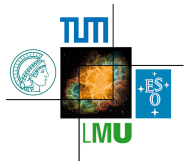
The COMPASS Hadron Program

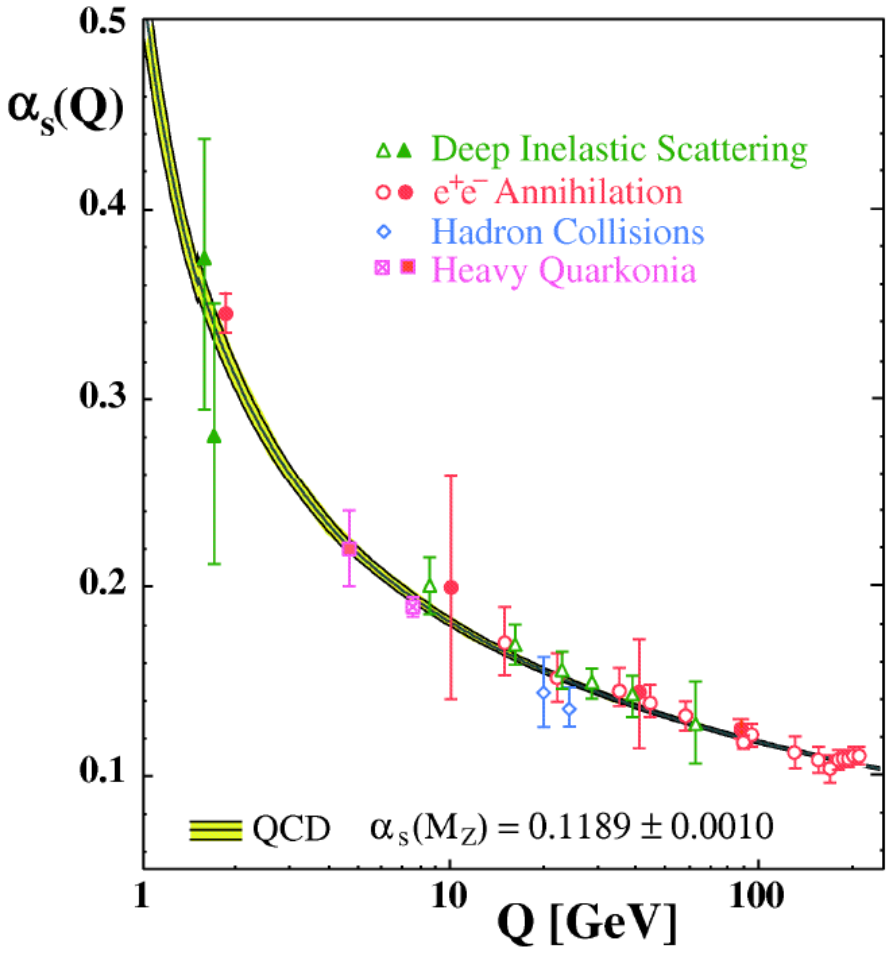
Sebastian Neubert

Technische Universität München

International Workshop on Hadron Structure and Spectroscopy 2012



From QCD to Hadron Physics



S. Bethke [arXiv:hep-ex/0606035v2]

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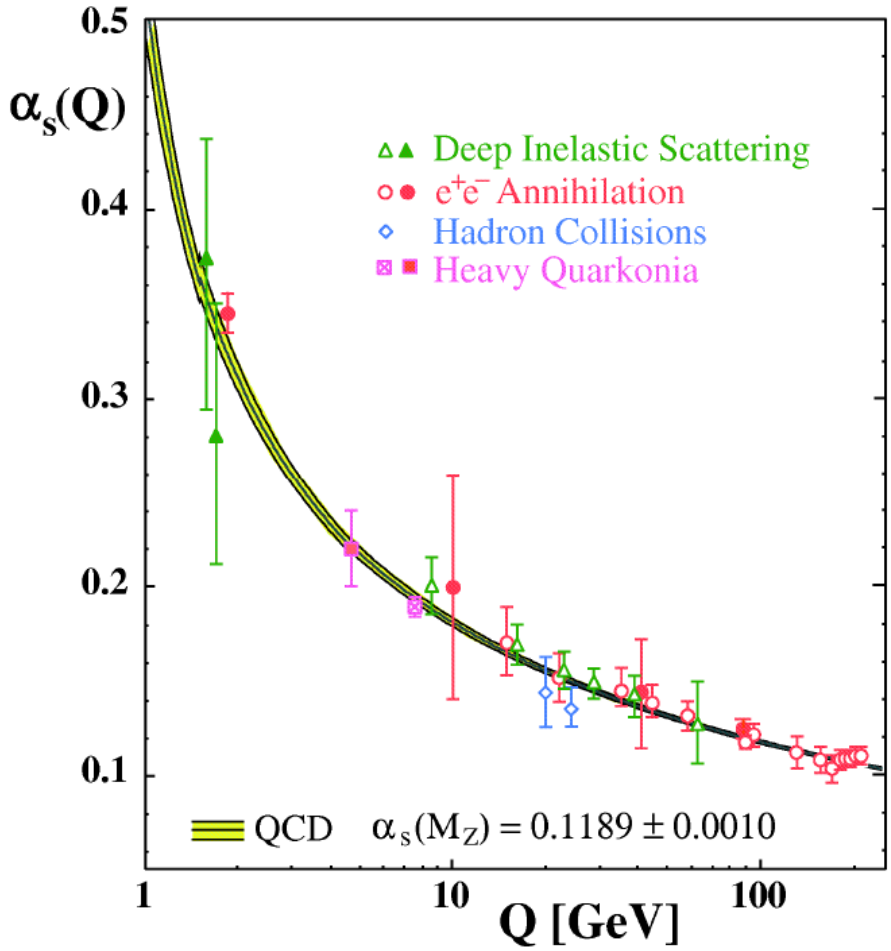

- Quarks&Gluons relevant DOF
- Perturbative QCD
- However: Hadronization

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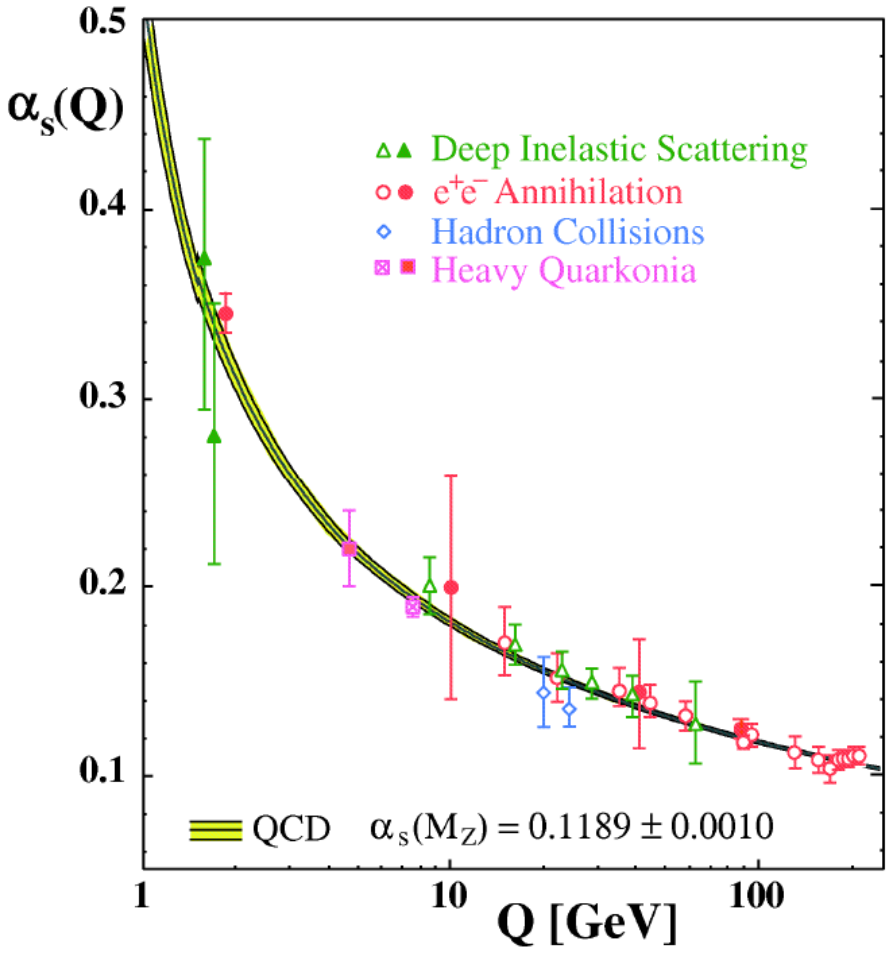
- Confinement
- Hadrons relevant DOF



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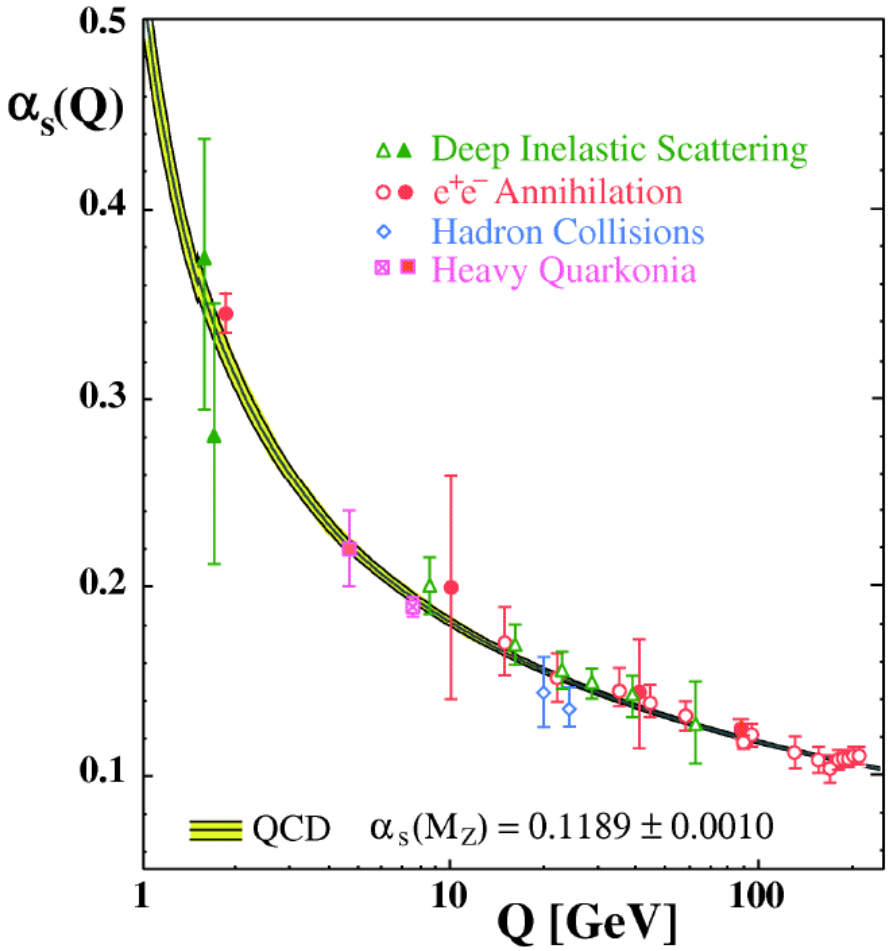
- Spontaneously broken chiral symmetry



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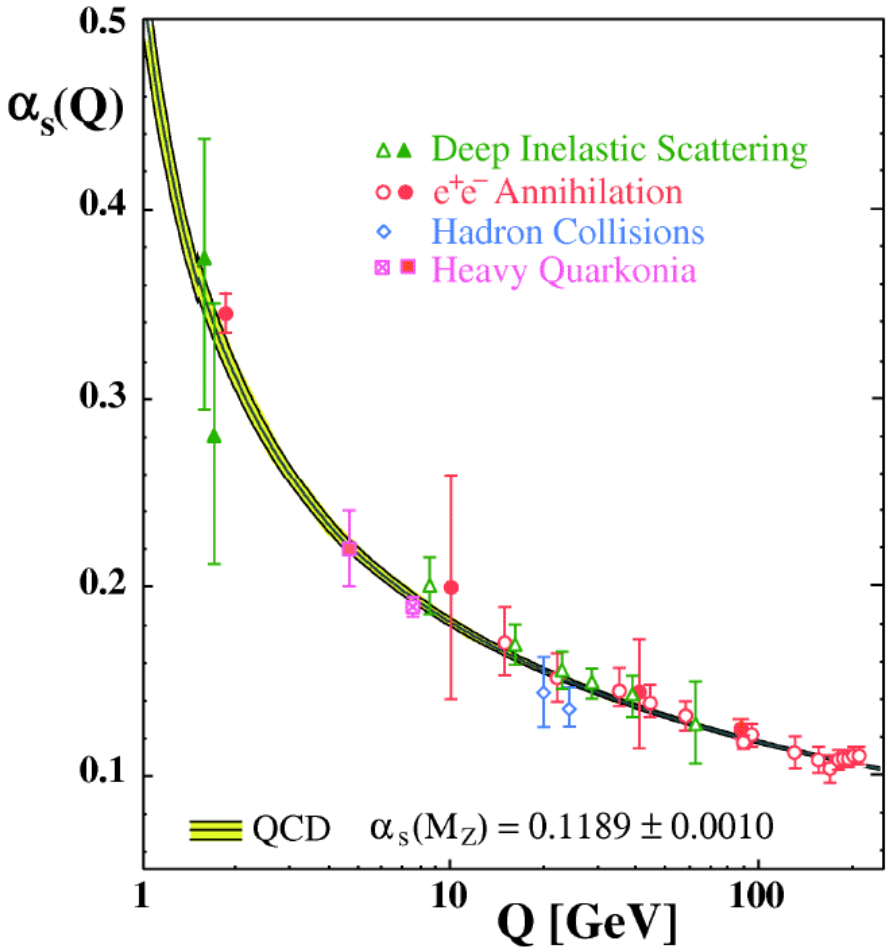

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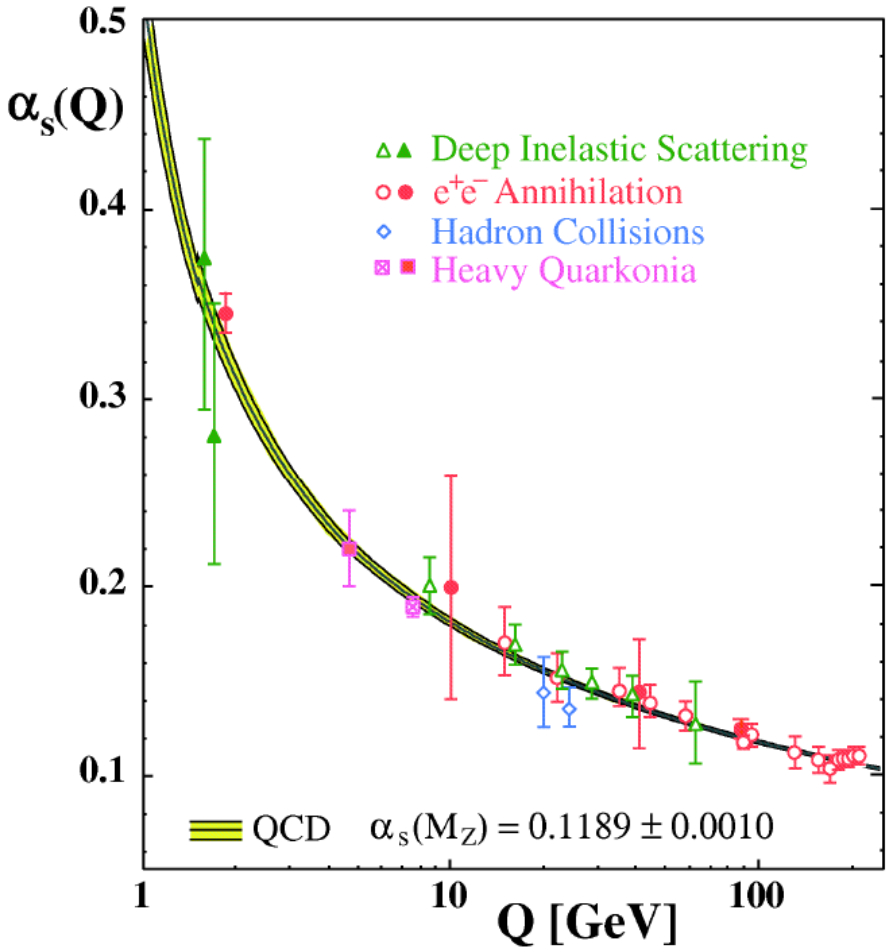
- Hadrons relevant DOF
- Spontaneously broken chiral symmetry
- χPT for slow pions
- Dynamics of excited states?



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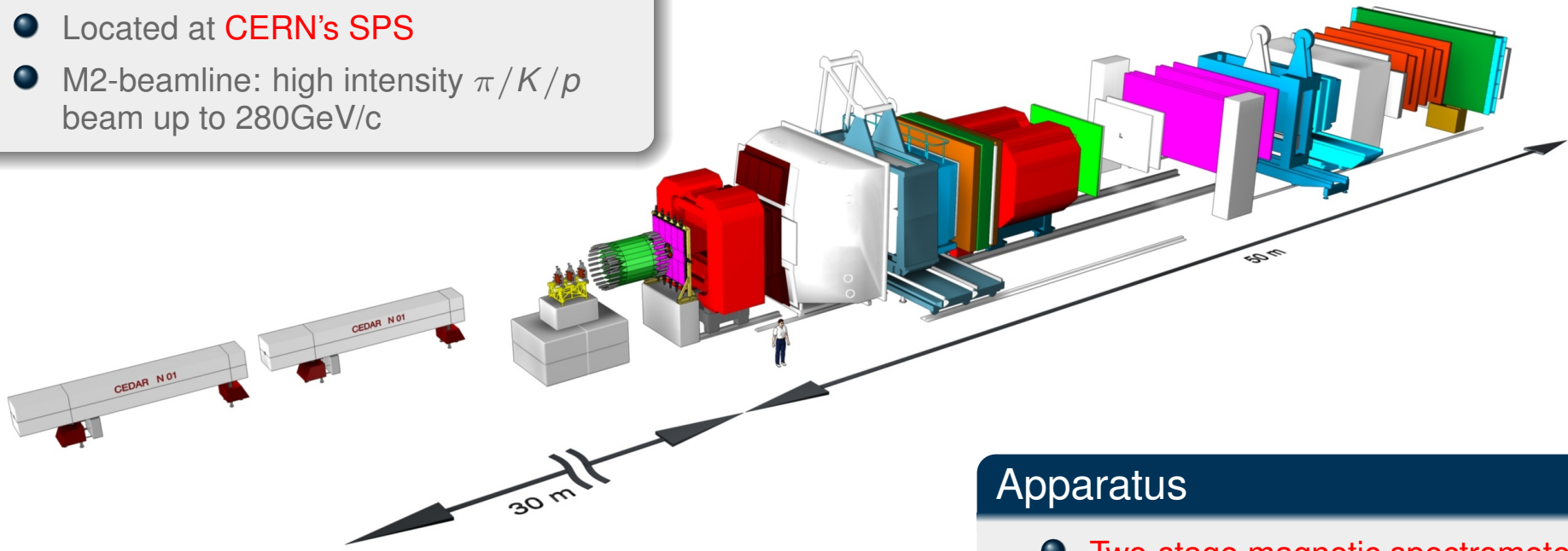
The COMPASS Hadron Setup

Spectrometer and Hadron Beam

Technische Universität München

Overview

- **CO**mmon **M**uon and **P**roton **A**pparatus for **S**tructure and **S**pectroscopy
- Located at CERN's SPS
- M2-beamline: high intensity $\pi/K/p$ beam up to 280GeV/c



Apparatus

- Two-stage magnetic spectrometer
- Large acceptance charged tracking
- Calorimetry (ECAL/HCAL)
- Kaon PID (CEDARs/RICH)

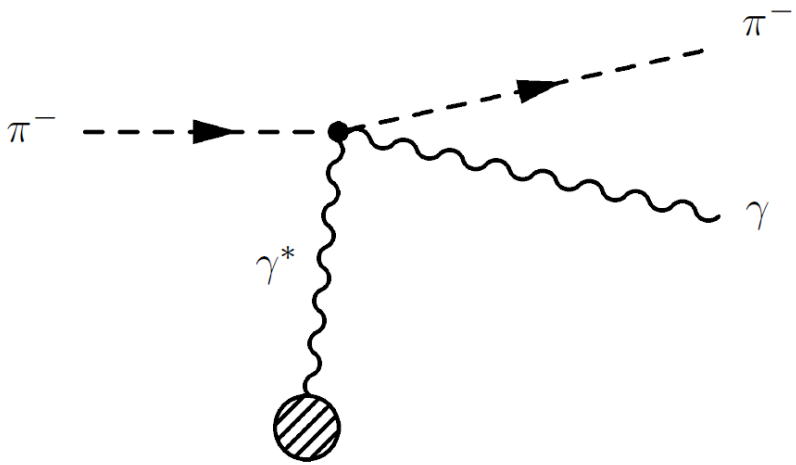
Tests of Chiral Dynamics

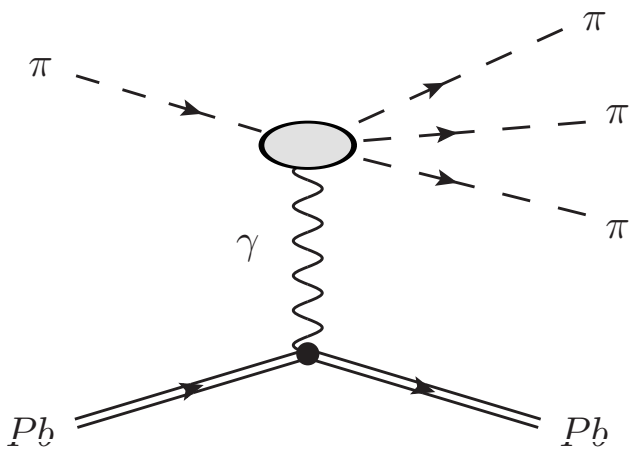
- Pion Polarizability
- **3π** Primakoff Production

Light Meson Spectroscopy

- $\pi^-\pi^-\pi^+$ and $\pi^-\pi^0\pi^0$
- $\eta\pi^-$ and $\eta'\pi^-$
- Status of the $J^{PC} = 1^{-+}$ Spin Exotic Partial Wave
- $\pi^-\pi^+\pi^-\pi^+\pi^-$
- $\pi\pi$ Production at Central Rapidities

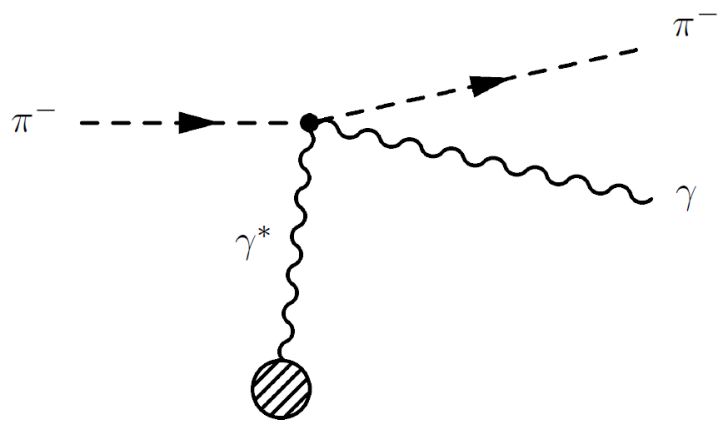
Tests of Chiral Dynamics





Pion Polarizability
in Primakoff–Compton Scattering

- Electric polarizability
  $\alpha_\pi \overset{\chi\text{pt}}{=} (2.93 \pm 0.5) \times 10^{-4}\,\text{fm}^3$
- Magnetic polarizability
  $\beta_\pi \overset{\chi\text{pt}}{=} (-2.77 \pm 0.5) \times 10^{-4}\,\text{fm}^3$
- Approx $\alpha_\pi + \beta_\pi = 0$



Pion Polarizability

in Primakoff–Compton Scattering

Dedicated Talk

J. Friedrich,
Wed. 18th, 09:00h

- Electric polarizability
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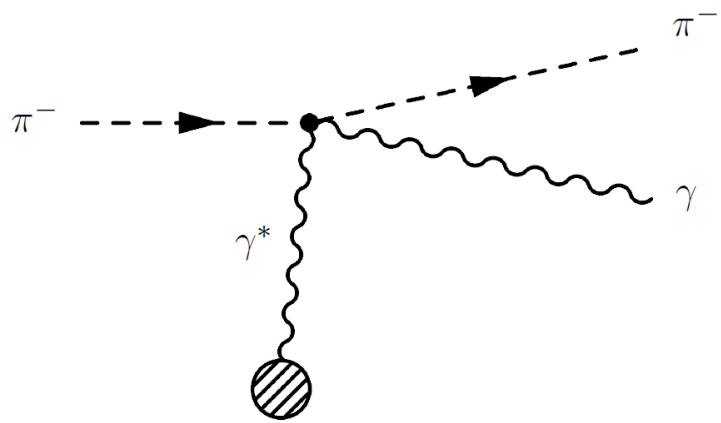
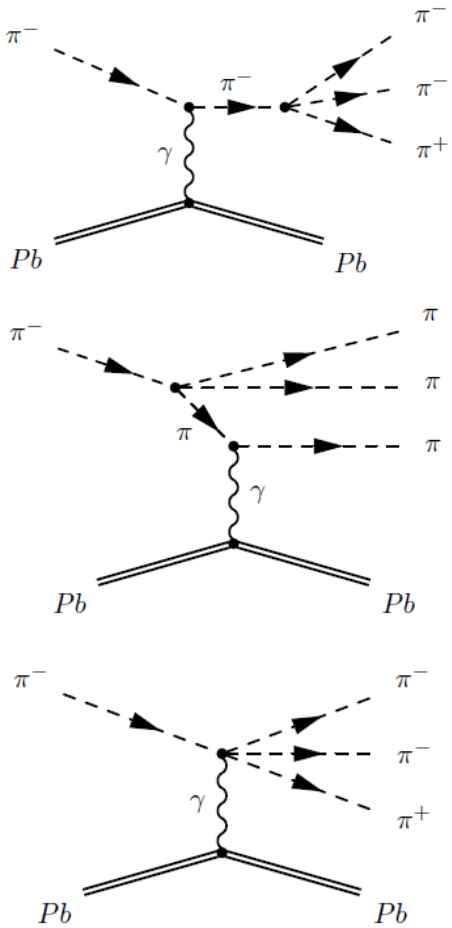


$$\frac{d\sigma_{\pi\gamma}}{dE_\gamma} = \frac{d\sigma_{\text{point}}}{dE_\gamma} + \frac{d\sigma(\beta_\pi)}{dE_\gamma}$$

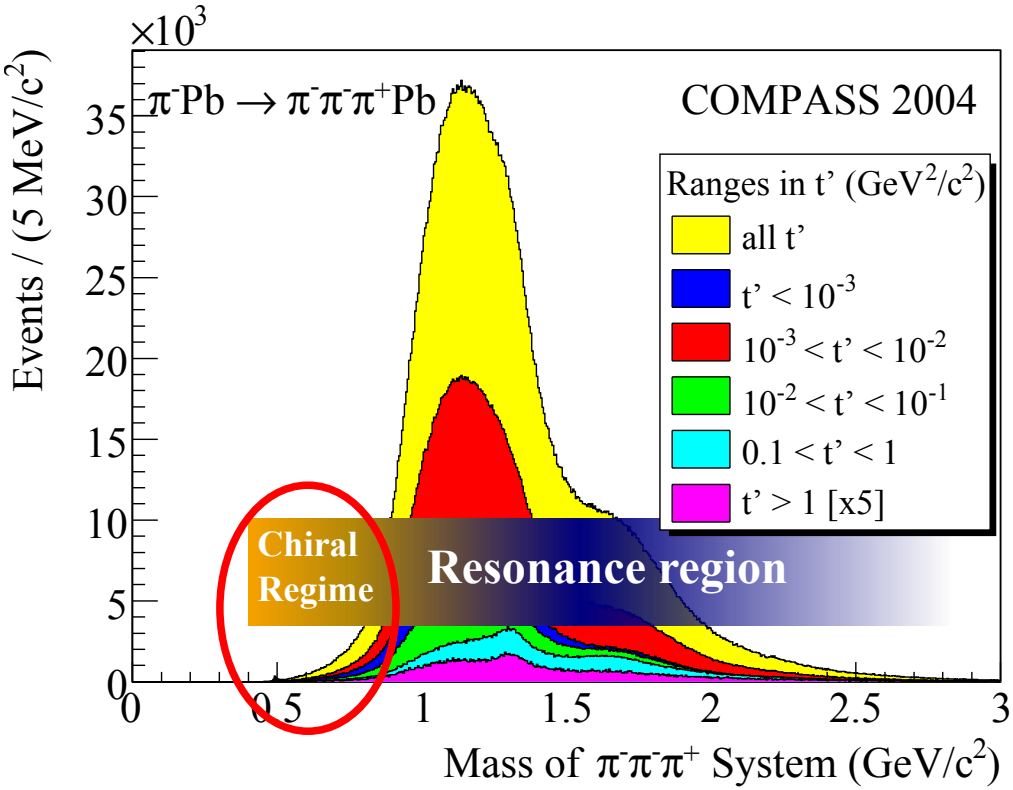
$$= C_1 \cdot \frac{E_{\text{beam}}}{E_\pi E_\gamma} + C_2 \cdot \frac{E_\gamma}{E_{\text{beam}}^2} \cdot \beta_\pi$$

Primakoff 3π Spectral Function from χPT

[arXiv:1111.5954v2] PRL accepted



- Weizsäcker-Williams:
  heavy nucleus acts as a quasi-real photon source
- χPT amplitude included in PWA
- $\Rightarrow \gamma\pi^- \to \pi^-\pi^+\pi^-$ absolute cross section



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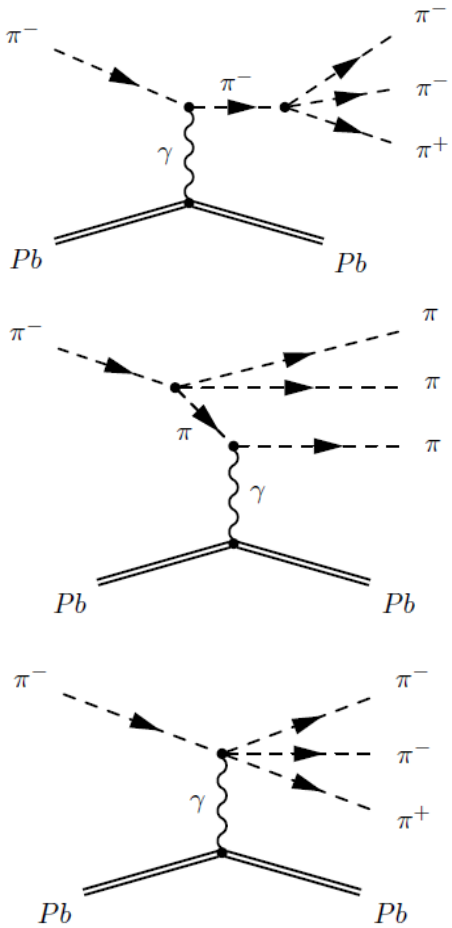

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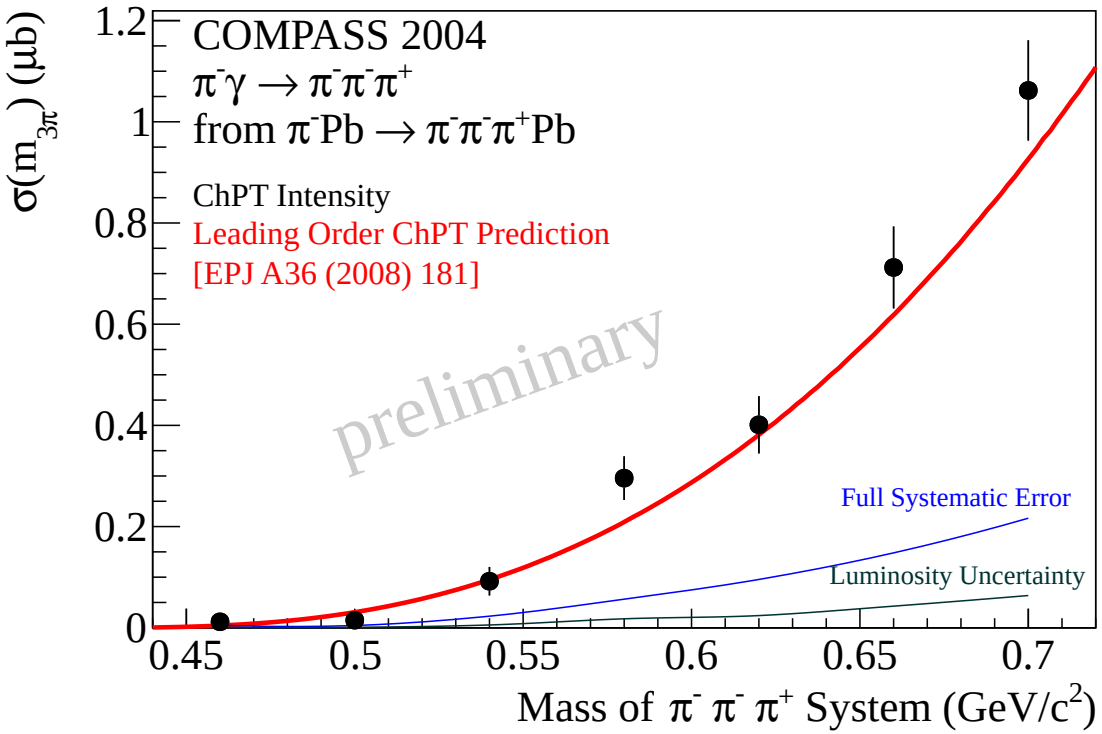
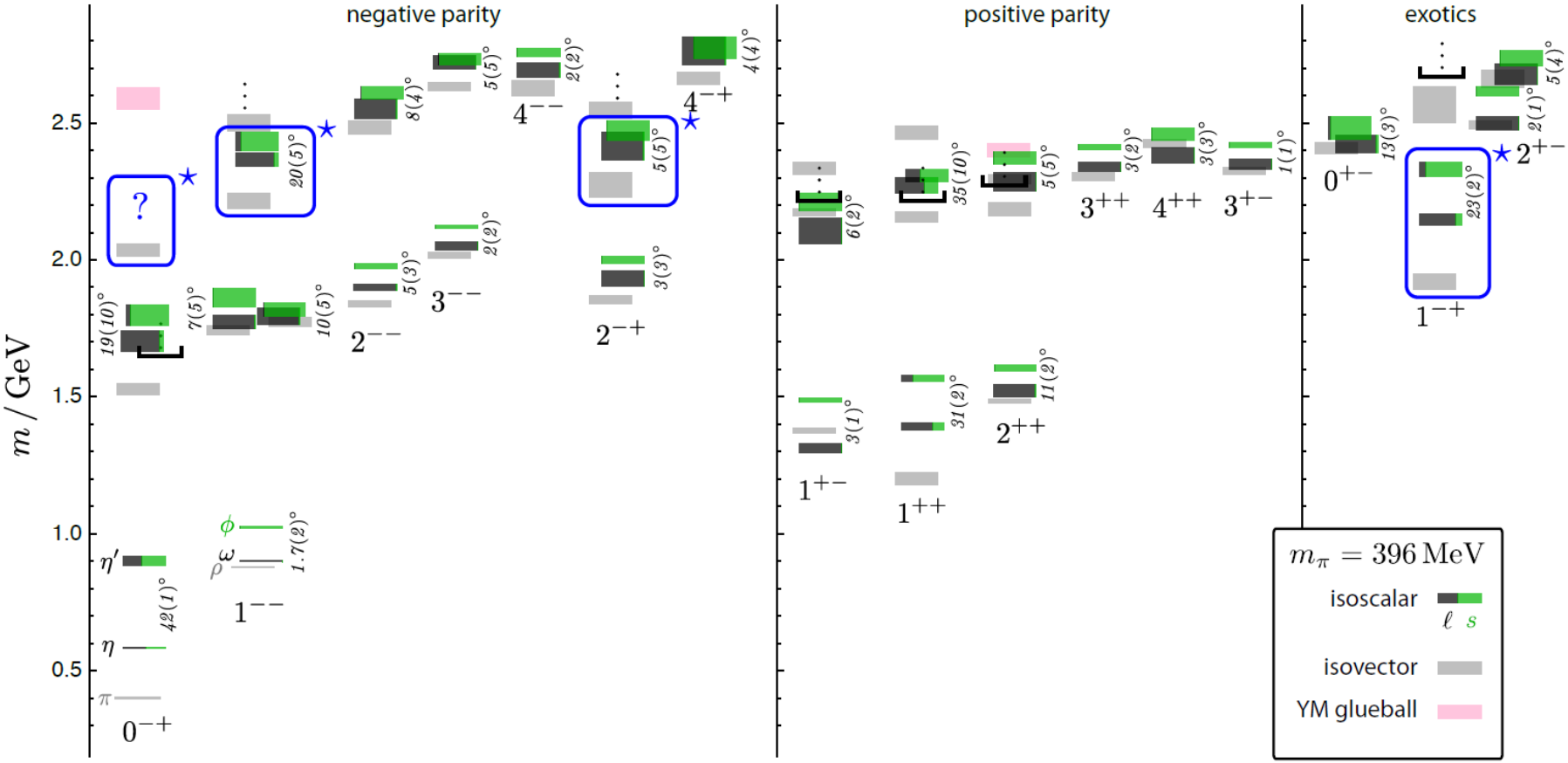


Light Meson Spectroscopy Search for States beyond the Constituent Quark Model

Light Meson Spectrum on the Lattice

Dudek et al. [arXiv:1106.5515v1] (@ $m_\pi = 400$MeV)



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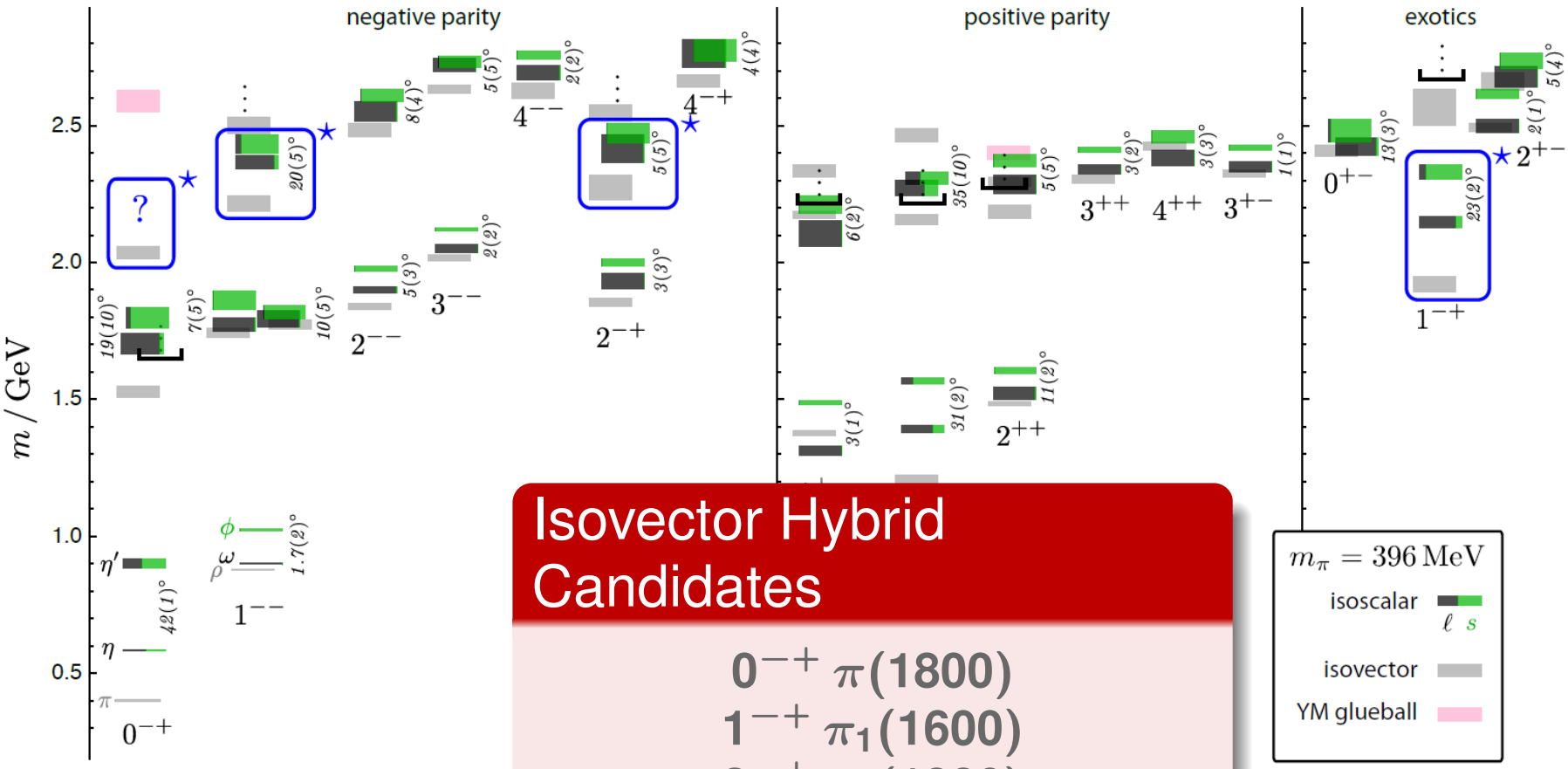


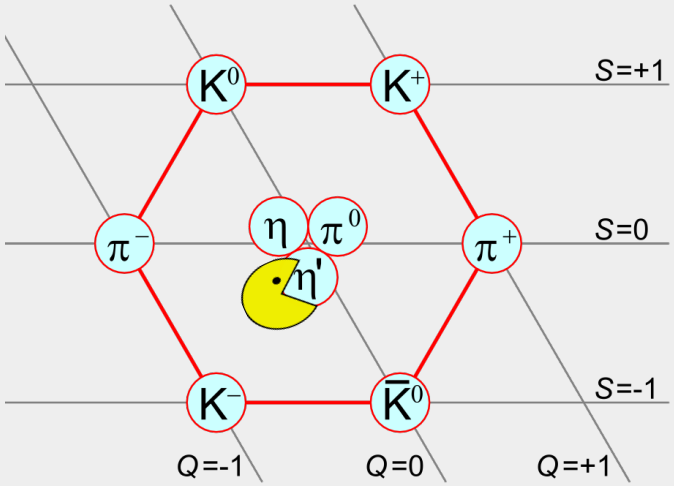
Isovector Hybrid Candidates

0^{-+} $\pi(1800)$
1^{-+} $\pi_1(1600)$
2^{-+} $\pi_2(1880)$

Perspectives beyond the Quark Model

Chiral Symmetry Breaking, Gluonic DoF ...

Octet: Goldstone-Bosons of Chiral Symmetry breaking



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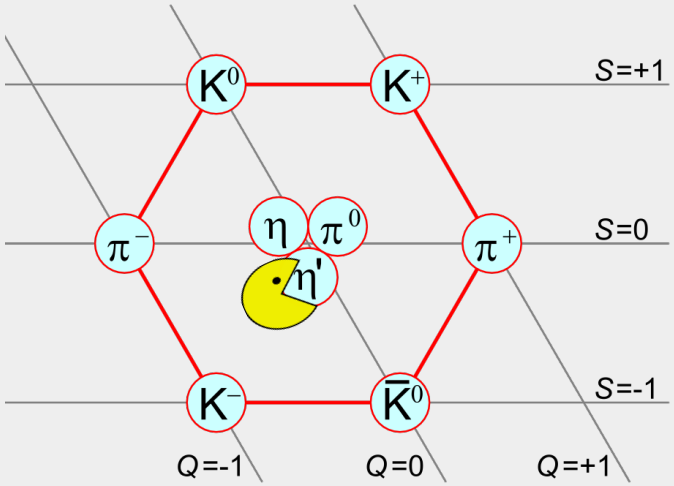
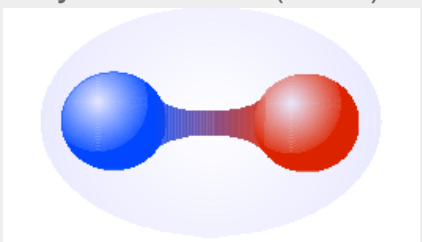


Exited States: Flux Tube Model

Isgur, Paton Phys. Rev. D31(1985)2910



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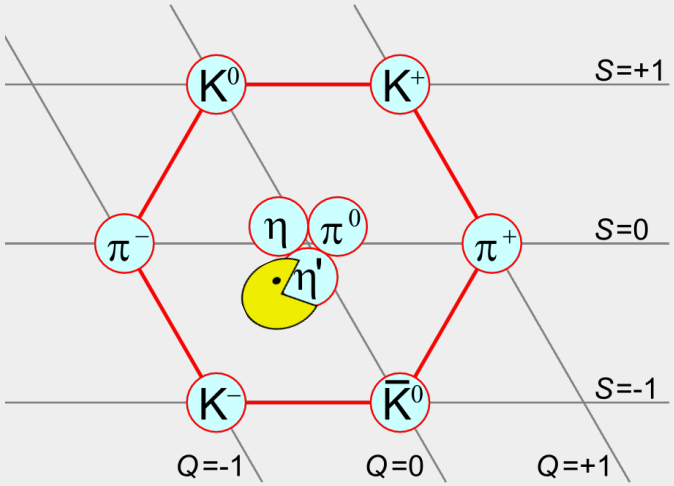

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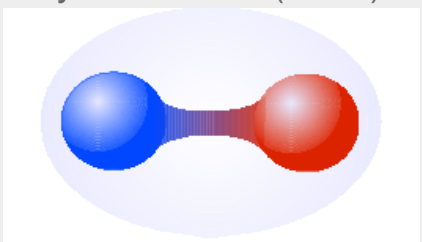
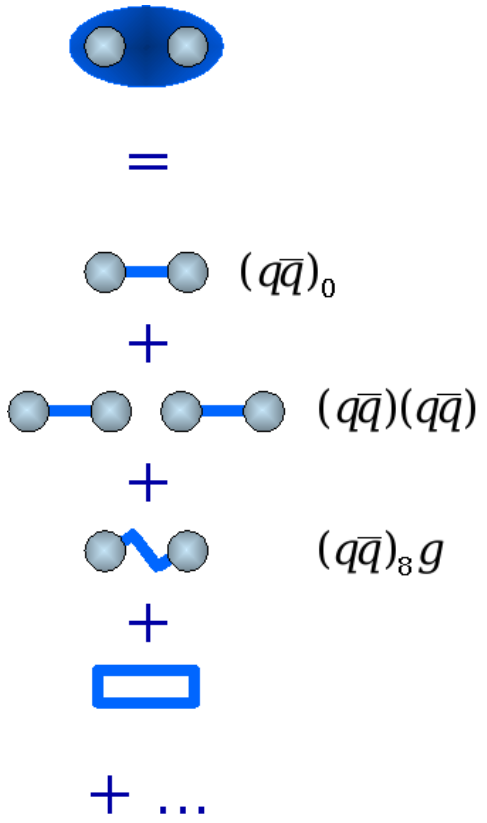
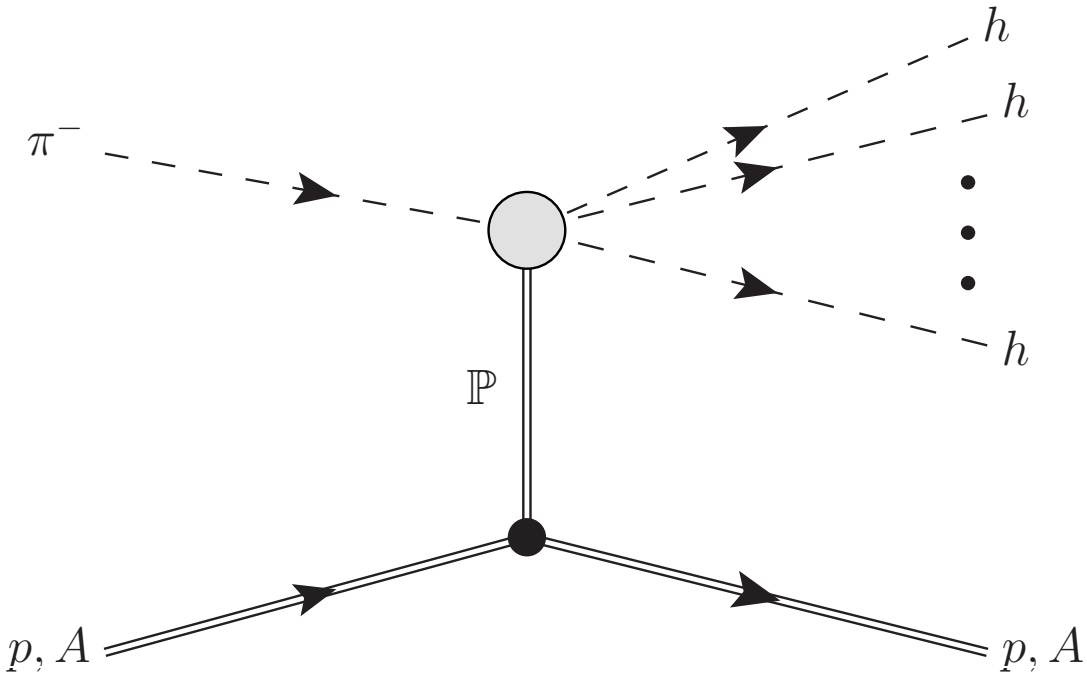


QCD-allowed contributions to meson spectrum:

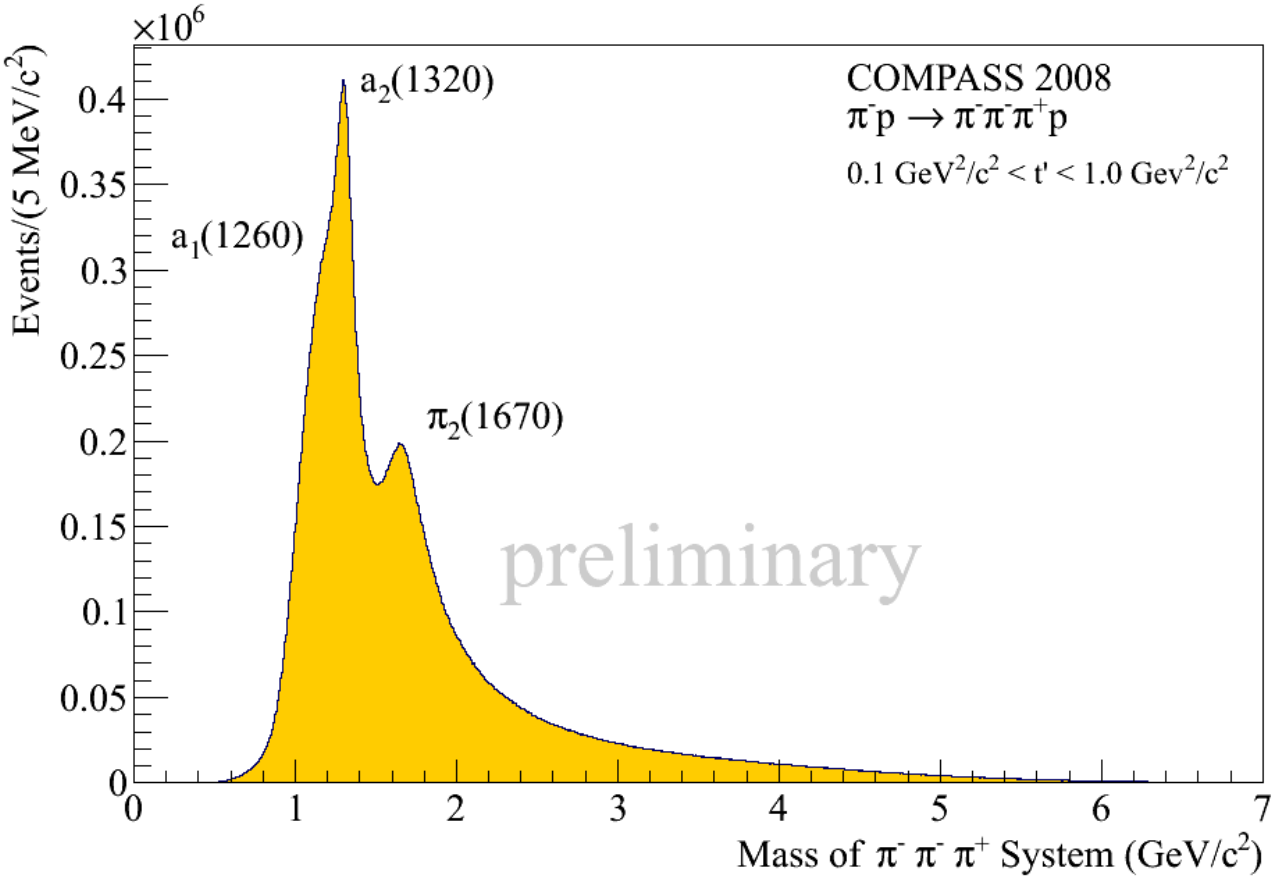


Isovector Mesons
Diffractive Pion Dissociation

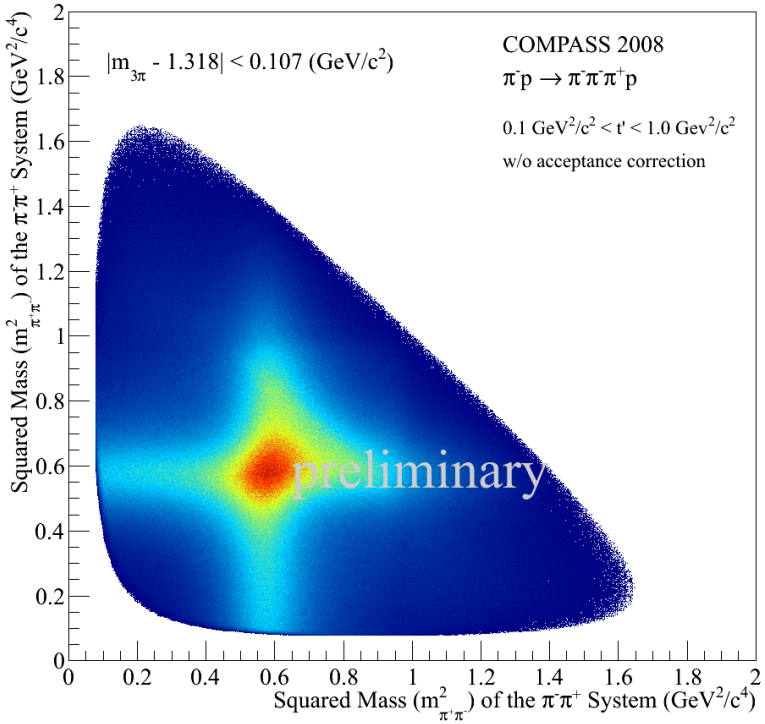


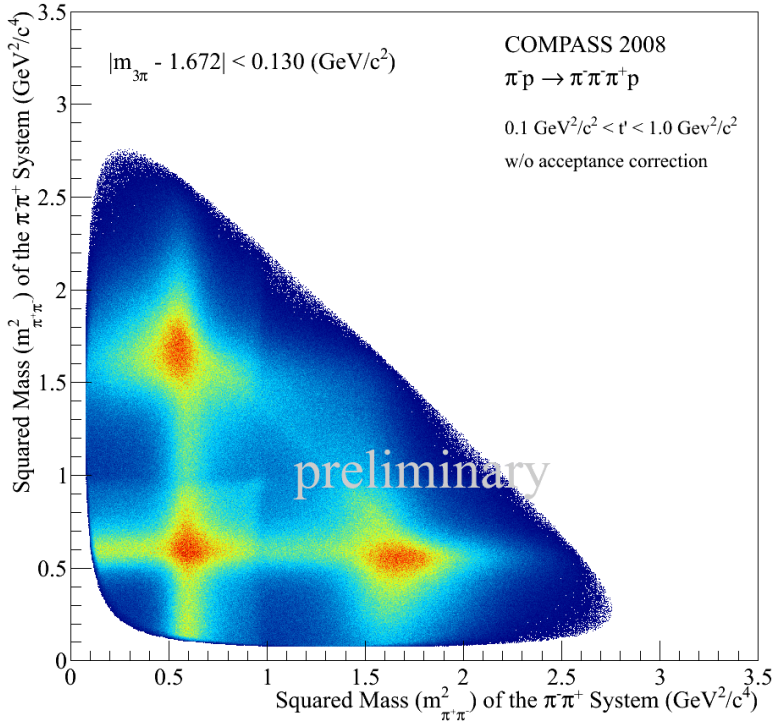
Invariant Mass Spectrum of $\pi^-\pi^-\pi^+$ (2008)

- 190 GeV/c hadron beam → $96\%\pi^-, 3.5\%K^-, 0.5\%\bar{p}$
- 40cm liquid hydrogen target
- $0.1\text{GeV}^2/c^2 < t' < 1.0\text{GeV}^2/c^2$
- ∼50M exclusive events (2008)



3π Dalitz Plots





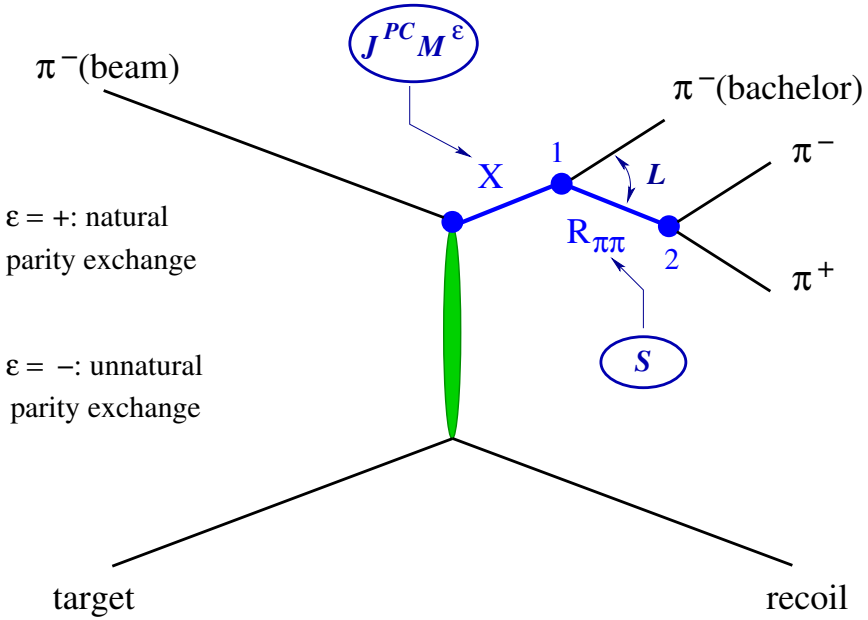
Left: Dalitz plot for $a_2(1320)$, events selected by $\pm\Gamma_0$ around a_2 mass.
Right: Dalitz plot for $\pi_2(1670)$ with $\pm 0.5\Gamma_0$.

- Input to PWA per mass bin: one Dalitz plot + 3 angles = 5 variables

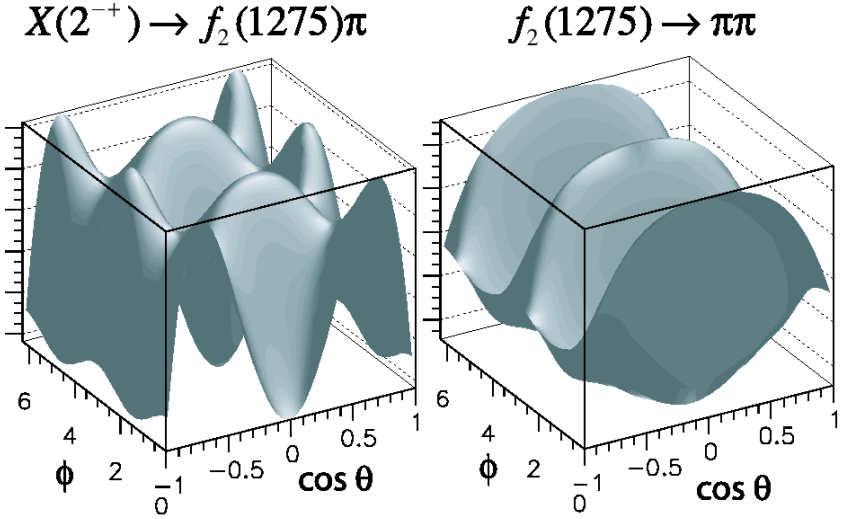
Decay Parameterization: The Isobar Model

Chain of successive 2-body decays

Model n-body decay by a chain of successive 2-body decays:



Example angular distributions:



- For fixed n-body mass m there are $3n - 4$ parameters (angles, intermediate state masses)
- Parameterization of isobar subsystems

PWA Formalism Redux

2-Stage Isobar-Model Fit

Mass-Independent PWA

- Fit angular distributions + isobar systems in independent mass bins

$$\mathcal{I}(\tau, m, t) = \sum_{\epsilon=\pm 1} \sum_{r=1}^{N_r} \left| \sum_i T_{ir}^{\epsilon}(m) \, f_i^{\epsilon}(t') \, \psi_i^{\epsilon}(\tau, m) \right|^2$$

- Production amplitude
- t'-dependence (or t'-binned analysis)
- Decay amplitude (Helicity formalism, reflectivity basis)

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- Production amplitude
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- Decay amplitude (Helicity formalism, reflectivity basis)

Mass-Dependent χ^2 fit $\rightarrow$ Extract Resonance Parameters

- Parameterization of spin-density matrix elements $\sum_r T_{ir}^{\epsilon} T_{jr}^{\epsilon *}(m_x)$
- Takes into account interference terms
- Coherent background for some waves

Intensities of dominant J^{PC} states

First results from mass independent PWA (2008)

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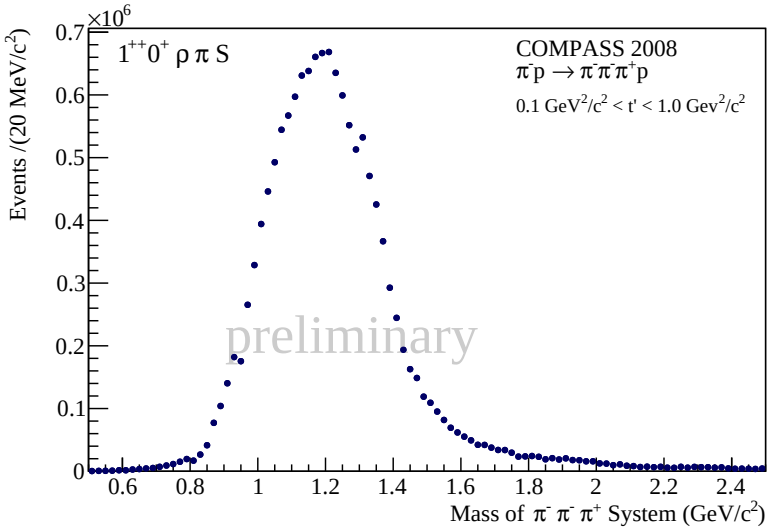

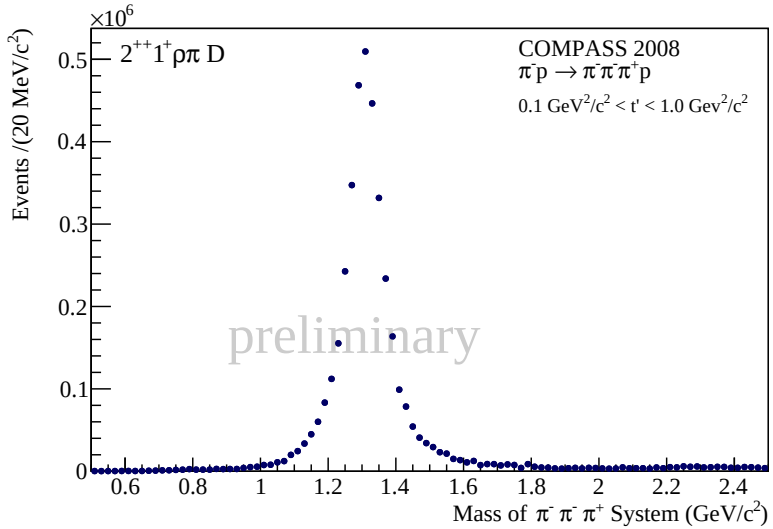

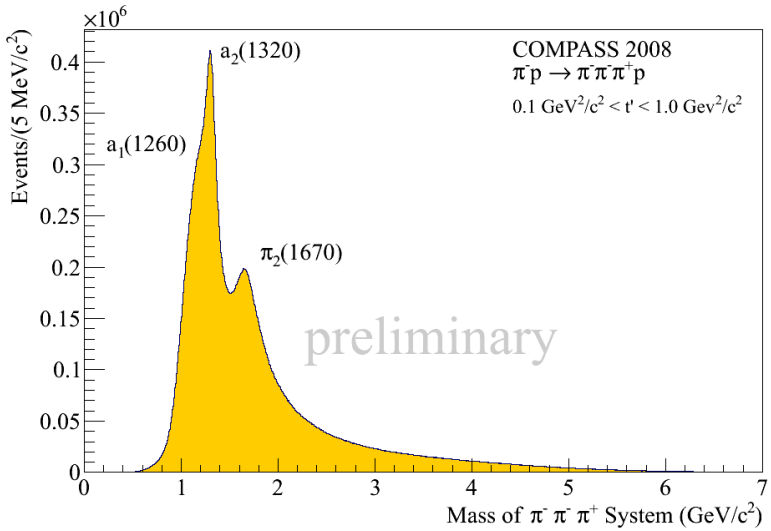

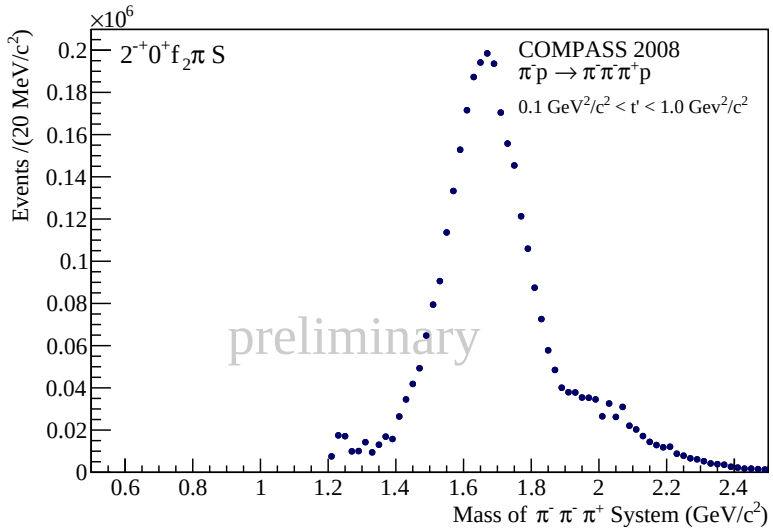

Comparison $\pi^-\pi^+\pi^-$ vs $\pi^-\pi^0\pi^0$

Partial wave intensities normalized to a_2

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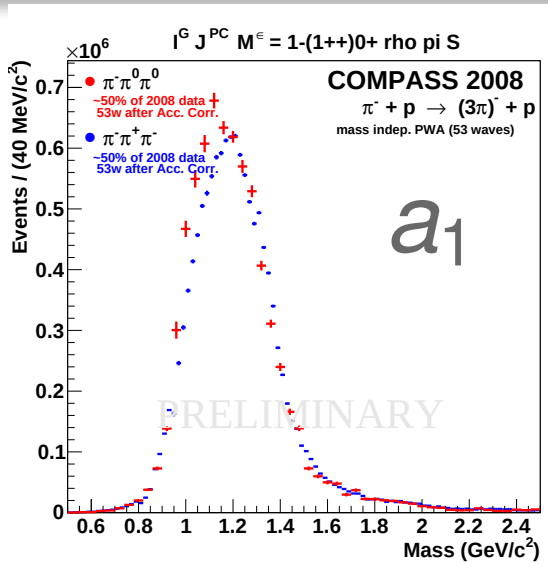
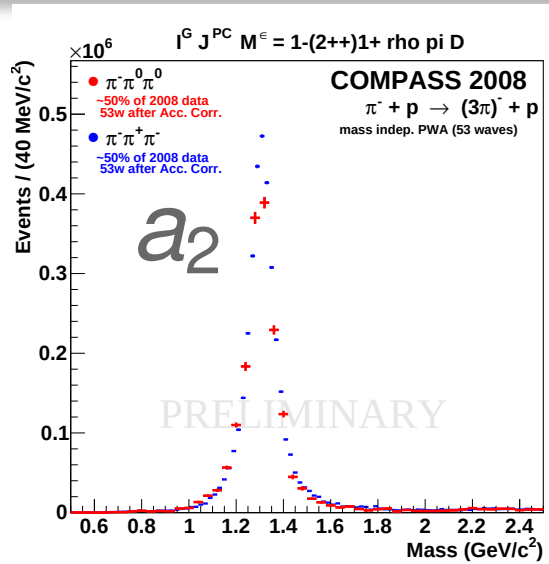
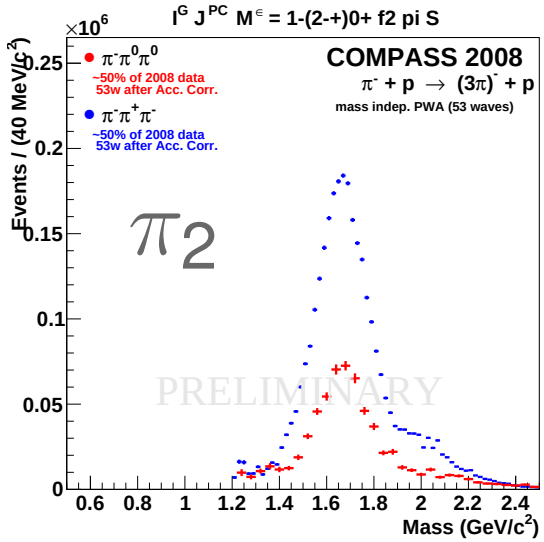




Charged/Neutral Pions

- Channels probe different parts of spectrometer
- Qualitative agreement
- Isospin symmetry



Systematic Improvements of the Model

WORK IN PROGRESS

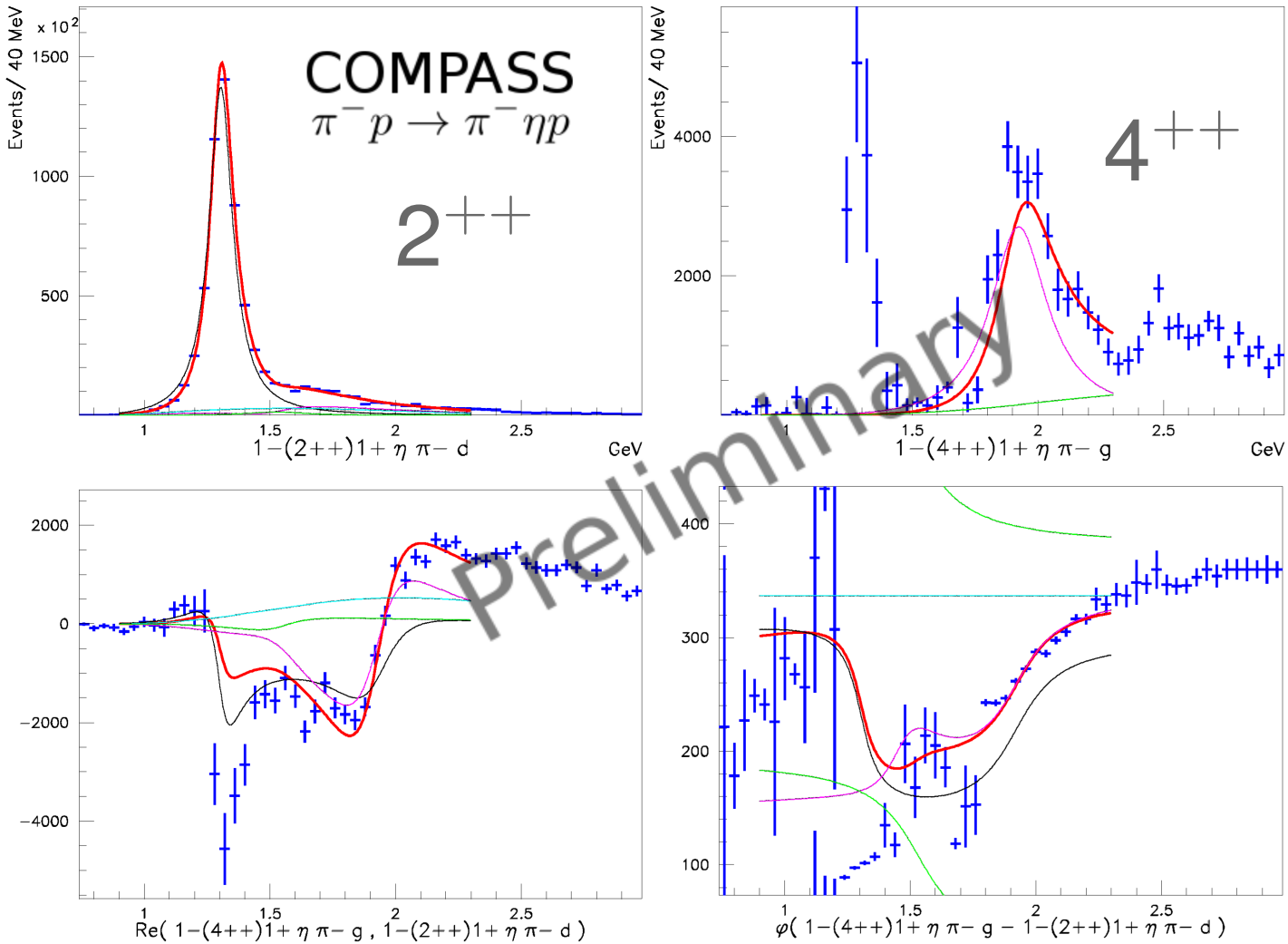
- Exploit full t'-dependence
  $\rightarrow$ 2D partial-wave decomposition in small m and t' bins
- Model non-resonant contributions (Deck effect)
- Semi-model-independent isobar parameterizations
  $\rightarrow$ extract $(\pi\pi)_{S-\mathrm{wave}}$ from 3π data
- Add chiral amplitudes where possible
- Improve fitting procedures (thresholds, model selection, ...)

$$\pi^- + p \to \eta\pi + p$$

$$\pi^- + p \to \eta'\pi + p$$

$\pi^- + p \to \eta\pi + p$ *D*- vs *G*-wave

Partial Wave Analysis



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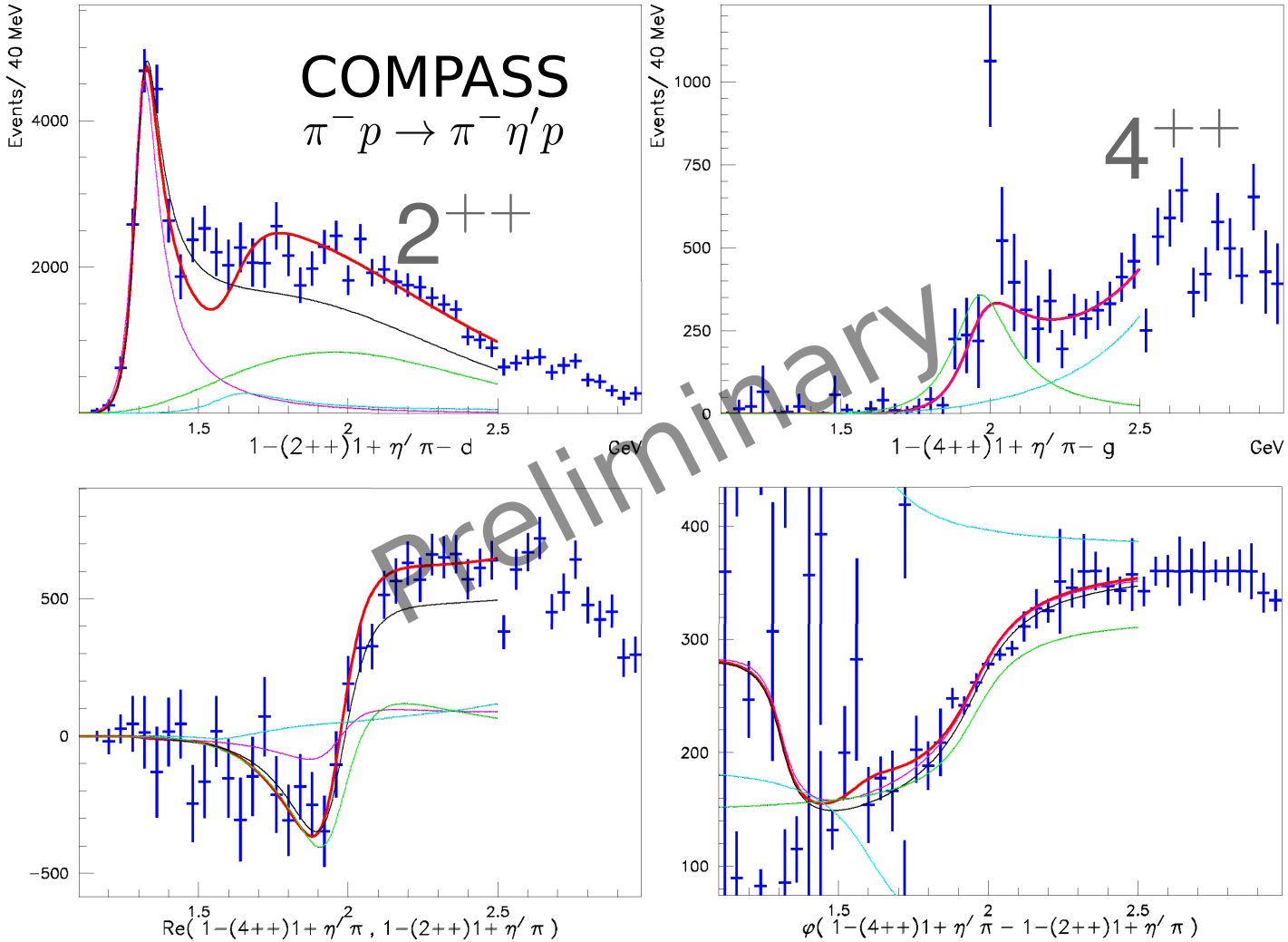
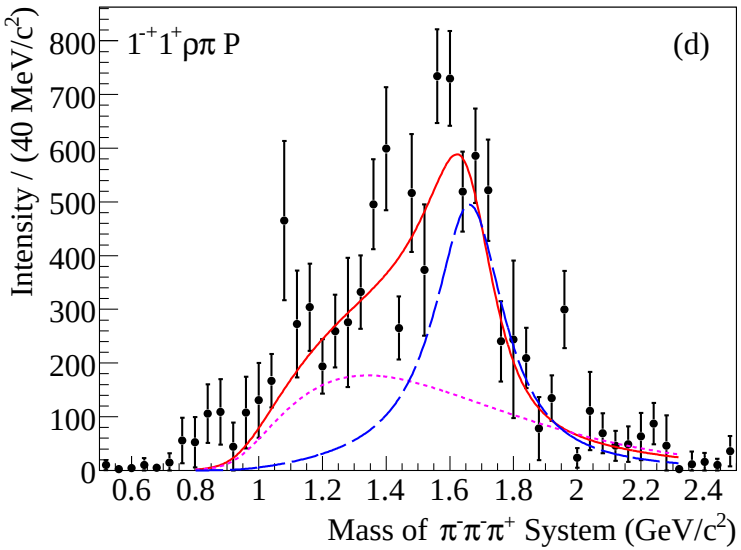


Status of the Spin Exotic $\pi_1(1600)$

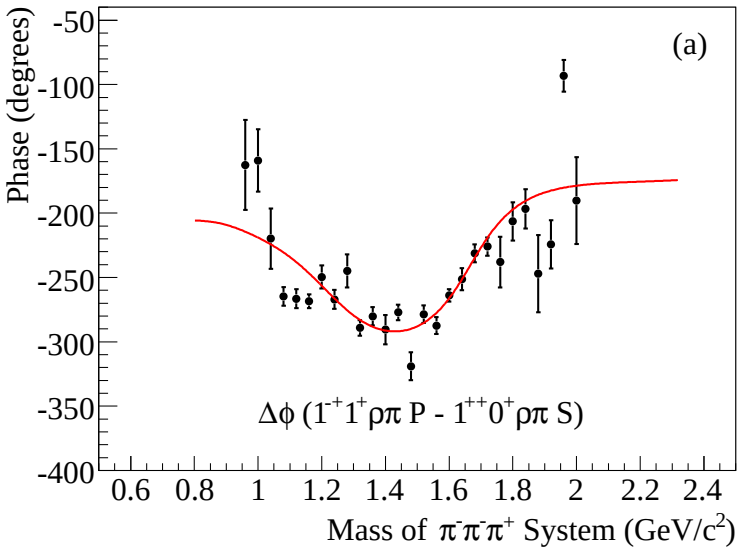
Exotic Signatures

- Over-filled multiplets (too many states)
- Isospin exotics: “forbidden” decays
- Spin exotics: $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-} \ldots$ forbidden in $q\bar{q}$

COMPASS (2004): $\pi^- \mathrm{Pb} \rightarrow \pi^-\pi^+\pi^- \mathrm{Pb}$ $\sim$ 400 000 events



[PRL **104** (2010) 241803]



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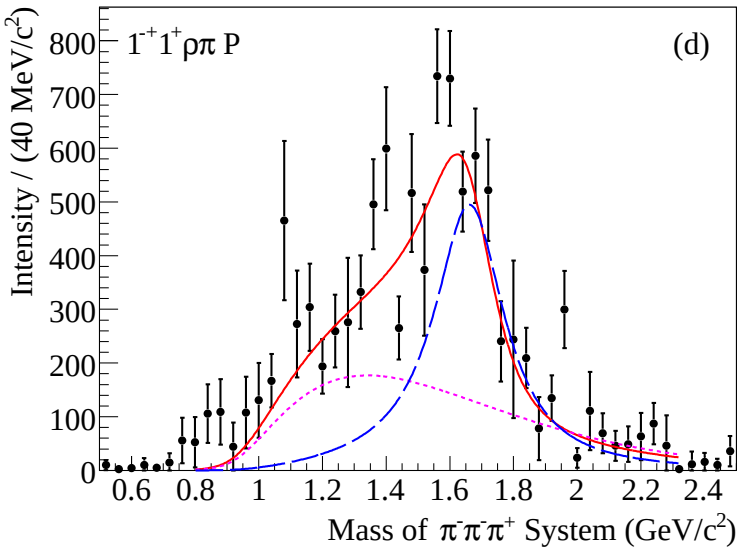
- Spin exotics: $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-} \ldots$ forbidden in $q\bar{q}$

COMPASS (2004): $\pi^- \mathrm{Pb} \rightarrow \pi^-\pi^+\pi^- \mathrm{Pb}$ $\sim 400\,000$ events



[PRL **104** (2010) 241803]

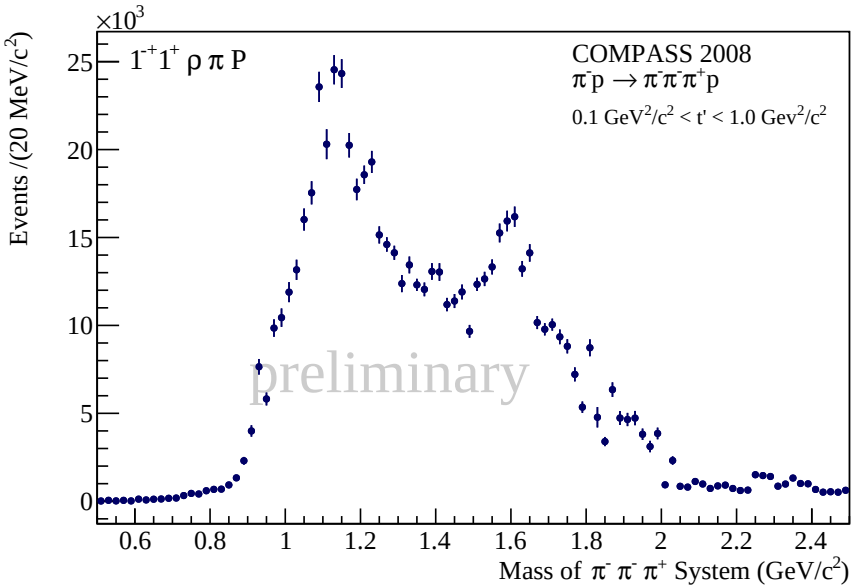
Spin Exotic $\pi_1(1600)$

- Significant 1^{-+} amplitude consistent with resonance at $\sim 1.7\,\mathrm{GeV}/c^2$
- No leakage observed ($< 5\%$)
- BW for $\pi_1(1600)$ + background:
  $M = (1.660 \pm 0.010\,^{+0.000}_{-0.064})\,\mathrm{GeV}/c^2$
  $\Gamma = (0.269 \pm 0.021\,^{+0.042}_{-0.064})\,\mathrm{GeV}/c^2$

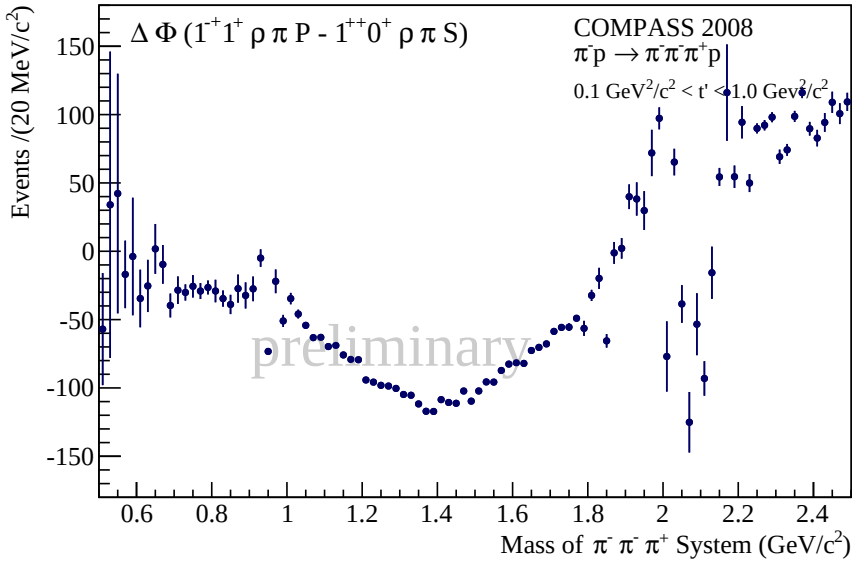
The spin exotic $J^{PC} = 1^{-+}$ $\rho\pi$ P-wave

1^{-+} in $\pi^-\pi^+\pi^-$

Intensity (statistical errors only)

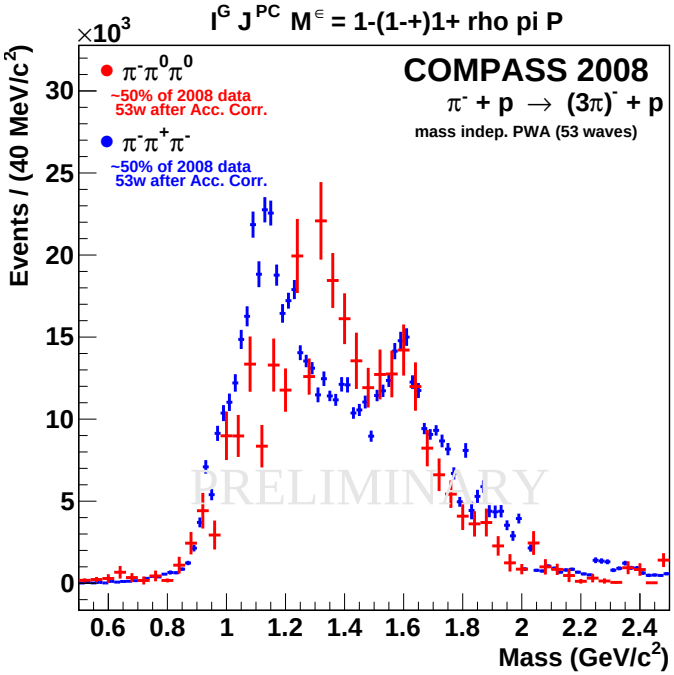


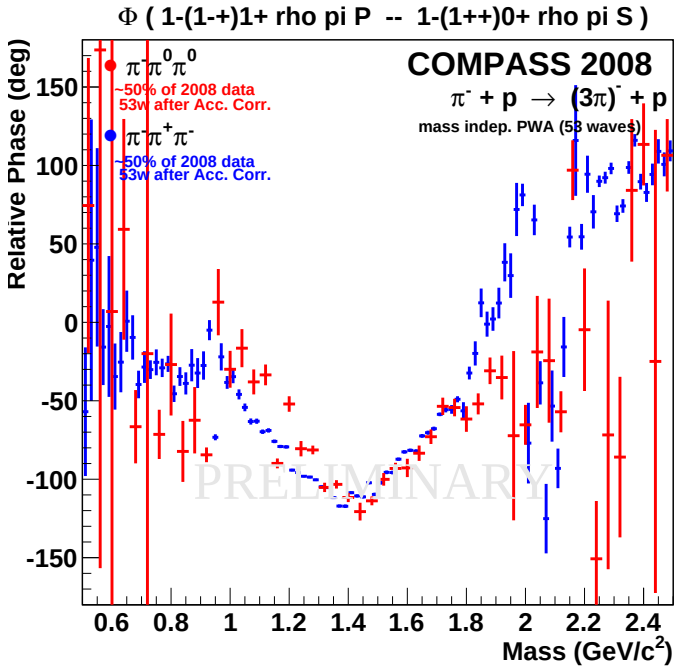
Phase motion vs 1^{++} $\rho\pi$ S-wave



$1^{-+}\rho\pi$ in $\pi^-\pi^+\pi^-$ vs $\pi^-\pi^0\pi^0$

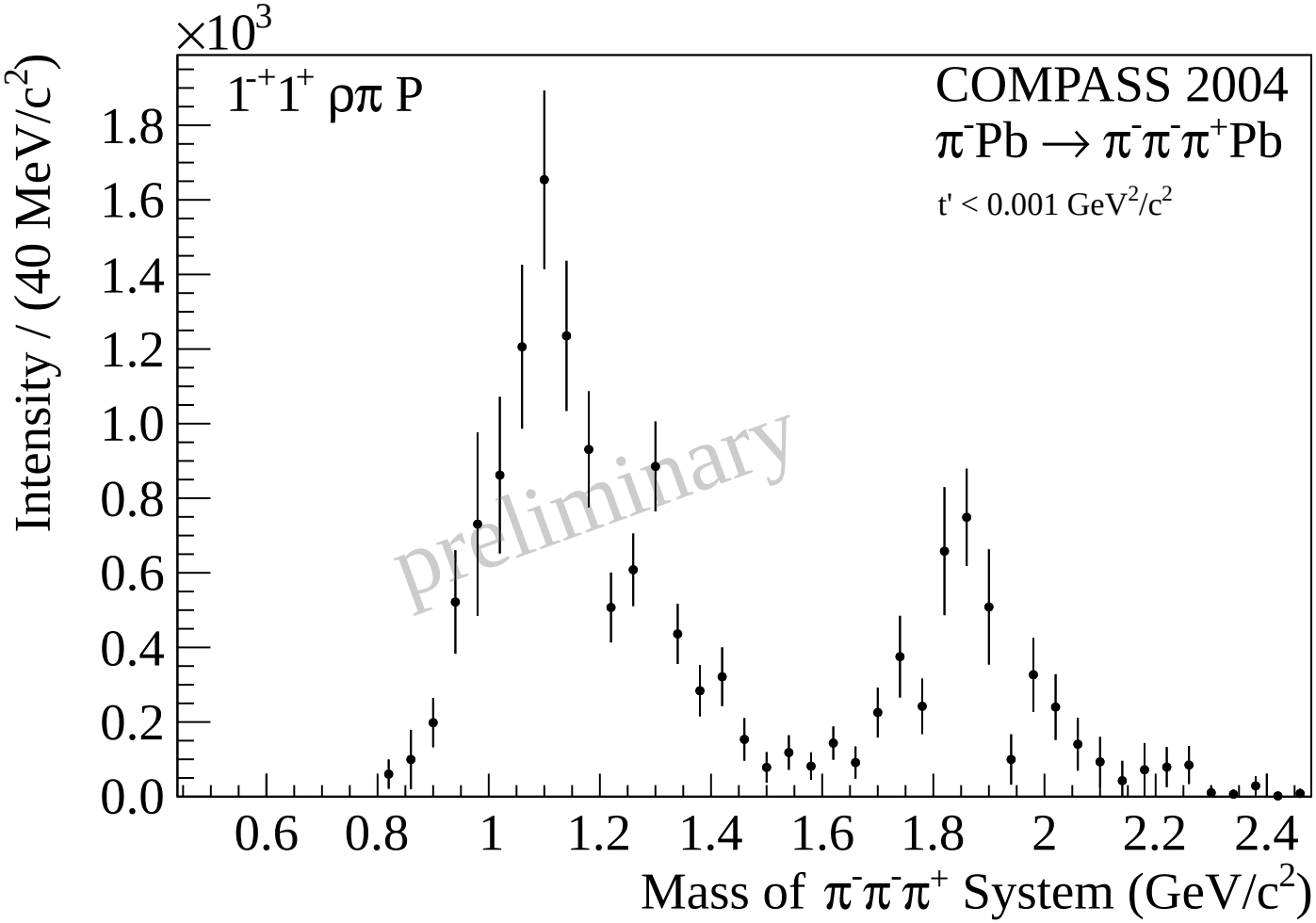
Partial wave intensities normalized to a_2





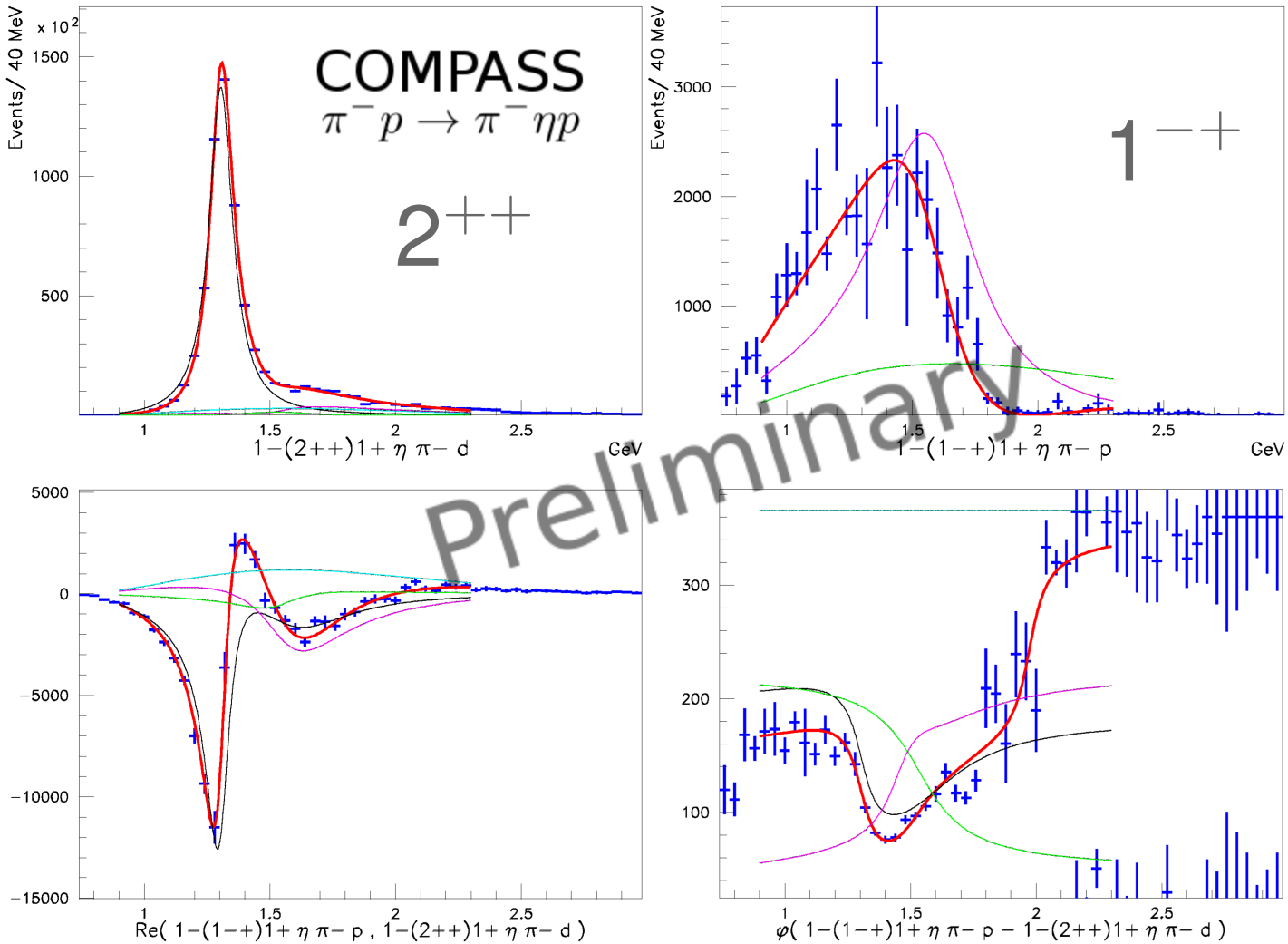
$1^{-+}\rho\pi$ in Primakoff Production

$\pi^- + Pb \to \pi^-\pi^+\pi^- + Pb$



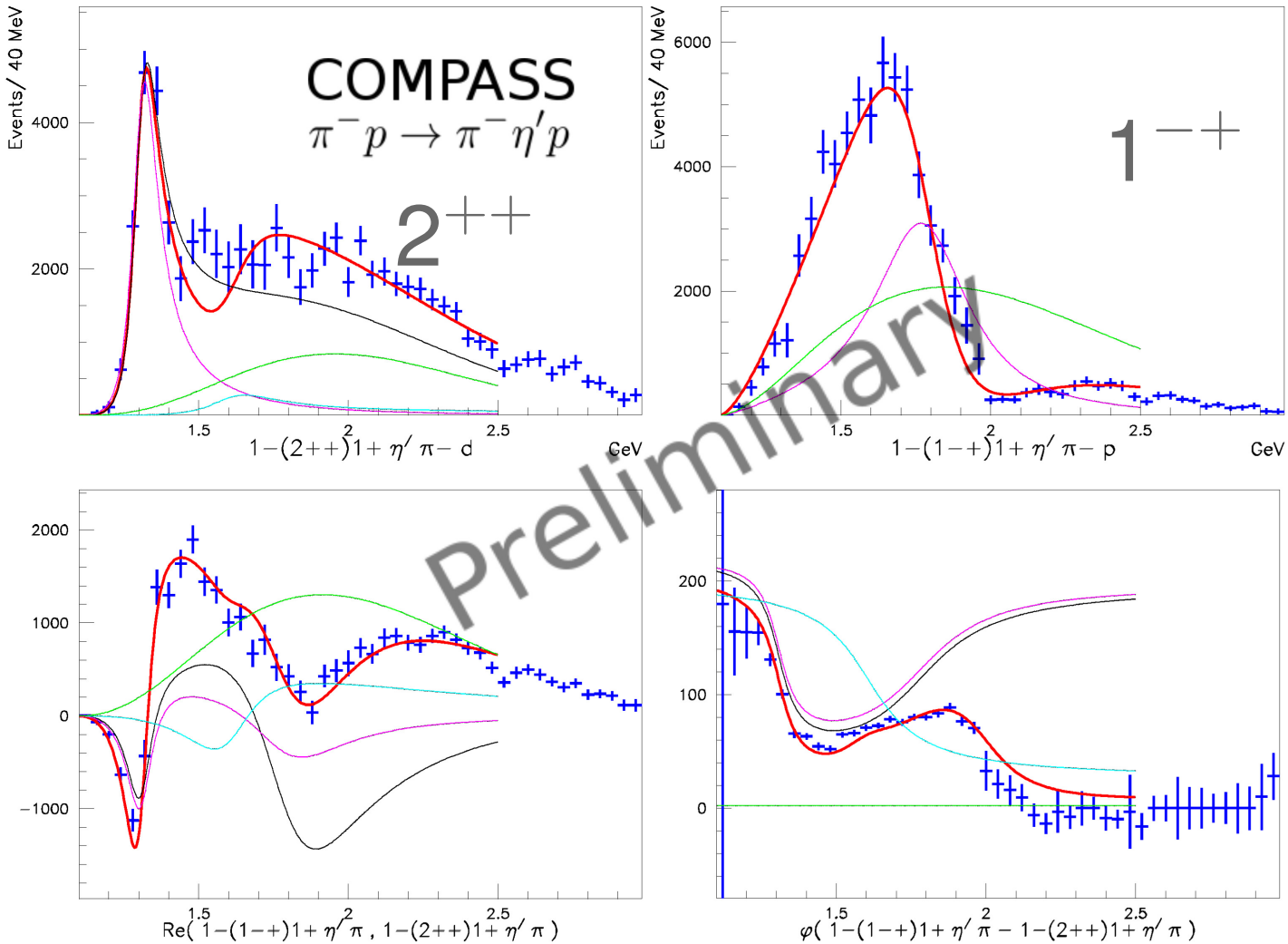
$\pi^- + p \to \eta\pi + p$ *D*- vs *P*-wave

Partial Wave Analysis



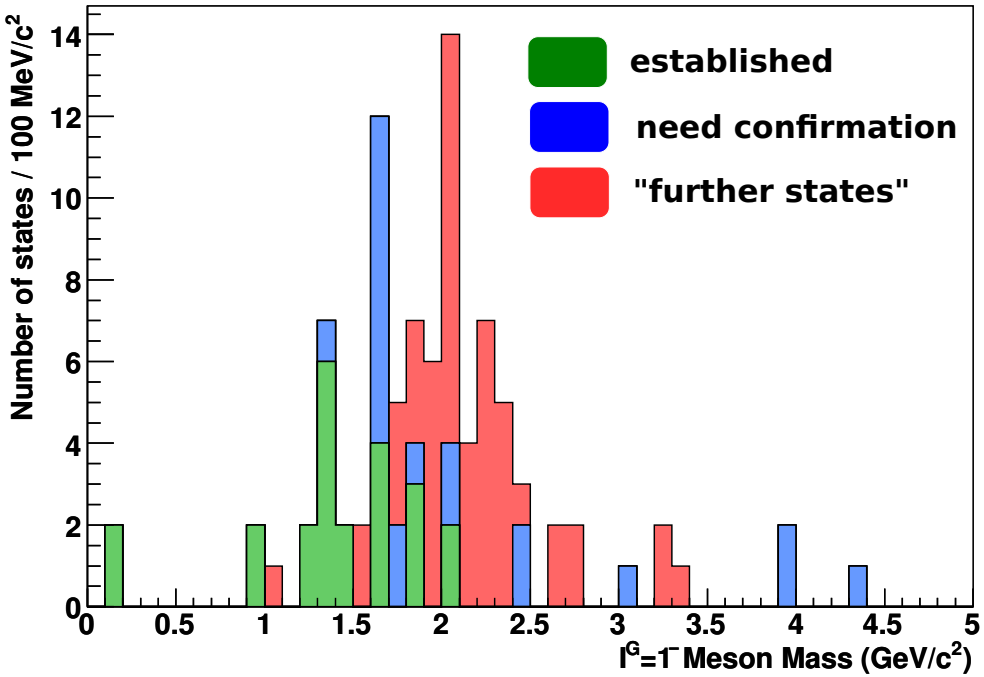
$\pi^- + p \to \eta'\pi + p$ *D*- vs *P*-wave

Partial Wave Analysis



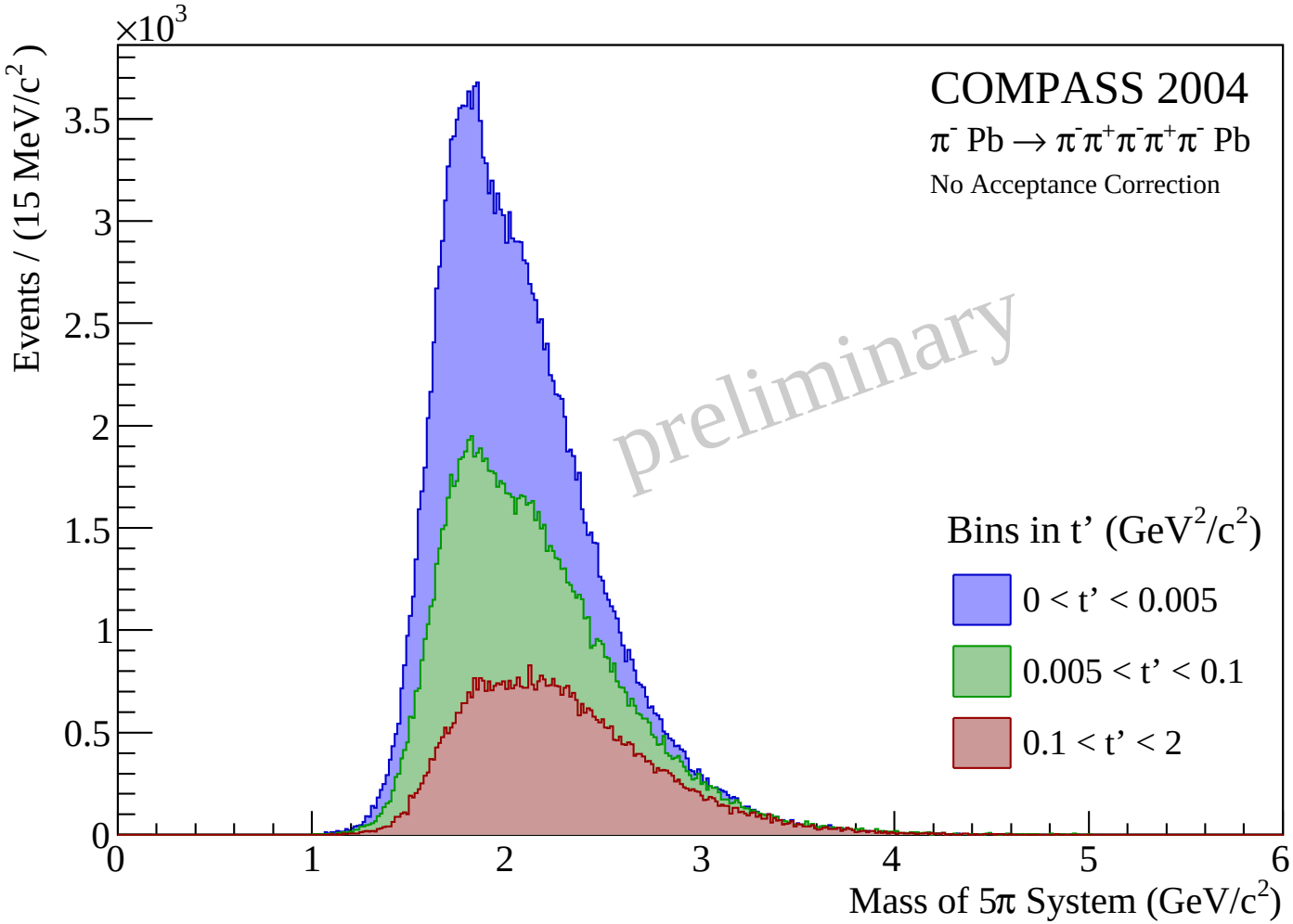
Exploring the light Meson Frontier

$$\pi^- + Pb \rightarrow \pi^-\pi^+\pi^-\pi^+\pi^- + Pb$$



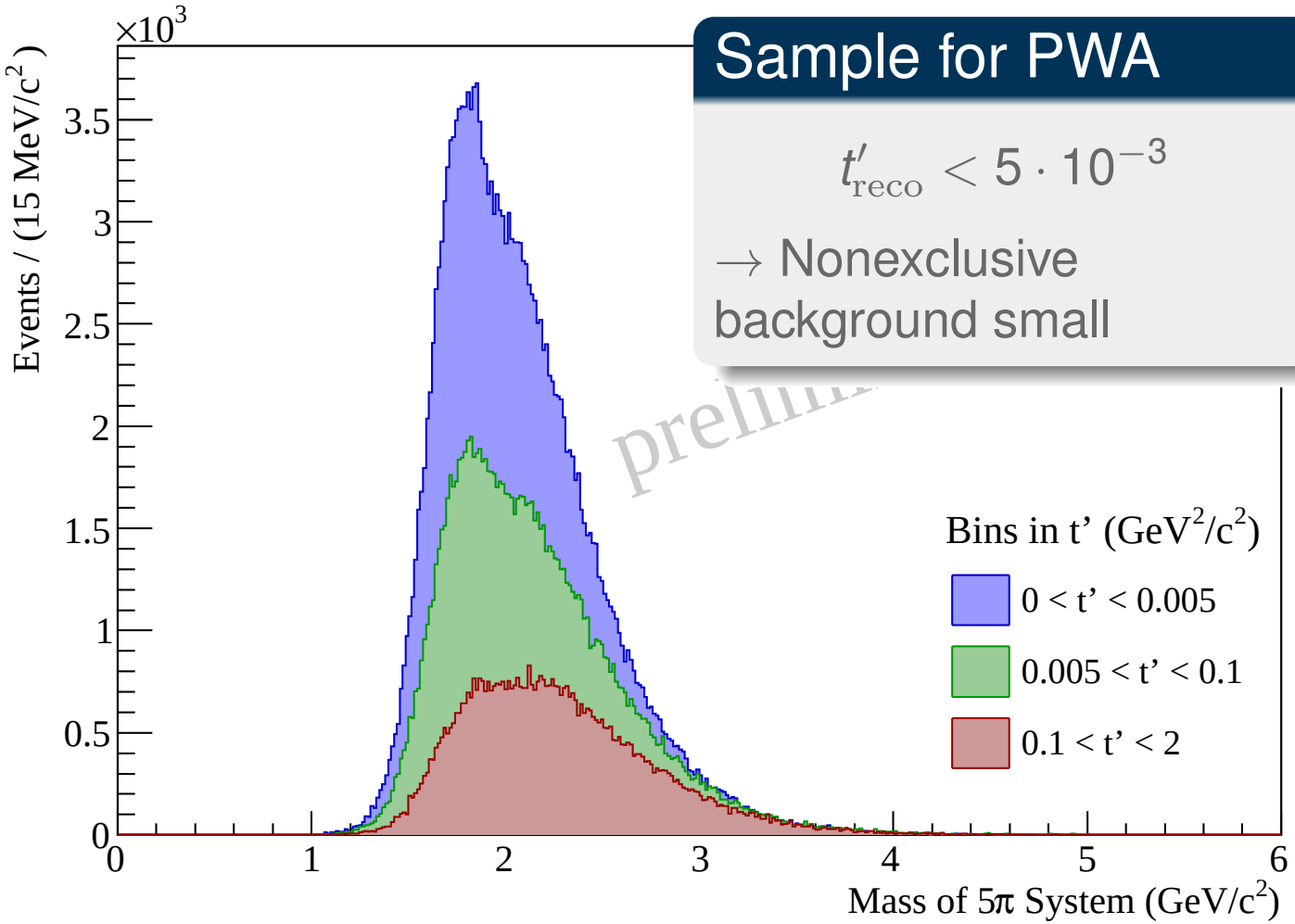
5π Data Sample

2004: $\pi + Pb \rightarrow 5\pi + Pb$



5π Data Sample

2004: $\pi + Pb \rightarrow 5\pi + Pb$



Sample for PWA

$$t'_{\text{reco}} < 5 \cdot 10^{-3}$$

$\rightarrow$ Nonexclusive background small

Spin Density Matrix (Subset)

7-Resonance Fit

$0^{-+}\pi^- f_0(1500)\ S$

$0^{-+}\rho a_1(1260)\ S$

$1^{++}\pi^- f_0(1370)\ P$

$1^{++}\pi^- f_1(1285)\ P$

$1^{++}\rho\pi(1300)\ S$

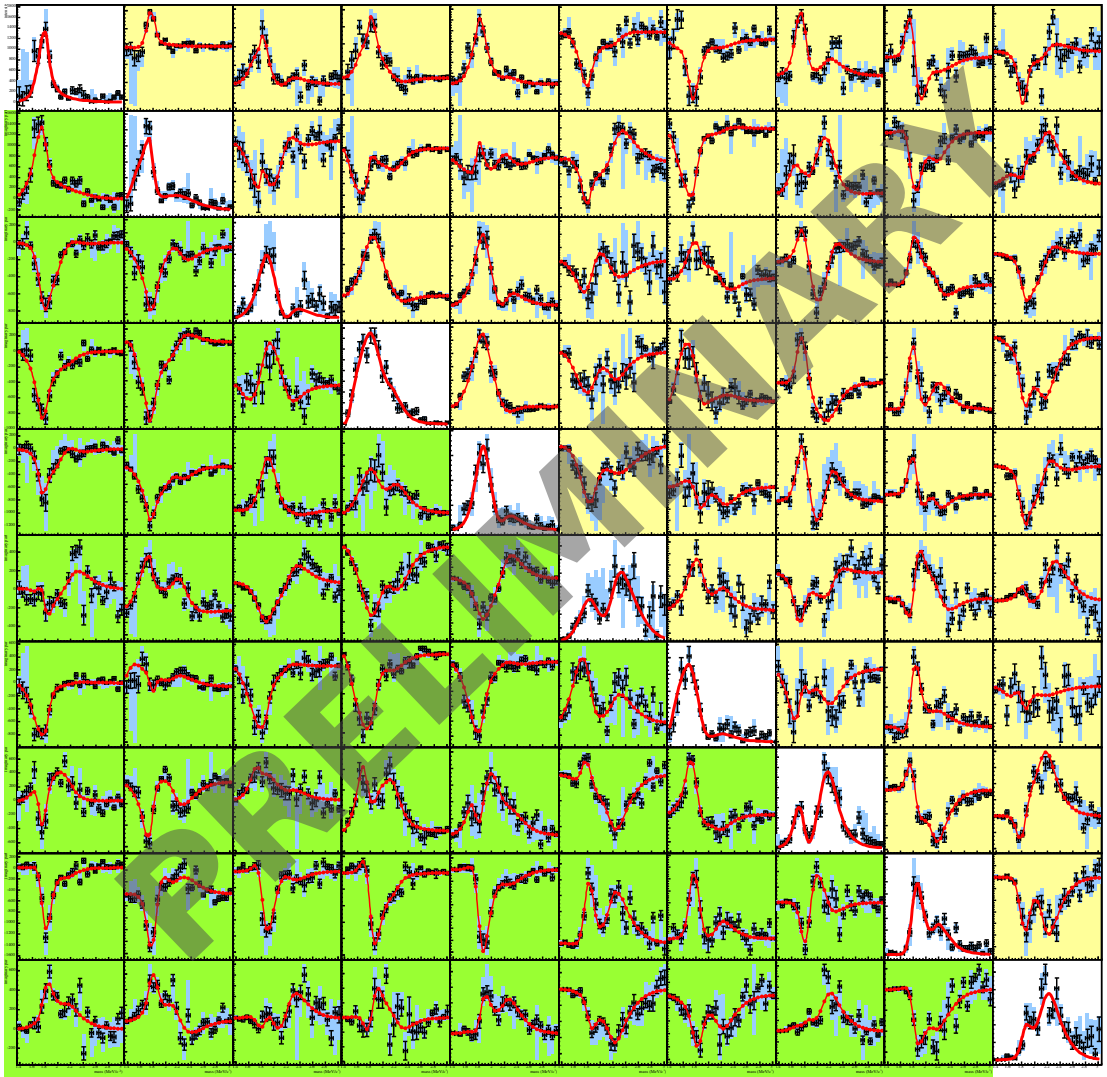
$1^{++}(\pi\pi)_S a_1\ D$

$2^{-+}\pi^- f_2(1270)\ S$

$2^{-+}\rho a_1(1260)\ S$

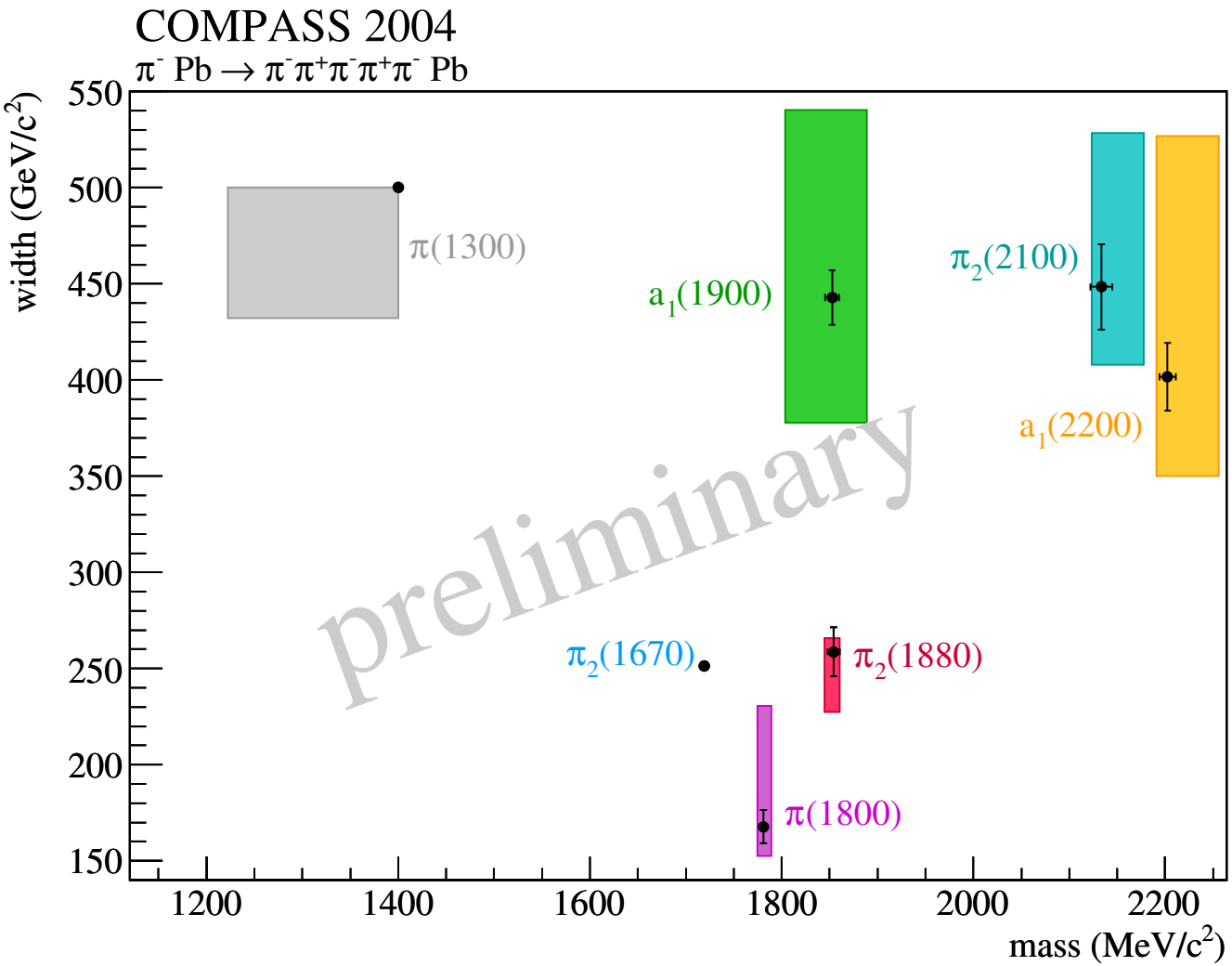
$2^{-+}\rho a_2(1320)\ S$

$2^{-+}\rho a_1(1260)\ D$



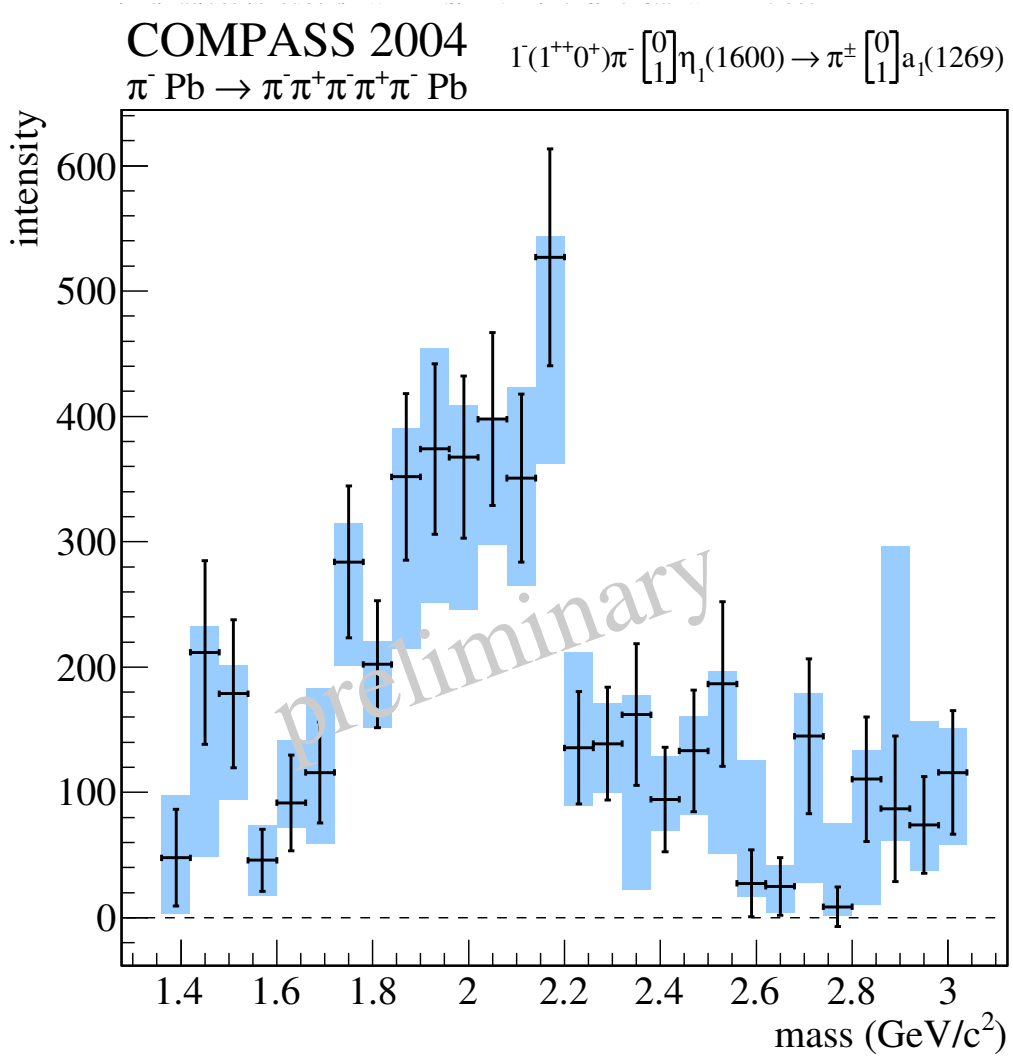
5π Resonances — Extracted Parameters

Summary of Resonance Parameters

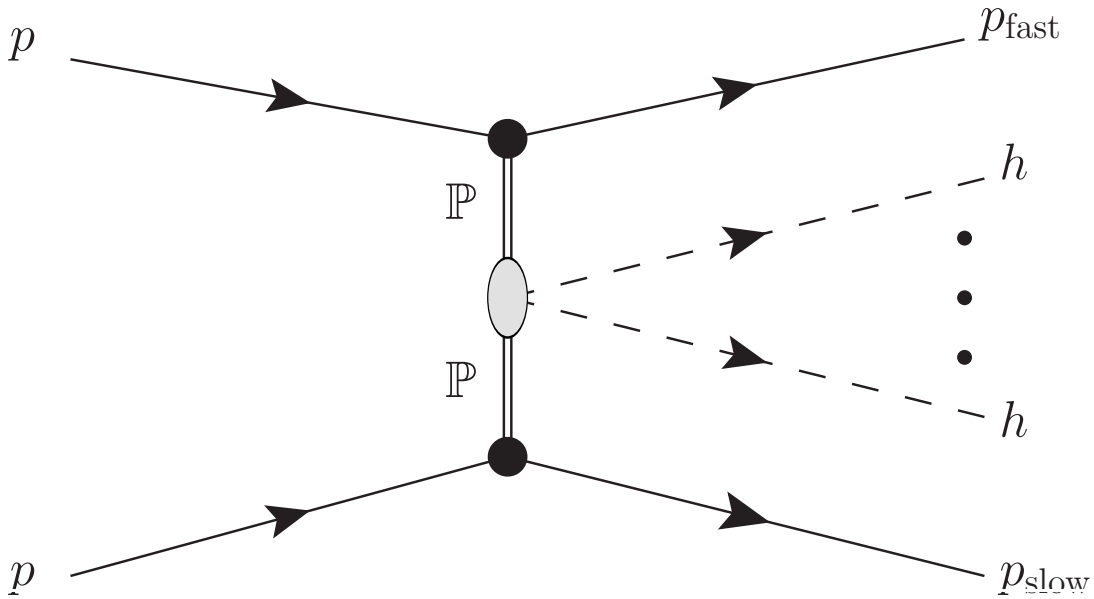


An Interesting Amplitude

With an exotic 4π isobar



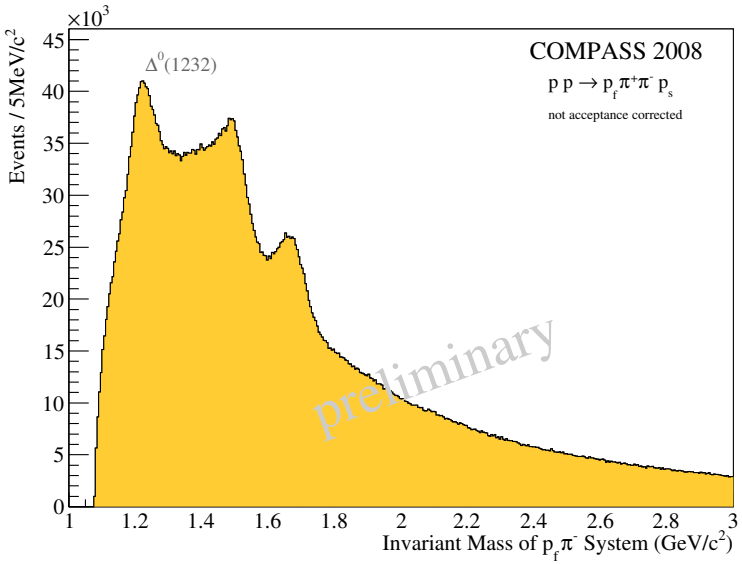
Isoscalar – Scalar Mesons
Meson Production at Central Rapidities in *pp* Scattering

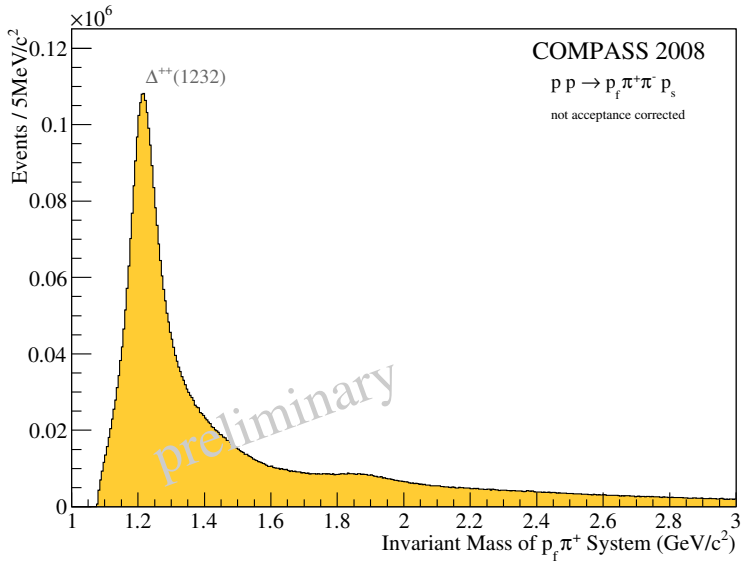


Proton Diffraction

$pp \to p_{\mathrm{fast}}\pi^+\pi^- + p_{\mathrm{slow}}$

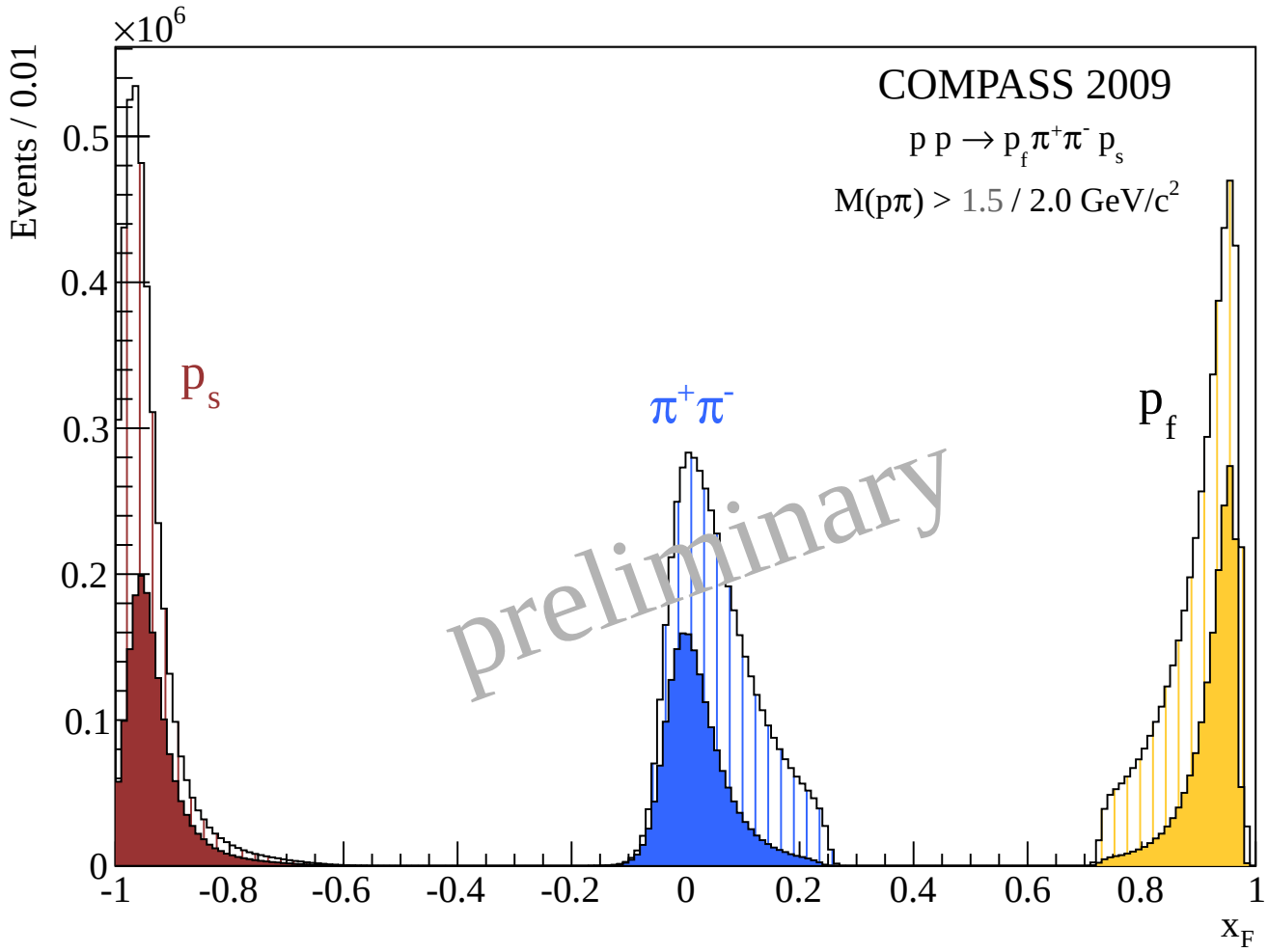
- Diffractive production of baryon resonances
- Kinematic overlap between production mechanisms
  $\to$ mass dependence



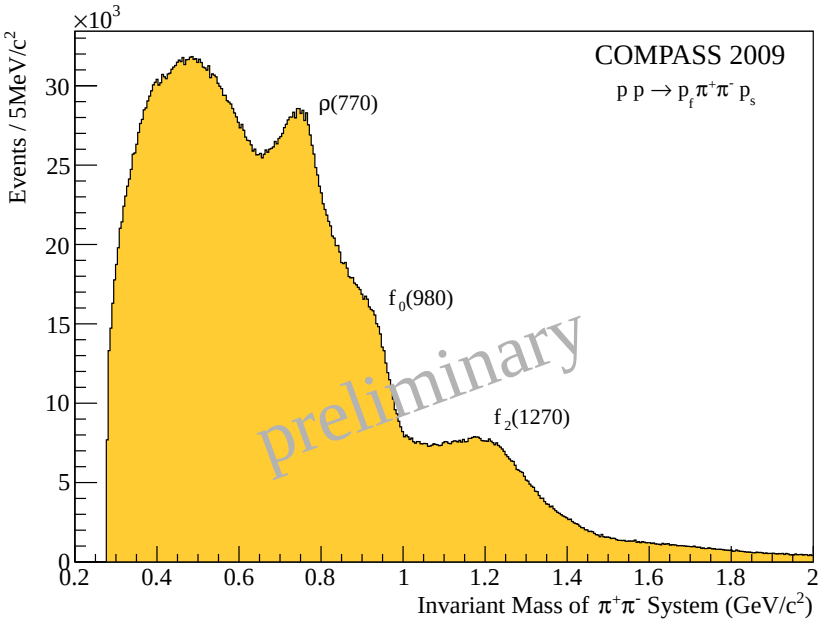


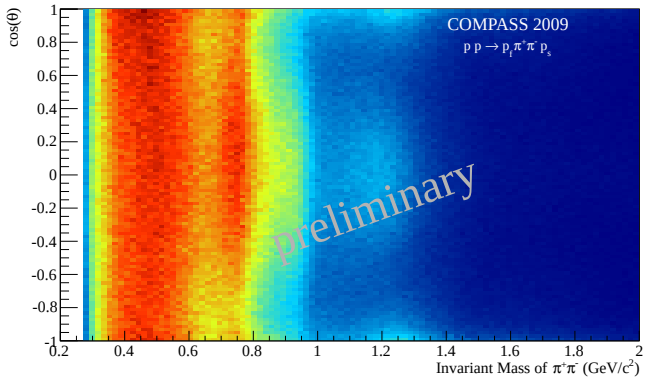
Central Production – $\pi^+\pi^-$ System

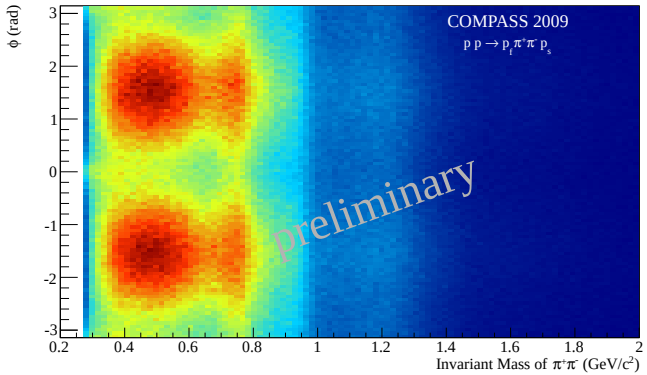
$x_{\mathrm{Feynman}} = 2p_l/\sqrt{s}$



Central Production – $\pi^+\pi^-$ System







Conclusion and Outlook

Conclusion

- COMPASS 2008/2009: large data sets in diffractive $\pi^-/K^-/p$ dissociation (up to 2 orders of magnitude improvement) and Primakoff
- Chiral dynamics: Pion polarizability / 3π–amplitude
- $\pi^-\pi^+\pi^-$, $\pi^-\pi^0\pi^0$, $\eta\pi^-$, $\eta'\pi^-$, $K^-\pi^+\pi^-$, 5π, $\pi^-\pi^+_{\mathrm{central/isobar}}$
  - Where is the $I = 0$ partner $\eta_1(1600)$ of the $\pi_1(1600)$? Hybrid Supermultiplet?

Outlook

- Dedicated Primakoff run 2012
- Further measurements: OZI-violation, Multi-particle final states...
- Study 2π and 4π systems, Isobar-fits, rescattering ...
- Consolidate Data $\rightarrow$ Global Meson Analysis Working Group

Backup

5π Resonance Parameters

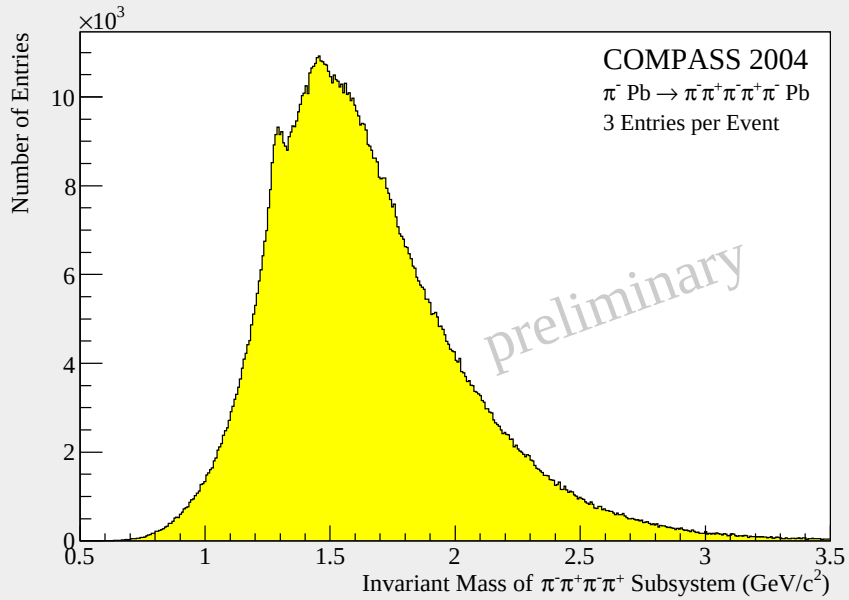
Comparison to PDG

	Parameter			Fit	PDG
	Resonance	J^{PC}		(MeV/c^2)	
	$\pi(1300)$	0^{-+}	M	1400^{*}	1300 ± 100
			Γ	$500^{\dagger}$	200…600
	$\pi(1800)$	0^{-+}	M	$1781 \pm 5^{+1(+8)}_{-6(-6)}$	1816 ± 14
			Γ	$168 \pm 9^{+5(+62)}_{-14(-15)}$	208 ± 12
○	$a_1(1900)$	1^{++}	M	$1853 \pm 7^{+36(+36)}_{-6(-49)}$	1930^{+30}_{-70}
			Γ	$443 \pm 14^{+12(+98)}_{-45(-65)}$	155 ± 45
○	$a_1(2200)$	1^{++}	M	$2202 \pm 8^{+15(+53)}_{-8(-11)}$	$2096 \pm 17 \pm 121$
			Γ	$402 \pm 17^{+41(+125)}_{-52(-51)}$	$451 \pm 41 \pm 81$
	$\pi_2(1670)$	2^{-+}	M	$1719.0^{\dagger}$	1672.4 ± 3.2
			Γ	$251.4^{\dagger}$	259 ± 9
	$\pi_2(1880)$	2^{-+}	M	$1854 \pm 6^{+6(+6)}_{-4(-9)}$	1895 ± 16
			Γ	$259 \pm 13^{+7(+7)}_{-17(-31)}$	235 ± 34
○	$\pi_2(2100)$	2^{-+}	M	$2133 \pm 12^{+7(+43)}_{-18(-18)}$	2090 ± 29

Isobars that have been used

4π Isobars ($G = +$)			3π Isobars ($G = -$)
Name	Mass / GeV	$I^G J^{PC}$	
f_0	1370 / 1500 / 1700	$0^+(0^{++})$	
η	1405	$0^+(0^{-+})$	
ρ'	1450 / 1700	$1^+(1^{--})$	
b_1	1235 / 1800	$1^+(1^{+-})$	
f_1	1285 / 1420	$0^+(1^{++})$	
f_2	1270 / 1565	$0^+(2^{++})$	
η_2'	1645	$0^+(2^{-+})$	
ρ_3	1690	$1^+(3^{--})$	
η_1	1600	$0^+(1^{-+})$	
b_0	1800	$1^+(0^{+-})$	
b_2	1800	$2^+(2^{+-})$	

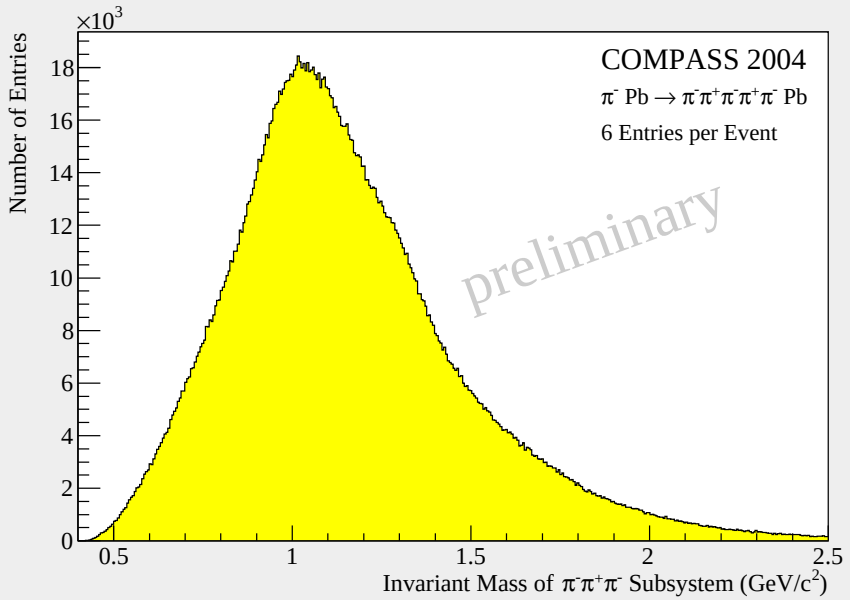
4π subsystem



Isobars that have been used

4π Isobars ($G = +$)			3π Isobars ($G = -$)		
			Name	Mass / GeV	$I^G J^{PC}$
			a_1	1270	$1^-(1^{++})$
			a_2	1320	$1^-(2^{++})$
			π'	1300	$1^-(0^{-+})$
			π_2	1670	$1^-(2^{-+})$
			π_1	1600	$1^-(1^{-+})$
b_2	1800	$2^+(2^{++})$			

3π subsystem



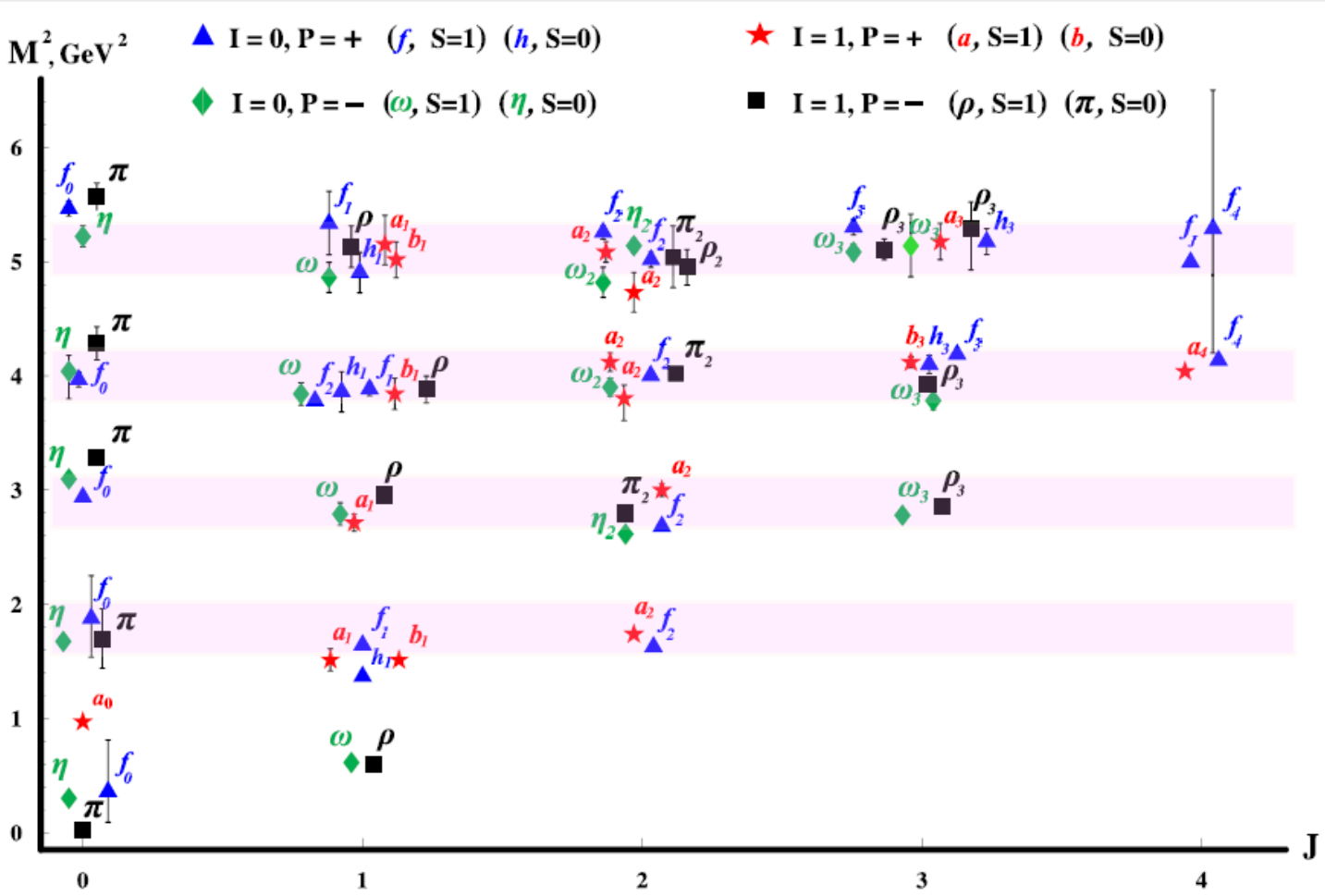
Isobars that have been used

4π Isobars ($G = +$)			3π Isobars ($G = -$)		
Name	Mass / GeV	$I^G J^{PC}$	Name	Mass / GeV	$I^G J^{PC}$
f_0	1370 / 1500 / 1700	$0^+(0^{++})$			
η	1405	$0^+(0^{-+})$	a_1	1270	$1^-(1^{++})$
ρ'	1450 / 1700	$1^+(1^{--})$	a_2	1320	$1^-(2^{++})$
b_1	1235 / 1800	$1^+(1^{+-})$	π'	1300	$1^-(0^{-+})$
f_1	1285 / 1420	$0^+(1^{++})$	π_2	1670	$1^-(2^{-+})$
f_2	1270 / 1565	$0^+(2^{++})$			
η_2'	1645	$0^+(2^{-+})$			
ρ_3	1690	$1^+(3^{--})$			
η_1	1600	$0^+(1^{-+})$			
b_0	1800	$1^+(0^{+-})$	π_1	1600	$1^-(1^{-+})$
b_2	1800	$2^+(2^{+-})$			

2π subsystem: σ, $\rho(770)$, $f_2(1270)$

Excited Mesons in Parity Doublets?

[PRD **77** (2008) 034002]



plot from M. Shifman and A. Vainshtein [PRD **77** (2008) 034002]

See also: R. F. Wagenbrunn and L. Ya. Glozman [PRD **75** (2007) 036007] and references therein

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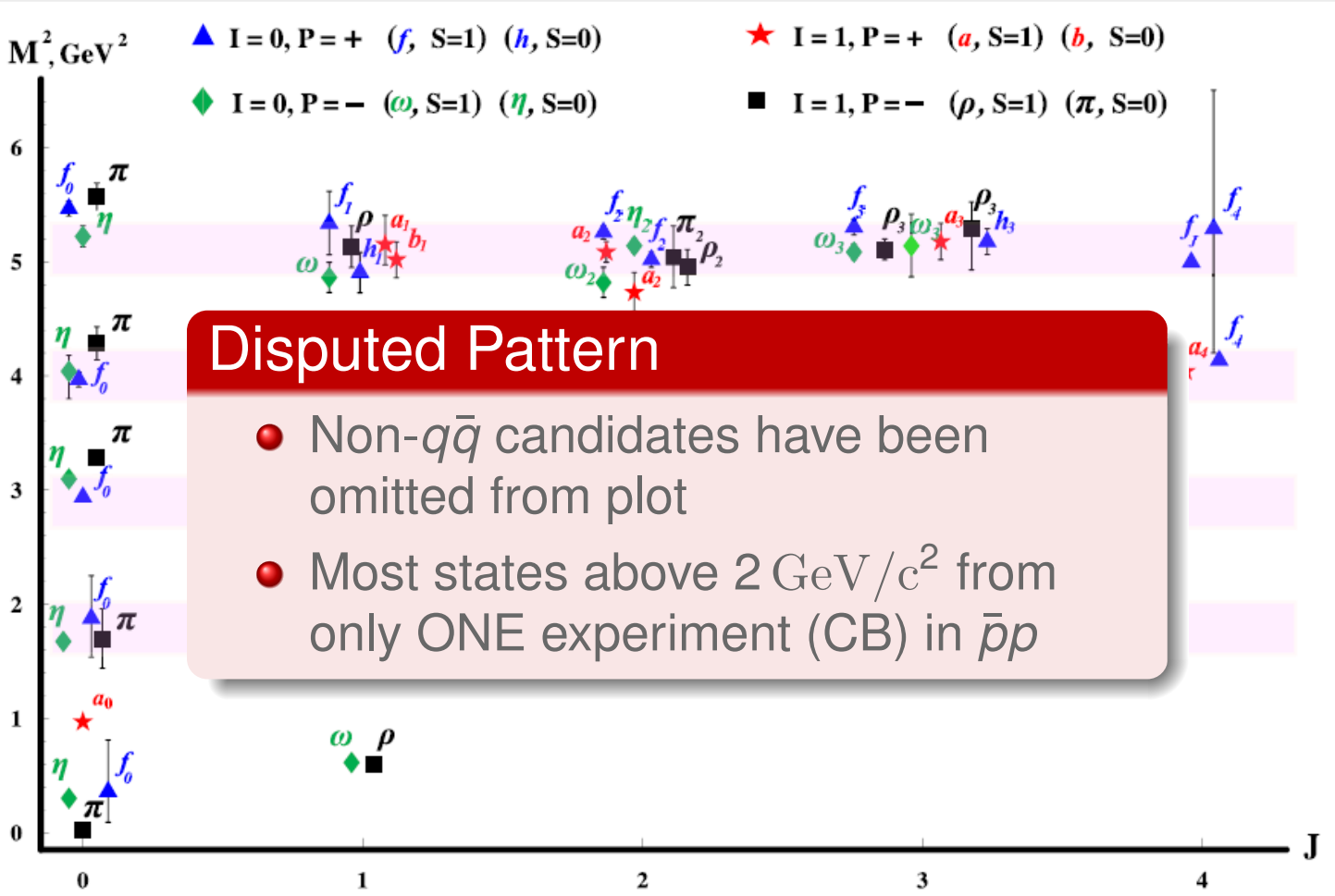


Disputed Pattern

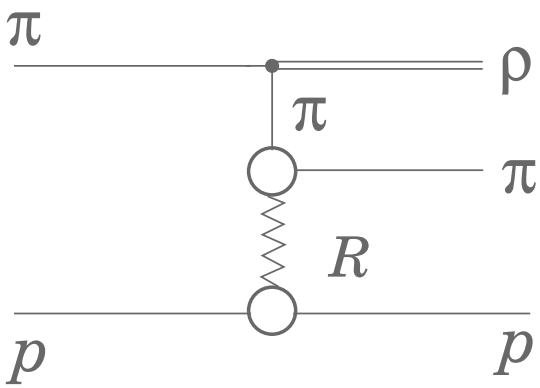
- Non-$q\bar{q}$ candidates have been omitted from plot
- Most states above 2 $\mathrm{GeV}/\mathrm{c}^2$ from only ONE experiment (CB) in $\bar{p}p$

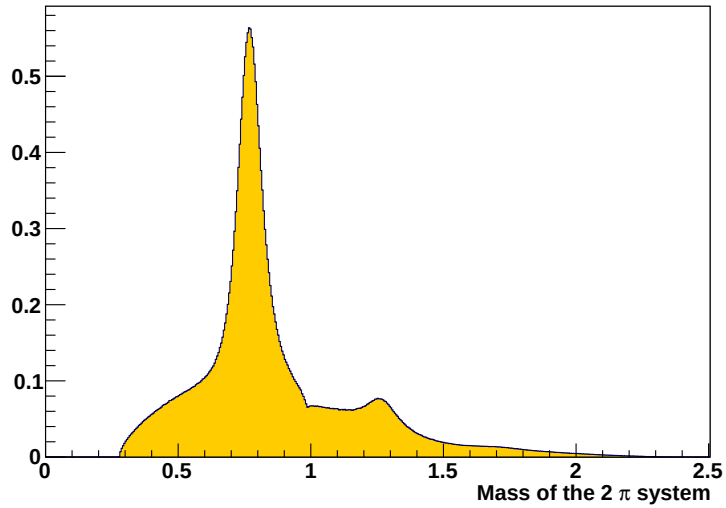
plot from M. Shifman and A. Vainshtein [PRD **77** (2008) 034002]

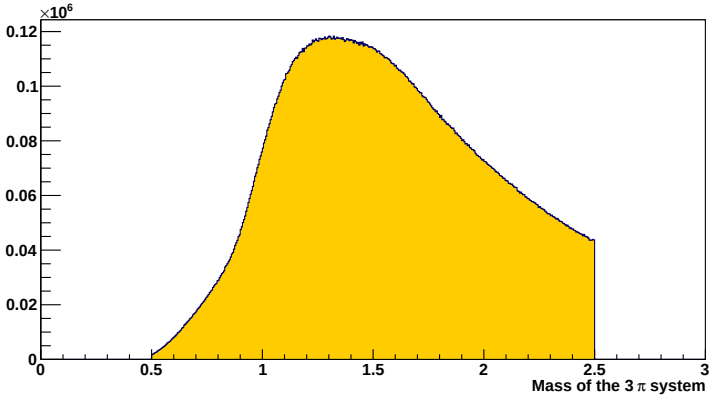
See also: R. F. Wagenbrunn and L. Ya. Glozman [PRD **75** (2007) 036007] and references therein

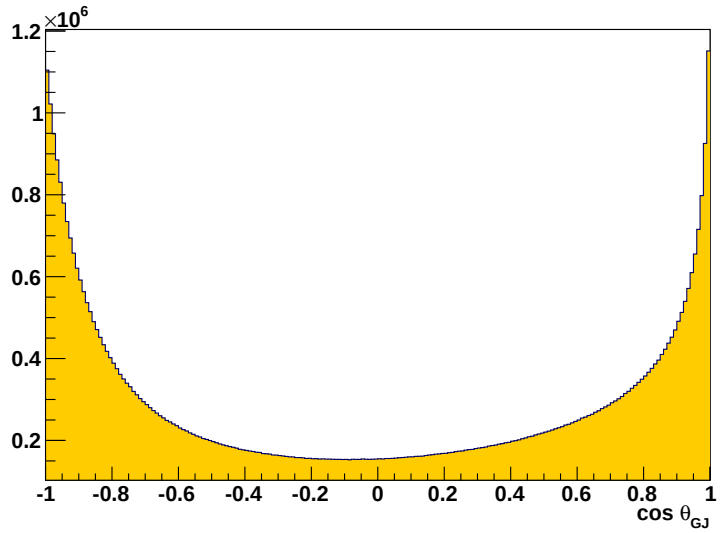
Nonresonant Scattering — Deck Effect

Monte Carlo Simulation



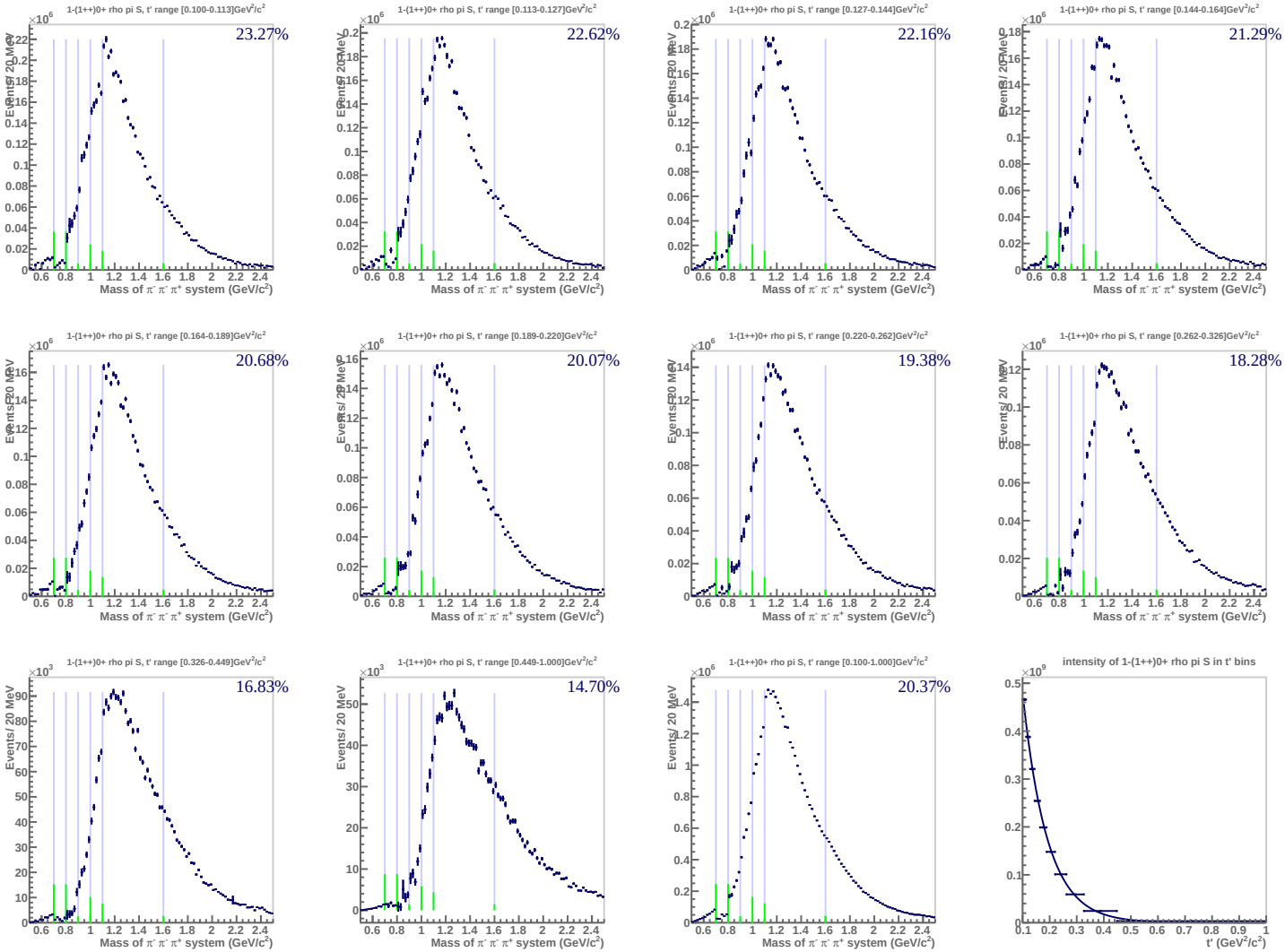






Deck Effect $J^{PC} = 1^{++}$ Component

Partial-Wave Decomposition in $m_{3\pi}$ and t' Bins



The Quark Model of (light) Mesons

Combining $q\bar{q}$ – there are some forbidden states!

Mesons:

- Color neutral objects,
- made from a fermion-antifermion ($q\bar{q}$) pair
- characterized by $I^G(J^{PC})$(mass)

Potential model:

$$V = H_{\text{conf}} + H_{\text{SS}} + H_{\text{LS}} + H_{\text{Annih}}$$

Godfrey, Isgur, Phys. Rev. D32(1985)189

$$G = (-1)^{I+\ell+s} \qquad P = (-1)^{\ell+1} \qquad C = (-1)^{\ell+s}$$

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Godfrey, Isgur, Phys. Rev. D32(1985)189

$$G = (-1)^{I+\ell+s} \qquad P = (-1)^{\ell+1} \qquad C = (-1)^{\ell+s}$$

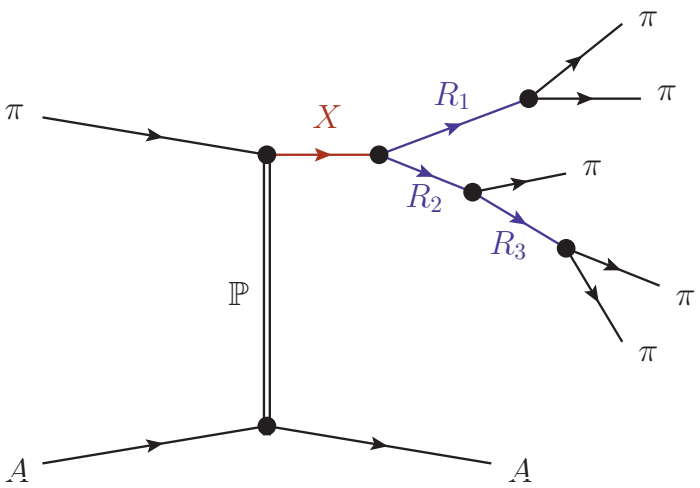
J^{PC} Multiplets

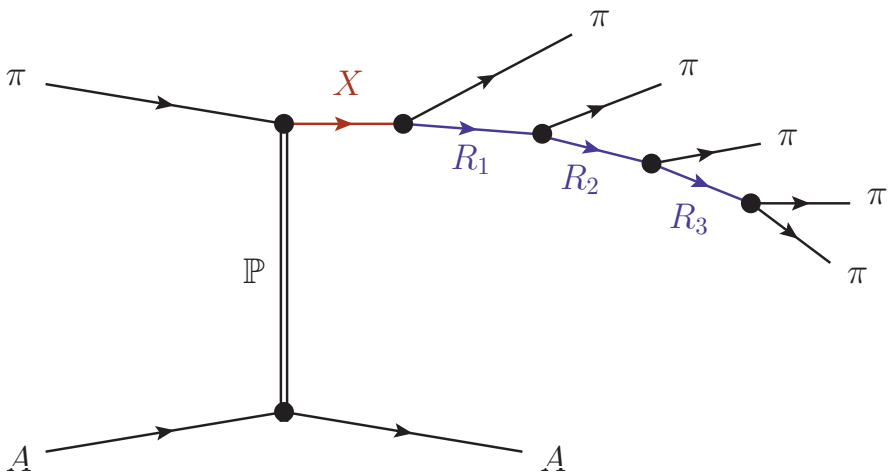
- $\ell = 0 \quad \Rightarrow \quad$ pseudoscalar 0^{-+}, vector 1^{--} states
- $\ell = 1 \quad \Rightarrow \quad$ scalar 0^{++}, axial vector 1^{+-}, 1^{++} and tensor 2^{++} states
- Same $J^{PC} \Rightarrow$ mixing!
- Forbidden: $0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, \ldots \rightarrow$ *spin exotic* states

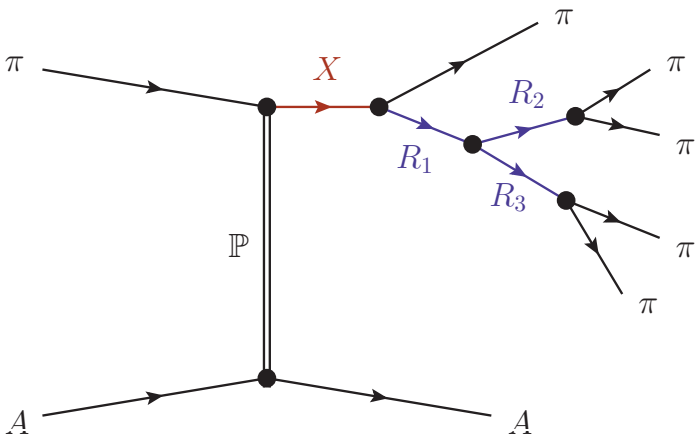
Isobar Model for 5π Final State

Challenges and Approaches

5-body isobar model







Isobar Decay Tree

- 11 indepependent variables τ:
  4 vertices $\times$ 2 angles $+$ 3 isobar masses
- Decay amplitudes $\psi(\tau)$
  in Helicity formalism
- Non-relativistic model

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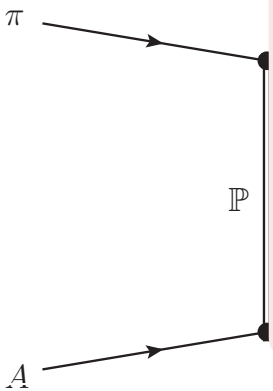
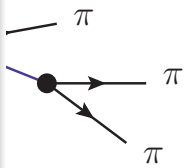
Challenges and Approaches

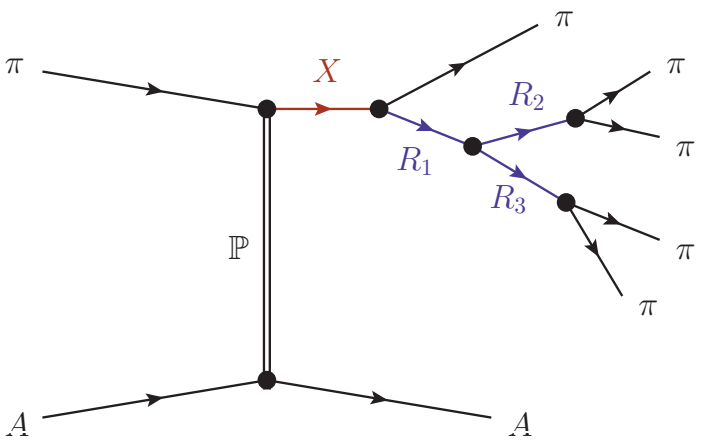
5-body isobar model



5-Body PWA Specials

- Decay topologies
- Many possible partial waves
- Assembly of waveset not possible by hand
- $\Rightarrow$ Waveset evolution
- 284 waves tested





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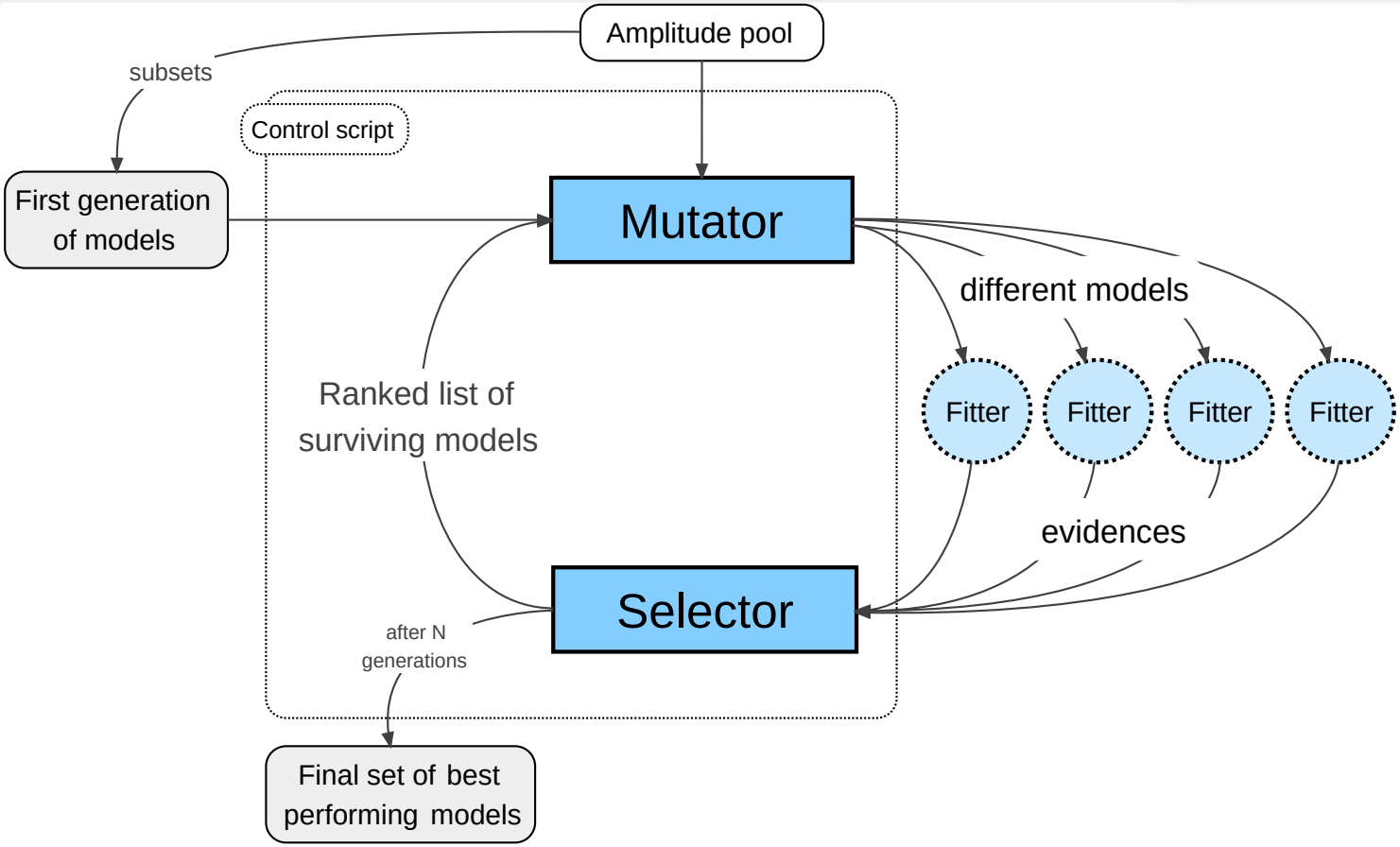
Evolutionary Waveset Exploration

Genetic Algorithm — 284 Waves in Pool



Evidence = Goodness of fit

- Bayesian Statistics → regularized Log-Likelihood
- Takes into account model complexity