



National Centre for  
Nuclear Research  
Warsaw, Poland

# Glucun Contribution to the Sivers Effect COMPASS Results

Adam Szabelski  
Krzysztof Kurek



XVI Workshop on High Energy Spin Physics, DSPIN-15  
Dubna, Russia, September 8-12



Beam:  $2 \cdot 10^8 \mu^+$  / spill (4.8s / 16.2s)

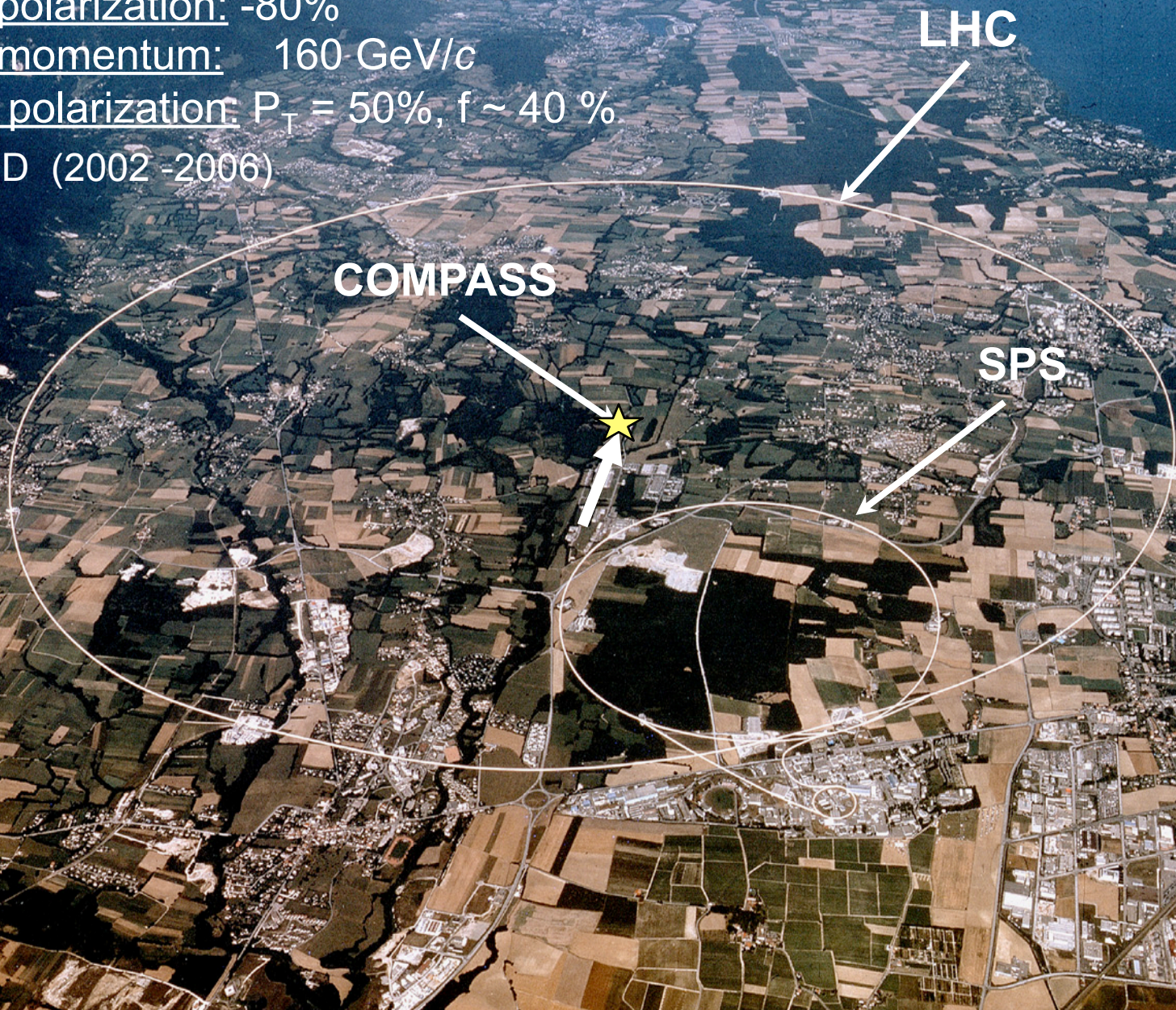
Luminosity  $\sim 5 \cdot 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$

Beam polarization: -80%

Beam momentum: 160 GeV/c

Target polarization:  $P_T = 50\%$ ,  $f \sim 40\%$

for  ${}^6\text{LiD}$  (2002 - 2006)

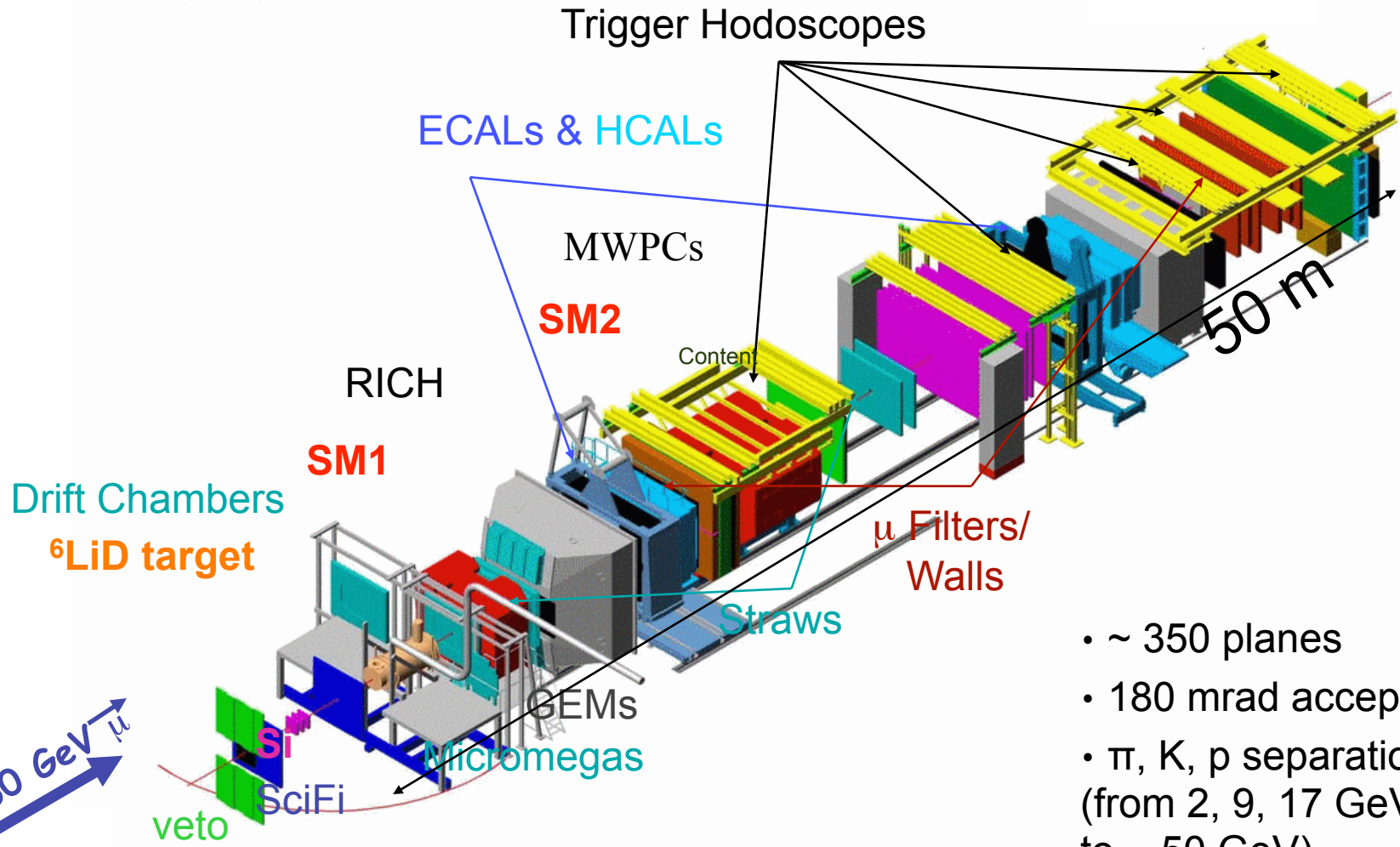




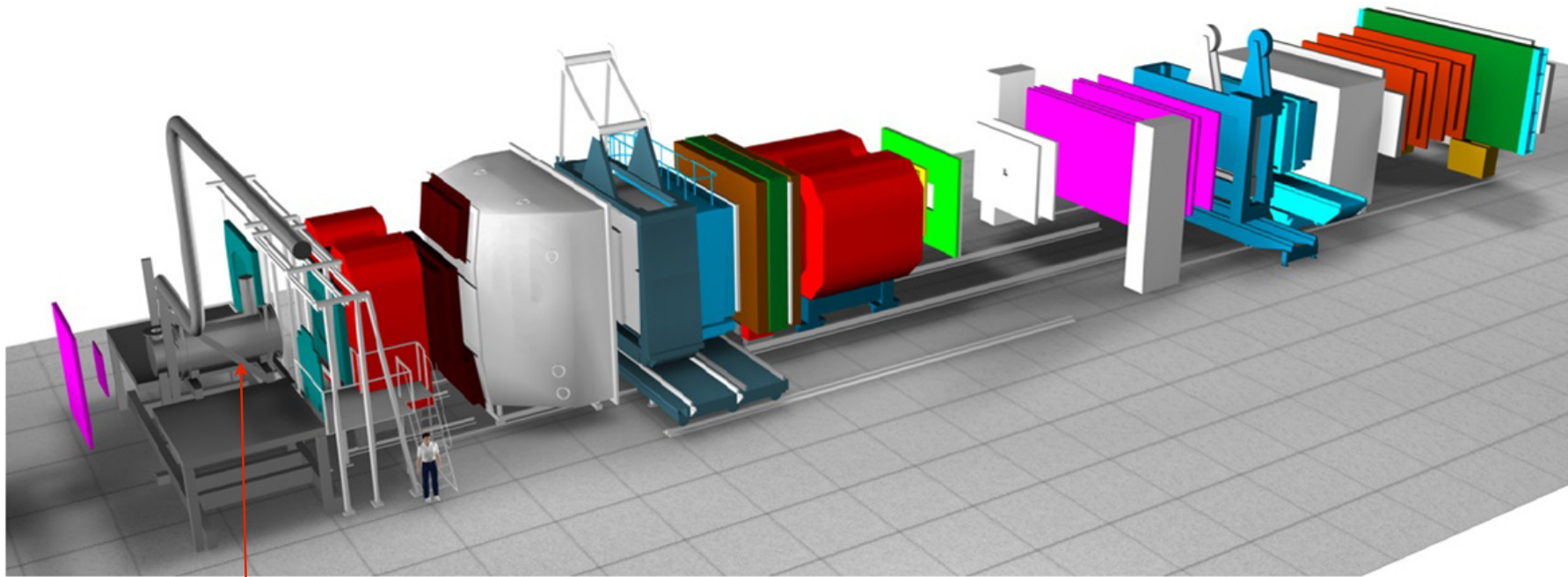
# The COMPASS spectrometer



COMPASS in muon run  
NIM A 577(2007) 455



- ~ 350 planes
- 180 mrad acceptance
- $\pi$ , K, p separation (from 2, 9, 17 GeV up to ~ 50 GeV)

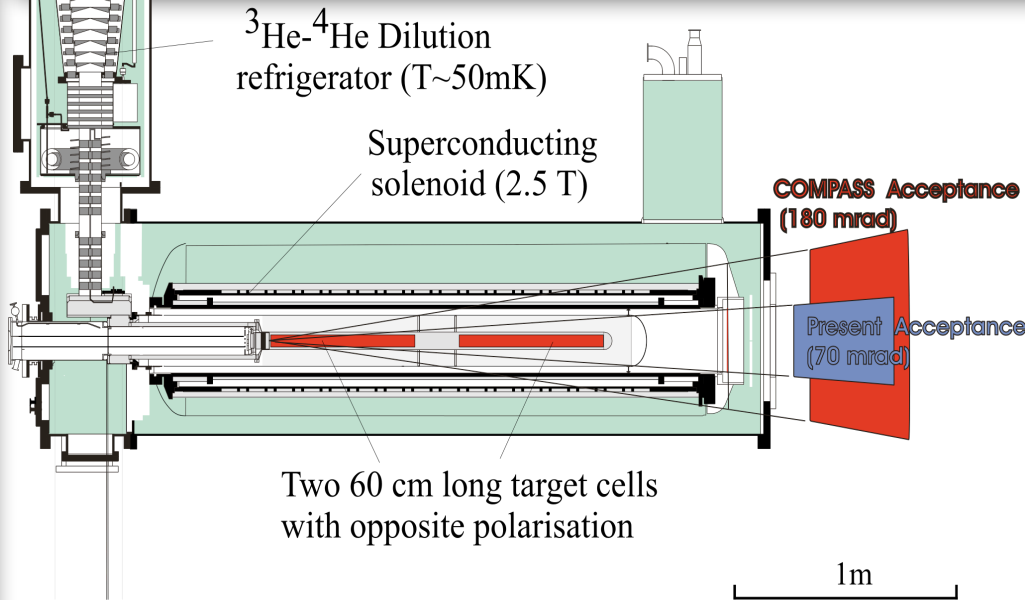


Spectrometer

polarised target

- ~ 350 planes
- 180 mrad acceptance
- $\pi$ , K, p separation  
(from 2, 9, 17 GeV up to ~ 50 GeV)

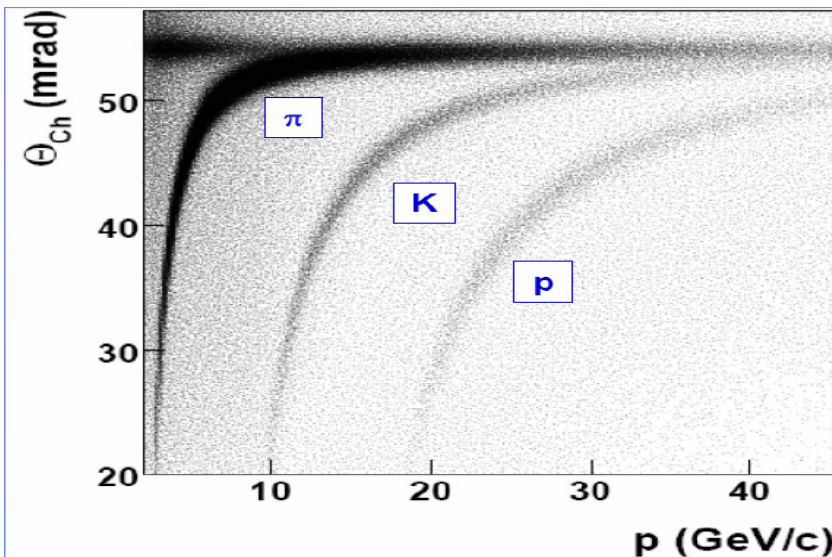
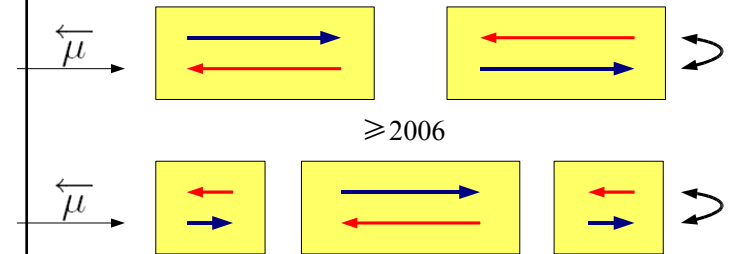
# The COMPASS polarized target and PID



Target material:  $^6\text{LiD}$   
 Polarisation:  $>50\%$   
 Dilution factor:  $\sim 0.4$   
 Dynamic Nuclear Polarization

2006 - new solenoid with acceptance 180 mrad  
 3 target cells  
 (reduce false asymmetries)

2002 - 2004



RICH 2006 upgrade : better PID

MAPMTs in central region

APV electronics in periphery



- Introduction
- Gluon „Sivers efect” measurement @ COMPASS
- Artificial Neural Network approach
- Data and Monte Carlo comparison
- Validation of the method - MC data
- Data selection
- Preliminary results on deuteron target
- Preliminary results on proton target
- Systematic studies
- Summary



## Beyond collinear approximation - $k_T$ dependence

		nucleon polarization		
		U	L	T
quark polarization	U	$f_1$ number density		$f_{1T}^\perp$ - <i>Sivers T-odd</i>
	L		$g_1$ - helicity	$g_{1T}$ - <i>Worm-gear</i>
	T	$h_1^\perp$ - <i>Boer-Mulders T-odd</i>	$h_{1L}^\perp$ - <i>transversity Pretzelosity</i>	$h_{1T}^\perp$ - <i>transversity Pretzelosity</i>

LO, twist-2 - 8 independent functions to parameterize structure



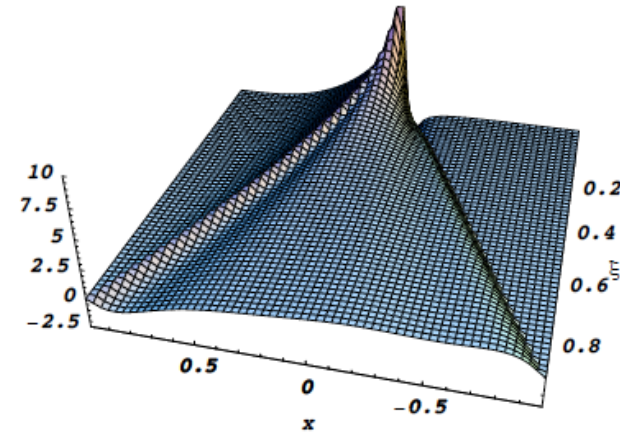
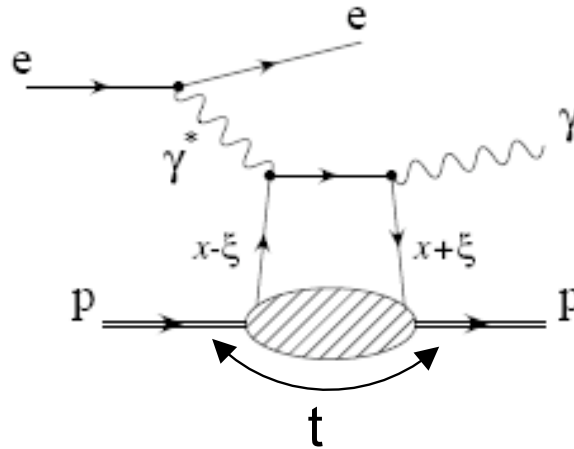


$$H(x, \xi, t), \tilde{H}, E, \tilde{E}$$

$$H(x, 0, 0) = q(x)$$

$$\tilde{H}(x, 0, 0) = \Delta q(x)$$

$$\int H(x, \xi, t) dx = F(t)$$

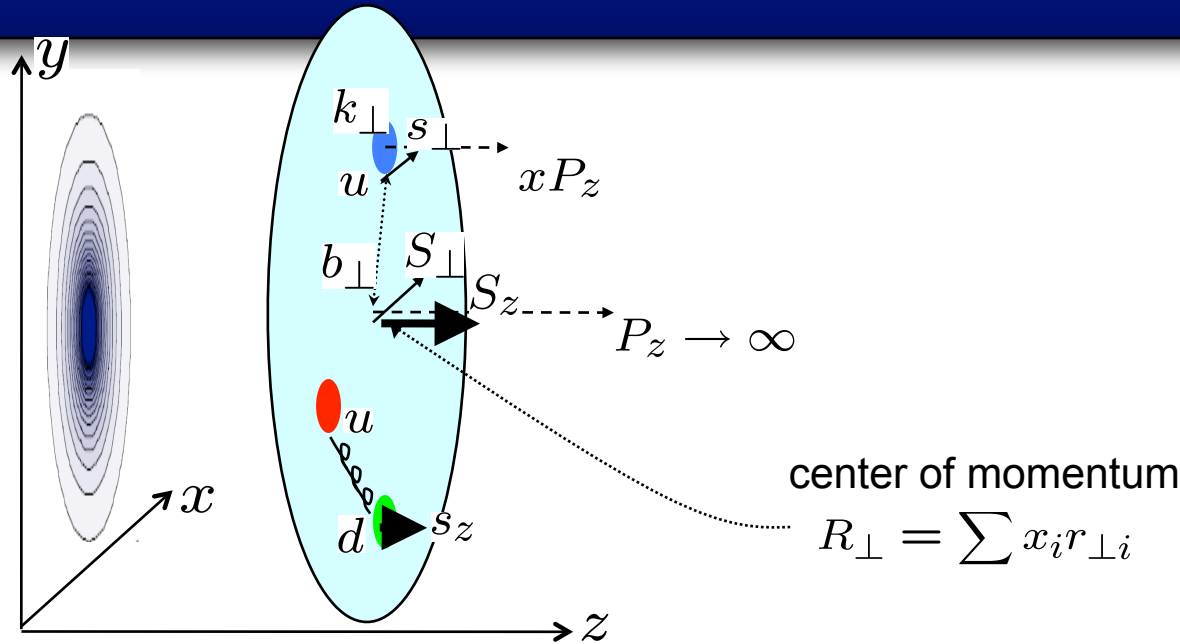


$$J^q(Q^2) = \frac{1}{2} \sum_{i=q, \bar{q}} \int_{-1}^1 x (H^i(Q^2, x, \xi, 0) + E^i(Q^2, x, \xi, 0)) dx$$

$$J^G(Q^2) = \frac{1}{2} \int_{-1}^1 x (H^G(Q^2, x, \xi, 0) + E^G(Q^2, x, \xi, 0)) dx$$

Ji's sum rule



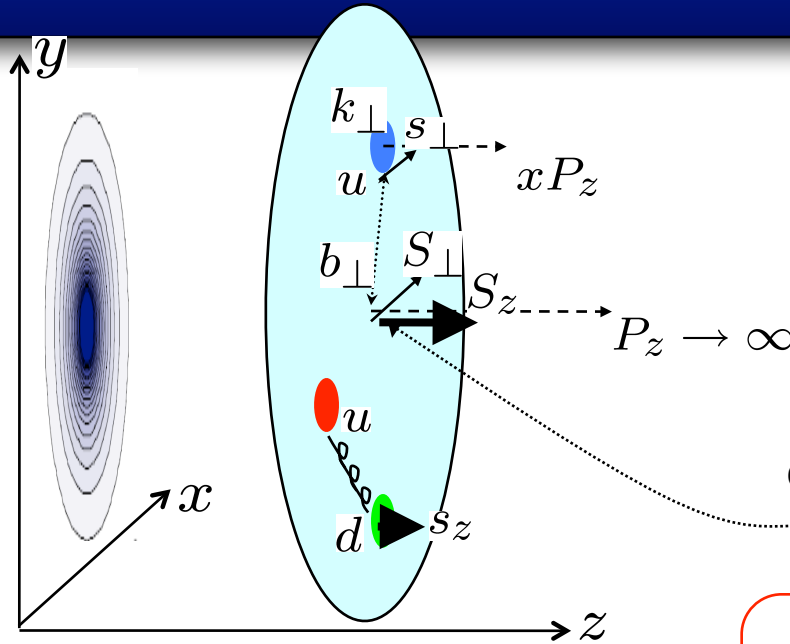


$$\mathcal{H}(x, \vec{b}_{\perp}) = \int \frac{d^2 \Delta_{\perp}}{2\pi} e^{i\vec{\Delta}_{\perp} \cdot \vec{b}_{\perp}} H(x, 0, -\vec{\Delta}_{\perp}^2).$$

$$\tilde{\mathcal{H}}(x, \vec{b}_{\perp}) = \int \frac{d^2 \Delta_{\perp}}{2\pi} e^{i\vec{\Delta}_{\perp} \cdot \vec{b}_{\perp}} \tilde{H}(x, 0, -\vec{\Delta}_{\perp}^2),$$

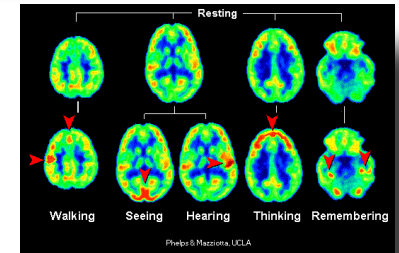
$$\mathcal{E}(x, \vec{b}_{\perp}) = \int \frac{d^2 \Delta_{\perp}}{2\pi} e^{i\vec{\Delta}_{\perp} \cdot \vec{b}_{\perp}} E(x, 0, -\vec{\Delta}_{\perp}^2).$$

3-Dimensional image of nucleon in the mixed transverse plane-longitudinal momentum space



center of momentum

$$R_{\perp} = \sum x_i r_{\perp i}$$



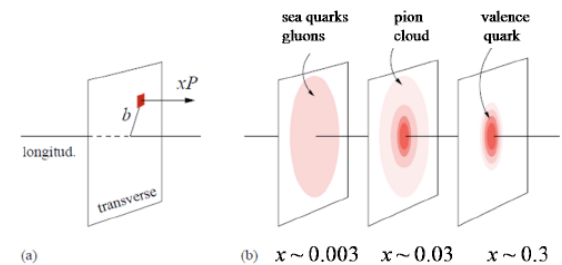
$$\mathcal{H}(x, \vec{b}_{\perp}) = \int \frac{d^2 \Delta_{\perp}}{2\pi} e^{i\vec{\Delta}_{\perp} \cdot \vec{b}_{\perp}} H(x, 0, -\vec{\Delta}_{\perp}^2).$$

$$\tilde{\mathcal{H}}(x, \vec{b}_{\perp}) = \int \frac{d^2 \Delta_{\perp}}{2\pi} e^{i\vec{\Delta}_{\perp} \cdot \vec{b}_{\perp}} \tilde{H}(x, 0, -\vec{\Delta}_{\perp}^2),$$

$$\mathcal{E}(x, \vec{b}_{\perp}) = \int \frac{d^2 \Delta_{\perp}}{2\pi} e^{i\vec{\Delta}_{\perp} \cdot \vec{b}_{\perp}} E(x, 0, -\vec{\Delta}_{\perp}^2).$$

3-Dimensional image of nucleon in the mixed transverse plane-longitudinal momentum space

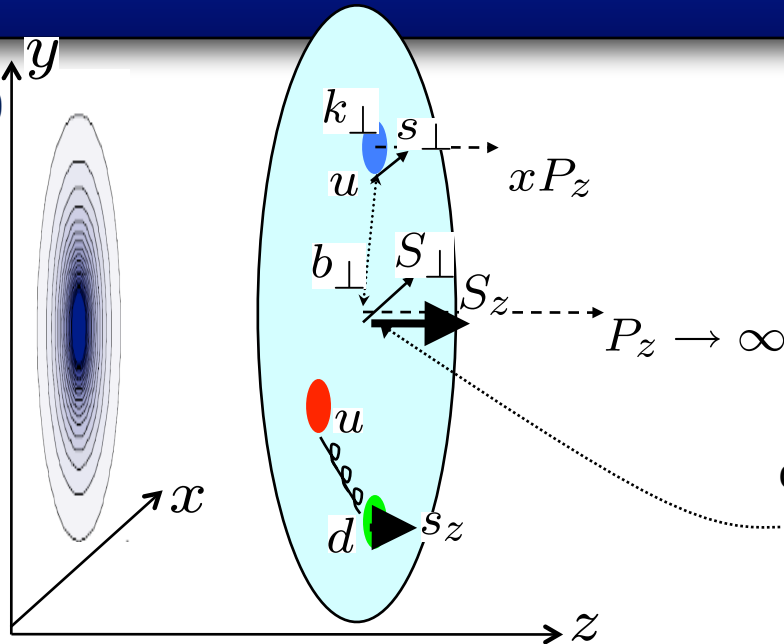
„Tomography”







Burhardt, 2000



center of momentum

$$R_{\perp} = \sum x_i r_{\perp i}$$

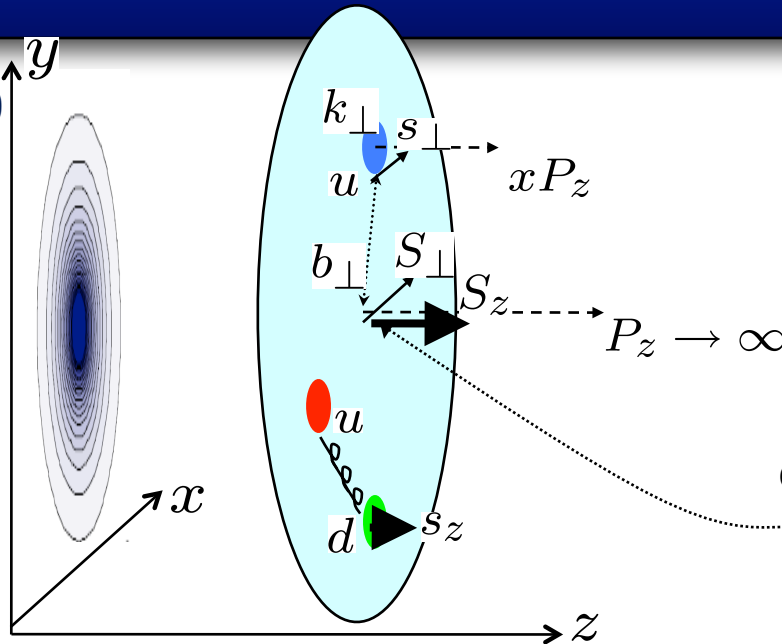
For a transversely polarized nucleon (e.g. polarized in the  $+\hat{x}$ -direction) the IPD  $q_{\hat{x}}(x, \vec{b}_{\perp})$  is no longer symmetric due to the non-zero value of the spin-flip GPD  $E$ . This deformation is described by the gradient of the Fourier transform of  $E$ :

$$q_{\hat{x}}(x, \vec{b}_{\perp}) = \mathcal{H}(x, \vec{b}_{\perp}) - \frac{1}{2M} \frac{\partial}{\partial b_y} \mathcal{E}(x, \vec{b}_{\perp}).$$

non-zero spin-flip GPD  $E$  - existence of non-zero orbital momentum



Burhardt, 2000



center of momentum

$$R_{\perp} = \sum x_i r_{\perp i}$$

For a transversely polarized nucleon (e.g. polarized in the  $+\hat{x}$ -direction) the IPD  $q_{\hat{x}}(x, \vec{b}_{\perp})$  is no longer symmetric due to the non-zero value of the spin-flip GPD  $E$ . This deformation is described by the gradient of the Fourier transform of  $E$ :

$$q_{\hat{x}}(x, \vec{b}_{\perp}) = \mathcal{H}(x, \vec{b}_{\perp}) - \frac{1}{2M} \frac{\partial}{\partial b_y} \mathcal{E}(x, \vec{b}_{\perp}).$$

non-zero spin-flip GPD  $E$  - existence of non-zero orbital momentum



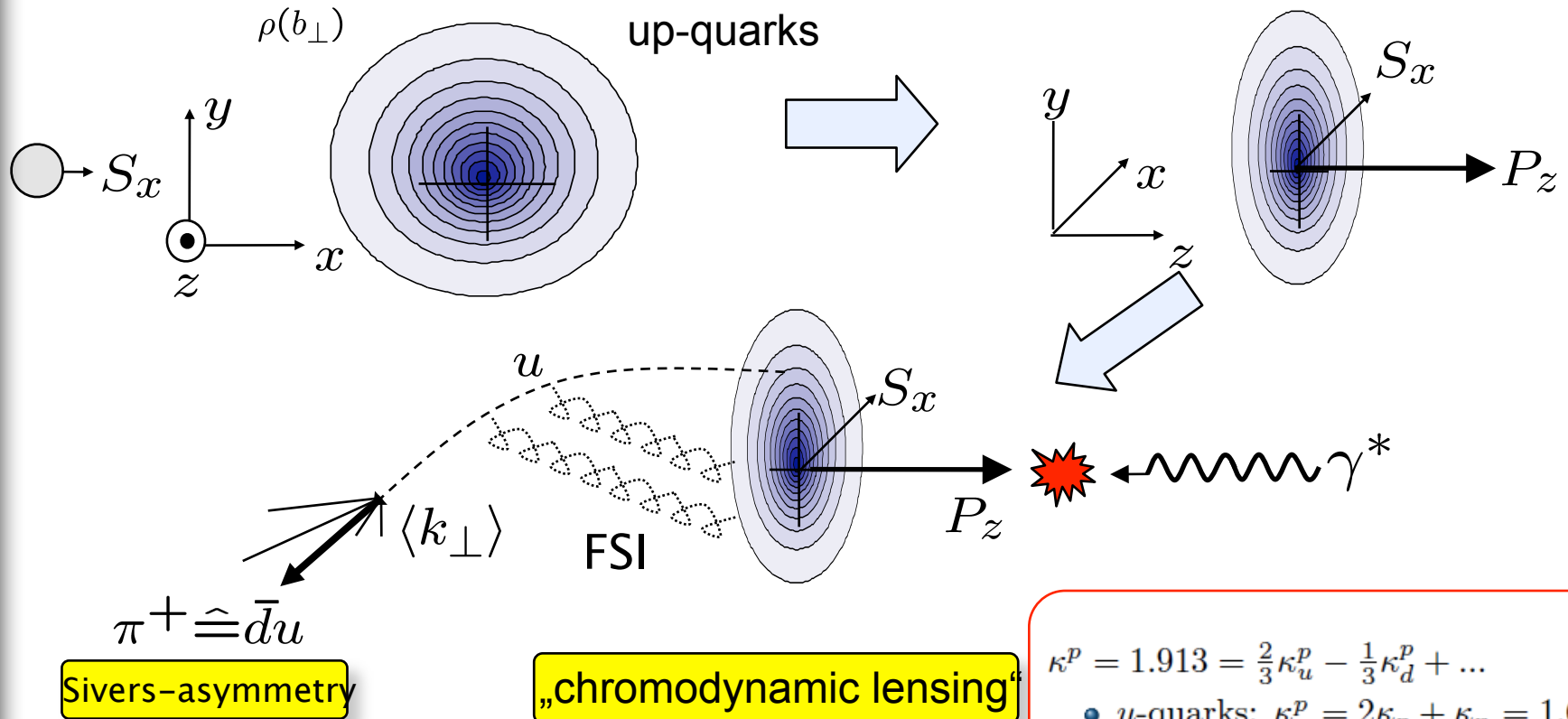


# Sivers function and spatial deformation



Dynamical origin of quark transverse momentum

M.Burhardt 2002/2003



$$\kappa^P = 1.913 = \frac{2}{3}\kappa_u^P - \frac{1}{3}\kappa_d^P + \dots$$

- $u$ -quarks:  $\kappa_u^P = 2\kappa_p + \kappa_n = 1.673$
- ↔ shift in  $+\hat{y}$  direction
- $d$ -quarks:  $\kappa_d^P = 2\kappa_n + \kappa_p = -2.033$
- ↔ shift in  $-\hat{y}$  direction
- $\langle b_y^q \rangle = \mathcal{O}(\pm 0.2 \text{ fm})$  !!!!

Deformation details are model-dependent but the size and directions is determined by anomalous magnetic moments of proton and neutron.

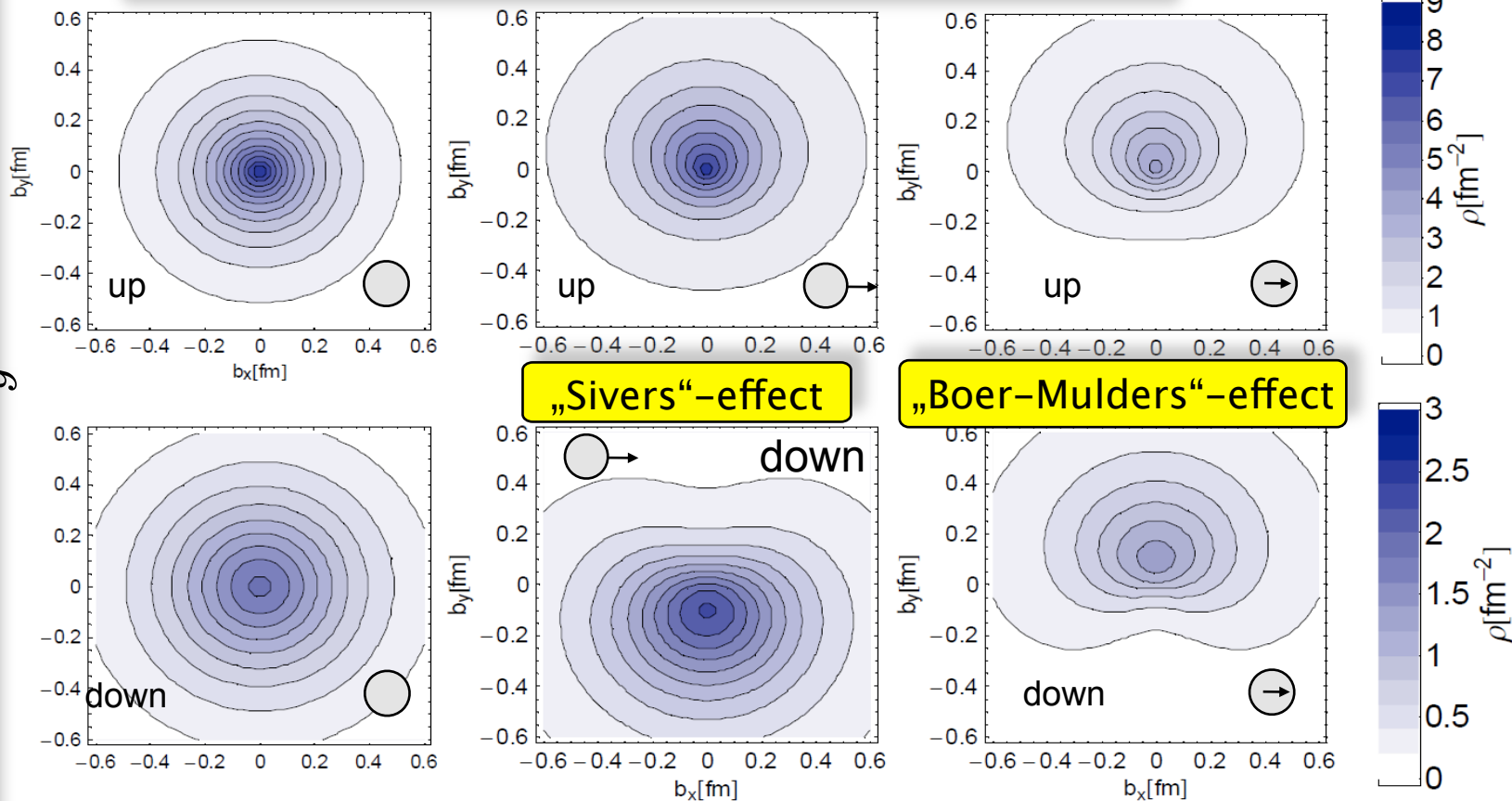


# Sivers function and spatial deformation



## Lowest x-moments of quark densities in coordinate space

QCDSF  $n_f=2$  Clover, Phys.Rev.Lett. 2007 [hep-lat/0612032]



„Sivers“-effect

„Boer-Mulders“-effect

strong deviations from spherical symmetry

$b_x$

Introduction

$b_y$





## Lowest x-moments of quark densities in coordinate space

QCDSF  $n_f=2$  Clover, Phys.Rev.Lett. 2007 [hep-lat/0612032]

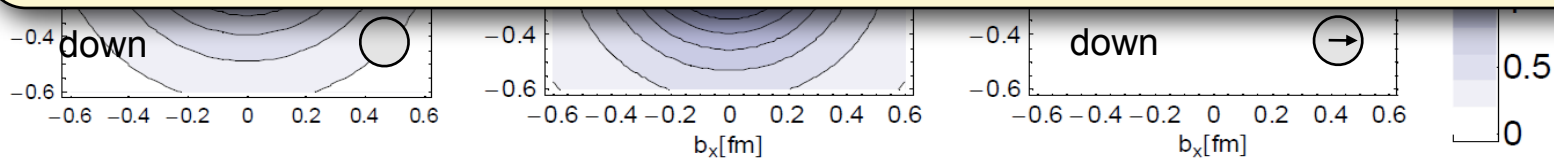


Q: Is gluon's spacial distribution in transverse plane also deformed?

Spin puzzle:

quarks  $\sim 1/3$ , gluon polarisation - still unclear but rather small,  
QCD Lattice calculations show significant orbital angular momenta of u  
and d quarks but in opposite directions - the contributions almost cancel.

Sivers effect for gluons can be related to gluon orbital motion



$b_x$

strong deviations from spherical symmetry

Introduction

$b_y$



The idea: selection of high- $p_T$  hadron pairs to increase photon-gluon fusion (PGF) similar to  $\Delta g$  determination with longitudinally polarised target.

difficulties comparing to  $\Delta g$

- asymmetry in azimuthal angle
- gluon „simulated” from pair of hadrons from PGF
- Sivers effect due to final interactions (no analyzing powers)

Single-spin asymmetry is measured:

$$A_T^h \sim \frac{d^6\sigma^\uparrow - d^6\sigma^\downarrow}{d^6\sigma^\uparrow + d^6\sigma^\downarrow}$$

8 asymmetries; concentrated on Sivers

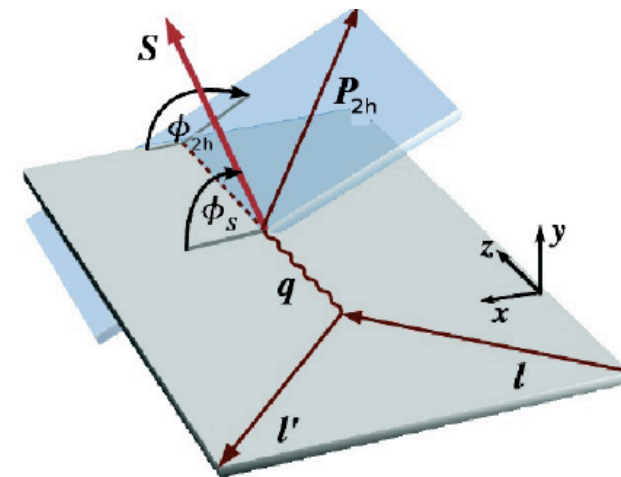
$\sigma$ - two-hadron cross-section integrated over  $\phi_R$ ;  
Phys.Rev.Lett.113, 062003 (2014), Phys. Rev. D 90, 074006 (2014)

The statistically weighted method was used, similar to open-charm and all- $p_T$  methods used in  $\Delta g$  determination

J.Pretz, J-M Le Goff NIM A 602 (2009) 594

COMPASS open-charm: Phys. Rev. D (2013) 052018

$$\begin{aligned} \mathbf{P}_{2h} &= \mathbf{p}_1 + \mathbf{p}_2 \\ \mathbf{R} &= \frac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2) \end{aligned}$$



$$\begin{aligned} \mathbf{P}_{2h} &= \mathbf{p}_1 + \mathbf{p}_2 \\ \text{Sivers angle:} \end{aligned}$$

$$\phi = \phi_{2h} - \phi_S$$



The number of events:

$$n_c(\vec{x}) = \alpha_c(\vec{x})(1 + \beta_c(\vec{x})A_{UT}^{\sin(\phi_{2h}-\phi_s)}(\vec{x}))$$

Every event is weighted by the weight  $\omega$

$$p_c := \int \omega(\vec{x})n_c(\vec{x})d\vec{x} = \int \omega(\vec{x})\alpha_c(\vec{x})d\vec{x} + \int \omega(\vec{x})\alpha_c(\vec{x})\beta_c(\vec{x})A_{UT}^{\sin(\phi_{2h}-\phi_s)}(\vec{x})d\vec{x} \approx \sum_{i=1}^{N_c} \omega_i, \\ \sum_{i=1}^{N_c} \omega_i = \tilde{\alpha}_c(1 + \{\beta_c\}_\omega \{A_{UT}^{\sin(\phi_{2h}-\phi_s)}\}_\omega \beta_c),$$

where

$$\tilde{\alpha}_c = \int \omega(\vec{x})\alpha_c(\vec{x})d\vec{x}, \\ \{A_{UT}^{\sin(\phi_{2h}-\phi_s)}\}_\omega \beta_c = \frac{\int A_{UT}^{\sin(\phi_{2h}-\phi_s)}(\vec{x})\omega(\vec{x})\beta_c\alpha_c(\vec{x})d\vec{x}}{\int \omega(\vec{x})\beta_c\alpha_c(\vec{x})d\vec{x}}, \\ \{\beta\}_\omega = \frac{\int \beta(\vec{x})\omega(\vec{x})\alpha_c(\vec{x})d\vec{x}}{\int \omega(\vec{x})\alpha_c(\vec{x})d\vec{x}} \approx \frac{\sum_i \beta_i \omega_i}{\sum_i \omega_i}.$$

where:

$$\vec{x} = (x_{Bj}, y, t, \phi, \dots)$$

$$\alpha_c(\vec{x}) = a_c \Phi n_c \sigma,$$

$$\beta_c(\vec{x}) = P_T f \sin(\phi_{2h} - \phi_s)$$

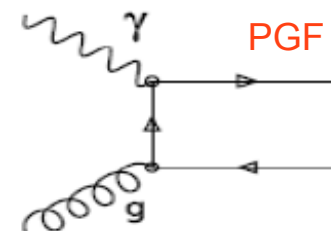
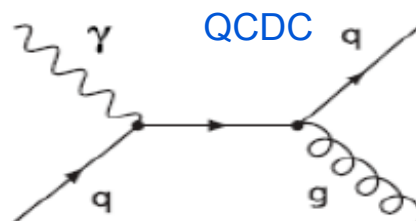
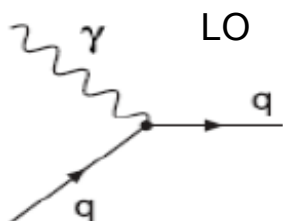
$$c = u, d, u', d'$$

$$\omega(\vec{x}) = \frac{\beta(\vec{x})}{P_T} = f \sin(\phi_{2h} - \phi_s)$$





## Physical model: three basic processes @LO



$$A_{UT}^{\sin(\phi_{2h} - \phi_s)} = R_{PGF} A_{PGF}^{\sin(\phi_{2h} - \phi_s)} (\langle x_G \rangle) + R_{LP} A_{LP}^{\sin(\phi_{2h} - \phi_s)} (\langle x_{Bj} \rangle) + R_{QCDC} A_{QCDC}^{\sin(\phi_{2h} - \phi_s)} (\langle x_C \rangle)$$

$$\begin{aligned} \omega_{PGF} &\equiv \omega^G = R_{PGF} f \sin(\phi_{2h} - \phi_s) = \frac{\beta^G}{P_T}, \\ \omega_{LP} &\equiv \omega^L = R_{LP} f \sin(\phi_{2h} - \phi_s) = \frac{\beta^L}{P_T}, \\ \omega_{QCDC} &\equiv \omega^C = R_{QCDC} f \sin(\phi_{2h} - \phi_s) = \frac{\beta^C}{P_T}. \end{aligned}$$



## Physical model: three basic processes @LO

leads to 12 eqs.:

$$\begin{aligned}
p_c^j &= \sum_{i=1}^{N_c} \omega_i^j = \tilde{\alpha}_c^j (1 + \{\beta_c^G\}_{\omega^j} A_{PGF}^{\sin(\phi_{2h}-\phi_s)}(\langle x_G \rangle)) \\
&+ \{\beta_c^L\}_{\omega^j} A_{LP}^{\sin(\phi_{2h}-\phi_s)}(\langle x_{Bj} \rangle) + \{\beta_c^C\}_{\omega^j} A_{QCDC}^{\sin(\phi_{2h}-\phi_s)}(\langle x_C \rangle)) \\
&= \tilde{\alpha}_c^j (1 + A_{PGF} \{\beta^G\}_{\omega^j} + A_{LP} \{\beta^L\}_{\omega^j} + A_{QCDC} \{\beta^C\}_{\omega^j})
\end{aligned}$$

with 15 unknowns: (3 asymmetries + 12 acceptances) but thanks to it is reduced to 12. \*Here j stands for LO, QCDC and PGF, respectively

$$\frac{\tilde{\alpha}_u^j \tilde{\alpha}_d^j}{\tilde{\alpha}_d^j \tilde{\alpha}_u^j} = 1,$$

To determine asymmetries the minimalization procedure has been used:

$$\chi^2 = (N_{exp}^{\vec{}} - N_{obs}^{\vec{}})^T COV^{-1} (N_{exp}^{\vec{}} - N_{obs}^{\vec{}})$$

$$\sim \sum_{N_c} \omega_x \omega_y.$$

$$N_{obs}^{\vec{}} = \left( \sum_{i=0}^{N_u} \omega_i^G, \dots, \sum_{i=0}^{N_d} \omega_i^C \right),$$

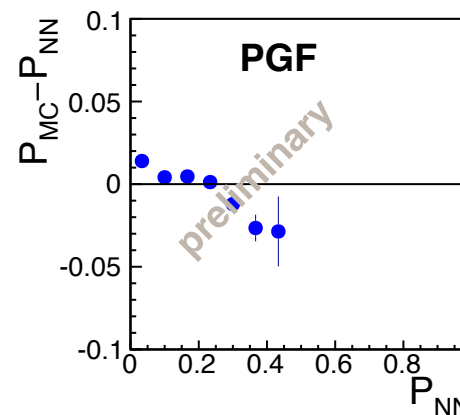
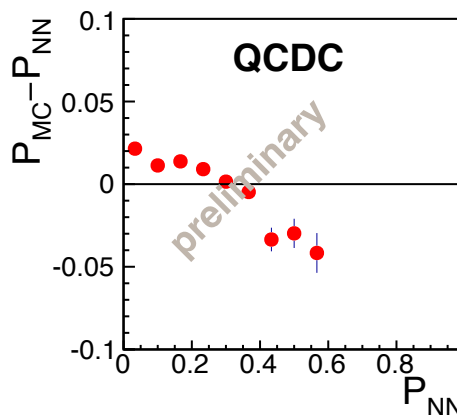
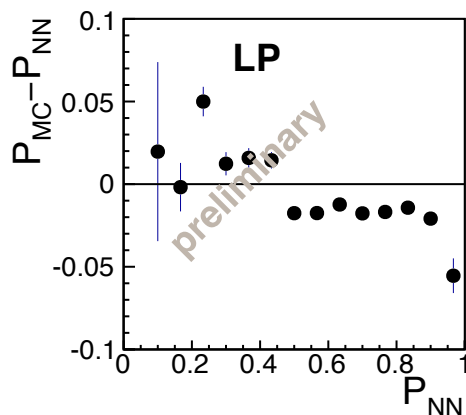
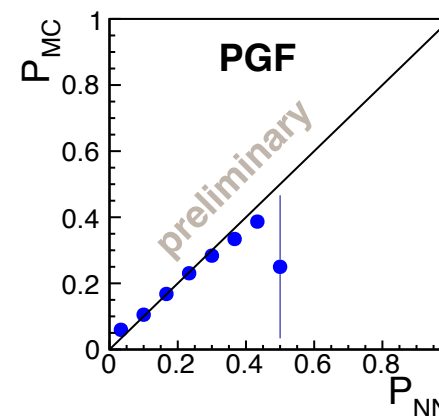
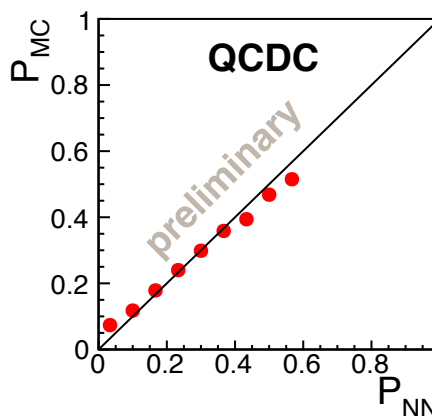
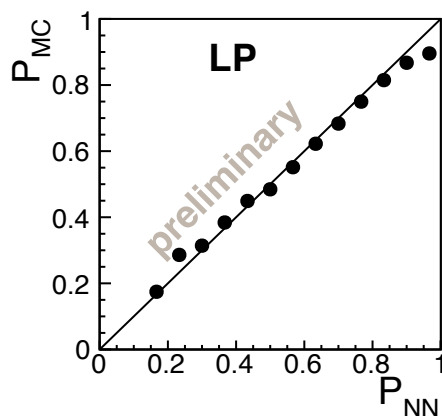
$$N_{exp}^{\vec{}} = (N_{exp,G}^u, \dots, N_{exp,C}^d),$$

$$N_{exp,i}^c = \tilde{\alpha}_c^j (1 + A_{PGF} \{\beta^G\}_{\omega^j} + A_{LP} \{\beta^L\}_{\omega^j} + A_{QCDC} \{\beta^C\}_{\omega^j})$$



To find R's (fractions or probabilities of three processes) the ANN approach has been used (as in longitudinal high- $p_T$  analysis for gluon polarisation measurement, see: *Phys. Lett. B* 718 (2013) 922 )

deuteron target



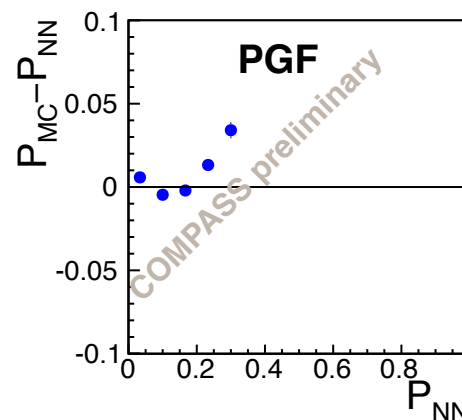
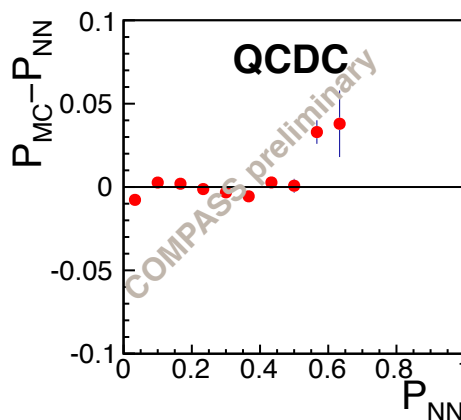
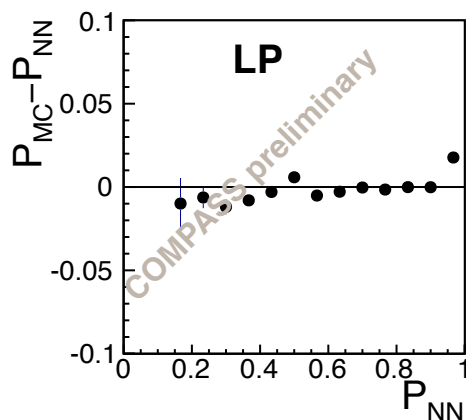
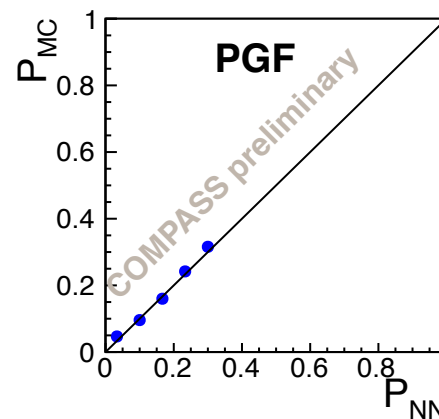
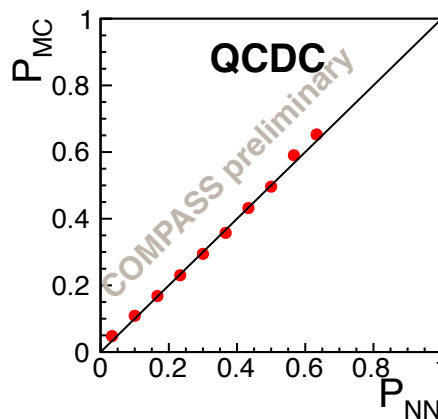
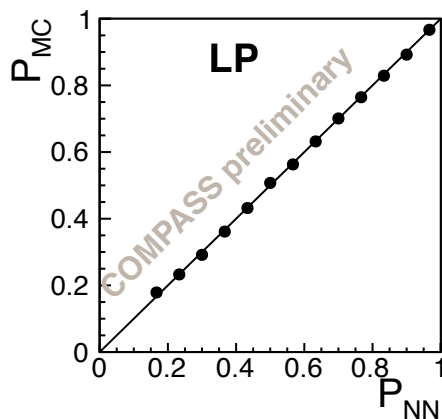
training vector:  $p_T, p_L, x_{Bjk}, Q^2$





To find R's (fractions or probabilities of three processes) the ANN approach has been used (as in longitudinal high- $p_T$  analysis for gluon polarisation measurement, see: *Phys. Lett. B* 718 (2013) 922 )

proton target



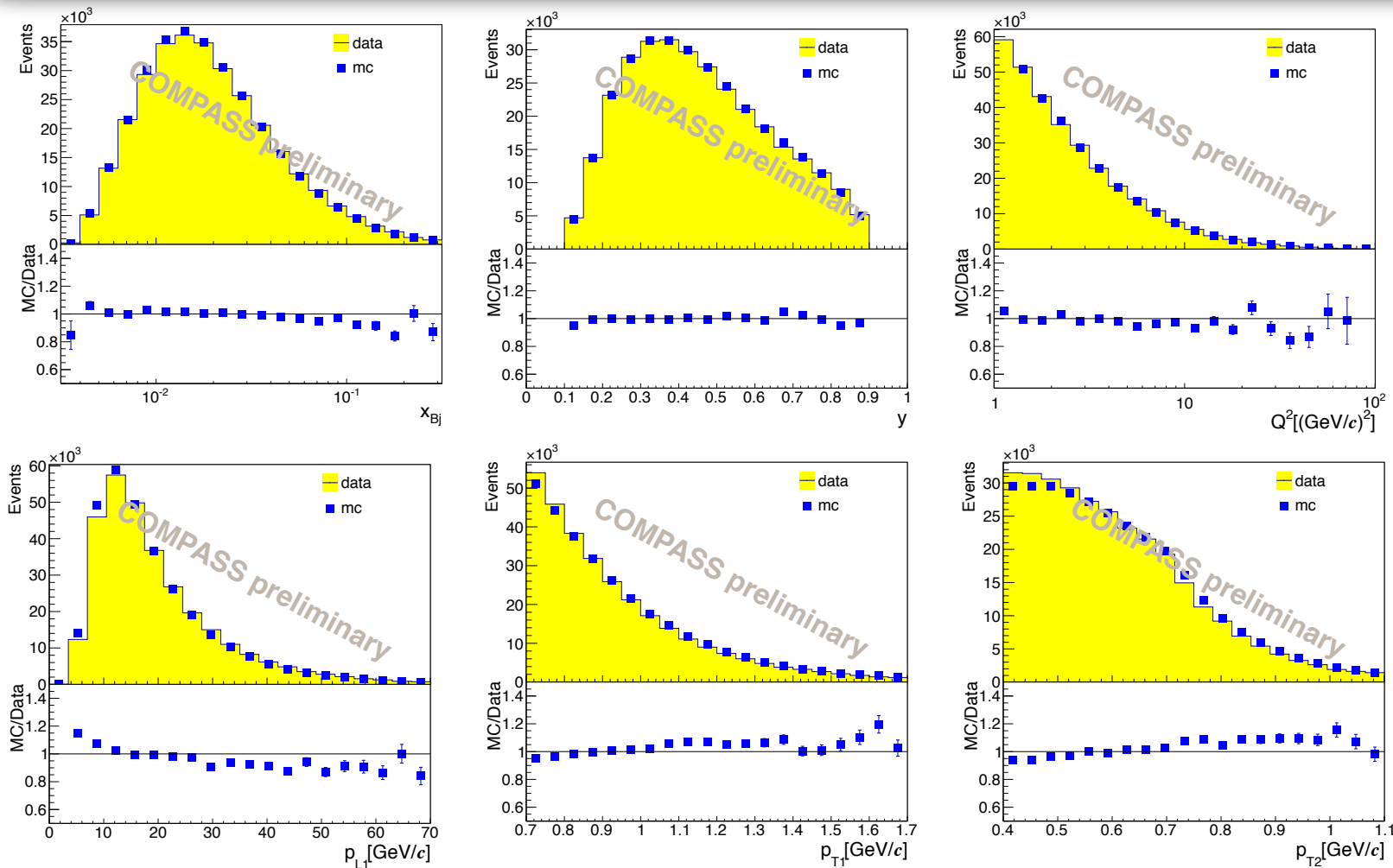
training vector:  $p_T, p_L, X_{Bjk}, Q^2$



# Data-MC comparison



Data and Monte Carlo comparison

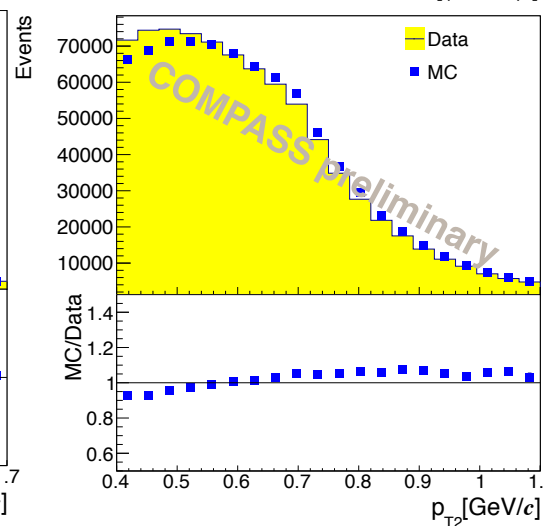
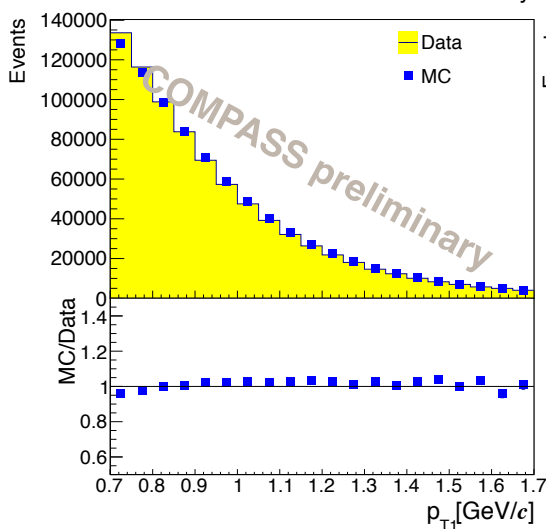
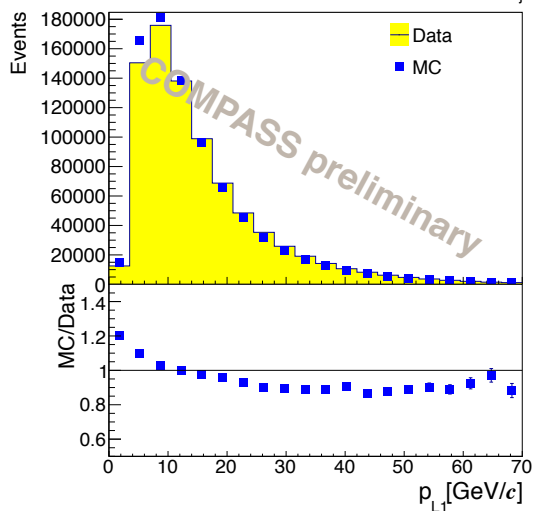
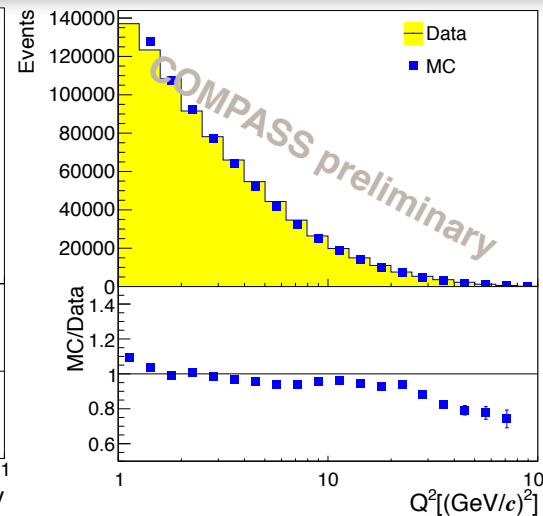
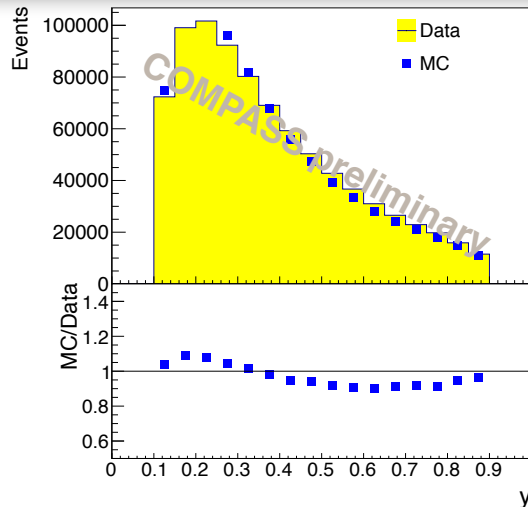
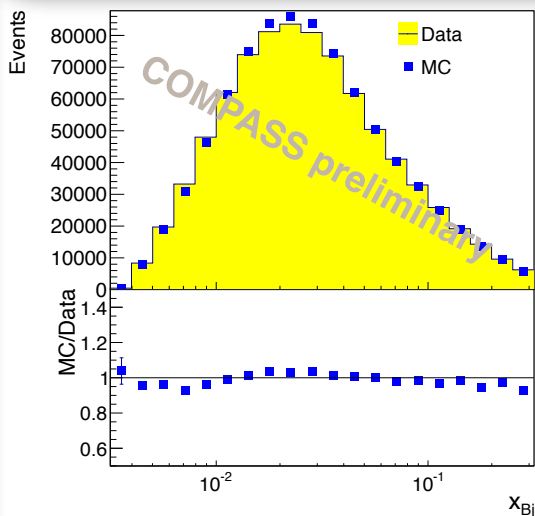
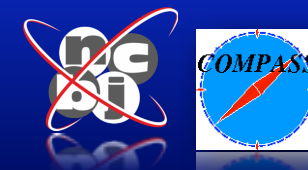


good agreement (COMPASS tuning high- $p_T$ )

deuteron target



# Data-MC comparison



good agreement (COMPASS tuning high- $p_T$ )

proton target





deuteron target

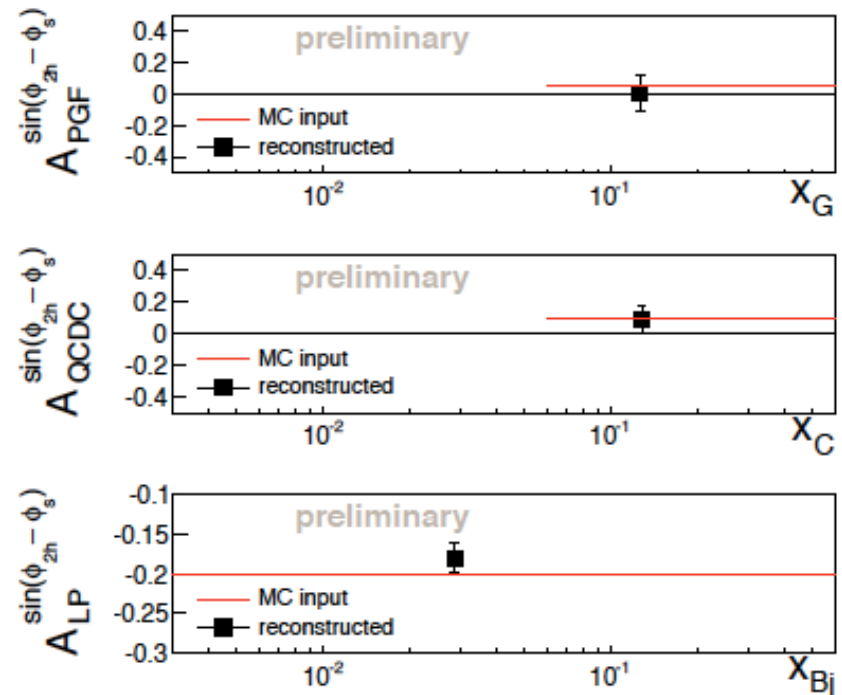
- MC (LEPTO + COMGEANT, high- $p_T$  tuning) events have no azimuthal asymmetries therefore we weight every event by  $1 + A \sin(\phi_{2h} - \phi_s)$ .

$\phi_{2h}$ - azimuthal angle of the vector sum of the 2 leading hadron momenta.

A - assumed asymmetry for LO, QCDC and PGF

- For each MC event we get  $R_{LP}$ ,  $R_{QCDC}$ ,  $R_{PGF}$  and  $x_C$ ,  $x_G$  from NN

## Sivers Asymmetry



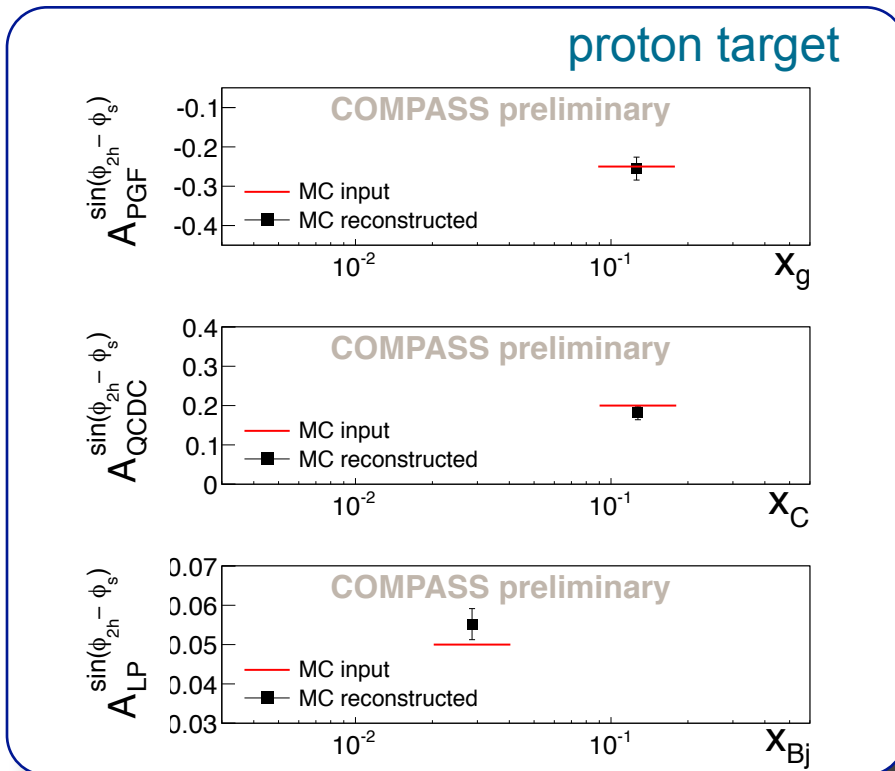


deuteron target

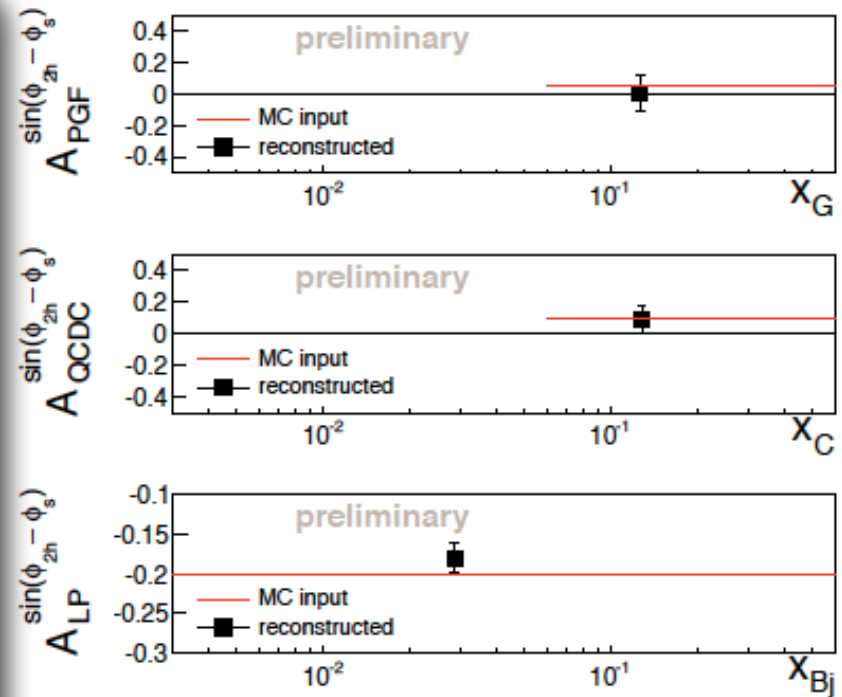
- MC (LEPTO + COMGEANT, high- $p_T$  tuning) events have no azimuthal asymmetries therefore we weight every event by  $1 + A \sin(\phi_{2h} - \phi_s)$ .

$\phi_{2h}$  - azimuthal angle of the vector sum of the 2 leading hadron momenta.

Validation of the method on MC data



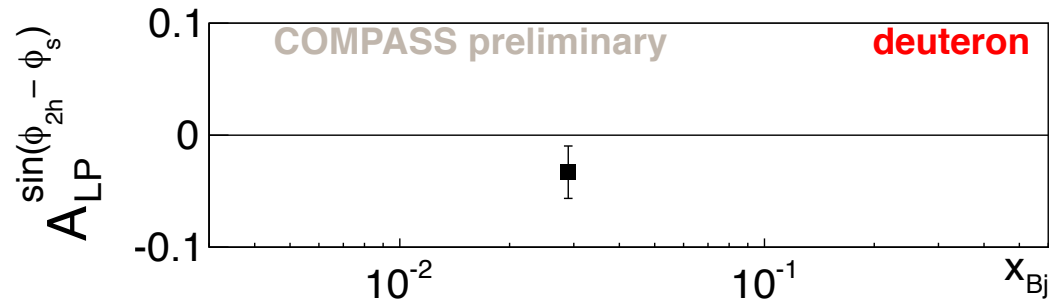
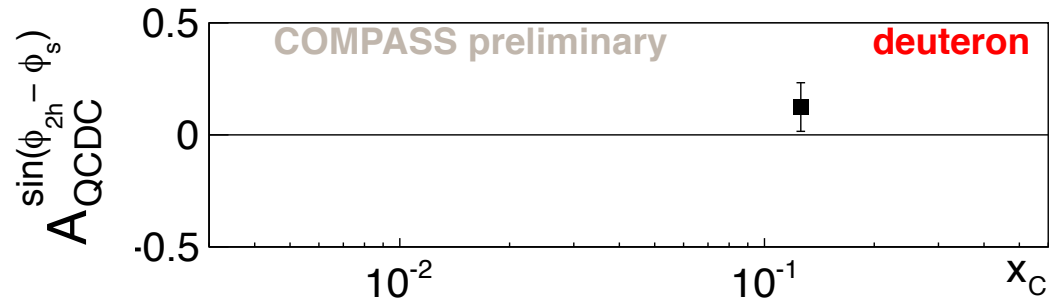
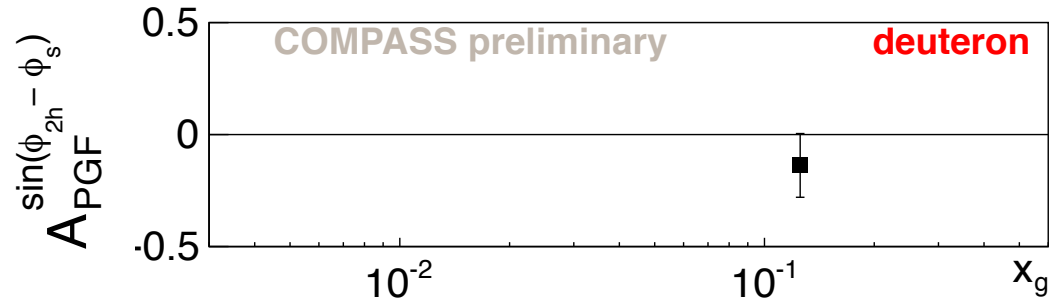
## Sivers Asymmetry





- Inclusive cuts:
  - $Q^2 > 1(\text{GeV}/c)^2$
  - $0.003 < x_{Bj} < 0.7$
  - $0.1 < y < 0.9$
- hadronic cuts
  - $p_{T1} > 0.7 \text{ GeV}/c$
  - $p_{T2} > 0.4 \text{ GeV}/c$
  - $z_1 > 0.1$
  - $z_2 > 0.1$

$$z_1 + z_2 < 0.9$$

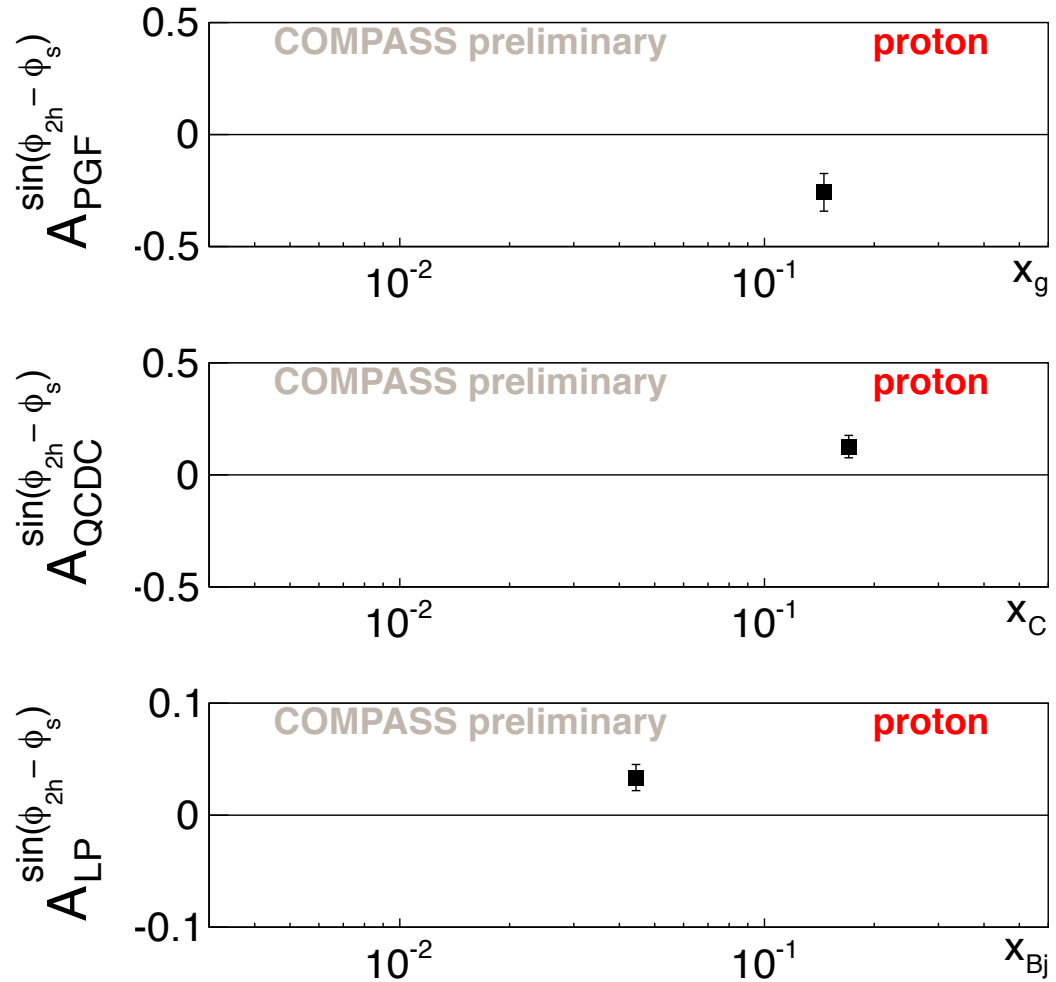


$$A_{PGF}^{\sin(\phi_{2h} - \phi_s)} = -0.14 \pm 0.15(\text{stat.}) \pm 0.06(\text{syst.}) \text{ at } \langle x_G \rangle = 0.13$$



- Inclusive cuts:
  - $Q^2 > 1(\text{GeV}/c)^2$
  - $0.003 < x_{Bj} < 0.7$
  - $0.1 < y < 0.9$
- hadronic cuts
  - $p_{T1} > 0.7 \text{ GeV}/c$
  - $p_{T2} > 0.4 \text{ GeV}/c$
  - $z_1 > 0.1$
  - $z_2 > 0.1$

$$z_1 + z_2 < 0.9$$

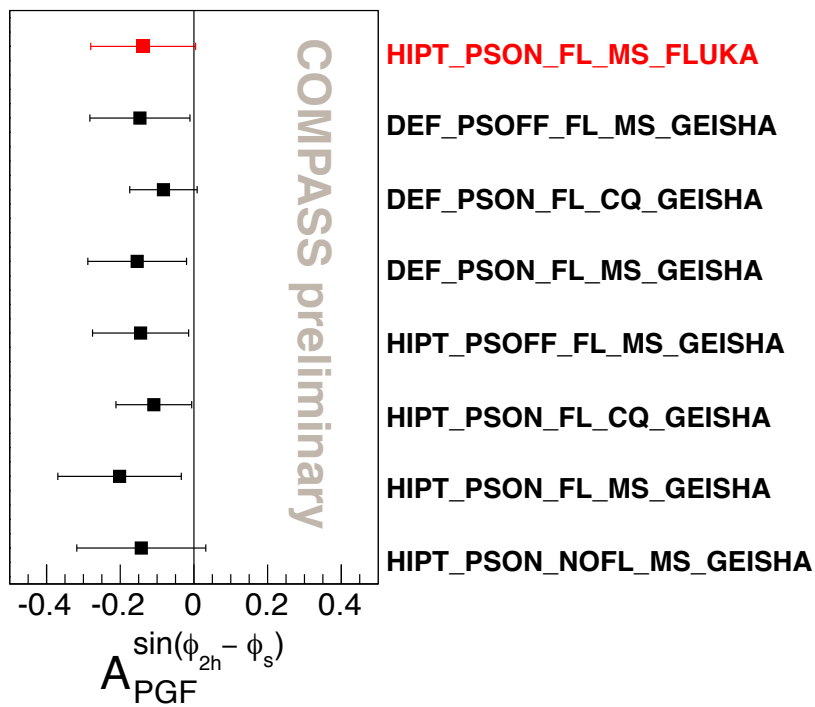


$$A_{PGF}^{\sin(\phi_{2h} - \phi_s)} = -0.26 \pm 0.09(\text{stat.}) \pm 0.08(\text{syst.}) \text{ at } \langle x_G \rangle = 0.15$$

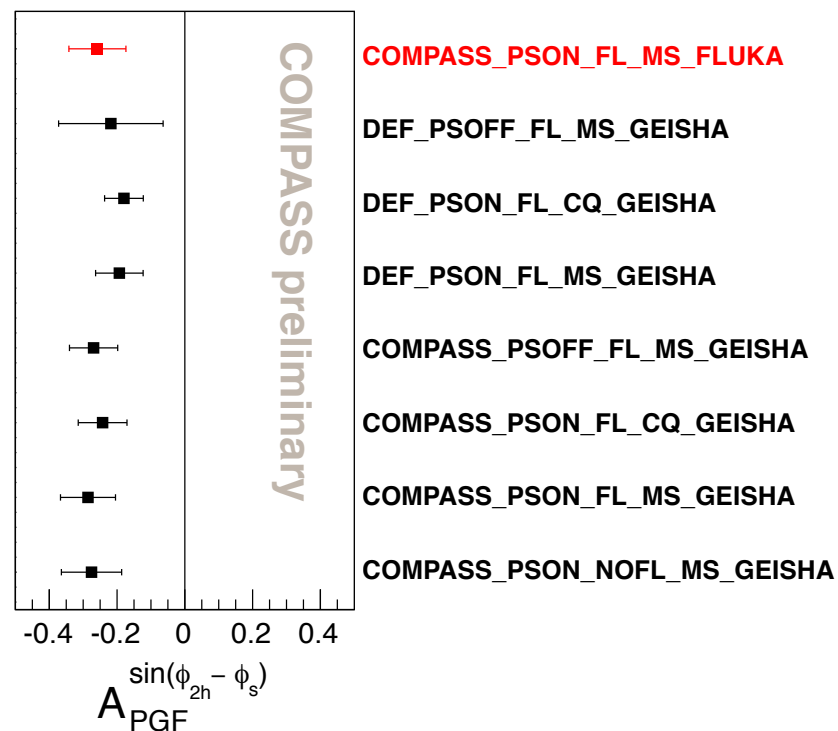




## deuteron target



## proton target





- For the first time preliminary result for Sivers asymmetry for gluons has been obtained from COMPASS on deuteron and on proton targets data:

$$A_{PGF}^{\sin(\phi_{2h}-\phi_s)} = -0.14 \pm 0.15(stat.) \pm 0.06(syst.) \text{ at } \langle x_G \rangle = 0.13$$

$$A_{PGF}^{\sin(\phi_{2h}-\phi_s)} = -0.26 \pm 0.09(stat.) \pm 0.08(syst.) \text{ at } \langle x_G \rangle = 0.15$$

- The result on deuteron is compatible with zero but the central value is negative with large error
- Proton data show a value which is negative,  $3\sigma$  below zero and statistically compatible with deuteron result

S.J.Brodsky & S. Gardner, Phys.Lett. B643 (2006) 22-28

# Thank you for your attention



National Centre for  
Nuclear Research  
Warsaw, Poland

## Gluon Contribution to the Sivers Effect COMPASS Results on Deuteron Target

Adam Szabelski  
Krzysztof Kurek



XVI Workshop on High Energy Spin Physics  
DSPIN-15  
Dubna, Russia, September 8-12

# Backup slides



Spares



## SIDIS cross section decomposition LO

$$\begin{aligned}
 \frac{d\sigma}{dx dy dd\phi_S d\phi_h dz_h dP_{hT}^2} = & \frac{e^4}{32\pi^2 x Q^2} \frac{y}{1-\varepsilon} \left(1 + \frac{\gamma^2}{2x}\right) \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} \right. \\
 & + \sqrt{2\varepsilon(1-\varepsilon)} (\cos\phi_h F_{UU}^{\cos\phi_h} + h_l \sin\phi_h F_{LU}^{\sin\phi_h}) \\
 & + S_{\parallel} \left[ \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \\
 & + S_{\parallel} h_l \left[ \sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h F_{LL}^{\cos\phi_h} \right] \\
 & + |S_{\perp}| \left[ \sin(\phi_h - \phi_S) (F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)}) \right. \\
 & \quad + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} \\
 & \quad \left. + \sqrt{2\varepsilon(1+\varepsilon)} (\sin\phi_S F_{UT}^{\sin\phi_S} + \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)}) \right] \\
 & + |S_{\perp}| h_l \left[ \sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} \right. \\
 & \quad \left. + \sqrt{2\varepsilon(1-\varepsilon)} (\cos\phi_S F_{LT}^{\cos\phi_S} + \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)}) \right] \left. \right\},
 \end{aligned}$$

## SIDIS cross section decomposition LO

$$\begin{aligned}
 \frac{d\sigma}{dx dy dd\phi_S d\phi_h dz_h dP_{hT}^2} = & \frac{e^4}{32\pi^2 x Q^2} \frac{y}{1-\varepsilon} \left(1 + \frac{\gamma^2}{2x}\right) \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} \right. \\
 & + \sqrt{2\varepsilon(1-\varepsilon)} (\cos\phi_h F_{UU}^{\leftarrow \cos\phi_h} + h_l \sin\phi_h F_{LU}^{\sin\phi_h}) \quad \text{unpolarised target} \quad \text{Cahn \& Boer-Mulders} \\
 & + S_{\parallel} \left[ \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \\
 & + S_{\parallel} h_l \left[ \sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h F_{LL}^{\cos\phi_h} \right] \\
 & + |S_{\perp}| \left[ \sin(\phi_h - \phi_S) (F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)}) \right. \\
 & \quad + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} \\
 & \quad \left. + \sqrt{2\varepsilon(1+\varepsilon)} (\sin\phi_S F_{UT}^{\sin\phi_S} + \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)}) \right] \\
 & + |S_{\perp}| h_l \left[ \sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} \right. \\
 & \quad \left. + \sqrt{2\varepsilon(1-\varepsilon)} (\cos\phi_S F_{LT}^{\cos\phi_S} + \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)}) \right] \left. \right\},
 \end{aligned}$$

## SIDIS cross section decomposition LO

$$\begin{aligned}
 \frac{d\sigma}{dx dy dd\phi_S d\phi_h dz_h dP_{hT}^2} = & \frac{e^4}{32\pi^2 x Q^2} \frac{y}{1-\varepsilon} \left(1 + \frac{\gamma^2}{2x}\right) \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} \right. \\
 & + \sqrt{2\varepsilon(1-\varepsilon)} (\cos\phi_h F_{UU}^{\leftarrow\cos\phi_h} + h_l \sin\phi_h F_{LU}^{\sin\phi_h}) \quad \text{unpolarised target} \quad \text{Cahn \& Boer-Mulders} \\
 & + S_{\parallel} \left[ \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \\
 & + S_{\parallel} h_l \left[ \sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h F_{LL}^{\cos\phi_h} \right] \quad \text{longitudinally polarised target} \\
 & + |S_{\perp}| \left[ \sin(\phi_h - \phi_S) (F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)}) \right. \\
 & \quad + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} \\
 & \quad \left. + \sqrt{2\varepsilon(1+\varepsilon)} (\sin\phi_S F_{UT}^{\sin\phi_S} + \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)}) \right] \\
 & + |S_{\perp}| h_l \left[ \sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} \right. \\
 & \quad \left. + \sqrt{2\varepsilon(1-\varepsilon)} (\cos\phi_S F_{LT}^{\cos\phi_S} + \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)}) \right] \left. \right\},
 \end{aligned}$$



## SIDIS cross section decomposition LO

$$\begin{aligned}
 \frac{d\sigma}{x dy d\phi_S d\phi_h dz_h dP_{hT}^2} = & \frac{e^4}{32\pi^2 x Q^2} \frac{y}{1-\varepsilon} \left(1 + \frac{\gamma^2}{2x}\right) \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} \right. \\
 & + \sqrt{2\varepsilon(1-\varepsilon)} (\cos\phi_h F_{UU}^{\leftarrow\cos\phi_h} + h_L \sin\phi_h F_{LU}^{\sin\phi_h}) \quad \text{unpolarised target} \quad \text{Cahn \& Boer-Mulders} \\
 & + S_{\parallel} \left[ \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \quad \text{longitudinally polarised target} \\
 & + S_{\parallel} h_L \left[ \sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h F_{LL}^{\cos\phi_h} \right] \\
 & + |S_{\perp}| \left[ \sin(\phi_h - \phi_S) (F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)}) \right. \\
 & \quad + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} \\
 & \quad \left. + \sqrt{2\varepsilon(1+\varepsilon)} (\sin\phi_S F_{UT}^{\sin\phi_S} + \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)}) \right] \quad \begin{array}{l} \text{transversely} \\ \text{polarised target} \end{array} \\
 & \quad \text{Sivers} \\
 & \quad \text{Collins} \\
 & \quad \text{Pretzelosity} \\
 & \quad \text{Worm Gear} \\
 & + |S_{\perp}| h_L \left[ \sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} \right. \\
 & \quad \left. + \sqrt{2\varepsilon(1-\varepsilon)} (\cos\phi_S F_{LT}^{\cos\phi_S} + \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)}) \right] \left. \right\},
 \end{aligned}$$

Spares



## SIDIS cross section decomposition LO

$$\frac{d\sigma}{x dy dd\phi_S d\phi_h dz_h dP_{hT}^2} = \frac{e^4}{32\pi^2 x Q^2} \frac{y}{1-\varepsilon} \left(1 + \frac{\gamma^2}{2x}\right) \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} \right. \\ \left. + \sqrt{2\varepsilon(1-\varepsilon)} (\cos\phi_h F_{UU}^{\leftarrow\phi_h} + h_U \sin\phi_h F_{LU}^{\sin\phi_h}) \right\} \text{ unpolarised target} \quad \text{Cahn \& Boer-Mulders}$$

$$+ S_{\parallel} \left[ \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right]$$

$$F_{UU,T} \sim \sum_q e_q^2 \cdot f_1^q \otimes D_q^h,$$

$$F_{LL} \sim \sum_q e_q^2 \cdot g_{1L}^q \otimes D_q^h,$$

$$F_{UU}^{\cos 2\phi_h} \sim \sum_q e_q^2 \cdot h_1^{\perp q} \otimes H_1^{\perp q},$$

$$F_{UL}^{\sin 2\phi_h} \sim \sum_q e_q^2 \cdot h_{1L}^{\perp q} \otimes H_1^{\perp q},$$

$$F_{LT}^{\cos(\phi_h - \phi_S)} \sim \sum_q e_q^2 \cdot g_{1T}^q \otimes D_q^h,$$

$$F_{UT,T}^{\sin(\phi_h - \phi_S)} \sim \sum_q e_q^2 \cdot f_{1T}^{\perp q} \otimes D_q^h,$$

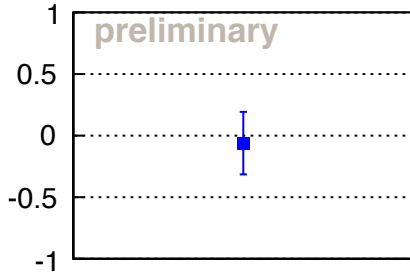
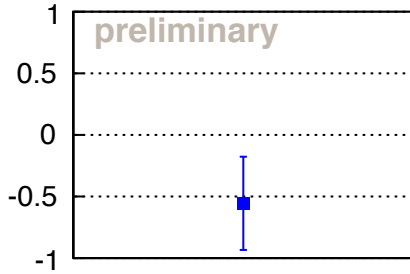
$$F_{UT}^{\sin(\phi_h + \phi_S)} \sim \sum_q e_q^2 \cdot h_1^q \otimes H_1^{\perp q},$$

$$F_{UT}^{\sin(3\phi_h - \phi_S)} \sim \sum_q e_q^2 \cdot h_{1T}^{\perp q} \otimes H_1^{\perp q}.$$

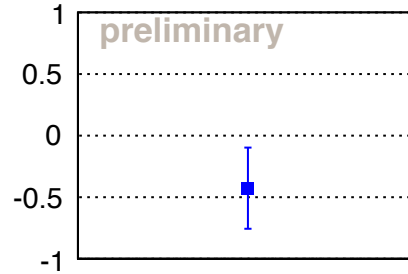
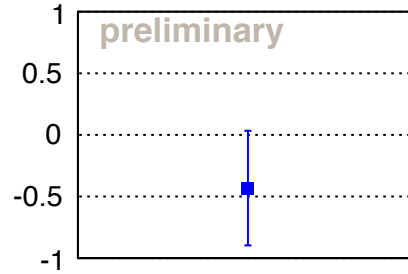


False PGF Sivers Asymmetry  
up  
down

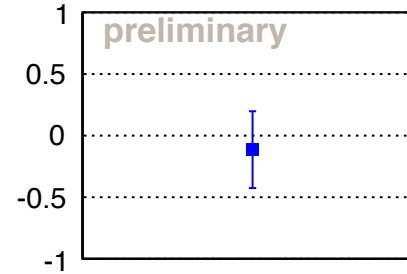
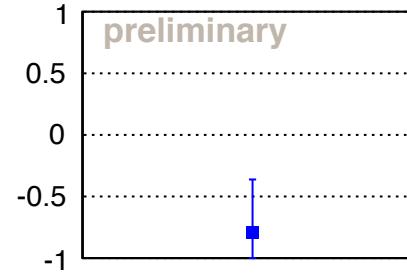
## P1H-P1G



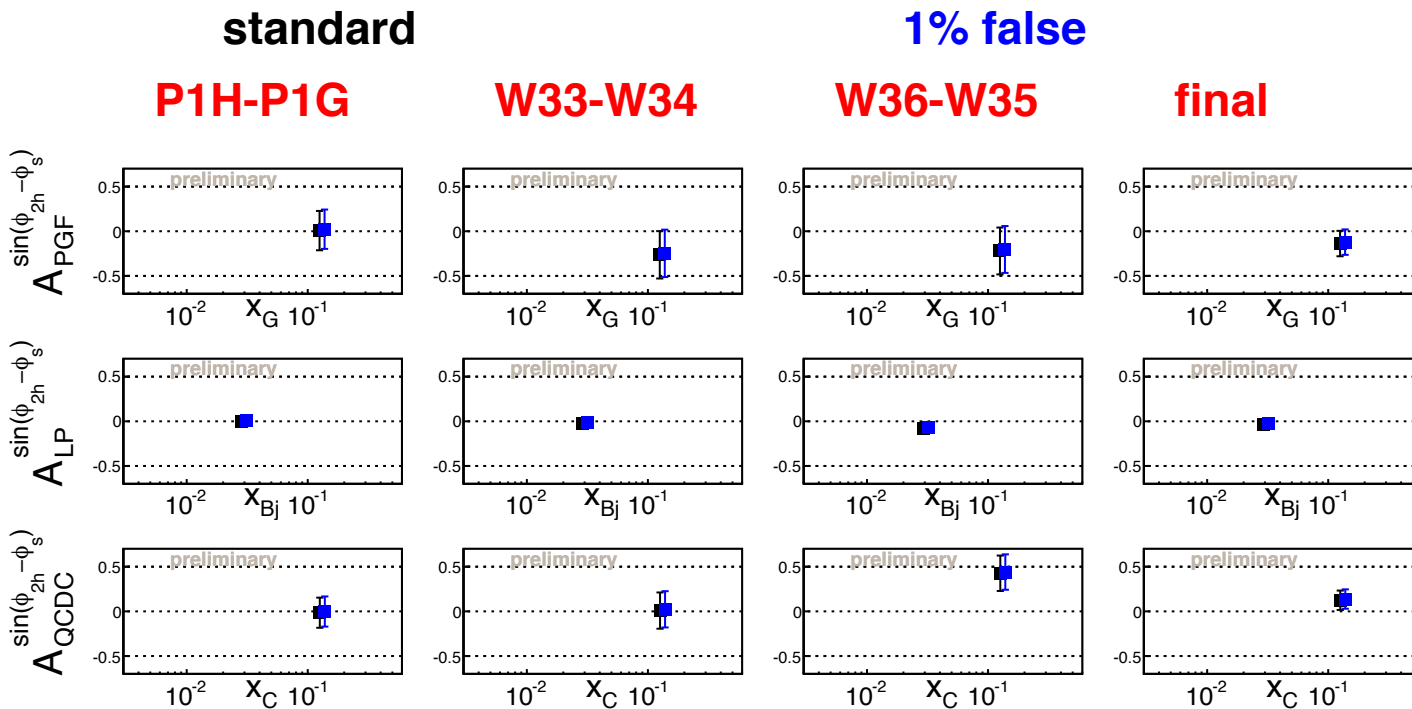
## W33-W34



## W36-W35



## Sivers Asymmetry



# Correlations

