Pion-photon Reactions and Chiral Dynamics in Primakoff Processes at COMPASS

Jan M. Friedrich

Physik-Department, TU München

for the COMPASS collaboration

St. Petersburg 8.-12.9.2014
How to understand quark-gluon dynamics?

complicated system of interacting quarks and gluons

ChPT $\rightarrow$ effective degrees of freedom at low energy: mass, charge, spin, effective (self-)coupling
pion scattering lengths: 2-loop predictions
- \( a_0^0 m_\pi = 0.220 \pm 0.005 \) confirmed by E865 in \( K^+ \rightarrow \pi^+ \pi^- e^+ \nu_e \)
- \((a_0^0 - a_2^0)m_\pi = 0.264 \pm 0.006\) confirmed by NA48 in \( 0.268 \pm 0.010 \) \( K^+ \rightarrow \pi^+ \pi^0 \pi^0 \)

pion polarisability: electric \( \alpha_\pi \), magnetic \( \beta_\pi \)
- contribution to Compton scattering
- ChPT prediction obtained by the relation to \( \pi^+ \rightarrow e^+ \nu_e \gamma \) [Gasser, Ivanov, Sainio, Nucl. Phys. B745, 2006]

\[
\begin{align*}
\alpha_\pi + \beta_\pi &= (0.2 \pm 0.1) \cdot 10^{-4} \text{fm}^3 \\
\alpha_\pi - \beta_\pi &= (5.7 \pm 1.0) \cdot 10^{-4} \text{fm}^3 \\
\alpha_\pi &= (2.9 \pm 0.5) \cdot 10^{-4} \text{fm}^3
\end{align*}
\]

ChPT prediction contradicting the experimental findings (prior to our analysis)
More pion-photon reactions

- **Pion scattering including a real photon**
  - Leading-order prediction from ChPT
  - Pion scattering lengths combined with photon coupling
  - *chiral loop contribution*
    - Theory prediction available, no measurement

- **Chiral anomaly $F_{3\pi}$**
  - Established on 10% level
  - Further development: inclusion of the $\rho$ resonance
  - Theoretical work by Kubis, Hoferichter, Sakkas
  - PRD86(2012)116009
ChPT prediction for the pion polarisability

\[ \pi + \gamma \rightarrow \pi + \gamma \]

Compton cross-section contains information about e.m. polarisability (as deviation from the expectation for a pointlike particle).

Polarisabilities \( \alpha_\pi, \beta_\pi [10^{-4} \text{ fm}^3] \)

ChPT (2-loop) prediction:
\[ \alpha_\pi - \beta_\pi = 5.7 \pm 1.0 \quad \alpha_\pi + \beta_\pi = 0.16 \]

Experiments: 4 — 14 \((\beta_\pi \approx -\alpha_\pi \text{ assumed})\)
ChPT prediction for the pion polarisability

\( \pi + \gamma \rightarrow \pi + \gamma \)

Compton cross-section contains information about e.m. polarisability
(as deviation from the expectation for a pointlike particle)

Polarisabilities \( \alpha_\pi, \beta_\pi \) [\(10^{-4} \text{ fm}^3\)]

ChPT (2-loop) prediction: \( \alpha_\pi = 2.93, \beta_\pi = -2.77 \)

Experiments: 2 – 7

(\( \beta_\pi \approx -\alpha_\pi \) assumed)
Polarisability effect in Primakoff technique

- Charged pion traversing the nuclear electric field
  - typical field strength at $r = 5R_{Ni}$: $E \sim 300$ kV/fm

- Bremsstrahlung emission
  - particle scatters off equivalent photons
  - tiny momentum transfer $Q^2 \approx 10^{-5}$ GeV$^2$/c$^2$
  - pion/muon (quasi-)real Compton scattering

- Polarisability contribution
  - Compton cross-section typically diminished
  - expected charge separation $\sim 10^{-5}$ fm $\cdot$ e
Polarisability effect in Primakoff technique

- Charged pion traversing the nuclear electric field
  - typical field strength at

\[ E \sim 300 \text{kV/fm} \]

- Bremsstrahlung emission
  - particle scatters off equivalent photons
  - tiny momentum transfer
  \[ Q^2 \approx 10^{-5} \text{GeV}^2/c^2 \]

- pion/muon (quasi-)real Compton scattering
  - Polarisability contribution
  - Compton cross-section typically diminished
  - expected charge separation
  \[ \sim 10^{-5} \text{fm} \cdot e \]
Pion Compton Scattering

\[ \pi \gamma \rightarrow \pi \gamma \]

- Two kinematic variables, in CM: total energy \( \sqrt{s} \), scattering angle \( \theta_{cm} \)

\[
\frac{d\sigma_{\pi\gamma}}{d\Omega_{cm}} = \frac{\alpha^2 (s^2 z_+^2 + m_\pi^4 z_-^2)}{s(sz_+ + m_\pi^2 z_-)^2} - \frac{\alpha m_\pi^3 (s - m_\pi^2)^2}{4s^2(sz_+ + m_\pi^2 z_-)} \cdot \mathcal{P}
\]

\[
\mathcal{P} = z_-^2 (\alpha_\pi - \beta_\pi) + \frac{s^2}{m_\pi^4} z_+^2 (\alpha_\pi + \beta_\pi) - \frac{(s - m_\pi^2)^2}{24s} z_-^3 (\alpha_2 - \beta_2)
\]

\[ z_\pm = 1 \pm \cos \theta_{cm} \]

- \( \sigma_{tot}(s) \) rather insensitive to pion’s low-energy structure
- Up to 20% effect on backward angular distributions of \( d\sigma/d\Omega_{cm} \)
**Pion Compton Scattering**

\[
\pi \gamma \rightarrow \pi \gamma
\]

- Two kinematic variables, in CM: total energy \( \sqrt{s} \), scattering angle \( \theta_{cm} \)

\[
\frac{d\sigma_{\pi \gamma}}{d\Omega_{cm}} = \frac{\alpha^2 (s^2 z_+^2 + m_\pi^4 z_-^2)}{s (sz_+ + m_\pi^2 z_-)^2} - \frac{\alpha m_\pi^3 (s - m_\pi^2)^2}{4s^2 (sz_+ + m_\pi^2 z_-)} \cdot \mathcal{P}
\]

\[
\mathcal{P} = z_-^2 (\alpha_\pi - \beta_\pi) + \frac{s^2}{m_\pi^4} z_+^2 (\alpha_\pi + \beta_\pi) - \frac{(s - m_\pi^2)^2}{24s} z_-^3 (\alpha_2 - \beta_2)
\]

\[
z_{\pm} = 1 \pm \cos \theta_{cm}
\]

- \( \sigma_{tot}(s) \) rather insensitive to pion’s low-energy structure
- Up to 20% effect on *backward* angular distributions of \( d\sigma/d\Omega_{cm} \)
Pion Compton scattering: embedding the process

Primakoff processes

Radiative pion photoproduction

Photon-Photon fusion
Intro: Pions & ChPT

COMPASS

Pion polarisability

Resonances in $\pi^- \pi^- \pi^+$

Summary and Outlook

Pion polarisability: world data before COMPASS

Primakoff processes

Radiative pion photoproduction

Photon-Photon fusion

world avg.: $12.7 \pm 2.5$

GIS'06: ChPT prediction, Gasser, Ivanov, Sainio, NPB745 (2006), plots: T. Nagel, PhD

Fil'kouv analysis objected by Pasquini, Drechsel, Scherer PRC81, 029802 (2010)
COmmon Muon and Proton Apparatus for Structure and Spectroscopy
CERN SPS: protons $\sim 400$ GeV  
(5 – 10 sec spills)

- secondary $\pi, K, \bar{p}$: up to $2 \cdot 10^7 / s$
- tertiary muons: $4 \cdot 10^7 / s$
  2002-04, 2006-07, 2010-11: spin structure of the nucleon
**Fixed-target experiment**

- two-stage magnetic spectrometer
- high-precision, high-rate tracking, PID, calorimetry
Fixed-target experiment

- two-stage magnetic spectrometer
- high-precision, high-rate tracking, PID, calorimetry


- 190 GeV $\pi^-$ beam on $p$ and nuclear targets (C, Ni, W, Pb)
- Silicon microstrip detectors for “vertexing”
- recoil and (digital) ECAL triggers
Principle of the measurement
Extraction of the pion polarisability

- Identify exclusive reactions
  \[ \pi \gamma \{ \text{Ni} \rightarrow \text{Ni} \}' \rightarrow \pi \gamma \]
  at smallest momentum transfer \(< 0.001 \text{ GeV}^2/c^2\)

- Assuming \(\alpha_\pi + \beta_\pi = 0\), from the cross-section

\[
R = \frac{\sigma(x_\gamma)}{\sigma_{\alpha_\pi=0}(x_\gamma)} = \frac{N_{\text{meas}}(x_\gamma)}{N_{\text{sim}}(x_\gamma)} = 1 - \frac{3}{2} \cdot \frac{m_\pi^3}{\alpha} \cdot \frac{x_\gamma^2}{1 - x_\gamma} \alpha_\pi
\]

is derived, depending on \(x_\gamma = E_\gamma(\text{lab})/E_{\text{Beam}}\).
Measuring \(R\) the polarisability \(\alpha_\pi\) can be concluded.

- Control systematics by
  \[ \mu \gamma \{ \text{Ni} \rightarrow \text{Ni} \}' \rightarrow \mu \gamma \]
  and
  \[ K^- \rightarrow \pi^- \pi^0 \rightarrow \pi \gamma \gamma \]
Extraction of the pion polarisability

- Identify exclusive reactions
  \[ \pi \gamma \{ \text{Ni} \rightarrow \text{Ni}' \} \rightarrow \pi \gamma \]
  at smallest momentum transfer \(< 0.001 \text{ GeV}^2/c^2\)

- Assuming \(\alpha_\pi + \beta_\pi = 0\), from the cross-section
  \[
  R = \frac{\sigma(x_\gamma)}{\sigma_{\alpha\pi=0}(x_\gamma)} = \frac{N_{\text{meas}}(x_\gamma)}{N_{\text{sim}}(x_\gamma)} = 1 - \frac{3}{2} \cdot \frac{m_\pi^3}{\alpha} \cdot \frac{x_\gamma^2}{1 - x_\gamma} \alpha_\pi
  \]
  is derived, depending on \(x_\gamma = E_\gamma(\text{lab})/E_{\text{Beam}}\).
  Measuring \(R\) the polarisability \(\alpha_\pi\) can be concluded.

- Control systematics by
  \[ \mu \gamma \{ \text{Ni} \rightarrow \text{Ni}' \} \rightarrow \mu \gamma \]
  and
  \[ K^- \rightarrow \pi^- \pi^0 \rightarrow \pi \gamma \gamma \]
Identifying the $\pi\gamma \rightarrow \pi\gamma$ reaction

- Energy balance $\Delta E = E_\pi + E_\gamma - E_{\text{Beam}}$
- Exclusivity peak $\sigma \approx 2.6$ GeV (1.4%)
- $\sim 63,000$ exclusive events ($x_\gamma > 0.4$) (Serpukhov $\sim 7000$ for $x_\gamma > 0.5$)
ΔQ_T ≈ 12 MeV/c (190 GeV/c beam → requires few-μrad angular resolution)

- first diffractive minimum on Ni nucleus at Q ≈ 190 MeV/c
Coulomb-nuclear interference

Photon density squared form factor

- calculation following G. Fäldt (Phys. Rev. C79, 014607)
- eikonal approximation: pions traverse Coulomb and strong-interaction potentials
Primakoff peak: muon data

- muon control measurement: pure electromagnetic interaction
Photon energy spectra for muon and pion beam

![Graph showing photon energy spectra for muon and pion beam](image)

J. M. Friedrich — Chiral Dynamics with COMPASS

arxiv:1405.6377v1 subm. to PRL
Pion polarisability: COMPASS result

\[ \alpha_\pi = (2.0 \pm 0.6_{\text{stat}}) \times 10^{-4} \text{ fm}^3 \]

(assuming \( \alpha_\pi = -\beta_\pi \))

“false polarisability” from muon data:

\[ (0.5 \pm 0.5_{\text{stat}}) \times 10^{-4} \text{ fm}^3 \]

arxiv:1405.6377v1 subm. to PRL
Radiative corrections (Compton scattering part)

Intro: Pions & ChPT

COMPASS

Pion polarisability

Resonances in $\pi^-$ $\pi^- \pi^+$

Summary and Outlook


<table>
<thead>
<tr>
<th>source of systematic uncertainty</th>
<th>estimated magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>tracking</td>
<td>0.6</td>
</tr>
<tr>
<td>radiative corrections</td>
<td>0.3</td>
</tr>
<tr>
<td>background subtraction in $Q$</td>
<td>0.4</td>
</tr>
<tr>
<td>pion electron scattering</td>
<td>0.2</td>
</tr>
<tr>
<td>quadratic sum</td>
<td>0.8</td>
</tr>
</tbody>
</table>

COMPASS result for the pion polarisability:

$$\alpha_\pi = \left( 2.00 \pm 0.60_{\text{stat}} \pm 0.70_{\text{syst}} \right) \times 10^{-4} \text{fm}^3$$

with $\alpha_\pi = -\beta_\pi$ assumed.
<table>
<thead>
<tr>
<th>source of systematic uncertainty</th>
<th>estimated magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>tracking</td>
<td>0.6</td>
</tr>
<tr>
<td>radiative corrections</td>
<td>0.3</td>
</tr>
<tr>
<td>background subtraction in $Q$</td>
<td>0.4</td>
</tr>
<tr>
<td>pion electron scattering</td>
<td>0.2</td>
</tr>
<tr>
<td>quadratic sum</td>
<td>0.8</td>
</tr>
</tbody>
</table>

COMPASS result for the pion polarisability:

$$\alpha_\pi = (2.0 \pm 0.6_{\text{stat}} \pm 0.7_{\text{syst}}) \times 10^{-4} \text{ fm}^3$$

with $\alpha_\pi = -\beta_\pi$ assumed
The new COMPASS result is in significant tension with the earlier measurements of the pion polarisability.

The expectation from ChPT is confirmed within the uncertainties.
Pion polarisability measurements at COMPASS

**Primakoff pilot run 2004**
- ~1 week
- ~10k events
- 0.3X₀ Ni

**Primakoff run 2009**
- ~3 weeks
- ~63k events
- 0.3X₀ Ni

**Primakoff run 2012**
- ~3 months
- ~200–400k events
- 0.3X₀ Ni

**Eγ/Ebeam > 0.4**
Access to $\pi + \gamma$ reactions via the Primakoff effect:

At smallest momentum transfers to the nucleus, high-energetic particles scatter predominantly off the electromagnetic field quanta ($\sim Z^2$)

$$\pi^- + \gamma \rightarrow \begin{cases} 
\pi^- + \gamma \\
\pi^- + \pi^0 / \eta \\
\pi^- + \pi^0 + \pi^0 \\
\pi^- + \pi^- + \pi^+ \\
\pi^- + \pi^- + \pi^+ + \pi^- + \pi^+ \\
\pi^- + \ldots 
\end{cases}$$

analogously: Kaon-induced reactions $K^- + \gamma \rightarrow \ldots$
2004 Primakoff results

\[ \pi^- \text{ Pb} \rightarrow \text{ Pb} \pi^- \pi^- \pi^+ \]

- "Low \( t' \)": \( 10^{-3} \text{ (GeV/c)}^2 < t' < 10^{-2} \text{ (GeV/c)}^2 \) \(\sim 2 \, 000 \, 000 \) events
- "Primakoff region": \( t' < 10^{-3} \text{ (GeV/c)}^2 \) \(\sim 1 \, 000 \, 000 \) events

Ranges in \( t' \) (GeV\(^2/c^2\)):
- All \( t' \)
- \( t' < 10^{-3} \)
- \( 10^{-3} < t' < 10^{-2} \)
- \( 10^{-2} < t' < 10^{-1} \)
- \( 0.1 < t' < 1 \)
- \( t' > 1 \) [x5]
First Measurement of $\pi \gamma \rightarrow 3\pi$ Absolute Cross-Section

Measured absolute cross-section of $\pi^- \gamma \rightarrow \pi^- \pi^- \pi^+$

COMPASS 2004
$\pi^- \gamma \rightarrow \pi^- \pi^- \pi^+$
from $\pi^- \text{Pb} \rightarrow \pi^- \pi^- \pi^+ \text{Pb}$

Fitted ChPT Intensity
Leading Order ChPT Prediction

Full Systematic Error
Luminosity Uncertainty

published in PRL 108 (2012) 192001
Higher-order effects

Chiral loops, e.g.
(N. Kaiser, NPA848 (2010) 198)

not (yet) included:
2004 Primakoff results

\[ \pi^- \text{Pb} \rightarrow \text{Pb} \pi^- \pi^- \pi^+ \]

- "Low \( t' \)": \( 10^{-3} \text{(GeV/c)}^2 < t' < 10^{-2} \text{(GeV/c)}^2 \) \( \sim 2 \, \text{million events} \)
- "Primakoff region": \( t' < 10^{-3} \text{(GeV/c)}^2 \) \( \sim 1 \, \text{million events} \)
2004 Primakoff results

\[ \pi^- \text{Pb} \rightarrow \text{Pb} \pi^- \pi^- \pi^+ \]

PWA of a1(1260), a2(1320) contributions in t slices

- "Low \( t' \): \( 10^{-3} \text{(GeV/c)}^2 < t' < 10^{-2} \text{(GeV/c)}^2 \) \( \sim 2 \times 10^6 \) events
- "Primakoff region": \( t' < 10^{-3} \text{(GeV/c)}^2 \) \( \sim 1 \times 10^6 \) events
PWA: $a_1$, $a_2$ and $\Delta \Phi$ in separated $t'$ regions

COMPASS 2004

$\pi \text{Pb} \rightarrow \pi \pi \pi^+ \text{Pb}$

$0.0015 < t' < 0.01 \text{ GeV}^2/c^2$

$t' < 0.0005 \text{ GeV}^2/c^2$

$1^{++0^+} \rho \pi S$

$2^{++1^+} \rho \pi D$
Radiative Coupling of $a_2(1320)$ and $\pi_2(1670)$

$\Gamma_0(a_2(1320) \rightarrow \pi\gamma)$ $M2$

$\Gamma_0(\pi_2(1670) \rightarrow \pi\gamma)$ $E2$

 ↔ meson w.f.'s: $\Gamma_{i \rightarrow f} \propto |\langle \Psi_f | e^{-i\vec{q} \cdot \hat{r}} \hat{c} \cdot \vec{p} | \Psi_i \rangle |^2$, VMD

- normalization via beam kaon decays
- large Coulomb correction

*published in EPJ A50 (2014) 79*
Summary and Outlook

- **Measurement of the pion polarisability at COMPASS**
  - Via the Primakoff reaction, COMPASS has determined
    \[ \alpha_\pi = (2.0 \pm 0.6_{\text{stat}} \pm 0.7_{\text{syst}}) \times 10^{-4} \text{ fm}^3 \]
    assuming \( \alpha_\pi + \beta_\pi = 0 \)
  - Most precise experimental determination
  - Systematic control: \( \mu \gamma \rightarrow \mu \gamma, \ K^- \rightarrow \pi^- \pi^0 \)

- **Chiral dynamics** in \( \pi^- \gamma \rightarrow \pi^- \pi^0 \) and \( \pi \gamma \rightarrow \pi \pi \pi \) reactions
  - Charged-channel \( \pi \gamma \rightarrow \pi^- \pi^- \pi^+ \) tree-level ChPT prediction confirmed
  - Neutral-channel \( \pi \gamma \rightarrow \pi^- \pi^0 \pi^0 \) analysis ongoing
  - Resonance properties, radiative couplings

- **High-statistics run 2012**
  - separate determination of \( \alpha_\pi \) and \( \beta_\pi \)
  - \( s \)-dependent quadrupole polarisabilities
  - First measurement of the kaon polarisability
Phase $a_2 - a_1$ in detail: $t'$ dependence

- transition of $\pi\gamma$ to $\pi IP \rightarrow a_2$ production
- work in progress
- interference can be used to map details of resonances and production mechanisms
Polarisability effect (LO ChPT values)

\[ \cos^{-1}(\theta) \]

\[ b \mu \]

\[ CM \]

\[ \Omega / \sigma_d \]

0.02 0.1 0.2 0.3 0.4

< 20 GeV

Primakoff

\[ E_\gamma \]

\[ < 20 \text{ GeV} \]

\[ \frac{d\sigma}{d\Omega_{cm}} \text{ [\mu b]} \]

\[ s=3m_\pi^2 \]

\[ s=5m_\pi^2 \]

\[ s=8m_\pi^2 \]

\[ s=15m_\pi^2 \]

\[ \alpha_\pi = 3.00, \beta_\pi = -3.00 \]

\[ \cos \theta_{cm} \]
Polarisability effect (NLO ChPT values)

Loop effects not shown

\[ \frac{d\sigma}{d\Omega_{\text{cm}}} \]
Polarisability effect (wrong sign $\alpha_\pi + \beta_\pi$)

\[
\cos \theta_{\text{cm}} \quad \begin{array}{c}
\begin{array}{cccccc}
& 0 & 0.2 & 0.4 & 0.6 & 0.8 & 1
\end{array}
\end{array}
\]

\[
\begin{array}{c}
\begin{array}{cccccc}
0.02 & 0.04 & 0.06 & 0.08 & 0.1 & 0.2 & 0.3 & 0.4
\end{array}
\end{array}
\]

$\frac{d\sigma}{d\Omega_{\text{cm}}} \, [\mu b]$

$s=3m^2_\pi$
$s=5m^2_\pi$
$s=8m^2_\pi$
$s=15m^2_\pi$

$\sim \alpha_\pi = 3.00, \beta_\pi = -3.14$

- Loop effects not shown
- $E_\gamma < 20$ GeV

Primakoff

J. M. Friedrich — Chiral Dynamics with COMPASS
Polarisability effect (Serpukhov values)

\[ \cos^{-1} \theta \]

\[ \cos \theta \text{ cm} \]

loop effects not shown

\[ d\sigma / d\Omega_{\text{cm}} [\mu b] \]

- - - \( \alpha_\pi = 6.10, \beta_\pi = -6.10 \)

\[ E_\gamma < 20 \text{ GeV} \]

\[ s = 3m_\pi^2 \]

\[ s = 5m_\pi^2 \]

\[ s = 8m_\pi^2 \]

\[ s = 15m_\pi^2 \]
• Radiative $\pi^+$ production on the proton:

$$\gamma \pi^* \rightarrow \pi \gamma \quad [\text{via } \gamma p \rightarrow n \pi^+ \gamma]$$

Mainz (2005) measurement: $\alpha_\pi - \beta_\pi = 11.6 \pm 1.5 \pm 3.0 \pm 0.5$

"$\pm 0.5$": model error only within the used ansatz, full systematics not under control

• Primakoff Compton reaction:

$$\gamma^* \pi \rightarrow \pi \gamma \quad [\text{via } \pi Z \rightarrow Z \pi \gamma]$$

tiny extrapolation $\gamma^* \rightarrow \gamma \ O(10^{-3} m^2_\pi)$

fully under theoretical control

Minimum transverse momentum of the charged particle

\[ p_T \] vs. \[ \text{counts / 2.5 MeV/c} \]

\[ \pi^- \text{Ni} \rightarrow \pi^- \gamma \text{Ni} \]

- Data
- Simulation (normalised)
CM energy in $\pi\gamma \rightarrow \pi\gamma$

$\rho$ contribution from $\pi\gamma \rightarrow \pi\pi^0$
\( \rho \) contribution from \( \pi \gamma \rightarrow \pi \pi^0 \)
Mandelstam $\{s,t\} \leftrightarrow \text{Laboratory } \{E_\gamma, \theta_\gamma\}$
for $\pi \gamma \rightarrow \pi \gamma$
M.R. Pennington in the 2nd DAΦNE Physics Handbook, “What we learn by measuring $\gamma\gamma \rightarrow \pi\pi$ at DAΦNE”:

All this means that the only way to measure the pion polarisabilities is in the Compton scattering process near threshold and not in $\gamma\gamma \rightarrow \pi\pi$. Though the low energy $\gamma\gamma \rightarrow \pi\pi$ scattering is seemingly close to the Compton threshold (...) and so the \textit{extrapolation} not very far, the dominance of the pion pole (...) means that the energy scale for this continuation is $m_\pi$. Thus the polarisabilities cannot be determined accurately from $\gamma\gamma$ experiments in a model-independent way and must be measured in the Compton scattering region.
No evidence for $a_1(1260) \rightarrow \pi\gamma$
Mass-independent PWA (narrow mass bins):

\[ \sigma_{\text{indep}}(\tau, m, t') = \sum_{\varepsilon=\pm 1} \sum_{r=1}^{N_R} |T_{ir}^\varepsilon f_i^\varepsilon(t') \psi_i^\varepsilon(\tau, m)|^2 \left| \int |f_i^\varepsilon(t')|^2 dt' \int |\psi_i^\varepsilon(\tau', m)|^2 d\tau' \right|^2 \]

- Production strength assumed constant in single bins
- Decay amplitudes \( \psi_i^\varepsilon(\tau, m) \), with \( t' \) dependence \( f_i^\varepsilon(t') \)
- Production amplitudes \( T_{ir}^\varepsilon \rightarrow \) Extended log-likelihood fit
- Acceptance corrections included

Spin-density matrix: \( \rho_{ij}^\varepsilon = \sum_r T_{ir}^\varepsilon T_{jr}^\varepsilon^* \)

→ Physical parameters:

\[ \text{Intens}^\varepsilon_i = \rho_{ii}^\varepsilon, \]

relative phase \( \Phi_{ij}^\varepsilon \)

\[ \text{Coh}_{i,j}^\varepsilon = \sqrt{\left( \Re \rho_{ij}^\varepsilon \right)^2 + \left( \Im \rho_{ij}^\varepsilon \right)^2} / \sqrt{\rho_{ii}^\varepsilon \rho_{jj}^\varepsilon} \]

Mass-dependent \( \chi^2 \)-fit (not presented here):

- \( X \) parameterized by Breit-Wigner (BW) functions
- Background can be added
Mass dependence of the diffractive slope

COMPASS 2004

$\pi^{-} \text{Pb} \rightarrow \pi^{-} \pi^{+} \pi^{-} \text{Pb}$

Diffractive slope $b_{\text{diff}}$ vs. Mass of $\pi^{-} \pi^{+} \pi^{-}$ system (GeV/c$^2$)
Isobar model:
Intermediate 2-particle decays

Partial wave in reflectivity basis:
\[ J^{PC} M^\epsilon [isobar] L \]

Mass-independent PWA (40 MeV/c^2 mass bins): 38 waves
Fit of angular dependence of partial waves, interferences

Mass-dependent \( \chi^2 \)-fit (Not presented here)
Major intensities in $m(3\pi)$-bins (acceptance corrected)

COMPASS 2004
$\pi^+\text{Pb} \rightarrow \pi\pi\pi^+\text{Pb}$
$t' < 0.001 \text{ GeV}^2/c^2$

$M=0$ Spin Total

$M=1$ Spin Total

$a_1(1260)$
$1^{++}0^+ \rho\pi S$

$a_2(1320)$

$2^{++}1$ Spin Total
PWA of data with low $t'$

Intensity of selected waves: $0^{-+} 0^+ f_0(980) \pi S$, $1^{++} 0^+ \rho \pi S$, $2^{++} 1^+ \rho \pi D$, $2^{-+} 0^+ f_2(1270) \pi S$
"Spin Totals": Sum of all contributions for given M (i.e. z-projection of J)

\( t' \)-dependent amplitudes:

Primakoff production: \( M=1: \sigma(t') \propto e^{-b_{\text{Prim}}t'} \rightarrow \text{arises at } t' \approx 0 \) (resolved shape!)

Diffractive production: \( M=0: \sigma(t') \propto e^{-b_{\text{diff}}(m)t'} \)

\( M=1: \sigma(t') \propto t' e^{-b_{\text{diff}}(m)t'} \rightarrow \text{vanishes for } t' \approx 0 \)
Theory: Phase \( a_2 \text{(strong+Coulomb)} - a_1 \text{(strong)} \)

![Graph showing phase vs. \( q_t^2 \) with Coulomb and strong \( \pi A \) interaction and difference.

Glauber model


Plot: N. Kaiser (TU München)

⇒ indicates confirmation of interference Coulomb-interaction - strong interaction
⇒ detailed studies of the nature of resonances
Primakoff contribution at \( t' < 10^{-3} \text{ (GeV/c)}^2 \)

Primakoff: \( \sigma(t') \propto e^{-b_{\text{Prim}}t'} \), \( b_{\text{Prim}} \approx 2000 \text{ (GeV/c)}^{-2} \) (mainly resolution)

Diffractive: \( \sigma(t') \propto e^{-b_{\text{diff}}t'} \), \( b_{\text{diff}} \approx 400 \text{ (GeV/c)}^{-2} \) for lead target

(Mass) spectrum of this Primakoff contribution?
⇒ Statistical subtraction of diffractive background (for bins of \( m_{3\pi} \))