

# Diffractive Dissociation into $\pi^- \pi^- \pi^+$ Final States at COMPASS

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for the COMPASS Collaboration

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## Introduction

## Diffractive Dissociation into $\pi^- \pi^- \pi^+$ Final States

## Deck Background Parametrization and Fit Improvements

## M-Dependence on the Target Material

## Conclusion and Outlook



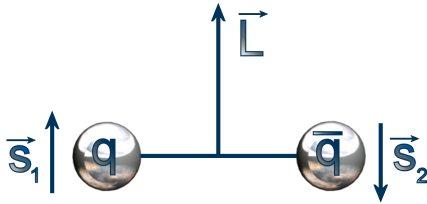
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## Quantum Numbers

- $|^G J^{PC}$
- Isospin I
- G-Parity:  
 $G = C \cdot e^{i\pi I_2}$
- LS-Coupling:  
 $J = \ell \oplus s = |\ell - s| \dots \ell + s$
- Parity:  
 $P = (-1)^{(\ell+1)}$
- Charge Conjugation:  
 $C = (-1)^{(\ell+s)}$

$J^{PC}(q\bar{q}) : 0^{++}, 0^{-+}, 1^{++}, 1^{+-}, 1^{--}, 2^{++}, 2^{-+}, 2^{--} \dots$

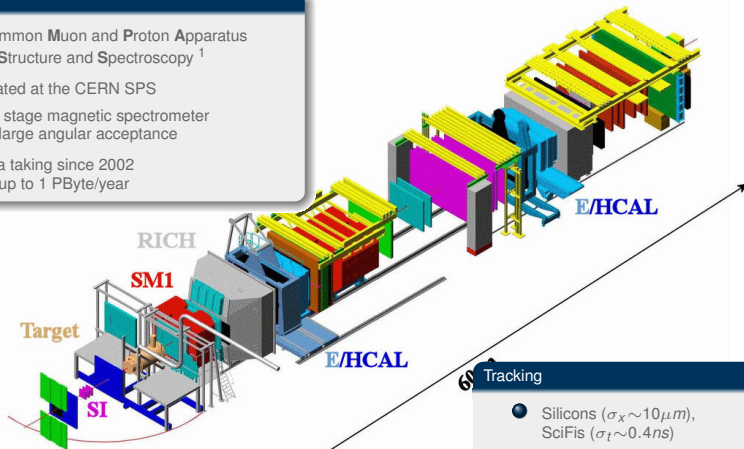
**exotic  $J^{PC} : 0^{+-}, 1^{-+}, 2^{+-}$**



# The COMPASS Experiment Overview

## Overview

- **CO**mmun **M**uon and **P**roton **A**pparatus for **S**tructure and **S**pectroscopy <sup>1</sup>
- located at the CERN SPS
- two stage magnetic spectrometer  
→ large angular acceptance
- data taking since 2002  
→ up to 1 PByte/year



## 6 Tracking

- Silicons ( $\sigma_x \sim 10 \mu m$ ),  
SciFis ( $\sigma_t \sim 0.4 ns$ )
- (Pixel)GEMs ( $\sigma_x \sim 70 \mu m$ ),  
MicroMegs ( $\sigma_x \sim 90 \mu m$ )
- Drift Chambers ( $\sigma_x \leq 200 \mu m$ )

<sup>1</sup>[Nucl. Instr. and Meth. A 577 (2007) 455]



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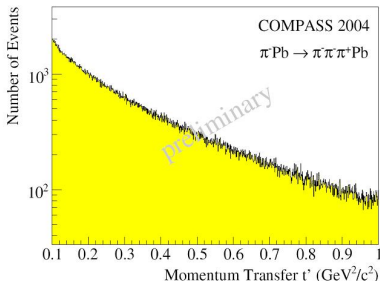
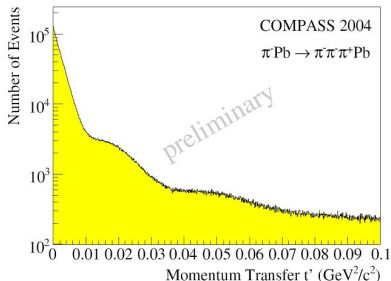
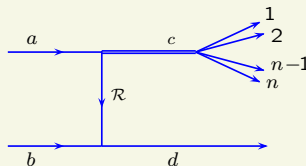
Conclusion and Outlook



## Diffractive Dissociation

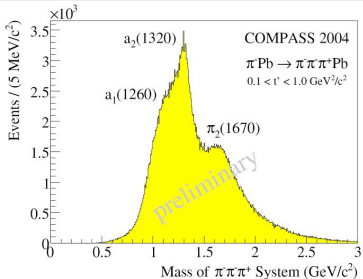
- Soft scattering of the beam  $\pi^-$  off the target
  - Pb (2004, 2009)
  - $\text{IH}_2$  (2008)
  - W, Ni (2009)
- Target particle remains intact
- Pomeron exchange

## Reaction



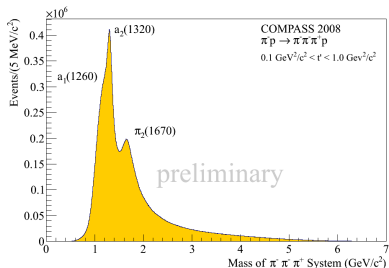


## $\pi^- \text{Pb} \rightarrow \pi^- \pi^- \pi^+ \text{Pb}$ (2004)



- $p_\pi = 190 \text{ GeV}/c$
- 4M events (full  $t$  range)
- 450k events in  $0.1 < t' < 1.0 \text{ GeV}^2/c^2$

## $\pi^- p \rightarrow \pi^- \pi^- \pi^+ p$ (2008)



- $p_\pi = 190 \text{ GeV}/c$
- $\sim 50\text{M}$  events in  $0.1 < t' < 1.0 \text{ GeV}^2/c^2$



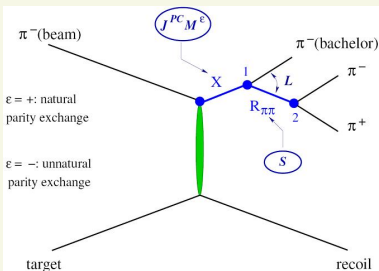


- Two-step approach:
  - Fit in mass bins

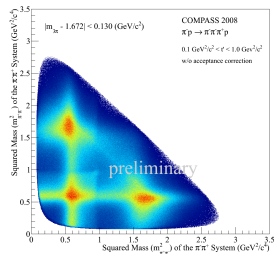
$$\mathcal{I}(\tau, t') = \sum_{\epsilon=\pm 1} \sum_{r=1}^{N_r} \left| \sum_i T_{ir}^{\epsilon} f(t') \psi_i^{\epsilon}(\tau) \right|$$

- Fit of the spin density matrix

## Isobar Model

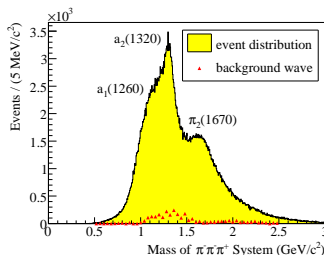
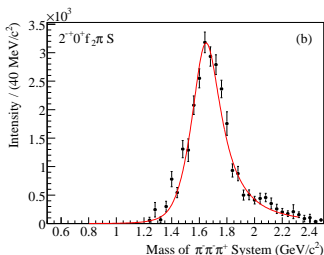
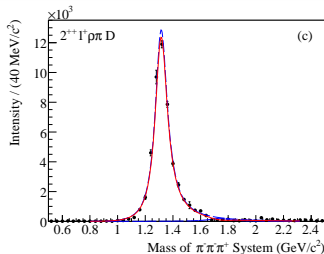
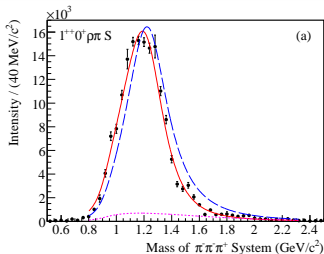


## Dalitz Plot $\pi_2(1670)$ region




 $\pi^- \text{Pb} \rightarrow \pi^- \pi^- \pi^+ \text{Pb} \text{ (2004)}$ 

# Intensities of Major Waves

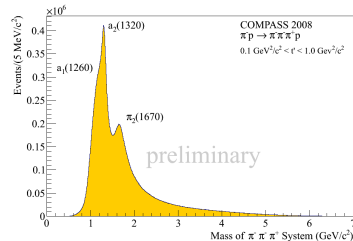
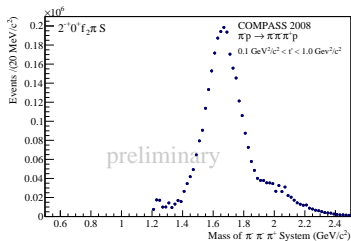
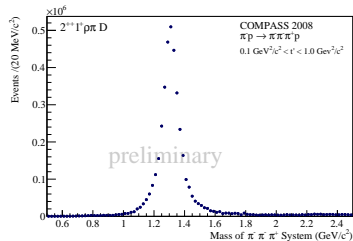
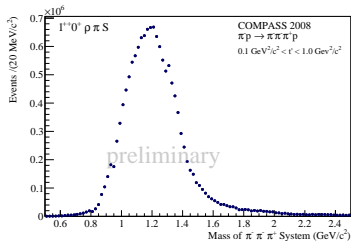



 $\pi^- p \rightarrow \pi^- \pi^- \pi^+ p$  (2008)

Intensities of Major Waves



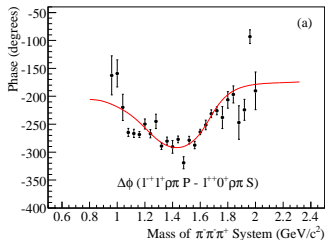
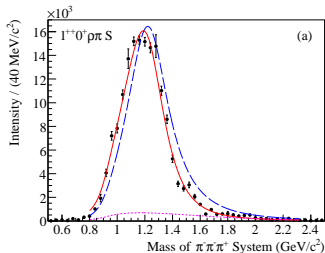
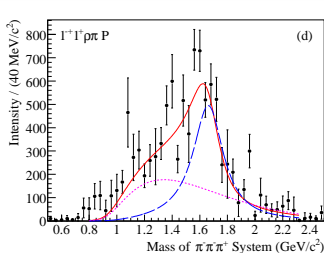
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# $\pi^- \text{Pb} \rightarrow \pi^- \pi^- \pi^+ \text{Pb}$ (2004)

## $J^{PC} = 1^{-+}$ Exotic Wave <sup>1</sup>

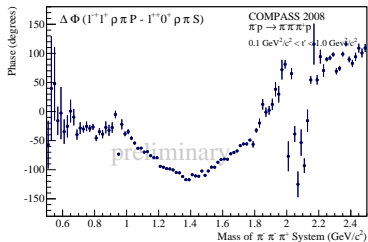
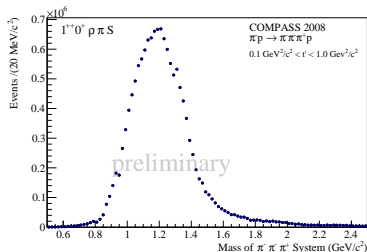
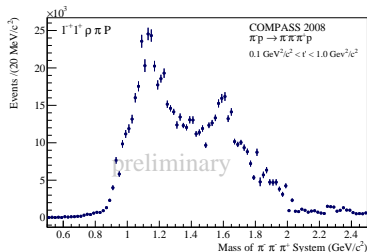


<sup>1</sup> A. Alekseev *et al.*, COMPASS Collaboration, Phys. Rev. Lett. 104, 241803 (2010)



# $\pi^- p \rightarrow \pi^- \pi^- \pi^+ p$ (2008)

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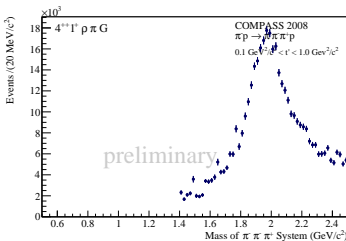
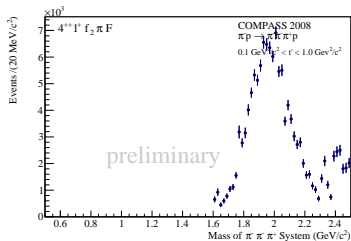
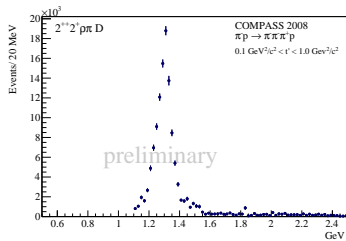
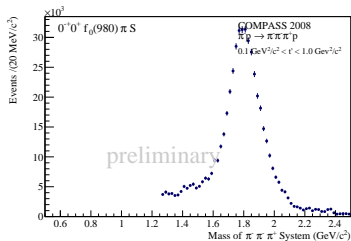


$$\pi^- p \rightarrow \pi^- \pi^- \pi^+ p \quad (2008)$$

# Additional Waves



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Diffractive Dissociation into  $\pi^- \pi^- \pi^+$  Final States

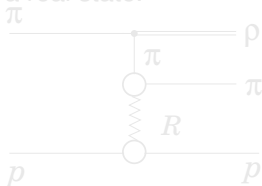
Deck Background Parametrization and Fit Improvements

M-Dependence on the Target Material

Conclusion and Outlook



- Additional production mechanism for the same final state  $\rightarrow$  background in amplitude analysis
- An incident beam pion dissociates into a  $\rho$  or  $f_2$  and a virtual  $\pi$ . The virtual  $\pi$  scatters diffractively from the target proton (via Pomeron) into a real state.



- Amplitude parametrisation:

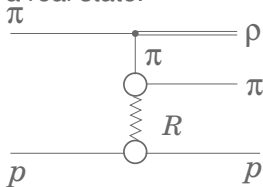
$$\Psi(M_{\pi\pi}, t_\pi, t) = \frac{A_{\pi\pi}(M_{\pi\pi}, t_\pi)A_{\pi p}(s_{\pi p}, t)}{m_\pi^2 - t_\pi}$$

- $A_{\pi\pi}$  scattering amplitude through the  $\rho$  or/and  $f_2$
- $A_{\pi p}$   $\pi^- p$  elastic scattering amplitude





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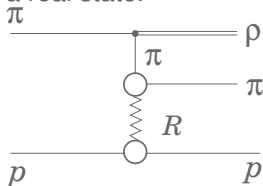
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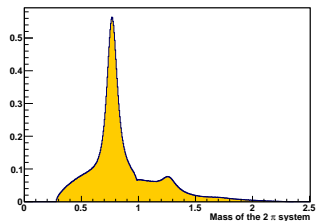
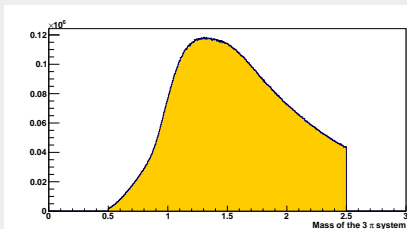
- Amplitude parametrisation:

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## Mass distributions of MC Deck Sample





- Poor knowledge of  $t'$  dependency, get rid of  $t'$  dependency  
→ Fit in mass **and**  $t'$  bins:

$$\mathcal{I}(\tau) = \sum_{\epsilon=\pm 1} \sum_{r=1}^{N_r} \left| \sum_i T_{ir}^{\epsilon} \psi_i^{\epsilon}(\tau) \right|$$

- Modify (lower) thresholds for each partial wave
- Modify parametrization of decay amplitudes



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- Modify (lower) thresholds for each partial wave
- Modify parametrization of decay amplitudes





- Same fit improvements applied to real data (not shown in this talk)
- $t'$  dependency different for diffractive data in respect to Deck produced data  
→ possibility to disentangle Deck background from diffractive data in fit to the spin density matrix





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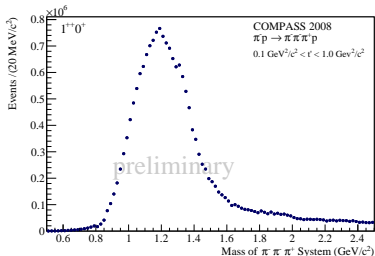
- 2008 IH<sub>2</sub> target, 2009 Pb target (W, Ni not shown)
- Different statistics
  - Normalisation to the integral of the  $a_2(1320)$  in the mass region between 1.1 GeV/c<sup>2</sup> and 1.6 GeV/c<sup>2</sup>
- Population of M = 1 states higher for lead target
- Population of M = 0 states higher for hydrogen target



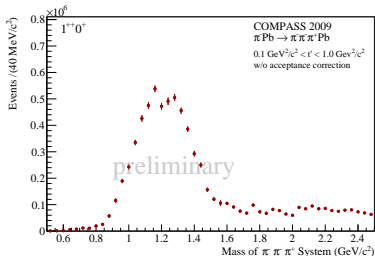
# Total Intensities for $J^{PC} = 1^{++}$ with $M = 0$



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$\pi^- p \rightarrow \pi^- \pi^- \pi^+ p$  (2008)



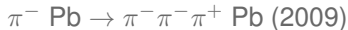
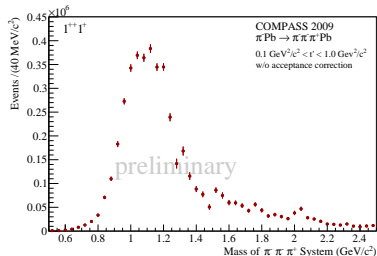
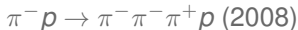
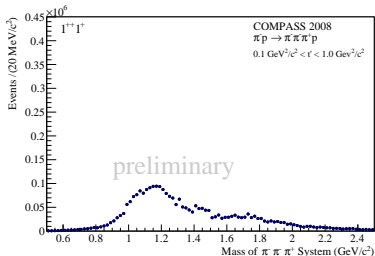
$\pi^- Pb \rightarrow \pi^- \pi^- \pi^+ Pb$  (2009)



# Total Intensities for $J^{PC} = 1^{++}$ with $M = 1$



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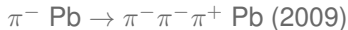
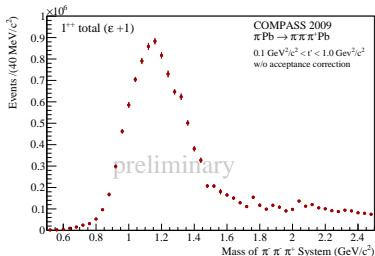
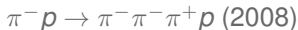
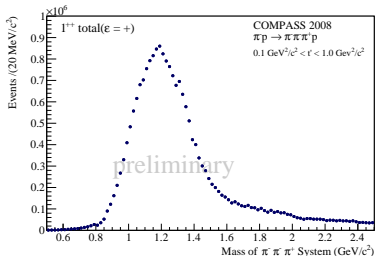


# Total Intensities for $J^{PC} = 1^{++}$

## Sum of M sub-states



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- Pilot Run 2004
  - Diffractive pion dissociation on Pb into  $\pi^- \pi^- \pi^+$
  - <sup>1</sup>Significant intensity of exotic wave  $1^{-+}$  at 1.66 GeV/c<sup>2</sup>
- COMPASS 2008/2009
  - Diffractive reactions: 10x BNL E852 statistics
  - Several different targets (p, Ni, W, Pb)
  - Consistency check with isospin partner channel  $\pi^- \pi^0 \pi^0$
  - Additional channels:  $5\pi$ ,  $\eta' \pi$
  - Analysis ongoing (more news probably this summer)
    - Enhancement of wave set
    - Study of Deck Effect
    - $t'$  dependence
    - Analysis of M-dependence
- Two independent PWA programs

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<sup>1</sup>A. Alekseev *et. al.*, COMPASS Collaboration, Phys. Rev. Lett. 104, 241803 (2010)



# Backup





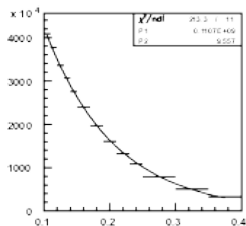
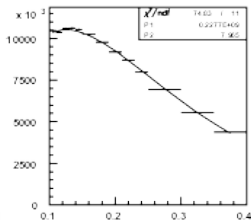
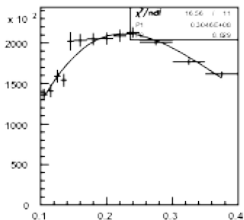
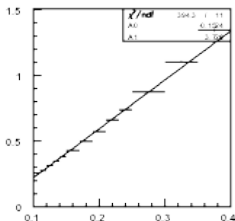
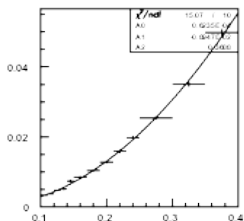


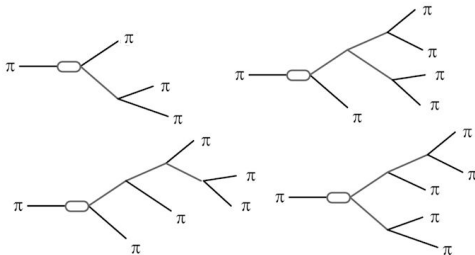
- Comparison of rank 1, 2, 3 mass independent fits
- Different Exclusivity Cut ( $189 \pm 3$  or 5 GeV)
- $\pi_1(1400)$  added as second Breit-Wigner resonance to describe  $1^{-+}$  wave, parameters of  $\pi_1(1400)$  fixed to PDG values
- 46 waves in mass-independent fit with four  $M = 2$  waves included, thresholds adjusted
- D-functions with relativistic factors instead of Zemach tensors used for mass-independent fit
- Dynamical width for  $a_4(2040)$  used instead of constant one



$$\pi^- p \rightarrow \pi^- \pi^- \pi^+ p$$

Is the  $2^{++}2^+$  real ?


 $J^{PC} M_1 = 1^{++}0+ \text{ total}$ 

 $J^{PC} M_1 = 2^{++}1+ \text{ total}$ 

 $J^{PC} M_1 = 2^{++}2+ \text{ total}$ 

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 $\pi_1(1600) 1^- 1^{--}$ 

- $(2\pi)^0\pi^-$ :  
 $\rho\pi^-, f_2(1270)\pi^-$
- $(4\pi)^0\pi^-$ :  
 $b_1(1235)\pi^-, f_1(1285)\pi^-$
- $\eta'(958)\pi^-$

COMPASS has access to all of these decay modes

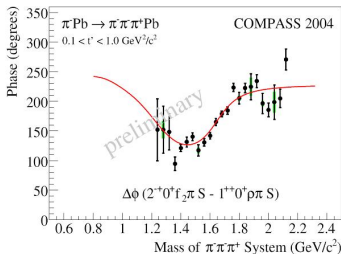
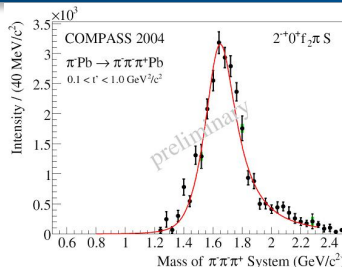
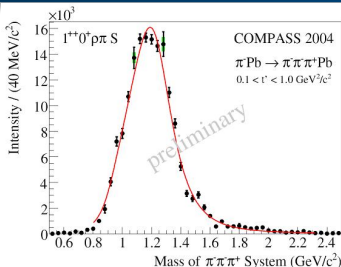


$J^{PC}M^{\epsilon}$	$L$	Isobar $\pi$	Thresh. [GeV]
$0^{-+}0^{+}$	<b>S</b>	$f_0\pi$	<b>1.40</b>
$0^{-+}0^{+}$	S	$(\pi\pi)_{S\pi}$	-
$0^{-+}0^{+}$	P	$\rho\pi$	-
$1^{-+}1^{+}$	<b>P</b>	$\rho\pi$	-
$1^{++}0^{+}$	<b>S</b>	$\rho\pi$	-
$1^{++}0^{+}$	P	$f_2\pi$	1.20
$1^{++}0^{+}$	P	$(\pi\pi)_{S\pi}$	0.84
$1^{++}0^{+}$	D	$\rho\pi$	1.30
$1^{++}1^{+}$	S	$\rho\pi$	-
$1^{++}1^{+}$	P	$f_2\pi$	1.40
$1^{++}1^{+}$	P	$(\pi\pi)_{S\pi}$	1.40
$1^{++}1^{+}$	D	$\rho\pi$	1.40
$2^{-+}0^{+}$	<b>S</b>	$f_2\pi$	<b>1.20</b>
$2^{-+}0^{+}$	P	$\rho\pi$	0.80
$2^{-+}0^{+}$	D	$f_2\pi$	1.50
$2^{-+}0^{+}$	D	$(\pi\pi)_{S\pi}$	0.80
$2^{-+}0^{+}$	F	$\rho\pi$	1.20
$2^{-+}1^{+}$	S	$f_2\pi$	1.20
$2^{-+}1^{+}$	P	$\rho\pi$	0.80
$2^{-+}1^{+}$	D	$f_2\pi$	1.50
$2^{-+}1^{+}$	D	$(\pi\pi)_{S\pi}$	1.20
$2^{-+}1^{+}$	F	$\rho\pi$	1.20

$J^{PC}M^{\epsilon}$	$L$	Isobar $\pi$	Thresh. [GeV]
$2^{++}1^{+}$	P	$f_2\pi$	1.50
$2^{++}1^{+}$	<b>D</b>	$\rho\pi$	-
$3^{++}0^{+}$	S	$\rho_3\pi$	1.50
$3^{++}0^{+}$	P	$f_2\pi$	1.20
$3^{++}0^{+}$	D	$\rho\pi$	1.50
$3^{++}1^{+}$	S	$\rho_3\pi$	1.50
$3^{++}1^{+}$	P	$f_2\pi$	1.20
$3^{++}1^{+}$	D	$\rho\pi$	1.50
$4^{-+}0^{+}$	F	$\rho\pi$	1.20
$4^{-+}1^{+}$	F	$\rho\pi$	1.20
$4^{++}1^{+}$	F	$f_2\pi$	1.60
$4^{++}1^{+}$	<b>G</b>	$\rho\pi$	<b>1.64</b>
$1^{-+}0^{-}$	P	$\rho\pi$	-
$1^{-+}1^{-}$	P	$\rho\pi$	-
$1^{++}1^{-}$	S	$\rho\pi$	-
$2^{-+}1^{-}$	S	$f_2\pi$	1.20
$2^{++}0^{-}$	P	$f_2\pi$	1.30
$2^{++}0^{-}$	D	$\rho\pi$	-
$2^{++}1^{-}$	P	$f_2\pi$	1.30
FLAT			



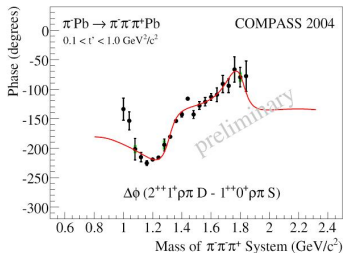
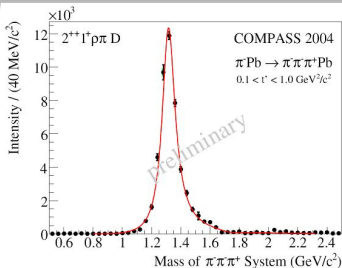
# $a_1(1260)$ und $\pi_2(1670)$



- BW für  $a_1(1260)$  + Untergrund  
 $M = (1255 \pm 6^{+7}_{-17}) \text{ MeV}/c^2$   
 $\Gamma = (367 \pm 9^{+28}_{-25}) \text{ MeV}/c^2$
- BW für  $\pi_2(1670)$   
 $M = (1658 \pm 3^{+24}_{-8}) \text{ MeV}/c^2$   
 $\Gamma = (271 \pm 9^{+22}_{-24}) \text{ MeV}/c^2$



# $a_2(1320)$



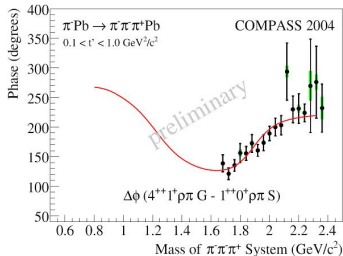
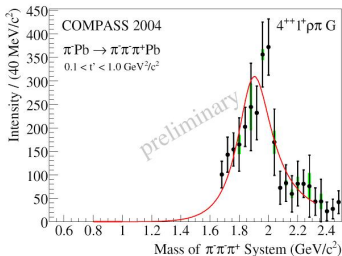
- Zwei Breit Wigner funktionen nötig um die Phasen Bewegung zu beschreiben
- BW1 für  $a_2(1320)$   
 $M = (1321 \pm 1_{-7}^{+0}) \text{ MeV}/c^2$   
 $\Gamma = (110 \pm 2_{-25}^{+2}) \text{ MeV}/c^2$
- BW2 für  $a_2(1700)$ :  $M = 1732 \text{ MeV}/c^2$ ,  $\Gamma = 194 \text{ MeV}/c^2$  (feste PDG Werte)



# $a_4(2040)$



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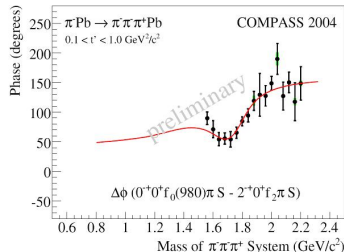
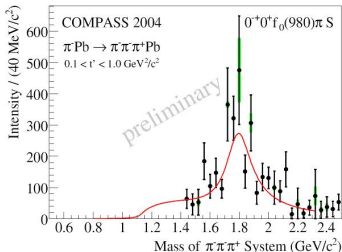
- BW mit konstanter Breite für  $a_4(2040)$  (Verweignungsverhältnis unbekannt)

- BW Parameter

$$M = (1885 \pm 13^{+50}_{-2}) \text{ MeV}/c^2$$

$$\Gamma = (294 \pm 25^{+46}_{-19}) \text{ MeV}/c^2$$





- BW mit konstanter Breite für  $\pi(1800)$  und Untergrund bei niedrigen Massen

- BW Parameter

$$M = (1785 \pm 9_{-6}^{+12}) \text{MeV}/c^2$$

$$\Gamma = (208 \pm 22_{-37}^{+21}) \text{MeV}/c^2$$



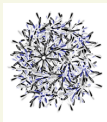
QuantumChromoDynamics predicts existence of meson states which are not foreseen within the quarkmodel:

## Tetraquarks



- Two  $q\bar{q}$  pairs
- Possible lightest candidates:  $f_0(600)$ ,  $f_0(980)$ ,  $a_0(980)$

## Glueballs



- Consists only of glue
- Lattice gauge theory: groundstate  $0^{++}$ , first excited state  $2^{++}$
- Mixing with nearby  $q\bar{q}$  states of same quantum numbers:  $f_0(1370)$ ,  $f_0(1500)$ ,  $f_0(1710)$

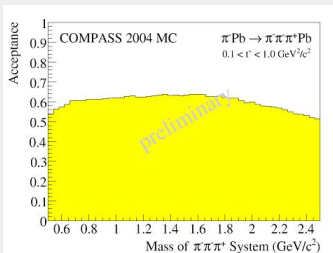
## Hybrids



- $q\bar{q}$  pair bound by excited gluons,  $q\bar{q}g$
- Lightest hybrid,  $J^{PC} = 1^{-+}$ , predicted in the mass region of 1.3-2.2 GeV/c<sup>2</sup>

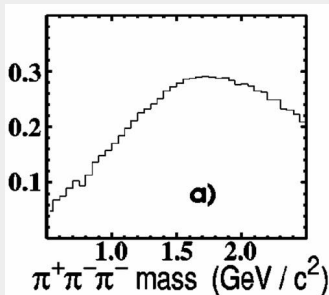


## COMPASS



- $p_{\pi} = 190 \text{ GeV}/c$
- 4M events (full  $t$  range)
- 450k events in  $0.1 < t' < 1.0 \text{ GeV}^2/c^2$

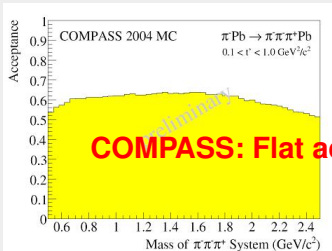
## BNL E852



- $p_{\pi} = 18 \text{ GeV}/c$
- 250k events in  $0.08 < t' < 1.0 \text{ GeV}^2/c^2$



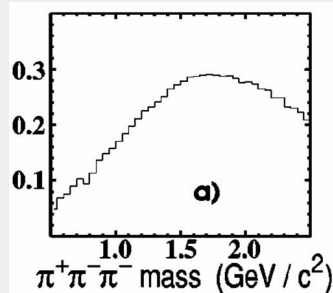
## COMPASS



**COMPASS: Flat acceptance**

- $p_{\pi} = 190 \text{ GeV}/c$
- 4M events (full  $t$  range)
- 450k events in  $0.1 < t' < 1.0 \text{ GeV}^2/c^2$

## BNL E852



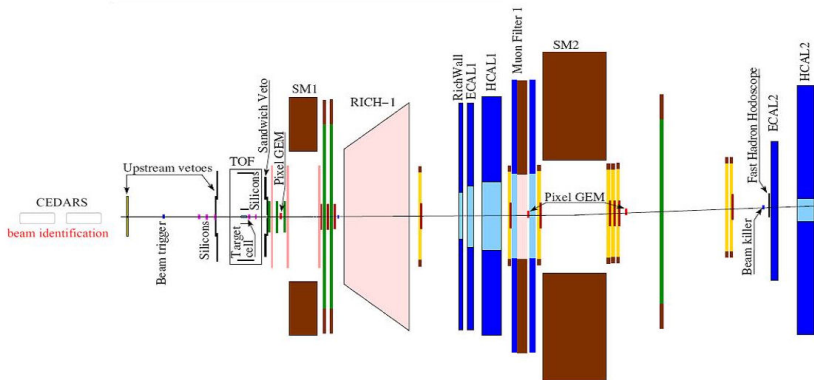
- $p_{\pi} = 18 \text{ GeV}/c$
- 250k events in  $0.08 < t' < 1.0 \text{ GeV}^2/c^2$

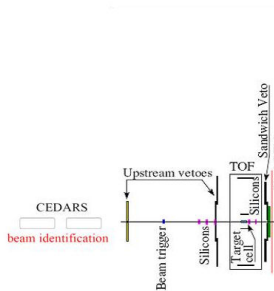


# Spectrometer Upgrade 2008



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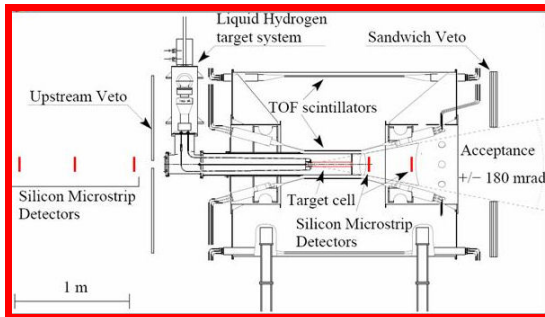
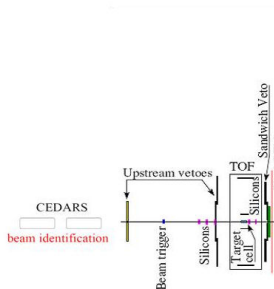




# Liquid Hydrogen Target - Proton Recoil Detector

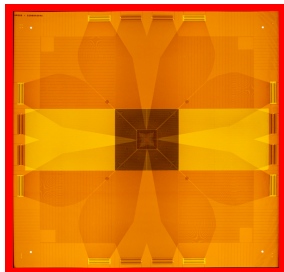
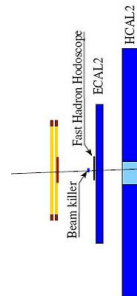
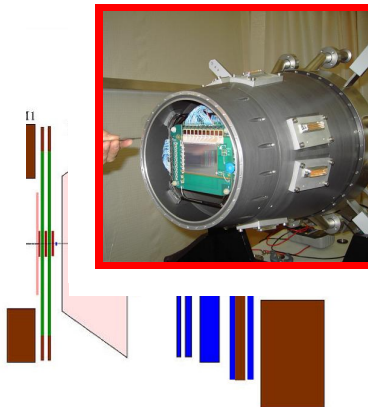


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# Additional Detectors

B<sub>x</sub>

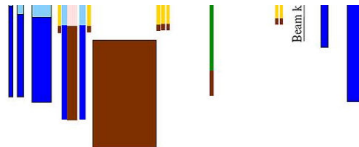
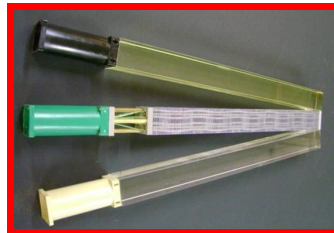
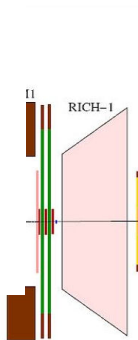


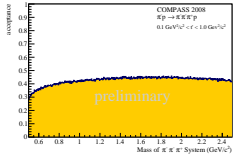
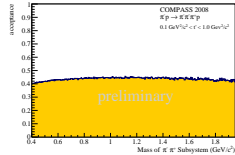
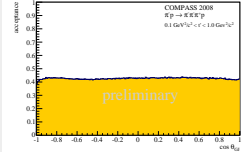
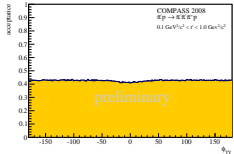
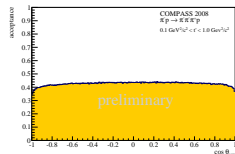
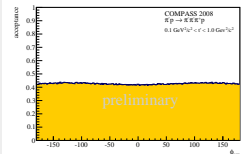


# Electromagnetic Calorimeter



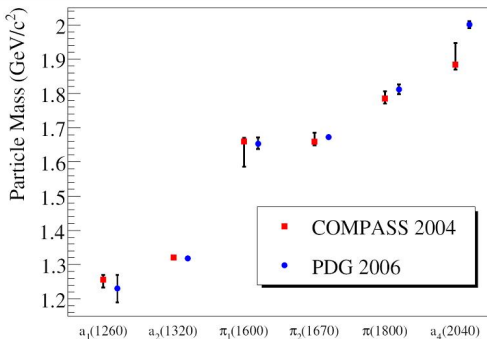
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 $\pi^- \pi^- \pi^+$  mass $\pi^- \pi^+$  mass $\cos\theta_{GJ}$  $\phi_{TY}$  $\cos\theta_{helicity}$  $\phi_{helicity}$ 



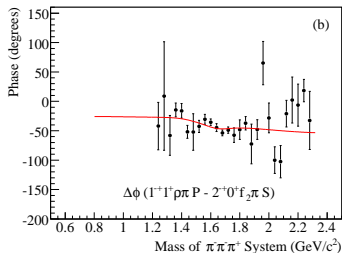
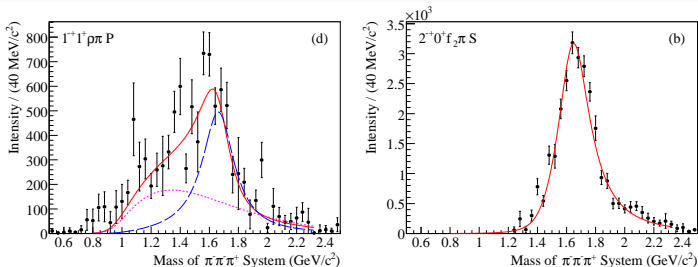
- Illinois/Protvino/Munich Program used.
- Enhanced wave set (53 partial waves).
- 20  $\text{MeV}/c^2$  mass bins.
- 30 fits per mass bin.
- D-Functions instead of Zemach-Tensors for parametrisation of decay amplitudes.
- Same mass range as for 2004 data:  $0.5\text{-}2.5 \text{ GeV}/c^2$ .

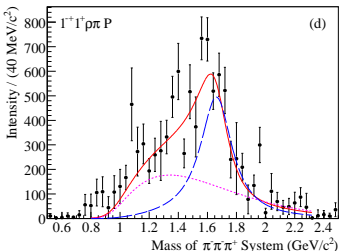


Resonance	Mass ( $\text{MeV}/c^2$ )	Width ( $\text{MeV}/c^2$ )	Intensity (%)	Channel $J^{PC}M^{\epsilon}[\text{isobar}]L$
$a_1(1260)$	$1255 \pm 6_{-17}^{+7}$	$367 \pm 9_{-25}^{+28}$	$67 \pm 3_{-20}^{+4}$	$1^{++}0^+ \rho\pi S$
$a_2(1320)$	$1321 \pm 1_{-7}^{+0}$	$110 \pm 2_{-15}^{+2}$	$19.2 \pm 0.6_{-2.2}^{+0.3}$	$2^{++}1^+ \rho\pi D$
$\pi_1(1600)$	$1660 \pm 10_{-64}^{+0}$	$269 \pm 21_{-64}^{+42}$	$1.7 \pm 0.2_{-0.1}^{+0.9}$	$1^{-+}1^+ \rho\pi P$
$\pi_2(1670)$	$1658 \pm 3_{-8}^{+24}$	$271 \pm 9_{-24}^{+22}$	$10.0 \pm 0.4_{-0.7}^{+0.7}$	$2^{-+}0^+ f_2\pi S$
$\pi(1800)$	$1785 \pm 9_{-6}^{+12}$	$208 \pm 22_{-37}^{+21}$	$0.8 \pm 0.1_{-0.1}^{+0.3}$	$0^{-+}0^+ f_0\pi S$
$a_4(2040)$	$1885 \pm 13_{-2}^{+50}$	$294 \pm 25_{-19}^{+46}$	$1.0 \pm 0.3_{-0.1}^{+0.1}$	$4^{++}1^+ \rho\pi G$



# $J^{PC} = 1^{-+}$ Exotic Wave





BW parameter<sup>1</sup> for  $\pi_1(1600)$

- $M = (1660 \pm 10_{-64}^{+0}) \text{ MeV}/c^2$

- $\Gamma = (269 \pm 21_{-64}^{+42}) \text{ MeV}/c^2$

<sup>1</sup>A. Alekseev *et. al.*, COMPASS Collaboration,  
Phys. Rev. Lett. 104, 241803 (2010)



## Step 1: Mass-Independent PWA

- Independent fits in 40 MeV mass bins

$$\sigma_{indep}(\tau, m, t') = \sum_{\epsilon=\pm 1} \sum_{r=1}^{N_r} \left| \sum_i T_{ir}^\epsilon f_i^\epsilon(t') \psi_i^\epsilon(\tau, m) / \sqrt{\int |\psi_i^\epsilon(\tau', m)|^2 d\tau'} \right|^2$$

- Production amplitudes  $T_{ir}^\epsilon \rightarrow$  extended maximum likelihood fit
- Decay amplitudes  $\psi_i^\epsilon(\tau, m)$  (Zemach tensors)
- 41 partial waves  $i = J^{PC} M^\epsilon [isobar]L$  + flat background
  - isobars:  $(\pi\pi)_S, \rho(770), f_0(980), f_2(1270), \rho_3(1690)$
  - 7 negative reflectivity waves included
  - more M=1 waves than previous (e.g. BNL E852) analyses

## Step 2: Mass-Dependent $\chi^2$ fit

- 6 waves
- Parameterized by Breit-Wigner
- Coherent background for some waves