Pion Polarisability and Related Physics at COMPASS

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COMPASS collaboration

International Workshop on
Hadron Structure and Spectroscopy

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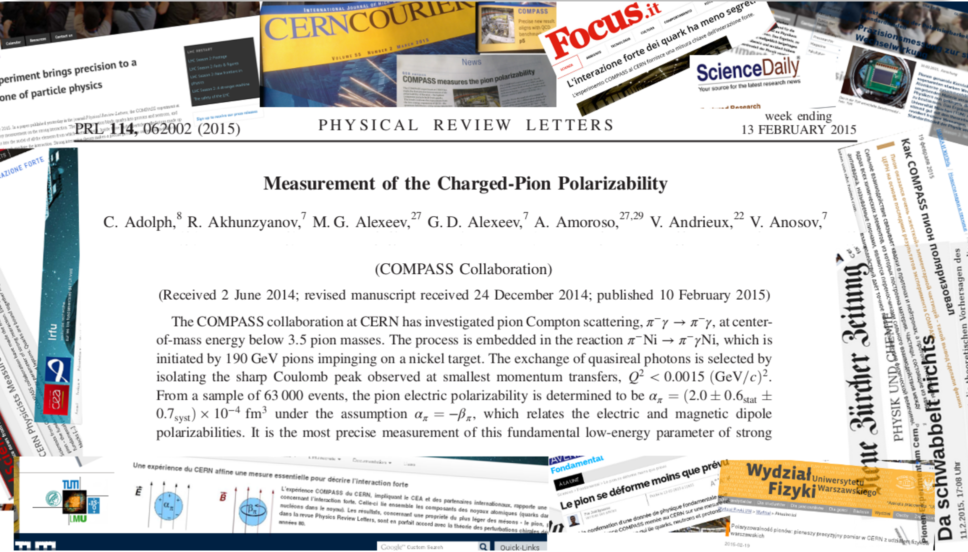
Suzdal, Russia, 18-20 May 2015

May 19, 2015

Bundesministerium
für Bildung
und Forschung

latest pubished data on the pion polarisability



PRL 114, 062002 (2015) PHYSICAL REVIEW LETTERS week ending 13 FEBRUARY 2015

Measurement of the Charged-Pion Polarizability

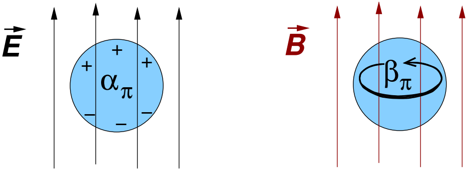
C. Adolph,[8] R. Akhunzyanov,[7] M. G. Alexeev,[27] G. D. Alexeev,[7] A. Amoroso,[27,29] V. Andrieux,[22] V. Anosov,[7]

(COMPASS Collaboration)

(Received 2 June 2014; revised manuscript received 24 December 2014; published 10 February 2015)

The COMPASS collaboration at CERN has investigated pion Compton scattering, $\pi^-\gamma \to \pi^-\gamma$, at center-of-mass energy below 3.5 pion masses. The process is embedded in the reaction $\pi^-\mathrm{Ni} \to \pi^-\gamma\mathrm{Ni}$, which is initiated by 190 GeV pions impinging on a nickel target. The exchange of quasireal photons is selected by isolating the sharp Coulomb peak observed at smallest momentum transfers, $Q^2 < 0.0015\ (\mathrm{GeV}/c)^2$. From a sample of 63 000 events, the pion electric polarizability is determined to be $\alpha_\pi = (2.0 \pm 0.6_{\mathrm{stat}} \pm 0.7_{\mathrm{syst}}) \times 10^{-4}\ \mathrm{fm}^3$ under the assumption $\alpha_\pi = -\beta_\pi$, which relates the electric and magnetic dipole polarizabilities. It is the most precise measurement of this fundamental low-energy parameter of strong

Pion polarisability and theoretical expectation



pion polarisabilities $\boldsymbol{\alpha_\pi}$, $\boldsymbol{\beta_\pi}$ in units of 10^{-4} fm^3

size of the pion $\sim$ 1 fm^3 [cf. atoms: polarisability $\approx$ size $\approx$ 1 Å^3]

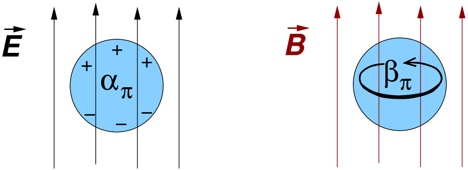
Theory: ChPT (2-loop) prediction:

$$\boldsymbol{\alpha_\pi - \beta_\pi} = 5.7 \pm 1.0$$
$$\boldsymbol{\alpha_\pi + \beta_\pi} = 0.16 \pm 0.1$$

experiments for $\boldsymbol{\alpha_\pi - \beta_\pi}$ lie in the range $4 \cdots 14$

($\alpha_\pi + \beta_\pi = 0$ assumed)

Pion polarisability and theoretical expectation



pion polarisabilities α_π, β_π in units of 10^{-4} fm^3

size of the pion $\sim$ 1 fm^3 [cf. atoms: polarisability $\approx$ size $\approx$ 1 Å^3]

Theory: ChPT (2-loop) prediction:

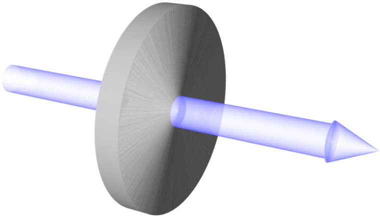
$$\alpha_\pi = 2.93 \pm 0.5$$
$$\beta_\pi = -2.77 \pm 0.5$$

experiments for α_π lie in the range $2 \cdots 7$

($\alpha_\pi + \beta_\pi = 0$ assumed)

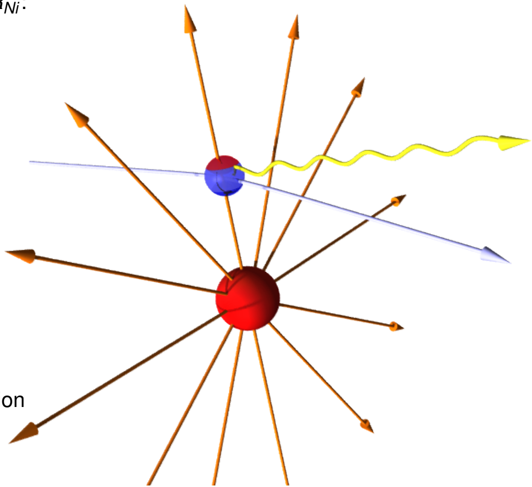
Principle of the COMPASS measurement

- high-energetic pion beam on 4mm nickel disk
- observe scattered pions in coincidence with produced hard photons
- study of cross-section shape



Polarisability effect in Primakoff technique

- Charged pions traverse the nuclear electric field
  - typical field strength at $d = 5R_{Ni}$: $E \approx 300\,\text{kV/fm}$
- Bremsstrahlung process:
  - particles scatter off *equivalent photons*
  - tiny momentum transfer $Q^2 \approx 10^{-5}\,\text{GeV}^2/c^2$
  - pion/muon (quasi-)real Compton scattering
- Polarisability contribution
  - Compton cross-section typically diminished
  - corresponding charge separation $\approx 10^{-5}\,\text{fm} \cdot e$



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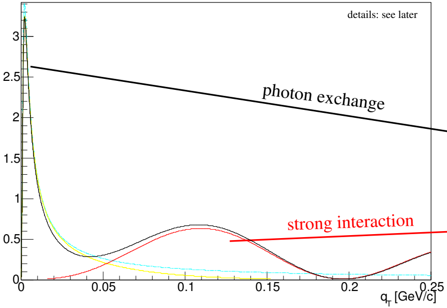

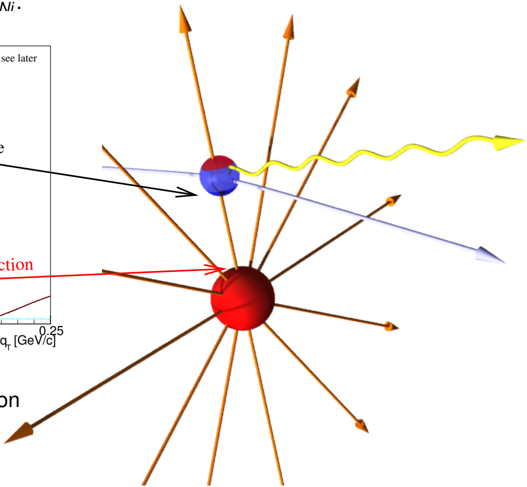

typically diminished

  - corresponding charge separation $\approx 10^{-5}$ fm $\cdot\ e$

Pion Compton scattering: embedding the process

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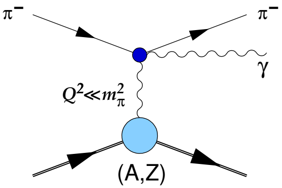
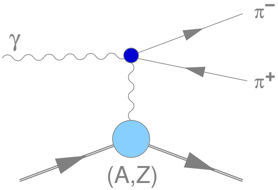
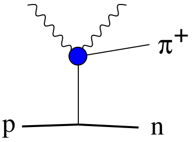
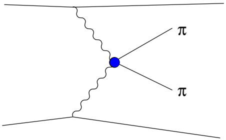




Primakoff processes

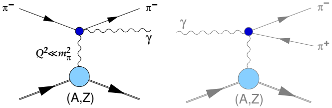


Radiative pion photoproduction



Photon-Photon fusion

Pion polarisability: world data before COMPASS



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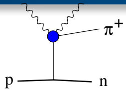

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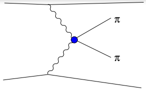
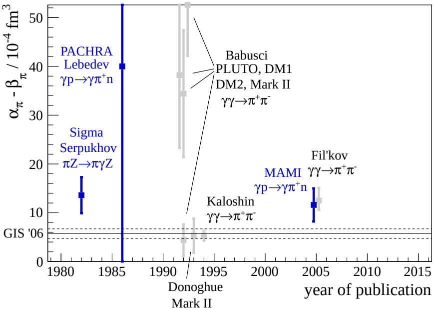
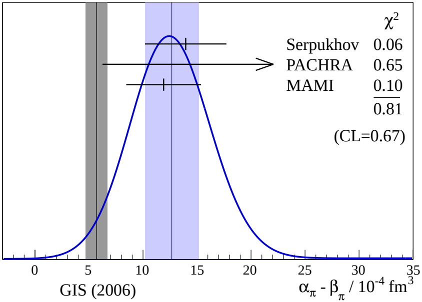


Photon-Photon fusion





GIS'06: ChPT prediction, Gasser, Ivanov, Sainio, NPB745 (2006), plots: T. Nagel, PhD
Fil'kov analysis objected by Pasquini, Drechsel, Scherer PRC81, 029802 (2010)

Pion Compton Scattering

$$\pi \ \gamma \ \rightarrow \ \pi \ \gamma$$

- Two kinematic variables, in CM: total energy $\sqrt{s}$, scattering angle θ_{cm}

$$\frac{d\sigma_{\pi\gamma}}{d\Omega_{cm}} = \frac{\alpha^2(s^2 z_+^2 + m_\pi^4 z_-^2)}{s(s z_+ + m_\pi^2 z_-)^2} - \frac{\alpha\, m_\pi^3\, (s - m_\pi^2)^2}{4s^2(s z_+ + m_\pi^2 z_-)} \cdot \mathcal{P}$$

$$\mathcal{P} = z_-^2(\alpha_\pi - \beta_\pi) + \frac{s^2}{m_\pi^4} z_+^2(\alpha_\pi + \beta_\pi) - \frac{(s - m_\pi^2)^2}{24s} z_-^3(\alpha_2 - \beta_2)$$

$$z_\pm = 1 \pm \cos\theta_{cm}$$

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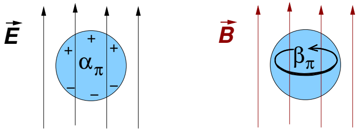

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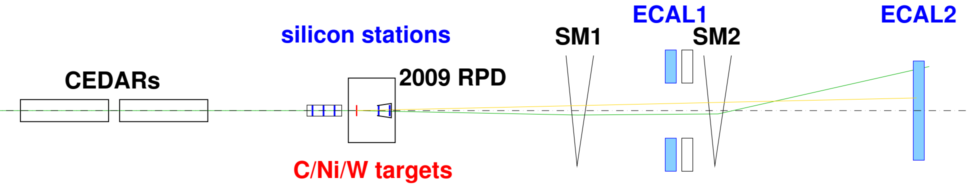
Principle of the measurement



Extraction of the pion polarisability

- Identify exclusive reactions

$$\pi\gamma_{\{Ni\to Ni'\}} \to \pi\gamma$$

  at smallest momentum transfer $< 0.001\,\text{GeV}^2/c^2$

- Assuming $\alpha_\pi + \beta_\pi = 0$, from the cross-section

$$R = \frac{\sigma(x_\gamma)}{\sigma_{\alpha_\pi=0}(x_\gamma)} = \frac{N_{meas}(x_\gamma)}{N_{sim}(x_\gamma)} = 1 - \frac{3}{2}\cdot\frac{m_\pi^3}{\alpha}\cdot\frac{x_\gamma^2}{1-x_\gamma}\,\alpha_\pi$$

  is derived, depending on $x_\gamma = E_{\gamma(lab)}/E_{Beam}$.
  Measuring R the polarisability α_π can be concluded.

- Control systematics by

$$\mu\gamma_{\{Ni\to Ni'\}} \to \mu\gamma$$

  and

$$K^- \to \pi^-\pi^0 \to \pi\gamma\gamma$$

Extraction of the pion polarisability

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$$\boldsymbol{\mu\gamma}_{\{\mathrm{Ni}\to\mathrm{Ni}'\}} \to \boldsymbol{\mu\gamma}$$

  and

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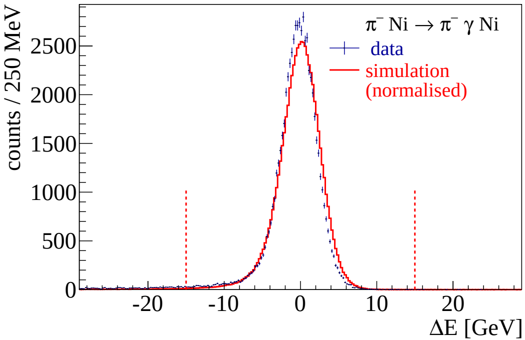
Identifying the $\pi\gamma \to \pi\gamma$ reaction

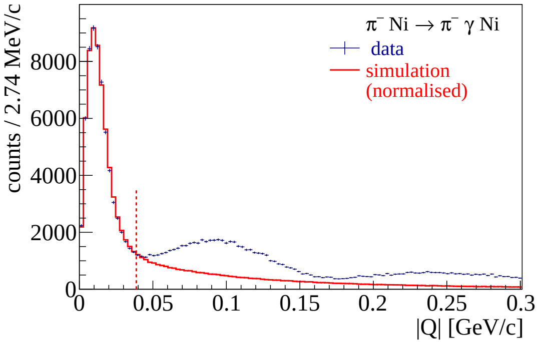
Phys. Rev. Lett. 114, 062002 (2015)



- Energy balance $\Delta E = E_\pi + E_\gamma - E_{\text{Beam}}$
- Exclusivity peak $\sigma \approx$ 2.6 GeV (1.4%)
- $\sim$ 63.000 exclusive events ($x_\gamma > 0.4$) (Serpukhov $\sim$ 7000 for $x_\gamma > 0.5$)

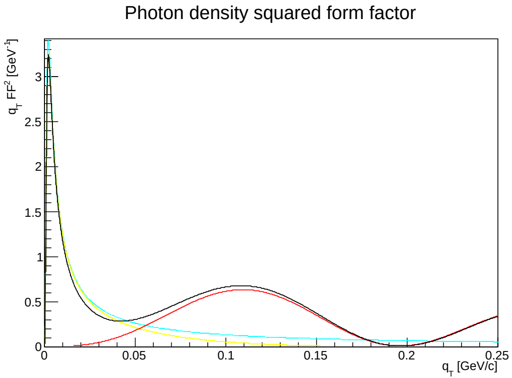
Primakoff peak

Phys. Rev. Lett. 114, 062002 (2015)



- $\Delta Q_T \approx$ 12 MeV/c (190 GeV/c beam $\rightarrow$ requires few-μrad angular resolution)
- first diffractive minimum on Ni nucleus at $Q \approx 190$ MeV$/c$
- data a little more narrow than simulation $\rightarrow$ negative interference?

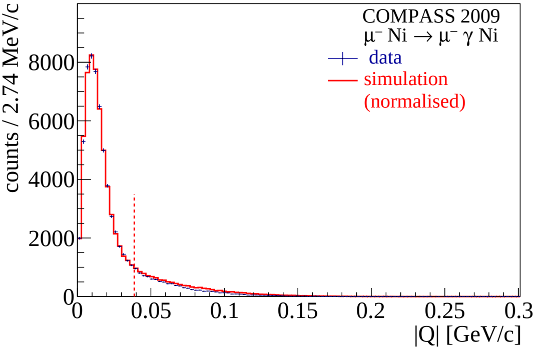
Coulomb-nuclear interference



- calculation following G. Fäldt (Phys. Rev. C79, 014607)
- eikonal approximation: pions traverse Coulomb and strong-interaction potentials

Primakoff peak: muon data

Phys. Rev. Lett. 114, 062002 (2015)



- muon control measurement: pure electromagnetic interaction
- e.m. nuclear effects well understood

Principle of the measurement

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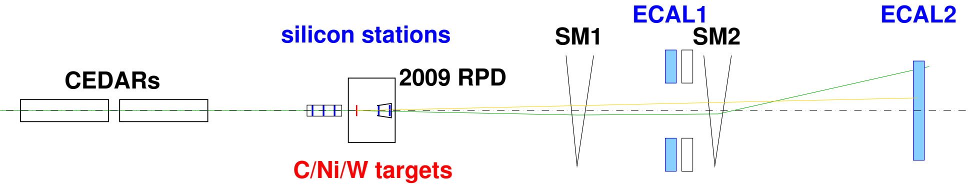

ECAL2: 3000 cells of different types

ECAL2: the quest for precision

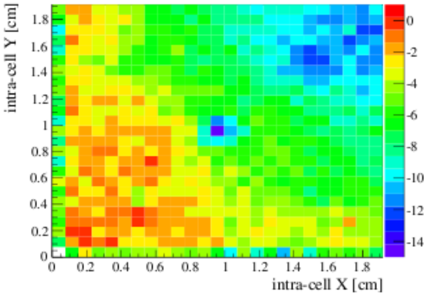

Figure 3.5: Profile of energy deviations shown for 1/4 of a shashlik block and for muon data photons within the range 133 GeV $< E_\gamma <$ 152 GeV .

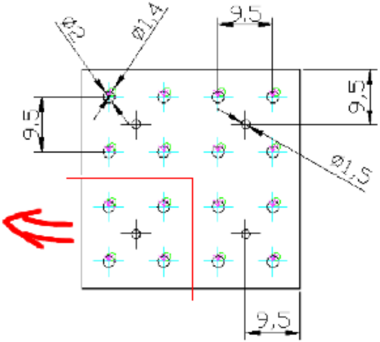

Figure 3.6: Technical drawing of a full shashlik cell to be compared with the figure to the left.

from: Th. Nagel, PhD thesis TUM 2012

Photon energy spectra for muon and pion beam

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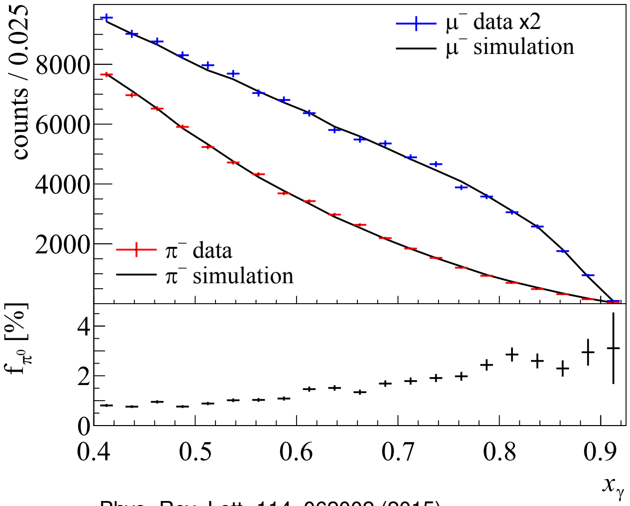
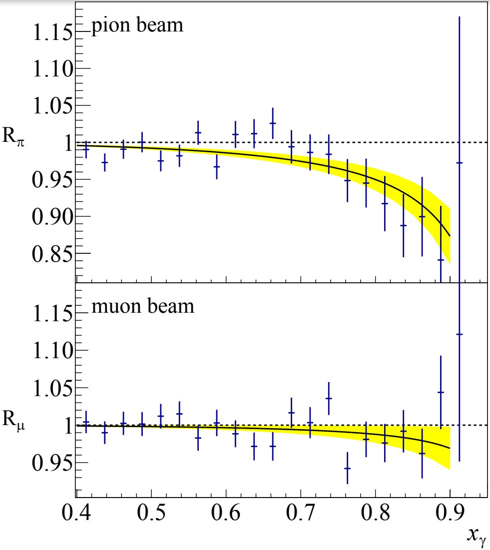


Phys. Rev. Lett. 114, 062002 (2015)

Pion polarisability: COMPASS result



$$\boldsymbol{\alpha_\pi} = (\ 2.0 \pm 0.6_{\text{stat}}\ ) \times 10^{-4}\,\text{fm}^3$$

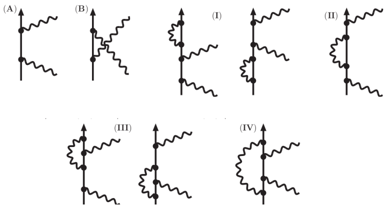
(assuming $\alpha_\pi = -\beta_\pi$)

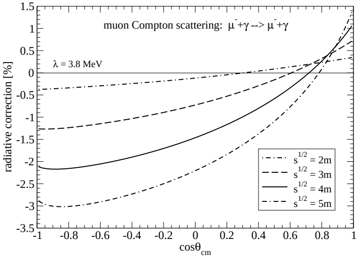
“false polarisability” from muon data:

$$(\ 0.5 \pm 0.5_{\text{stat}}\ ) \times 10^{-4}\,\text{fm}^3$$

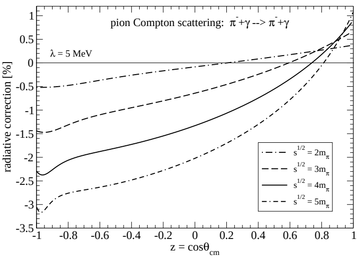
Phys. Rev. Lett. 114, 062002 (2015)

Radiative corrections (Compton scattering part)





Nucl.Phys. A837 (2010)



Eur.Phys.J. A39 (2009) 71

source of systematic uncertainty	estimated magnitude CL $= 68\,\%$ $[10^{-4}\,\mathrm{fm}^3]$
tracking	0.5
radiative corrections	0.3
background subtraction in Q	0.4
pion electron scattering	0.2
quadratic sum	0.7

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COMPASS result for the pion polarisability:

$$\boldsymbol{\alpha_\pi} = (\,\mathbf{2.0} \pm \mathbf{0.6}_{\text{stat}} \pm \mathbf{0.7}_{\text{syst}}\,) \times \mathbf{10^{-4}}\,\text{fm}^3$$

with $\alpha_\pi = -\beta_\pi$ assumed

Pion polarisability: world data including COMPASS

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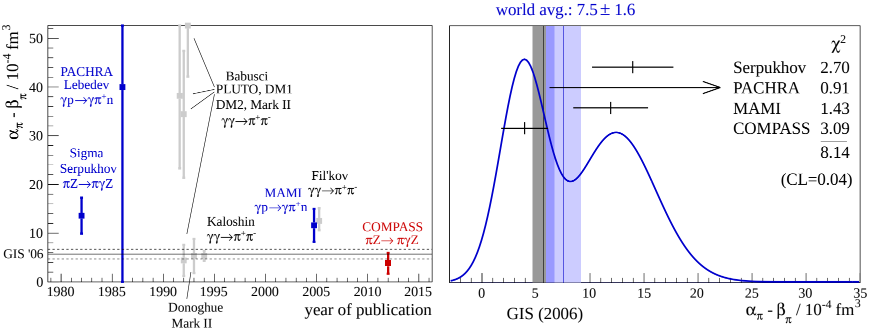

- The new COMPASS result is in significant tension with the earlier measurements of the pion polarisability
- The expectation from ChPT is confirmed within the uncertainties

About crossing

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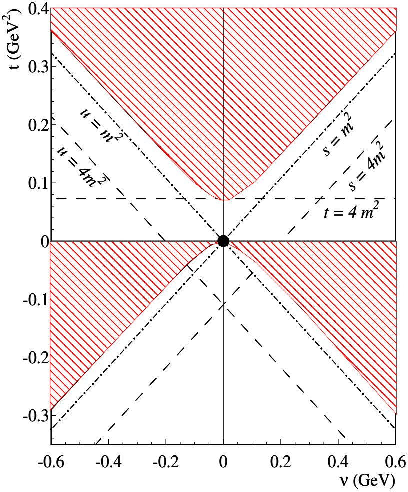

- red hatched: physical regions
  $\gamma + \gamma \to \pi + \pi$
  $\gamma + \pi \to \gamma + \pi$
- two-pion thresholds at $s = 4m_\pi^2$, $u = 4m_\pi^2$, $t = 4m_\pi^2$
- DR integration paths
  $t = 0$ (forward),
  $\theta = 180°$ (backward)
  $u = m_\pi^2$, $s = m_\pi^2$, ...

from: D. Drechsel, talk at IWHSS 2011 Paris

Photon-photon fusion process $\gamma\gamma \to \pi^+\pi^-$

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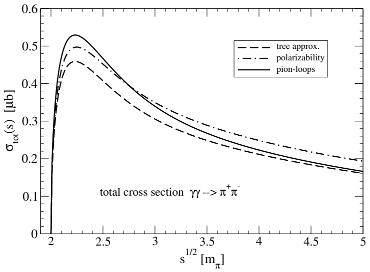
- Planned measurements at ALICE and JLab

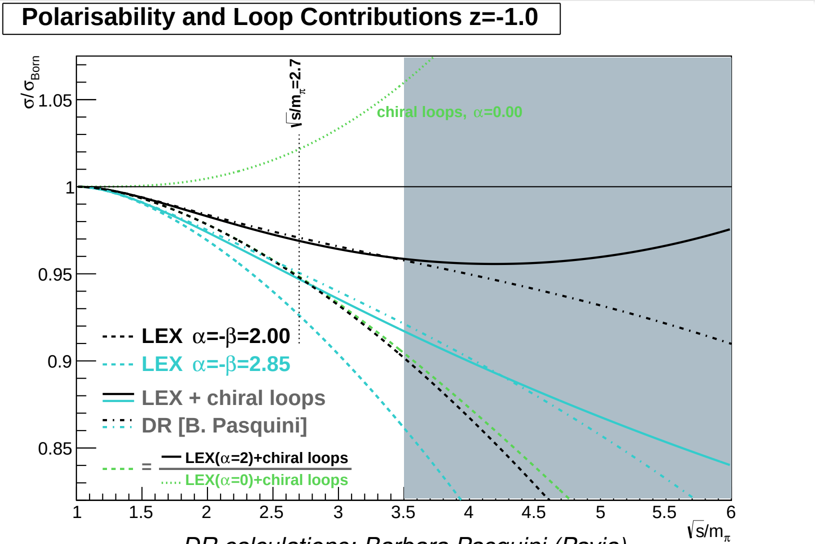
$$\sigma_{tot}(s) = \frac{2\pi\alpha^2}{\hat{s}^3 m_\pi^2}\left\{\left[4+\hat{s}+\hat{s}|C(\hat{s})|^2\right]\sqrt{\hat{s}(\hat{s}-4)} + 8\left[2-\hat{s}+\hat{s}\,\mathrm{Re}\,C(\hat{s})\right]\ln\frac{\sqrt{\hat{s}}+\sqrt{\hat{s}-4}}{2}\right\},$$

$$C(\hat{s}) = -\beta_\pi \frac{m_\pi^3}{2\alpha}\hat{s} - \frac{m_\pi^2}{(4\pi f_\pi)^2}\left\{\frac{\hat{s}}{2} + 2\left[\ln\frac{\sqrt{\hat{s}}+\sqrt{\hat{s}-4}}{2} - \frac{i\pi}{2}\right]^2\right\}$$



courtesy Norbert Kaiser (TUM)

Dispersion relations and ChPT



DR calculations: Barbara Pasquini (Pavia)

Pion polarisability on the lattice

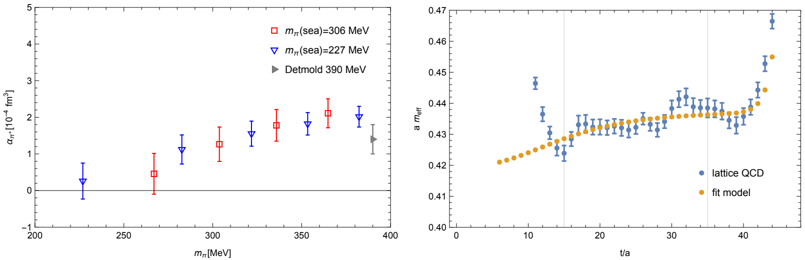
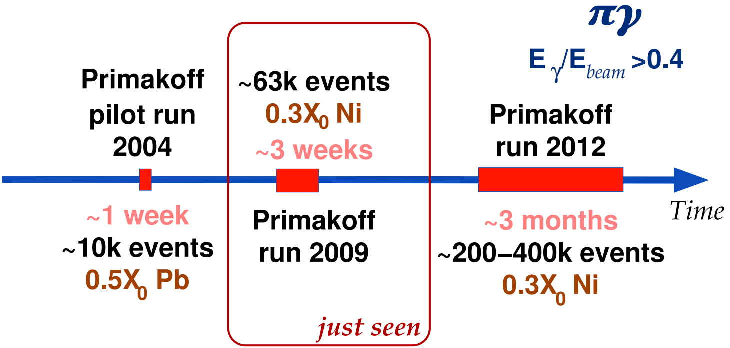


FIGURE 3. Left: electric polarizability for the charged pions as a function of the valence quark mass. The data for $m_\pi = 390\,\text{MeV}$ is taken from [5]. Right: effective mass for a charged pion correlator together with the scalar particle correlator determined from the fit. The fitting range is indicated by the vertical bars.

Alexandru *et al.*, Pion electric polarizability from lattice QCD, arXiv:1501.06516

Pion polarisability measurements at COMPASS

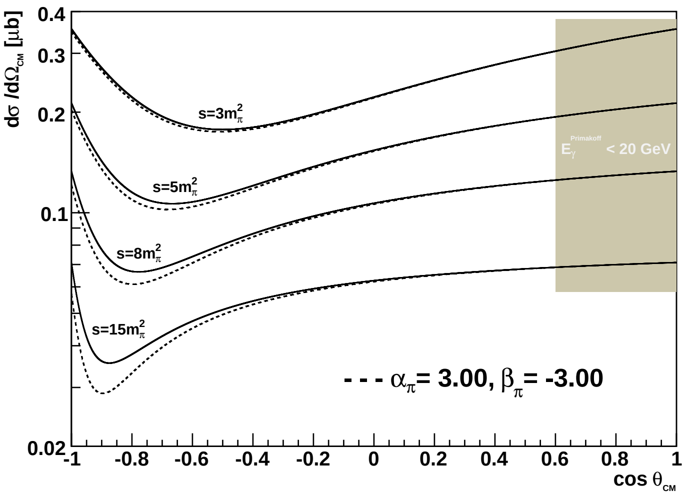


$$\mathcal{P} = z_-^2(\alpha_\pi - \beta_\pi) + \frac{s^2}{m_\pi^4} z_+^2(\alpha_\pi + \beta_\pi) - \frac{(s - m_\pi^2)^2}{24s} z_-^3(\alpha_2 - \beta_2)$$

$$z_\pm = 1 \pm \cos\theta_{cm}$$

Polarisability effect (LO ChPT values)

loop effects not shown



Polarisability effect (NLO ChPT values)

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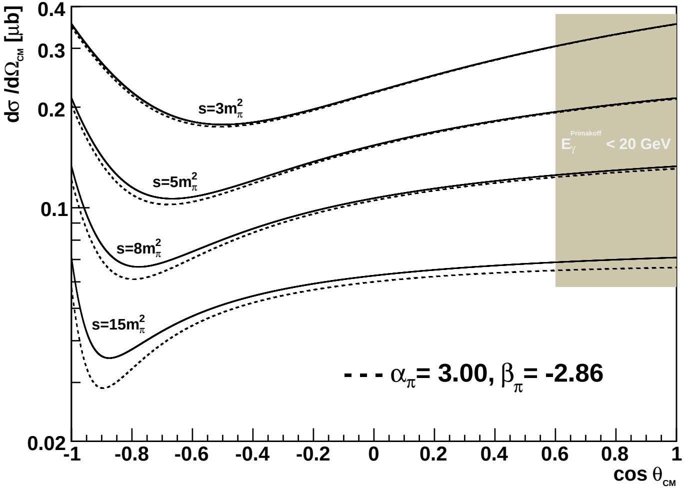
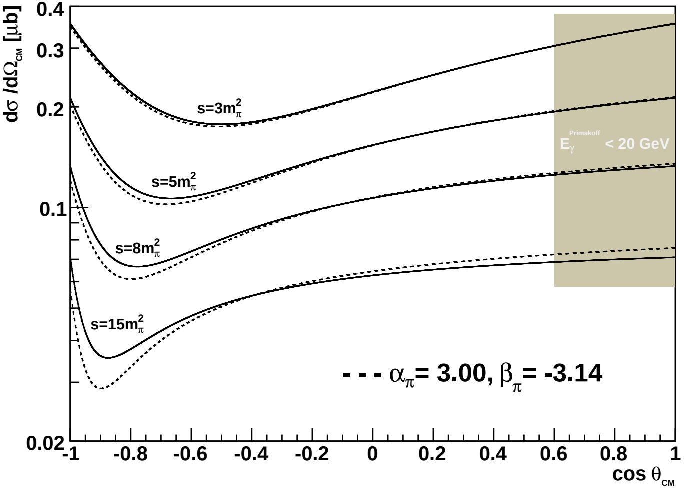
loop effects not shown



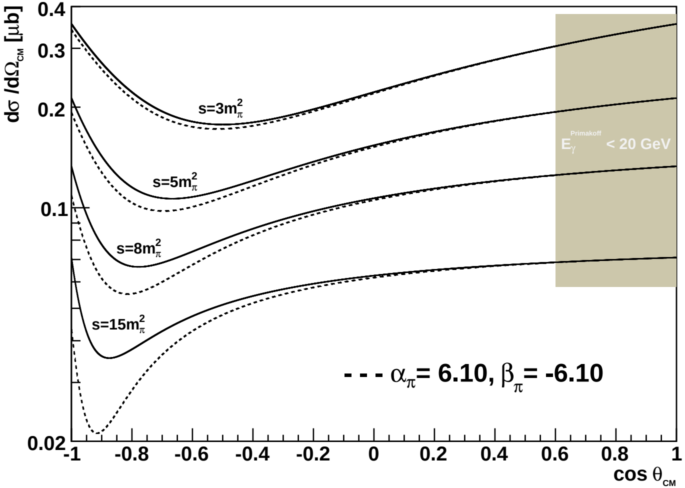
Polarisability effect with "wrong-sign" $\alpha_\pi + \beta_\pi < 0$

loop effects not shown



Polarisability effect (Serpukhov values)

loop effects not shown



Primakoff reactions accessible at COMPASS

Access to $\pi + \gamma$ reactions via the Primakoff effect:

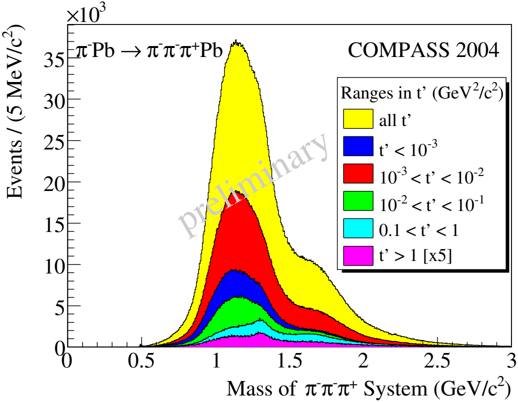
At smallest momentum transfers to the nucleus, high-energetic particles scatter predominantly off the electromagnetic field quanta ($\sim Z^2$)

$$\pi^- + \gamma \rightarrow \begin{cases} \pi^- + \gamma \\ \pi^- + \pi^0 \;/\; \eta \\ \pi^- + \pi^0 + \pi^0 \\ \pi^- + \pi^- + \pi^+ \qquad \Longleftarrow \\ \pi^- + \pi^- + \pi^+ + \pi^- + \pi^+ \\ \pi^- + \ldots \end{cases}$$

analogously: Kaon-induced reactions $K^- + \gamma \rightarrow \cdots$

2004 Primakoff results

π^- Pb $\rightarrow$ Pb $\pi^- \pi^- \pi^+$



- "Low t'": $10^{-3}\,(\mathrm{GeV}/c)^2 < t' < 10^{-2}\,(\mathrm{GeV}/c)^2$ $\sim$ 2 000 000 events
- "Primakoff region": $t' < 10^{-3}\,(\mathrm{GeV}/c)^2$ $\sim$ 1 000 000 events

First Measurement of $\pi\gamma \to 3\pi$ Absolute Cross-Section

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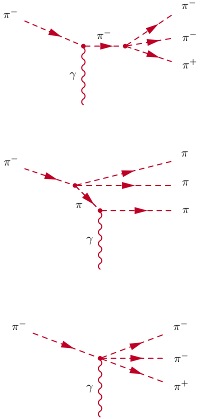
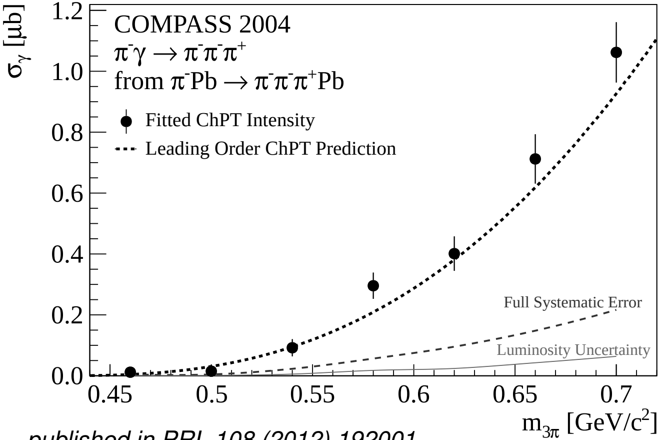
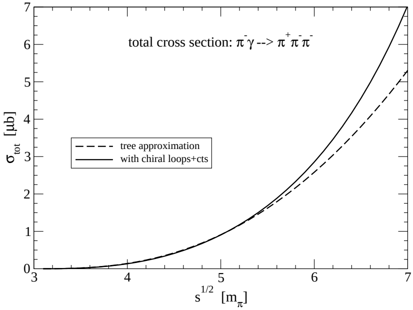
Measured absolute cross-section of $\pi^-\gamma \to \pi^-\pi^-\pi^+$

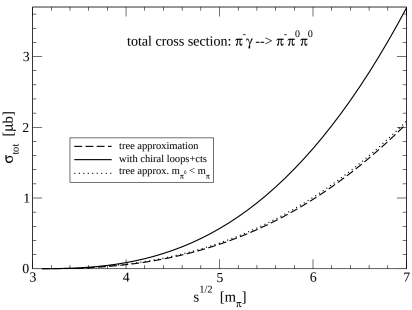




published in PRL 108 (2012) 192001

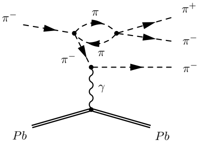
Higher-order effects

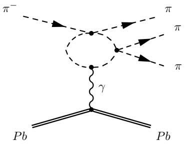




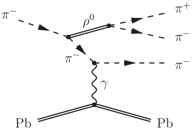
Chiral loops, e.g.

(N. Kaiser, NPA848 (2010) 198)





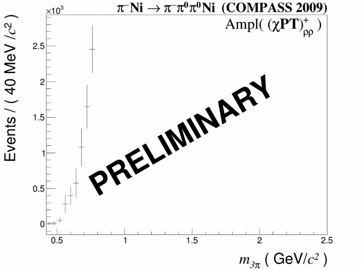
ρ terms:

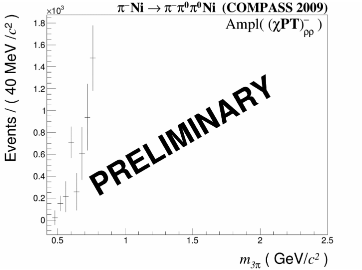


$\pi^-\gamma \rightarrow \pi^-\pi^0\pi^0$

Partial Wave Analysis

Isobaric Model – Chiral Wave

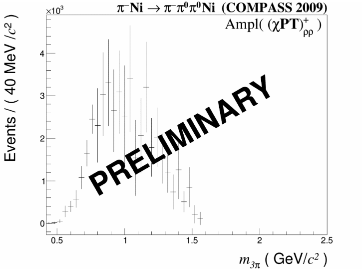


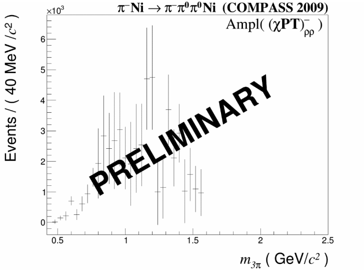


$\pi^-\gamma \to \pi^-\pi^0\pi^0$

Partial Wave Analysis

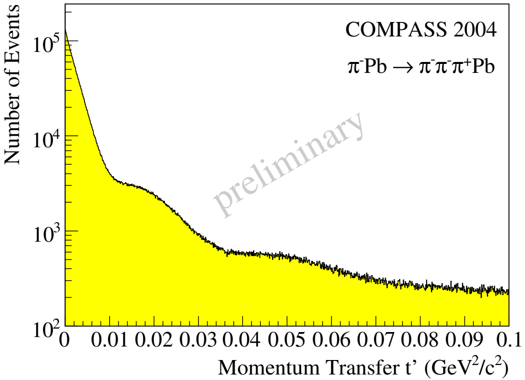
Chiral Model - Amplitudes





2004 Primakoff results

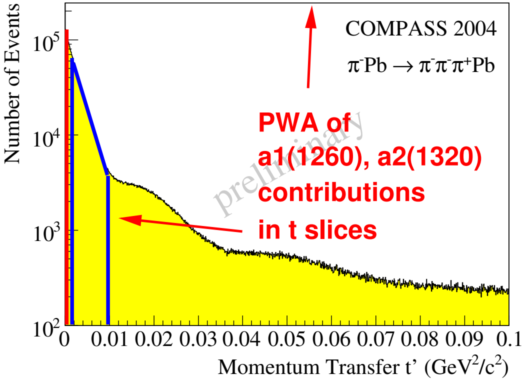
$$\pi^- \text{ Pb} \rightarrow \text{Pb } \pi^- \ \pi^- \ \pi^+$$



- "Low t'": $10^{-3}\,(\text{GeV}/c)^2 < t' < 10^{-2}\,(\text{GeV}/c)^2$ $\sim$ 2 000 000 events
- "Primakoff region": $t' < 10^{-3}\,(\text{GeV}/c)^2$ $\sim$ 1 000 000 events

2004 Primakoff results

$$\pi^- \text{Pb} \to \text{Pb}\, \pi^- \pi^- \pi^+$$



- "Low t'": $10^{-3}\,(\text{GeV}/c)^2 < t' < 10^{-2}\,(\text{GeV}/c)^2$ $\sim$ 2 000 000 events
- "Primakoff region": $t' < 10^{-3}\,(\text{GeV}/c)^2$ $\sim$ 1 000 000 events

PWA: a_1, a_2 and $\Delta\Phi$ in separated t' regions

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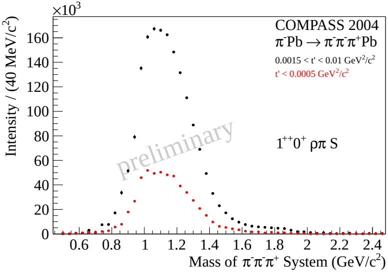
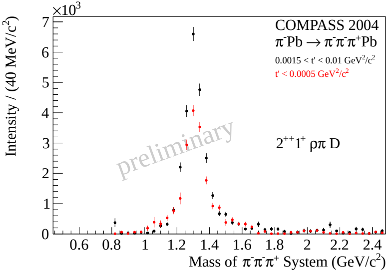
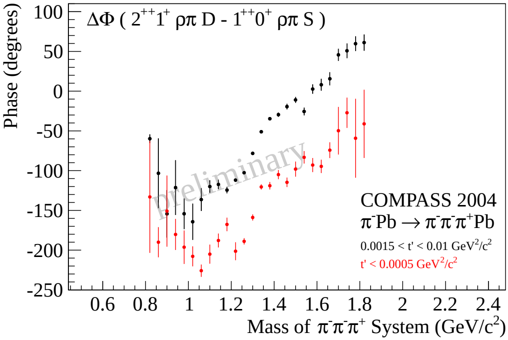
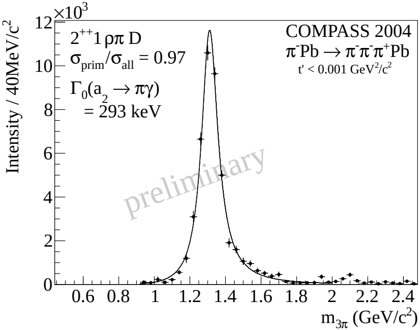
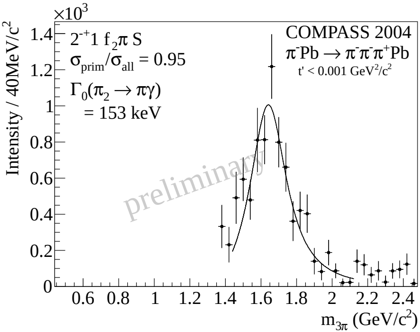






Radiative Coupling of $a_2(1320)$ and $\pi_2(1670)$





$\Gamma_0(a_2(1320) \to \pi\gamma)$ *M2* $\quad$ $\Gamma_0(\pi_2(1670) \to \pi\gamma)$ *E2*

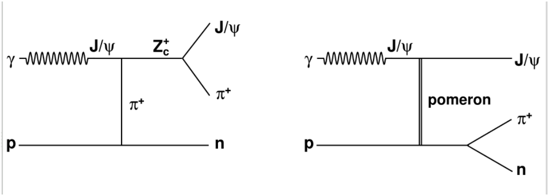
$\Leftrightarrow$ meson w.f.'s: $\Gamma_{i\to f} \propto |\langle\Psi_f|\, e^{-i\vec{q}\cdot\vec{r}}\, \hat{\epsilon}\cdot\vec{p}\,|\Psi_i\rangle|^2$, VMD

- normalization via beam kaon decays
- large Coulomb correction

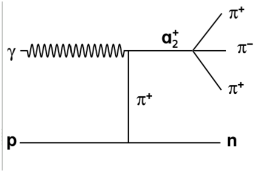
published in EPJ A50 (2014) 79

Charge-exchange reactions and radiative widths

- does (charged-)pion exchange play a role at COMPASS collision energies?
- search for Z_c photo-production:



- a_2 photo-production via pion exchange:



- constrain contribution from pionic trajectory

Summary and Outlook

- Measurement of the pion polarisability at COMPASS
  - Via the Primakoff reaction, COMPASS has determined

$$\boxed{\alpha_\pi = (\,2.0 \pm 0.6_{\text{stat}} \pm 0.7_{\text{syst}}\,) \times 10^{-4}\,\text{fm}^3}$$ assuming $\alpha_\pi + \beta_\pi = 0$

  - most direct access to the $\pi\gamma \to \pi\gamma$ process
  - Most precise experimental determination
  - Systematic control: $\mu\gamma \to \mu\gamma,\ K^- \to \pi^-\pi^0$
- COMPASS measures more aspectes of chiral dynamics in $\pi^-\gamma \to \pi^-\pi^0$ and $\pi\gamma \to \pi\pi\pi$ reactions
- High-statistics run 2012
  - separate determination of α_π and β_π
  - $s-$dependent quadrupole polarisabilities
  - First measurement of the kaon polarisability

Thank you for your attention!

Backup

Phase $a_2 - a_1$ in detail: t' dependence

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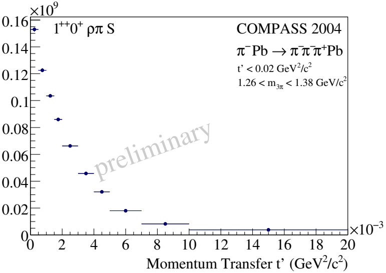
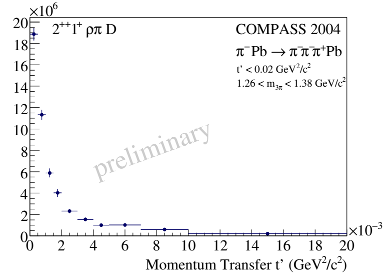
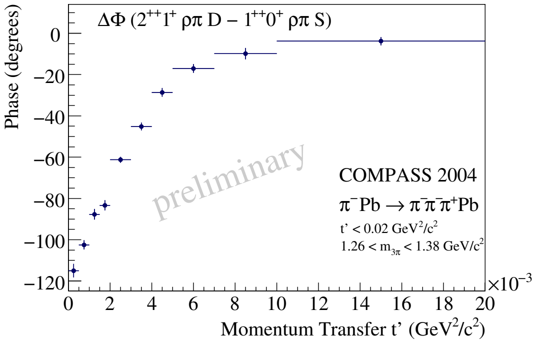






- transition of $\pi\gamma$ to $\pi I\!P \to a_2$ production
- work in progress
- interference can be used to map details of resonances and production mechanisms

- Radiative π^+ production on the proton:

$$\gamma\,\pi^{*} \longrightarrow \pi\,\gamma \qquad [\text{via } \gamma\, p \to n\, \pi^+\, \gamma]$$

Mainz (2005) measurement: $\alpha_\pi - \beta_\pi = 11.6 \pm 1.5 \pm 3.0 \pm 0.5$

"± 0.5": model error *only within the used ansatz*,

full systematics not under control

- Primakoff Compton reaction:

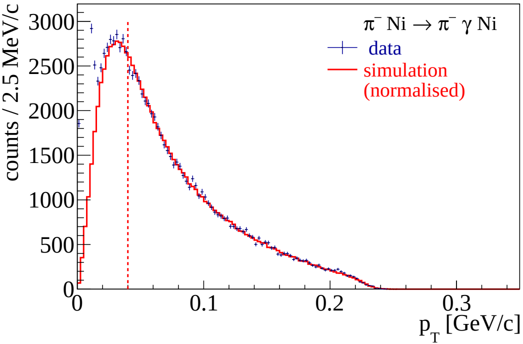
$$\gamma^{*}\,\pi \longrightarrow \pi\,\gamma \qquad [\text{via } \pi\, Z \to Z\, \pi\, \gamma]$$

tiny extrapolation $\gamma^* \to \gamma$ $\mathcal{O}(10^{-3} m_\pi^2)$

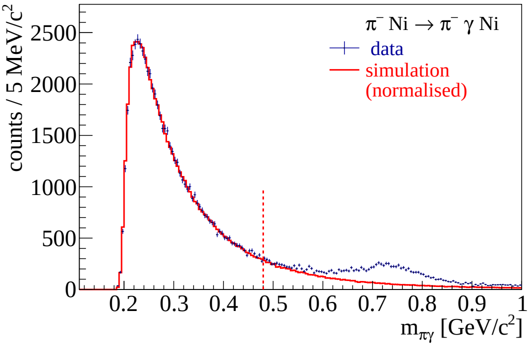
fully under theoretical control

[N. Kaiser, J.F., Nucl. Phys. A 812 (2008) 186]

Minimum transverse momentum of the charged particle

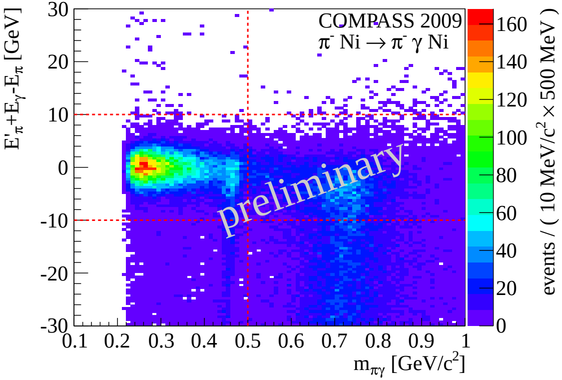


CM energy in $\pi\gamma \to \pi\gamma$



- ρ contribution from $\pi\gamma \to \pi\pi^0$

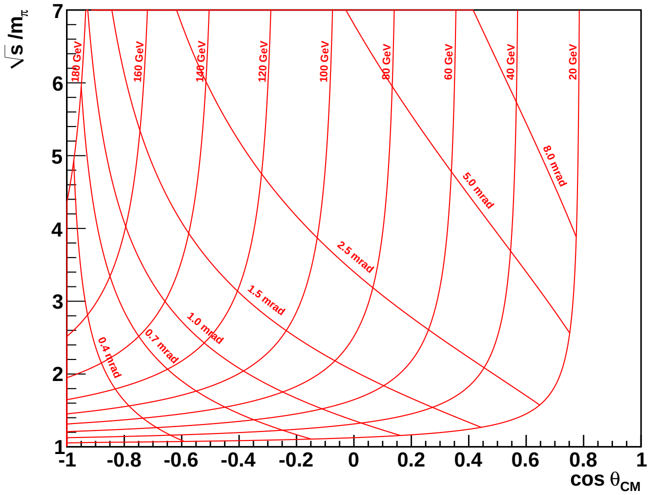
Exclusivity vs. $\sqrt{s}$



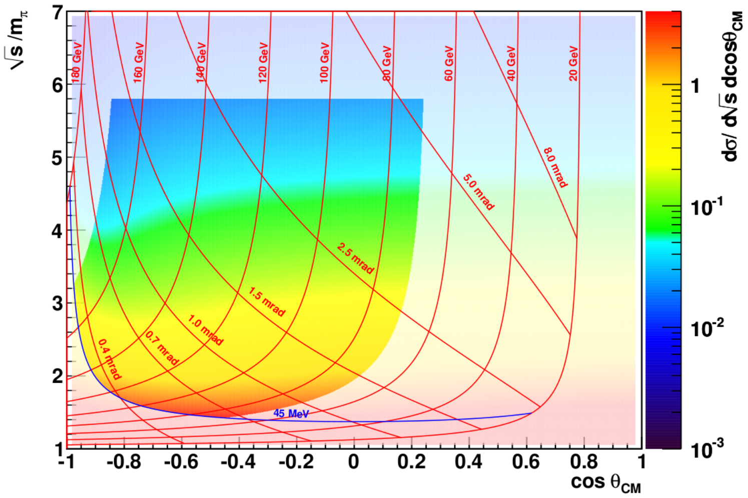
- ρ contribution from $\pi\gamma \to \pi\pi^0$

Mandelstam $\{s,t\} \leftrightarrow$ Laboratory $\{E_\gamma, \theta_\gamma\}$

for $\pi\gamma \rightarrow \pi\gamma$



Cross section



M.R. Pennington in the 2nd DAΦNE Physics Handbook,
"What we learn by measuring $\gamma\gamma \to \pi\pi$ at DAΦNE":

> All this means that the only way to measure the pion polarisabilities is in the Compton scattering process near threshold and not in $\gamma\gamma \to \pi\pi$. Though the low energy $\gamma\gamma \to \pi\pi$ scattering is seemingly close to the Compton threshold (...) and so the *extrapolation* not very far, the dominance of the pion pole (...) means that the energy scale for this continuation is m_π. Thus the polarisabilities cannot be determined accurately from $\gamma\gamma$ experiments in a model-independent way and must be measured in the Compton scattering region.

Primakoff production of a1(1260) vs. E272 result

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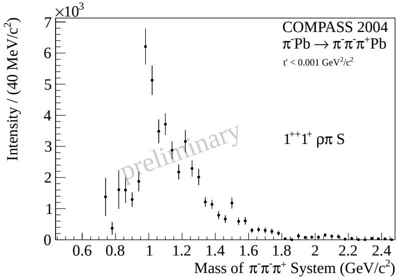
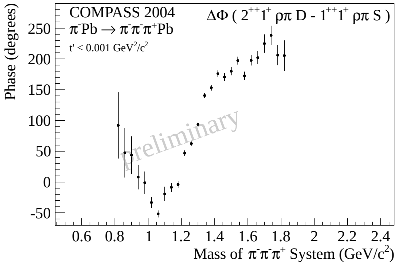
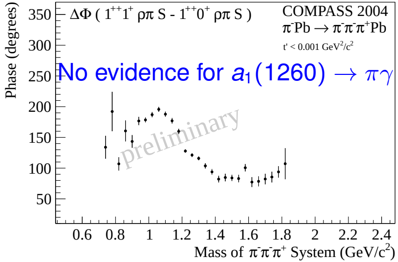
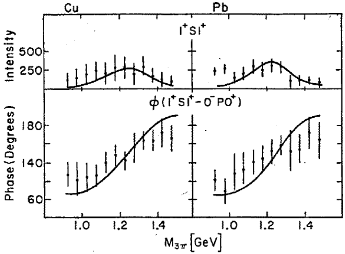




No evidence for $a_1(1260) \to \pi\gamma$





M. Zielinski et al, Phys. Rev. Lett 52 (1984) 1195

BACKUP: Partial Wave Analysis Formalism

- Mass-independent PWA (narrow mass bins):

$$\sigma_{\rm indep}(\tau, m, t') = \sum_{\epsilon=\pm 1} \sum_{r=1}^{N_r} \left| \sum_i T_{ir}^{\epsilon} f_i^{\epsilon}(t') \psi_i^{\epsilon}(\tau, m) \Big/ \sqrt{\int |f_i^{\epsilon}(t')|^2 \mathrm{d}t'} \sqrt{\int |\psi_i^{\epsilon}(\tau', m)|^2 \mathrm{d}\tau'} \right|^2$$

  - Production strenght assumed constant in single bins
  - Decay amplitudes $\psi_i^{\epsilon}(\tau, m)$, with t' dependence $f_i^{\epsilon}(t')$
  - Production amplitudes T_{ir}^{ϵ} $\rightarrow$ Extended log-likelihood fit
  - Acceptance corrections included
- Spin-density matrix: $\rho_{ij}^{\epsilon} = \sum_r T_{ir}^{\epsilon} T_{jr}^{\epsilon *}$

$\rightarrow$ Physical parameters:

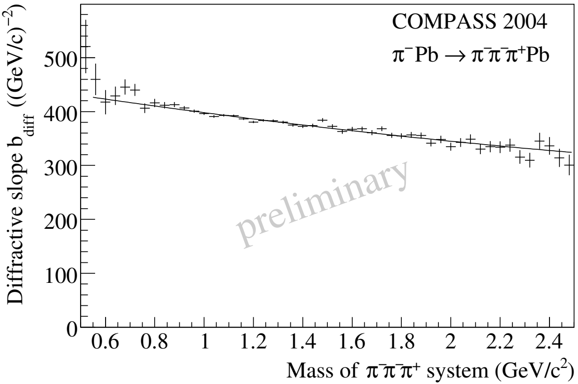
$$\mathrm{Intens}_i^{\epsilon} = \rho_{ii}^{\epsilon},$$

relative phase Φ_{ij}^{e}

$$\mathrm{Coh}\ {}_{i,j}^{\epsilon} = \sqrt{(\ \mathrm{Re}\ \rho_{ij}^{\epsilon})^2 + (\ \mathrm{Im}\ \rho_{ij}^{\epsilon})^2} \Big/ \sqrt{\rho_{ii}^{\epsilon} \rho_{jj}^{\epsilon}}$$

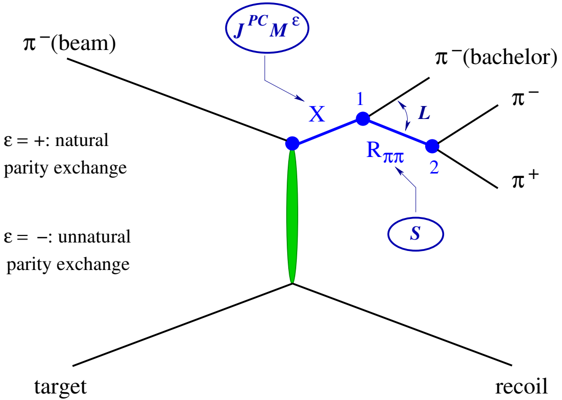
- Mass-dependent χ^2-fit (not presented here):
  - X parameterized by Breit-Wigner (BW) functions
  - Background can be added

Mass dependence of the diffractive slope



Partial Wave Analysis Formalism

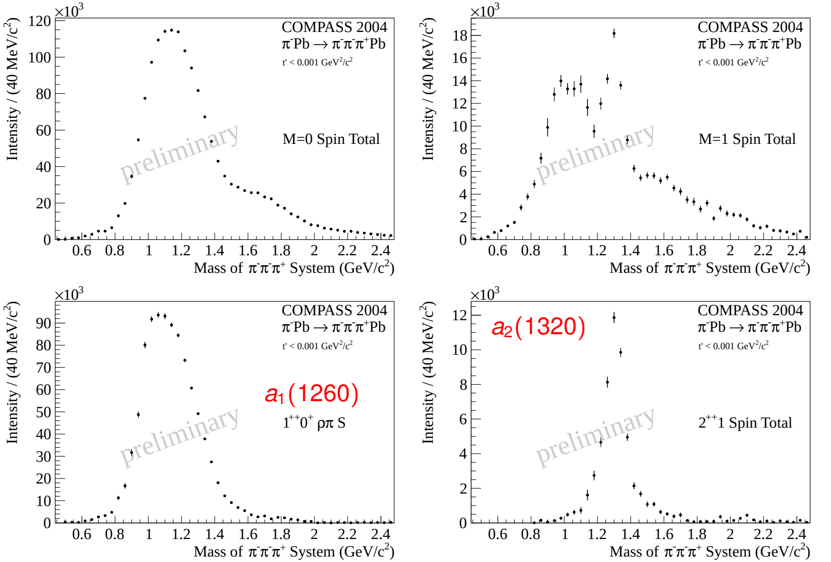
Isobar Model



- Isobar model: Intermediate 2-particle decays
- Partial wave in reflectivity basis: $J^{PC}M^{\epsilon}[isobar]L$

- Mass-independent PWA ($40\,\mathrm{MeV}/c^2$ mass bins): 38 waves
  Fit of angular dependence of partial waves, interferences
- Mass-dependent χ^2-fit (Not presented here)

Major intensities in m(3π)-bins (acceptance corrected)



PWA of data with low t'

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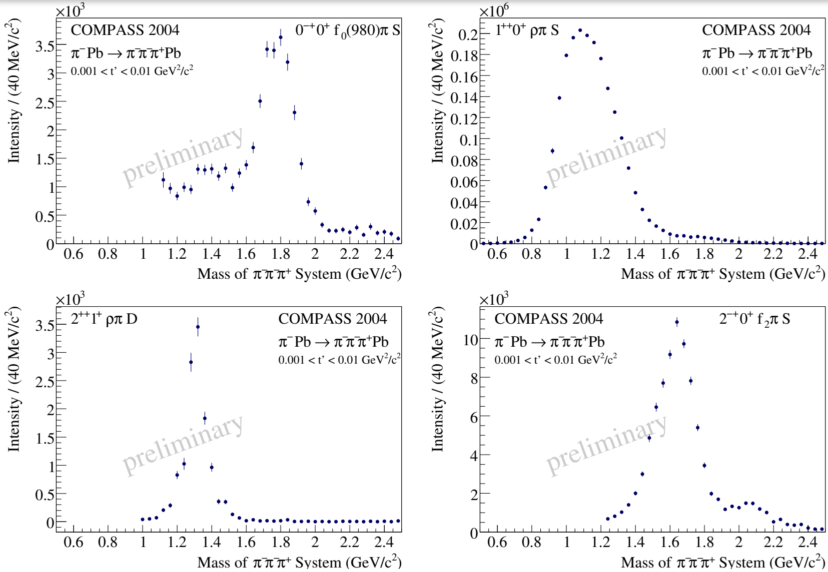
Intensity of selected waves: $0^{-+}0^{+} f_0(980)\pi S$, $1^{++}0^{+} \rho\pi S$, $2^{++}1^{+} \rho\pi D$, $2^{-+}0^{+} f_2(1270)\pi S$



Spin Totals for $t' < 10^{-3}\,(\mathrm{GeV}/c)^2$

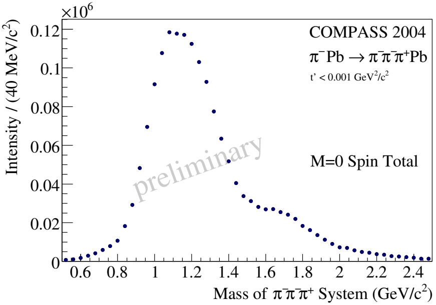
"Spin Totals": Sum of all contributions for given M (i.e. z-projection of J)

t'-dependent amplitudes:

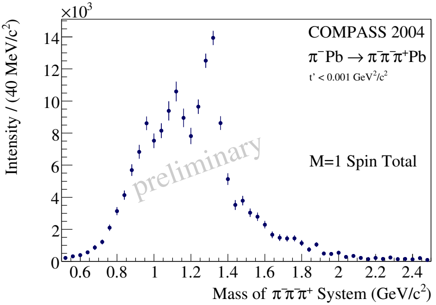
Primakoff production: M=1: $\sigma(t') \propto e^{-b_{\mathrm{Prim}} t'}$ $\rightarrow$ arises at $t' \approx 0$ (resoluted shape!)

Diffractive production: M=0: $\sigma(t') \propto e^{-b_{\mathrm{diff}}(m) t'}$

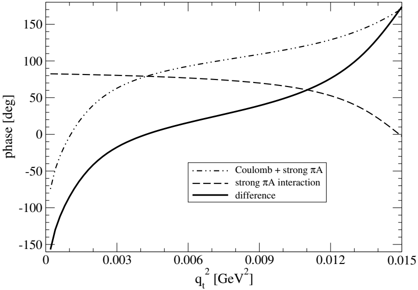
M=1: $\sigma(t') \propto t' e^{-b_{\mathrm{diff}}(m) t'}$ $\rightarrow$ vanishes for $t' \approx 0$

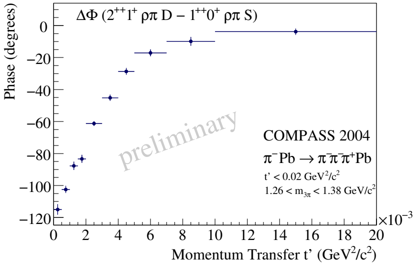


M=0



M=1





Glauber modell

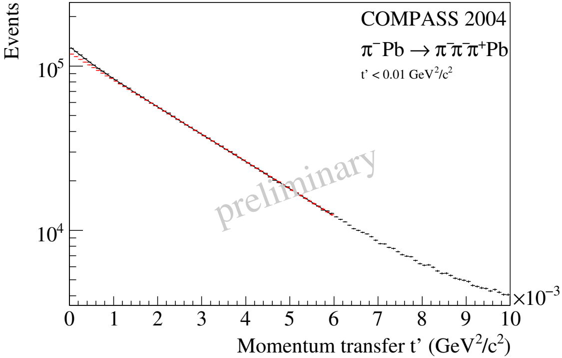
G. Fäldt and U. Tengblad, Phys. Rev. C79, 014607 (2009)

Plot: N. Kaiser (TU München)

$\Rightarrow$ indicates confirmation of interference Coulomb-interaction - strong interaction

$\Rightarrow$ detailed studies of the nature of resonances

Primakoff: $\sigma(t') \propto e^{-b_{\mathrm{Prim}} t'}$, $b_{\mathrm{Prim}} \approx 2000\,(\mathrm{GeV}/c)^{-2}$ (mainly resolution)
Diffractive: $\sigma(t') \propto e^{-b_{\mathrm{diff}} t'}$, $b_{\mathrm{diff}} \approx 400\,(\mathrm{GeV}/c)^{-2}$ for lead target



(Mass) spectrum of this Primakoff contribution?
⇒ Statistical subtraction of diffractive background (for bins of $m_{3\pi}$)