

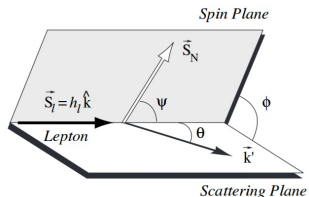
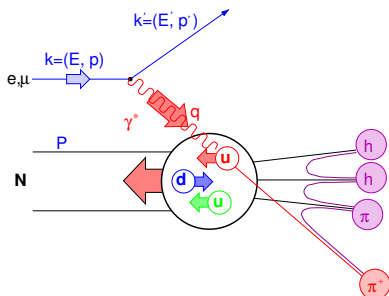
Polarised low x physics presently and on planned e-p machines

Barbara Badelek
University of Warsaw

LOW x

Kyoto, June 17 – June 21, 2014

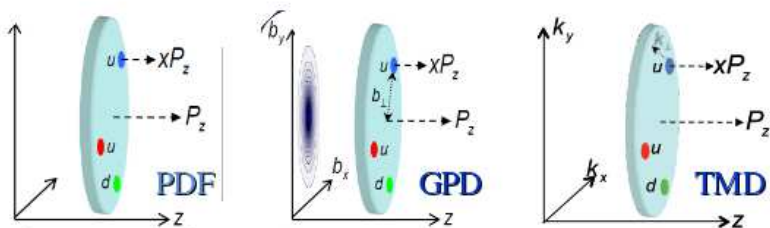
Nucleon spin structure in the electroproduction



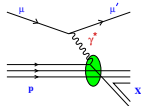
- $\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2}{2Mq^4} \frac{E'}{E} L_{\mu\nu} W^{\mu\nu}$
- Symmetric part of $W^{\mu\nu}$ – unpol. DIS, antisymmetric – polarised DIS
- Nominally $F_{1,2}$, $q(x) \rightarrow g_{1,2}$, $\Delta q(x)$ but...
- ...anomalous gluon contribution to $g_1(x)$
- ... $g_2(x)$ has no interpretation in terms of partons.

$$\sigma = \bar{\sigma} - \frac{1}{2} h_l (\cos \psi \Delta\sigma_{\parallel} + \sin \psi \cos \phi \Delta\sigma_{\perp})$$

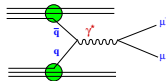
Transverse Momentum Dependent (TMD) distributions



- parton intrinsic k_T taken into account
- related to quark angular momentum, L !
- at COMPASS studied in 2 ways:
 - semi-inclusive DIS (polarised muons on unpolarised/transversely polarised target)
 - **In the future:** Drell-Yan process (π beam on unpolarised/transversely polarised tgt.)



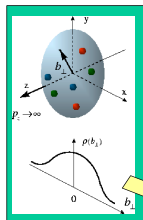
SIDIS



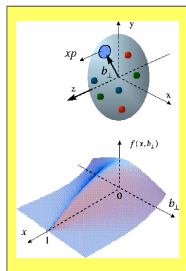
DY

3D picturing of the proton *via* GPD

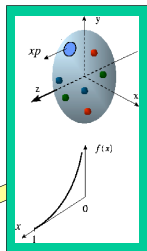
D. Mueller, X. Ji, A. Radyushkin, A. Belitsky, ...
M. Burkardt, ... Interpretation in impact parameter space



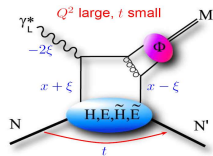
Proton form factors,
transverse charge &
current densities



Correlated quark momentum
and helicity distributions in
transverse space - **GPDs**

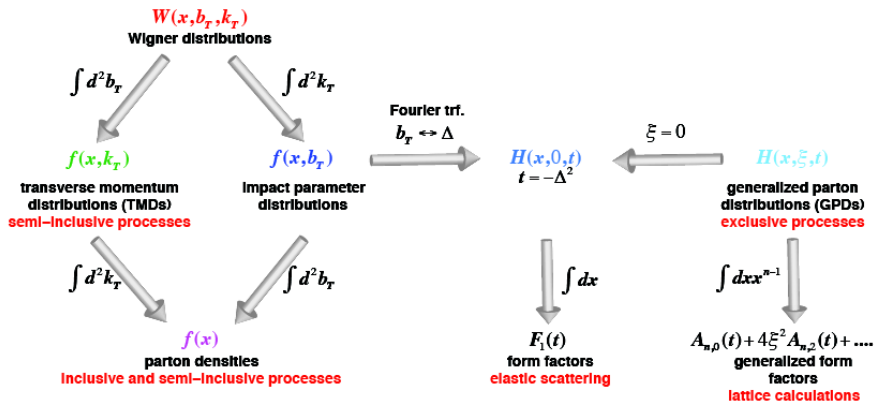


Structure functions,
quark **longitudinal**
momentum & helicity
distributions



- Four GPDs (H , E , \tilde{H} , \tilde{E}) for each flavour and for gluons
- All depend on 3 variables: x , ξ , t ; DIS @ $\xi = t = 0$
- H , \tilde{H} conserve nucleon helicity; E , \tilde{E} flip nucleon helicity
- H , E refer to unpolarised distributions; \tilde{H} , \tilde{E} refer to polarised distr.

Descriptions of pdf^s in the nucleon



From "White paper", arXiv:1212.1701

Partonic structure of the nucleon; distribution functions



- In LT and considering k_T , 8 PDF describe the nucleon
- QCD-TMD approach valid $k_T \ll \sqrt{Q^2}$
- After integrating over k_T only 3 survive: f_1, g_1, h_1
- TMD accessed in SIDIS and DY by measuring azimuthal asymmetries
- SIDIS: e.g. $A_{\text{Sivers}} \propto \text{PDF} \otimes \text{FF}$
- DY: e.g. $A_{\text{Sivers}} \propto \text{PDF}^{\text{beam}} \otimes \text{PDF}^{\text{target}}$
- OBS! Boer-Mulders and Sivers PDF are T-odd, i.e. process dependent

$$h_1^\perp(\text{SIDIS}) = -h_1^\perp(\text{DY})$$

$$f_{1T}^\perp(\text{SIDIS}) = -f_{1T}^\perp(\text{DY})$$

- OBS! transversity PDF is chiral-odd (may only be measured with another chiral-odd partner, e.g. fragmentation function).
- Boer-Mulders, Sivers and transversity ($h_1^\perp, f_{1T}^\perp, h_1$) will be measured in COMPASS II

		NUCLEON		
		unpolarized	longitudinally pol.	transversely pol.
QUARK	unpolarized	f_1 number density		f_{1T}^\perp Sivers
	longitudinally pol.		g_{1L} helicity	g_{1T} helicity
	transversely pol.	h_1^\perp Boer-Mulders		h_1 transversity
	longitudinally pol.		h_{1L}^\perp helicity	h_{1T}^\perp pretzelocity

Nucleon spin structure: observables in $\vec{\mu}\vec{N}$ scattering

- Inclusive asymmetry, A_{meas} :

$$A_{meas} = \frac{1}{fP_T P_B} \left(\frac{N^{\leftarrow} - N^{\leftarrow}}{N^{\leftarrow} + N^{\leftarrow}} \right) \approx DA_1 = D \frac{g_1(x, Q^2)}{F_1(x, Q^2)} = D \frac{\sum_q e_q^2 \Delta q(x, Q^2)}{\sum_q e_q^2 q(x, Q^2)}$$

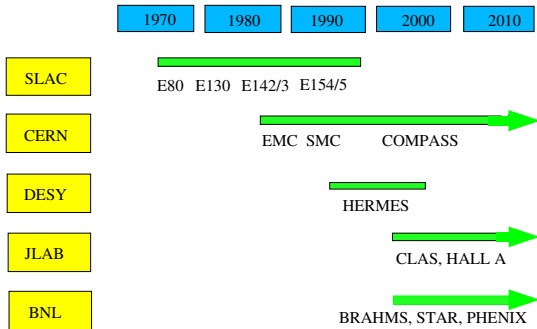
$$\Delta q = q^+ - q^-, \quad q = q^+ + q^-, \quad g_1^d = g_1^N \left(1 - \frac{3}{2}\omega_D\right) = \frac{g_1^p + g_1^n}{2} \left(1 - \frac{3}{2}\omega_D\right);$$

$$\omega_D = 0.05 \pm 0.01$$

- At LO, semi-inclusive asymmetry, A_1^h :

$$A_1^h(x, z, Q^2) \approx \frac{\sum_q e_q^2 \Delta q(x, Q^2) D_q^h(z, Q^2)}{\sum_q e_q^2 q(x, Q^2) D_q^h(z, Q^2)} \quad z = \frac{E_h}{\nu} \quad D_q^h \neq D_{\bar{q}}^h$$

Experiments



Experiment	Polarised beam	Polarised target	Energy (GeV)
SLAC	e	p, n, d	$\lesssim 50$
CERN/EMC	μ	p	100–200
CERN/SMC	μ	p, d	100, 190
DESY/HERMES	e	p, n, d	27.5
CERN/COMPASS	μ	p, d	160, 200
JLAB	e	p, n, d	$\lesssim 6$
BNL/RHIC	p	p	$\lesssim 250+250$

Acceptance of high energy electroproduction experiments

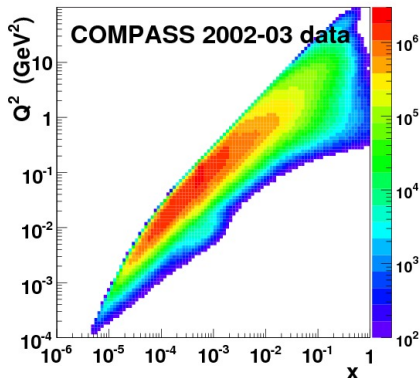
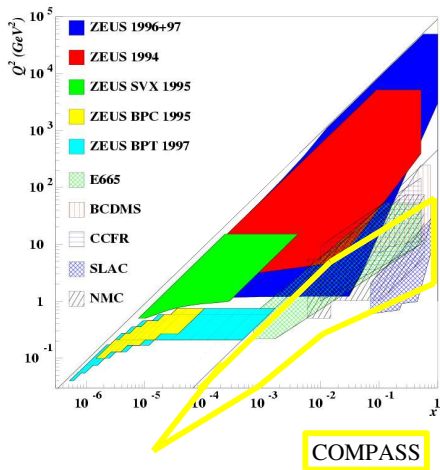
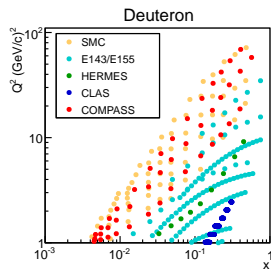
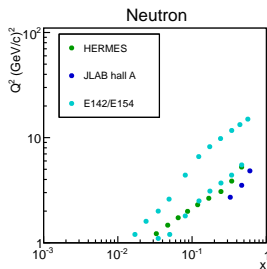
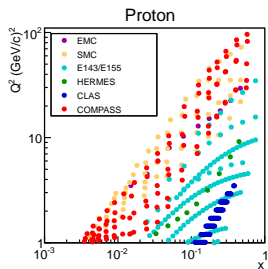


Figure from: N. D'Hose, Villars 2004

Acceptance of spin experiments: different targets

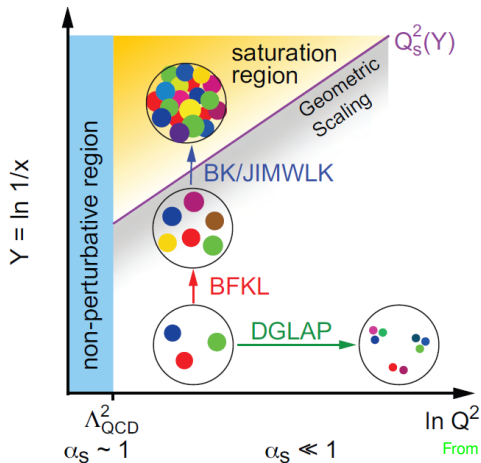
$$Q^2 > 1 \text{ GeV}^2$$



From M. Wilfert, DIS2014

Why is low x needed?

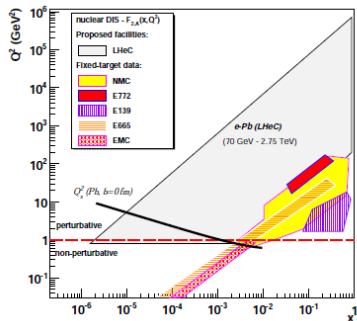
- Manifestation of new dynamics
- Problem with truncated sum rules...



From "White paper" arXiv:1212.1701

A Large Hadron Electron Collider (LHeC) at CERN

- Symposium on the European Strategy for Particle Physics, Cracow, 2012
arXiv:12111.483
- Two options: ring–ring (RR) and linac–ring (LR). Basic beam design:



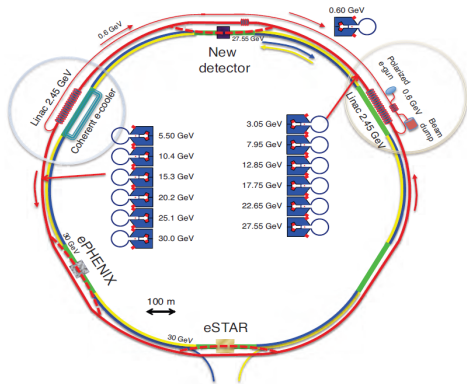
electron beam 60 GeV	Ring	Linac
e^- (e^+) per bunch N_e [10^{11}]	20 (20)	1 (0.1)
e^- (e^+) polarisation [%]	40 (40)	90 (0)
bunch length [mm]	6	0.6
tr. emittance at IP $\gamma e_{x,y}^e$ [mm]	0.59, 0.29	0.05
IP β function $\beta_{x,y}^*$ [m]	0.4, 0.2	0.12
beam current [mA]	100	6.6
energy recovery efficiency [%]	–	94
proton beam 7 TeV		
protons per bunch N_p [10^{11}]	1.7	1.7
transverse emittance $\gamma e_{x,y}^p$ [μm]	3.75	3.75
collider		
Lum e^-p (e^+p) [$10^{32}\text{cm}^{-2}\text{s}^{-1}$]	9 (9)	10 (1)
bunch spacing [ns]	25	25
rms beam spot size $\sigma_{x,y}$ [μm]	45, 22	7
crossing angle θ [mrad]	1	0
$L_{eN} = A L_{eA}$ [$10^{32}\text{cm}^{-2}\text{s}^{-1}$]	0.45	1

- The “Strategy” has not recommended a continuation of R&D for LHeC!

e-p machine, EIC, planned at BNL or JLab

BNL

Electron beam facility needed
(inside RHIC tunnel)



JLab

ELIC + injector needed

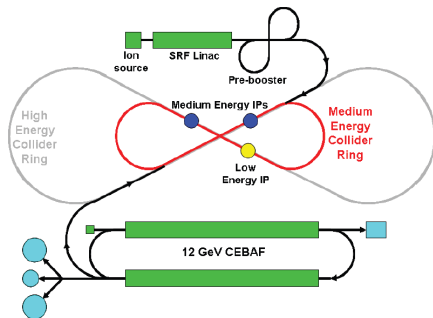
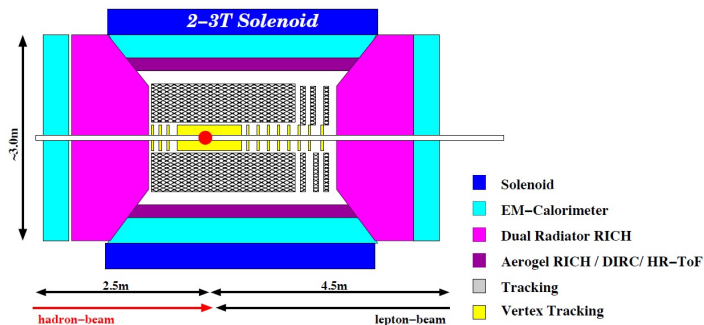


figure from The White Paper, arXiv:1212.1701



A dedicated EIC detector



- Acceptance $-5 < \eta < 5$ (large, comparable to CMS forward)
- PID: π , K, p, leptons
- Low material density (minimal multiple scattering and bremsstrahlung)
- Hadron beams: proton to lead

From "White paper", arXiv:1212.1701

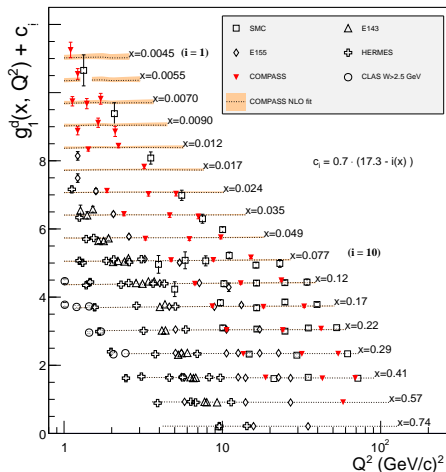
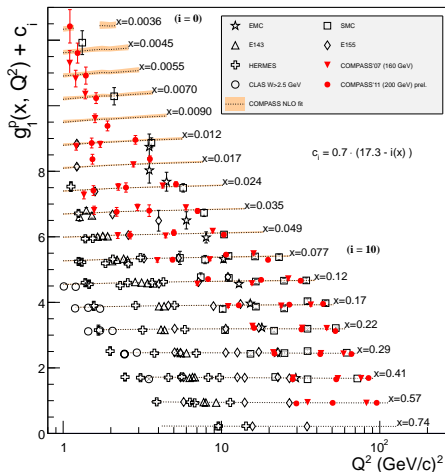
EIC: main features

- Highly polarised ($\sim 70\%$) e, N beams
- ions from deuteron to uranium (lead ?)
- variable \sqrt{s} from ~ 20 GeV to ~ 100 (150) GeV
- high luminosity: $\sim 10^{33-34} \text{ cm}^{-2} \text{ s}^{-1}$ (cooling of hadronic beam !)
- more than one interaction region
- limits of current technology \implies R & D!
- staged realisation; first stage: $\sqrt{s} = 60 - 100$ GeV and high luminosity.

Perturbative ($Q^2 > 1 \text{ GeV}^2$) region

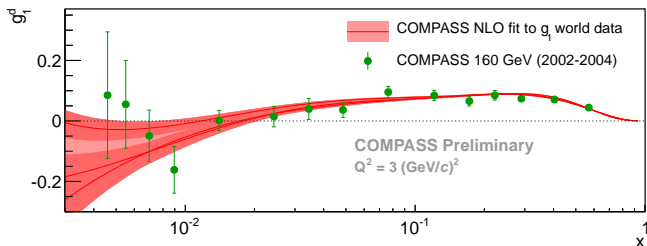
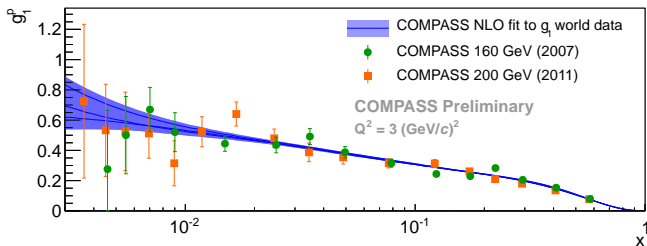
$g_1^p(x)$ and $g_1^d(x)$

NEW: COMPASS proton data 2011 (prelim.); full deuteron statistics



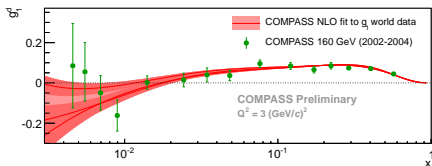
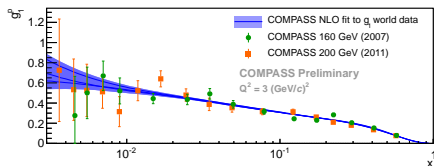
COMPASS measurements at high Q^2 important for the QCD analysis! but little sensitive to Δg

COMPASS NLO fit to g_1 world data; $Q^2 = 3 \text{ (GeV/c)}^2$



DGLAP at low x

NLO QCD analysis of g_1^p and g_1^d world data:



At LO

$$g_1(x, Q^2) \sim \exp \left[A \sqrt{\xi(Q^2) \ln(1/x)} \right], \quad \xi(Q^2) = \int_{\mu_0^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi}, \quad A_s \neq A_{n_s}$$

A possible singular low x behaviour of gluon and sea distributions may originate from

- parametrisation of the starting distributions at $Q_0^2 \sim 4 \text{ GeV}^2$
- Evolution starting from non-singular “valence-like” parton distributions at very low scale, $\mu_0 \sim 0.35 \text{ GeV}^2$

$\ln^2(1/x)$ corrections to $g_1^{ns}(x, Q^2)$

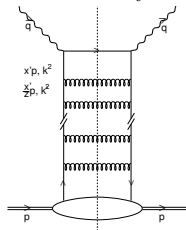
- Leading low x behaviour of g_1 (g_1^s and g_1^{ns}) generated by powers of $\alpha_s \ln^2(1/x)$; a standard DGLAP for spin dependent pdf generate only $\ln(1/x)$ terms.

- A way of including the above to QCD evolution: through $f(x, k_t^2)$

where conventional parton distributions:
$$p(x, Q^2) = \int^{Q^2} \frac{dk_t^2}{k_t^2} f(x, k_t^2)$$

This formalism permits
an easy extrapolation to $Q^2 = 0$
(for fixed W^2).

- $\ln^2(1/x)$ corrections to g_1^{ns}
are generated by ladder diagrams \implies



$\ln^2(1/x)$ corrections to $g_1^{ns}(x, Q^2)$... cont'd

- $\ln^2(1/x)$ corrections to g_1^{ns} are generated mathematically by equation:

$$f(x', k) = f^{(0)}(x', k) + \bar{\alpha}_s(k^2) \int_{x'}^1 \frac{dz}{z} \int_{k_0^2}^{k^2/z} \frac{dk'^2}{k'^2} f\left(\frac{x'}{z}, k'^2\right)$$

and

$$g_1^{ns}(x, Q^2) = g_1^{ns(0)}(x) + \int_{k_0^2}^{W^2} \frac{dk^2}{k^2} f(x' = x(1 + \frac{k^2}{Q^2}), k^2)$$

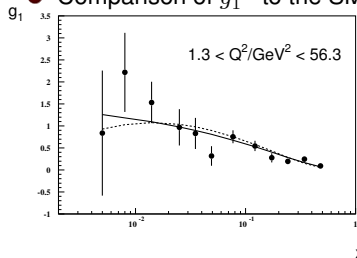
where $\bar{\alpha}_s(k^2) = 2\alpha_s(k^2)/3\pi$ and

$g_1^{(0)}(x)$ is a nonperturbative part, corresponding to $k^2 < k_0^2$.

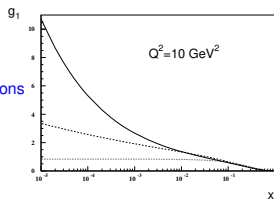
- $\ln^2(1/x)$ terms originate from the z -dependent limit of the $\int dk'^2/k'^2$ and x -dependent limit in $W^2(x)$.
- For fixed (i.e. non-running) $\bar{\alpha}_s(k^2) \rightarrow \tilde{\alpha}_s$, small x behaviour is $g_1^{ns}(x, Q^2) \sim x^{-\lambda}$ where $\lambda = 2\sqrt{\tilde{\alpha}_s}$

$\ln^2(1/x)$ corrections to $g_1^{ns}(x, Q^2)$... cont'd

- A unified equation which incorporates the complete LO DGLAP at finite x and $\ln^2(1/x)$ effects at $x \rightarrow 0$ was formulated.
- Potentially large $\ln Q^2$ and $\ln(1/x)$ treated on equal footing.
- Assumed values $g_1^{ns(0)} = 2g_A(1-x)^3/3$, $g_A = 1.257$ (axial vector coupling). At $x \rightarrow 0$, $g_1^{ns(0)} \rightarrow \text{const}$, in agreement with the Regge expectation.
- The $g_1^{ns(0)}$ satisfies the Bjorken sum rule at LO: $\int_0^1 dx g_1^{ns(0)}(x) = g_A/6$.
- Parameter $k_0^2 = 1 \text{ GeV}^2$.
- Comparison of g_1^{ns} to the SMC data (COMPASS results ready soon)

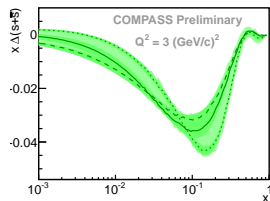
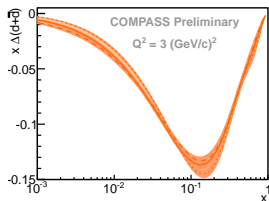
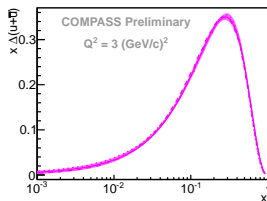
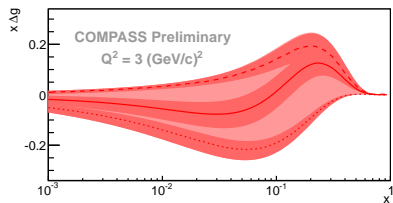
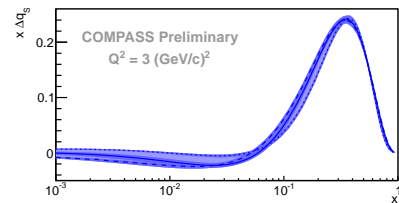


g_1^{ns} vs x
continuous – full calculations
broken – LO DGLAP



From J. Kwieciński and BB, Phys.Lett. B418 (1998) 229

COMPASS NLO fit to g_1 world data... cont'd



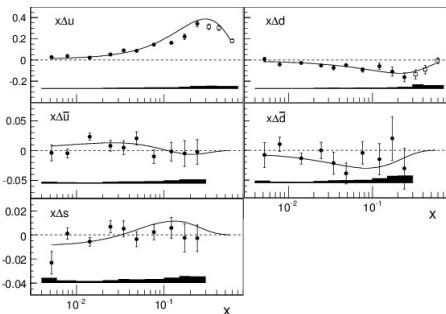
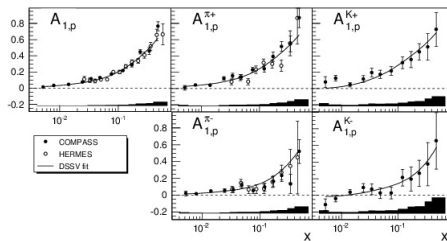
From M. Wilfert, DIS2014

Semi-inclusive asymmetries and parton distributions

- Measured on both proton and deuteron targets
- for identified, positive and negative pions and (for the first time) kaons

COMPASS, Phys. Lett. B **680** (2009) 217

DSSV, Phys. Rev. D **80** (2009) 034030



- LO DSS fragmentation functions and LO unpolarised MRST pdf assumed here.
- NLO parameterisation of DSSV describes the data well.

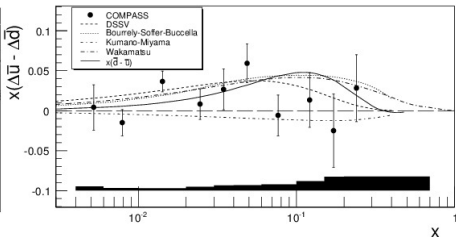
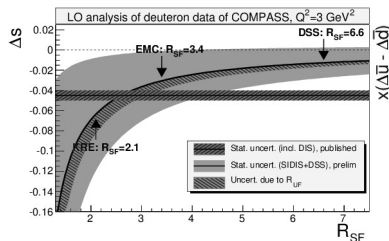
Polarisation of quark sea

- Δs puzzle. Strange quark polarisation:

$2\Delta S = \int_0^1 (\Delta s(x) + \Delta \bar{s}(x)) dx = -0.09 \pm 0.01 \pm 0.02$ from incl. asymmetries + SU_3 ,
while from semi-inclusive asymmetries it is compatible with zero

but depends upon chosen fragmentation functions. **Most critical:** $R_{SF} = \frac{\int D_{\bar{s}}^{K^+}(z) dz}{\int D_u^{K^+}(z) dz}$

\Rightarrow plan to extract it from COMPASS data on multiplicities.

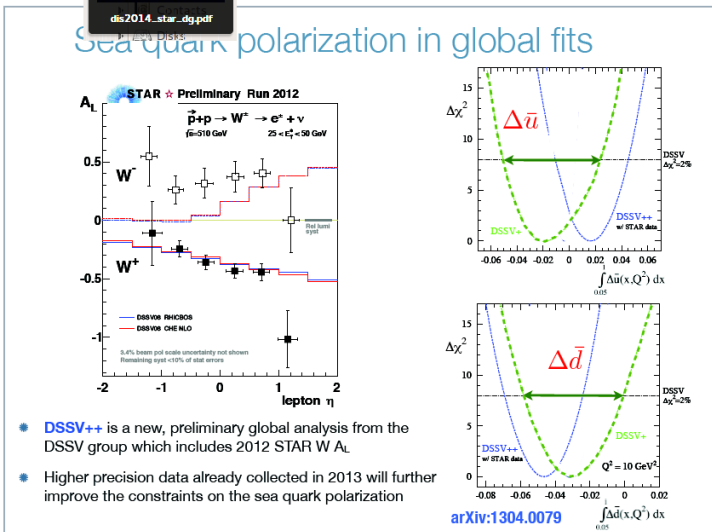


- The sea is not unsymmetric: COMPASS, *Phys. Lett. B*, **680** (2009) 217; *ibid.*, **693** (2010) 227.

$$\int_{0.004}^{0.3} [\Delta \bar{u}(x, Q^2) - \Delta \bar{d}(x, Q^2)] dx = 0.06 \pm 0.04 \pm 0.02 \text{ @ } Q^2 = 3 \text{ (GeV/c)}^2$$

Thus the data disfavour models predicting $\Delta \bar{u} - \Delta \bar{d} \gg \bar{d} - \bar{u}$

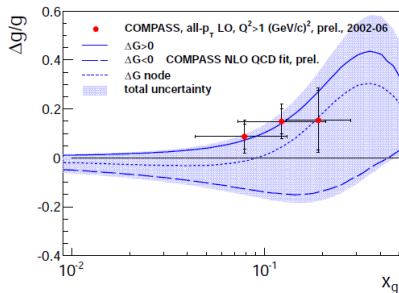
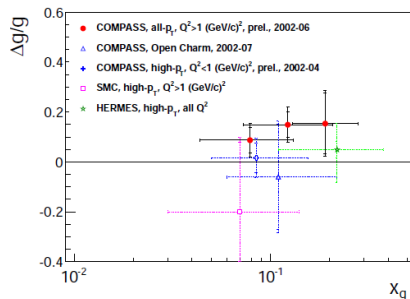
RHIC results on sea polarization and on Δg



QCD Frontier 2013: 10.21.13

Justin Stevens, III 19

New results on Δg from COMPASS



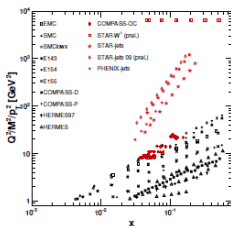
From M.Stolarski, DIS2014

Other QCD fits to world data

NNPDF fits: only DIS data (no SIDIS); recently included RHIC data

DSSV fits: DIS + SIDIS (fragmentation functions!)

Old and new experimental data set



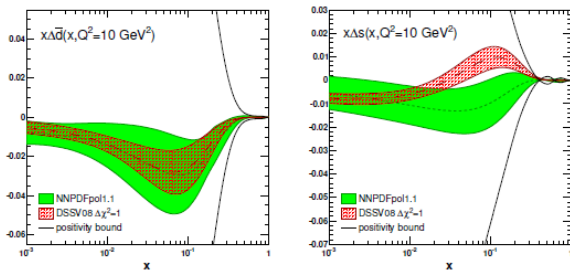
- Include the experimental information from:
 - jet and W production data (collider)
 - STAR, PHENIX
- Limited kinematic coverage at low- x
- Large Q^2 reached by collider data
- New data sets are included in NNPDFp11.0 (DIS-only fit) via Bayesian reweighting

REACTION	PARTONIC SUBPROCESS	PDF PROBED	x	Q^2 [GeV 2]
$e^\pm \{p, d, n\} \rightarrow e^\pm X$	$\gamma^* q \rightarrow q$	$\Delta q + \Delta \bar{q}$ Δg	$0.003 \lesssim x \lesssim 0.8$	$1 \lesssim Q^2 \lesssim 70$
$e^\pm \{p, d\} \rightarrow e^\pm DX$	$\gamma^* g \rightarrow c\bar{c}$	Δg	$0.06 \lesssim x \lesssim 0.2$	$Q^2 \sim 10$
$\vec{p} \vec{p} \rightarrow jet(s)X$	$gg \rightarrow q\bar{q}$ $q\bar{q} \rightarrow q\bar{q}$	Δg	$0.05 \lesssim x \lesssim 0.2$	$30 \lesssim p_T^2 \lesssim 800$
$\vec{p} p \rightarrow W^\pm X$	$u_L \bar{d}_R \rightarrow W^+$ $d_L \bar{u}_R \rightarrow W^-$	$\Delta u \Delta \bar{d}$ $\Delta d \Delta \bar{u}$	$0.05 \lesssim x \lesssim 0.4$	$\sim M_W^2$

Other QCD fits to world data... cont'd

Comparison with DSSV

$$\Delta \bar{d} \text{ and } \Delta s = \Delta \bar{s}$$

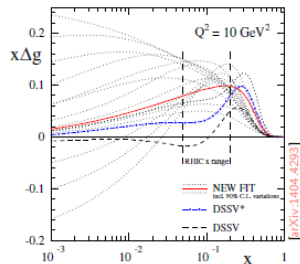
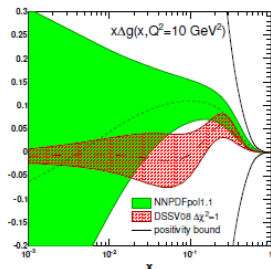


- Good agreement for the $\Delta \bar{d}$ distribution, but with large uncertainty
- Discrepancy between NNPDF and DSSV determinations of Δs (bias from the kaon fragmentation function used to include SIDIS data?)

Other QCD fits to world data... cont'd

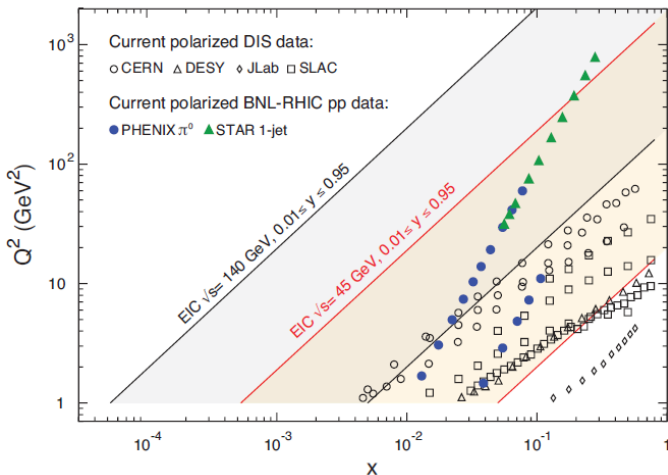
Comparison with DSSV

Δg



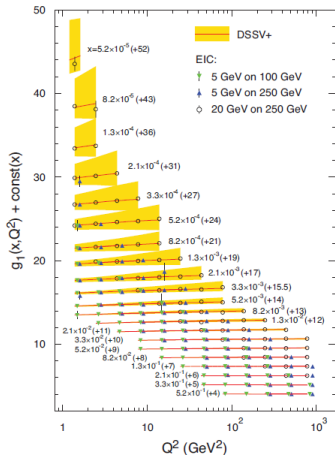
- For $x < 0.2$ the gluon has a node in DSSV08, while it is definitely positive in NNPDFpol1.1
- NNPDFpol1.1 and DSSV are in perfect agreement once recent jet data are included
- First evidence of gluon polarization in the proton in the region covered by data, $x \gtrsim 0.2$
- The uncertainty of the gluon blows up in the unmeasured small- x region

Acceptance of present spin experiments and EIC



From "White paper" arXiv:1212.1701

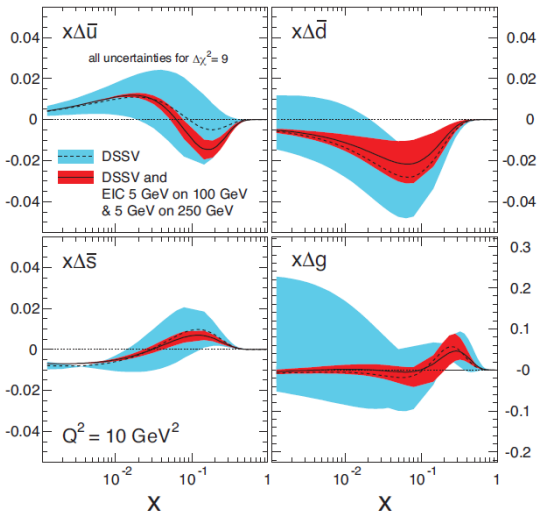
Inclusive $g_1(x, Q^2)$ at EIC (pseudo-data)



Errors statistical; bands mark current uncertainties (from DSSV+)

From "White paper" arXiv:1212.1701

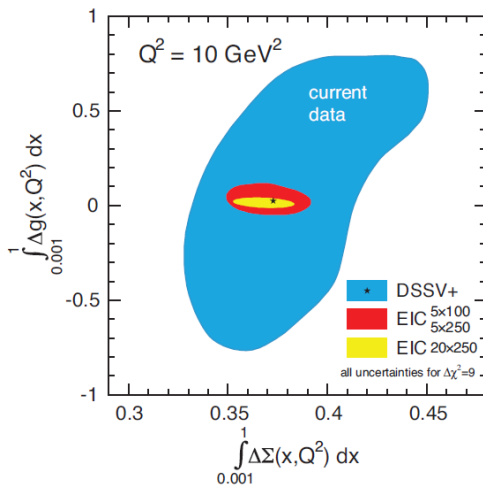
EIC pseudo-data (inclusive and semi-inclusive)



From "White paper" arXiv:1212.1701

EIC pseudo-data (inclusive and semi-inclusive)...

cont'd



Measurements on a transversely polarised target

Properties of $\Delta_T q(x)$:

- is chiral-odd \implies hadron(s) in final state needed to be observed
- simple QCD evolution since no gluons involved
- related to GPD
- sum rule for transverse spin
- first moment gives “tensor charge” (now being studied on the lattice)

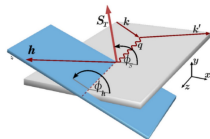
Transversity measured e.g. via the Collins asymmetry: \perp polarised $q \implies$ unpolarised h (asymmetry in the distribution of hadrons):

$$N_h^\pm(\phi_C) = N_h^0 [1 \pm p_T D_{NN} A_{Coll} \sin \phi_C]$$

$$\phi_C = \phi_h + \phi_S$$

which in turn gives at LO:

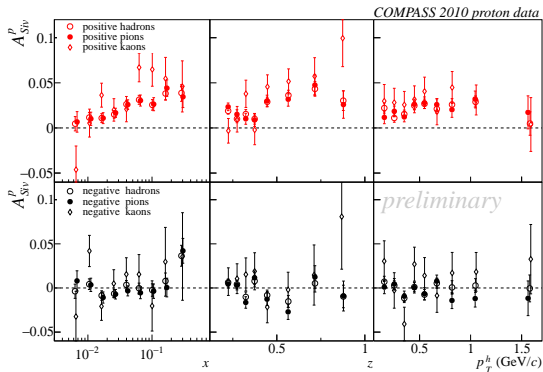
$$A_{Coll} \sim \frac{\sum_q e_q^2 \cdot \Delta_T q \cdot \Delta_T^0 D_q^h}{\sum_q e_q^2 \cdot q \cdot D_q^h}$$



But **transverse fragmentation functions $\Delta_T^0 D_q^h$** needed to extract $\Delta_T q(x)$ from the Collins asymmetry! Recently those FF measured by BELLE.

Properties of the Sivers process ($\phi_S = \phi_h - \phi_S$, correlation of \perp nucleon spin with k_T of unpolarised q): it is related to L_q in the proton. **Fundamental!**

Results for the Sivers asymmetry for protons

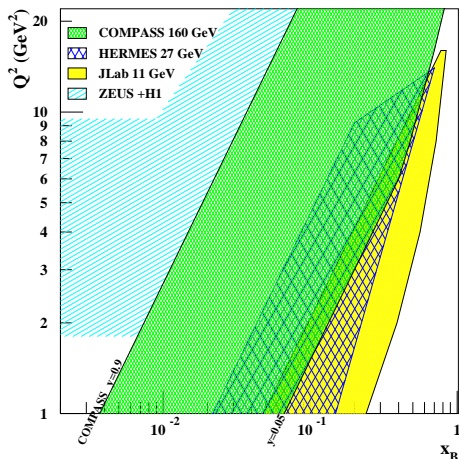


- Sivers asymmetries for proton measured for +/- unidentified and identified hadrons...
- ...are strongly dependent on Q^2 (compare with HERMES)
- COMPASS deuteron data show very small asymmetry
- Sivers functions (f_{1T}^\perp) for d and u quarks have opposite signs

COMPASS II Proposal

- CERN–SPSC–2010–014 (SPSC–P–340) of May 17, 2010
wwwcompass.cern.ch/compass/proposal/compass-II_proposal/compass-II_proposal.pdf
- Approved in December 2010 initially 3 years data taking (Phase 1)
- Flavour separation and fragmentation in spin-averaged SIDIS (strange sector !)
- Focus on transverse structure of the nucleon
 - T-odd TMD (Sivers, Boer-Mulders distributions)
 - Drell-Yan process and TMD sign change SIDIS \longleftrightarrow DY
 - GPD, transverse size and parton orbital angular momentum
- π/K polarisabilities and tests of ChPT in the Compton scattering via Primakoff reaction.
- Addendum foreseen (spin-dependent GPD), Phase 2.

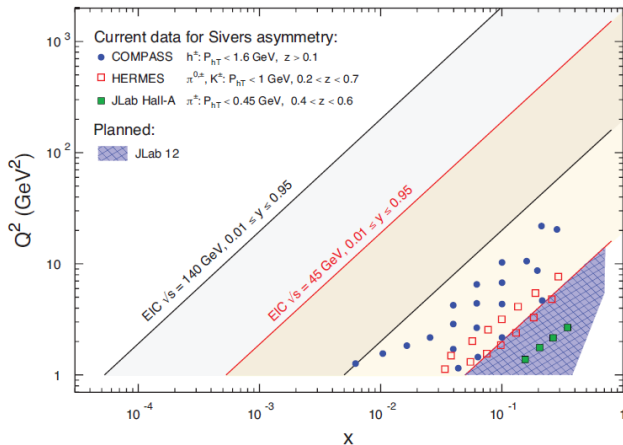
GPD at COMPASS: data taking in 2016-17



- CERN high energy muon beam
 - 100 - 190 GeV
 - 80% polarisation
 - $\mu^+ \leftarrow$ and $\mu^- \rightarrow$ beams
- Kinematic range
 - between HERA and HERMES/JLab12
 - intermediate x (sea and valence)
- Separation
 - pure B-H @ low x_B
 - predominant DVCS @ high x_B
- Plans
 - DVCS
 - DVMP
- Goals
 - from unpolarised target: H (Phase 1)
 - from \perp polarised target: E (Phase 2)

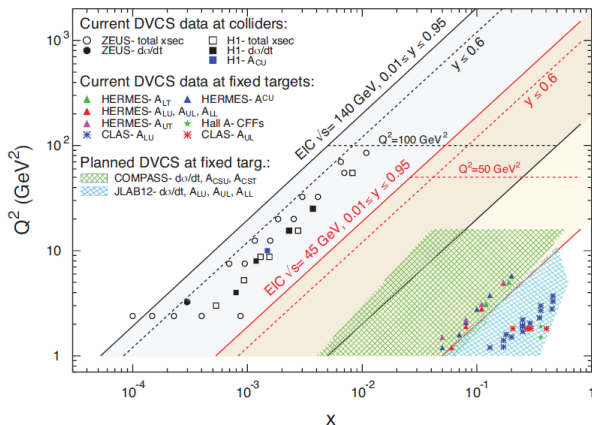
Test runs: 2008-9 and 2012; DVCS signal seen, full setup evaluated

Sivers measurements



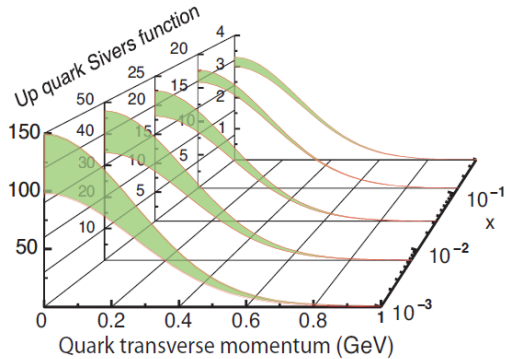
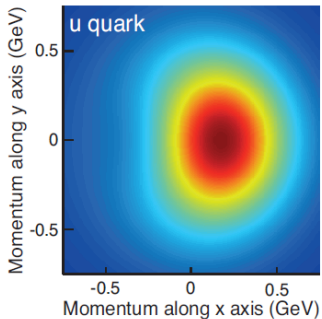
From "White paper" arXiv:1212.1701

Acceptance of present and EIC DVCS



From "White paper", arXiv:1212.1701

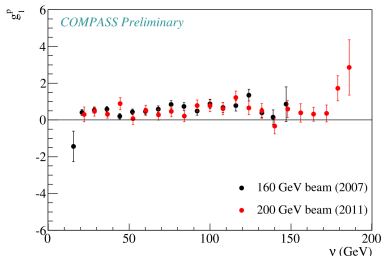
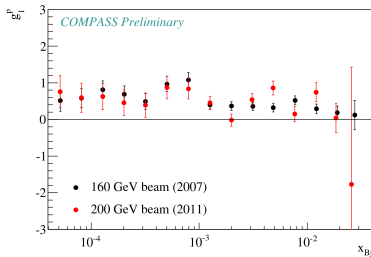
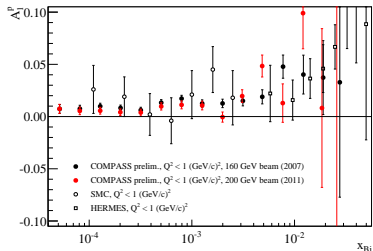
EIC foreseen results on the Sivers function



From "White paper", arXiv:1212.1701

Nonperturbative ($Q^2 < 1 \text{ GeV}^2$) region

g_1^p and g_1^d measurements: COMPASS very precise



$g_1^p(x)$, $g_1^p(\nu)$ significantly positive, $g_1^d(x) \approx 0$

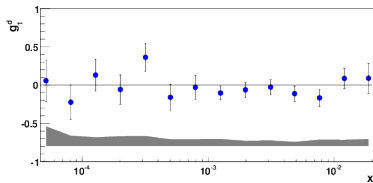


Figure above is from Phys.Lett. B647 (2007) 330

Regge model predictions for g_1

- Regge limit of DIS: $Q^2 \ll W^2 = M^2 + Q^2(1/x - 1)$ at fixed $Q^2 \implies$ low x
- For $g_1(x, Q^2)$ it gives: $g_1^i(x, Q^2) \sim \beta(Q^2)x^{-\alpha_i(0)}$, $i = \text{singlet, nonsinglet}$
($g_1^s = g_1^p + g_1^n$, $g_1^{\text{ns}} = g_1^p - g_1^n$)
- Intercepts $\alpha_i(0)$ correspond to axial vector mesons:
 g_1^s : f_1 ($I=0$)
 g_1^{ns} : a_1 ($I=1$)
where the intercepts: $\alpha_{s,\text{ns}}(0) \lesssim 0$, $\alpha_s(0) \approx \alpha_{\text{ns}}(0)$
- Consequence of the above: $g_1(W^2) \sim W^{2\alpha(0)}$ dependence at $Q^2 \rightarrow 0$
- At large Q^2 , Regge behaviour of g_1 unstable against DGLAP
(and against $\ln^2(1/x)$ resummation) \implies more singular x dependence
- Testing Regge behaviour of g_1 : through x -dependence at $Q^2 = \text{const}$
Not possible in SMC, not in COMPASS either
- Assuming $g_1 \sim 0$ to get $x \rightarrow 0$ extrapolation of g_1 is not correct!
Evolve to a common Q^2 first!

Towards the nonperturbative region

- For $Q^2 \rightarrow 0$ (for fixed W^2), g_1 should be a finite function of W^2 , free from kinematical singularities or zeroes at $Q^2 = 0$.
- If $g_1^{ns(0)}(x)$ has a singularity then it should be replaced by $g_1^{ns(0)}(\bar{x})$ where $\bar{x} = x(1 + k_0^2/Q^2)$. Remaining parts left unchanged.
- Then g_1^{ns} can be extrapolated to the low Q^2 for fixed $2M\nu = Q^2/x$ including $Q^2 = 0$. **Observe!** That is just the partonic contribution to the low Q^2 region; it may not be the only one there.
- Data on $g_1(x, Q^2)$ extend to low $Q^2 \sim 0.0001 \text{ GeV}^2$
- Nonperturbative mechanisms dominate the particle dynamics there; transition from “soft” to “hard” physics may be studied
- Partonic contribution to g_1 has to be suitably extrapolated to low Q^2 and complemented by a nonperturbative component
- Low Q^2 , spin-independent electroproduction well described by the GVMD \implies GVMD should be used to describe the g_1

Method I

$$g_1(x, Q^2) = g_1^{VMD}(x, Q^2) + g_1^{part}(x, Q^2)$$

g_1^{part} at low x is controlled by the $\ln^2(1/x)$ terms [Kwiecinski, Ziaja, Phys. Rev. D60 \(1999\) 054004](#).

$$g_1^{VMD}(x, Q^2) = \frac{M\nu}{4\pi} \sum_{V=\rho,\omega,\phi} \frac{M_V^4 \Delta\sigma_V(W^2)}{\gamma_V^2(Q^2 + M_V^2)^2}$$

The unknown $\Delta\sigma_V(W^2)$ are combinations of the total cross sections for the scattering of polarised vector mesons and nucleons. At high W^2 : $\Delta\sigma_V = (\sigma_{1/2} - \sigma_{3/2})/2$

$$\frac{M\nu}{4\pi} \sum_{V=\rho,\omega} \frac{M_V^4 \Delta\sigma_V}{\gamma_V^2(Q^2 + M_V^2)^2} =$$

$$C \left[\frac{4}{9} (\Delta u_{val}^0(x) + 2\Delta \bar{u}^0(x)) + \frac{1}{9} (\Delta d_{val}^0(x) + 2\Delta \bar{d}^0(x)) \right] \frac{M_\rho^4}{(Q^2 + M_\rho^2)^2},$$

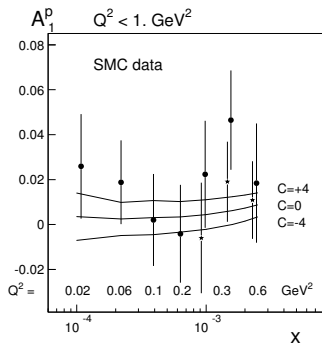
$$\frac{M\nu}{4\pi} \frac{M_\phi^4 \Delta\sigma_{\phi p}}{\gamma_\phi^2(Q^2 + M_\phi^2)^2} = C \frac{2}{9} \Delta \bar{s}^0(x) \frac{M_\phi^4}{(Q^2 + M_\phi^2)^2},$$

Nonperturbative effects in $g_1 \dots$ cont'd

Each $\Delta p_j^0(x) \rightarrow x^0$ for $x \rightarrow 0$. Thus $\Delta\sigma_V \rightarrow 1/W^2$ at large W^2 , i.e. zero intercept of the appropriate Regge trajectories.

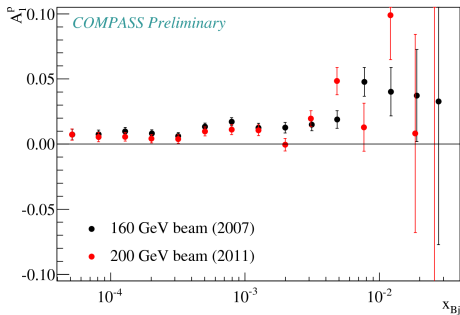
Results for the proton spin asymmetry, $A_1 = g_1/F_1$ and for different C :

$C ??$



SMC data

$C > 0 ?$



COMPASS preliminary data

Method II (GVMD)

For fixed $W^2 \gg Q^2$, i.e. small $x = Q^2/(Q^2 + W^2 - M^2)$ assume:

$$g_1(x, Q^2) = g_1^L(x, Q^2) + g_1^H(x, Q^2) = \frac{M\nu}{4\pi} \sum_V \frac{M_V^4 \Delta\sigma_V(W^2)}{\gamma_V^2 (Q^2 + M_V^2)^2} + g_1^H(x, Q^2)$$

g_1^L : light vector mesons, $M_V < Q_0$, $Q_0^2 \sim 1 \text{ GeV}^2$. Unknown $\Delta\sigma_V$: from combinations of nonperturbative parton distributions, evaluated at fixed Q_0^2 , similar to method I.

g_1^H : heavy vector mesons, $M_V > Q_0$

$$g_1^H(x, Q^2) = M\nu \int_{Q_0^2}^{\infty} dQ'^2 \frac{\Phi(Q'^2, W^2)}{(Q^2 + Q'^2)^2} \quad x = \frac{Q^2}{(Q^2 + W^2 - M^2)}$$

Here:

$$\Phi(Q^2, W^2) = -\frac{1}{\pi} \Im \int^{-Q^2} \frac{dQ'^2}{Q'^2} g_1^{AS}(x' = \frac{Q'^2}{(Q^2 + W^2 - M^2)}, Q'^2)$$

By construction:

$$\lim_{Q^2 \rightarrow \infty} g_1^H(x, Q^2) = g_1^{AS}(x, Q^2)$$

Nonperturbative effects in $g_1 \dots$ cont'd

Representation of Φ can also be treated as an extrapolation of the QCD improved parton model structure function, $g_1^{AS}(x, Q^2)$, to arbitrary values of Q^2 :

$g_1^H(x, Q^2) = g_1^{AS}(\bar{x}, Q^2 + Q_0^2)$. Here $\bar{x} = (Q^2 + Q_0^2)/(Q^2 + Q_0^2 + W^2 - M^2)$.

$$g_1(x, Q^2) = C \left[\frac{4}{9}(\Delta u_{val}^0(x) + 2\Delta \bar{u}^0(x)) + \frac{1}{9}(\Delta d_{val}^0(x) + 2\Delta \bar{d}^0(x)) \right] \frac{M_\rho^4}{(Q^2 + M_\rho^2)^2} \\ + C \left[\frac{1}{9}(2\Delta \bar{s}^0(x)) \right] \frac{M_\phi^4}{(Q^2 + M_\phi^2)^2} + g_1^{AS}(\bar{x}, Q^2 + Q_0^2).$$

Here $Q_0^2 = 1.2 \text{ GeV}^2$ and the only free parameter C is fixed in the $Q^2 = 0$ limit via the DHGHY sum rule:

$$I(0) = I_{res}(0) + M \int_{\nu_t(0)}^{\infty} \frac{d\nu}{\nu^2} g_1(x(\nu), 0) = -\frac{\kappa_{p(n)}^2}{4}, \quad \kappa_{p(n)} = 1.79 \text{ } (-1.91)$$

First moment of g_1 , Γ_1 is related to DHGHY moment, $I(Q^2)$: $\lim_{Q^2 \rightarrow 0} \Gamma_1 \rightarrow \frac{Q^2}{2M^2} I(Q^2)$

and I_{res} = contribution from resonances, **measured!** Coefficient $C \sim -0.3$

Outlook

- A wealth of data on polarised (SI)DIS collected by fixed target experiments and by RHIC
- COMPASS II running starts fall 2014
- Great hopes for EIC (BNL or JLab)
⇒ higher energy and low(er) x
- Long Range Planning in Nuclear Physics just started; final report will be submitted to DOE fall of 2015?
- Do not underestimate the fixed target results. Remember NMC vs HERA!