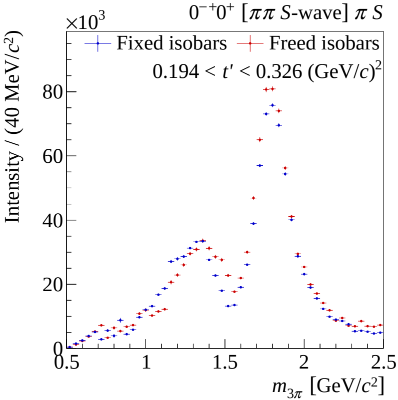
- Sum up all amplitudes in $m_{\pi^+\pi^-}$
- Compare with sum of conventional $f_0(\ldots)\pi^-$ amplitudes
- $\pi(1800)$ peak visible



$0^{-+}0^{+}[\pi\pi]_{0^{++}}\,\pi S$

Comparison with conventional analysis

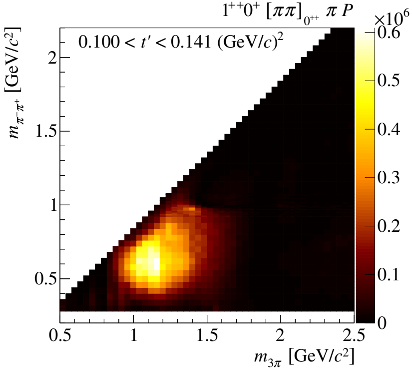
- Sum up all amplitudes in $m_{\pi^+\pi^-}$
- Compare with sum of conventional $f_0(\ldots)\pi^-$ amplitudes
- $\pi(1800)$ peak visible
- Novel method reproduces shape in $m_{3\pi}$



$1^{++}0^{+}[\pi\pi]_{0^{++}}\,\pi P$

Different t' regions

$0.10 < t' < 0.14\,(\text{GeV}/c)^2$



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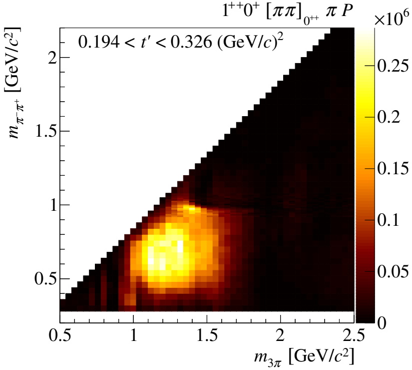

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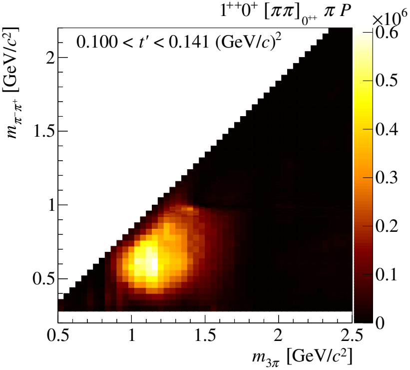

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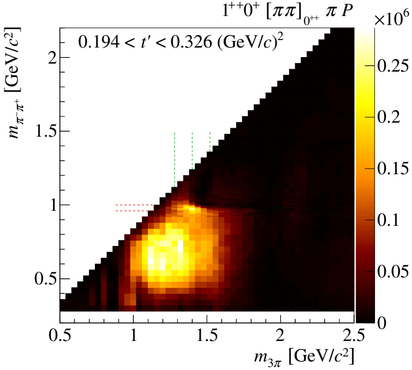
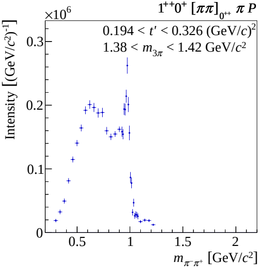


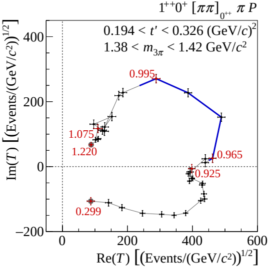
$1^{++}0^{+}[\pi\pi]_{0^{++}}\,\pi P$

Slices in $m_{3\pi}$

$0.19 < t' < 0.33 (\mathrm{GeV}/c)^2$

$1.38 < m_{3\pi} < 1.42$ GeV/c^2



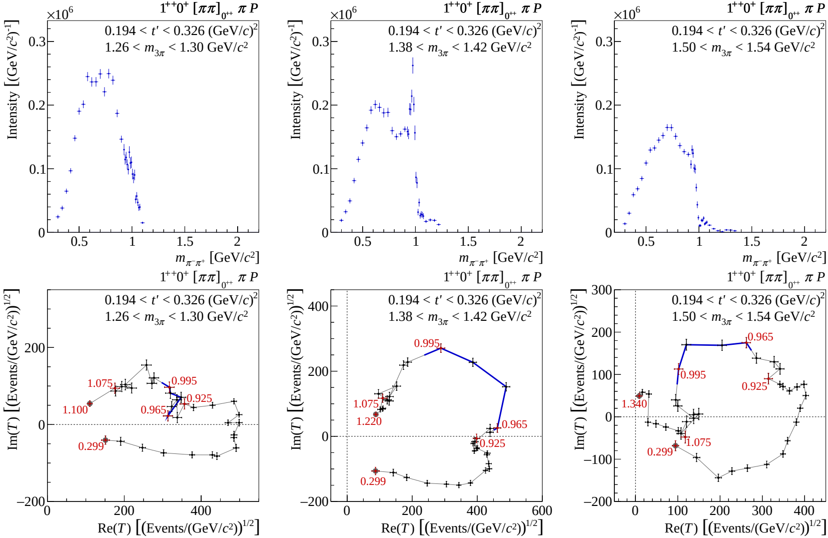


$1.26 < m_{3\pi} < 1.30$ GeV/c^2

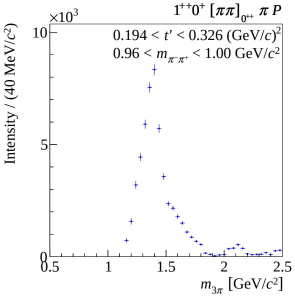
$1.38 < m_{3\pi} < 1.42$ GeV/c^2

$1.50 < m_{3\pi} < 1.54$ GeV/c^2

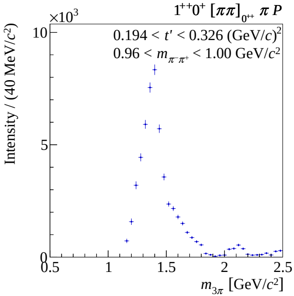
$0.19 < t' < 0.33 (\text{GeV}/c)^2$

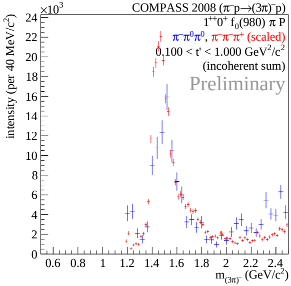


- Sum up amplitudes in the $f_0(980)$ region



- Sum up amplitudes in the $f_0(980)$ region
- Compare with $1^{++}0^{+} f_0(980)\pi P$ wave from established PWA

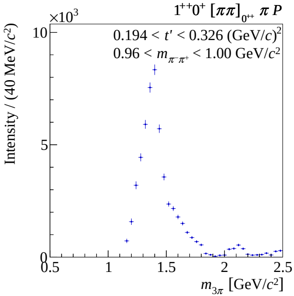


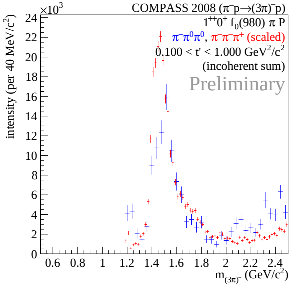


$1^{++}0^{+}[\pi\pi]_{0^{++}}\,\pi P$

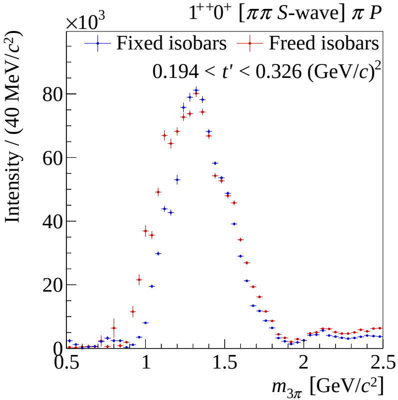
Comparison of $f_0(980)$ region

- Sum up amplitudes in the $f_0(980)$ region
- Compare with $1^{++}0^{+} f_0(980)\pi P$ wave from established PWA
- New resonance $a_1(1420)$ reproduced
- Not an artifact of isobar parametrizations





- Sum up all amplitudes in $m_{\pi^+\pi^-}$
- Compare with sum of conventional $f_0(\ldots)\pi^-$ amplitudes
  - $1^{++}0^{+}f_0(500)\pi P$
  - $1^{++}0^{+}f_0(980)\pi P$



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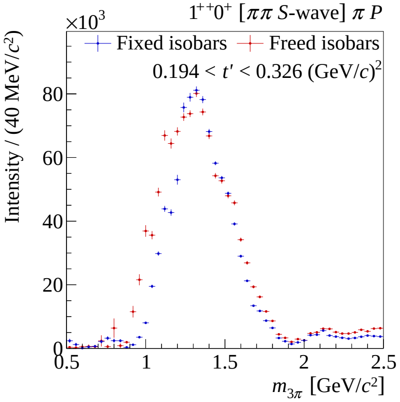
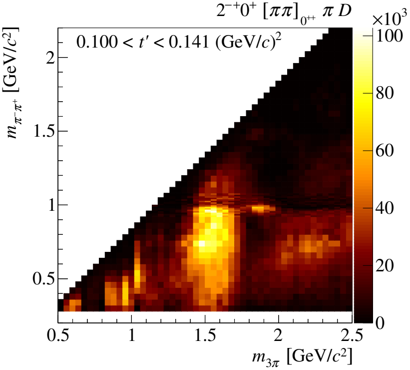
- Compatible shapes
- Isobar model works



$0.10 < t' < 0.14\,(\text{GeV}/c)^2$



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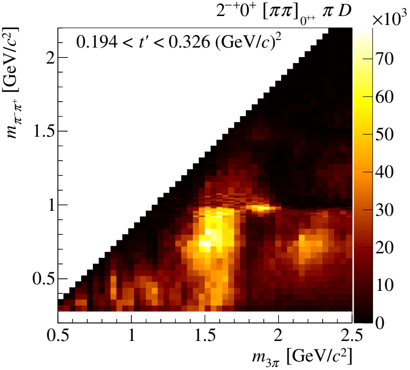
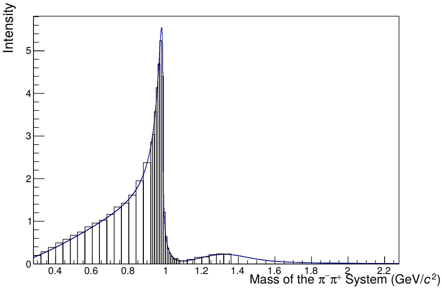


Conclusions

- Novel method:
  Fixed isobar amplitudes replaced
  by sets of binned functions $[\pi\pi]_{J^{PC}}$



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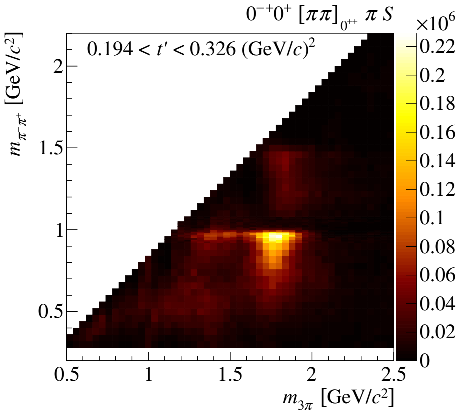
- Study resonance production in three dimensions: $m_{3\pi}$, $m_{\pi^+\pi^-}$ and t'



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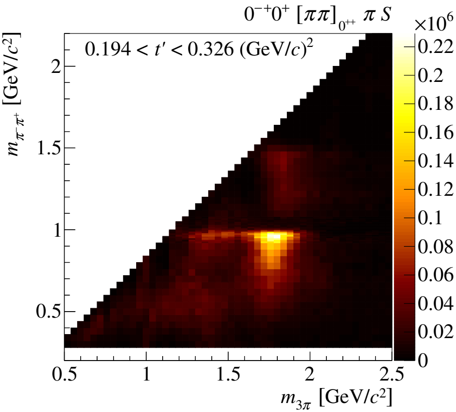

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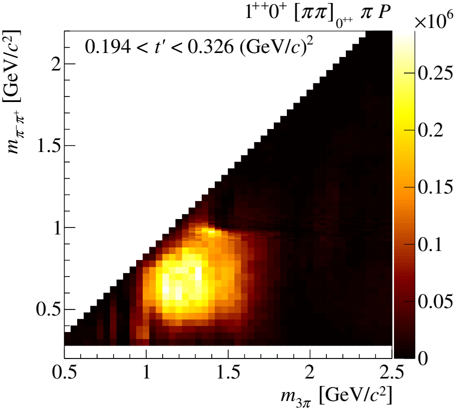

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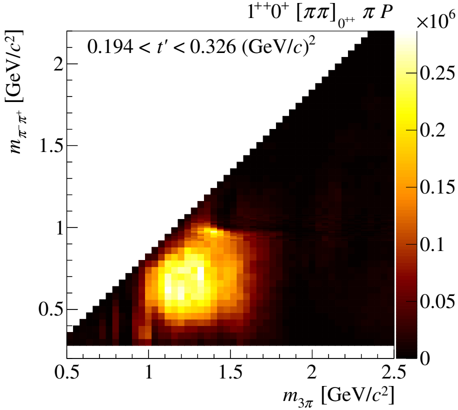
- The new $a_1(1420) \to f_0(980)\pi^-$ is confirmed
- t' dependent, broad structures at small $m_{3\pi}$, $m_{\pi^+\pi^-}$
  $\to$ Possible non-resonant processes



Outlook

- Effects from imperfect parametrizations in other waves

  $\rightarrow$ Free isobar-amplitudes for all large waves

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- Goal: Free 11 waves

$0^{-+}\,0^{+}\,f_0(980)\pi S$

$0^{-+}\,0^{+}\,\rho(770)\pi P$

$1^{++}\,0^{+}\,f_0(980)\pi P$

$1^{++}\,0^{+}\,\rho(770)\pi S$

$1^{++}\,1^{+}\,\rho(770)\pi S$

$2^{-+}\,0^{+}\,f_0(980)\pi D$

$2^{-+}\,0^{+}\,\rho(770)\pi P$

$2^{-+}\,0^{+}\,\rho(770)\pi F$

$2^{-+}\,0^{+}\,\rho(770)\pi P$

$2^{-+}\,1^{+}\,f_2(1270)\pi S$

$2^{++}\,1^{+}\,\rho(770)\pi S$

Outlook

- Effects from imperfect parametrizations in other waves

  $\rightarrow$ Free isobar-amplitudes for all large waves

- Goal: Free 11 waves
  - 75% of the total intensity
  - All waves that contribute more than 1% to the intensity

$0^{-+}0^{+}f_0(980)\pi S$

$0^{-+}0^{+}\rho(770)\pi P$

$1^{++}0^{+}f_0(980)\pi P$

$1^{++}0^{+}\rho(770)\pi S$

$1^{++}1^{+}\rho(770)\pi S$

$2^{-+}0^{+}f_0(980)\pi D$

$2^{-+}0^{+}\rho(770)\pi P$

$2^{-+}0^{+}\rho(770)\pi F$

$2^{-+}0^{+}\rho(770)\pi P$

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Outlook

- Effects from imperfect parametrizations in other waves

  $\rightarrow$ Free isobar-amplitudes for all large waves

- Goal: Free 11 waves
  - 75% of the total intensity
  - All waves that contribute more than 1% to the intensity
- Challenges:
  - Drastic increase in number of parameters
  - Appearance of linear dependences, which cause ambiguities

$0^{-+}\,0^{+}\,f_0(980)\pi S$

$0^{-+}\,0^{+}\,\rho(770)\pi P$

$1^{++}\,0^{+}\,f_0(980)\pi P$

$1^{++}\,0^{+}\,\rho(770)\pi S$

$1^{++}\,1^{+}\,\rho(770)\pi S$

$2^{-+}\,0^{+}\,f_0(980)\pi D$

$2^{-+}\,0^{+}\,\rho(770)\pi P$

$2^{-+}\,0^{+}\,\rho(770)\pi F$

$2^{-+}\,0^{+}\,\rho(770)\pi P$

$2^{-+}\,1^{+}\,f_2(1270)\pi S$

$2^{++}\,1^{+}\,\rho(770)\pi S$