Measurement of the Charged-Pion Polarisability at COMPASS

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COMPASS collaboration

EP Seminar
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Yukawa 1935: hypothesis of $\sim 100$ MeV massive exchange particle “$\mu$” for the strong interaction between protons and neutrons

Discovery of muons 1936
Short story of the pion

- Yukawa 1935: hypothesis of \( \sim 100 \text{ MeV} \) massive exchange particle “\( \mu \)” for the strong interaction between protons and neutrons
- Discovery of muons 1936
- Discovery of pions 1947
Short story of the pion

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- 1958: decay $\pi^+ \rightarrow \mu^+ \nu_\mu$ dominant, small branching $\pi^+ \rightarrow e^+ \nu_e$
  (CERN CycloSynchrotron) $\Rightarrow$ $V - A$ theory of weak interaction
- 1961: Spin-1 mesonic excitation of the pion ($\rho$-resonance)
- 1964: quark hypothesis
- 1966: pion scattering lengths
Short story of the pion

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1961: Spin-1 mesonic excitation of the pion ($\rho$-resonance)
1964: quark hypothesis
1966: pion scattering lengths

1982: first data on the pion polarisability
Measurement of the Charged-Pion Polarizability

C. Adolph,8 R. Akhunzyanov,7 M. G. Alexeev,27 G. D. Alexeev,7 A. Amoroso,27,29 V. Andrieux,22 V. Anosov,7
... [213 authors]
(COMPASS Collaboration)

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The COMPASS collaboration at CERN has investigated pion Compton scattering, $\pi^- \gamma \rightarrow \pi^- \gamma$, at center-of-mass energy below 3.5 pion masses. The process is embedded in the reaction $\pi^- \text{Ni} \rightarrow \pi^- \gamma \text{Ni}$, which is initiated by 190 GeV pions impinging on a nickel target. The exchange of quasireal photons is selected by isolating the sharp Coulomb peak observed at smallest momentum transfers, $Q^2 < 0.0015 \text{ (GeV/c)}^2$. From a sample of 63 000 events, the pion electric polarizability is determined to be $\alpha_\pi = (2.0 \pm 0.6_{\text{stat}} \pm 0.6_{\text{syst}}) \times 10^{-4} \text{ fm}^3$ under the assumption $\alpha_\pi = -\beta_\pi$, which relates the electric and magnetic dipole polarizabilities. It is the most precise measurement of this fundamental low-energy parameter of strong
CERN experiment brings precision to a cornerstone of particle physics

11 Feb 2015

Geneva, 11 February 2015. In a paper published yesterday in the journal Physical Review Letters, the COMPASS experiment at CERN reports a key measurement on the strong interaction. The strong interaction binds quarks into protons and neutrons, and protons and neutrons into the nuclei of all the elements from which matter is built. Inside those nuclei, particles called pions made up of a quark and an antiquark mediate the interaction. Strong interaction theory makes a precise prediction on the polarisability of
The COMPASS experiment at CERN has made the first precise measurement of the polarizability of the pion – the lightest composite particle built from quarks. The result confirms the expectation from low-energy expansions of QCD – the quantum field theory of the strong interaction between quarks – but is at variance with the previously published values, which overestimated the pion polarizability by more than a factor of two.

Every composite system made from charged particles can be polarized by an external electromagnetic field, which acts to separate positive and negative charges. The size of this charge separation – the induced dipole moment – is related to the external field by the polarizability. As a measure of the response of a complex system to an external force, polarizability is directly related to the system’s stiffness against deformability, and hence the binding force between the constituents.

The pion, made up of a quark and an antiquark, is the lightest object bound by the strong force and has a size of about 0.6 femtometers, or $6 \times 10^{-15}$ m. So, to observe a measurable effect, the particle must be subjected to electric fields in the order of 100 V/cm across a distance that is about 10^{-5} times its diameter – that is, about 10^{-10} V/m. To achieve this, the COMPASS experiment used an electric field around nuclei. To high-energy pions, this field acts as a source of atomic real transitions, primarily electron-positron pair production.

The COMPASS experiment is the North Area on the Protokoll site at CERN, and it has been designed to be a powerful combination, with main beams on a powerful combination, with main beams on a powerful combination, with main beams on a powerful combination.
Press echo in spring 2015

Intro: Pions & ChPT

COMPASS Pion polarisability Summary and Outlook

Press echo in spring 2015

J. M. Friedrich — Pion Polarisability with COMPASS
The COMPASS experiment at CERN has conducted a key measurement on the strong interaction. The strong interaction is responsible for holding particles together and ensuring stability. This interaction is essential for understanding the structure of matter. In the COMPASS experiment, scientists have achieved high precision in the measurement by using a combination of advanced techniques and technologies.

The results of this experiment will provide insights into the nature of quarks and their interactions. This knowledge is crucial for developing a more comprehensive understanding of the fundamental forces that govern the universe. The findings will have implications for various fields, including particle physics, astrophysics, and cosmology.

Press echo in spring 2015

J. M. Friedrich — Pion Polarisability with COMPASS
Press echo in spring 2015
CERN Physicists Measure Polarizability of Pion

Feb 16, 2015 by Sci-News.com

Scientists from CERN's COMPASS collaboration have made the most precise measurement ever of the polarizability of pion — the fundamental low-energy parameter of strong interaction.

Everything we see in the Universe is made up of fundamental particles called...
How to understand quark-gluon dynamics?

complicated system of interacting quarks and gluons

ChPT

effective degrees of freedom at low energy: mass, charge, spin, effective (self-)coupling

\begin{align*}
\pi & \rightarrow \pi \\
\pi & \rightarrow \gamma \\
\pi & \rightarrow \pi \rightarrow \gamma \\
\pi & \rightarrow \gamma \\
\pi & \rightarrow \pi \rightarrow \gamma
\end{align*}

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\pi & \rightarrow \pi \rightarrow \gamma
\end{align*}
pion scattering lengths: 2-loop predictions

- \(a_0^0 m_\pi = 0.220 \pm 0.005\) confirmed by E865 in \(K^+ \rightarrow \pi^+ \pi^- e^+ \nu_e\)
- \((a_0^0 - a_0^2) m_\pi = 0.264 \pm 0.006\) confirmed by NA48 in \(0.268 \pm 0.010\) \(K^+ \rightarrow \pi^+ \pi^0 \pi^0\)

pion polarisability: electric \(\alpha_\pi\), magnetic \(\beta_\pi\)

- Leading structure-dependent contribution to Compton scattering
- **ChPT prediction** obtained by the relation to \(\pi^+ \rightarrow e^+ \nu_e \gamma\) [Gasser, Ivanov, Sainio, Nucl. Phys. B745, 2006]
  
  [PIBETA, M. Bychkov et al., PRL 103, 051802, 2009]

**ChPT prediction contradicts** the experimental findings (prior to this analysis)
More pion-photon reactions

- Pion scattering including a real photon
  - Leading-order prediction from ChPT
  - Pion scattering lengths + coupled photon
  - Chiral loop contribution
    - Theory prediction available

- Radiative widths of meson resonances

- Chiral anomaly $F_{3\pi}$
  - Established on 10% level
  - Further development: inclusion of the $\rho$ resonance, theoretical work by Kubis, Hoferichter, Sakkas PRD86(2012)116009

\[ \gamma \rightarrow \pi^0, \pi^0, \pi^- \]

\[ \pi^- \rightarrow \pi^- \]

COMPASS 2004
\[ \pi \gamma \rightarrow \pi \pi \pi^- \]
from $\pi \text{Pb} \rightarrow \pi \pi \pi^- \text{Pb}$

Fitted ChPT Intensity
Leading Order ChPT Prediction

Full Systematic Error
Luminosity Uncertainty

Intensity / 40MeV/c
\[ \times 10^3 \]

\[ 2^+ f_2 \pi S \]
\[ \sigma_{\text{prim}} / \sigma_{\text{all}} = 0.95 \]
\[ \Gamma(\pi_0 \rightarrow \pi \gamma) = 153 \text{ keV} \]

COMPASS 2004
$\pi \text{Pb} \rightarrow \pi \pi \pi^- \text{Pb}$
$\epsilon < 0.001 \text{ GeV}^2/c^2$

Intensity / 40MeV/c
\[ \times 10^3 \]

\[ 2^+ f_2 \pi S \]
\[ \sigma_{\text{prim}} / \sigma_{\text{all}} = 0.95 \]
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[EPJA 50, 79 (2014)]
ChPT prediction for the pion polarisability

Pion polarisabilities $\alpha_\pi, \beta_\pi$ in units of $10^{-4}$ fm$^3$

ChPT (2-loop) prediction:

$$\alpha_\pi - \beta_\pi = 5.7 \pm 1.0$$
$$\alpha_\pi + \beta_\pi = 0.16 \pm 0.1$$

Experiments for $\alpha_\pi - \beta_\pi$ lie in the range $4 \cdots 14$

($\alpha_\pi + \beta_\pi = 0$ assumed)
ChPT prediction for the pion polarisability

pion polarisabilities $\alpha_\pi, \beta_\pi$ in units of $10^{-4} \text{ fm}^3$

ChPT (2-loop) prediction: $\alpha_\pi = 2.93 \pm 0.5$
$\beta_\pi = -2.77 \pm 0.5$

experiments for $\alpha_\pi$ lie in the range 2 $\cdots$ 7

($\alpha_\pi + \beta_\pi = 0$ assumed)
Principle of the COMPASS measurement

- high-energetic pion beam on 4mm nickel disk
- observe scattered pions in coincidence with produced hard photons
- study of cross-section shape
• Charged pions traverse the nuclear electric field
  - typical field strength at $d = 5R_{Ni}$:
    \[ E \approx 300 \text{ kV/fm} \]

• Bremsstrahlung process:
  - particles scatter off equivalent photons
  - tiny momentum transfer
    \[ Q^2 \approx 10^{-5} \text{ GeV}^2/c^2 \]
  - pion/muon (quasi-)real Compton scattering

• Polarisability contribution
  - Compton cross-section typically diminished
  - equivalent charge separation
    \[ \approx 10^{-5} \text{ fm} \cdot e \]
Charged pions traverse the nuclear electric field
- typical field strength at $d = 5R_{Ni}$:
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Bremsstrahlung process:
- particles scatter off equivalent photons
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Compton scattering
- Polarisability contribution
- Compton cross-section typically diminished
- equivalent charge separation
  - $\approx 10^{-5} \text{ fm} \cdot e$

(details: see later)
Pion Compton Scattering

\[ \pi \gamma \rightarrow \pi \gamma \]

- Two kinematic variables, in CM: total energy \( \sqrt{s} \), scattering angle \( \theta_{cm} \)

\[
\frac{d\sigma_{\pi\gamma}}{d\Omega_{cm}} = \frac{\alpha^2 (s^2 z_+^2 + m_{\pi}^4 z_-^2)}{s(sz_+ + m_{\pi}^2 z_-)^2} - \frac{\alpha m_{\pi}^3 (s - m_{\pi}^2)^2}{4s^2(sz_+ + m_{\pi}^2 z_-)} \cdot \mathcal{P}
\]

\[
\mathcal{P} = z_-^2(\alpha_{\pi} - \beta_{\pi}) + \frac{s^2}{m_{\pi}^4} z_+^2(\alpha_{\pi} + \beta_{\pi}) - \frac{(s - m_{\pi}^2)^2}{24s} z_-^3(\alpha_2 - \beta_2)
\]

\[
z_\pm = 1 \pm \cos \theta_{cm}
\]
Pion Compton Scattering

Two kinematic variables, in CM: total energy $\sqrt{s}$, scattering angle $\theta_{cm}$

$$\frac{d\sigma_{\pi\gamma}}{d\Omega_{cm}} = \frac{\alpha^2 (s^2 z_+^2 + m_{\pi}^4 z_-^2)}{s(sz_+ + m_{\pi}^2 z_-)^2} - \frac{\alpha m_{\pi}^3 (s - m_{\pi}^2)^2}{4s^2(sz_+ + m_{\pi}^2 z_-)^2} \cdot \mathcal{P}$$

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$$z_\pm = 1 \pm \cos \theta_{cm}$$
Pion Compton scattering: embedding the process

\[ Q^2 \ll m^2_\pi \]

Primakoff processes

Radiative pion photoproduction

Photon-Photon fusion
Pion polarisability: world data before COMPASS

Primakoff processes

Radiative pion photoproduction

Photon-Photon fusion

GIS'06: ChPT prediction, Gasser, Ivanov, Sainio, NPB745 (2006), plots: T. Nagel, PhD
Fil'kov analysis objected by Pasquini, Drechsel, Scherer PRC81, 029802 (2010)
CERN SPS: protons $\sim$ 400 GeV (5 – 10 sec spills)

- secondary $\pi, K, (\bar{p})$: up to $2 \cdot 10^7$/s (typ. $5 \cdot 10^6$/s)
- tertiary muons: $4 \cdot 10^7$/s
  2002-04, 2006-07, 2010-11: spin structure of the nucleon
Fixed-target experiment

- two-stage magnetic spectrometer
- high-precision, high-rate tracking, PID, calorimetry
Fixed-target experiment

- two-stage magnetic spectrometer
- high-precision, high-rate tracking, PID, calorimetry


- 190 GeV $\pi^-$ beam on $p$ and nuclear targets (C, Ni, W, Pb)
- Silicon microstrip detectors for “vertexing”
- recoil and (digital) ECAL triggers
Principle of the measurement
Silicon detector module

double sided
\(\ell N_2\) cooled 200K
\(\sigma_{x,y} \sim 8 \mu m\)
Extracting the pion polarisability

- Identify exclusive reactions
  \[ \pi \gamma \{ \text{Ni} \to \text{Ni}' \} \to \pi \gamma \]
  at smallest momentum transfer \(< 0.001 \, \text{GeV}^2/c^2 \)

- Assuming \( \alpha_\pi + \beta_\pi = 0 \), from the cross-section
  \[
  R = \frac{\sigma(x_\gamma)}{\sigma_{\alpha_\pi=0}(x_\gamma)} = \frac{N_{\text{meas}}(x_\gamma)}{N_{\text{sim}}(x_\gamma)} = 1 - \frac{3}{2} \cdot \frac{m_\pi^3}{\alpha} \cdot \frac{x_\gamma^2}{1 - x_\gamma} \alpha_\pi
  \]
  is derived, depending on \( x_\gamma = E_\gamma(\text{lab})/E_{\text{Beam}} \).
  Measuring \( R \) the polarisability \( \alpha_\pi \) can be concluded.

- Control systematics by
  \[ \mu \gamma \{ \text{Ni} \to \text{Ni}' \} \to \mu \gamma \]
  and
  \[ K^- \to \pi^- \pi^0 \to \pi \gamma \gamma \]
Extraction of the pion polarisability

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  \[ \mu \gamma \{|Ni \rightarrow Ni'\} \rightarrow \mu \gamma \]
  and
  \[ K^- \rightarrow \pi^- \pi^0 \rightarrow \pi \gamma \gamma \]
Identifying the $\pi \gamma \rightarrow \pi \gamma$ reaction


- Energy balance $\Delta E = E_\pi + E_\gamma - E_{\text{Beam}}$
- Exclusivity peak $\sigma \approx 2.6$ GeV (1.4%)
- $\sim 63.000$ exclusive events ($x_\gamma > 0.4$) (Serpukhov $\sim 7000$ for $x_\gamma > 0.5$)
Primakoff peak


- $\Delta Q_T \approx 12 \text{ MeV/c}$ (190 GeV/c beam → requires few-$\mu$rad angular resolution)
- first diffractive minimum on Ni nucleus at $Q \approx 190 \text{ MeV/c}$
- data a little more narrow than simulation → negative interference?
Coulomb-nuclear interference

Photon density squared form factor

calculation following G. Fäl dt (Phys. Rev. C79, 014607)
eikonal approximation: pions traverse Coulomb and strong-interaction potentials
Primakoff peak: muon data


- muon control measurement: pure electromagnetic interaction
- e.m. nuclear effects well understood
Principle of the measurement

CEDARs  C/Ni/W targets  silicon stations  2009 RPD  SM1  SM2  ECAL1  ECAL2
ECAL2: 3000 cells of different types
Figure 3.5: Profile of energy deviations shown for 1/4 of a shashlik block and for muon data photons within the range $133 \text{ GeV} < E_\gamma < 152 \text{ GeV}$.

Figure 3.6: Technical drawing of a full shashlik cell to be compared with the figure to the left.

from: Th. Nagel, PhD thesis TUM 2012
Photon energy spectra for muon and pion beam

Counts / 0.025

$\gamma x$

$0.4 \ 0.5 \ 0.6 \ 0.7 \ 0.8 \ 0.9$

$\text{data}$ $x^2 - \mu$

$\text{simulation} - \mu$

$\text{data} - \pi$

$\text{simulation} - \pi$

\[ \alpha_\pi = \left( 2.0 \pm 0.6_{\text{stat}} \right) \times 10^{-4} \text{ fm}^3 \]

(assuming \( \alpha_\pi = -\beta_\pi \))

“false polarisability” from muon data:

\[ \left( 0.5 \pm 0.5_{\text{stat}} \right) \times 10^{-4} \text{ fm}^3 \]

Radiative corrections (Compton scattering part)

\[ z = \cos \theta_{cm} \]

\[ \lambda = 3.8 \text{ MeV} \]

\[ s^{1/2} = \text{2m} \]
\[ s^{1/2} = \text{3m} \]
\[ s^{1/2} = \text{4m} \]
\[ s^{1/2} = \text{5m} \]

\[ \mu^- + \gamma \rightarrow \mu^- + \gamma \]

\[ \pi^- + \gamma \rightarrow \pi^- + \gamma \]


<table>
<thead>
<tr>
<th>source of systematic uncertainty</th>
<th>estimated magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>tracking</td>
<td>0.5</td>
</tr>
<tr>
<td>radiative corrections</td>
<td>0.3</td>
</tr>
<tr>
<td>background subtraction in $Q$</td>
<td>0.4</td>
</tr>
<tr>
<td>pion electron scattering</td>
<td>0.2</td>
</tr>
<tr>
<td><strong>quadratic sum</strong></td>
<td><strong>0.7</strong></td>
</tr>
</tbody>
</table>

COMPASS result for the pion polarisability:

$$\alpha_\pi = (2.0 \pm 0.6_{\text{stat}} \pm 0.7_{\text{syst}}) \times 10^{-4} \text{ fm}^3$$
source of systematic uncertainty | estimated magnitude
---|---
tracking | 0.5
radiative corrections | 0.3
background subtraction in $Q$ | 0.4
pion electron scattering | 0.2

quadratic sum | 0.7

COMPASS result for the pion polarisability:

\[ \alpha_\pi = (2.0 \pm 0.6_{\text{stat}} \pm 0.7_{\text{syst}}) \times 10^{-4} \text{ fm}^3 \]

with $\alpha_\pi = -\beta_\pi$ assumed
The new COMPASS result is in significant tension with the earlier measurements of the pion polarisability.

The expectation from ChPT is confirmed within the uncertainties.
About crossing

- **red hatched:** physical regions
  \[ \gamma + \gamma \rightarrow \pi + \pi \]
  \[ \gamma + \pi \rightarrow \gamma + \pi \]

- two-pion thresholds at \( s = 4m^2, u = 4m^2, t = 4m^2 \)

- DR integration paths
  \( t = 0 \) (forward), \( \theta = 180^\circ \) (backward)
  \( u = m^2, s = m^2, \ldots \)

from: D. Drechsel, talk at IWHSS 2011 Paris
Dispersioon relations and ChPT

Polarisability and Loop Contributions $z=-1.0$

$\frac{\sigma}{\sigma_{\text{Born}}}$

- **LEX $\alpha=-\beta=2.00$**
- **LEX $\alpha=-\beta=2.85$**
- **LEX + chiral loops**
- **DR [B. Pasquini]**

$\sqrt{s/m_\pi}$

DR calculations: Barbara Pasquini (Pavia)
FIGURE 3. Left: electric polarizability for the charged pions as a function of the valence quark mass. The data for $m_\pi = 390$ MeV is taken from [5]. Right: effective mass for a charged pion correlator together with the scalar particle correlator determined from the fit. The fitting range is indicated by the vertical bars.

Alexandru et al., Pion electric polarizability from lattice QCD, arXiv:1501.06516
Pion polarisability measurements at COMPASS

- Primakoff pilot run 2004
  - ~1 week
  - ~10k events
  - 0.5X₀ Ni

- Primakoff run 2009
  - ~63k events
  - 0.3X₀ Ni
  - ~3 weeks

- Primakoff run 2012
  - ~3 months
  - ~200–400k events
  - 0.3X₀ Ni

- Eγ/Ebeam > 0.4

*just seen*
Measurement of the pion polarisability at COMPASS

Via the Primakoff reaction, COMPASS has determined

\[
\alpha_\pi = (2.0 \pm 0.6_{\text{stat}} \pm 0.7_{\text{syst}}) \times 10^{-4} \text{ fm}^3
\]

assuming \( \alpha_\pi + \beta_\pi = 0 \)

- most direct access to the \( \pi\gamma \to \pi\gamma \) process
- Most precise experimental determination
- Systematic control: \( \mu\gamma \to \mu\gamma, \ K^- \to \pi^-\pi^0 \)

(not shown today:) COMPASS measures other aspects of chiral dynamics in \( \pi^-\gamma \to \pi^-\pi^0 \) and \( \pi\gamma \to \pi\pi\pi \) reactions

High-statistics run 2012

- separate determination of \( \alpha_\pi \) and \( \beta_\pi \)
- \( s \)-dependent quadrupole polarisabilities
- First measurement of the kaon polarisability
Thank you for your attention!
Access to $\pi + \gamma$ reactions via the Primakoff effect:

At smallest momentum transfers to the nucleus, high-energetic particles scatter predominantly off the electromagnetic field quanta ($\sim Z^2$)

$$\pi^- + \gamma \rightarrow \begin{cases} 
\pi^- + \gamma \\
\pi^- + \pi^0 / \eta \\
\pi^- + \pi^0 + \pi^0 \\
\pi^- + \pi^- + \pi^+ \\
\pi^- + \pi^- + \pi^+ + \pi^- + \pi^+ \\
\pi^- + \ldots 
\end{cases}$$

analogously: Kaon-induced reactions $K^- + \gamma \rightarrow \cdots$
ChPT & Resonances in $\pi^-\pi^-\pi^+$

2004 Primakoff results

$\pi^-\text{Pb} \rightarrow \text{Pb} \, \pi^-\pi^-\pi^+$

- **"Low $t'$":** $10^{-3} (\text{GeV}/c)^2 < t' < 10^{-2} (\text{GeV}/c)^2 \sim 2\,000\,000$ events
- **"Primakoff region":** $t' < 10^{-3} (\text{GeV}/c)^2 \sim 1\,000\,000$ events
First Measurement of $\pi \gamma \rightarrow 3\pi$ Absolute Cross-Section

Measured absolute cross-section of $\pi^- \gamma \rightarrow \pi^- \pi^- \pi^+$

COMPASS 2004

$\pi^- \gamma \rightarrow \pi^- \pi^- \pi^+$
from $\pi^- \text{Pb} \rightarrow \pi^- \pi^- \pi^+ \text{Pb}$

- Fitted ChPT Intensity
- Leading Order ChPT Prediction

Full Systematic Error

Luminosity Uncertainty

published in PRL 108 (2012) 192001
Chiral loops, e.g. (N. Kaiser, NPA848 (2010) 198)

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2004 Primakoff results

\[ \pi^- \text{ Pb} \rightarrow \text{ Pb} \pi^- \pi^- \pi^+ \]

- "Low \( t' \): \( 10^{-3} \text{ (GeV/c)}^2 < t' < 10^{-2} \text{ (GeV/c)}^2 \) \( \sim 2 000 000 \) events
- "Primakoff region": \( t' < 10^{-3} \text{ (GeV/c)}^2 \) \( \sim 1 000 000 \) events
2004 Primakoff results

\( \pi^- \text{Pb} \rightarrow \text{Pb} \pi^- \pi^- \pi^+ \)

PWA of \(a_1(1260), a_2(1320)\) contributions in \(t\) slices

- "Low \(t'\)": \(10^{-3} \text{(GeV/c)}^2 < t' < 10^{-2} \text{(GeV/c)}^2\) \(\sim 2\,000\,000\) events
- "Primakoff region": \(t' < 10^{-3} \text{(GeV/c)}^2\) \(\sim 1\,000\,000\) events
ChPT & Resonances in $\pi^{-}\pi^{-}\pi^{+}$

PWA: $a_1$, $a_2$ and $\Delta\Phi$ in separated $t'$ regions

COMPASS 2004
$\pi\text{Pb} \rightarrow \pi\pi\pi^{+}\text{Pb}$
$0.0015 < t' < 0.01 \text{ GeV}^2/c^2$
$t' < 0.0005 \text{ GeV}^2/c^2$

$1^{++0^+}\rho\pi S$

Mass of $\pi\pi\pi^{+}$ System (GeV/c^2)

Intensity / (40 MeV/c)^2

COMPASS 2004
$\pi\text{Pb} \rightarrow \pi\pi\pi^{+}\text{Pb}$
$0.0015 < t' < 0.01 \text{ GeV}^2/c^2$
$t' < 0.0005 \text{ GeV}^2/c^2$

$2^{++1^+}\rho\pi D$

Mass of $\pi\pi\pi^{+}$ System (GeV/c^2)

Intensity / (40 MeV/c)^2

$\Delta\Phi \left( 2^{++1^+}\rho\pi D - 1^{++0^+}\rho\pi S \right)$

Phase (degrees)

COMPASS 2004
$\pi\text{Pb} \rightarrow \pi\pi\pi^{+}\text{Pb}$
$0.0015 < t' < 0.01 \text{ GeV}^2/c^2$
$t' < 0.0005 \text{ GeV}^2/c^2$

Mass of $\pi\pi\pi^{+}$ System (GeV/c^2)
Radiative Coupling of $a_2(1320)$ and $\pi_2(1670)$

$\Gamma_0(a_2(1320) \rightarrow \pi\gamma) \quad M2$

$\Gamma_0(\pi_2(1670) \rightarrow \pi\gamma) \quad E2$

$\Leftrightarrow$ meson w.f.'s: $\Gamma_{i \rightarrow f} \propto |\langle \psi_f | e^{-i\vec{q} \cdot \vec{r}} \hat{\epsilon} \cdot \vec{p} | \psi_i \rangle|^2$, VMD

- normalization via beam kaon decays
- large Coulomb correction

*published in EPJ A50 (2014) 79*
ChPT & Resonances in $\pi^- \pi^- \pi^+$

**Phase $a_2 - a_1$ in detail: $t'$ dependence**

- Transition of $\pi\gamma$ to $\pi IP \rightarrow a_2$ production
- Work in progress
- Interference can be used to map details of resonances and production mechanisms
ChPT & Resonances in $\pi^-\pi^-\pi^+$

**Photon-photon fusion process** $\gamma\gamma \rightarrow \pi^+\pi^-$

- Planned measurements at ALICE and JLab

\[
\sigma_{tot}(s) = \frac{2\pi\alpha^2}{\hat{s}^3 m_{\pi}^2} \left\{ 4 + \hat{s} + \hat{s} |C(\hat{s})|^2 \right\} \sqrt{\hat{s}(\hat{s} - 4)} \\
+ 8 \left[ 2 - \hat{s} + \hat{s} \text{Re}C(\hat{s}) \right] \ln \frac{\sqrt{\hat{s}} + \sqrt{\hat{s} - 4}}{2} \\
C(\hat{s}) = -\beta_\pi \frac{m_{\pi}^3}{2\alpha} \hat{s} - \frac{m_{\pi}^2}{(4\pi f_{\pi})^2} \left\{ \hat{s} + 2 \left[ \ln \frac{\sqrt{\hat{s}} + \sqrt{\hat{s} - 4}}{2} - \frac{i\pi}{2} \right]^2 \right\}
\]

courtesy Norbert Kaiser (TUM)
Polarisability effect (LO ChPT values)

- - - $\alpha_\pi = 3.00$, $\beta_\pi = -3.00$

$\cos \theta_{\text{cm}}$

$\frac{d\sigma}{d\Omega_{\text{cm}}} \ [\mu b]$

loop effects not shown

$E_\gamma < 20 \ \text{GeV}$

$\gamma E \ \pi \ s=3m_\pi^2$

$\gamma E \ \pi \ s=5m_\pi^2$

$\gamma E \ \pi \ s=8m_\pi^2$

$\gamma E \ \pi \ s=15m_\pi^2$

$\gamma E \ \pi$
Polarisability effect (NLO ChPT values)

\[ \alpha_\pi = 3.00, \beta_\pi = -2.86 \]
ChPT & Resonances in $\pi^- \pi^- \pi^+$

Polarisability effect (wrong sign $\alpha_{\pi} + \beta_{\pi}$)

- - - $\alpha_{\pi} = 3.00$, $\beta_{\pi} = -3.14$

loop effects not shown

$E_\gamma < 20 \text{ GeV}$
Polarisability effect (Serpukhov values)

- - - $\alpha_\pi = 6.10$, $\beta_\pi = -6.10$

Loop effects not shown

$\frac{d\sigma}{d\Omega_{\text{CM}}}$ [µb]

$\cos \theta_{\text{CM}}$

$s=3m^2_\pi$

$s=5m^2_\pi$

$s=8m^2_\pi$

$s=15m^2_\pi$

$E_\gamma < 20$ GeV

Primakoff
Radiative $\pi^+$ production on the proton:

$$\gamma \, \pi^* \rightarrow \pi \, \gamma$$

[via $\gamma \, p \rightarrow n \, \pi^+ \, \gamma$]

Mainz (2005) measurement: $\alpha_\pi - \beta_\pi = 11.6 \pm 1.5 \pm 3.0 \pm 0.5$

“$\pm 0.5$”: model error only within the used ansatz, full systematics not under control

Primakoff Compton reaction:

$$\gamma^* \, \pi \rightarrow \pi \, \gamma$$

[via $\pi \, Z \rightarrow Z \, \pi \, \gamma$]

tiny extrapolation $\gamma^* \rightarrow \gamma \, \mathcal{O}(10^{-3} \, m_{\pi}^2)$

fully under theoretical control

Minimum transverse momentum of the charged particle

\[ \pi^- \text{Ni} \rightarrow \pi^- \gamma \text{Ni} \]

- **Data**
- **Simulation (normalised)**

Counts / 2.5 MeV/c

Minimum transverse momentum of the charged particle

$T_p$

0.1

0.2

0.3

$p_T$ [GeV/c]
ChPT & Resonances in $\pi^- \pi^- \pi^+$

CM energy in $\pi \gamma \rightarrow \pi \gamma$

- $\rho$ contribution from $\pi \gamma \rightarrow \pi \pi^0$
Exclusivity vs. $\sqrt{s}$

- $\rho$ contribution from $\pi\gamma \rightarrow \pi\pi^0$
Mandelstam \{s,t\} ↔ Laboratory \{E_\gamma, \theta_\gamma\}
for \pi\gamma \rightarrow \pi\gamma

\sqrt{s}/m_\pi

\cos \theta_{CM}
ChPT & Resonances in $\pi^-\pi^-\pi^+$

Cross section

$\sqrt{s}/m_{\pi}$

$\cos \theta_{\text{CM}}$

$\frac{d^2 \sigma}{d\sqrt{s} d\cos \theta_{\text{CM}}}$
M.R. Pennington in the 2\textsuperscript{nd} DAΦNE Physics Handbook, “What we learn by measuring $\gamma\gamma \rightarrow \pi\pi$ at DAΦNE”:

All this means that the only way to measure the pion polarisabilities is in the Compton scattering process near threshold and not in $\gamma\gamma \rightarrow \pi\pi$. Though the low energy $\gamma\gamma \rightarrow \pi\pi$ scattering is seemingly close to the Compton threshold (...) and so the \textit{extrapolation} not very far, the dominance of the pion pole (...) means that the energy scale for this continuation is $m_\pi$. Thus the polarisabilities cannot be determined accurately from $\gamma\gamma$ experiments in a model-independent way and must be measured in the Compton scattering region.
Primakoff production of $a_1(1260)$ vs. E272 result

No evidence for $a_1(1260) \rightarrow \pi \gamma$
**Mass-independent PWA (narrow mass bins):**

\[ \sigma_{\text{indep}}(\tau, m, t') = \sum_{\epsilon=\pm 1} \sum_{r=1}^{N_r} \left| \sum_i T_{ir}^\epsilon f_i^\epsilon(t') \psi_i^\epsilon(\tau, m) \right| \sqrt{\int |f_i^\epsilon(t')|^2 dt'} \sqrt{\int |\psi_i^\epsilon(\tau', m)|^2 d\tau'} \]

- Production strength assumed constant in single bins
- Decay amplitudes \( \psi_i^\epsilon(\tau, m) \), with \( t' \) dependence \( f_i^\epsilon(t') \)
- Production amplitudes \( T_{ir}^\epsilon \rightarrow \) Extended log-likelihood fit
- Acceptance corrections included

**Spin-density matrix:**

\[ \rho_{ij}^\epsilon = \sum_r T_{ir}^\epsilon T_{jr}^\epsilon^* \]

\[ \rightarrow \text{Physical parameters:} \]

\[ \text{Intens}^\epsilon_i = \rho_{ii}^\epsilon, \]

relative phase \( \Phi_{ij}^\epsilon \)

\[ \text{Coh}_{i,j}^\epsilon = \sqrt{\left( \text{Re} \rho_{ij}^\epsilon \right)^2 + \left( \text{Im} \rho_{ij}^\epsilon \right)^2} / \sqrt{\rho_{ii}^\epsilon \rho_{jj}^\epsilon} \]

**Mass-dependent \( \chi^2 \)-fit** (not presented here):

- \( X \) parameterized by Breit-Wigner (BW) functions
- Background can be added
Mass dependence of the diffractive slope

\[ \text{Diffractive slope } b_{\text{diff}} \]

\[ \left( \text{GeV/c} \right)^{-2} \]

\[ \pi^- \text{Pb} \rightarrow \pi^- \pi^- \pi^+ \text{Pb} \]

COMPASS 2004

Preliminary
Partial Wave Analysis Formalism

Isobar Model

- Isobar model: Intermediate 2-particle decays
- Partial wave in reflectivity basis: $J^{PC}M^\epsilon[isobar]L$

- Mass-independent PWA (40 MeV/$c^2$ mass bins): 38 waves
  Fit of angular dependence of partial waves, interferences

- Mass-dependent $\chi^2$-fit (Not presented here)
Major intensities in $m(3\pi)$-bins (acceptance corrected)

\[ \times 10^3 \]

\begin{align*}
\text{Intensity / (40 MeV/c}^2) \\
0 & 20 & 40 & 60 & 80 & 100 & 120 \\
0.6 & 0.8 & 1 & 1.2 & 1.4 & 1.6 & 1.8 & 2 & 2.2 & 2.4
\end{align*}

Mass of $\pi\pi\pi^*$ System (GeV/c$^2$)

\[ \times 10^3 \]

\begin{align*}
\text{Intensity / (40 MeV/c}^2) \\
0 & 2 & 4 & 6 & 8 & 10 & 12 \\
0.6 & 0.8 & 1 & 1.2 & 1.4 & 1.6 & 1.8 & 2 & 2.2 & 2.4
\end{align*}

Mass of $\pi\pi\pi^*$ System (GeV/c$^2$)

COMPASS 2004
\[ \pi \text{Pb} \rightarrow \pi\pi\pi^*\text{Pb} \]
\[ t' < 0.001 \text{ GeV}^2c^2 \]

M=0 Spin Total

COMPASS 2004
\[ \pi \text{Pb} \rightarrow \pi\pi\pi^*\text{Pb} \]
\[ t' < 0.001 \text{ GeV}^2c^2 \]

M=1 Spin Total

COMPASS 2004
\[ \pi \text{Pb} \rightarrow \pi\pi\pi^*\text{Pb} \]
\[ t' < 0.001 \text{ GeV}^2c^2 \]

$S_{\pi\rho} + 0^{++} 1$

COMPASS 2004
\[ \pi \text{Pb} \rightarrow \pi\pi\pi^*\text{Pb} \]
\[ t' < 0.001 \text{ GeV}^2c^2 \]

$1^{++} 0^{+} \rho\pi S$

a$_2$(1320)

J. M. Friedrich — Pion Polarisability with COMPASS
PWA of data with low $t'$

Intensity of selected waves: $0^{-+}0^+ f_0(980)\pi S$, $1^{++}0^+ \rho\pi S$, $2^{++}1^+ \rho\pi D$, $2^{-+}0^+ f_2(1270)\pi S$

<table>
<thead>
<tr>
<th>Mass of $\pi\pi\pi^+$ System (GeV/$c^2$)</th>
<th>Intensity $(40 \text{ MeV}/c^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6 - 0.8</td>
<td>0.6 - 0.8</td>
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<tr>
<td>0.8 - 1.0</td>
<td>0.8 - 1.0</td>
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<tr>
<td>1.0 - 1.2</td>
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<td>1.2 - 1.4</td>
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<td>1.4 - 1.6</td>
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<tr>
<td>2.0 - 2.2</td>
<td>2.0 - 2.2</td>
</tr>
<tr>
<td>2.2 - 2.4</td>
<td>2.2 - 2.4</td>
</tr>
</tbody>
</table>

**COMPASS 2004**

$\pi^-\text{Pb} \rightarrow \pi\pi\pi^+\text{Pb}$

$0.001 < t' < 0.01 \text{ GeV}^2/c^2$

**Preliminary**

$1^{++}0^+ \rho\pi S$

$2^{++}1^+ \rho\pi D$

$2^{-+}0^+ f_2(1270)\pi S$
"Spin Totals": Sum of all contributions for given M (i.e. z-projection of J)

$t'$-dependent amplitudes:

Primakoff production: \( M=1: \sigma(t') \propto e^{-b_{Prim}t'} \rightarrow \text{arises at } t' \approx 0 \) (resoluted shape!)

Diffractive production: \( M=0: \sigma(t') \propto e^{-b_{diff}(m)t'} \)
\( M=1: \sigma(t') \propto t'e^{-b_{diff}(m)t'} \rightarrow \text{vanishes for } t' \approx 0 \)
ChPT & Resonances in $\pi^-\pi^-\pi^+$

Theory: Phase $a_2$(strong+Coulomb)-$a_1$(strong)

![Graph showing phase vs. momentum transfer]

Glauber modell


Plot: N. Kaiser (TU München)

⇒ indicates confirmation of interference Coulomb-interaction - strong interaction
⇒ detailed studies of the nature of resonances
Primakoff contribution at \( t' < 10^{-3} \text{ (GeV/c)}^2 \)

\begin{align*}
\text{Primakoff:} \quad & \sigma(t') \propto e^{-b_{\text{Prim}}t'}, \quad b_{\text{Prim}} \approx 2000 \text{ (GeV/c)}^{-2} \text{ (mainly resolution)} \\
\text{Diffractive:} \quad & \sigma(t') \propto e^{-b_{\text{diff}}t'}, \quad b_{\text{diff}} \approx 400 \text{ (GeV/c)}^{-2} \text{ for lead target}
\end{align*}

(Mass) spectrum of this Primakoff contribution?
⇒ Statistical subtraction of diffractive background (for bins of \( m_{3\pi} \))