Experimental results on nucleon structure
Lecture I

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Outline

Course literature

1. D. H. Perkins, “Introduction to high energy physics”, CUP 2000 (4th edition or later).
2. B.R. Martin and G. Shaw, “Particle Physics” Wiley 1997 or later.
3. A. W. Thomas and W. Weise, “The structure of the nucleon”, Wiley-VCH 2001.
4. B. Povh, et al., “Particles and Nuclei”, Springer 2008 (6th edition or later)
5. R. G. Roberts, “The structure of the proton: Deep inelastic scattering”, CUP 1990.
6. and original papers, e.g. for spin see C. A. Aidala et al., arXiv: 1209.2803 v2 (1 April 2013)

Outline

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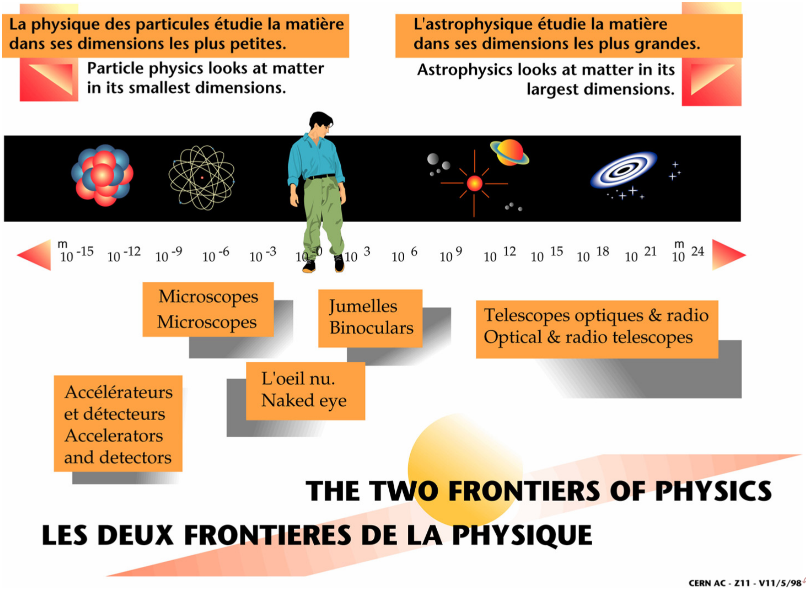
Two limits of research



Reminder: scales (distance, energy, mass,...) and constants

- $r \sim$ 1 fm (presently: an object is pointlike if its dimensions $\lesssim$ 0.001 fm = 10^{-18} m)
  $E \sim$ 1 GeV
  $m \sim$ 1 GeV/c^2

- Important constants:
  - Planck constant, $h \approx 6 \cdot 10^{-34}$ J·s (quantum physics must be applied),
  - speed of light, $c \approx 3 \cdot 10^8$ m/s (relativistic physics must be applied),
  - fine structure constant, $\alpha = \dfrac{e^2}{4\pi\epsilon_0\hbar c} \approx \dfrac{1}{137}$.
  - Heaviside – Lorentz system: $1 = \hbar = c = \epsilon_0 = \mu_0 \implies \alpha = \dfrac{e^2}{4\pi}$ will be used

- Very useful quantity: $\hbar c = 1 \approx 0.197$ GeV·fm

Elementary building blocks of matter

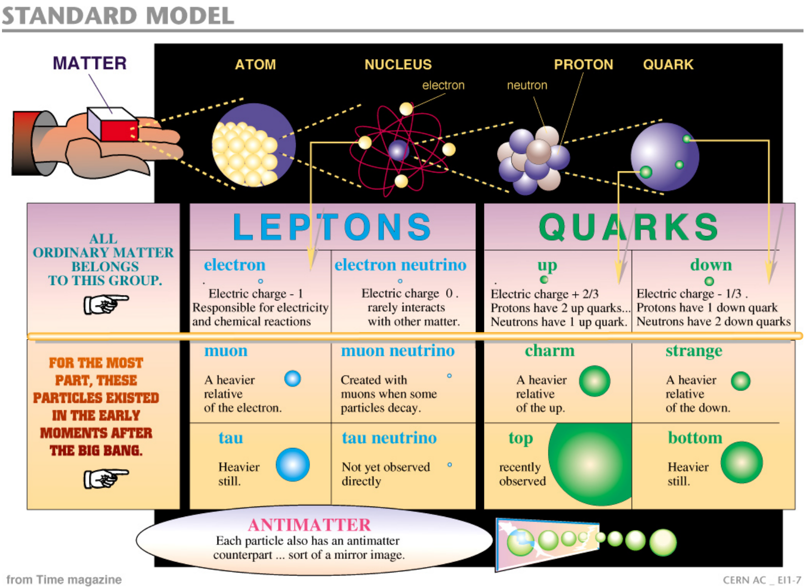
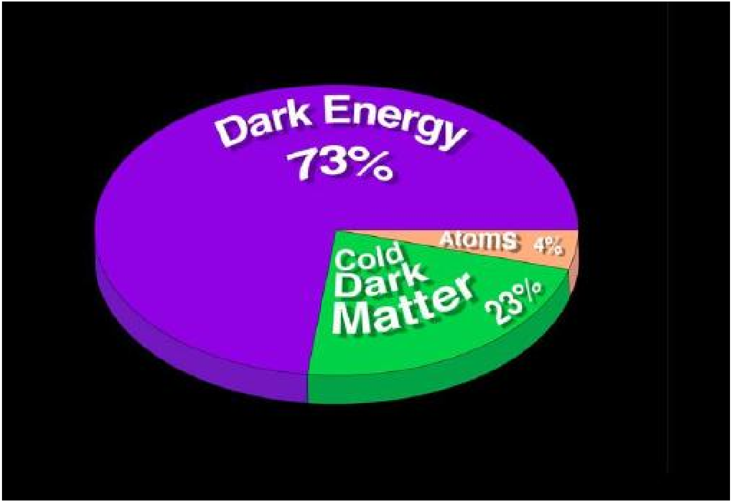
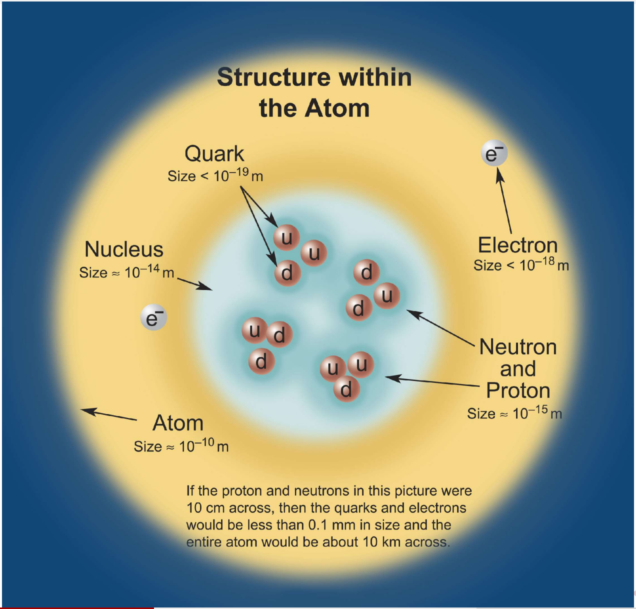


Photo: BB, 2012

Do we REALLY understand the structure of matter?



Reminder: dimensions of atom and its constituents



Baryons: nucleons & Co.

Baryons qqq and Antibaryons $\bar{q}\bar{q}\bar{q}$

Baryons are fermionic hadrons.

These are a few of the many types of baryons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
p	proton	**uud**	1	0.938	1/2
$\bar{p}$	antiproton	$\bar{u}\bar{u}\bar{d}$	–1	0.938	1/2
n	neutron	**udd**	0	0.940	1/2
Λ	lambda	**uds**	0	1.116	1/2
Ω^-	omega	**sss**	–1	1.672	3/2

Mesons

Mesons $q\bar{q}$ Mesons are bosonic hadrons These are a few of the many types of mesons.					
Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
π^+	pion	$u\bar{d}$	+1	0.140	0
K^-	kaon	$s\bar{u}$	–1	0.494	0
ρ^+	rho	$u\bar{d}$	+1	0.776	1
B^0	B-zero	$d\bar{b}$	0	5.279	0
η_c	eta-c	$c\bar{c}$	0	2.980	0

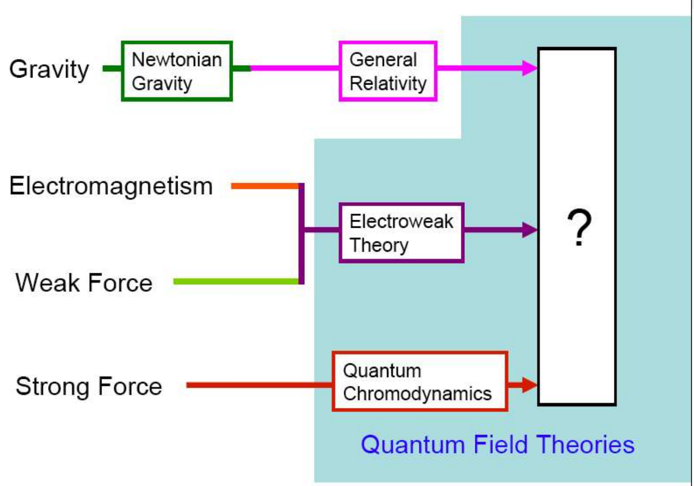
Types and properties of interactions (forces)

Properties of the Interactions

The strengths of the interactions (forces) are shown relative to the strength of the electromagnetic force for two u quarks separated by the specified distances.

Property	Gravitational Interaction	Weak Interaction (Electroweak)	Electromagnetic Interaction	Strong Interaction
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge
Particles experiencing:	All	Quarks, Leptons	Electrically Charged	Quarks, Gluons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons
Strength at 10^{-18} m	10^{-41}	0.8	1	25
Strength at 3×10^{-17} m	10^{-41}	10^{-4}	1	60
mass (GeV)	0	80-90	0	0
range (m)	∞	10^{-18}	∞	$\leq 10^{-15}$
coupling constant	10^{-38}	10^{-5}	1/137	1
time (s)	–	$10^{-8} - 10^{-10}$	10^{-20}	10^{-23}

Unification of interactions at energy of 10^{15} GeV ???



– > ENERGY

Add supersymmetry: fermions ⇆ bosons

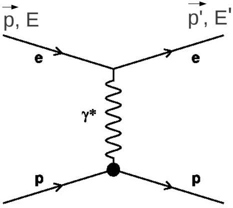
Standard Model of elementary interactions

- Family of elementary objects: at least 36 members of which at least 12 are interaction (or force) carriers.
- In our conditions we see at least 4 interactions; their relative strength changes with energy:
  - strong $\searrow$
  - electromagnetic $\nearrow$

  May be that immediately after the Big Bang all interactions had similar strength $\longrightarrow$ Grand Unification Theories (GUT), at E $\gtrsim$ 10^{15} GeV (proton mass: $\sim$ 1 GeV; largest proton energy in an accelerator (LHC) now: 4 TeV, soon: 7 TeV).
- Standard Model: perfectly agrees with experiment but DOES NOT predict several parameters, e.g. particle masses and features of forces (about 20 “free” parameters). Also: gravitation???

Interactions; probability amplitude; cross section

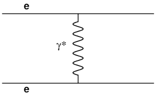
- (Electromagnetic) interaction = emission and absorption of a virtual photon, γ^*.
- Momentum transfer: $\vec{k} = (\vec{p} - \vec{p}\,')$
  Energy transfer: $\nu = (E - E')$.
- Define (negative) 4-momentum transfer squared (photon virtuality):
  $Q^2 = -q^2 = (\vec{p} - \vec{p}\,')^2 - (E - E')^2 = -M^2_{\gamma^*} \neq 0$!
- Define cross-section, σ: $\sigma \sim \mathrm{probability} = |f|^2$
- Observe that $f \sim \Delta t$ (time of the emission process)
  so that $f \sim \dfrac{1}{\sqrt{-M^2_{\gamma^*}}} = \dfrac{1}{Q}$
  or: $f = \dfrac{e}{Q} \Longrightarrow$ whole interaction: $f \sim \dfrac{e_1 e_2}{Q^2}$ and $\dfrac{\mathrm{d}\sigma}{\mathrm{d}Q^2} \sim \dfrac{e_1^2 e_2^2}{Q^4}$



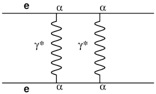
This probability amplitude is universal, i.e. describes several processes.

Feynman diagrams in Coulomb interactions

- Scattering amplitude: $f \sim \frac{ee}{Q^2} \Longrightarrow \frac{d\sigma}{dQ^2} \sim \frac{e^4}{Q^4} \sim \frac{\alpha^2}{Q^4}$



- For $2\gamma^*$ exchange $\sigma \sim \alpha^4$,
  i.e. σ is $\alpha^2 \approx \left(\frac{1}{137}\right)^2$ smaller than for $1\gamma^*$ exchange.



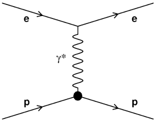
- Scattering from an effective charge eF:

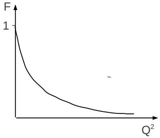
$$\frac{d\sigma}{dQ^2} \sim \frac{\alpha^2 F^2(Q^2)}{Q^4}$$

with limiting conditions:

$$\lim_{Q^2 \to \infty} F(Q^2) = 0 \quad \text{and} \quad \lim_{Q^2 \to 0} F(Q^2) = 1$$

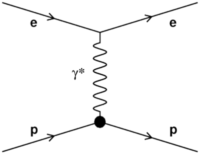
where $F(Q^2)$ – elastic nucleon (target) form factor.



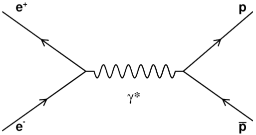


Electrons in nucleon structure experiments

- Electron – nucleon (nucleus) scattering; electrons point-like, $r \lesssim 10^{-18}$m
- Background of ee scattering easy to separate (except from forward scattering).
- (Electromagnetic) processes which yield information on proton structure:



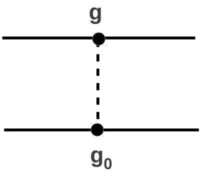
Rutherford scattering, $e^- p \to e^- p$
$M^2_{\gamma^*} < 0, \quad Q^2 > 0$



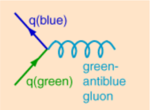
Annihilation: $e^+e^- \to p\bar{p}$,
$M^2_{\gamma^*} > 0, \quad Q^2 < 0$

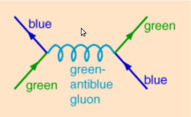
Strong interactions (between quarks)

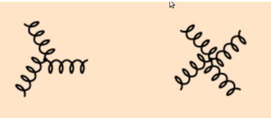
- Generally an interaction between 2 particles is an exchange of a boson of mass m.



Broken line = a boson
$g, \; g_0$ = “charges”
Scattering amplitude: $f(Q^2) = \dfrac{g g_0}{Q^2 + m^2}$

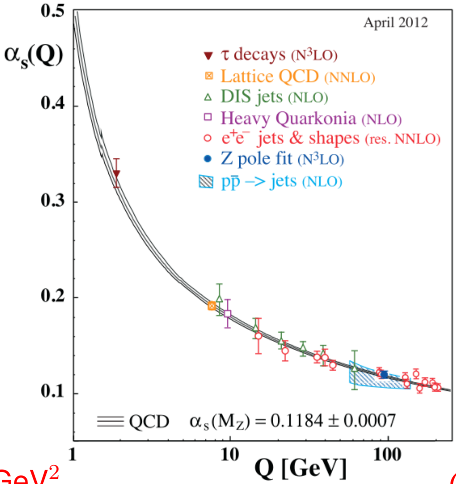






BUT $\alpha_s \sim 1$!!!!
multigluon exchanges!!!

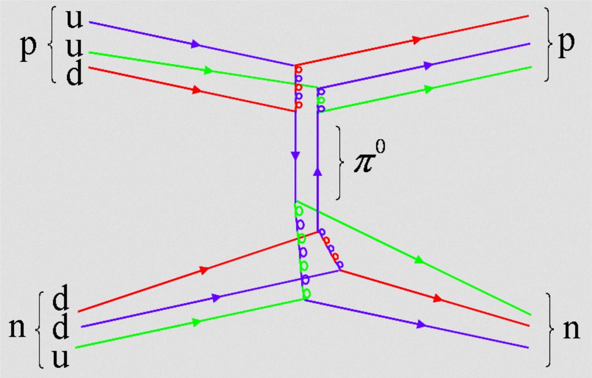
Strong coupling “constant”



$Q^2 \lesssim$ 1 GeV^2
Confinement

$Q^2 \to \infty$
“Asymptotic freedom”!!

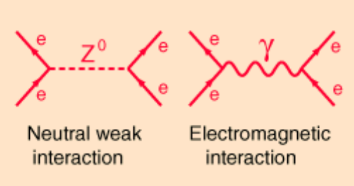
Residual strong interaction (in a nucleus)

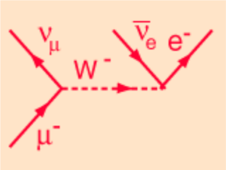


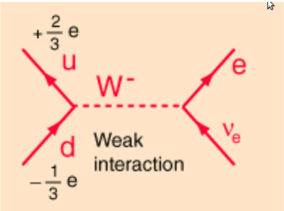
Final state quarks “dress up” into hadrons $\Longrightarrow$ fragmentation.

Factorization theorem: physics particles’ cross section
= (calculable QCD parton cross-section) $\otimes$ (universal long-distance functions)

Weak interactions







Scattering amplitude: $f(Q^2) = \dfrac{g^2}{Q^2 + m^2_{Z(W)}}$

Outline

Why high energies?

Searching for elementary components demands using high–energy beams since:

- some elementary particles are heavy (e.g. $m_{\mathrm{Z}^0} \sim 90 m_{\mathrm{p}}$), and energy, $E = mc^2$, is needed to produce them;
- goal is to investigate small distances, $\Delta x \sim$ 1 fm, and since $\Delta x \Delta p \sim \hbar$ then Δp large and $\Longrightarrow p$ large too. Another argument: $\lambda \sim$ small $\Longrightarrow p$ large since $\lambda \sim h/p$.

Example 1: electrons of $\lambda \sim$ 1 fm have $E \sim$ 0.2 GeV.

Example 2: investigating protons, $\lesssim$ 1 fm, demands $Q^2 \gtrsim$ 1 GeV^2.

Reminder: centre-of-mass vs laboratory systems

- A beam particle A hits a target particle B:

$$p^2 = (\vec{p}_A + \vec{p}_B)^2 - (E_A + E_B)^2 = -m_A^2 - m_B^2 + 2(\vec{p}_A\vec{p}_B - E_A E_B) = -(E^{cms})^2$$

- Consider a fixed target experiment, i.e. $\vec{p}_B$ = 0 ($E_B = m_B$); here

$$p^2 = -(E^{cms})^2 = -m_A^2 - m_B^2 - 2\, E_A\, m_B$$

or, if particles masses are negligible with respect to their energies (momenta):

$$E_A = \frac{(E^{cms})^2}{2m_B}$$

- Consider a collider experiment, i.e. $\vec{p}_A \uparrow\downarrow \vec{p}_B$ (or: $\sphericalangle(\vec{p}_A, \vec{p}_B) = \pi$):

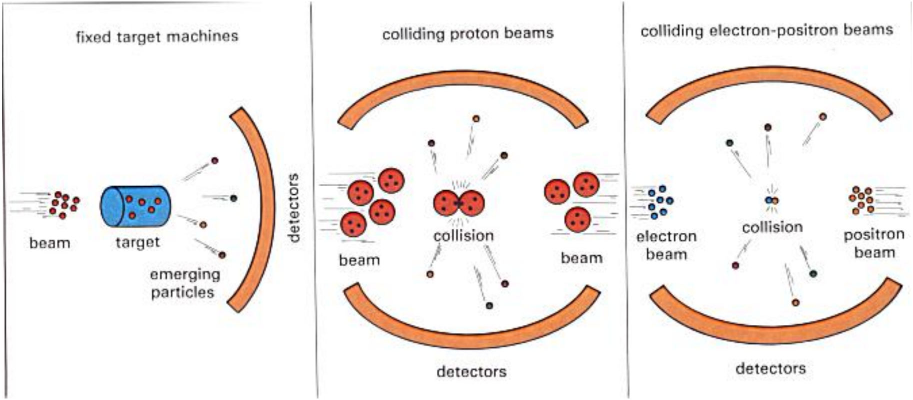
$$p^2 = -(E^{cms})^2 = -m_A^2 - m_B^2 + 2(-|\vec{p}_A|\,|\vec{p}_B| - E_A E_B)$$

or, if particle masses are negligible with respect to their energies (momenta):

$$p^2 = -(E^{cms})^2 \approx -4E_A E_B$$

- Important example: LHC operating at 7 TeV per proton beam: $E^{cms} = 2 \cdot 7$ TeV = 14 TeV
If such E^{cms} were to achieve in a fixed-target experiment then
a beam of $E_A \approx$ 100 000 TeV had to be prepared !!!! Not possible...
(Compare: highest observed energy of cosmic rays: $\sim 10^9$ TeV)

Types of high energy experiments



e.g. COMPASS/CERN e.g. LHC/CERN e.g. ILC (planned)

How many variables needed to describe a reaction?

Consider elastic ($ep \to ep$) and inelastic ($ep \to eX$) interactions where the initial state (i.e. masses and energies) is known.

	$ep \to ep$	$ep \to eX$
initial state	known	known
final state		
2 particles x 4 variables	8 variables	8 variables
-4 eqs (en.-mom. conservation)	4	4
-1 (azimuthal angle, φ)	3	3
known masses in the final state	1 variable	2 variables

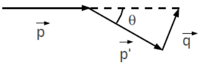
Thus for elastic scattering: 1 variable is enough, e.g. Q^2;
here also: $W = M$ (W - effective mass of the X system, M - proton mass).

Inelastic electron–proton scattering

For the inelastic scattering 2 variables needed, e.g. Q^2 and ν.
Try to find a relation $W \longleftrightarrow Q^2, \nu$. In the bottom vertex:

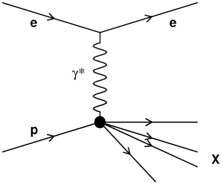
Energy conservation: $\nu + M = E_X$

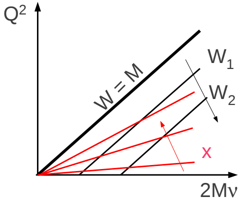
Momentum conservation: $Q^2 = \vec{k}^2 - \nu^2 = p_X^2 - \nu^2$



Result: $$W^2 = 2M\nu + M^2 - Q^2 \quad (1)$$

$$Q^2 = (\vec{p} - \vec{p}\,')^2 - (E - E')^2 = -2m^2 - 2pp' \cos\vartheta + 2EE' \approx 4EE' \sin^2 \frac{\vartheta}{2} \quad (2)$$





Define: $$x = \frac{Q^2}{2M\nu} \quad (3)$$

so that for elastic: $x = 1$ or $W = M$ and for inelastic: $x < 1$ or $W > M$

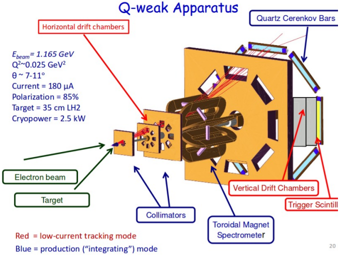
Nucleon structure main research centres

In red – running experiments, in green –future ones.

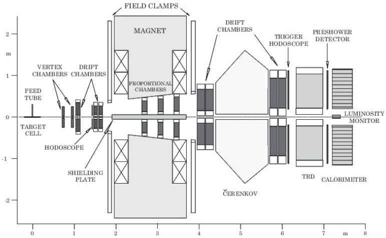
- SLAC (closed): several experiments, $E_e \lesssim$ 50 GeV, also polarised.
- CERN: μ, E_μ: 90 – 300 GeV, naturally polarised; proton and deuteron targets.
  - BCDMS (completed)
  - EMC (completed)
  - NMC (completed)
  - SMC (spin, completed)
  - COMPASS (spin)
- FNAL: exp. E665, μ, E_μ = 470 GeV.
- HERA (closed): e–p collider, 28 GeV + 300 GeV
  - H1 (being analysed)
  - ZEUS (being analysed)
  - HERMES, electrons, E_e = 27 GeV on fixed-target (spin, being analysed)
- RHIC: p-p, 250 GeV + 250 GeV, polarised
  - STAR (also spin)
  - PHENIX (also spin)
- JLAB: several experiments, $E_e \lesssim$ 6 GeV (also spin); soon $E_e \lesssim$ 12 GeV.
- LHC (CMS, ATLAS): p-p, 4 TeV + 4 TeV; soon: 7 TeV + 7 TeV.
- Large Hadron-electron Collider, LHeC and/or Electron Ion Collider, EIC: e–p and e–A

Examples of detectors

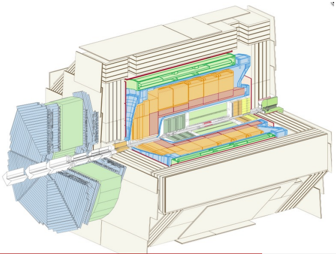
QWEAK/JLAB



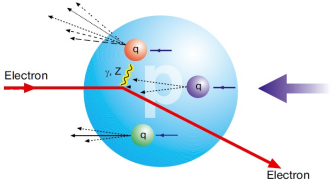
HERMES/DESY



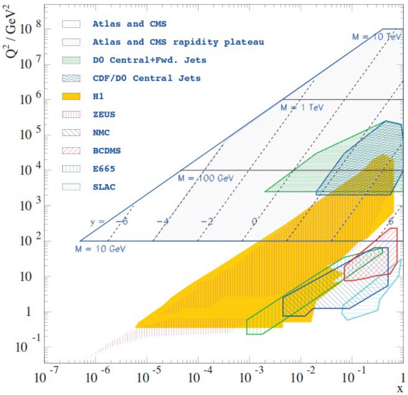
H1/DESY

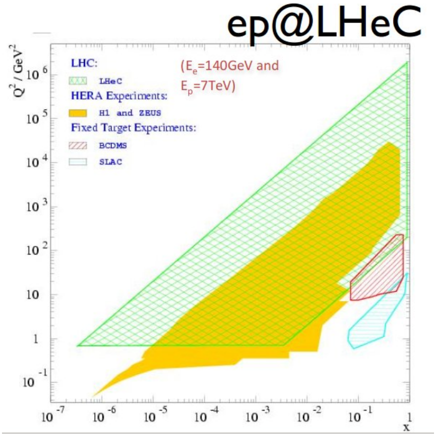


H1 kinematics



Acceptance of nucleon structure experiments





Electron beams: high statistics, high systematics (radiative processes), “cheap”

Muon beams: low statistics, low systematics, “expensive”

Proton beams: complicated analysis.

Outline

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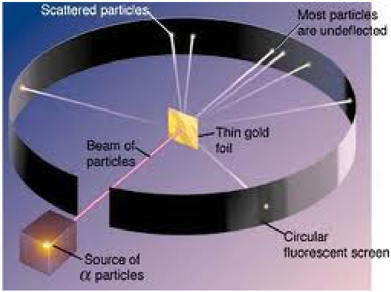
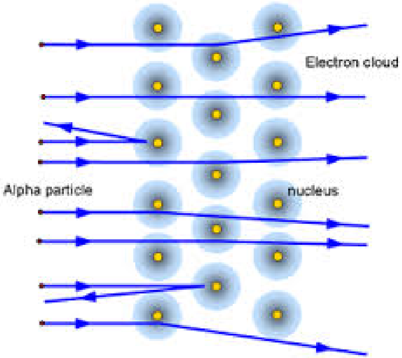
Rutherford scattering

In 1910 – 1911, Ernest Rutherford + students: H. Geiger and E. Marsden



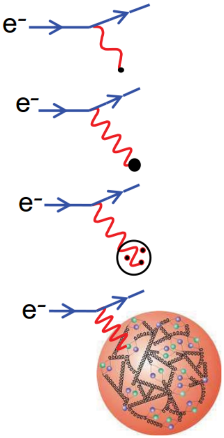


First exploration of atomic structure:

small, massive, positive nucleus and negative charge around it

Probing the structure of the proton

- **At very low electron energies** $\lambda \gg r_p$ **:** **the scattering is equivalent to that from a "point-like" spin-less object**
- **At low electron energies** $\lambda \sim r_p$ **:** **the scattering is equivalent to that from a extended charged object**
- **At high electron energies** $\lambda < r_p$ **:** **the wavelength is sufficiently short to resolve sub-structure. Scattering from constituent quarks**
- **At very high electron energies** $\lambda \ll r_p$ **:** **the proton appears to be a sea of quarks and gluons.**



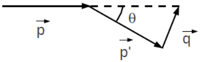
From: M.A. Thomson, Michaelmas Term 2011

Elastic electron – nucleon scattering

- Rutherford scattering: a particle ze of E scatters off Ze at rest and changes its momentum vector by ϑ:

$$\left(\frac{d\sigma}{d\Omega}\right)_R = \frac{(zeZe)^2}{(4\pi\epsilon_0)^2(4E)^2\sin^4\frac{\vartheta}{2}}$$

This formula is nonrelativistic; target recoils is neglected (= target is very heavy), i.e.
$E = E', \quad |\vec{p}| = |\vec{p}'|, \quad |\vec{q}| = |\vec{k}| = 2|\vec{p}|\sin\vartheta/2$



A relativistic formula ($E \approx |\vec{p}|c$) and $z = 1$:

$$\left(\frac{d\sigma}{d\Omega}\right)_R = \frac{Z^2\alpha^2(\hbar c)^2}{4E^2\sin^4\frac{\vartheta}{2}} \qquad (4)$$

At relativistic energies, Rutherford formula is modified by spin effects.

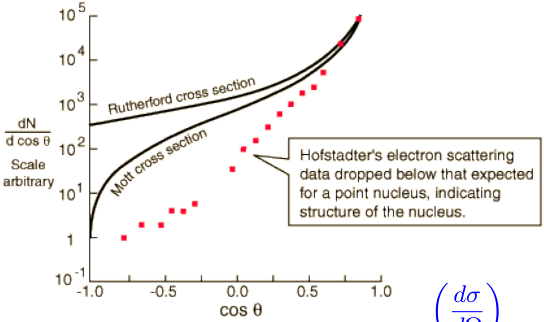
- If electron relativistic and its spin included $\Longrightarrow$ Mott cross-section (still no recoil):

$$\left(\frac{d\sigma}{d\Omega}\right)^*_{Mott} = \left(\frac{d\sigma}{d\Omega}\right)_R \cdot \left(1-\beta^2\sin^2\frac{\vartheta}{2}\right) = \frac{4Z^2\alpha^2(\hbar c)^2E'^2}{|\vec{q}c|^4}\cos^2\frac{\vartheta}{2} \qquad (5)$$

(an asterisk means the recoil of the nucleus is neglected)
For $\beta \rightarrow$1, $\vartheta = \pi$ supressed on spinless target
(a consequence of a helicity, h, conservation, $h = \vec{s}\vec{p}/|\vec{s}||\vec{p}|$).

Elastic electron – nucleon scattering...cont'd

- Mott expression agrees with the data for $|\vec{q}| \to 0 (\vartheta \to 0)$ but at higher $|\vec{q}|$ experimental cross-sections are smaller: a form factor!



$$\left(\frac{d\sigma}{d\Omega}\right)_{exp} = \left(\frac{d\sigma}{d\Omega}\right)^{*}_{Mott} \cdot |F(\vec{q}^{\,2})|^2 \quad (6)$$

(for spherically symmetric systems, the form factor depends on $\vec{q}$ only!

- Determination of a form factor: measure of $d\sigma/d\Omega$ at fixed E and diferent ϑ (= various $|\vec{q}|$) and divide by the Mott cross- section.

Elastic electron – nucleon scattering...cont'd

- First measurements at SLAC in early 50-ties, E_e = 0.5 GeV.
- Define a charge distribution function f by $\rho(\vec{r}) = Zef(\vec{r})$ so that $\int f(\vec{r})d^3r = 1$. Then the form factor:

$$F(\vec{q}^{\,2}) = \int e^{i\vec{q}\vec{r}/\hbar} f(\vec{r})d^3r \tag{7}$$

but only under conditions: no recoil, Ze small (or $Z\alpha \ll$1)!

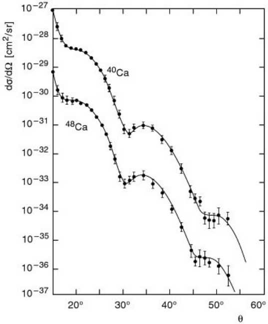
- For spherically symmetric cases f depends only on $r = |\vec{r}|$. Then

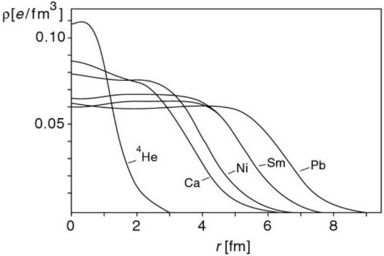
$$F(\vec{q}^{\,2}) = 4\pi \int f(r)\frac{\sin(|\vec{q}|r/\hbar)}{|\vec{q}|r/\hbar}r^2dr \qquad 1 = 4\pi \int_0^\infty f(r)r^2dr$$

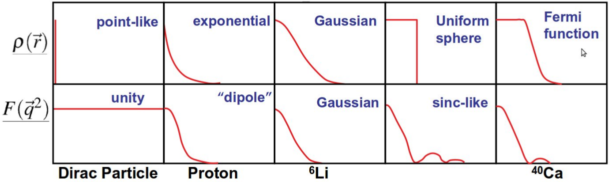
- The radial charge distribution, $f(r)$ cannot be determined from the inverse Fourier transform of $F(\vec{q}^2)$ due to limited interval of measured values of $|\vec{q}|$.
- Thus a procedure of finding $F(\vec{q}^{\,2})$: choose parameterisation of $f(r)$, calculate $F(\vec{q}^{\,2})$ and vary its parameters to get best fit to data.
- Observe:

$$\langle r^2 \rangle = 4\pi \int_0^\infty r^2 f(r)dr = -6\hbar^2 \frac{dF(\vec{q}^2)}{d\vec{q}^2}\bigg|_{\vec{q}^2=0} \tag{8}$$

Examples of form factors and charge densities







From: M.A. Thomson, Michaelmas Term 2011

Form factors of the nucleons

- Studies of nucleon structure ($r_n \sim$ 0.8 fm) demand $E \gtrsim$ 1 GeV; thus nucleon recoil can no longer be neglected.

- With recoil:

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \left(\frac{d\sigma}{d\Omega}\right)^{*}_{Mott} \cdot \frac{E'}{E} \tag{9}$$

  Also: a $Q^2 = -q^2 = 4EE' \sin^2 \vartheta/2$ needed instead of $\vec{q}^{\,2}$ in the Mott cross-section

- Apart of e-N Coulomb interaction, now also electron current - nucleon's magnetic moment. Reminder: a pointlike, charged particle of spin 1/2 has a magnetic moment: $\mu = g\frac{e}{2M}\frac{\hbar}{2}$ (g=2 from relativistic Q.M.). Magnetic interaction associated with a flip of the nucleon spin.

- Scattering at ϑ = 0 not consistent with helicity (and angular momentum) conservation; scattering at $\vartheta = \pi$ is preferred. Thus:

$$\left(\frac{d\sigma}{d\Omega}\right)_{point;spin1/2} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left[1 + 2\tau \tan^2 \frac{\vartheta}{2}\right], \qquad \tau = \frac{Q^2}{4M^2} \tag{10}$$

  New, magnetic term (above) is large at large Q^2 and ϑ.

Form factors of the nucleons...cont'd

- Anomalous magnetic moments for nucleons:

$$\mu_p = +2.79... \cdot \frac{e\hbar}{2M} = +2.79... \cdot \mu_N \qquad \mu_n = -1.91... \cdot \frac{e\hbar}{2M} = -1.91... \cdot \mu_N \tag{11}$$

where $\mu_N = 3.1525 \cdot 10^{-14}$ MeV/T = nuclear magneton.

- Charge and currrent distributions described by two (Sachs) form factors (Rosenbluth):

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot \left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2)\tan^2\frac{\vartheta}{2}\right] \tag{12}$$

Here: $G_E(Q^2)$ and $G_M(Q^2)$ are the electric and magnetic form factors.
At very low Q^2, $G_E(Q^2)$ and $G_M(Q^2)$ are Fourier transforms of the charge and magnetization current densities inside the nucleon.

- At $Q^2 \to 0$:

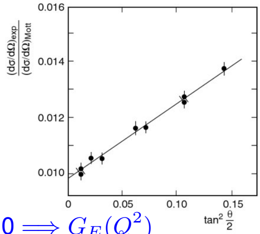
$$G_E^p = 1, \qquad G_E^n = 0$$
$$G_M^p = 2.79, \qquad G_M^n = -1.91 \tag{13}$$

- How do we measure (separate) $G_E(Q^2)$ and $G_M(Q^2)$?

Outline

Measurement of form factors – Rosenbluth method

- Independent measurement of $G_E(Q^2), G_M(Q^2)$ needed. This is done at fixed Q^2 and different ϑ (or energies E). The measured cross-section is then divided by σ_{Mott}. Example of results:



Slope of the line yields $G_M(Q^2)$, intercept at $\vartheta = 0 \Longrightarrow G_E(Q^2)$

- For a long time it seemed that:

$$G_E^p(Q^2) = \frac{G_M^p(Q^2)}{2.79} = \frac{G_M^n(Q^2)}{-1.91} =$$
$$= G_D(Q^2) = \left(1 + \frac{Q^2}{0.71(\text{GeV}/c)^2}\right)^{-2}$$

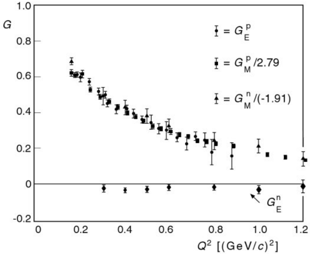
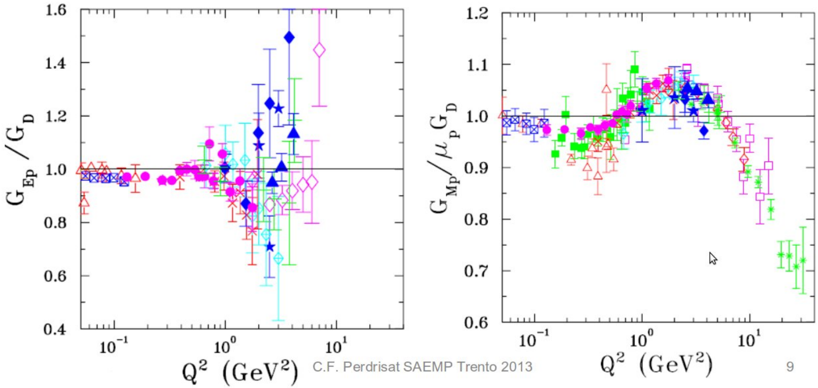


Fig. from the book of B.Povh et al.

Measurement of form factors – Rosenbluth method...cont'd

All published Rosenbluth separation data for the proton:



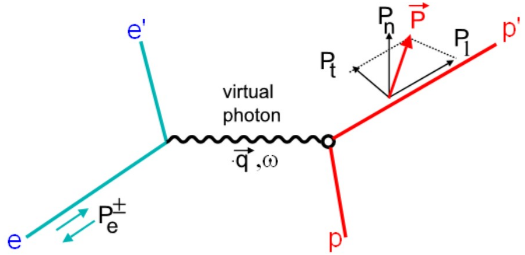
- At $Q^2 \gtrsim$1 GeV^2, G_E^p inaccurately determined...
- ...but G_M^p errors small up to $Q^2 \sim 30$ GeV^2.
- Neutron data (from elastic and break-up ed ecattering) of poorer quality.
- Old paradigm (until 1998 → JLAB): proton from factors similar, and close to G_D.

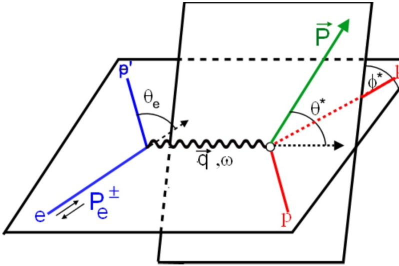
www.scholarpedia.org/article/Nuclear_Form_factors

Measurement of form factors – polarisation transfer method

1998: a new method of form factors determination (coincided with opening of the JLAB): polarisation observables instead of cross-sections.

1) $\vec{e}p \to e\vec{p}$ or 2) $\vec{e}\vec{p} \to ep$





www.scholarpedia.org/article/Nuclear_Form_factors

Measurement of form factors – polarisation transfer...cont'd

1998: a new method of form factors determination (coincided with opening of the JLAB): polarisation observables instead of cross-sections.

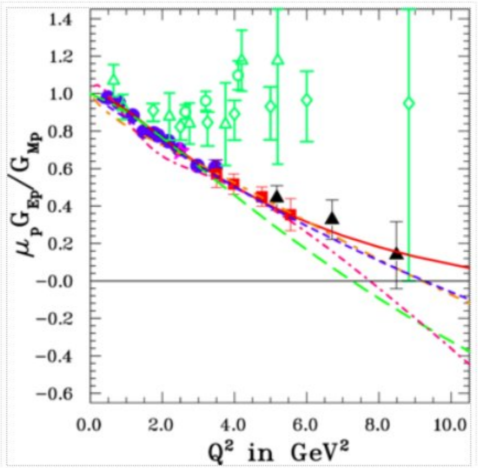
$$1)\ \ \vec{e}p \to e\vec{p} \qquad \text{or} \qquad 2)\ \ \vec{e}\vec{p} \to ep$$

- Polarisation of the recoil proton contains terms proportional to $G_E^p G_M^p$ so that G_E^p may be determined even when it is small. Also radiative corrections minimised (polarisation observables are ratios of cross sections).
- In reaction (1): measurement of 2 components of the proton polarisation, e.g. longitudinal (P_l) and transverse (P_t) to the proton momentum in the scattering plane.
- If only polarisations measured in reaction (1) then only

$$\frac{G_E^p}{G_M^p} = -\frac{P_t}{P_l}\frac{(E+E')}{2M}\tan\frac{\vartheta}{2}$$

  determined; separation of form factors need cross-section mesurements.
- Independently of beam polarisation, a small normal component, P_n, is introduced by a double-photon exchange.

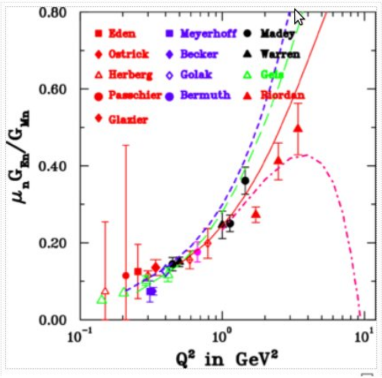
Measurement of form factors – polarisation method results



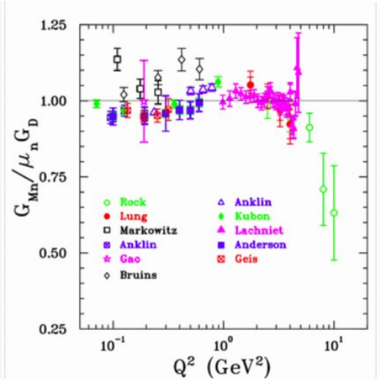
Green points - Rosenbluth method; other colours - recoil polarisation results.

www.scholarpedia.org/article/Nuclear_Form_factors

Measurement of form factors – results for neutron



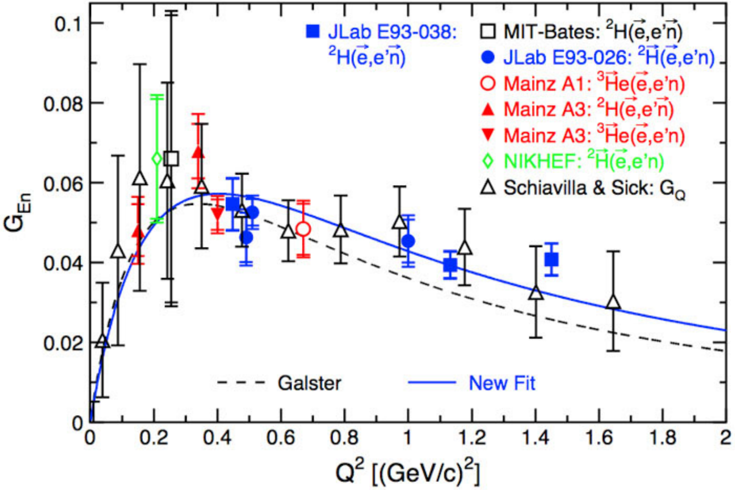
Double polarisation measurements



Cross-section measurements

www.scholarpedia.org/article/Nuclear_Form_factors

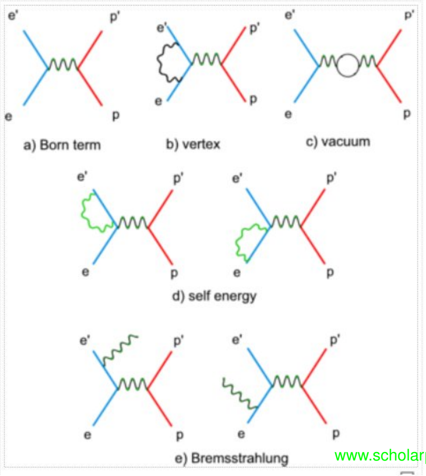
Measurement of form factors – results for neutron,...cont'd



Outline

Corrections to data

Lowest order QED corrections to data ("radiative corrections"):

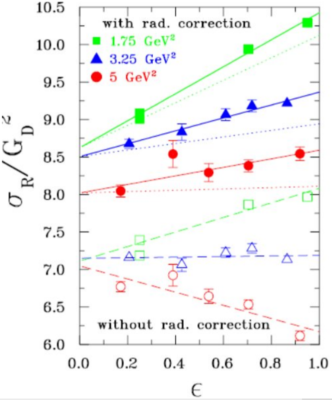


www.scholarpedia.org/article/Nuclear_Form_factors

Change of paradigm ???

- Which approach is correct?
- And what is a reason of discrepancy?

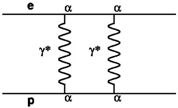
$$R = \mu \frac{G_E}{G_M} \qquad \text{vs} \qquad \epsilon = \left[1 + 2(1+\tau) \tan^2 \frac{\vartheta}{2} \right]^{-1}$$



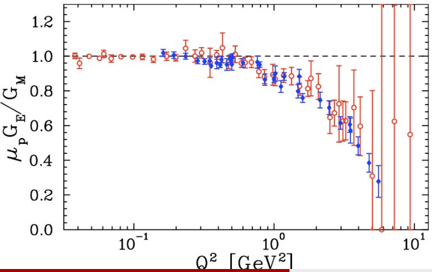
C.F. Perdrisat, Trento 2013

Change of paradigm ???...cont'd

- Which approach is correct?
- And what is a reason of discrepancy? Missing physics?



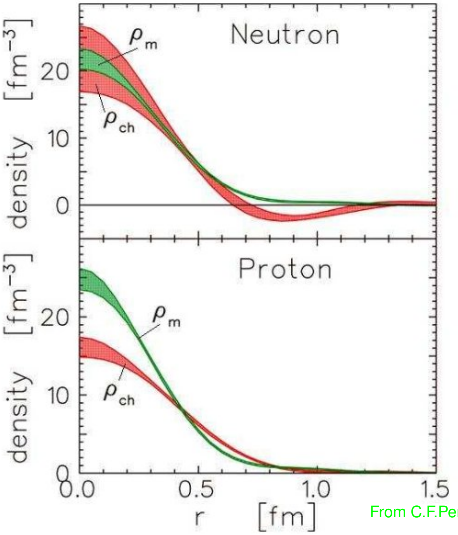
- Most probable cuprit: neglecting 2γ exchange in radiative corrections!
- Rosenbluth method: σ very sensitive to ϑ dependence $\Longrightarrow$ dramatic effect; polarisation (ratio) method: few percent effect.
- Results from both methods agree if 2γ exchange contribution accounted for:



red – Rosenbluth method
blue – polarisation transfer method

J. Arrington et al.,Phys.Rev. C76 (2007) 035205

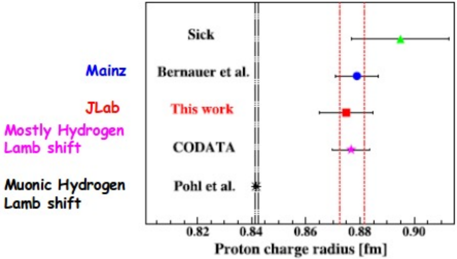
From form factors to charge/magnetisation densities



neutron charge distr. multiplied by a factor of 6 !

From C.F.Perdrisat *et al.*, Prog.Part.Nucl.Phys. 59 (2007) 694

Proton Charge Radius Puzzle



The figure is from X. Zhan et al., PLB 705, 59 (2011)

Dotted red lines combined CODATA, Bernauer (Mainz).

From the New York Times, July 13, 2010.

"For a Proton, a Little Off the Top (or Side) Could Be Big Trouble"

It went from 0.8768±0.0069 fm to 0.8418±0.0007 fm.

2/17/2013 C.F. Perdrisat SAEMP Trento 2013 47

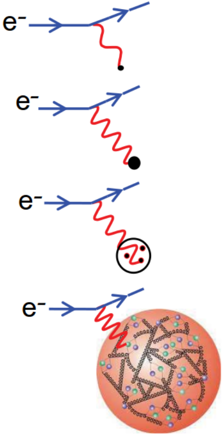
Outline

1. Course literature
2. Introduction
   - Scales, elementary particles, interactions
   - Kinematics, experiments and observables
3. Nucleon elastic form factors
   - Basic formulae
   - Form factor measurements
   - Radiative corrections
4. Parton structure of the nucleon
   - Feynman parton model
   - Partons vs quarks

Outline

Probing the structure of the proton

- At very low electron energies $\lambda \gg r_p$:
  the scattering is equivalent to that from a "point-like" spin-less object
- At low electron energies $\lambda \sim r_p$:
  the scattering is equivalent to that from a extended charged object
- At high electron energies $\lambda < r_p$:
  the wavelength is sufficiently short to resolve sub-structure. Scattering from constituent quarks
- At very high electron energies $\lambda \ll r_p$:
  the proton appears to be a sea of quarks and gluons.

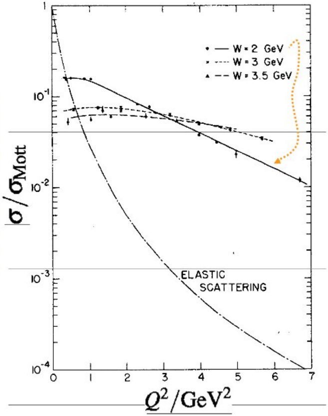


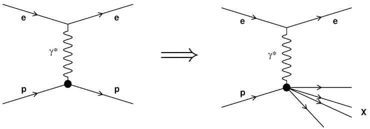
From: M.A. Thomson, Michaelmas Term 2011

Towards inelastic electron – nucleon scattering

- At large scattering angles ϑ (i.e. large Q^2 or large ν): $F(Q^2) \to 0$ and inelastic scattering becomes more probable than the elastic.
- Now $Q^2 \neq 2M\nu$ (or $x \neq 1$) or: $Q^2 = M^2 + 2M\nu - W^2$ and a second variable, apart of Q^2 is needed, e.g. ν or x.

M.Breidenbach et al.,
Phys. Rev. Lett. 23 (1969) 935

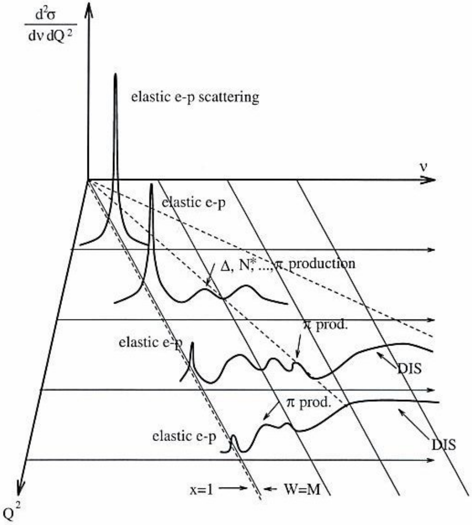




Scattering from point-like components in the proton!

From: M.A. Thomson, Michaelmas Term 2011

Towards inelastic electron – nucleon scattering,...cont'd



Radial, broken lines: x =const.
Parallel, continuous lines: W =const.

Low x – large parton (gluon) densities.
Low Q^2 – nonperturbative effects.

DIS = Deep Inelastic Scattering
(large Q^2, ν)

Inelastic electron – nucleon scattering

From Eq. (12):

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot \left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2)\tan^2\frac{\vartheta}{2}\right] \tag{14}$$

But

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{dQ^2} \cdot \frac{EE'}{\pi}$$

Then:

$$\left(\frac{d\sigma}{dQ^2}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot \frac{\pi}{EE'} \cdot \left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2)\tan^2\frac{\vartheta}{2}\right] \tag{15}$$

Therefore the result in Eq. (12) may be written as:

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1 - y - \frac{M^2y^2}{Q^2}\right) f_2(Q^2) + \frac{1}{2}y^2 f_1(Q^2)\right] \tag{16}$$

This may be compared with an inelastic cross-section:

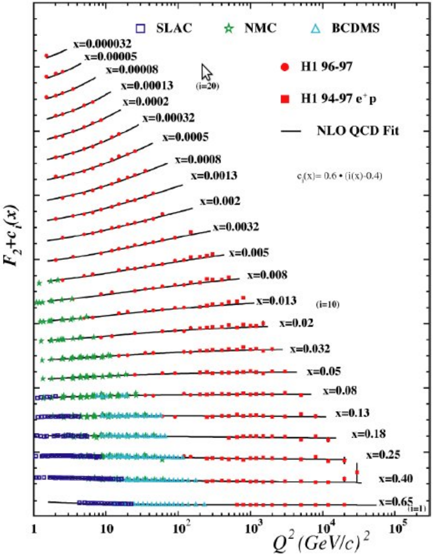
$$\frac{d^2\sigma}{dQ^2dx} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1 - y - \frac{M^2y^2}{Q^2}\right) \frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2)\right] \tag{17}$$

where $y = \nu/E$

Structure functions $F_1(x,Q^2)$ and $F_2(x,Q^2)$

- Instead of two elastic form factors, $G_E(Q^2), G_M(Q^2)$ we have two structure functions $F_1(x,Q^2), F_2(x,Q^2)$.
- As the form factors, the structure functions cannot be obtained from theory; must be measured.
- Experimentally: both F_1 and F_2 are only weakly dependent on Q^2.
- To determine F_1 and F_2 and for a given x and Q^2 need measurements of the differential cross section at several different scattering angles and incoming electron beam energies.

Structure functions,... cont'd



Bjorken scaling hypothesis

- If leptons scatter from point-like components then structure functions cannot depend on any dimensioned variable, e.g. Q^2 or ν.
- Bjorken: if for $Q^2 \to \infty$ and $\nu \to \infty$, $F_2(Q^2, \nu)$ is finite then it may depend only on dimensionless, finite ratio of these variabes, i.e. on $x = \dfrac{Q^2}{2M\nu}$.
  This is called scale invariance or "scaling".
- Scaling holds already for $Q^2 \approx$ few M^2 or ~ 1 GeV^2...
- ...but it is slightly ($\sim 10\%$) violated, especially at low x.
- SLAC experiments 1967 – 1973; luckily run at $x \sim$ 0.2.
- Physics interpretation of scaling
  $\Longrightarrow$ Feynman's parton model (1969).

Feynman parton model

- Assume a coordinate system where a proton target has ∞ momentum. There: $M \sim 0$ and all 4-momenta: $P = (p, 0, 0, p)$.
- Every parton in a proton has 4-momentum uP where $0 < u < 1$.
- At large P, masses (m_p) and $\perp$ momenta of partons are ≈ 0.
- Thus a proton $\equiv$ a parallel stream of partons, each with 4-momentum uP.
- A scattered parton absorbs Q^2; thus:

$$(uP + Q)^2 = -m_p^2 \approx 0$$

- If $u^2 P^2 = u^2 M^2 \ll Q^2$ then we get: $u^2 P^2 + 2uPQ + Q^2 \approx 0$

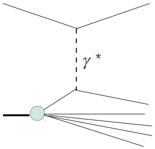
$$2uPQ + Q^2 = 0 \Longrightarrow u = \frac{-Q^2}{2PQ}$$

- In the lab. system: $P = (0, 0, 0, M)$ and $Q = (\bar{q}, \nu)$ and

$$u = \frac{-Q^2}{-2M\nu} = \frac{Q^2}{2M\nu} \equiv x$$

- Thus a meaning of x: a fraction of proton (three-)momentum carried by a struck quark (in an infinite proton momentum frame).

Feynman parton model,... cont'd



- However, free partons are not observed in nature; therefore it is assumed that a scattering is a two-step process
  1. a photon-parton collision which occurs in $t_1 \sim \hbar/\nu$
  2. a final state parton recombination into a hadron of mass W which occurs in

$$t_2 \sim \frac{\hbar}{W} \Longrightarrow \gamma_L \frac{\hbar}{W} = \frac{\nu}{W}\frac{\hbar}{W} = \frac{\nu\hbar}{W^2} = (W^2 = M^2 + 2M\nu - Q^2 \approx 2M\nu) = \frac{\nu\hbar}{2M\nu} = \frac{\hbar}{2M} \gg \frac{\hbar}{\nu} = t_1$$

- We then assume that the cross-section will depend on the initial state dynamics and will be almost independent of the final state interaction (a good approximation except when $\nu \sim M$).
- To summarise: in the Feynman model, the $ep \to eX$ interaction is an incoherent sum of electron-parton interactions. Scaling is a direct consequence of that (elastic scattering is described by one variable only).
- Important: during the photon-parton interaction, remaining (spectator) partons do not interact with each other!

How do we measure structure functions $F_{1,2}(x, Q^2)$

- Observables: E, E', ϑ measured in detectors.
- From the above observables we reconstruct kinematic variables, e.g.:

$$Q^2 \approx 4EE' \sin^2 \vartheta/2 \qquad\qquad x = \frac{Q^2}{2M\nu} = \frac{4EE' \sin^2 \frac{\vartheta}{2}}{2M(E - E')}$$

  These formulae valid for a fixed-target experiment; in a collider, definition of ν is different.

- $F_{1,2}(x, Q^2)$ is be determined for fixed (x, Q^2) values by a method similar to the Rosenbluth method of separating two form factors: measurements must be done at different values of energy, E.
- Traditionally, instead of F_1 one rather measures a function $R(x, Q^2)$ defined as:

$$R(x, Q^2) = \frac{F_2(x, Q^2)}{2xF_1(x, Q^2)} \left(1 + \frac{4M^2x^2}{Q^2}\right) - 1 = \frac{\sigma_L}{\sigma_T} \tag{18}$$

  where $\sigma_{L,T}$ are cross sections for γ^*- parton interactions for longitudinal/transverse polarised virtual photons. Observe that $\lim_{Q^2 \to 0} \sigma_L = 0$ and $\lim_{Q^2 \to 0} \sigma_T = \sigma^{\text{tot}}(\gamma\text{p})$ and that 50% of transverse photons is left- and 50% right polarised (electromagnetic interactions do not tell between left and right (they conserve parity)).

Outline

Partons – what are they? a) partons' spin

- Compare two equations: for scattering of electron on a point-like target of spin 1/2, charge ze and mass m, Eq. (10):

$$\left(\frac{d\sigma}{dQ^2}\right) = \frac{4\pi^2 z^2}{q^4}\left(\frac{E'}{E}\right)^2 \left(\cos^2\frac{\vartheta}{2} + \frac{Q^2}{2m^2}\sin^2\frac{\vartheta}{2}\right)$$

and for the inelastic electron scattering on a target of mass M, Eq. (17):

$$\frac{d^2\sigma}{dQ^2 dx} = \frac{4\pi\alpha^2}{Q^4}\frac{E'}{Ex}\left[F_2\cos^2\frac{\vartheta}{2} + \frac{Q^2}{2M^2x^2}2xF_1\sin^2\frac{\vartheta}{2}\right]$$

- Coefficients in front of $\cos^2\frac{\vartheta}{2}$ and $\sin^2\frac{\vartheta}{2}$ should be the same, thus:

$$z^2\frac{E'}{E} = \frac{1}{x}F_2, \qquad z^2\frac{E'}{E}\frac{Q^2}{2m^2} = \frac{1}{x}\frac{Q^2}{2M^2x^2}2xF_1$$

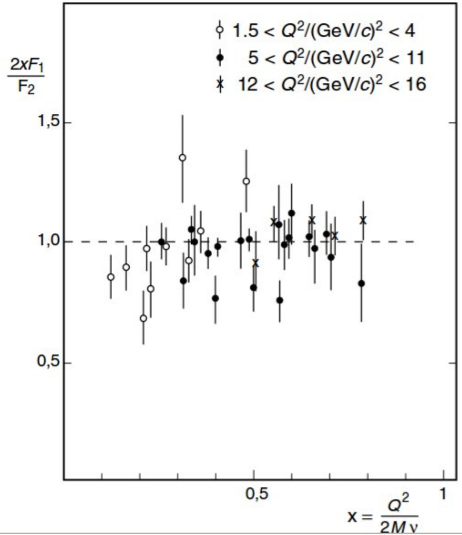
dividing the two equations by each other:

$$\frac{Q^2}{2m^2} = \frac{Q^2}{2M^2x^2}\frac{2xF_1}{F_2} \quad \Longrightarrow \quad \frac{2xF_1}{F_2} = 1 \quad (\text{if} \quad m = Mx) \tag{19}$$

This is the Callan–Gross relation, valid if scattering occurs on a point-like nucleon components, of spin 1/2 and "normal" magnetic moments: $\mu = (ze\hbar)/(2mc)$.

- Zero spin partons would have: $(2xF_1)/(F_2) = 0$
(F_1 = 0 and it corresponds to a magnetic interaction).

Partons – what are they? a) partons' spin,...cont'd



Evidence for partons' spin = 1/2

Figure from the book by B. Povh et al.

Partons – what are they? b) partons' charge

Let us now use the formula for an inelastic cross-section, Eq. (17):

$$\frac{d^2\sigma}{dQ^2dx} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1 - y - \frac{M^2y^2}{Q^2}\right)\frac{F_2(x,Q^2)}{x} + y^2F_1(x,Q^2)\right]$$

noticing that $\frac{M^2y^2}{Q^2} \approx 0$:

$$\frac{d^2\sigma}{dQ^2dx} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2(x,Q^2)}{x} + \frac{y^2}{2}\frac{2xF_1(x,Q^2)}{x}\right]$$

Now we take the $\vartheta \to 0$ limit of it:

$$\frac{d^2\sigma}{dQ^2dx} \to \frac{4\pi\alpha^2}{Q^4}\frac{F_2}{x} \qquad \Longrightarrow \qquad \frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\int\frac{F_2}{x}dx$$

But in the Rutherford scattering:

$$\left(\frac{d\sigma}{dQ^2}\right)_{\rm Ruth} \sim \frac{(Ze\cdot e)^2}{Q^4}$$

which means that $\int\frac{F_2}{x}dx$ must have a meaning of a sum of squares of parton charges. Thus $F_2^{\rm ep}(x)/x$ is expressed through quark densities in the proton, weighted by squares of charges.
Therefore:

$$F_2(x) = x\sum_{i=1}^{6}e_i^2q_i(x) \qquad (20)$$

Partons – what are they? b) partons' charge,...cont'd

Why only quarks and not gluons? And WHICH quarks (remember quark ($q\bar{q}$) pair sea)?

E.g. proton: p $\equiv (u, d, u\bar{u}, d\bar{d}, s\bar{s}, ...)$

$$F_2^{\rm ep}(x) = x\left\{\frac{4}{9}\left[u^p(x)+\bar{u}^p(x)\right]+\frac{1}{9}\left[d^p(x)+\bar{d}^p(x)\right]+\frac{1}{9}\left[s^p(x)+\bar{s}^p(x)\right]+...\right\}$$

Strong interactions do not see electric charges, i.e. for them a proton $\equiv$ a neutron or a "u" quark $\equiv$ a "d" quark; thus:

$$u^p \equiv d^n = u$$
$$d^p \equiv u^n = d$$
$$s^p \equiv s^n = s$$

(same for antiquarks).

Thus we get:

$$\frac{1}{x}F_2^{\rm ep} = \frac{4}{9}(u+\bar{u})+\frac{1}{9}(d+\bar{d}+s+\bar{s})...$$
$$\frac{1}{x}F_2^{\rm en} = \frac{4}{9}(d+\bar{d})+\frac{1}{9}(u+\bar{u}+s+\bar{s})... \qquad (21)$$

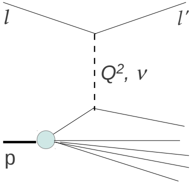
For clarity we neglect a contribution from $s\bar{s}$ (at $x \sim 0.03$ it is about 6% error):

$$\frac{1}{x}F_2^{\rm eN} = \frac{1}{x}\frac{\left(F_2^{\rm ep}+F_2^{\rm en}\right)}{2} = \frac{5}{18}(u+\bar{u}+d+\bar{d})$$

Partons – what are they? b) partons' charge,...cont'd

To determine separately a charge of a “u” and “d” quark, from the F_2 measurements we need another piece of information $\Longrightarrow$ neutrino scattering.

Summarising lepton–nucleon scattering:



l	l'	exchanged boson	interaction	example
$e^{\pm}$ $\mu^{\pm}$	$e^{\pm}$ $\mu^{\pm}$	γ γ	electromagnetic	$e^- p \to e^- X$ $\mu^+ p \to \mu^+ X$
ν_μ $\bar{\nu}_\mu$	μ^- μ^+	$W^{\pm}$ $W^{\pm}$	weak, charge currents (CC)	$\nu_\mu d \to \mu^- u$ $\bar{\nu}_\mu u \to \mu^+ d$
ν_μ $\bar{\nu}_\mu$	ν_μ $\bar{\nu}_\mu$	Z^0 Z^0	weak, neutral currents (NC)	$\nu_\mu d \to \nu_\mu d$ $\bar{\nu}_\mu u \to \bar{\nu}_\mu u$

In weak interactions, $W^{\pm}$, Z^0 do not couple to electric charges.
This means that:

$$F_2^{\nu \mathrm{p}} = 2x(d + \bar{u}) \qquad F_2^{\nu \mathrm{n}} = 2x(u + \bar{d})$$

or

$$F_2^{\nu \mathrm{N}} = x\left[u + \bar{u} + d + \bar{d}\right] \tag{22}$$

Partons – what are they? b) partons' charge,...cont'd

Finally we get for the nucleon:

$$\frac{F_2^{\mathrm{eN}}}{F_2^{\nu\mathrm{N}}} = \frac{1}{2}\left(e_u^2 + e_d^2\right) = \frac{5}{18} \approx 0.28 \quad \text{or more accurately :} \quad F_2^{\mathrm{eN}} \geq \frac{5}{18} F_2^{\nu\mathrm{N}}$$

EMC measurements @ CERN gave:

$$\frac{F_2^{\mathrm{eN}}}{F_2^{\nu\mathrm{N}}} = 0.29 \pm 0.02$$

We need to separately determine e_u and e_d. We have to use neutron (or deuteron) and assume that sea distributions ar the same for proton and neutron. Then any difference will result from valence partons.

$$\frac{F_2^{\mathrm{p}} - F_2^{\mathrm{n}}}{x} = e_u^2 u_v^p + e_d^2 d_v^p - e_u^2 u_v^n - e_d^2 d_v^n = \left(e_u^2 - e_d^2\right)\left(u_v - d_v\right)$$

$$\int \frac{F_2^{\mathrm{p}} - F_2^{\mathrm{n}}}{x} dx = \left(e_u^2 - e_d^2\right)\left[\int u_v dx - \int d_v dx\right]$$

EMC gave: $\left(e_u^2 - e_d^2\right) = 0.24 \pm 0.11$; but we also have: $\int u_v dx = 2$ and $\int d_v dx = 1$ and thus:

$$e_u = 0.64 \pm 0.05 \qquad e_d = 0.41 \pm 0.09$$

Partons – what are they?

If we identify partons with quarks, then the following integral:

$$\frac{18}{5}\int F_2^{\mathrm{eN}}(x)dx = \int F_2^{\nu \mathrm{N}}(x)dx = \int \left[u(x)+\bar{u}(x)+d(x)+\bar{d}(x)\right] x dx \qquad (23)$$

should be ≈ 1. Yet measurements give $\approx$ 0.5 !!! $\Longrightarrow$ gluons (a POSTULATE!)

Summary of basic quark properties

1. Approximate scale invariance, $F_2(x,Q^2) \approx F_2(x) \Longrightarrow$ the nucleon has point-like components
2. $2xF_1 \approx F_2 \Longrightarrow$ these components have spin (1/2)$\hbar$
3. Electromagnetic and weak interaction cross-sections point towards identyfying active partons with quarks of fractional charges
4. $\frac{18}{5}\int F_2^{\mathrm{eN}}(x)dx = \int F_2^{\nu \mathrm{N}}(x)dx \approx 0.5 \Longrightarrow$ quarks carry about 50% of nucleon momentum; the rest is attributed to gluons.

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{\nu^2+Q^2}} \approx \frac{h}{\nu} = \frac{h2Mx}{Q^2} \approx 10^{-3}\ \mathrm{fm}$$

(for $x = 0.2, Q^2$ = 100 (GeV/c)^2)

Quark model of hadrons

- All hadron properties should be reproducible from quark properties
- Charges - OK, np.: proton≡ (uud), $\frac{2}{3}e + \frac{2}{3}e - \frac{1}{3}e = +1e$
- Magnetic moments - OK:

TABLE 6.5 A comparison of the observed magnetic moments of the $\frac{1}{2}^+$ baryon octet, and the predictions of the simple quark model, Eqs. (6.25a) and (6.26), for $m_u = m_d = 336\ \text{MeV}/c^2$ and $m_s = 510\ \text{MeV}/c^2$

Particle	Prediction (μ_N)	Experiment (μ_N)
p(938)	2.79	2.793[a]
n(940)	−1.86	−1.913[a]
Λ(1116)	−0.61	−0.613 ± 0.004
Σ^+(1189)	2.69	2.458 ± 0.010
Σ^-(1197)	−1.04	−1.160 ± 0.025
Ξ^0(1315)	−1.44	−1.250 ± 0.014
Ξ^-(1321)	−0.51	−0.651 ± 0.003

[a] The errors on the proton and neutron magnetic moments are of the order 6×10^{-8} and 5×10^{-7} respectively.

- What about SPINS ?

Table from the book of Martin and Shaw