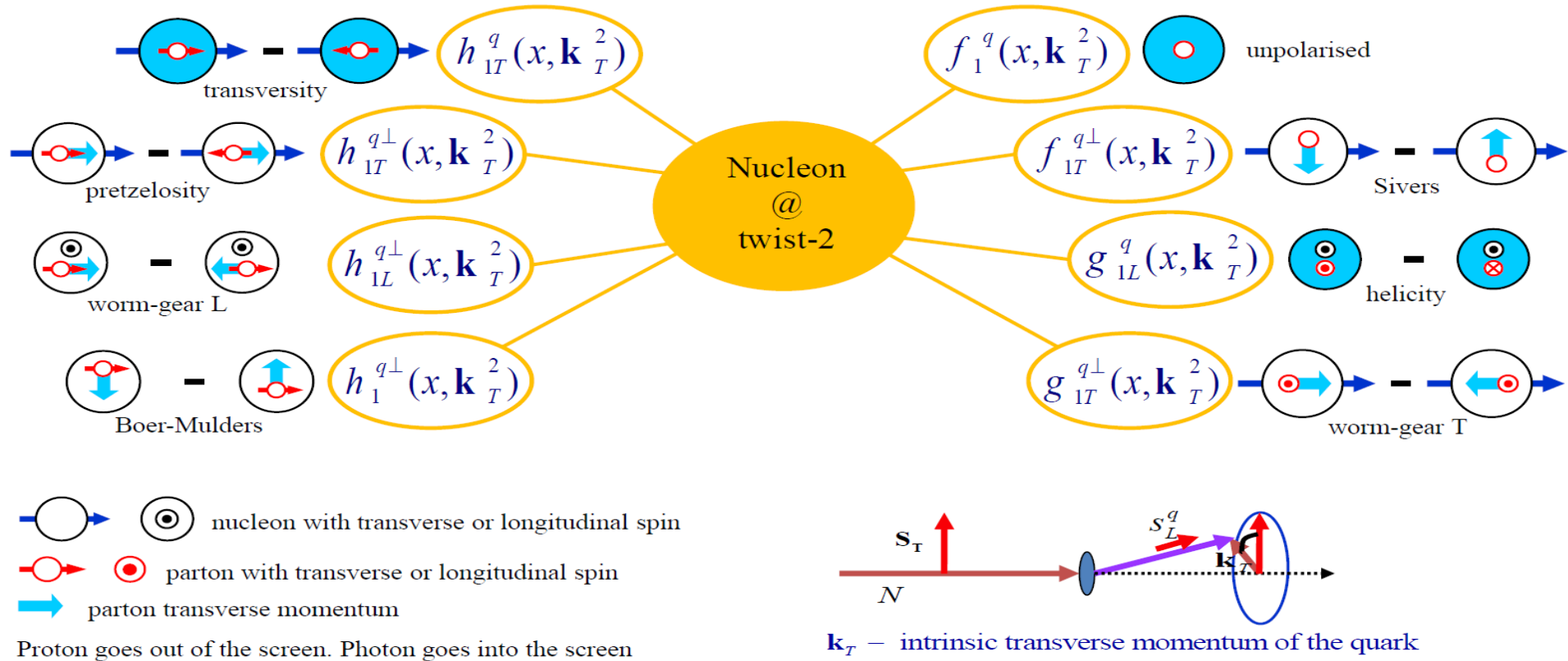




Andrea Bressan (University of Trieste and INFN)
On behalf of the COMPASS Collaboration

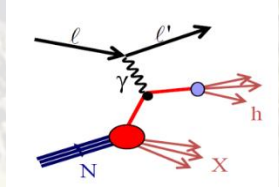
Recent COMPASS results on Transverse Spin and Momentum dependent distributions and fragmentation functions

TMD Distribution Functions



Accessing Spin and TMD PDFs and FFs

- SIDIS off polarized p, d, n targets

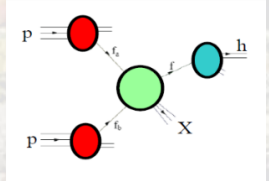


HERMES
COMPASS
JLab

$$\sigma^{\ell p \rightarrow \ell' h X} \sim q(x) \otimes \hat{\sigma}^{\gamma q \rightarrow q} \otimes D_q^h(z)$$

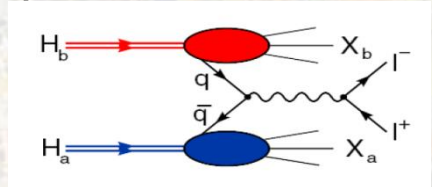
future: **eN colliders?**

- hard polarised pp scattering



RHIC

- polarised Drell-Yan

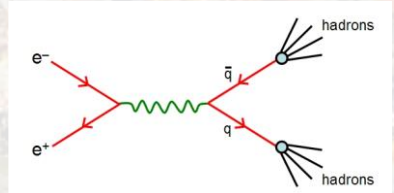


COMPASS
RHIC
FNAL

$$\sigma^{hp \rightarrow \mu\mu} \sim \bar{q}_h(x_1) \otimes q_p(x_2) \otimes \hat{\sigma}^{\bar{q}q \rightarrow \mu\mu}(\hat{s})$$

future: **FAIR, JPark, NICA**

- $e^+ e^- \rightarrow h_1 h_2$



BaBar
Belle
Bes III

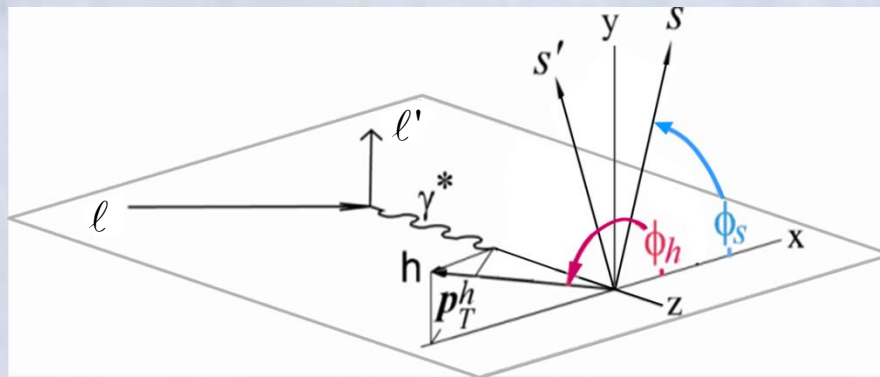
$$\sigma^{e^+ e^- \rightarrow h_1 h_2} \sim \hat{\sigma}^{\ell\ell \rightarrow \bar{q}q}(\hat{s}) \otimes D_q^{h_1}(z_1) \otimes D_q^{h_2}(z_2)$$

Azimuthal asymmetries

SIDIS:

Azimuthal asymmetries in the
Gamma-Nucleon System (GNS)

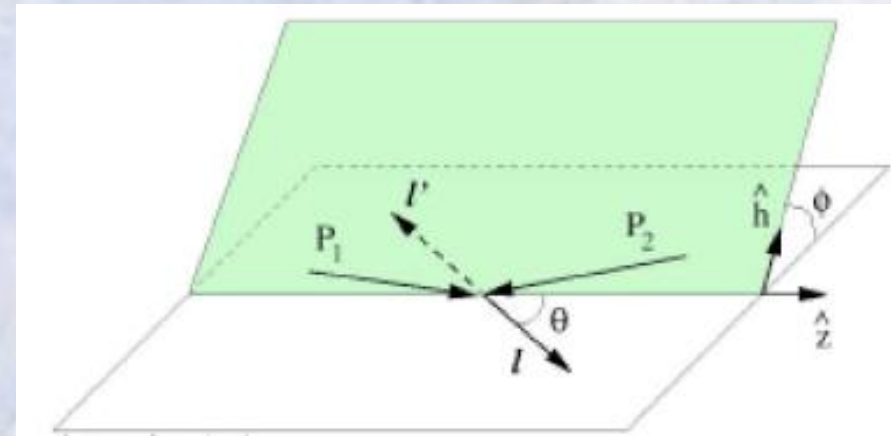
Needs 2π acceptance with respect
To the γ^* direction (over the γ^* range)



Collins-Soper frame (of virtual photon)

θ, φ lepton plane wrt hadron plane
target rest frame

φ_S target transverse spin vector /virtual
photon



LO content

SIDIS

$$A_{UU}^{\cos \phi_h} \propto \frac{1}{Q} \left(f_1^q \otimes D_{1q}^h - h_1^{\perp q} \otimes H_{1q}^{\perp h} + \dots \right)$$

$$A_{LT}^{\cos(\phi_h - \phi_S)} \propto g_{1T}^q \otimes D_{1q}^h$$

$$A_{UU}^{\cos 2\phi_h} \propto h_1^{\perp q} \otimes H_{1q}^{\perp h} + \frac{1}{Q} \left(f_1^q \otimes D_{1q}^h + \dots \right)$$

$$A_{UT}^{\sin \phi_S} \propto \frac{1}{Q} \left(h_1^q \otimes H_{1q}^{\perp h} + f_{1T}^{\perp q} \otimes D_{1q}^h + \dots \right)$$

$$A_{UT}^{\sin(\phi_h - \phi_S)} \propto f_{1T}^{\perp q} \otimes D_{1q}^h$$

$$A_{UT}^{\sin(2\phi_h - \phi_S)} \propto \frac{1}{Q} \left(h_1^{\perp q} \otimes H_{1q}^{\perp h} + f_{1T}^{\perp q} \otimes D_{1q}^h + \dots \right)$$

$$A_{UT}^{\sin(\phi_h + \phi_S)} \propto h_1^q \otimes H_{1q}^{\perp h}$$

$$A_{LT}^{\cos \phi_S} \propto \frac{1}{Q} \left(g_{1T}^q \otimes D_{1q}^h + \dots \right)$$

$$A_{UT}^{\sin(3\phi_h - \phi_S)} \propto h_{1T}^{\perp q} \otimes H_{1q}^{\perp h}$$

$$A_{LT}^{\cos(2\phi_h - \phi_S)} \propto \frac{1}{Q} \left(g_{1T}^q \otimes D_{1q}^h + \dots \right)$$

DY

$$A_U^{\cos 2\varphi_{CS}} \propto h_{1,\pi}^{\perp q} \otimes h_{1,p}^{\perp q}$$

$$A_T^{\sin \varphi_{CS}} \propto f_{1,\pi}^q \otimes f_{1T,p}^{\perp q}$$

$$A_T^{\sin(2\varphi_{CS} - \varphi_S)} \propto h_{1,\pi}^{\perp q} \otimes h_1^q$$

$$A_T^{\sin(2\varphi_{CS} + \varphi_S)} \propto h_{1,\pi}^{\perp q} \otimes h_{1T,p}^{\perp q}$$

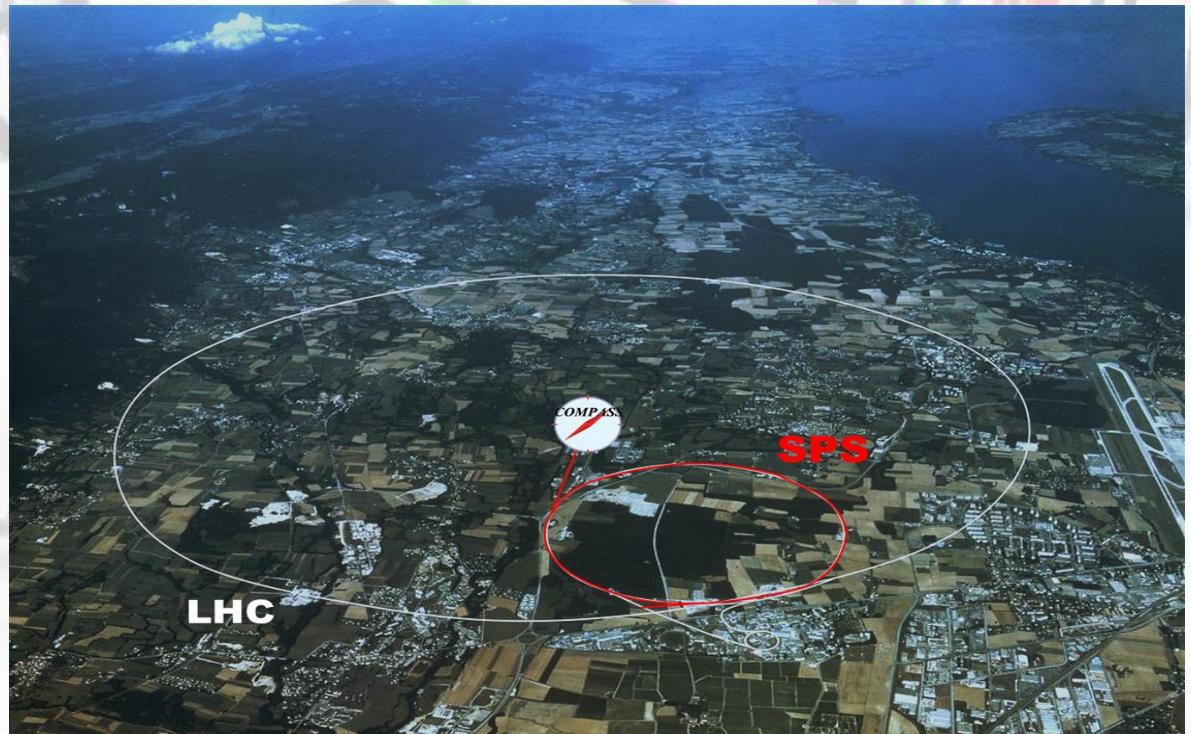
COmmun
Muon and
Proton
Apparatus for
Structure and
Spectroscopy

Collaboration

~ 250 physicists
from 24 Institutions
of 13 Countries

- fixed target
- experiment
- at the CERN SPS

data taking: since 2002





Дубна (LPP and LNP),
Москва (INR, LPI, State
University),
Протвино

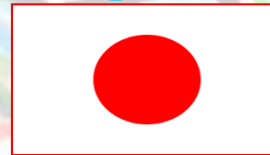


CERN

Bochum, Bonn
(ISKP & PI),
Erlangen,
Freiburg, Mainz,
München TU



Warsawa (NCBJ),
Warsawa (TU)
Warsawa (U)



Yamagata

USA (UIUC)

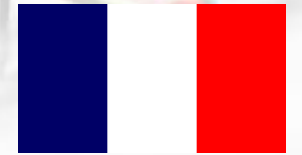


Praha (CU/CTU)
Liberec (TU)
Brno (ISI-ASCR)



Lisboa/Aveiro

Saclay

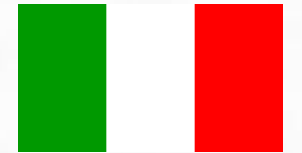


Calcutta (Matrivian)



Tel Aviv

Torino
(University, INFN),
Trieste
(University, INFN)



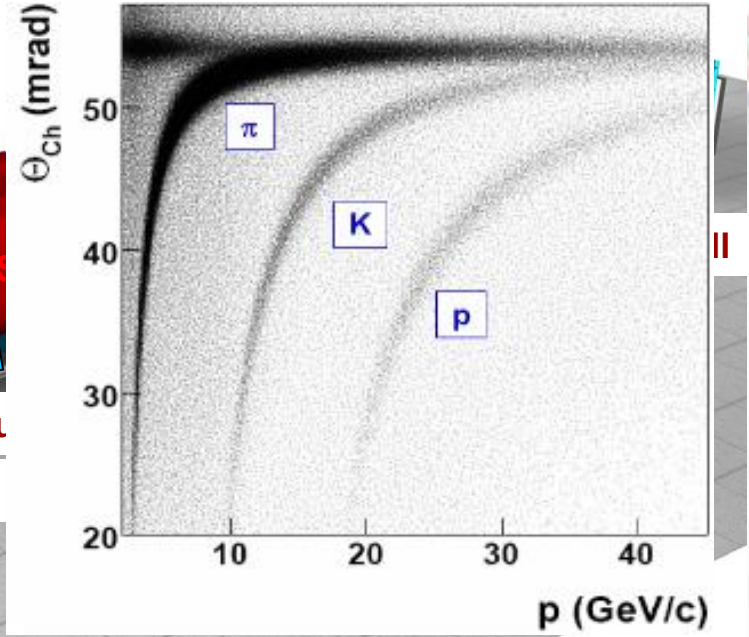
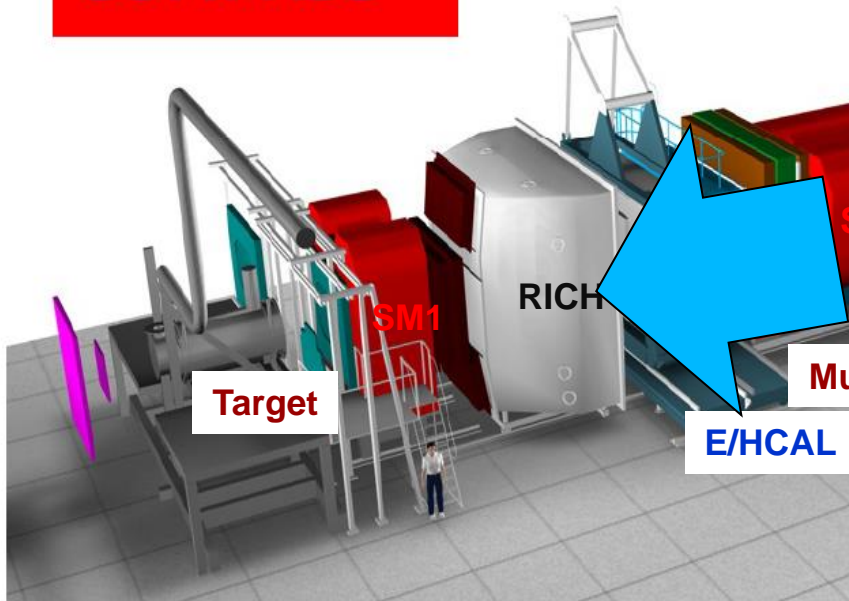
Taipei (AS)

- high energy beam
- large angular acceptance
- broad kinematical range

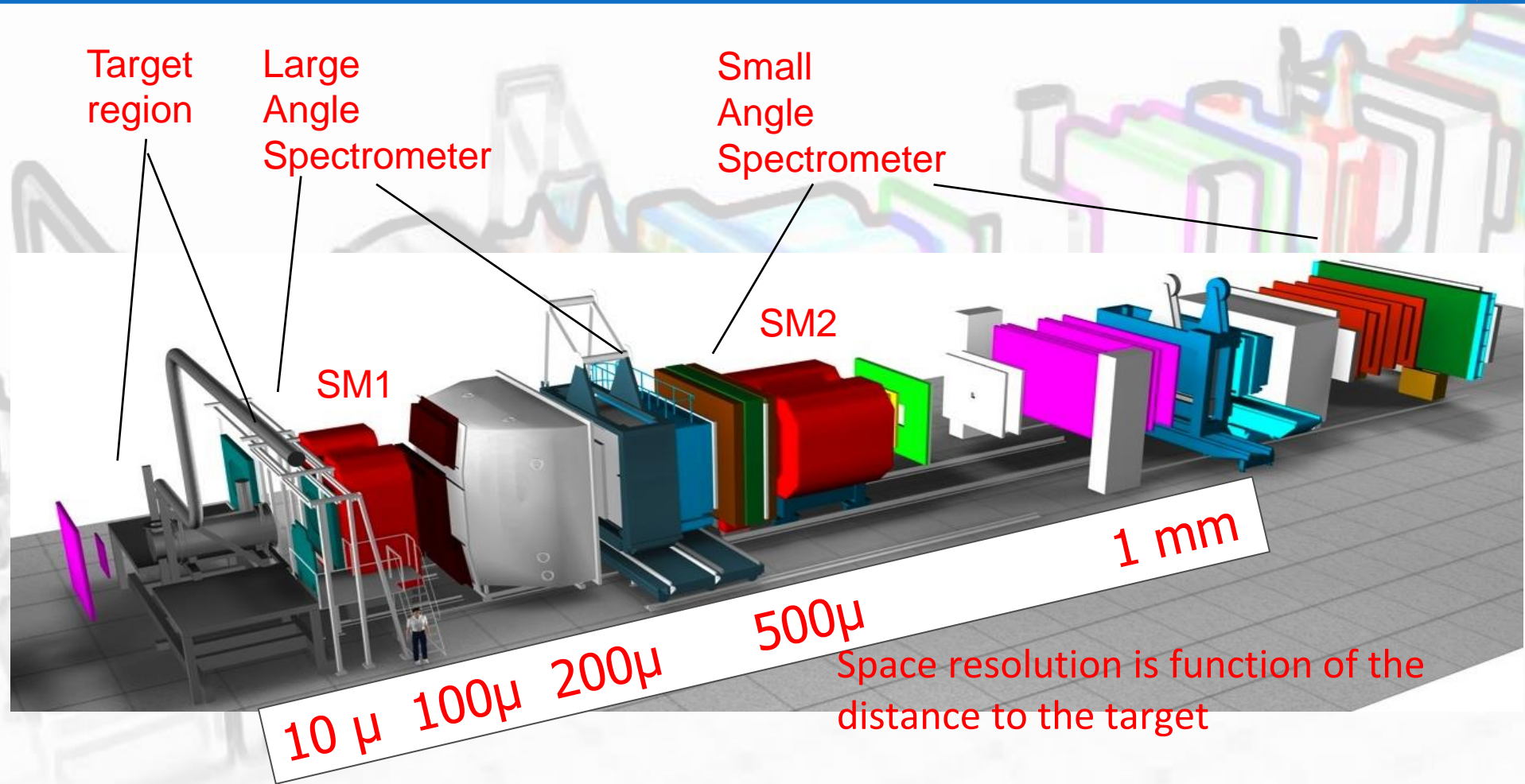
two stages spectrometer

Large Angle Spectrometer (SM1)
 Small Angle Spectrometer (SM2)

COMPASS

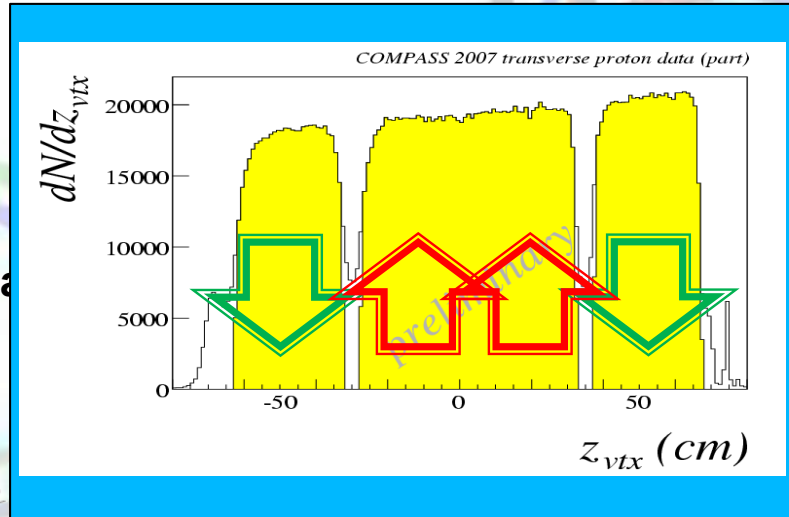
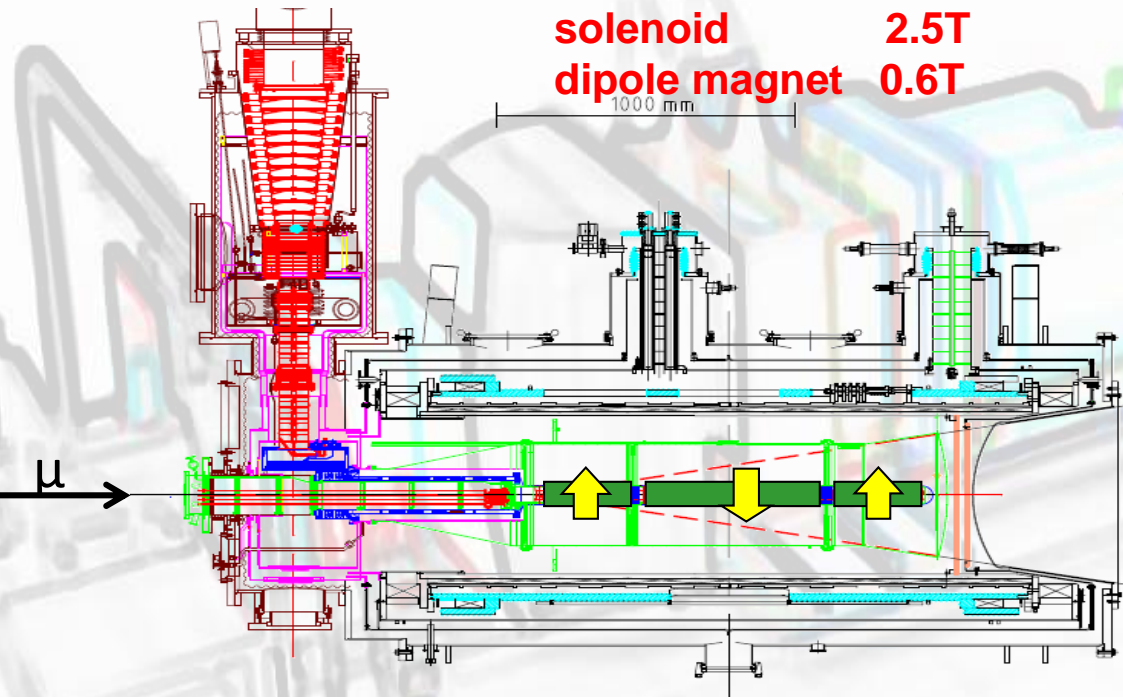


Space resolution



the polarized target system (>2005)

$^3\text{He} - ^4\text{He}$ dilution refrigerator ($T \sim 50\text{mK}$)



opposite polarisation

	d (^6LiD)	p (NH_3)
polarization	50%	90%
dilution factor	40%	16%

no evidence for relevant nuclear effects (160 GeV)

Few facts:

- Transverse Spin and Momentum effects were put under scrutiny by the COMPASS Proposal in 1996, starting with transversity via the Collins mechanism

We propose to measure in semi-inclusive DIS on transversely polarised proton and deuterium targets the transverse spin distribution functions $\Delta_T q(x) = q_\uparrow(x) - q_\downarrow(x)$, where \uparrow (\downarrow) indicates a quark polarisation parallel (antiparallel) to the transverse polarisation of the nucleon. Hadron identification allows to tag the quark flavour.

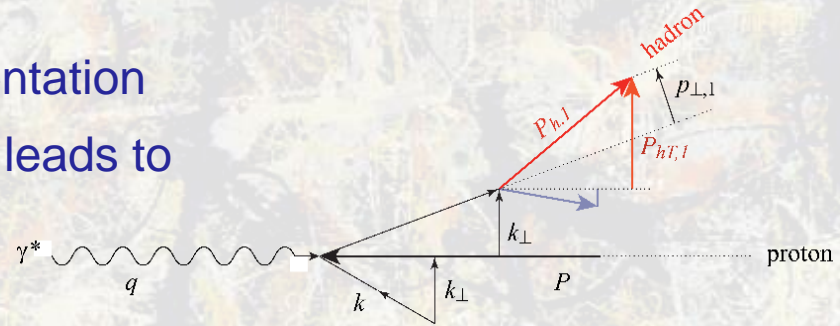
As suggested by J. Collins [71], the fragmentation function for transversely polarised quarks should exhibit a specific azimuthal dependence. The transversely polarised quark fragmentation function D_q^h should be built up from two pieces, a spin-independent part D_q^h , and a spin-dependent part ΔD_q^h :

$$D_q^h(z, \vec{p}_q^h) = D_q^h(z, p_q^h) + \Delta D_q^h(z, p_q^h) \cdot \sin(\phi_h - \phi_{S'}), \quad (3.23)$$

- The measurement of the Sivers PDF was added to the program soon after ... the other TMD with the developments over the years
- Measurements started in 2002 by HERMES (p) and COMPASS (d)
- This field has grown considerably in the last years and comes one of high priority measurements for the JLab12 program and for the planned polarized lepton nucleon colliders.

Unpolarized SIDIS

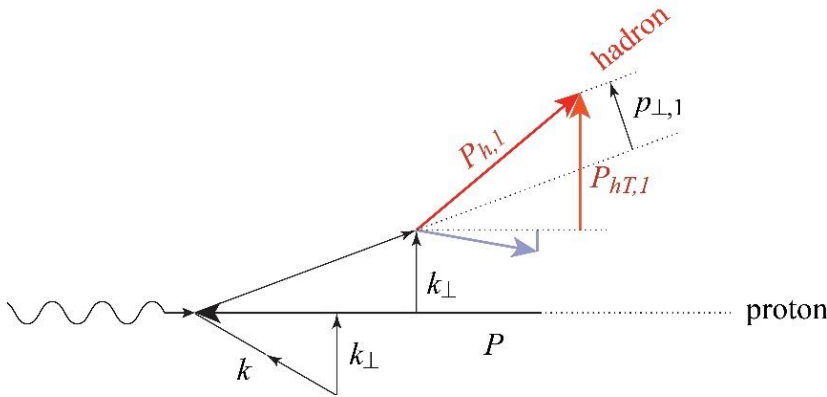
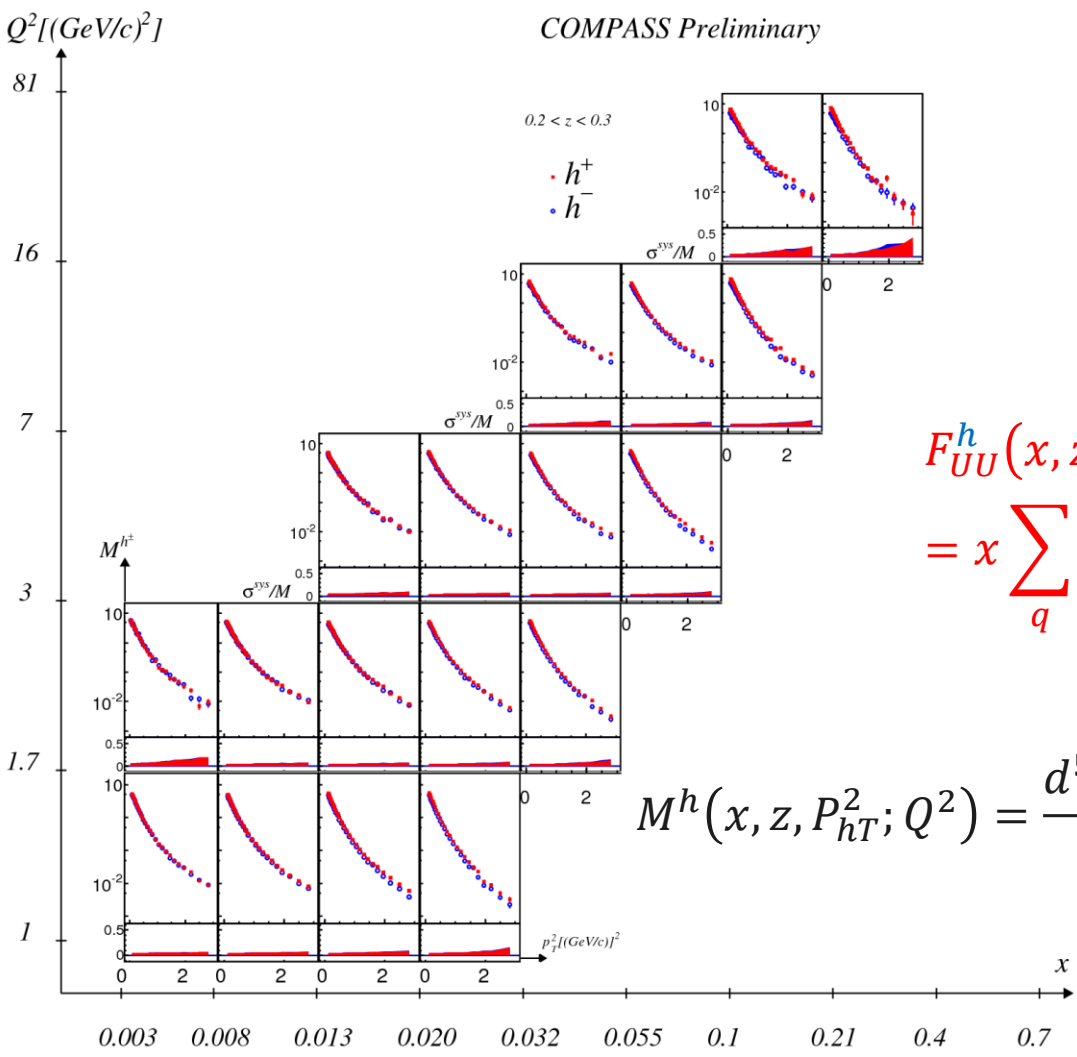
- The cross-section dependence from P_{hT} results from:
 - intrinsic k_{\perp} of the quarks
 - p_{\perp} generated in the quark fragmentation
 - A Gaussian ansatz for k_{\perp} and p_{\perp} leads to
 - $\langle P_{hT}^2 \rangle = z^2 \langle k_{\perp}^2 \rangle + \langle p_{\perp}^2 \rangle$



- The azimuthal modulations in the unpolarized cross-sections comes from:
 - Intrinsic k_{\perp} of the quarks
 - The Boer-Mulders PDF
 - ...

Difficult measurements were one has to correct for the apparatus acceptance

Unpolarized SIDIS

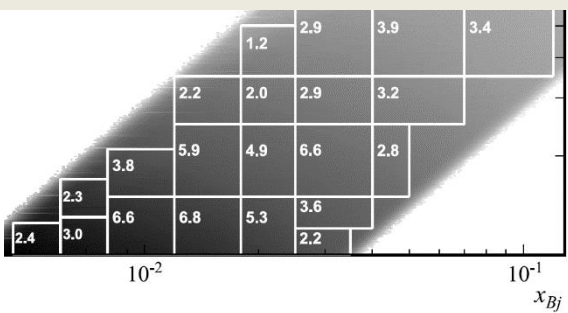
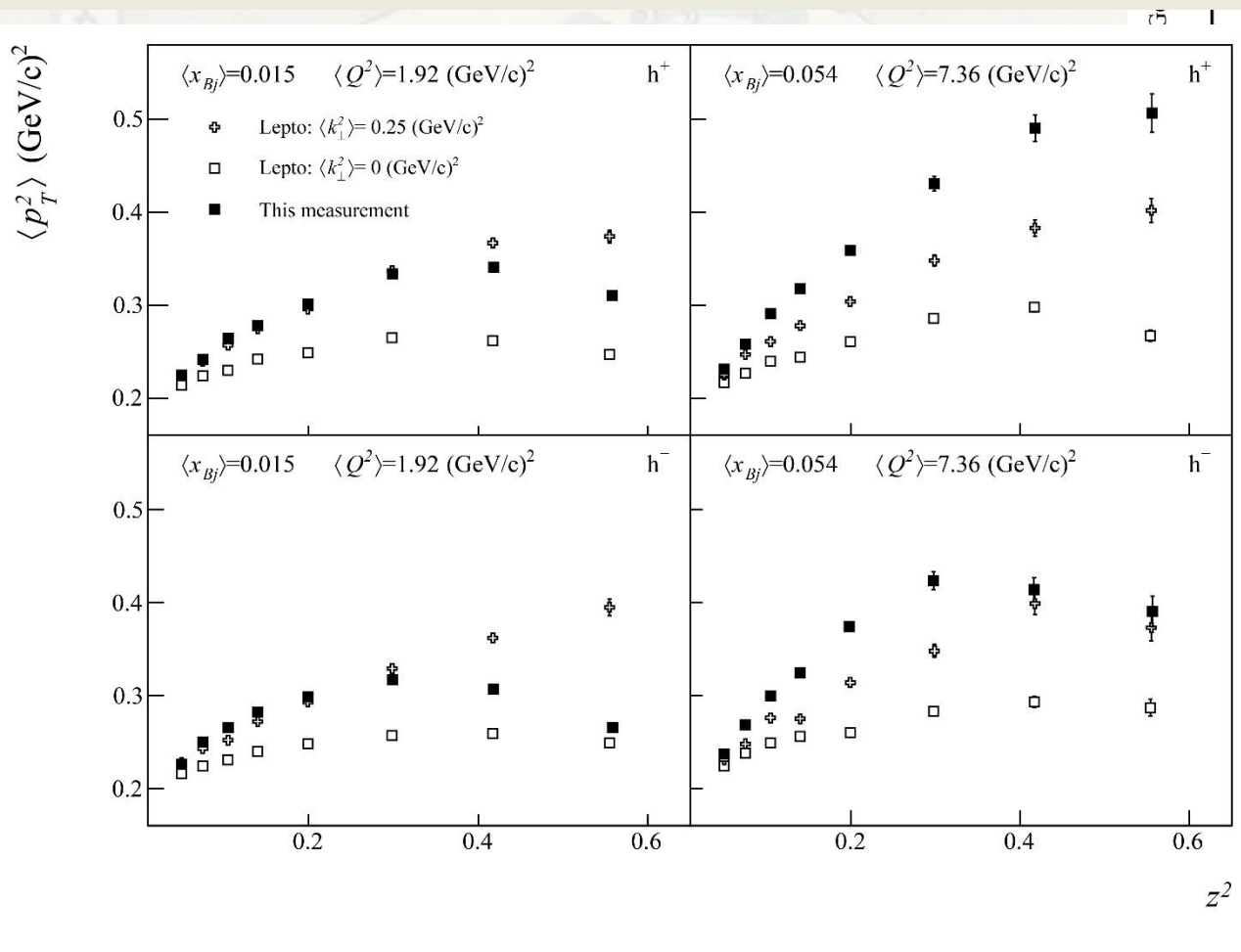


$$F_{UU}^h(x, z, P_{hT}^2; Q^2)$$

$$= x \sum_q e_q^2 \int d^2\vec{k}_{\perp} d^2\vec{p}_{\perp} \delta(\vec{p}_{\perp} - z\vec{k}_{\perp})$$

$$M^h(x, z, P_{hT}^2; Q^2) = \frac{d^5\sigma^h/dxdQ^2 dzd^2\vec{p}_T}{d^2\sigma^{DIS}/dxdQ^2} \sim \frac{F_{UU}^h(x, z, P_{hT}^2; Q^2)}{F_{UU,T} + \epsilon F_{UU,L}}$$

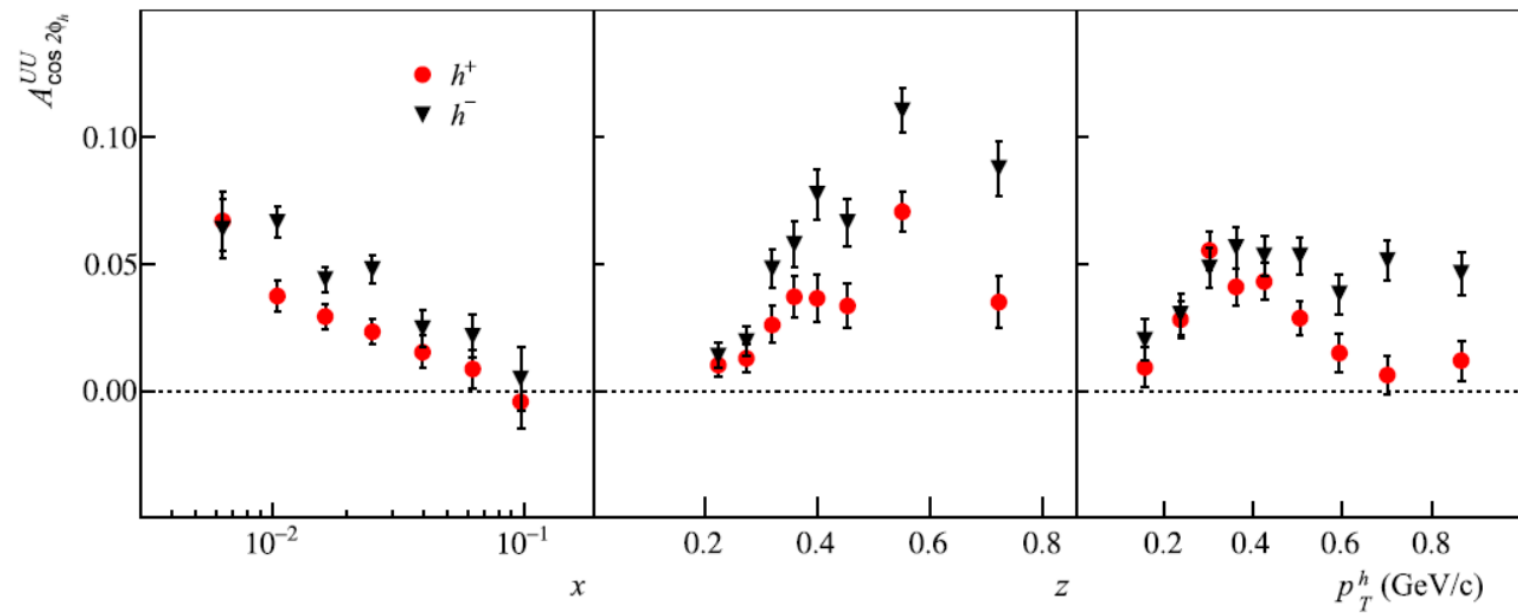
Mean values



Boer-Mulders in $\cos 2\phi$

1064

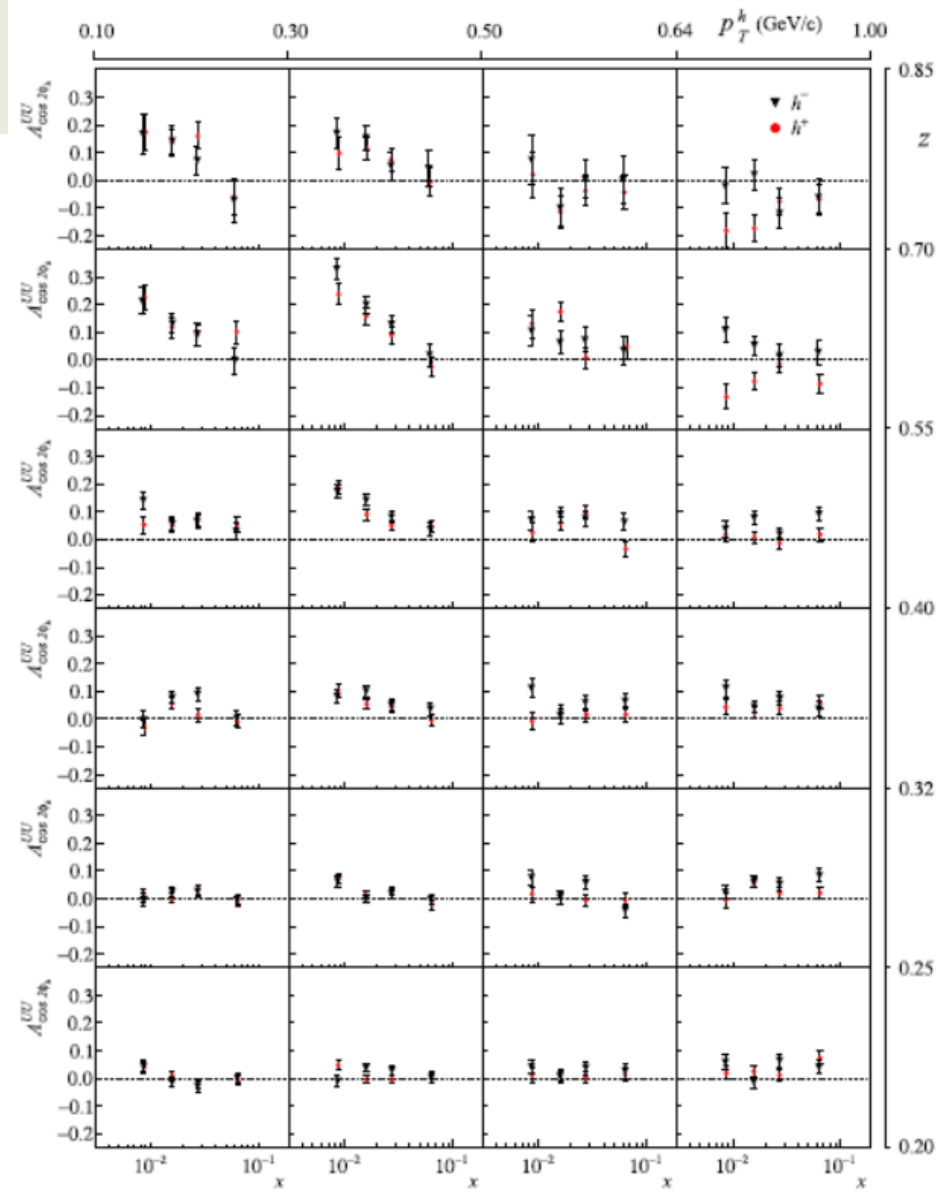
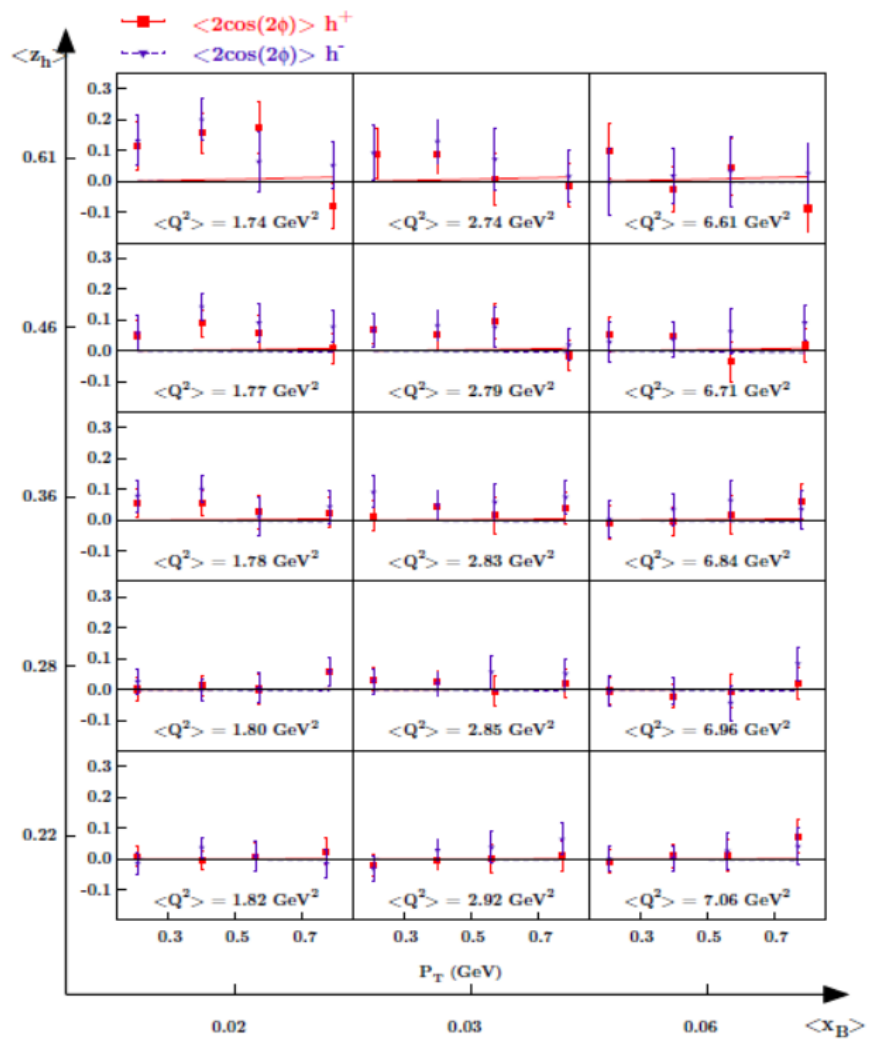
C. Adolph et al. / Nuclear Physics B 886 (2014) 1046–1077



$$F_{UU}^{\cos 2\phi}(x, z, P_{hT}^2; Q^2)$$

$$= -x \sum_q e_q^2 \int d^2\vec{k}_\perp d^2\vec{p}_\perp \frac{2(\hat{h} \cdot \vec{k}_\perp)(\hat{h} \cdot \vec{k}_\perp) - \vec{k}_\perp \cdot \vec{p}_\perp}{Mm_h} h_1^{\perp,q}(x, k_\perp^2; Q^2) H_1^{\perp,q \rightarrow h}(z, p_\perp^2; Q^2)$$

Boer-Mulders in $\cos 2\phi$



Transversity

is chiral-odd:

observable effects are given only by the product of $h_1^q(\mathbf{x})$ and an other chiral-odd function can be measured in SIDIS on a transversely polarised target via “quark polarimetry”

$$l N^\uparrow \rightarrow l' h X$$

$$l N^\uparrow \rightarrow l' h h X$$

$$l N^\uparrow \rightarrow l' \Lambda X$$

“Collins” asymmetry

“Collins” Fragmentation Function

“two-hadron” asymmetry

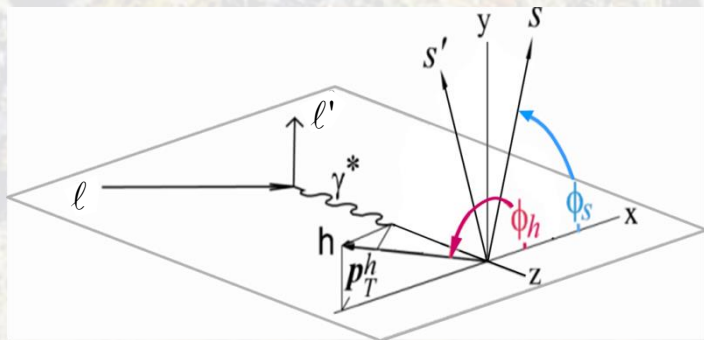
“Interference” Fragmentation Function

Λ polarisation

Fragmentation Function of $q^\uparrow \rightarrow \Lambda$

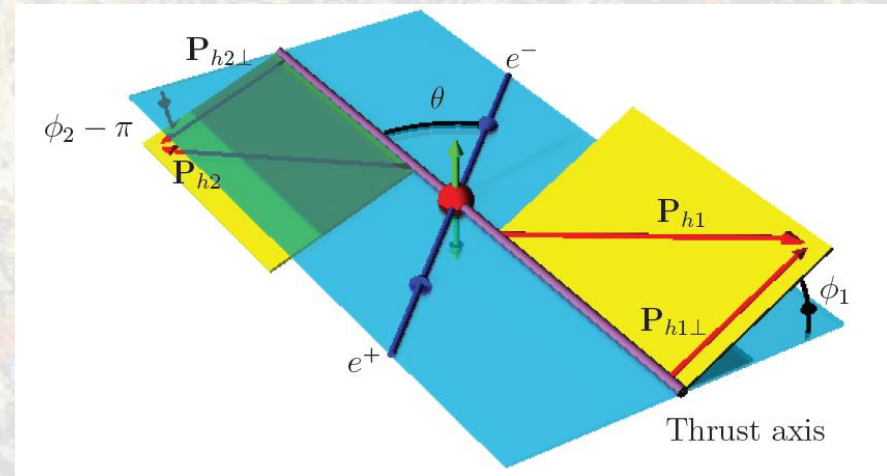
Transversity from Collins SSA and Collins FF

$$A_{UT}^{\sin(\phi_h + \phi_S - \pi), h} = \frac{\sum_q e_q^2 h_1^q(k_\perp) \otimes H_1^{\perp q \rightarrow h}(p_\perp)}{\sum_q e_q^2 f_1^q \otimes D_1^{q \rightarrow h}}$$

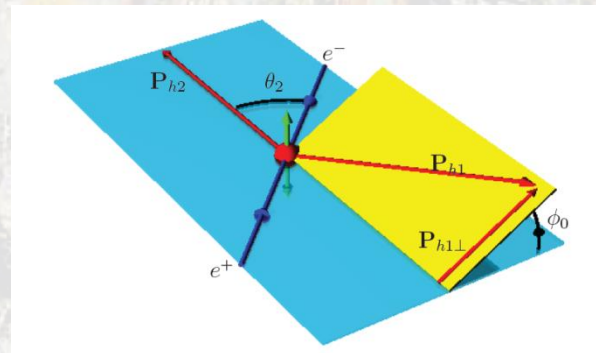


Collins effect:

a quark with an upward (downward) polarization, perpendicular to the motion, prefers to emit the leading meson to the left (right) side with respect to the quark direction



$$A_{12}^{h_1 h_2} = \frac{\sin^2 \theta}{1 + \cos^2 \theta} \frac{\sum_q e_q^2 H_1^{\perp(1/2)q \rightarrow h_{1/2}} H_1^{\perp(1/2)\bar{q} \rightarrow h_{1/2}}}{\sum_q e_q^2 D_1^{q \rightarrow h_{1/2}} D_1^{\bar{q} \rightarrow h_{1/2}}}$$

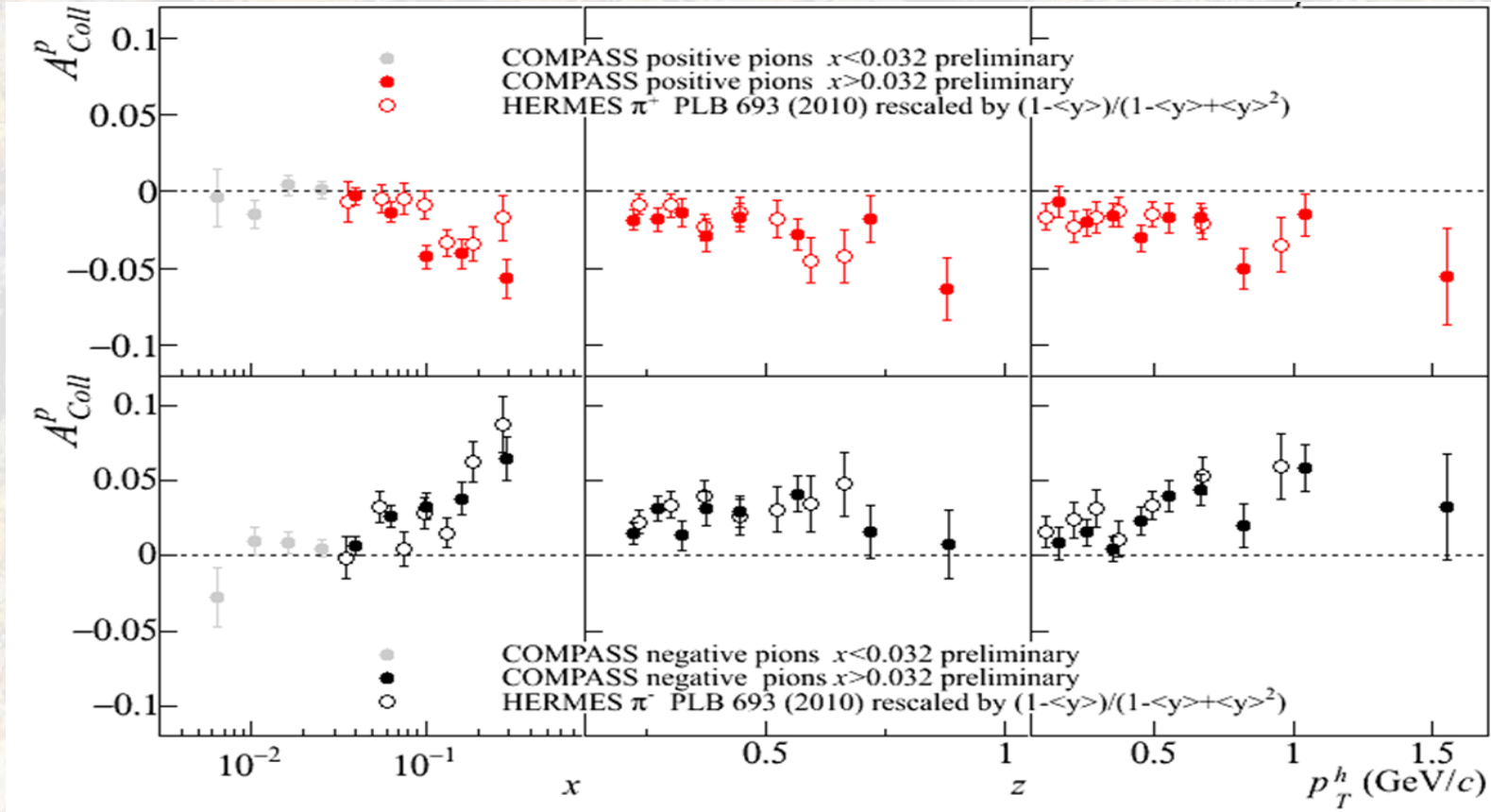
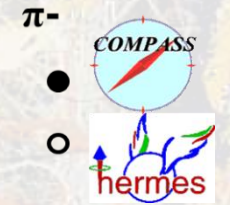
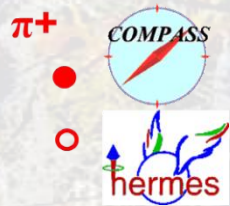


Collins asymmetry on proton

$x > 0.032$ region

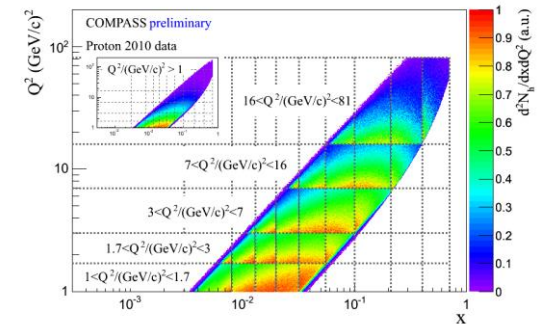
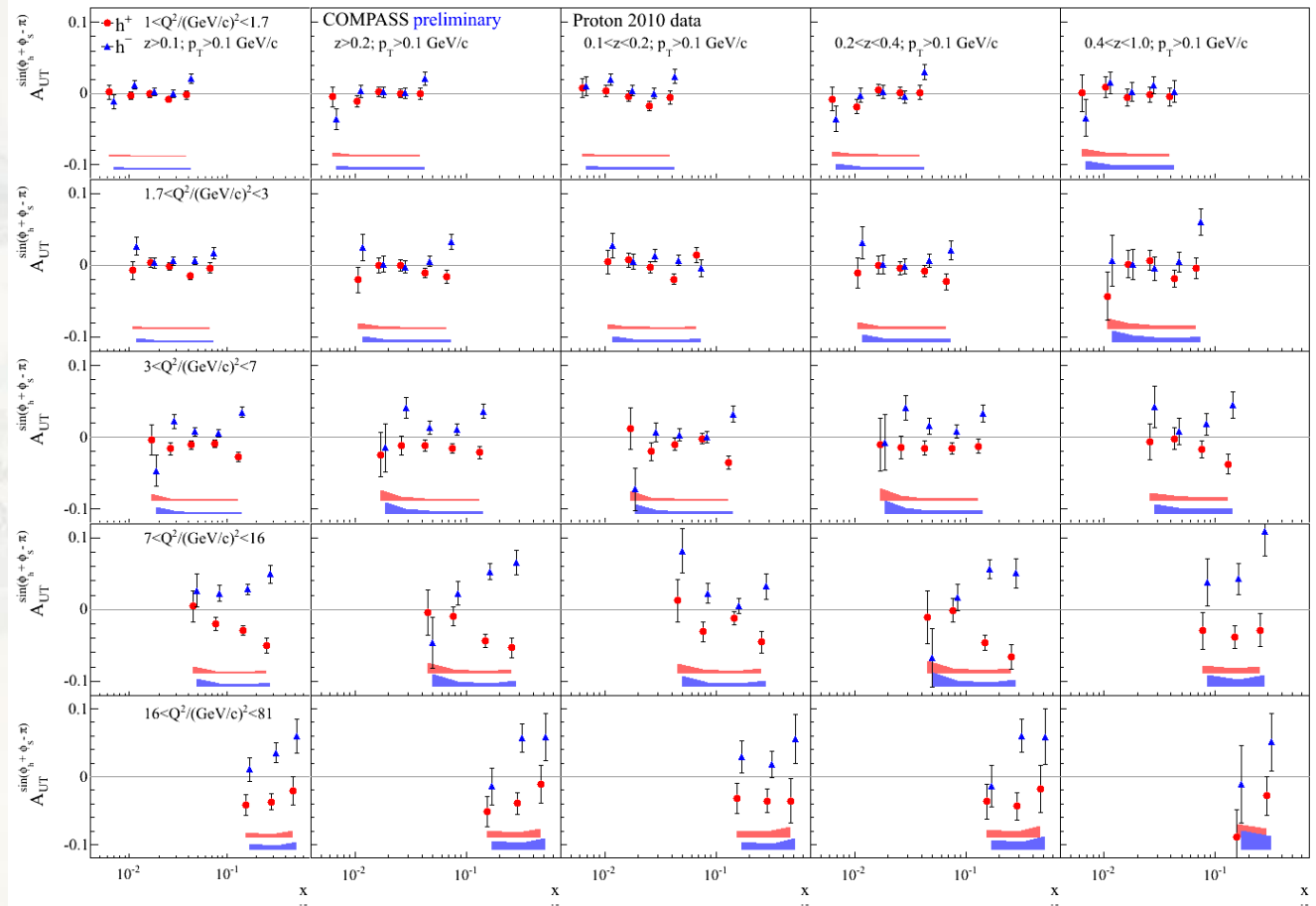
charged pions

COMPASS and HERMES results

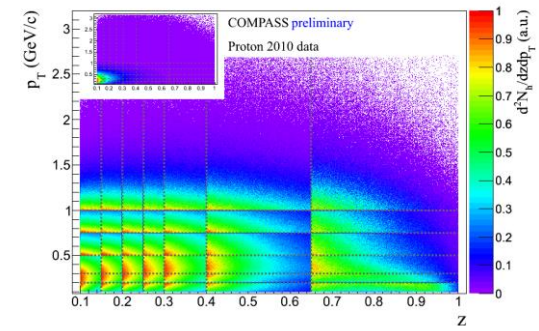


Collins asymmetry on proton. Multidimensional

First extraction of TSAs within a Multi-D ($x: Q^2: z: p_T$) approach

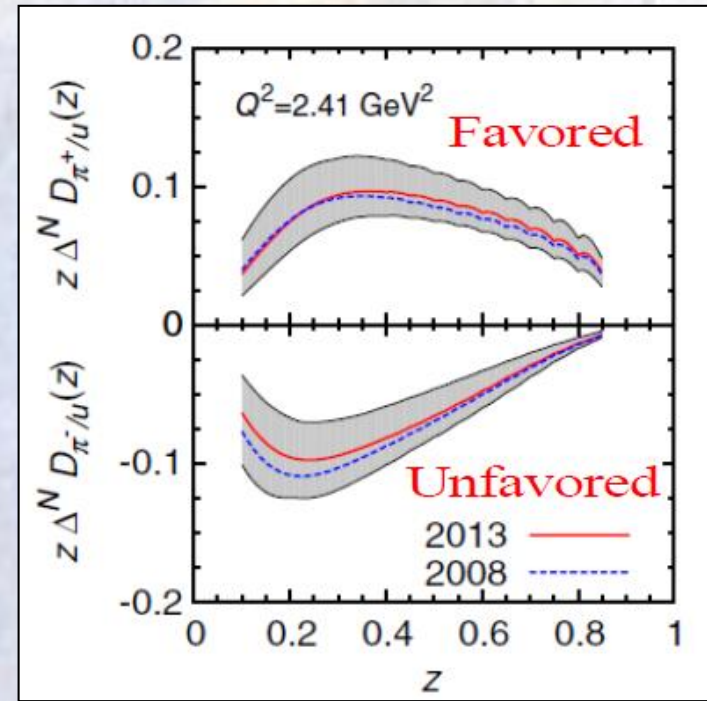
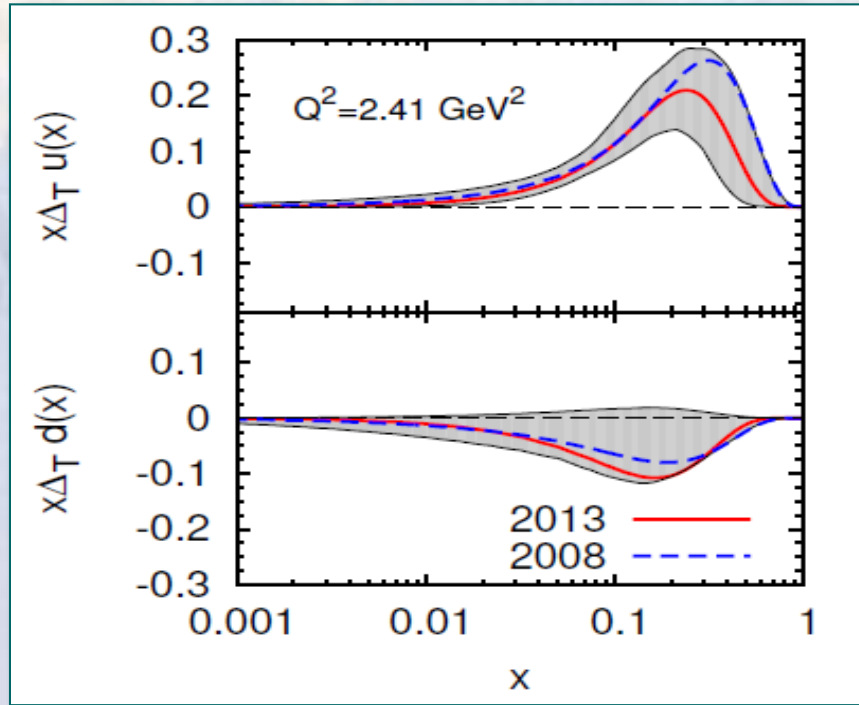


One dense plot out of many



Transversity from Collins

Combined analyses of **HERMES**, **COMPASS** and **BELLE fragm.fct.** data



Anselmino et al. arXiv: 1303.3822

From Collins asymmetries to transversity

- Following Physical Review D 91, 014034 (2015), in the valence region

$$xh_1^u = \frac{1}{5} \frac{1}{\tilde{\alpha}_p^h(1 - \tilde{\alpha})} \left[(xf_p^+ A_p^+ - xf_p^- A_p^-) + \frac{1}{3} (xf_d^+ A_d^+ - xf_d^- A_d^-) \right]$$

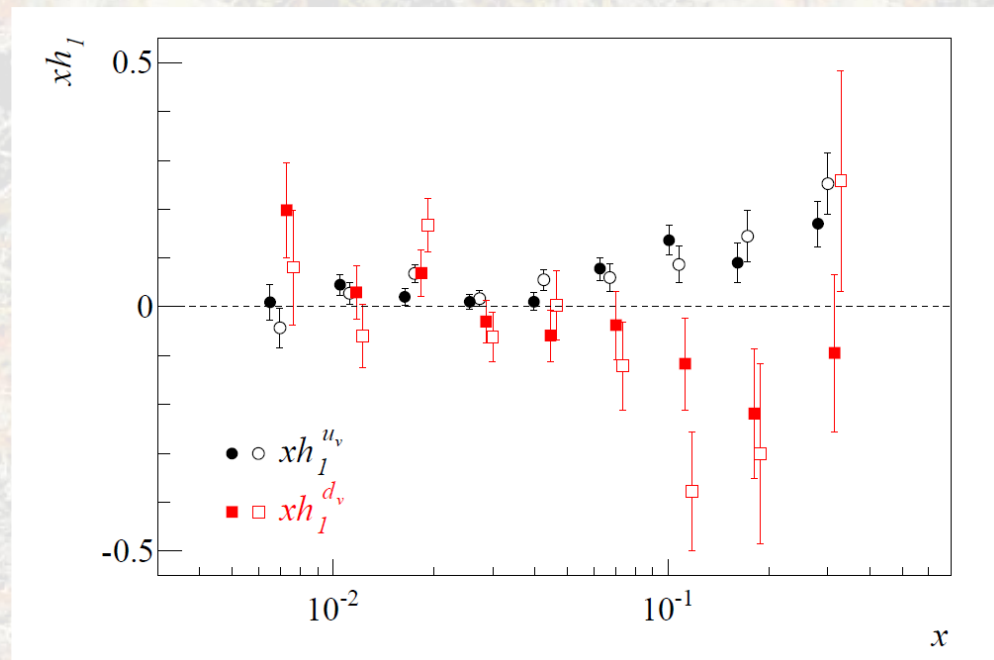
$$xh_1^d = \frac{1}{5} \frac{1}{\tilde{\alpha}_p^h(1 - \tilde{\alpha})} \left[\frac{4}{3} (xf_d^+ A_d^+ - xf_d^- A_d^-) - (xf_p^+ A_p^+ - xf_p^- A_p^-) \right]$$

With $\tilde{\alpha}_p^h$ and $\tilde{\alpha}$ constants

Transversity from our data

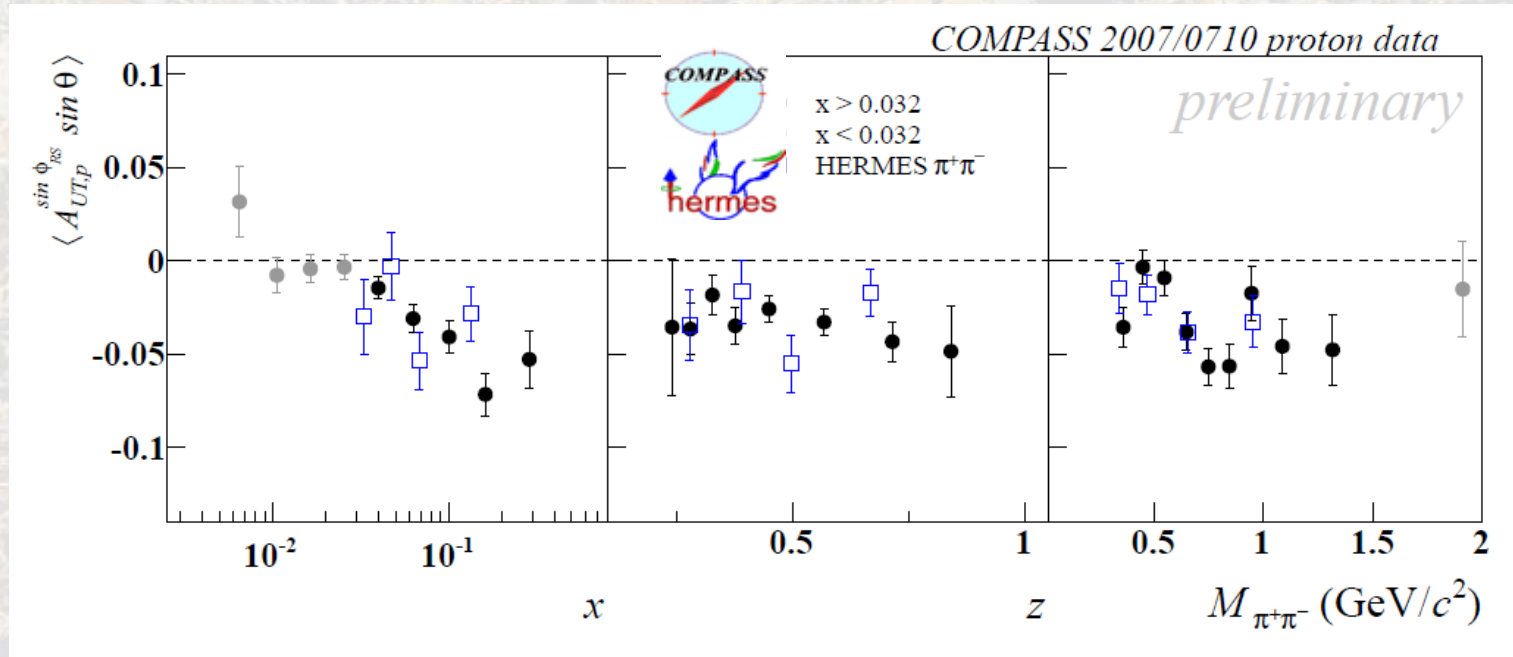
- Poin-to-poiny extraction [Physical Review D 91, 014034 (2015)]
- Keep in mind that we are the only one to have measured TSA on deuteron

Open points/squares – from dihadron
Closed points/squares – from Collins



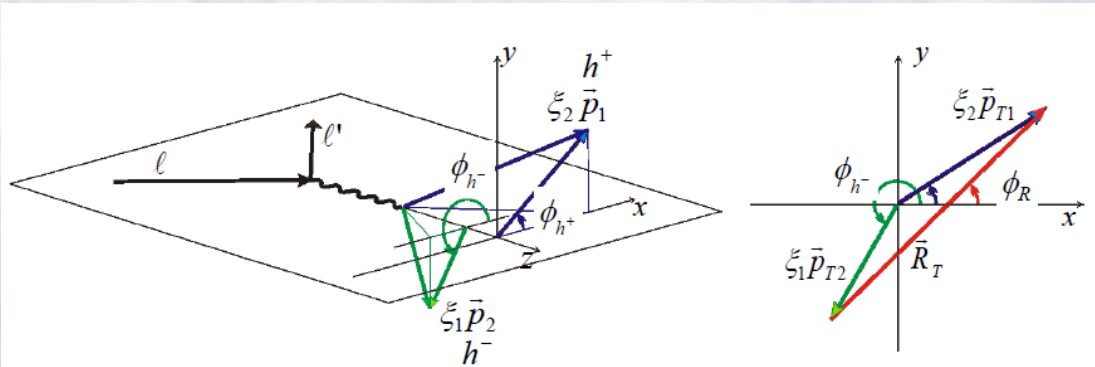
**ERRORS ON h_1^d ARE A FACTOR 4 LARGER THAT THE ONES ON h_1^u
NEED OF MORE D DATA (1 Y. AS 2010 P DATA)**

2h asymmetries on p



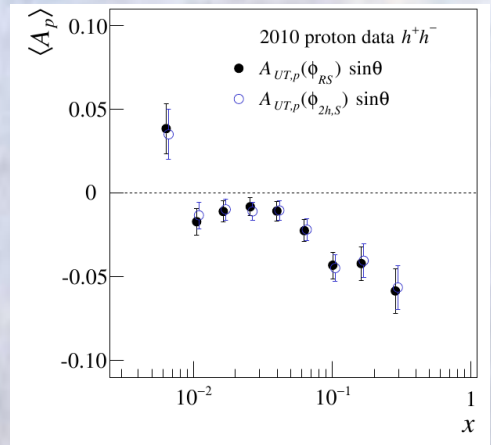
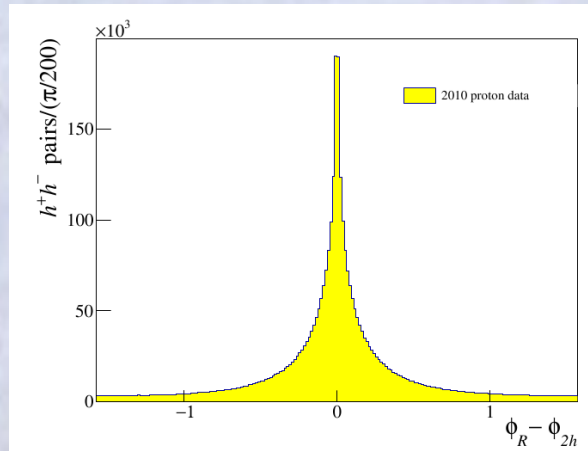
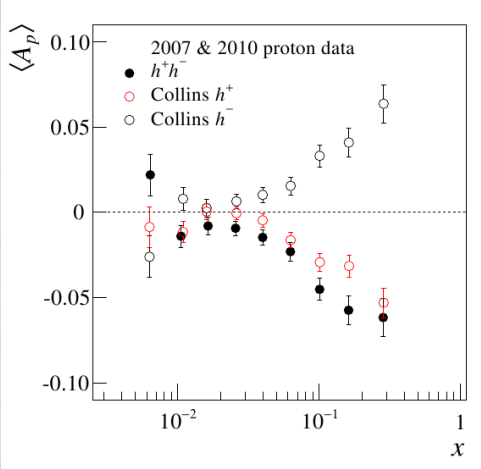
$$A_{UT}^{\sin(\phi_R+\phi_S-\pi)} = \frac{\sum_q e_q^2 h_1^q(x) H_{q \rightarrow h_1 h_2}^{\mathbb{Z}}(z, \mathcal{M}_{h_1 h_2}^2)}{\sum_q e_q^2 q(x) D_q^{h_1 h_2}(z, \mathcal{M}_{h_1 h_2}^2)}$$

Hadron correlations

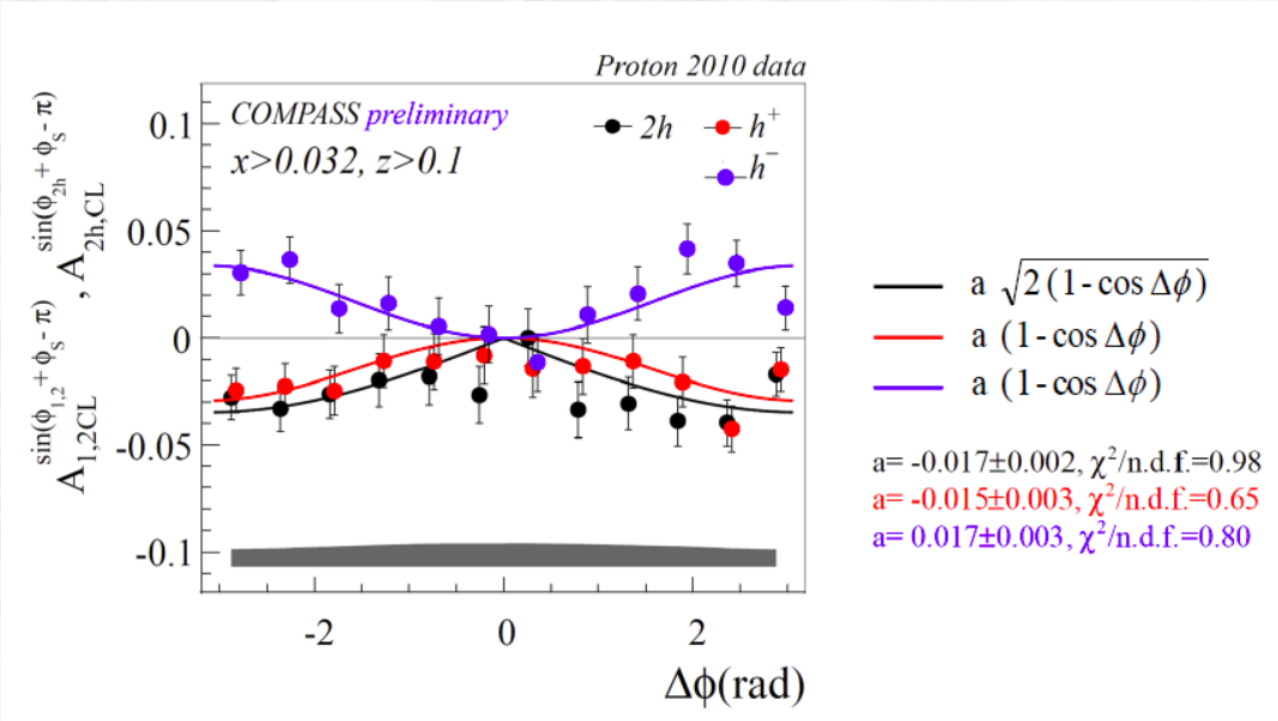


Interplay between Collins and IFF asymmetries

common hadron sample for Collins and 2h analysis



Asymmetries for $x > 0.032$ vs $\Delta\phi = \phi_{h^+} - \phi_{h^-}$



$$a = \frac{\sigma_{1C}^{h^+h^-}(\Delta\phi)}{\sigma_U(\Delta\phi)}$$

$$= - \frac{\sigma_{2C}^{h^+h^-}(\Delta\phi)}{\sigma_U(\Delta\phi)}$$

ratio of the integrals compatible with $4/\pi$

Hints for a common origin of 1h and 2h mechanisms

Sivers Asymmetry

Sivers: correlates nucleon spin & quark transverse momentum k_T / T-ODD

at LO:

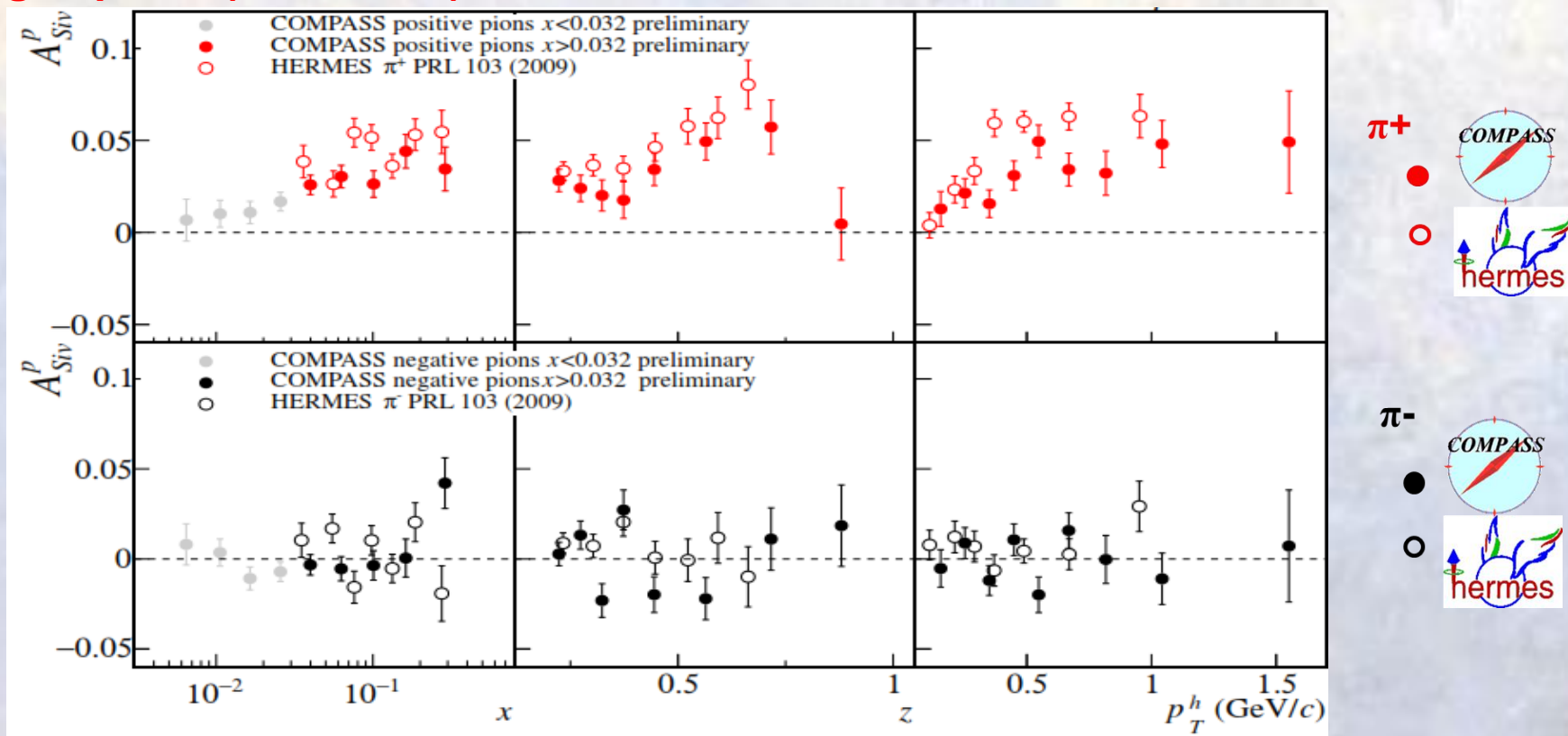
$$A_{Siv} = \frac{\sum_q e_q^2 f_{1Tq}^\perp \otimes D_q^h}{\sum_q e_q^2 q \otimes D_q^h}$$

$$\mu p^\uparrow \rightarrow \mu X h^\pm$$

The Sivers PDF	
1992	Sivers proposes f_{1T}^\perp
1993	J. Collins proofs $f_{1T}^\perp = 0$ for T invariance
2002	S. Brodsky, Hwang and Schmidt demonstrate that f_{1Tq}^\perp may be $\neq 0$ due to FSI
2002	J. Collins shows that $(f_{1T}^\perp)_{DY} = -(f_{1T}^\perp)_{SIDIS}$
2004	HERMES on p: $A_{Siv}^{\pi^+} \neq 0$ and $A_{Siv}^{\pi^-} = 0$
2004	COMPASS on d: $A_{Siv}^{\pi^+} = 0$ and $A_{Siv}^{\pi^-} = 0$
2008	COMPASS on p: $A_{Siv}^{\pi^+} \neq 0$ and $A_{Siv}^{\pi^-} = 0$

Sivers asymmetry on p

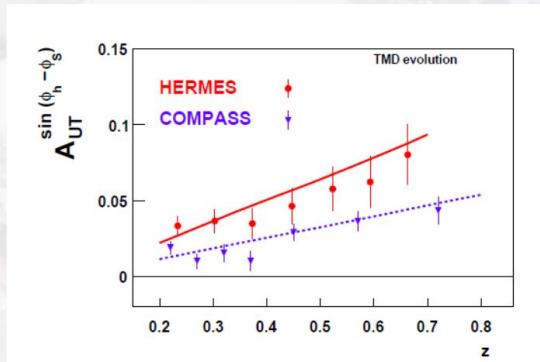
charged pions (and kaons), HERMES and COMPASS



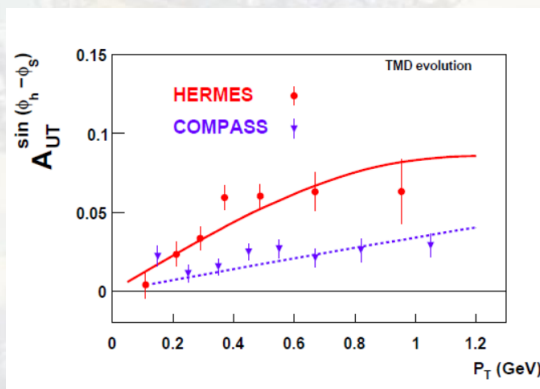
Sivers asymmetry on proton

charged hadrons, 2010 data - Q^2 evolution
comparison with

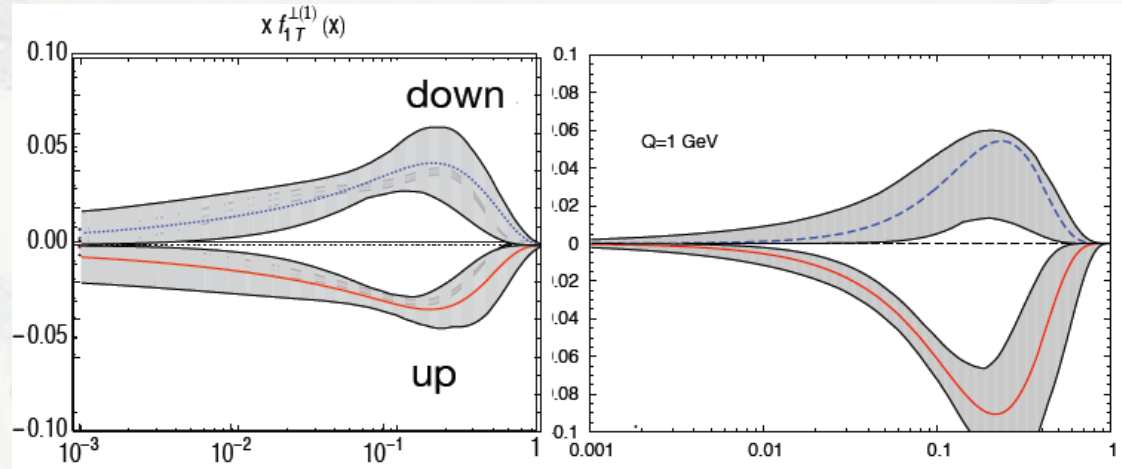
S. M. Aybat, A. Prokudin and T. C. Rogers calculations PRL 108 (2012) 242003



No TMD
evolution

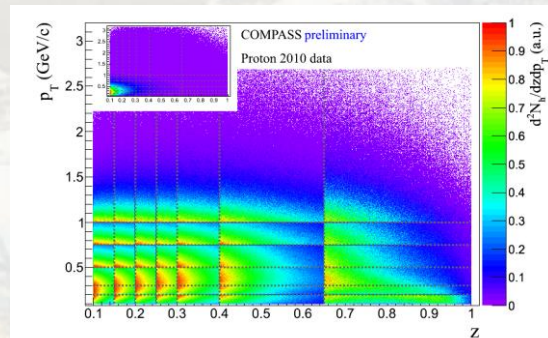
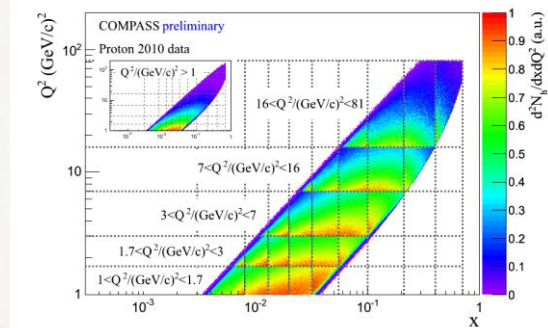
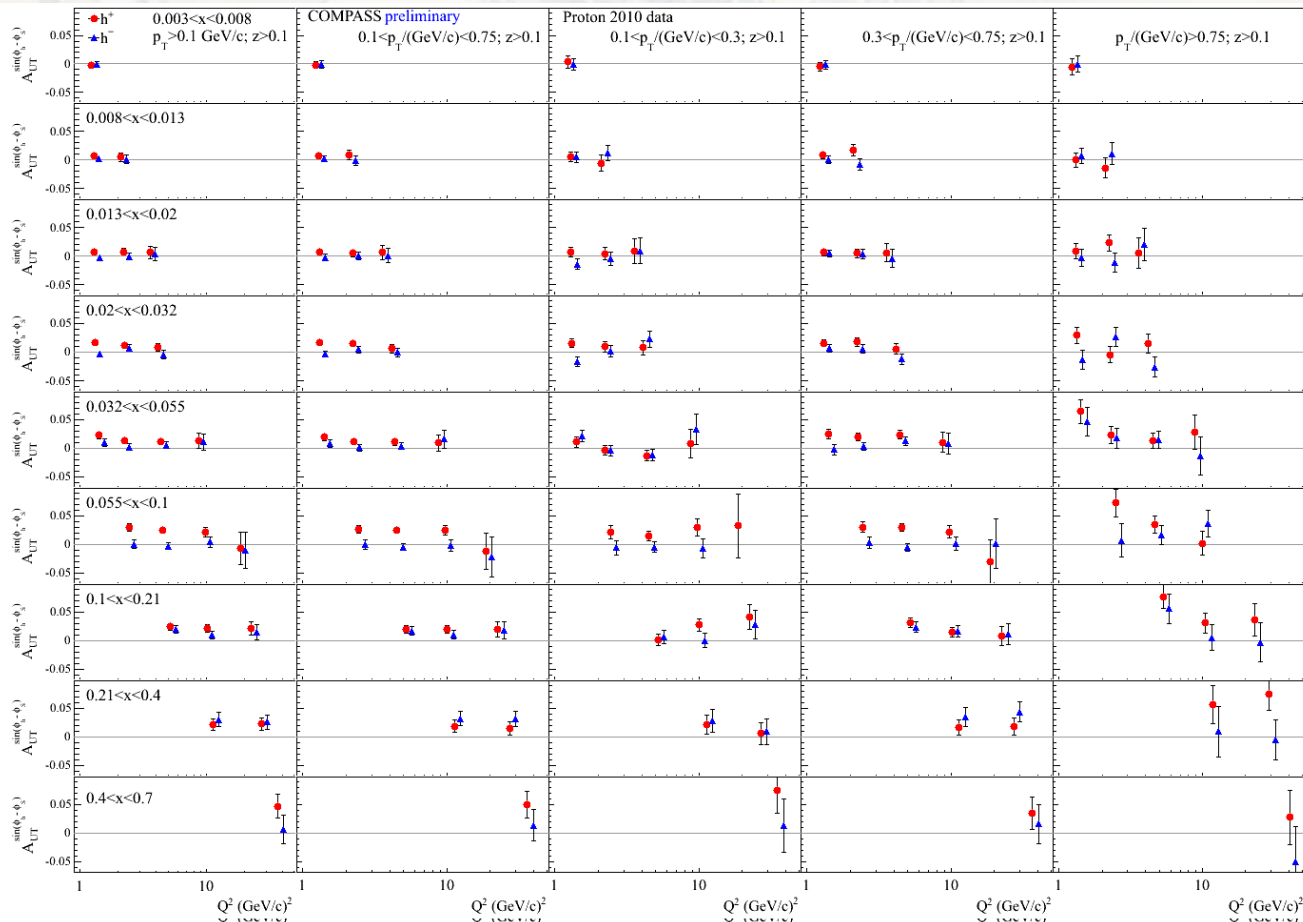


with TMD
evolution

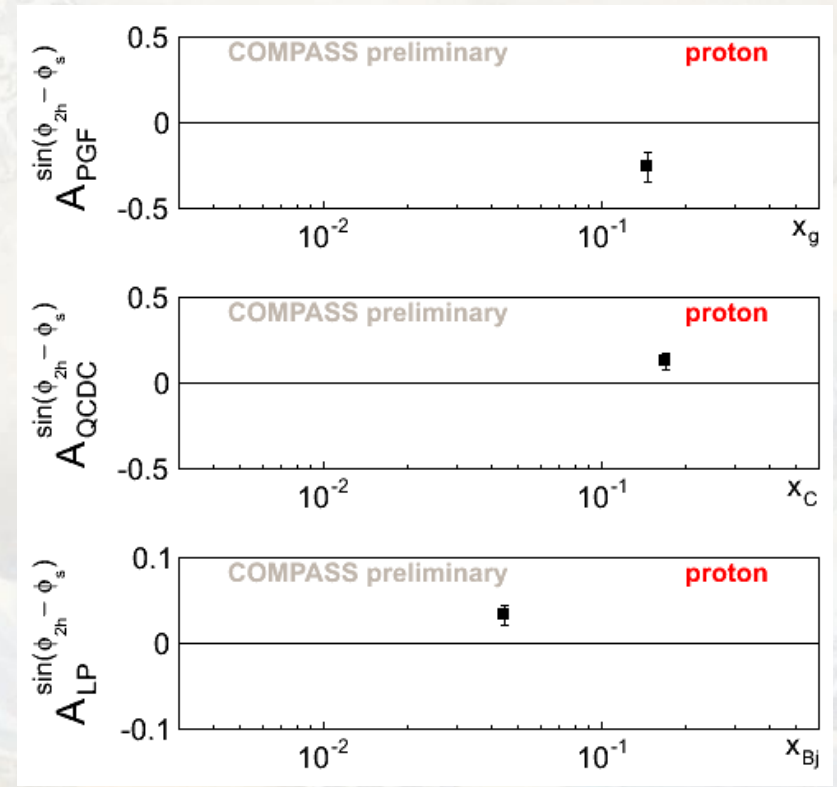
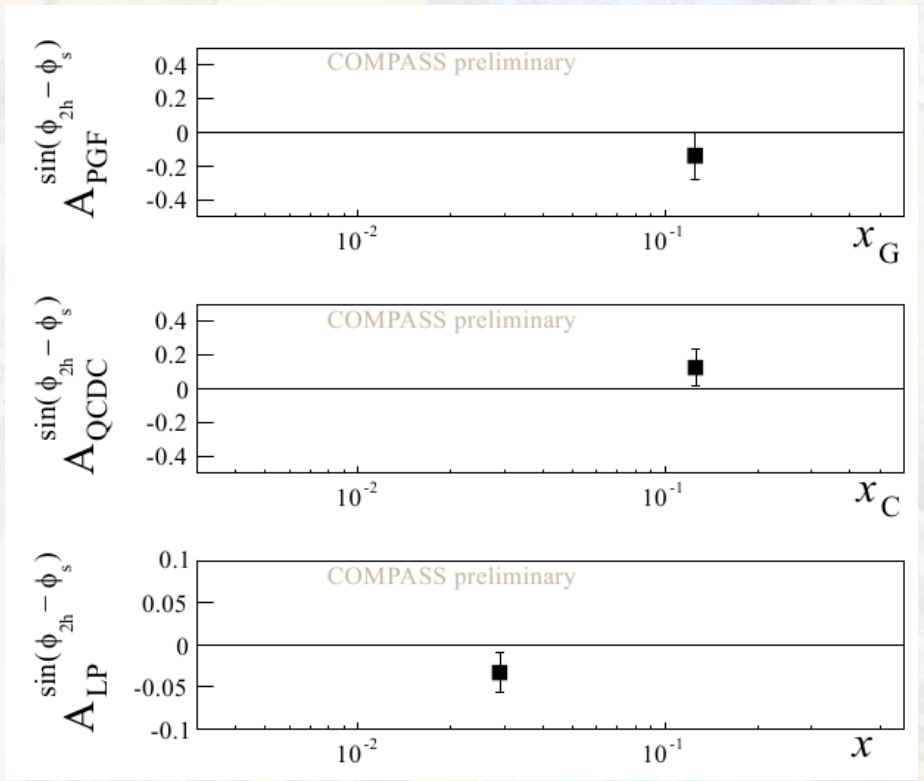


Sivers asymmetry on proton. Multidimensional

First ever extraction of TSAs within such a Multi-D ($x: Q^2: z: p_T$) approach



Sivers asymmetry on deuteron and proton for Gluons



Other SSAs - proton data

$$F_{LT}^{\cos(\phi_h - \phi_s)} \propto g_{1T}^q \otimes D_{1q}^h$$

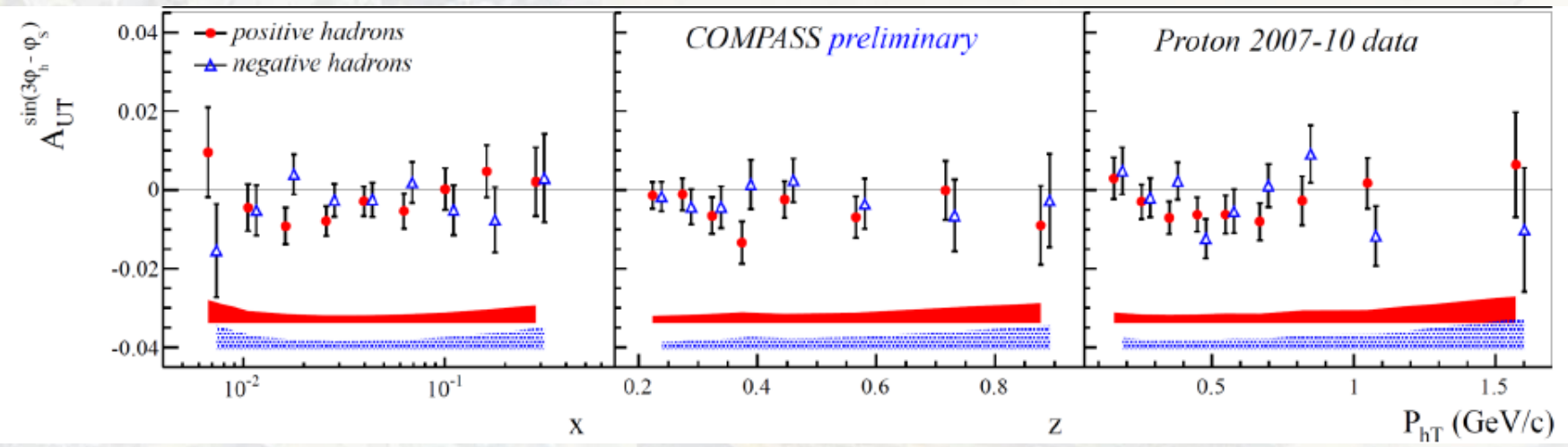
$$F_{UT}^{\sin(3\phi_h - \phi_s)} \propto h_{1T}^{\perp q} \otimes H_{1q}^{\perp h}$$

two twist-2 asymmetries can be interpreted in QCD parton



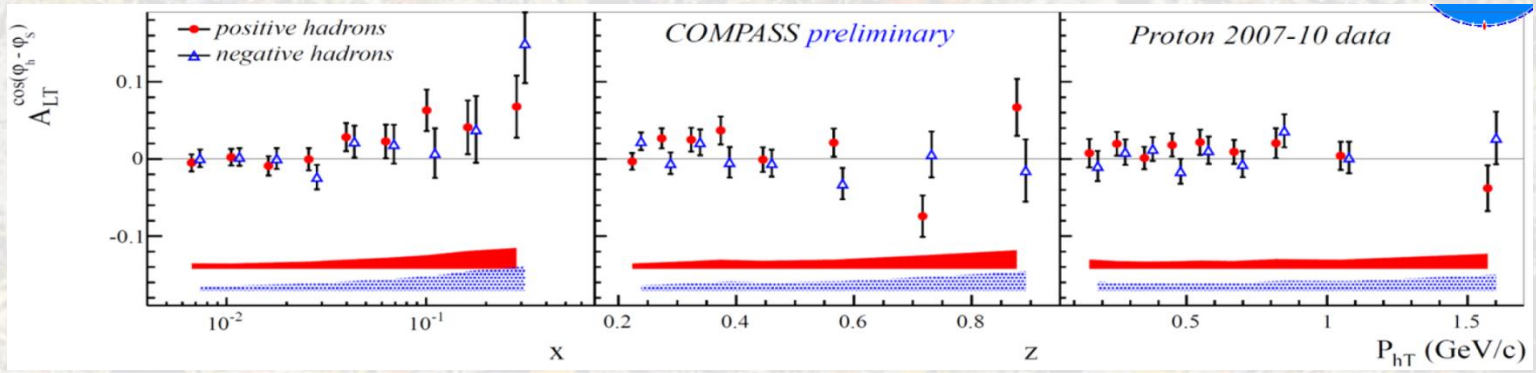
“pretzelosity” \otimes Collins FF

In some models $h_{1T}^{\perp} = g_1 - h_1$

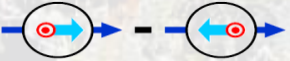


Other Transverse Target spin asymmetries on p

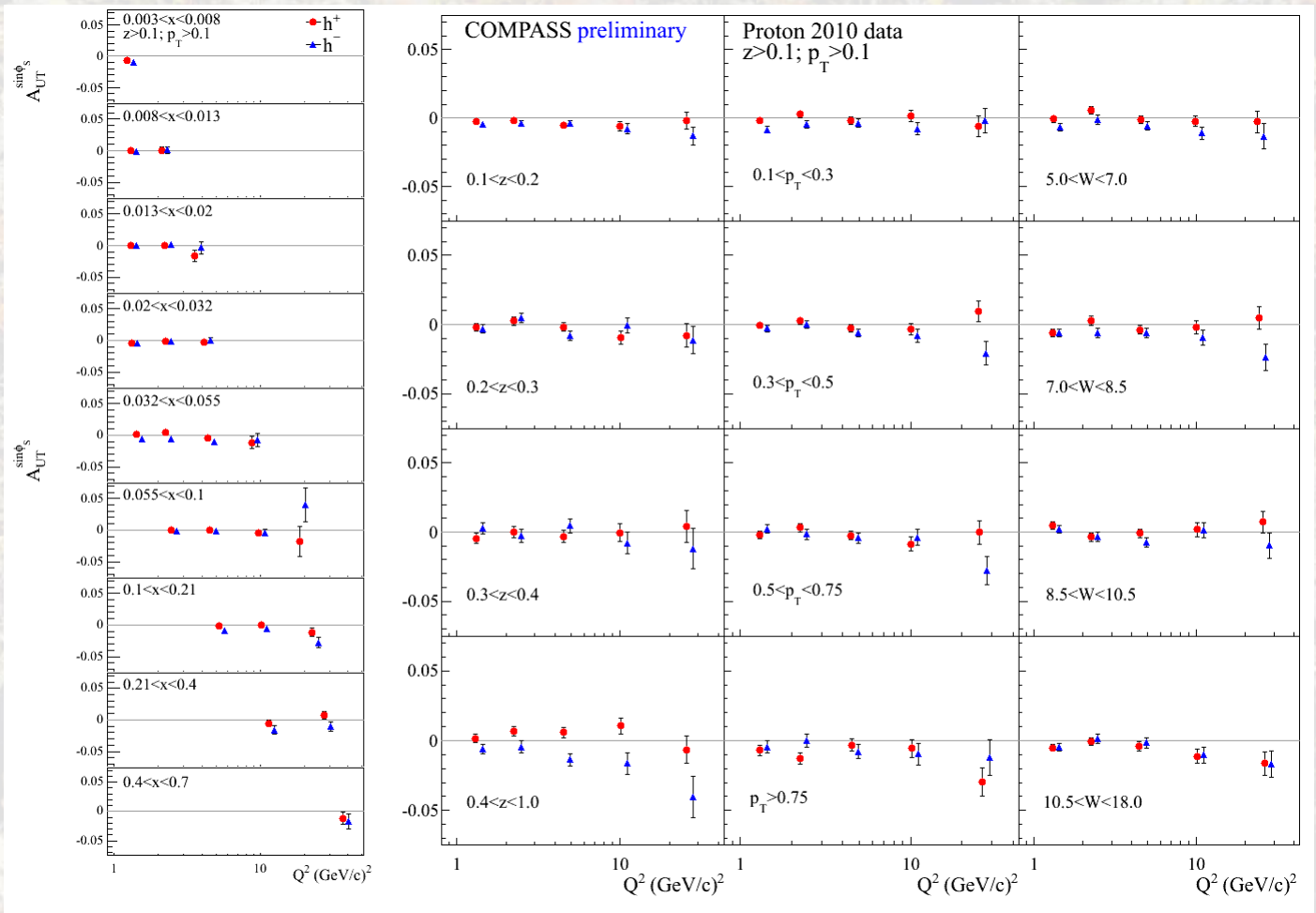
$$A_{LT}^{\cos(\phi_h - \phi_s)}$$



$$A_{LT}^{\cos(\phi_h - \phi_s)} \propto g_{1T}^q \otimes D_{1q}^h, \text{ "Worm Gear" PDF } g_{1T}^q :$$



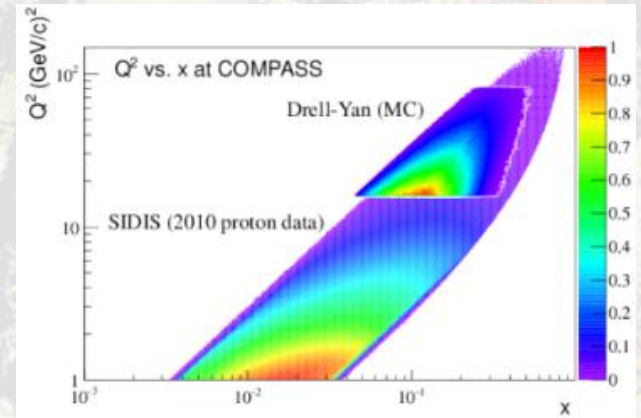
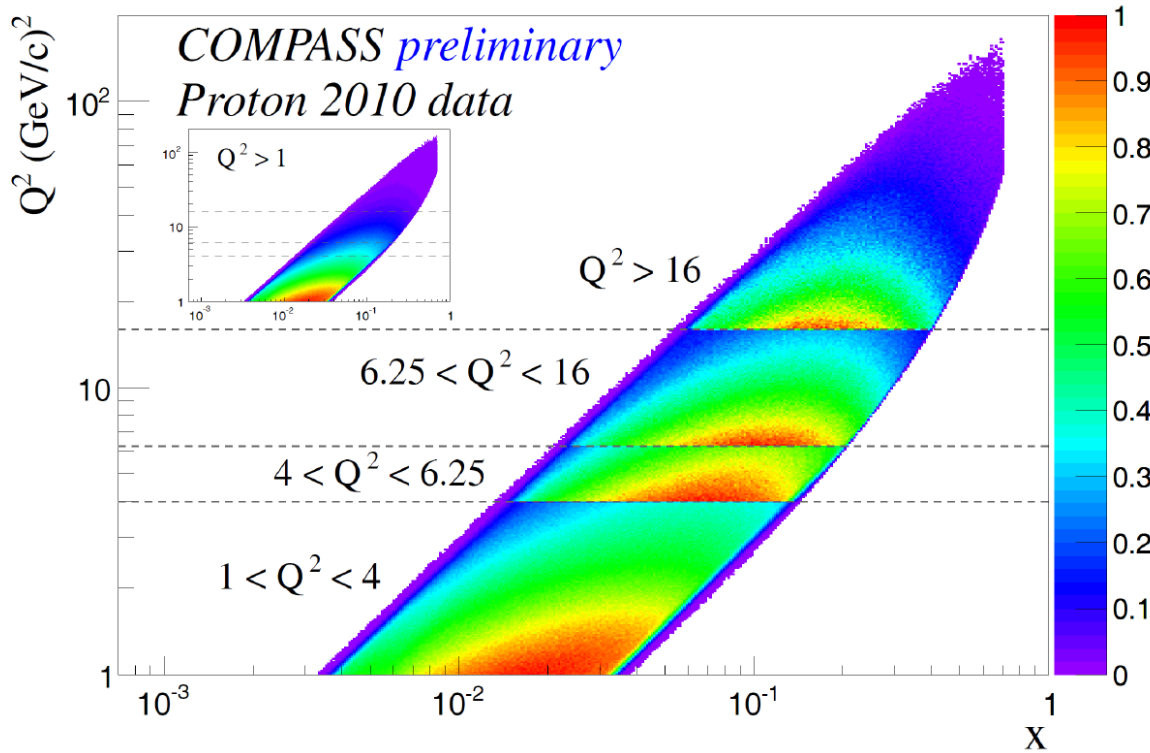
Other Transverse Target spin asymmetries on p



Near COMPASS future is defined:

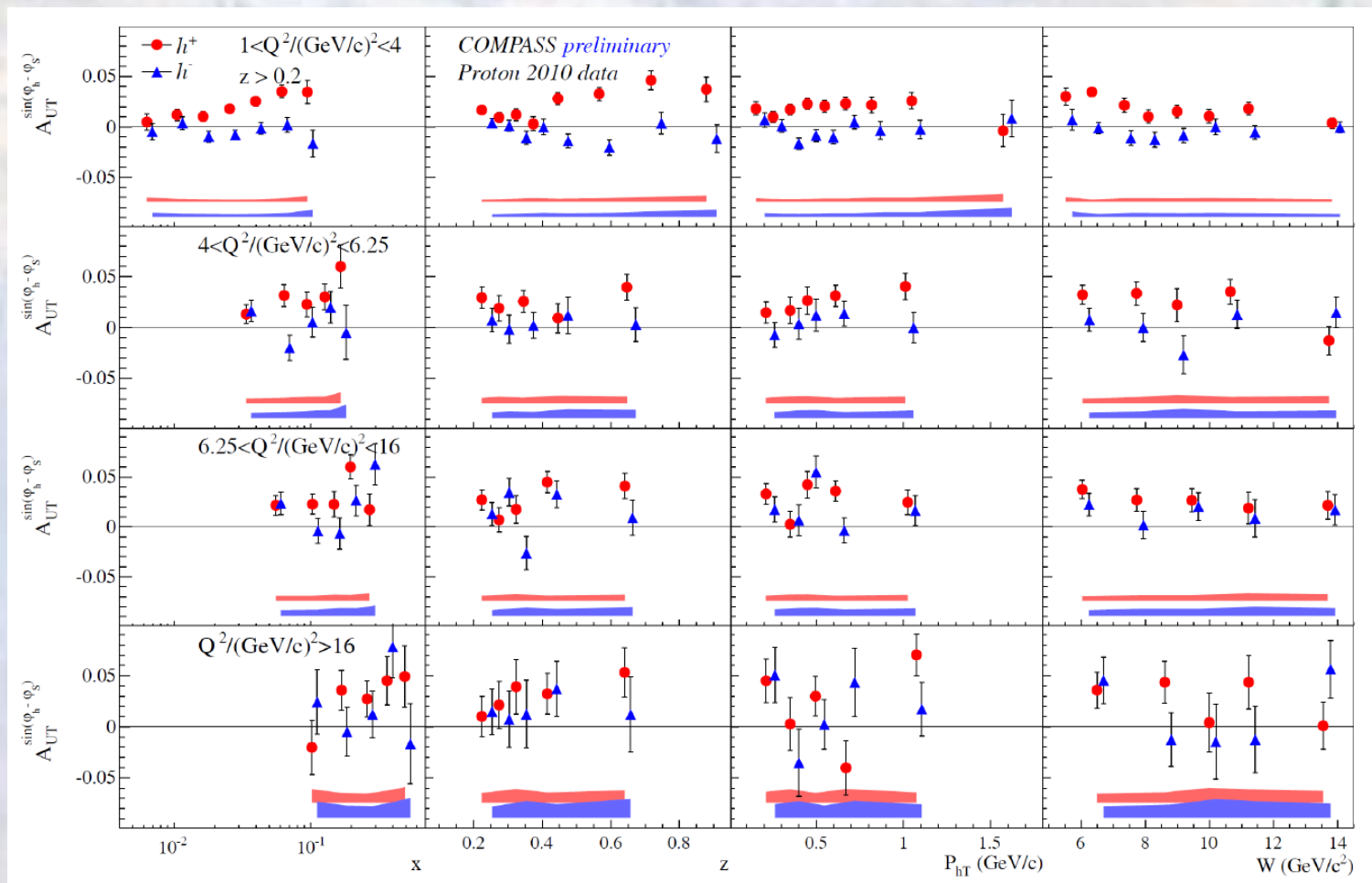
- 2014-2015: Transversely polarized DY
 - to check pseudo-universality ($[f_{1T}^\perp(x, Q^2)]_{DY} \approx -[f_{1T}^\perp(x, Q^2)]_{SIDIS}$)
- 2016-2017: Unpolarised DVCS/HVMP
 - (B slope and GPD H)
 - and unpolarised SIDIS on LH_2
 - $dn^h / (dN^\mu dz dp_T^2)$ i.e. p_T dependent multiplicities, and h_{1T}^\perp Boer-Mulders TMD PDF

Q^2 vs x phase space at COMPASS



The phase spaces of the two processes overlap at COMPASS
→ Consistent extraction of TMD DPFs in the same region

Sivers in DY range



More in the FUTURE:

	physics item	key aspects of the measurement
Hadron	glueballs	280 GeV beam, higher intensity, π , K and \bar{p} separation
GPD	E	transversely polarized proton target
SIDIS	h_1^d with same accuracy as h_1^u f_1^\perp evolution	transversely polarized deuteron target 100 GeV and transversely polarized proton target
DY	universality of TMD PDFs flavor separation test of the Lam-Tung relation EMC effect in DY	higher statistics with transversely polarized proton target transversely polarized deuteron target hydrogen target different nuclear targets

For the next 10 years

- **before any collider is available,**
- **and complementary to Jlab 12 GeV**

COMPASS@CERN can be a major player in QCD physics using its unique high energy both:

- **hadron beam and**
- **positive and negative muon beams**

Looking even further...a polarized lepton-nucleon collider will be a mandatory tool

Thank You



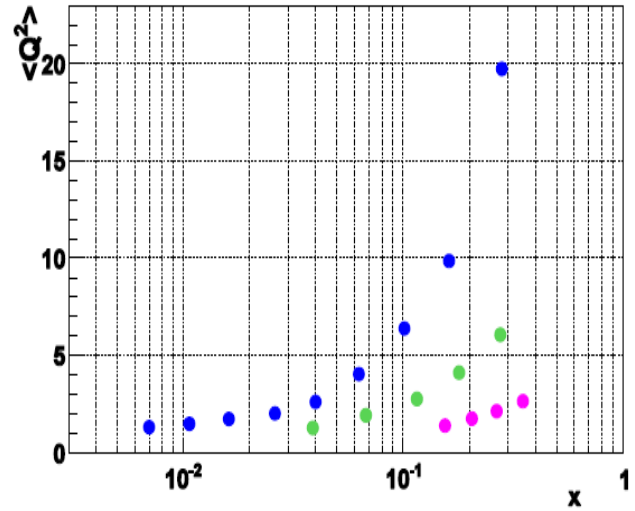
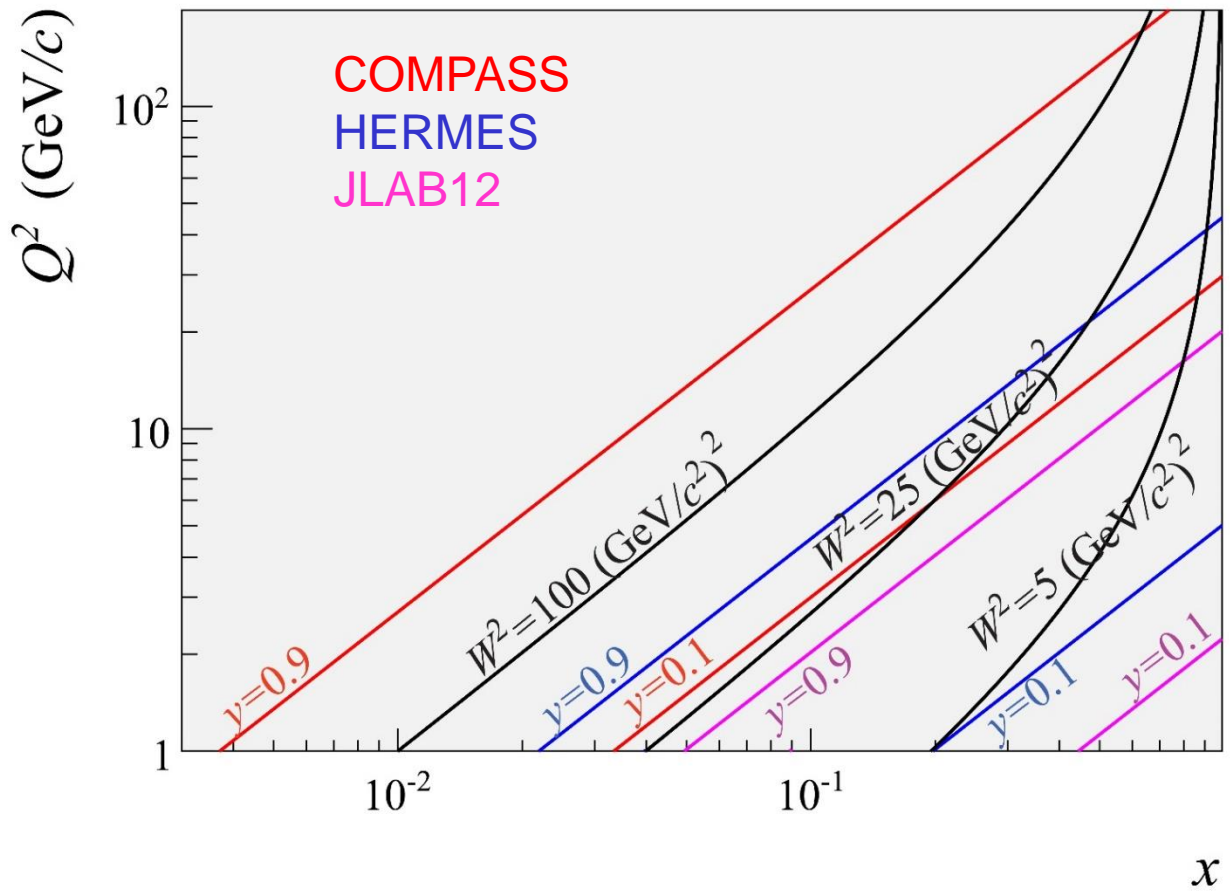
Measurements with the target transversely polarized:

Year	Obs	
2005	$A_{Siv,d}^h, A_{Col,d}^h$	First ${}^6\text{LiD}$ data
2006	$A_{Siv,d}^h, A_{Col,d}^h$	Full ${}^6\text{LiD}$ statistics
2009	$A_{Siv,d}^{\pi^\pm, K^\pm, K_S^0}, A_{Col,d}^{\pi^\pm, K^\pm, K_S^0}$	Full ${}^6\text{LiD}$ statistics
2010	$A_{Siv,p}^h, A_{Col,p}^h$	2007 NH_3 data
2012	$A_{UT,d}^{\sin\phi_{RS}}, A_{UT,p}^{\sin\phi_{RS}}$	Full ${}^6\text{LiD}$
2012	$A_{Siv,p}^h, A_{Col,p}^h$	Full NH_3 statistics
2012	$A_{UT,d}^{\sin(\phi_\rho - \phi_S)}, A_{UT,p}^{\sin(\phi_\rho - \phi_S)}$	Exclusive ρ^0
2013	$A_{UT,d}^{(\phi_\rho, \phi_S)}, A_{UT,p}^{(\phi_\rho, \phi_S)}$	Exclusive ρ^0 , all asyms.
2014	$A_{UT,d}^{\sin\phi_{RS}}, A_{UT,p}^{\sin\phi_{RS}}$	Full ${}^6\text{LiD}$ and NH_3
2014	$A_{Siv,d}^{\pi^\pm, K^\pm, K_S^0}, A_{Col,d}^{\pi^\pm, K^\pm, K_S^0}$	Full NH_3 statistics
2015	Interplay $A_{UT,p}^{\sin\phi_{RS}}$ vs $A_{Col,p}^h$	Full NH_3 statistics

Measurements with unpolarised targets:

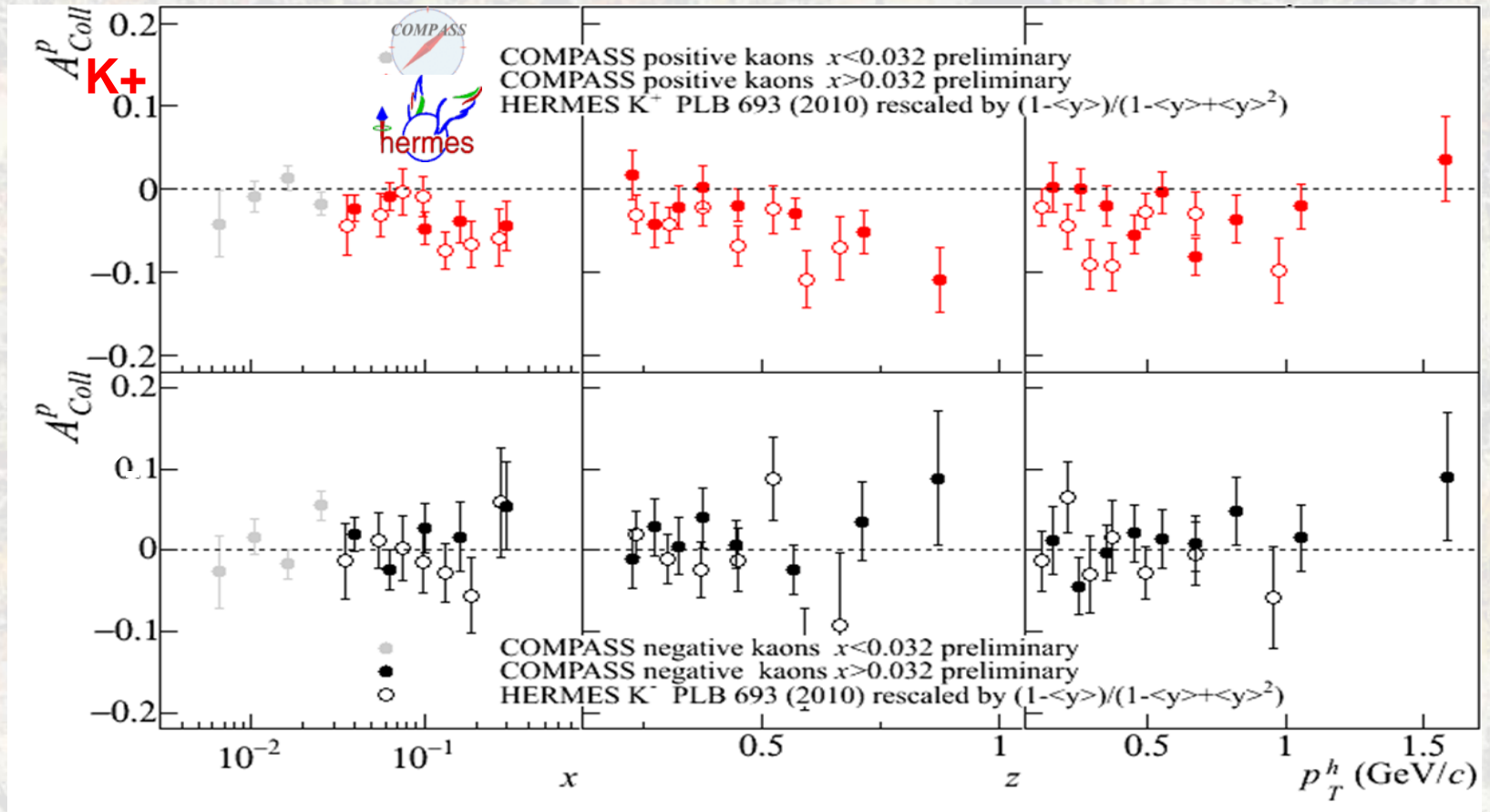
Year	Obs	
2013	$dn^h / (dN^\mu dz dp_T^2)$	Unpolarized multiplicities on d, 2004
2014	$A_{UU,d}^{\cos \phi_h}, A_{UU,d}^{\cos 2\phi_h}, A_{LU,d}^{\sin \phi_h}$	2004, part
2016	$dn^\pi / (dN^\mu dz)$	Unpolarized multiplicities on d, 2006
2016	$dn^h / (dN^\mu dz dp_T^2)$	Unpolarized multiplicities on d, 2006
2016	$dn^K / (dN^\mu dz)$	Unpolarized multiplicities on d, 2006

Kinematic coverage

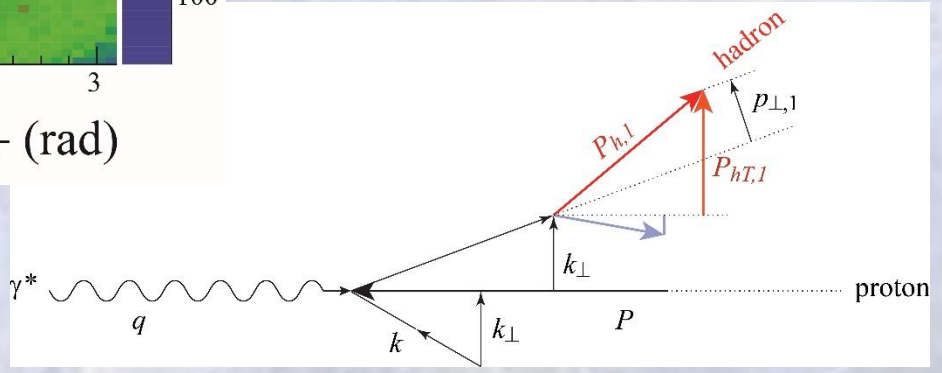
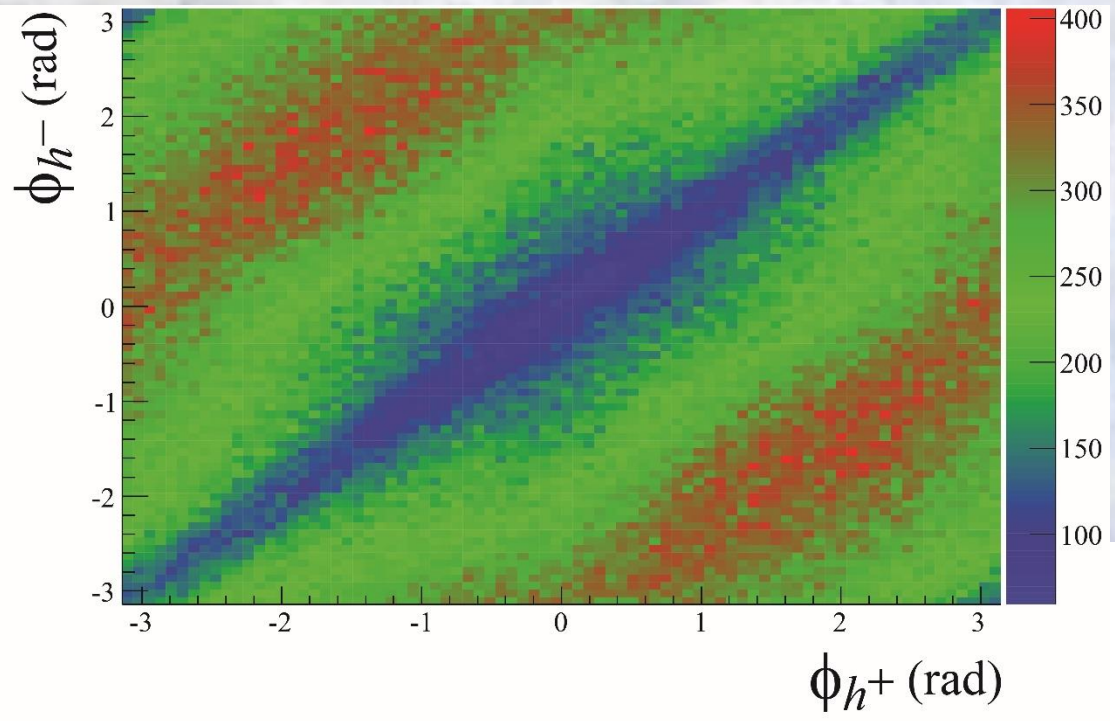


Collins asymmetry on proton $x > 0.032$ region

charged kaons COMPASS and HERMES results



Is correlation having an impact?

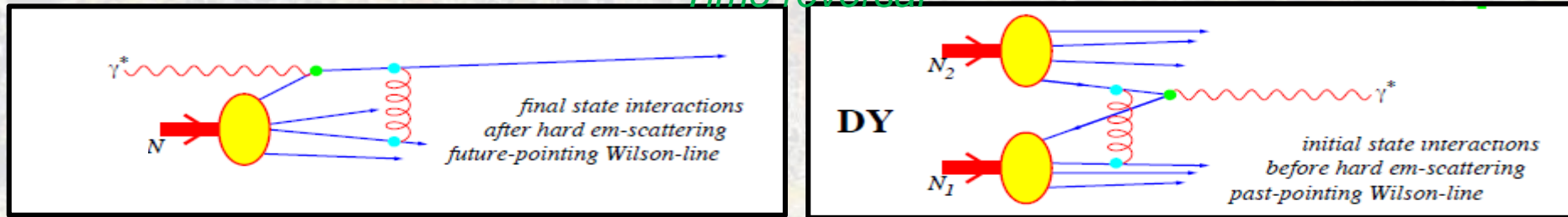


Test of universality

T-odd character of the Boer-Mulders and Sivers functions

In order not to vanish by time-reversal invariance T-odd SSA require an interaction phase generated by a rescattering of the struck parton in the field of the hadron remnant

Time reversal



these functions are process dependent, they change sign to provide the gauge invariance

$$h_1^\perp(\text{SIDIS}) = -h_1^\perp(\text{DY})$$

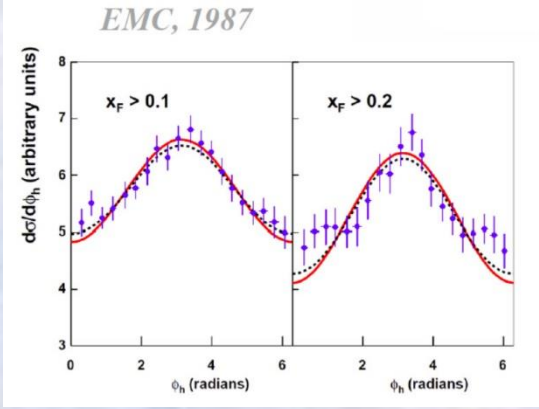
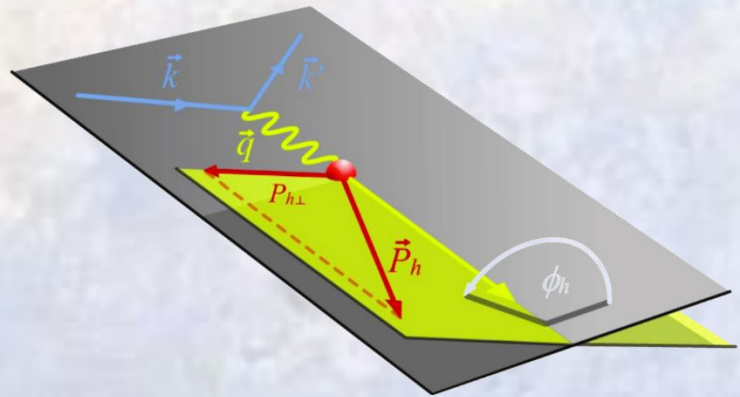
Boer-Mulders

Sivers

$$f_{1T}^\perp(\text{SIDIS}) = -f_{1T}^\perp(\text{DY})$$

Unpolarised Azimuthal Modulation

Huge azimuthal ϕ modulation on unpolarised target measured by EMC in 1987



$d\sigma^{\ell p \rightarrow \ell' h X} = \sum_q f_q(x, Q^2) \otimes d\sigma^{\ell q \rightarrow \ell' q} \otimes D_q^h(z, Q^2)$ where, in collinear PM $d\sigma^{\ell q \rightarrow \ell' q} = \hat{s}^2 + \hat{u}^2 = x[1 + (1 - y)^2]$, i.e. no ϕ_h dependence. Taking into account the parton transverse momentum in the kinematics leads to:

$$\hat{s} = sx \left[1 - \frac{2k_{\perp}}{Q} \sqrt{1-y} \cos \phi_h \right] + \mathcal{O} \left(\frac{k_{\perp}^2}{Q} \right) \quad \hat{u} = sx(1-y) \left[1 - \frac{2k_{\perp}}{Q\sqrt{1-y}} \cos \phi_h \right] + \mathcal{O} \left(\frac{k_{\perp}^2}{Q} \right)$$

Resulting in the $\cos \phi_h$ and $\cos 2\phi_h$ modulations observed in the azimuthal distributions

SIDIS access to TMDs

$$\sigma(\ell p \rightarrow \ell' h X) \sim q(x) \otimes \hat{\sigma}^{\gamma q \rightarrow q} \otimes D_q^h(z)$$

TMDs
(x, \vec{k}_\perp)

FFs
(z, \vec{p}_\perp)

Nucleon polarization

		U	T	L
Parton polarization	U	f_1	f_{1T}^\perp	
	T	h_1^\perp	h_1, h_{1T}^\perp	h_{1L}^\perp
	L		g_{1T}	g_{1L}

Hadron polarization

		U	T	L
Parton polarization	U	D_1	D_{1T}^\perp	
	T	H_1^\perp	H_1, H_{1T}^\perp	H_{1L}^\perp
	L		G_{1T}	G_{1L}

T odd

chiral odd

Factorisation (Collins & Soper, Ji, Ma, Yuan, Qiu & Vogelsang, Collins & Metz...)

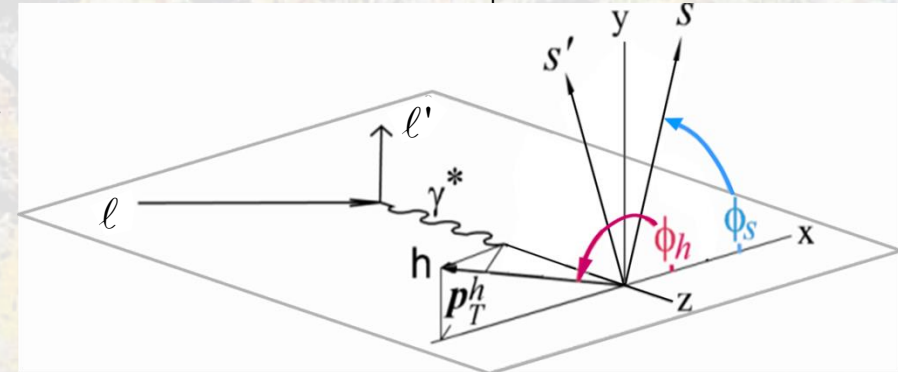
SIDIS 1h x-section

$$A_{U(L),T}^{w(\varphi_h, \varphi_S)} = \frac{F_{U(L),T}^{w(\varphi_h, \varphi_S)}}{F_{UU,T} + \varepsilon F_{UU,L}}$$

$$\frac{d\sigma}{dx dy dz dP_{h\perp}^2 d\varphi_h d\psi} = \left[\frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x} \right) \right] \times (F_{UU,T} + \varepsilon F_{UU,L}) \times$$

$$\varepsilon = \frac{1 - y - \frac{1}{4} y^2 \gamma^2}{1 - y + \frac{1}{2} y^2 + \frac{1}{4} y^2 \gamma^2}, \quad \gamma = \frac{2xM}{Q}$$

$$\left[\begin{array}{l} 1 + \cos \varphi_h \times \sqrt{2\varepsilon(1+\varepsilon)} A_{UU}^{\cos \varphi_h} + \cos(2\varphi_h) \times \varepsilon A_{UU}^{\cos(2\varphi_h)} + \lambda \sin \varphi_h \times \sqrt{2\varepsilon(1-\varepsilon)} A_{LU}^{\sin \varphi_h} + \\ S_L \left[\sin \varphi_h \times \sqrt{2\varepsilon(1+\varepsilon)} A_{UL}^{\sin \varphi_h} + \sin(2\varphi_h) \times \varepsilon A_{UL}^{\sin(2\varphi_h)} \right] + \\ S_L \lambda \left[\sqrt{1-\varepsilon^2} A_{LL} + \cos \varphi_h \sqrt{2\varepsilon(1-\varepsilon)} A_{LL}^{\cos \varphi_h} \right] + \\ S_T \left[\begin{array}{l} \sin \varphi_S \times \left(\sqrt{2\varepsilon(1+\varepsilon)} A_{UT}^{\sin \varphi_S} \right) + \\ \sin(\varphi_h - \varphi_S) \times \left(A_{UT}^{\sin(\varphi_h - \varphi_S)} \right) + \\ \sin(\varphi_h + \varphi_S) \times \left(\varepsilon A_{UT}^{\sin(\varphi_h + \varphi_S)} \right) + \\ \sin(2\varphi_h - \varphi_S) \times \left(\sqrt{2\varepsilon(1+\varepsilon)} A_{UT}^{\sin(2\varphi_h - \varphi_S)} \right) + \\ \sin(3\varphi_h - \varphi_S) \times \left(\varepsilon A_{UT}^{\sin(3\varphi_h - \varphi_S)} \right) \end{array} \right] + \\ S_T \lambda \left[\begin{array}{l} \cos \varphi_S \times \left(\sqrt{2\varepsilon(1-\varepsilon)} A_{LT}^{\cos \varphi_S} \right) + \\ \cos(\varphi_h - \varphi_S) \times \left(\sqrt{(1-\varepsilon^2)} A_{UT}^{\cos(\varphi_h - \varphi_S)} \right) + \\ \cos(2\varphi_h - \varphi_S) \times \left(\sqrt{2\varepsilon(1-\varepsilon)} A_{UT}^{\cos(2\varphi_h - \varphi_S)} \right) \end{array} \right] \end{array} \right]$$

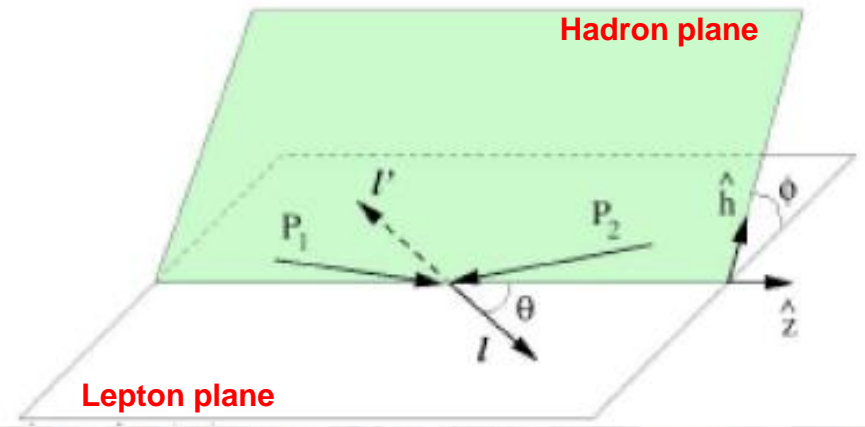


The polarized Drell-Yan process in $\pi^- p$

$$\frac{d\sigma}{d^4q d\Omega} = \left[\frac{\alpha^2}{Fq^2} (F_{UU}^1 + F_{UU}^1) (1 + A_{UU}^1 \cos^2 \theta) \right] \times$$

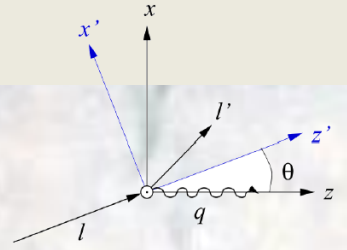
$$\left\{ \begin{aligned} & 1 + \cos \varphi \times D_{[\sin 2\theta]} A_{UU}^{\cos \varphi} + \cos(2\varphi) \times D_{[\sin^2 \theta]} A_{UU}^{\cos(2\varphi_h)} + \\ & S_L \left[\sin \varphi \times D_{[\sin 2\theta]} A_{UL}^{\sin \varphi} + \sin(2\varphi) \times D_{[\sin^2 \theta]} A_{UL}^{\sin(2\varphi)} \right] + \\ & S_T \left[\begin{aligned} & \sin \varphi_S \times \left(D_{[1]} A_{UT}^{\sin \varphi_S} + D_{[\cos^2 \theta]} \tilde{A}_{UT}^{\sin \varphi_S} \right) + \\ & \sin(\varphi - \varphi_S) \times \left(D_{[\sin 2\theta]} A_{UT}^{\sin(\varphi - \varphi_S)} \right) + \\ & \sin(\varphi + \varphi_S) \times \left(D_{[\sin 2\theta]} A_{UT}^{\sin(\varphi + \varphi_S)} \right) + \\ & \sin(2\varphi - \varphi_S) \times \left(D_{[\sin^2 \theta]} A_{UT}^{\sin(2\varphi - \varphi_S)} \right) + \\ & \sin(2\varphi + \varphi_S) \times \left(D_{[\sin^2 \theta]} A_{UU}^{\sin(2\varphi_h + \varphi_S)} \right) \end{aligned} \right] + \end{aligned} \right.$$

Collins-Soper frame (of virtual photon)
 θ, φ lepton plane wrt hadron plane
 target rest frame
 φ_S target transverse spin vector / virtual photon

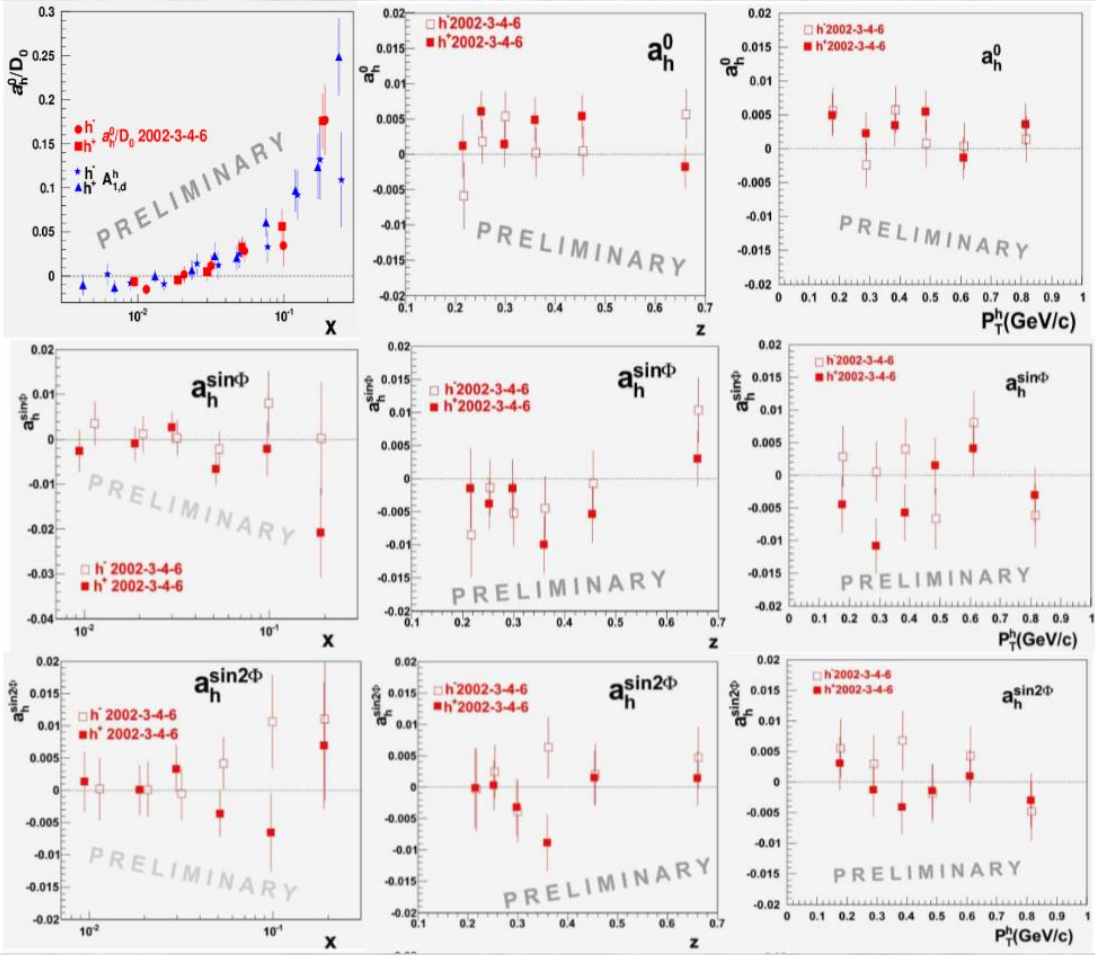


SIDIS 1h x-section

$$\begin{aligned}
 \frac{d\sigma}{dx dy dz dP_{h\perp}^2 d\varphi_h d\varphi_S} &= \left[\frac{\cos\theta}{1 - \sin^2\theta \sin^2\varphi_S} \right] \left[\frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x} \right) \right] \times (F_{UU,T} + \varepsilon F_{UU,L}) \times \\
 &\left\{ \begin{aligned}
 &1 + \cos\varphi_h \times \sqrt{2\varepsilon(1+\varepsilon)} A_{UU}^{\cos\varphi_h} + \cos(2\varphi_h) \times \varepsilon A_{UU}^{\cos(2\varphi_h)} + \lambda \sin\varphi_h \times \sqrt{2\varepsilon(1-\varepsilon)} A_{LU}^{\sin\varphi_h} + \\
 &\left. \begin{aligned}
 &\sin\varphi_S \times \left(\cos\theta \sqrt{2\varepsilon(1+\varepsilon)} A_{UT}^{\sin\varphi_S} \right) + \\
 &\sin(\varphi_h - \varphi_S) \times \left(\cos\theta A_{UT}^{\sin(\varphi_h - \varphi_S)} + \frac{1}{2} \sin\theta \sqrt{2\varepsilon(1+\varepsilon)} A_{UL}^{\sin 2\varphi_h} \right) + \\
 &\sin(\varphi_h + \varphi_S) \times \left(\cos\theta \varepsilon A_{UT}^{\sin(\varphi_h + \varphi_S)} + \frac{1}{2} \sin\theta \sqrt{2\varepsilon(1+\varepsilon)} A_{UL}^{\sin 2\varphi_h} \right) + \\
 &\sin(2\varphi_h - \varphi_S) \times \left(\cos\theta \sqrt{2\varepsilon(1+\varepsilon)} A_{UT}^{\sin(2\varphi_h - \varphi_S)} + \frac{1}{2} \sin\theta \varepsilon A_{UL}^{\sin 2\varphi_h} \right) + \\
 &\sin(3\varphi_h - \varphi_S) \times \left(\cos\theta \varepsilon A_{UT}^{\sin(3\varphi_h - \varphi_S)} \right) + \sin(2\varphi_h + \varphi_S) \times \left(\frac{1}{2} \sin\theta \varepsilon A_{UL}^{\sin 2\varphi_h} \right) + \\
 &\cos\varphi_S \times \left(\cos\theta \sqrt{2\varepsilon(1-\varepsilon)} A_{LT}^{\cos\varphi_S} + \sin\theta \sqrt{(1-\varepsilon^2)} A_{LL} \right) + \\
 &\cos(\varphi_h - \varphi_S) \times \left(\cos\theta \sqrt{(1-\varepsilon^2)} A_{UT}^{\cos(\varphi_h - \varphi_S)} + \frac{1}{2} \sin\theta \sqrt{2\varepsilon(1-\varepsilon)} A_{LL}^{\cos\varphi_h} \right) + \\
 &\cos(2\varphi_h - \varphi_S) \times \left(\cos\theta \sqrt{2\varepsilon(1-\varepsilon)} A_{UT}^{\cos(2\varphi_h - \varphi_S)} \right) + \cos(\varphi_h + \varphi_S) \times \left(\frac{1}{2} \sin\theta \sqrt{2\varepsilon(1-\varepsilon)} A_{LL}^{\cos\varphi_h} \right)
 \end{aligned} \right\} \\
 &\left. \begin{aligned}
 &\frac{\mathbf{P}_T}{\sqrt{1 - \sin^2\theta \sin^2\varphi_S}} \\
 &\frac{\mathbf{P}_T \lambda}{\sqrt{1 - \sin^2\theta \sin^2\varphi_S}}
 \end{aligned} \right\} \text{lepton plane}
 \end{aligned}$$



Longitudinal modulations



The asymmetries

- The asymmetries are:

$$A_{U(L),T}^{w(\phi_h,\phi_s)}(x, z, p_T; Q^2) = \frac{F_{U(L),T}^{w(\phi_h,\phi_s)}}{F_{UU,T} + \varepsilon F_{UU,L}}$$

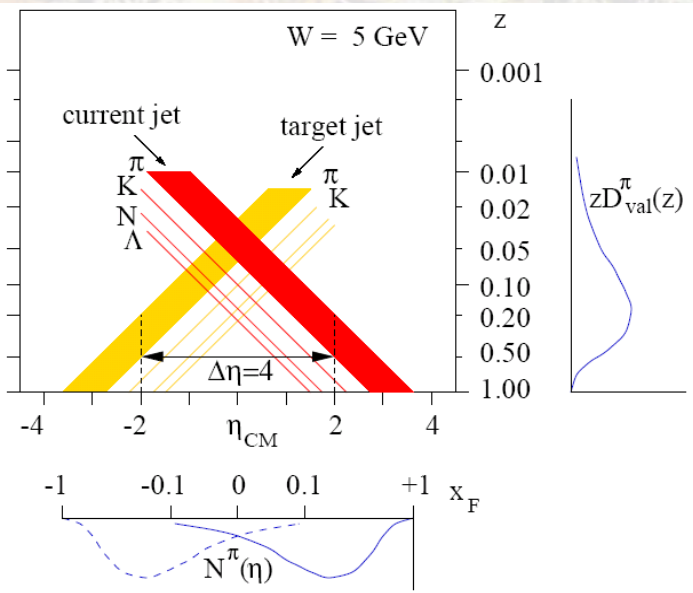
- When we measure on 1D

$$A_{U(L),T}^{w(\phi_h,\phi_s)}(x) = \frac{\int_{Q_{min}^2}^{Q_{max}^2} dQ^2 \int_{z_{min}}^{z_{max}} dz \int_{p_{T,min}}^{p_{T,max}} d^2\vec{p}_T F_{U(L),T}^{w(\phi_h,\phi_s)}}{\int_{Q_{min}^2}^{Q_{max}^2} dQ^2 \int_{z_{min}}^{z_{max}} dz \int_{p_{T,min}}^{p_{T,max}} d^2\vec{p}_T (F_{UU,T} + \varepsilon F_{UU,L})}$$

Ed. Berger criterion (separation of CFR & TFR)

The typical hadronic correlation length in rapidity is

$$\Delta y_h \simeq 2$$



if the dynamics of quark fragmentation is to be studied independently of “contamination” from target fragmentation, it is necessary that $Y \gtrsim 4$, or, equivalently, that

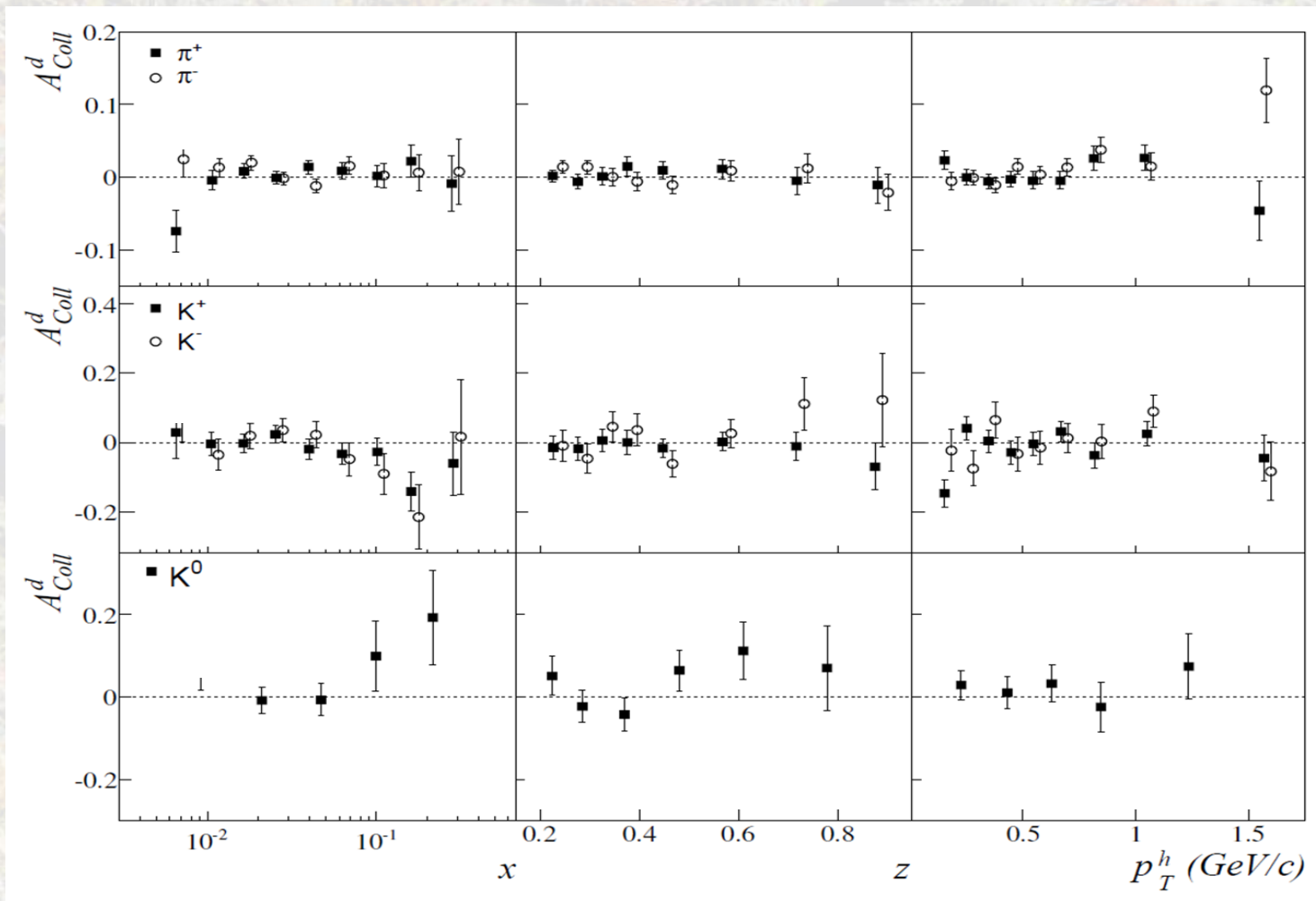
$$W_X = \left[\frac{Q^2(1-x)}{x} \right]^{1/2} \gtrsim 7.4 \text{ GeV}. \quad (17)$$

If the inequality Eq. (17) is satisfied, it should be possible to measure fragmentation functions $D(z, Q^2)$ over essentially the full range of z , $0 < z < 1$. Somewhat smaller values of W_X may be adequate if attention is restricted to the large z region. As Y is increased above 2, or

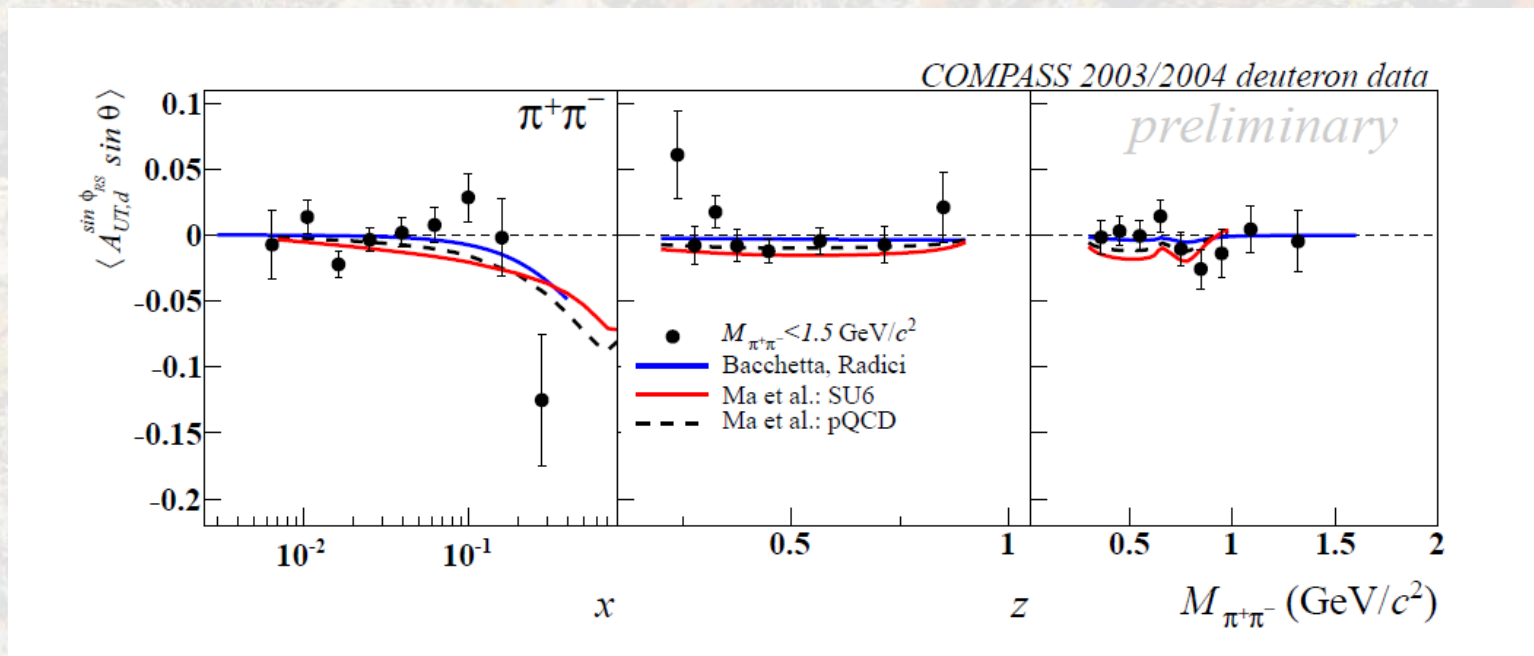
$$W_X \gtrsim 3 \text{ GeV}, \quad (18)$$

the quark and target fragmentation regions begin to separate. As long as $Y \gtrsim 2$, the hadrons with the largest values of z are most likely quark fragments. Data¹⁴ from $e^+e^- \rightarrow hX$ show that a distinct function $D(z)$ may have developed for $z \gtrsim 0.5$ at $W = 3 \text{ GeV}$. The region extends to $z \simeq 0.2$ for $W = 4.8 \text{ GeV}$, and to $z \simeq 0.1$ for $W = 7.4 \text{ GeV}$. For $z > 0.3$, fragmentation functions have been obtained from data¹⁵ on $ep \rightarrow e'\pi^\pm X$ at $E = 11.5 \text{ GeV}$, with $3 < W_X < 4 \text{ GeV}$.

Collins asymmetry on deuteron



2h asymmetries on d



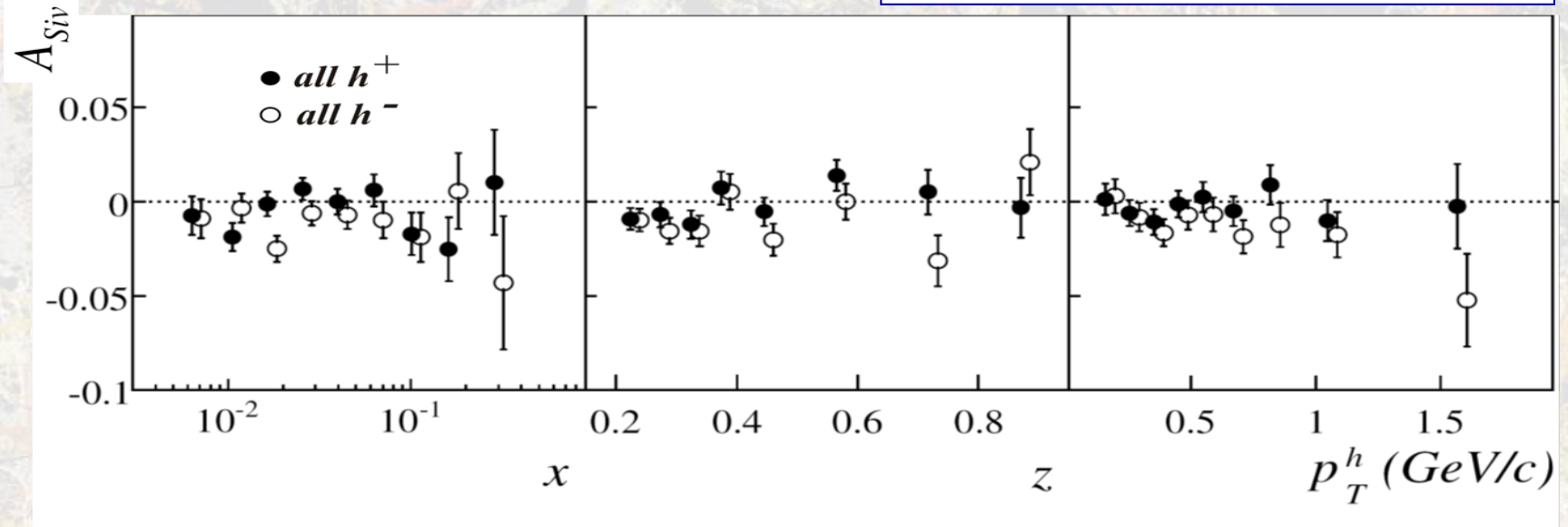
$$A_{UT}^{\sin(\phi_R + \phi_S - \pi)} = \frac{\sum_q e_q^2 h_1^q(x) H_{q \rightarrow h_1 h_2}^Z(z, \mathcal{M}_{h_1 h_2}^2)}{\sum_q e_q^2 q(x) D_q^{h_1 h_2}(z, \mathcal{M}_{h_1 h_2}^2)}$$

Sivers asymmetry on deuteron

PLB 673 (2009) 127

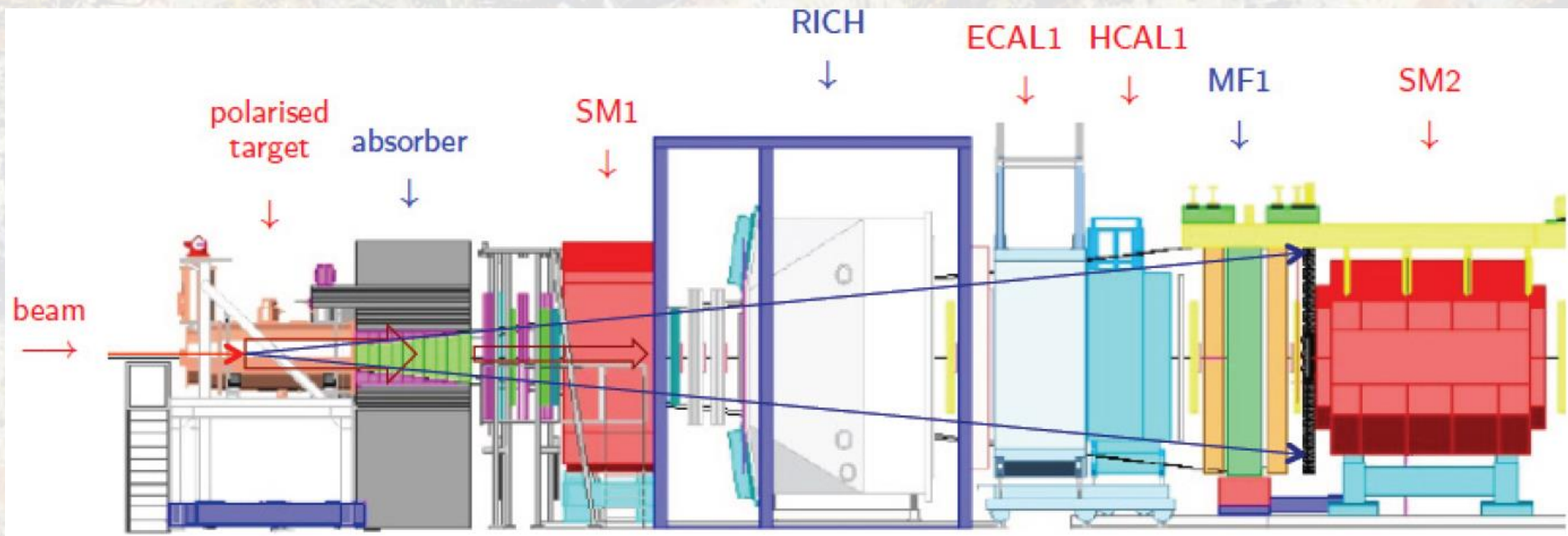


understood as
u – d cancellation



$$f_{1T,u}^\perp \approx -f_{1T,d}^\perp$$

Hadron beam: Drell-Yan setup



GPDs with Hard Exclusive γ and Meson Production

COMPASS-II 2016-17: with LH₂ target + RPD (phase 1) $\mu^{\downarrow}, \mu^{\uparrow}$ 160 GeV

- ✓ the t-slope of the DVCS and HEMP cross section
→ transverse distribution of partons
- ✓ the Beam Charge and Spin Sum and Difference
→ $\text{Re } T^{DVCS}$ and $\text{Im } T^{DVCS}$ for the GPD H determination
- ✓ Vector Meson $\rho^0, \rho^+, \omega, \Phi$
- ✓ Pseudo-scalar π^0

(Using the 2007-10 data: transv. polarized NH₃ target without RPD)

Chromodynamic lensing

Use SIDIS Sivers asymmetry data to constrain shape

Use anomalous magnetic moments to constrain integral

$$f_{1T}^{\perp(0)q}(x, Q_L^2) = -L(x)E^q(x, 0, 0, Q_L^2)$$

$L(x)$ – Lensing function (from Burkart)

E^q – GPD related to quark OAM

n -th moment of a TMD with respect to k_{\perp}

$$f_{1T}^{\perp(n)q}(x, Q^2) = \int d^2k_{\perp} \left(\frac{k_{\perp}^2}{2M^2} \right)^n f_{1T}^{\perp(0)q}(x, k_{\perp}^2, Q_L^2)$$

