The COMPASS Hadron Program

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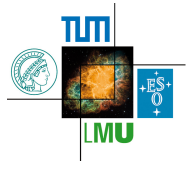
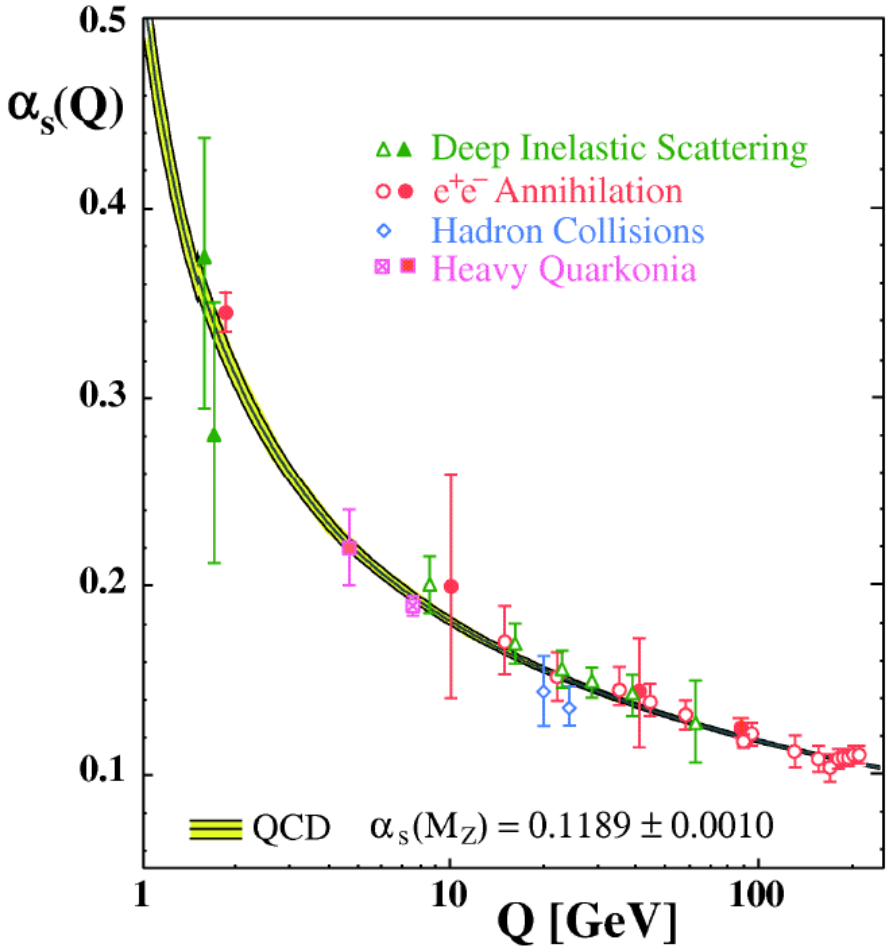
Cluster of Excellence: Origin and Structure of the Universe, BMBF



From QCD to Hadron Physics



S. Bethke

[arXiv:hep-ex/0606035v2]

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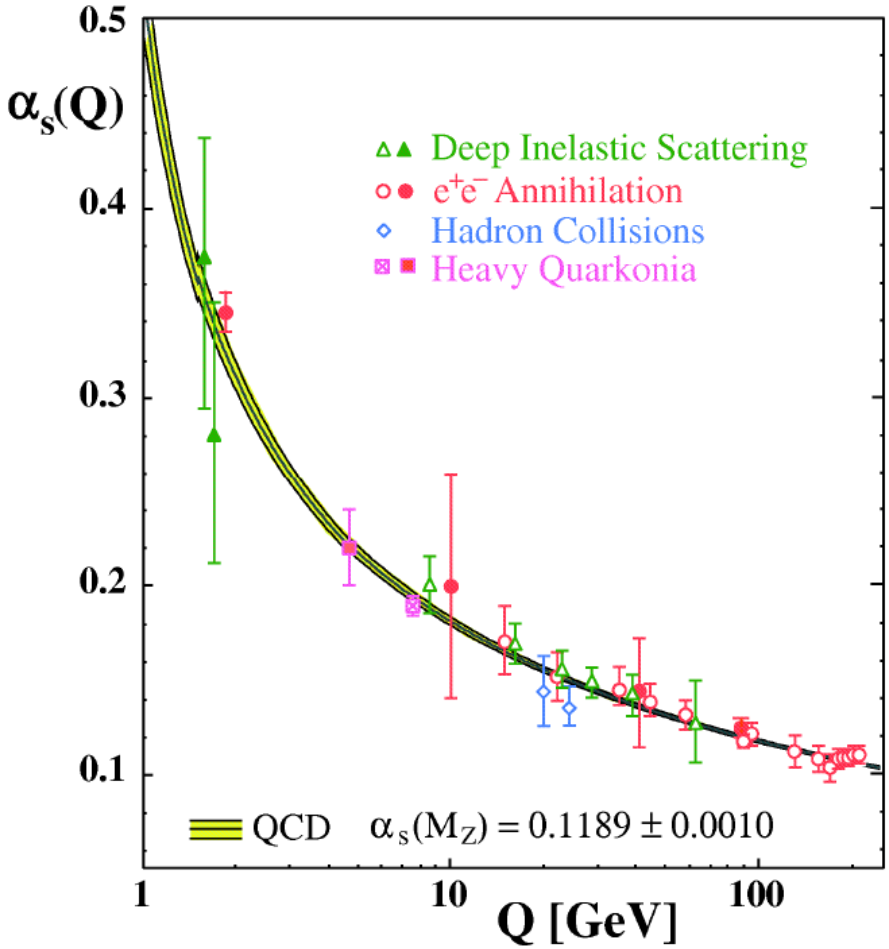

- Asymptotic Freedom
- Quarks & Gluons relevant DOF
- Perturbative QCD
- Hadronization, Jets

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- Confinement
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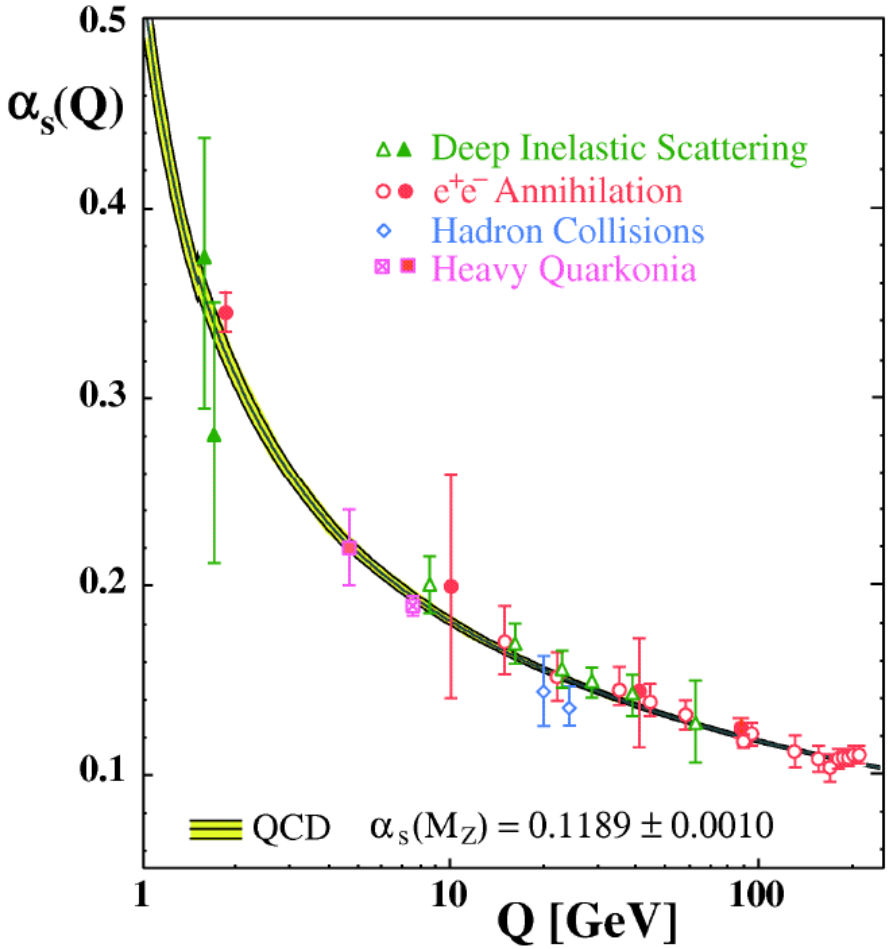

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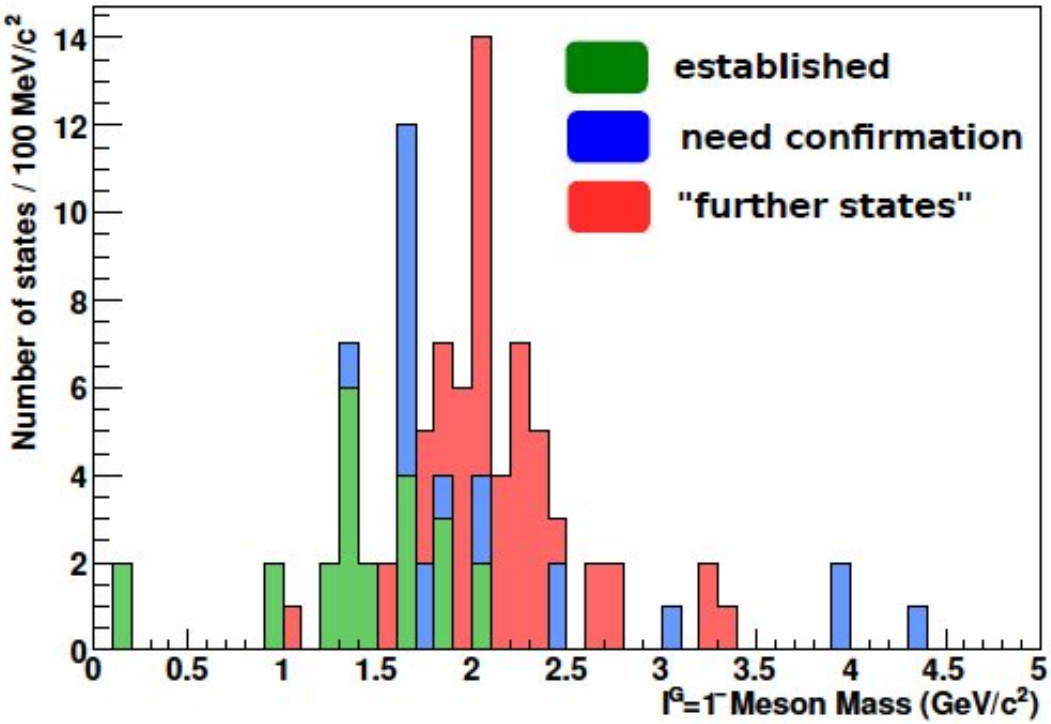
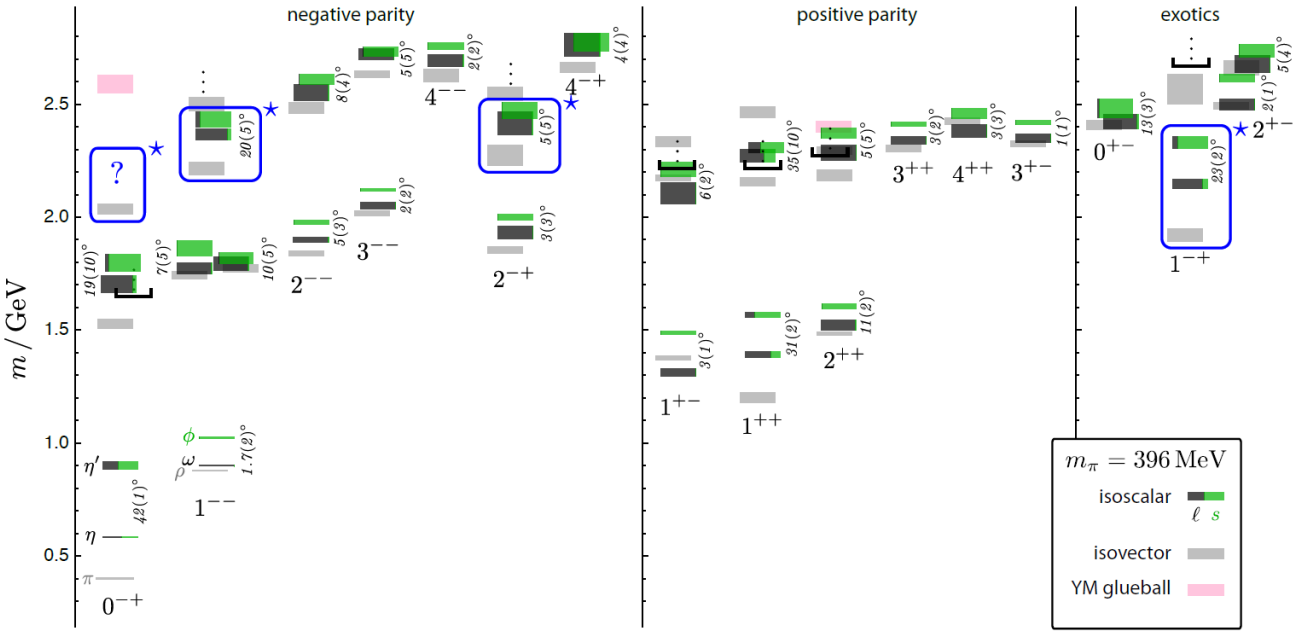
- Dynamics of excited states?



From QCD to Hadron Physics

- Confinement
- Hadrons
  relevant DOF
- Dynamics of excited states?
- Models and theories
  - Quark model
  - Bag model
  - Flux tube model
  - χPT for slow pions
  - Lattice QCD



Dudek et al. [arXiv:1106.5515v1]

From QCD to Hadron Physics

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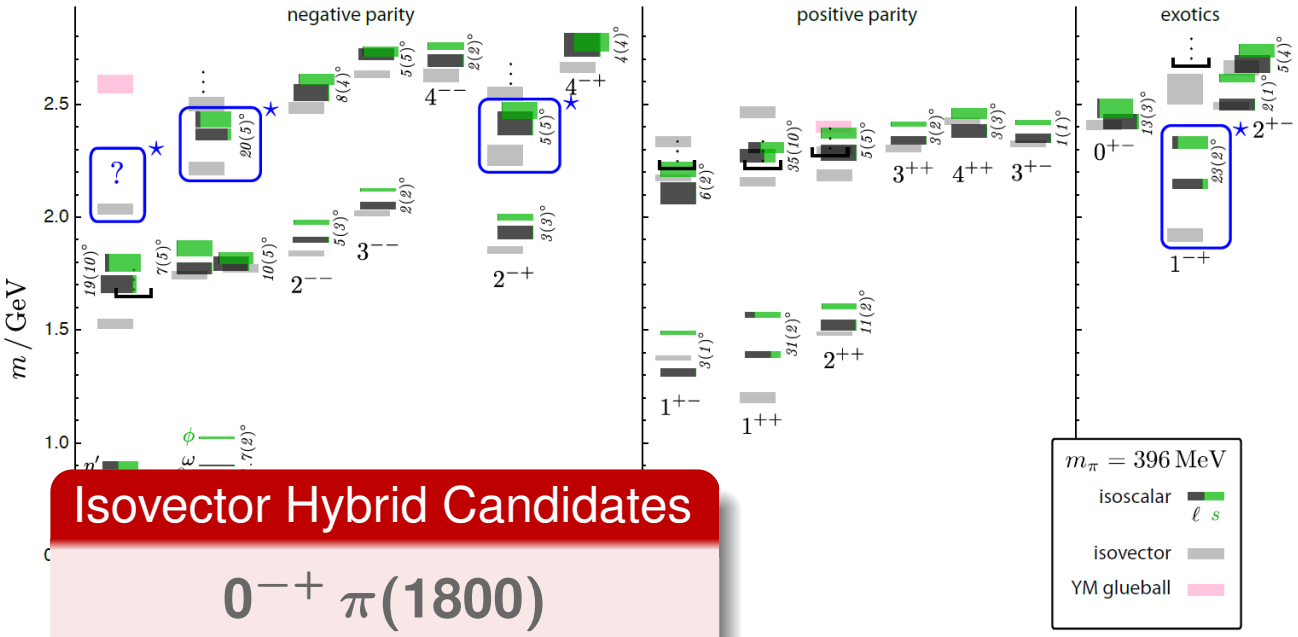

Isovector Hybrid Candidates

0^{-+} $\pi(1800)$
1^{-+} $\pi_1(1600)$
2^{-+} $\pi_2(1880)$

06.5515v1]

The COMPASS Hadron Setup

Spectrometer and Hadron Beam

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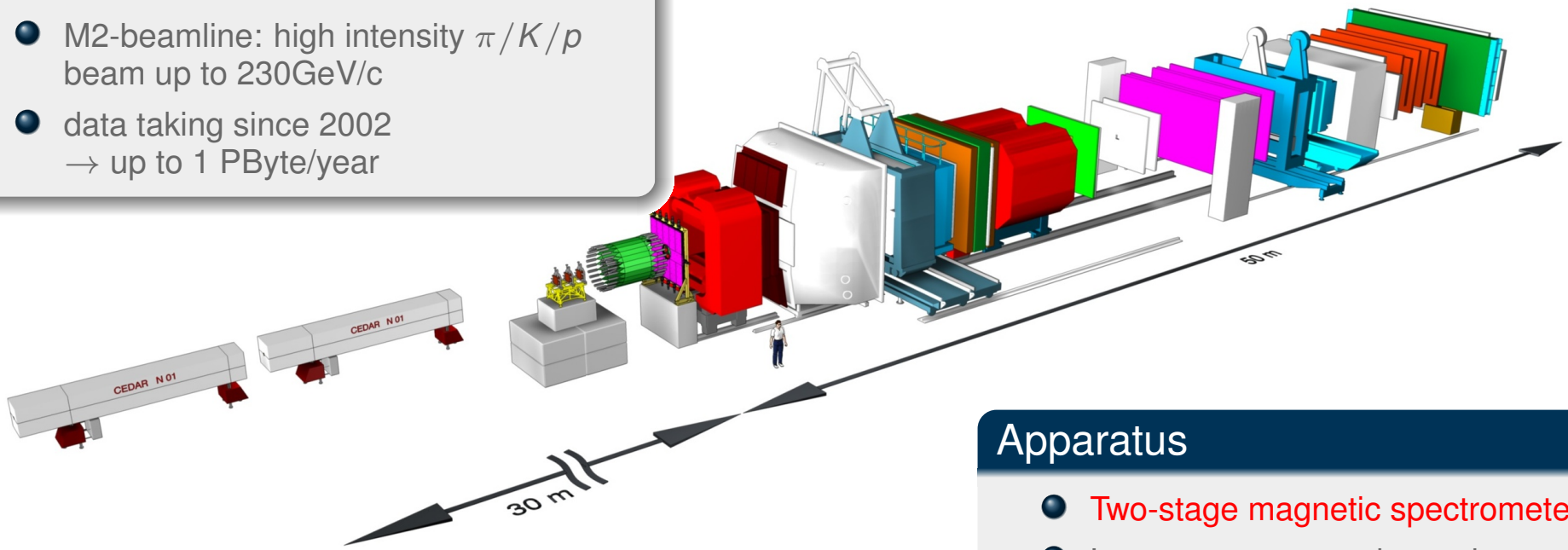
Overview

- **CO**mmon **M**uon and **P**roton **A**pparatus for **S**tructure and **S**pectroscopy [1]
- Located at CERN SPS
- M2-beamline: high intensity $\pi/K/p$ beam up to 230GeV/c
- data taking since 2002
  $\rightarrow$ up to 1 PByte/year



Apparatus

- Two-stage magnetic spectrometer
- Large acceptance charged tracking
- Calorimetry (ECAL/HCAL)
- Kaon PID (CEDARs/RICH)

[1] [Nucl. Instr. and Meth. A 577 (2007) 455]

Light Meson Spectroscopy

- $\pi^-\pi^-\pi^+$ and $\pi^-\pi^0\pi^0$
- $\eta\pi^-$ and $\eta'\pi^-$
- $\pi^-\pi^+\pi^-\pi^+\pi^-$
- Status of the $J^{PC} = 1^{-+}$ Spin Exotic Partial Wave
- $\pi\pi$ Production at Central Rapidities

Tests of Chiral Dynamics

- **3**π Primakoff Production

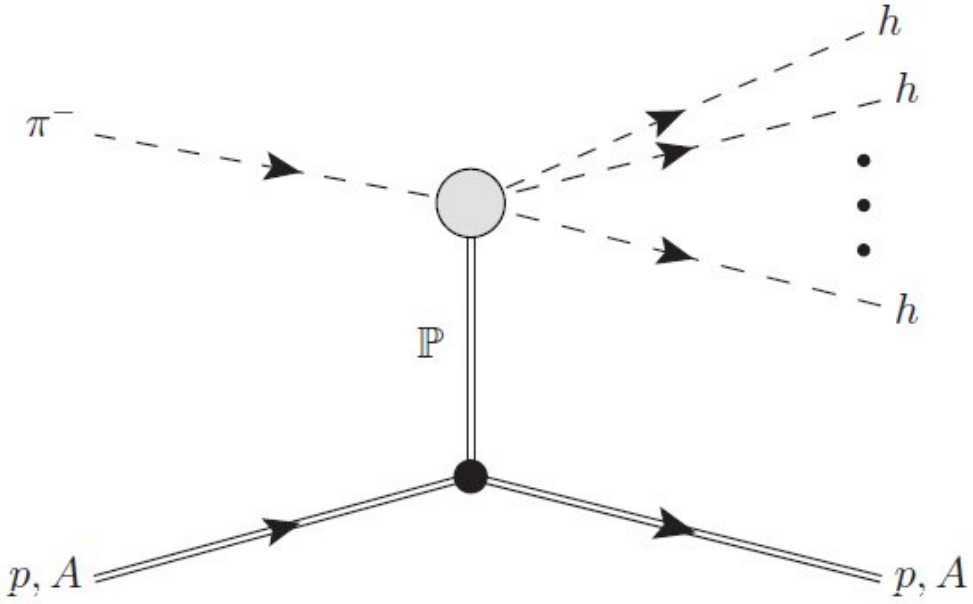
Conclusions and Outlook

- Pion Polarizability

Light Meson Spectroscopy

Isovector Mesons

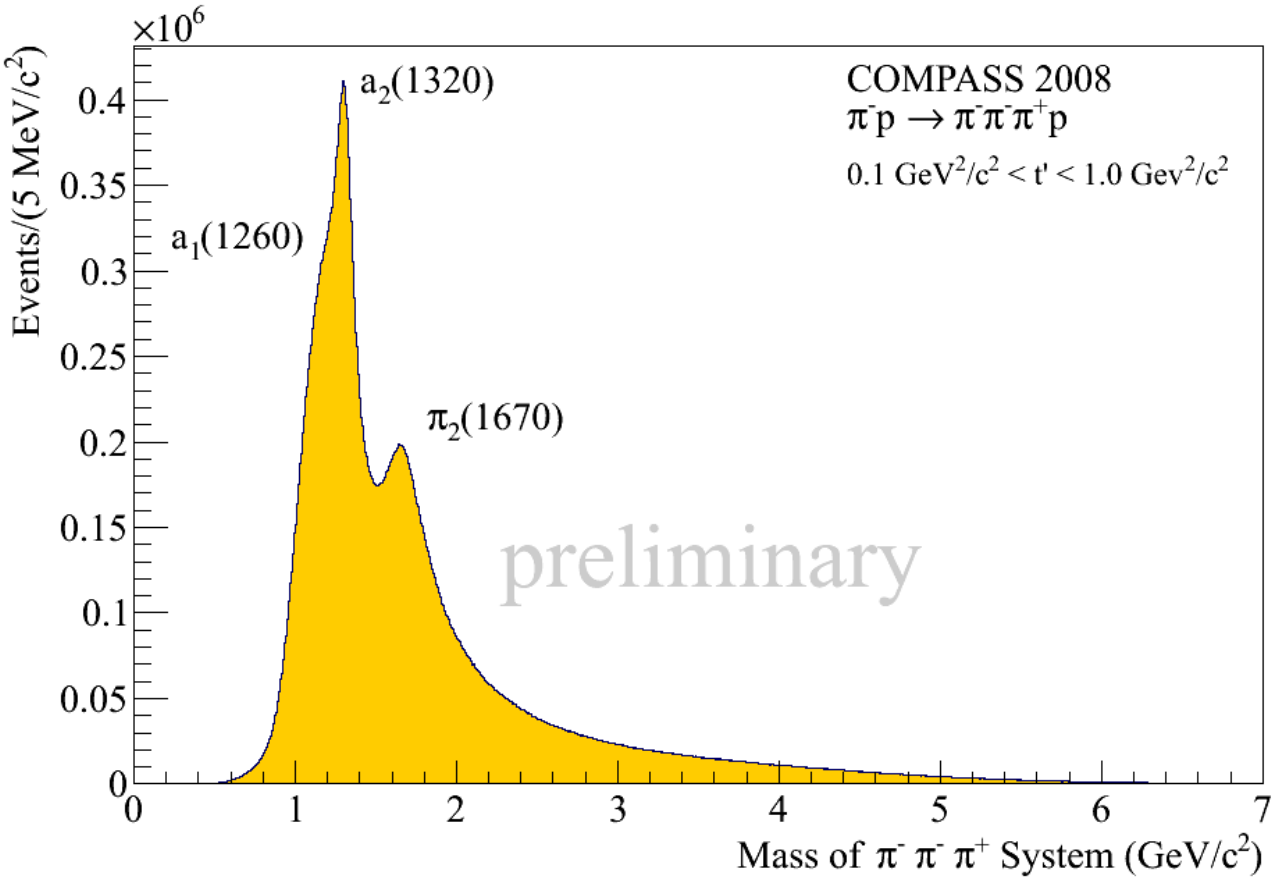
Diffractive Pion Dissociation



$\pi^- p \to \pi^- \pi^- \pi^+ p$ (2008)

Invariant Mass Spectrum

- 190 GeV/c hadron beam → $96\%\pi^-, 3.5\%K^-, 0.5\%\overline{p}$
- 40cm liquid hydrogen target
- $0.1\text{GeV}^2/c^2 < t' < 1.0\text{GeV}^2/c^2$
- ∼50M exclusive events (2008)



Partial Wave Analysis - Formalism

Two-Step Approach

- Fit in mass bins (Decomposition in Partial Waves)

$$\mathcal{I}(\tau, t') = \sum_{\epsilon=\pm 1} \sum_{r=1}^{N_r} \left| \sum_i T^{\epsilon}_{ir} f(t') \psi^{\epsilon}_i(\tau) \right|^2$$

  - T: Transition amplitude $\in \mathbb{C}$(to be fitted)
  - $f(t')$: t'-dependence $\in \mathbb{R}$(or t' binned analysis)
  - ψ: Decay amplitude $\in \mathbb{C}$(Helicity formalism, reflectivity basis)

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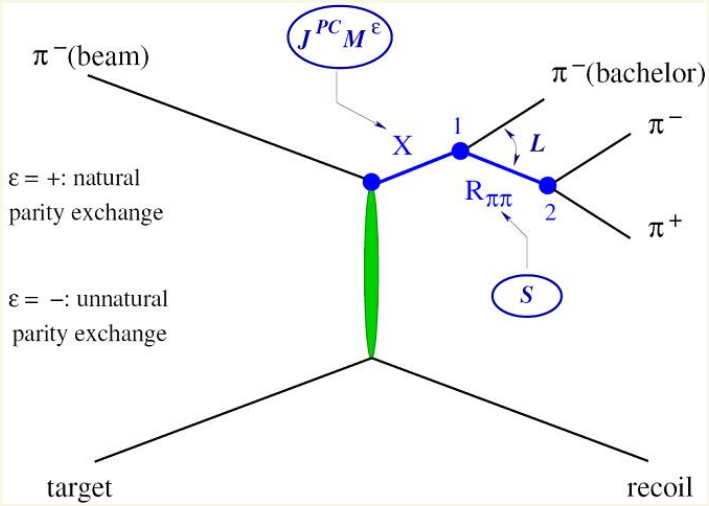
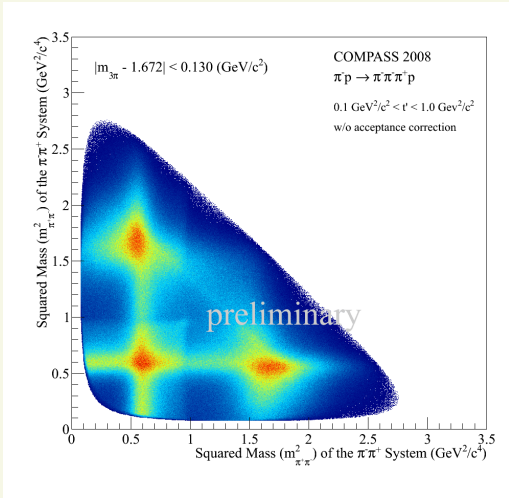
Isobar Model



Dalitz Plot $\pi_2(1670)$ region



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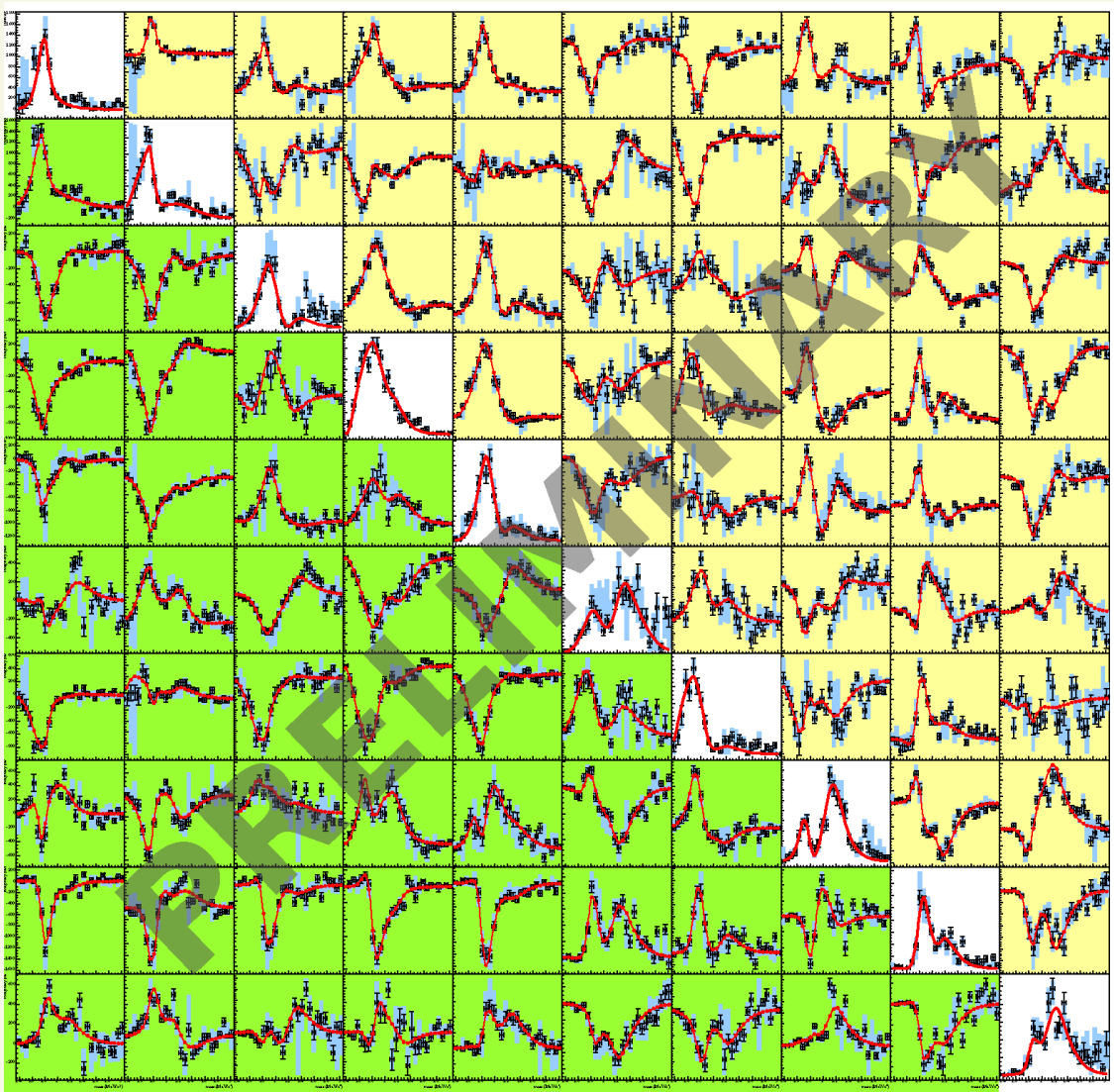
- χ^2 fit of the spin-density matrix (Extraction of Resonance Parameters)
  - Parametrization of spin-density matrix elements, $\sum_r T^{\epsilon}_{ir} T^{\epsilon *}_{jr}(m_x)$
  - Takes into account interference terms
  - Coherent background for some waves

Partial Wave Analysis - Formalism

Two-Step Approach

Example: Spin density matrix of 10 waves



)

$\Big|^2$

)

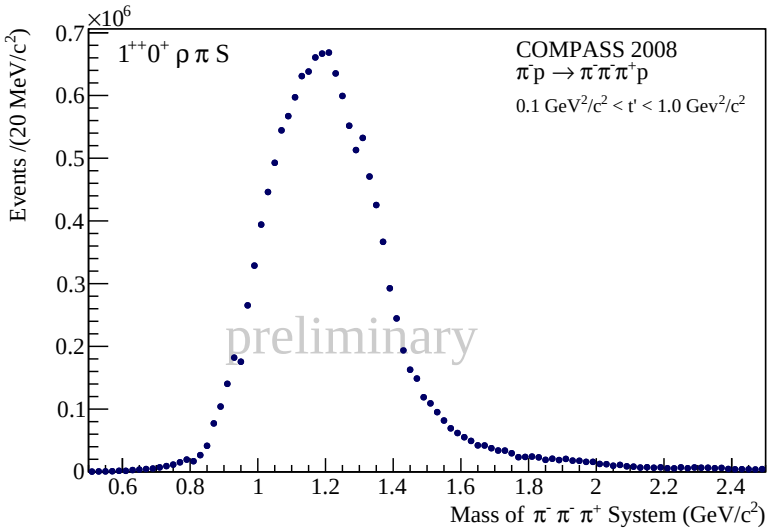
tivity basis)

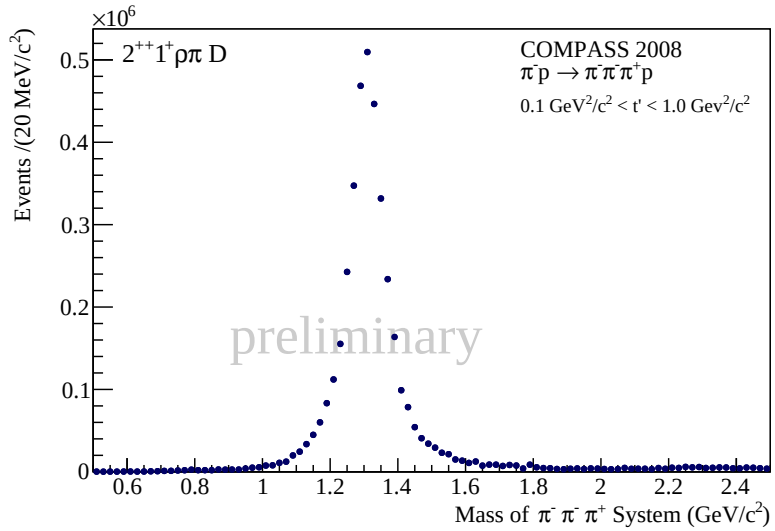
sonance

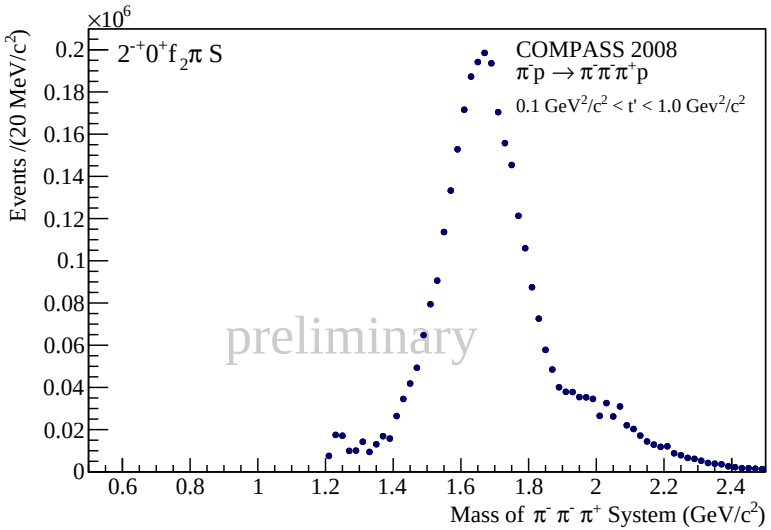
$\sum_r T^{\epsilon}_{ir} T^{\epsilon *}_{jr}(m_x)$

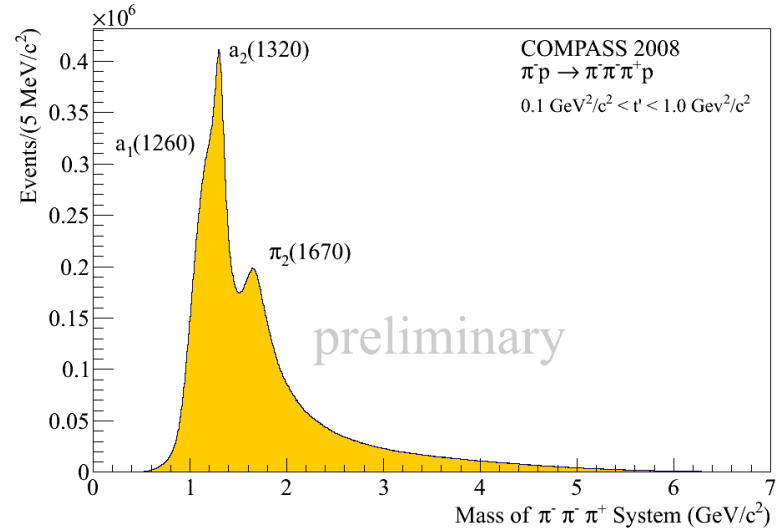
$\pi^- p \to \pi^- \pi^- \pi^+ p$ (2008)

Intensities of dominant J^{PC} states



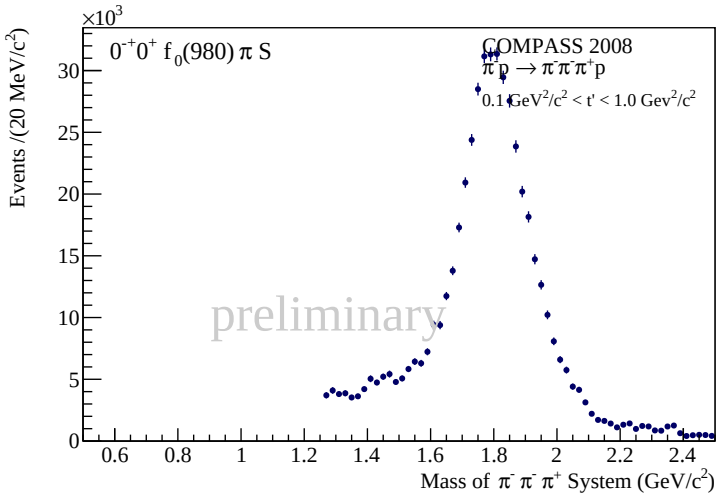


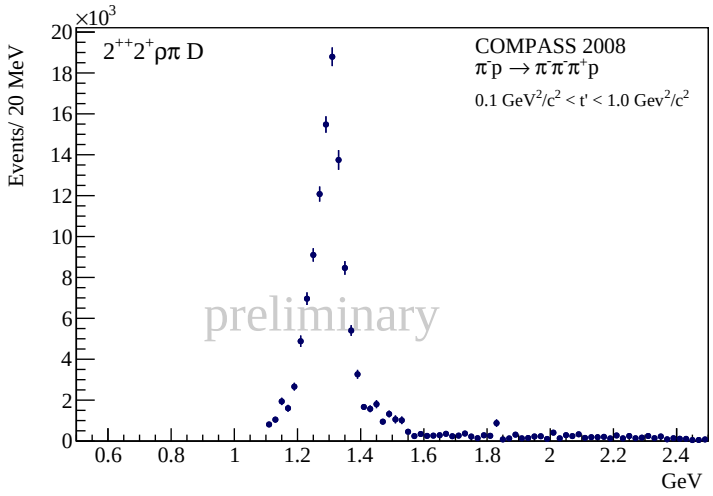


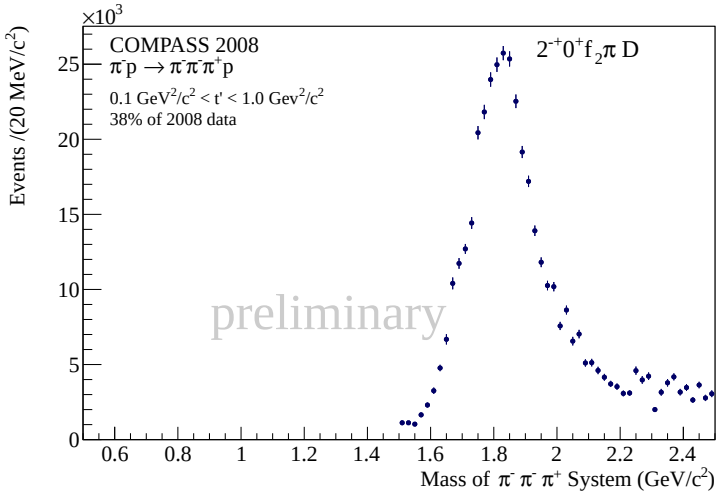


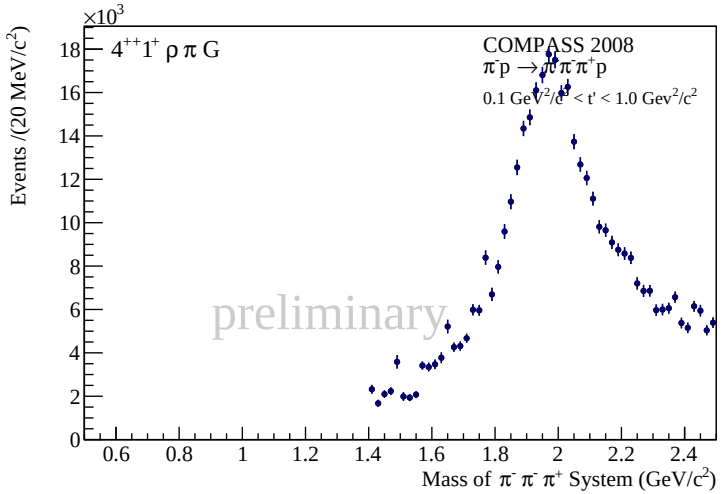
$\pi^- p \to \pi^- \pi^- \pi^+ p$ (2008)

Additional Waves

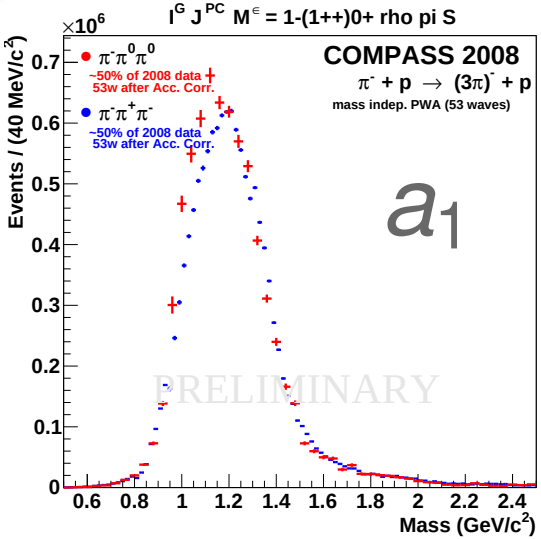


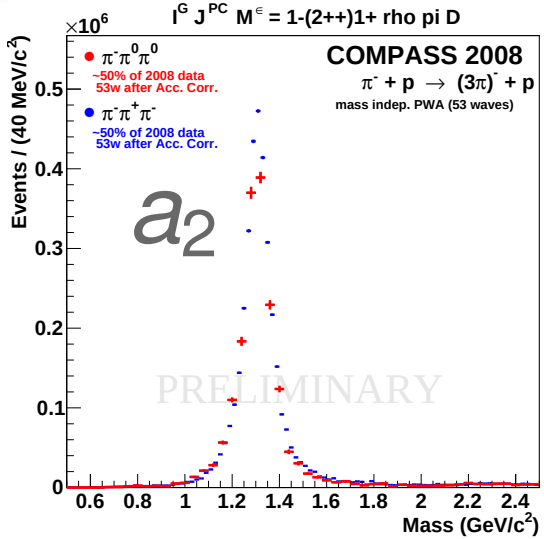






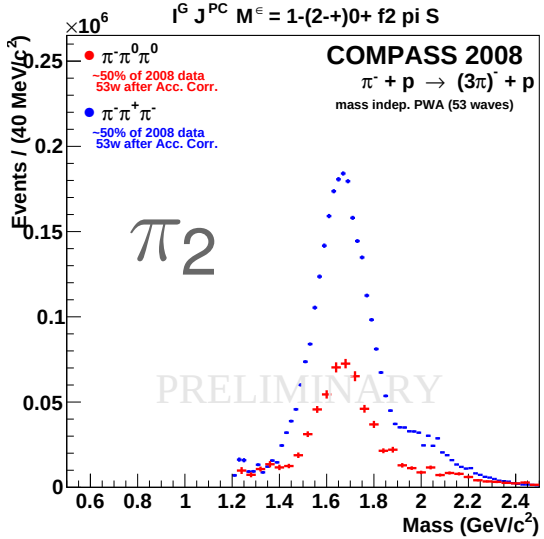
Comparison $\pi^- p \to \pi^- \pi^- \pi^+ p$ vs $\pi^- p \to \pi^- \pi^0 \pi^0 p$ (2008)





Charged/Neutral Pions

- Channels probe different parts of spectrometer
- Qualitative agreement
- Isospin symmetry



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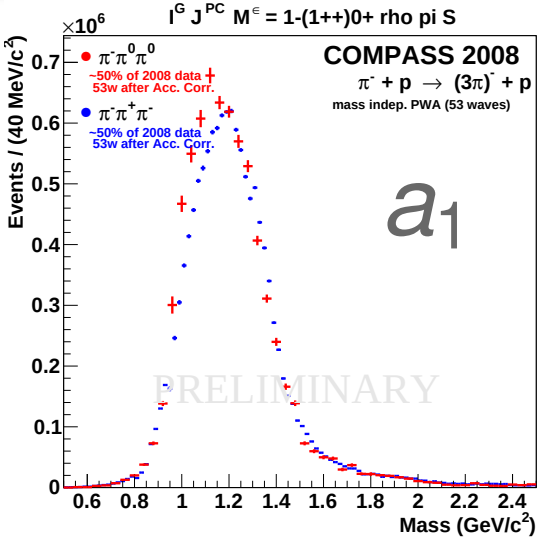
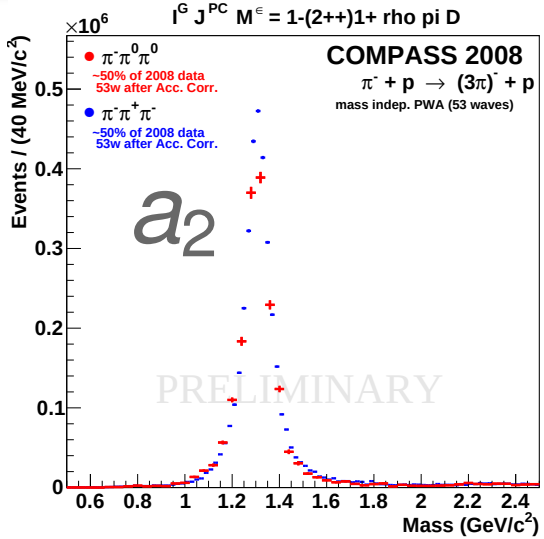
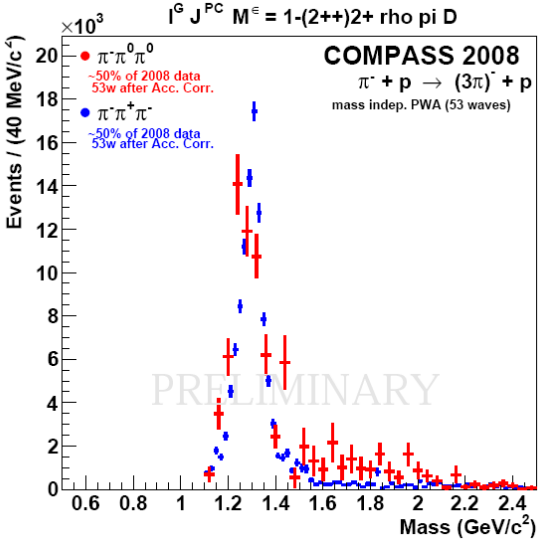
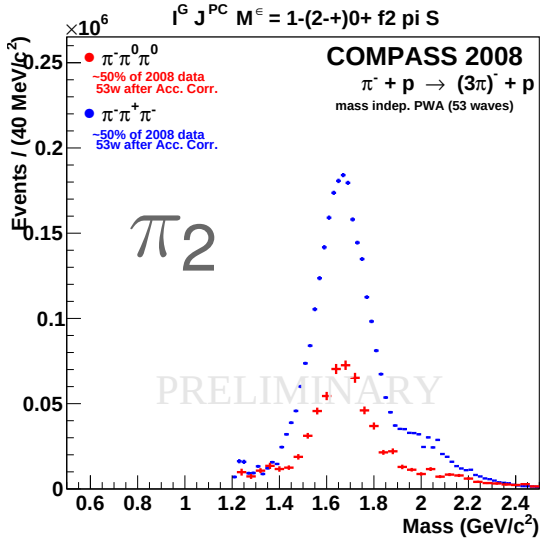






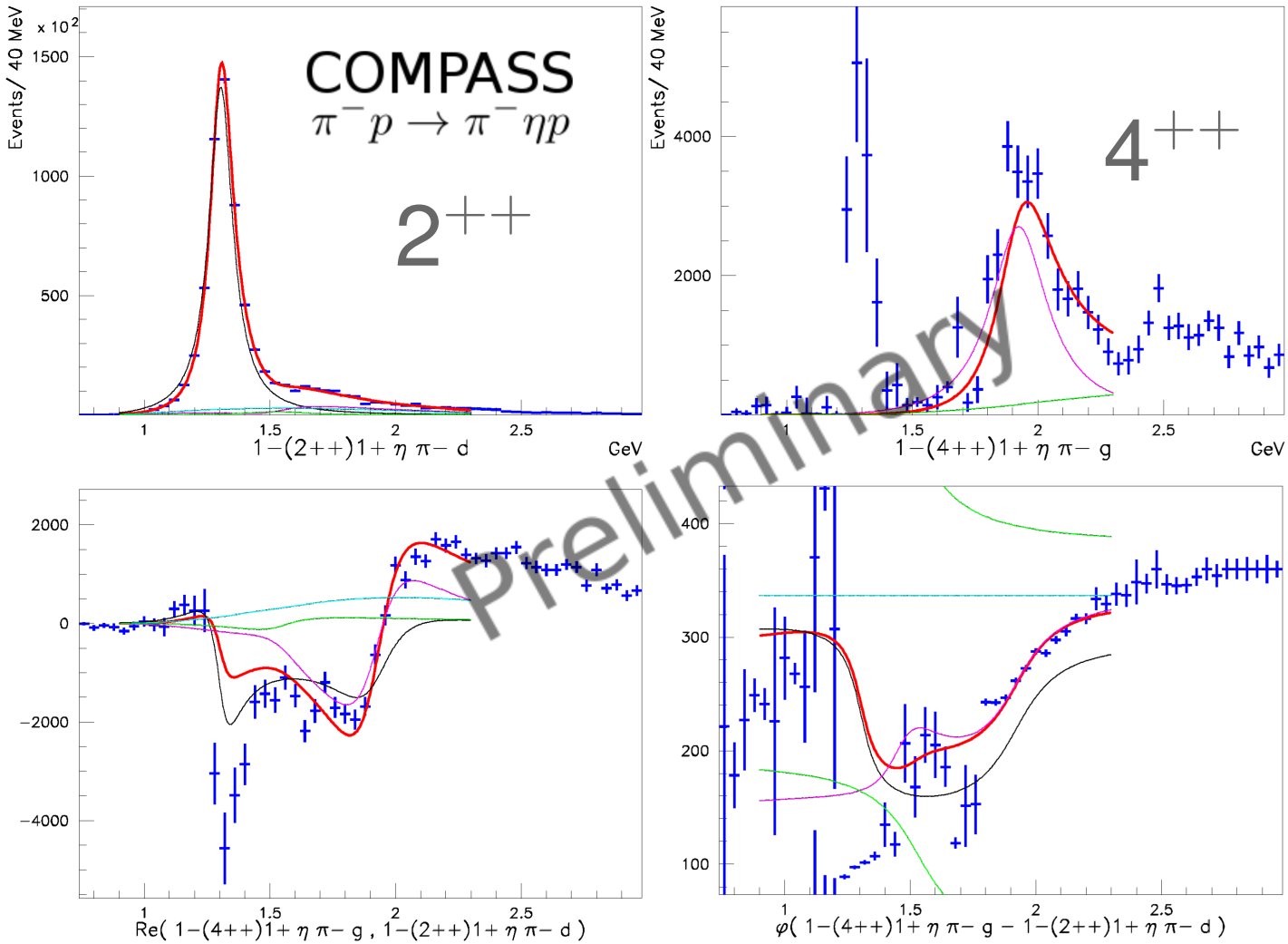


$$\pi^- + p \to \eta\pi + p$$

$$\pi^- + p \to \eta'\pi + p$$

$\pi^- + p \to \eta\pi + p$ (2008)

D_+- vs G_+-wave



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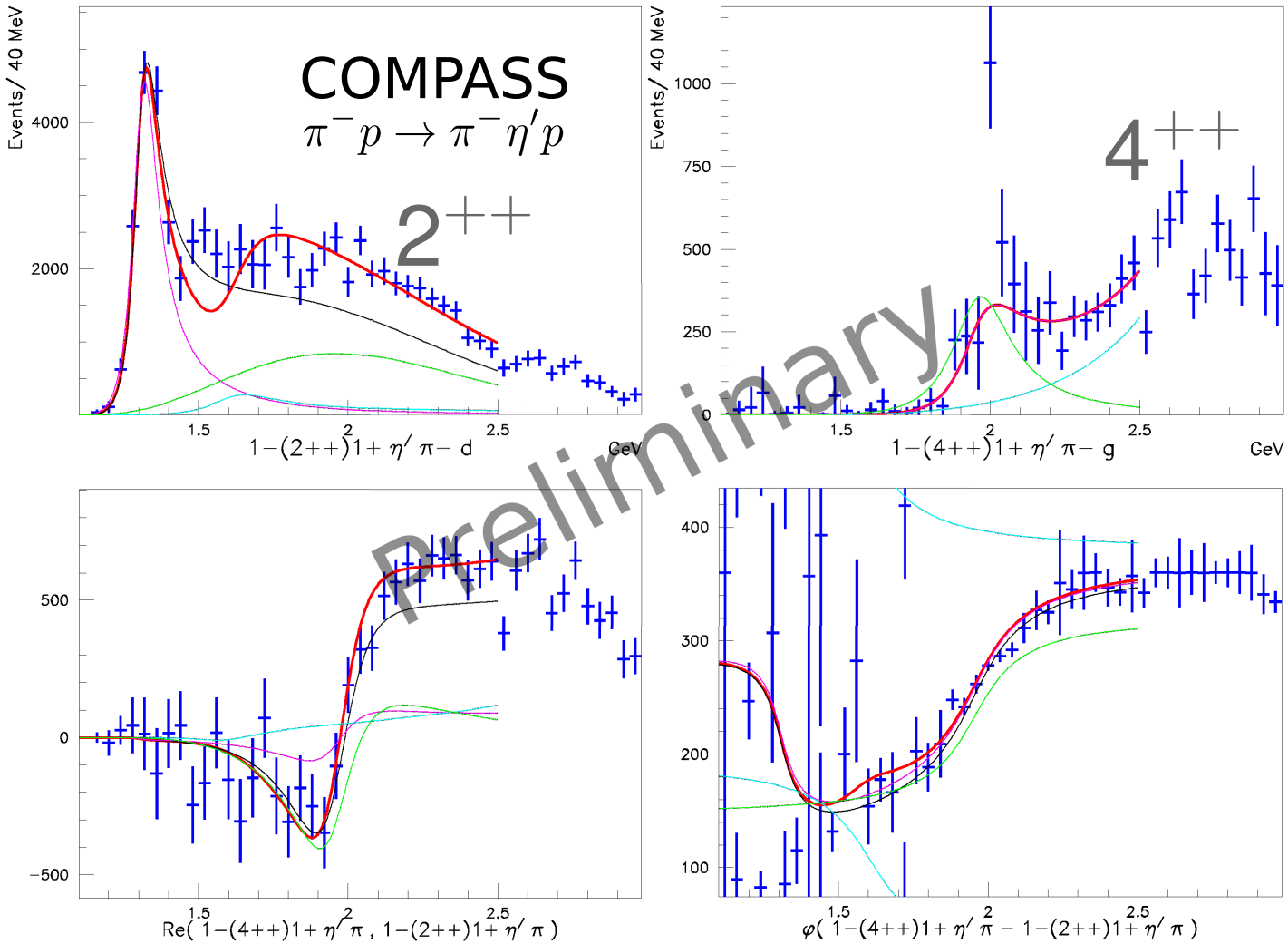
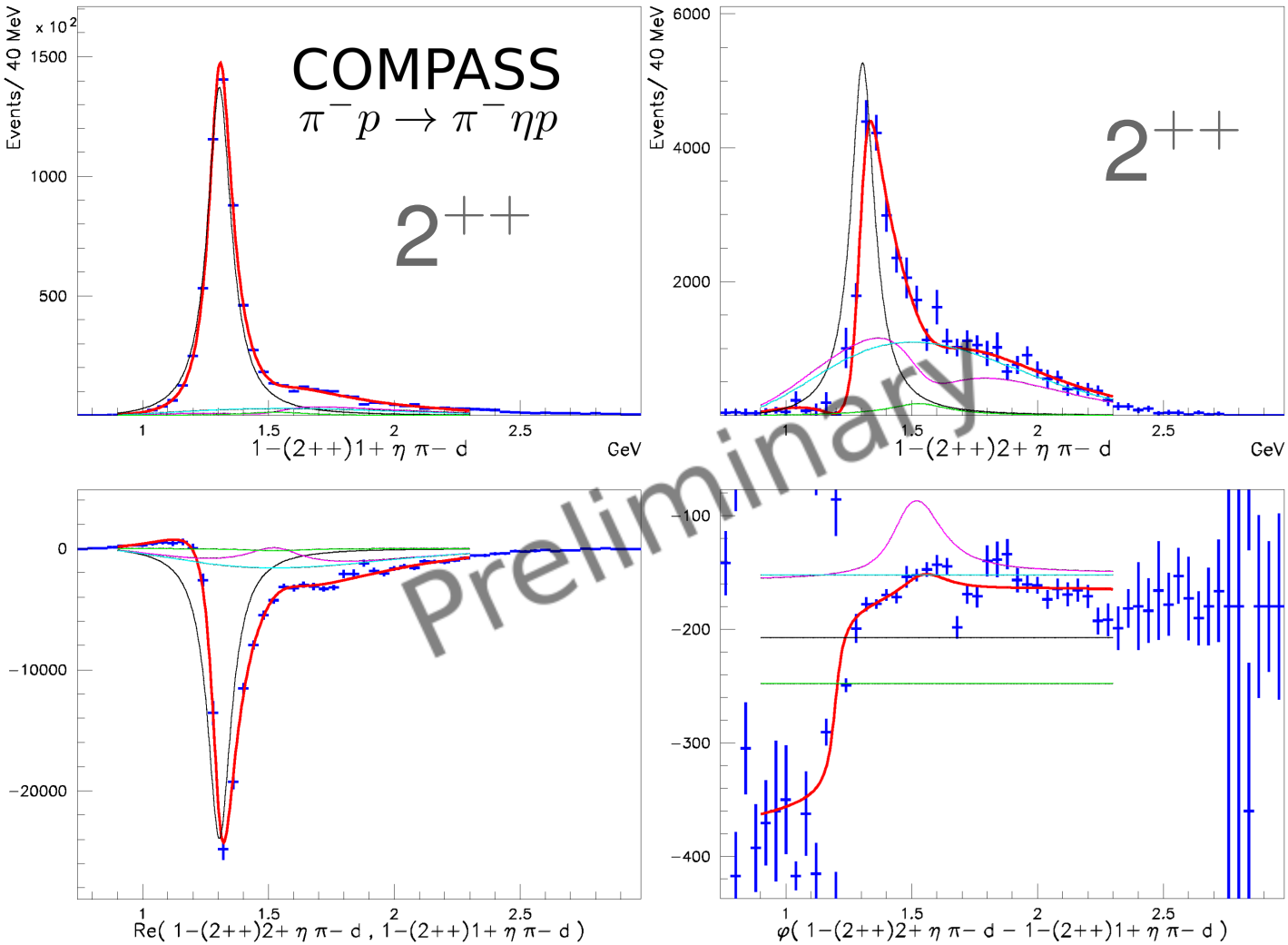


$\pi^- + p \to \eta\pi + p$ (2008)

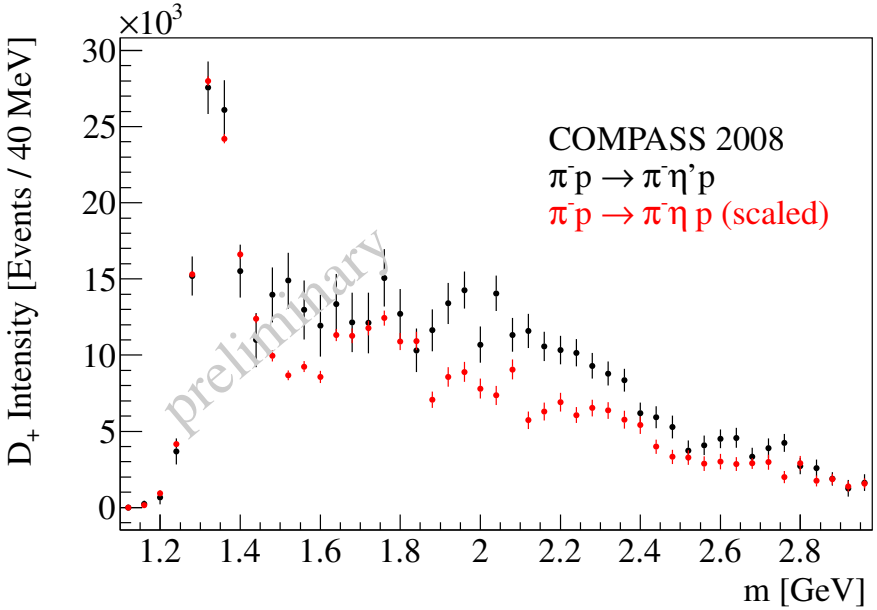
D_+- vs D_{++}-wave



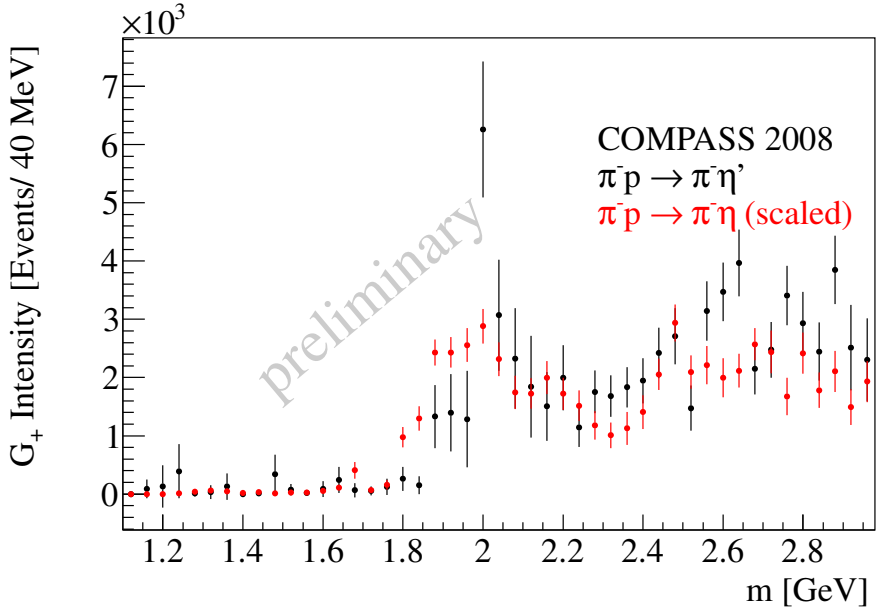
Comparison $\pi^- + p \to \eta'\pi + p$ vs $\pi^- + p \to \eta\pi + p$ (2008)

Scaling: Adjustment for branching and phase space

Comparison of D_+ waves



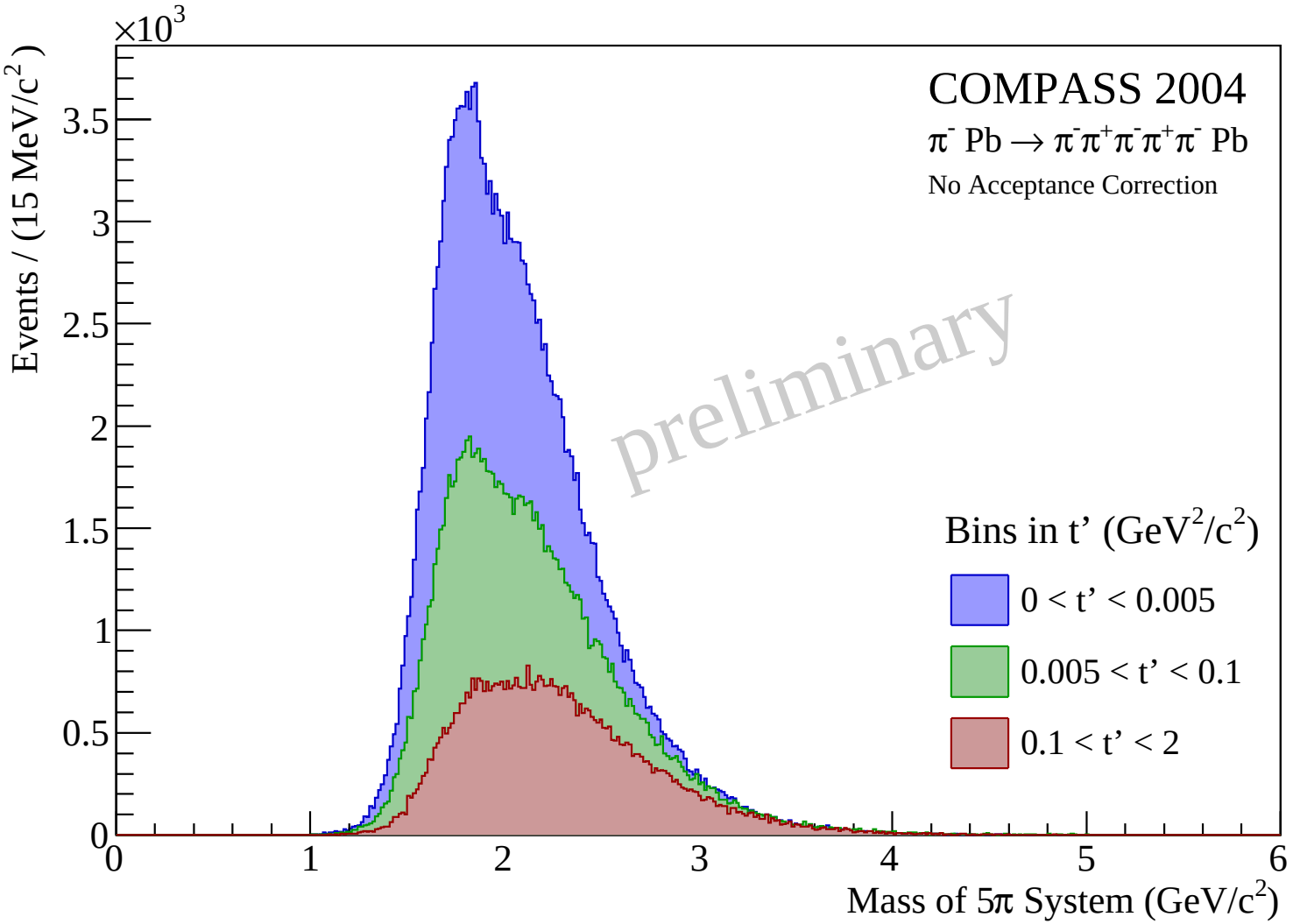
Comparison of G_+ waves



Exploring the light Meson Frontier

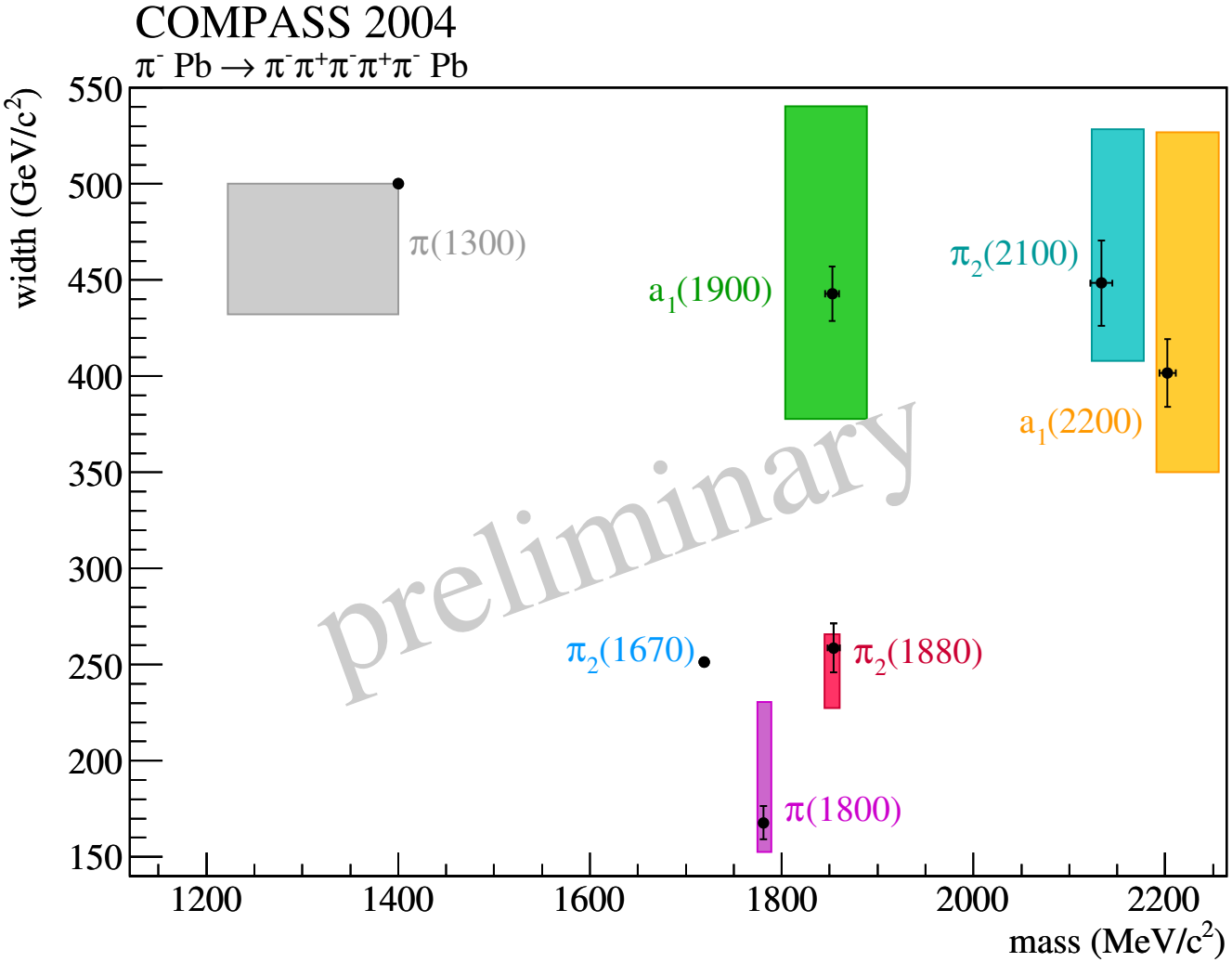
$$\pi^- + Pb \rightarrow \pi^-\pi^+\pi^-\pi^+\pi^- + Pb$$

$\pi^- Pb \to \pi^-\pi^+\pi^-\pi^+\pi^- Pb$ (2004)



5π Resonances — Extracted Parameters

Summary of Resonance Parameters



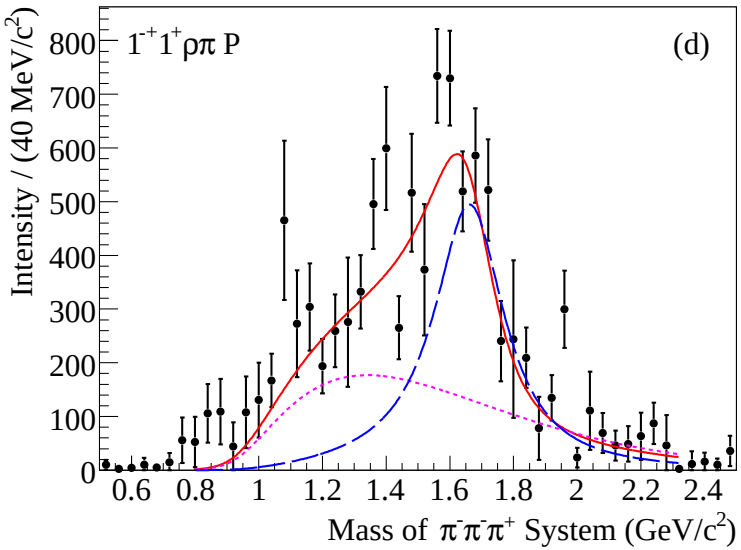
$\pi^- Pb \to \pi^-\pi^+\pi^- Pb$ (2004)

The spin exotic $J^{PC} = 1^{-+}\rho\pi$ P-wave

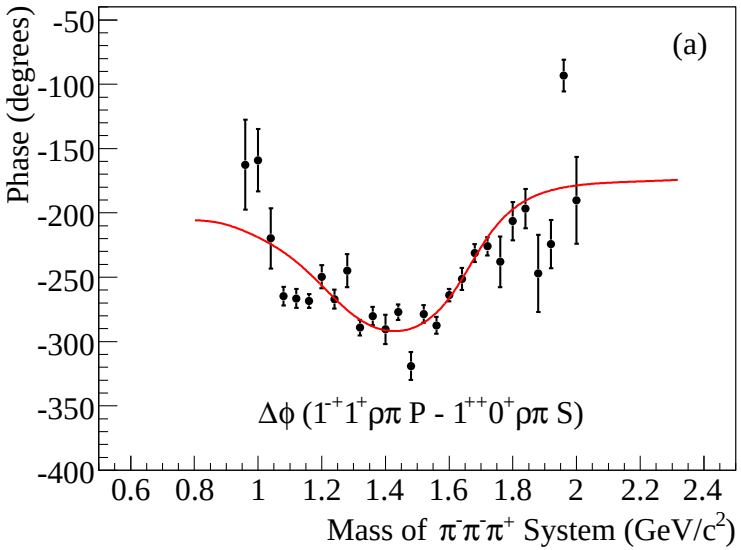
Exotic Signatures

- Isospin exotics: “forbidden” decays
- Spin exotics: $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$... forbidden in $q\bar{q}$
- Proof of existence $\rightarrow$ strong hint for physics beyond the quark model

COMPASS (2004): π^-Pb$\rightarrow \pi^-\pi^+\pi^-$Pb $\sim$ 400 000 events



[PRL **104** (2010) 241803]



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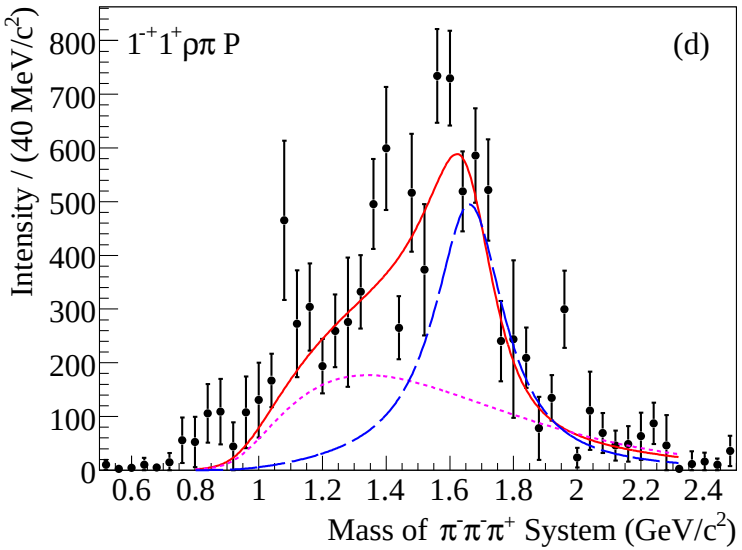


[PRL **104** (2010) 241803]

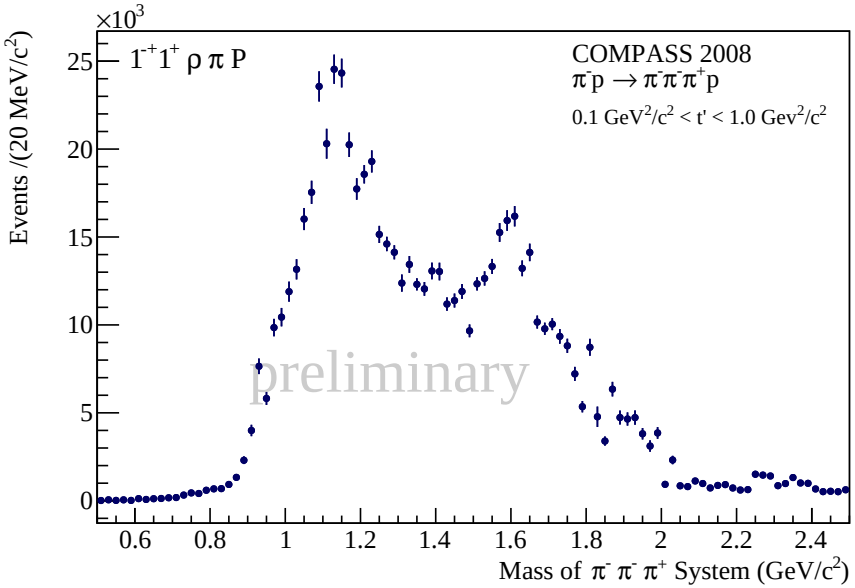
Spin Exotic $\pi_1(1600)$

- Significant 1^{-+} amplitude consistent with resonance at $\sim 1.7\,\mathrm{GeV/c^2}$
- No leakage observed ($< 5\%$)
- BW for $\pi_1(1600)$ + background:
  $M = (1.660 \pm 0.010\ ^{+0.000}_{-0.064})\,\mathrm{GeV/c^2}$
  $\Gamma = (0.269 \pm 0.021\ ^{+0.042}_{-0.064})\,\mathrm{GeV/c^2}$

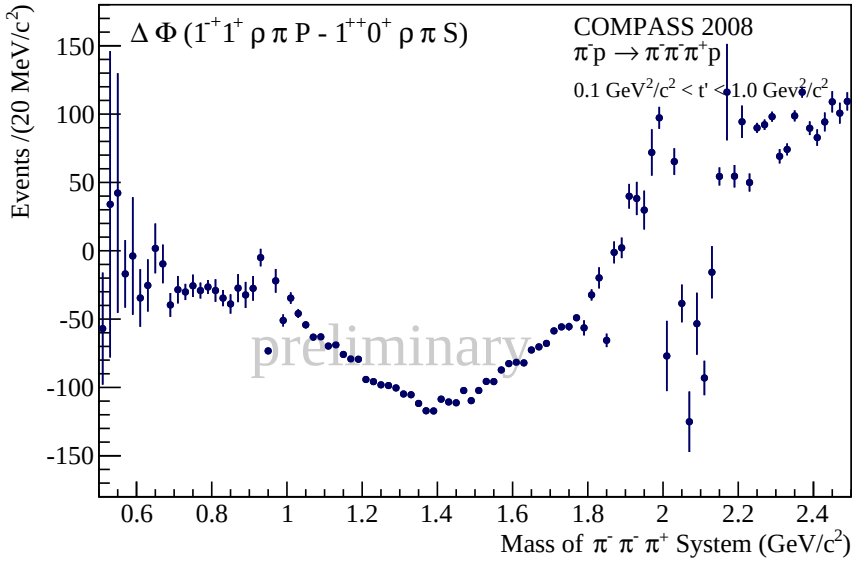
$\pi^- p \to \pi^- \pi^+ \pi^- p$ 2008)

The spin exotic $J^{PC} = 1^{-+}\rho\pi$ P-wave

Intensity (statistical errors only)



Phase motion vs $1^{++}\rho\pi$ S-wave



Comparison $\pi^- p \to \pi^- \pi^- \pi^+ p$ vs $\pi^- p \to \pi^- \pi^0 \pi^0 p$ (2008)

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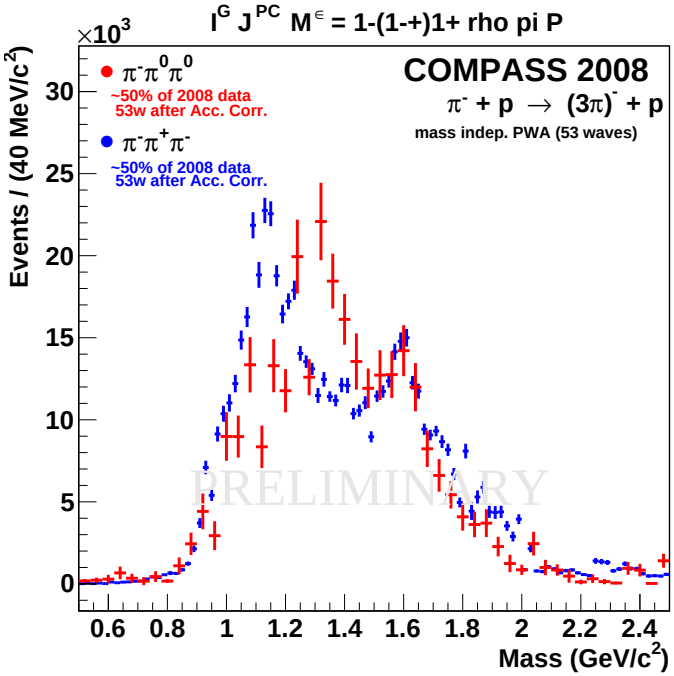
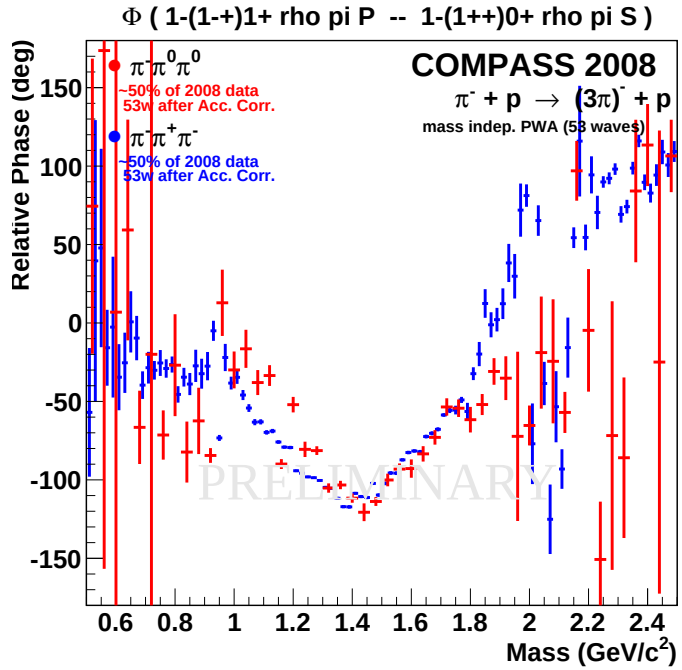
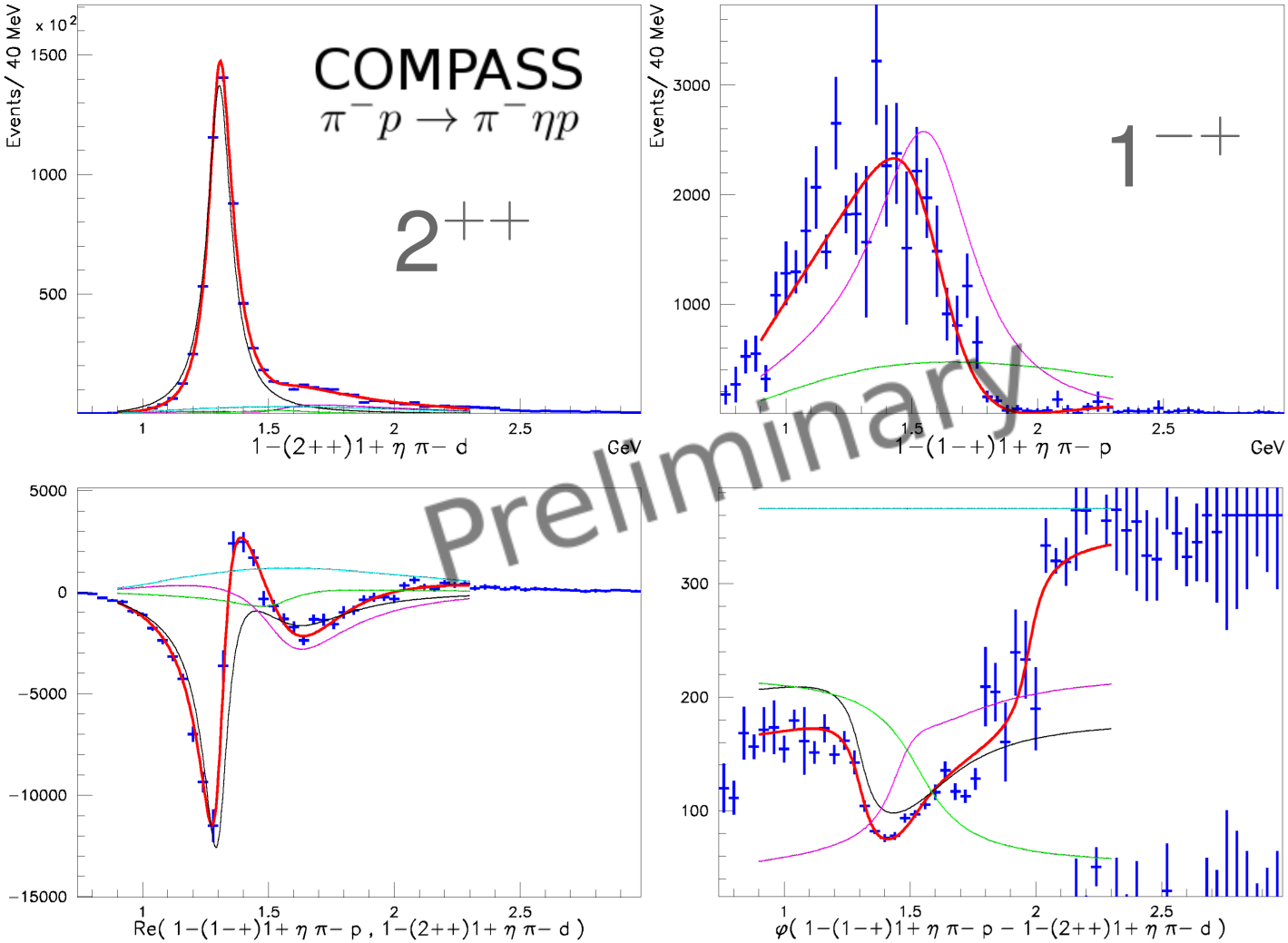




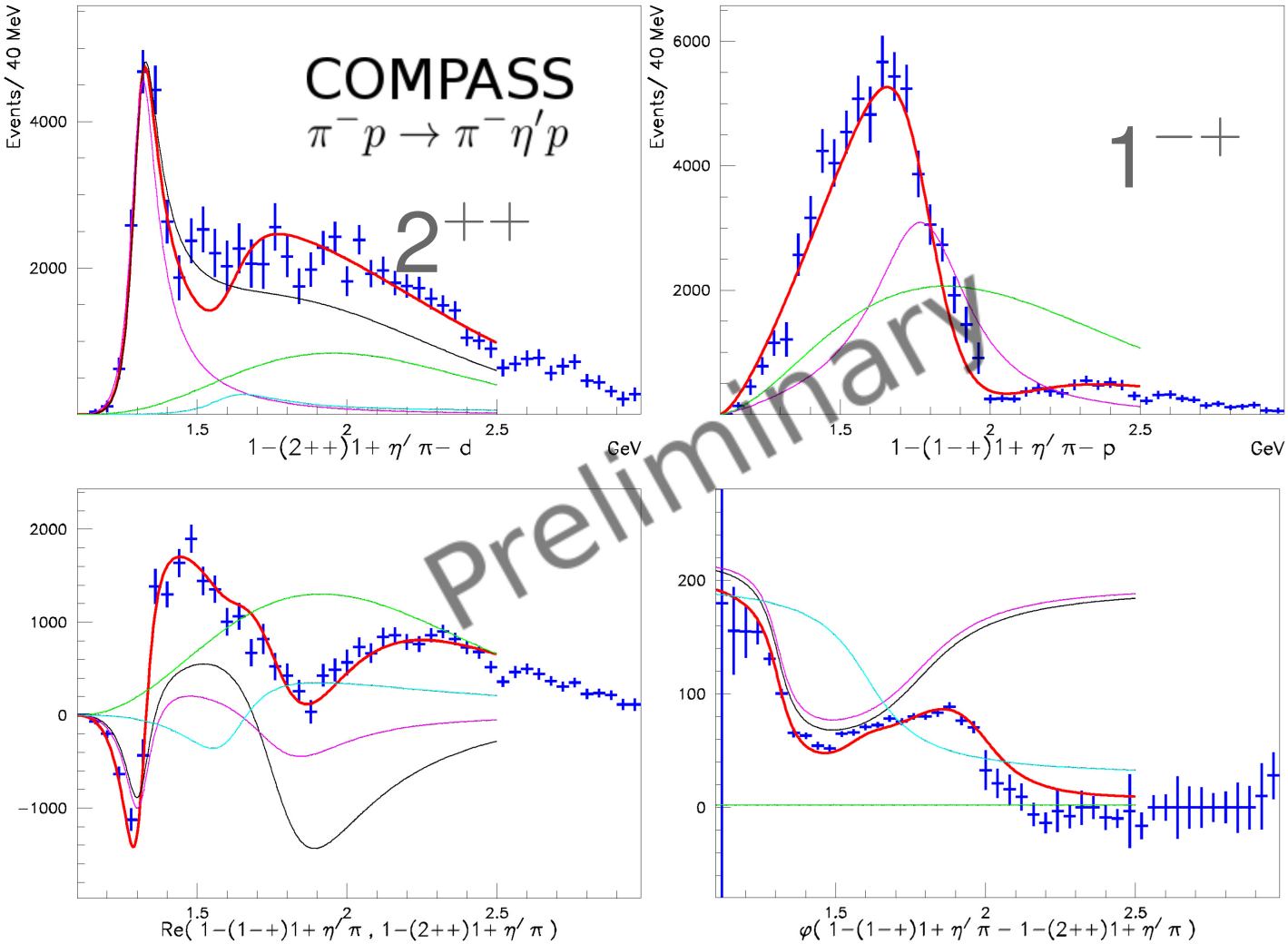
$\pi^- + p \to \eta\pi + p$ (2008)

D- vs *P*-wave



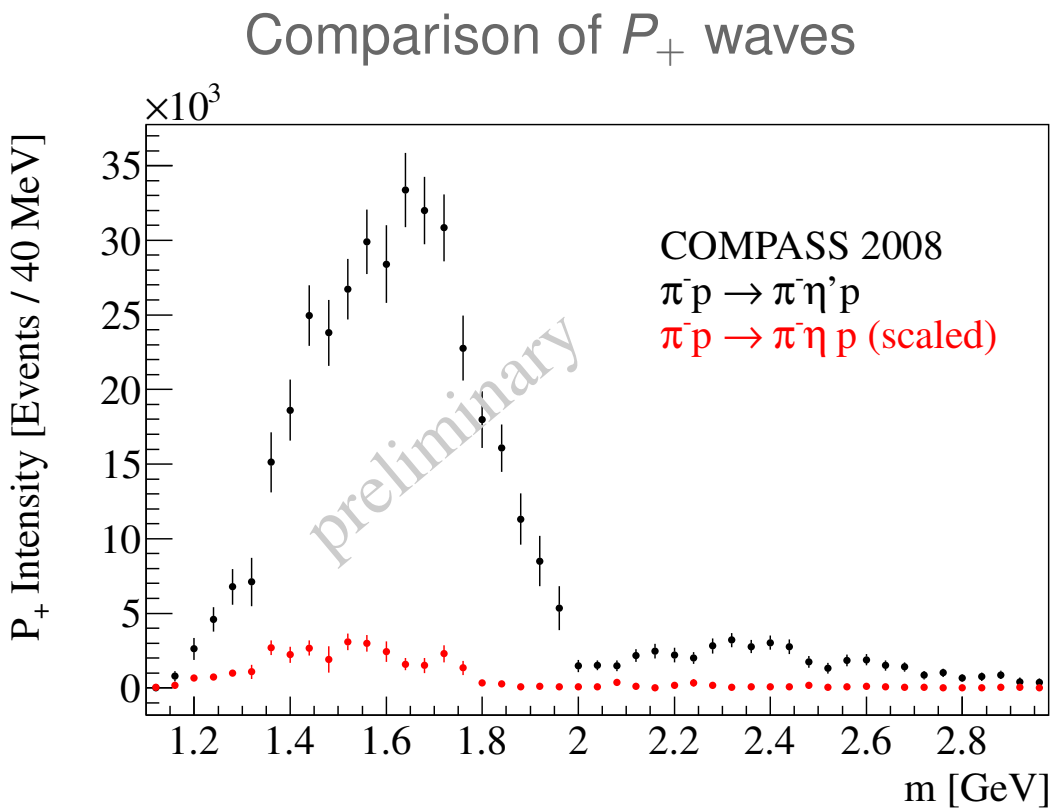
$\pi^- + p \to \eta'\pi + p$ (2008)

D- vs *P*-wave



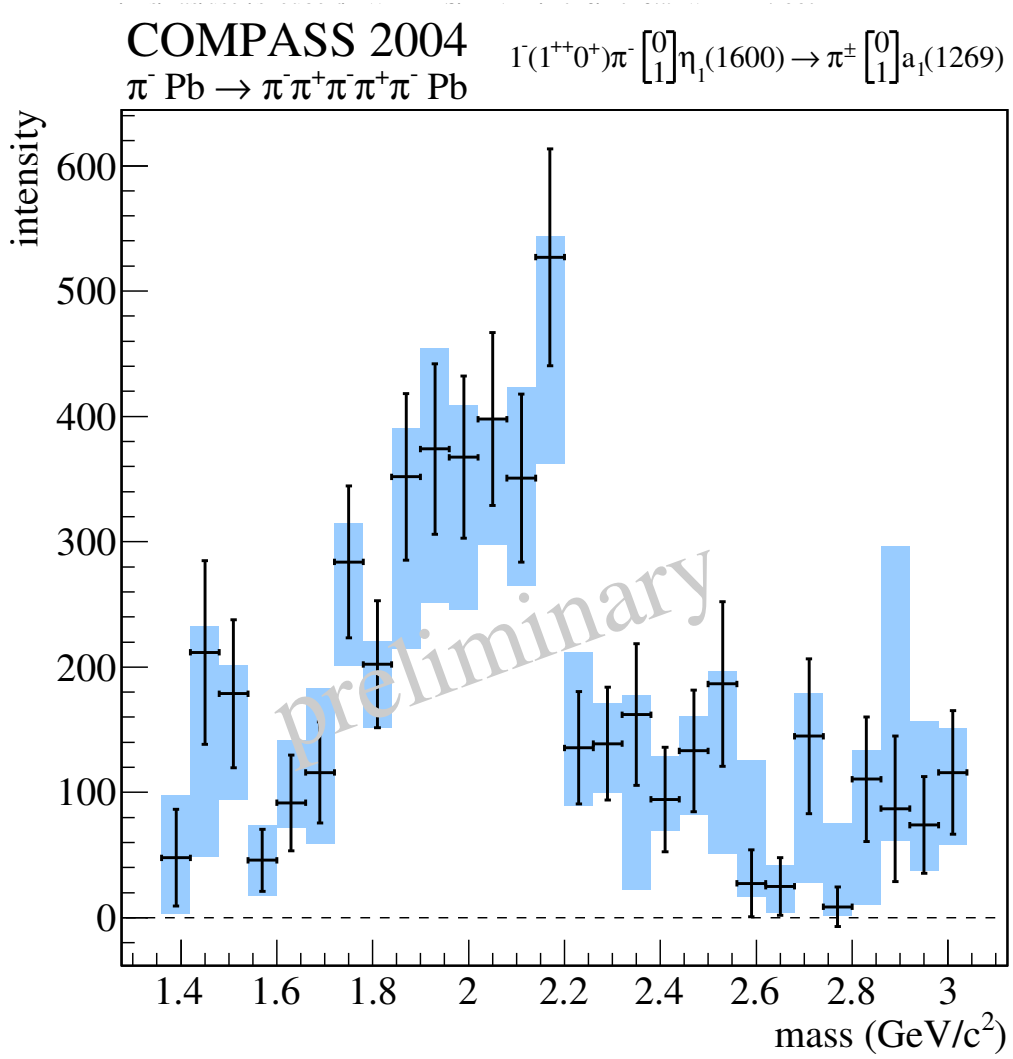
Comparison $\pi^- + p \to \eta'\pi + p$ vs $\pi^- + p \to \eta\pi + p$ (2008)

Scaling: Adjustment for branching and phase space



$\pi^- Pb \to \pi^-\pi^+\pi^-\pi^+\pi^- Pb$ (2004)

An exotic 4π isobar

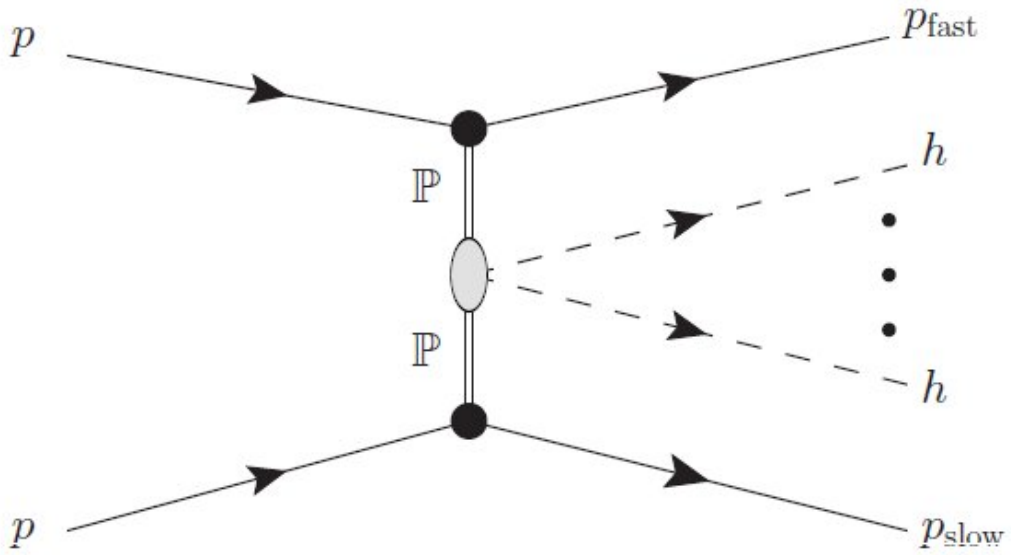


Systematic Improvements of the Model

WORK IN PROGRESS

- Exploit full t'-dependence
  $\rightarrow$ 2D partial-wave decomposition in small m and t' bins
- Model non-resonant contributions (Deck effect)
- Semi-model-independent isobar parameterizations
  $\rightarrow$ extract $(\pi\pi)_{S-\mathrm{wave}}$ from 3π data
- Improve fitting procedures
  - thresholds
  - objective model selection (genetic algorithm)

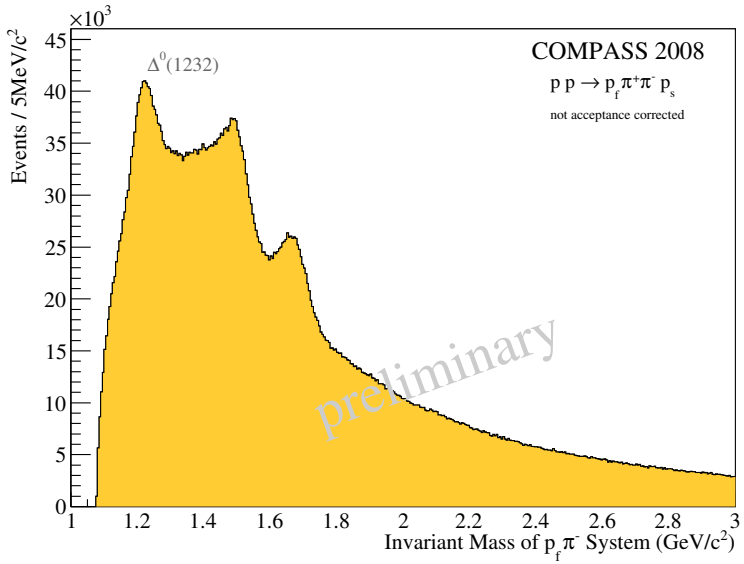
Isoscalar – Scalar Mesons
Meson Production at Central Rapidities in *pp* Scattering

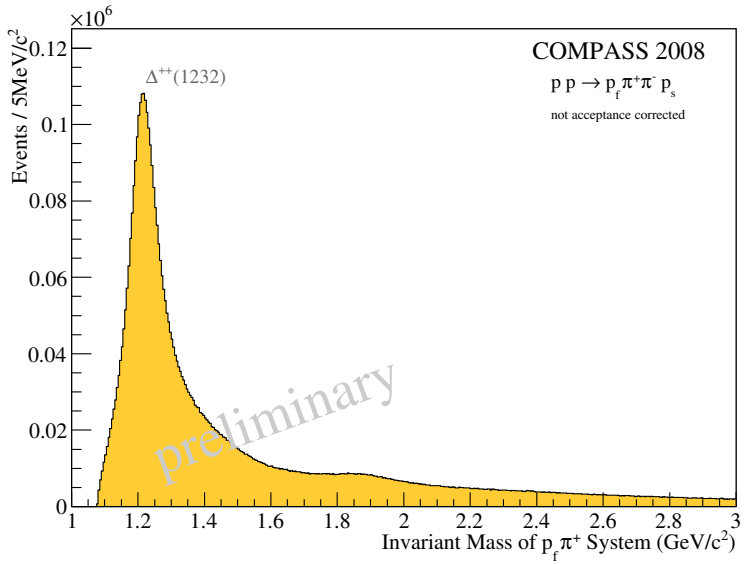


$pp \to p_{\text{fast}}\pi^+\pi^- + p_{\text{slow}}$ (2008)

Proton Diffraction

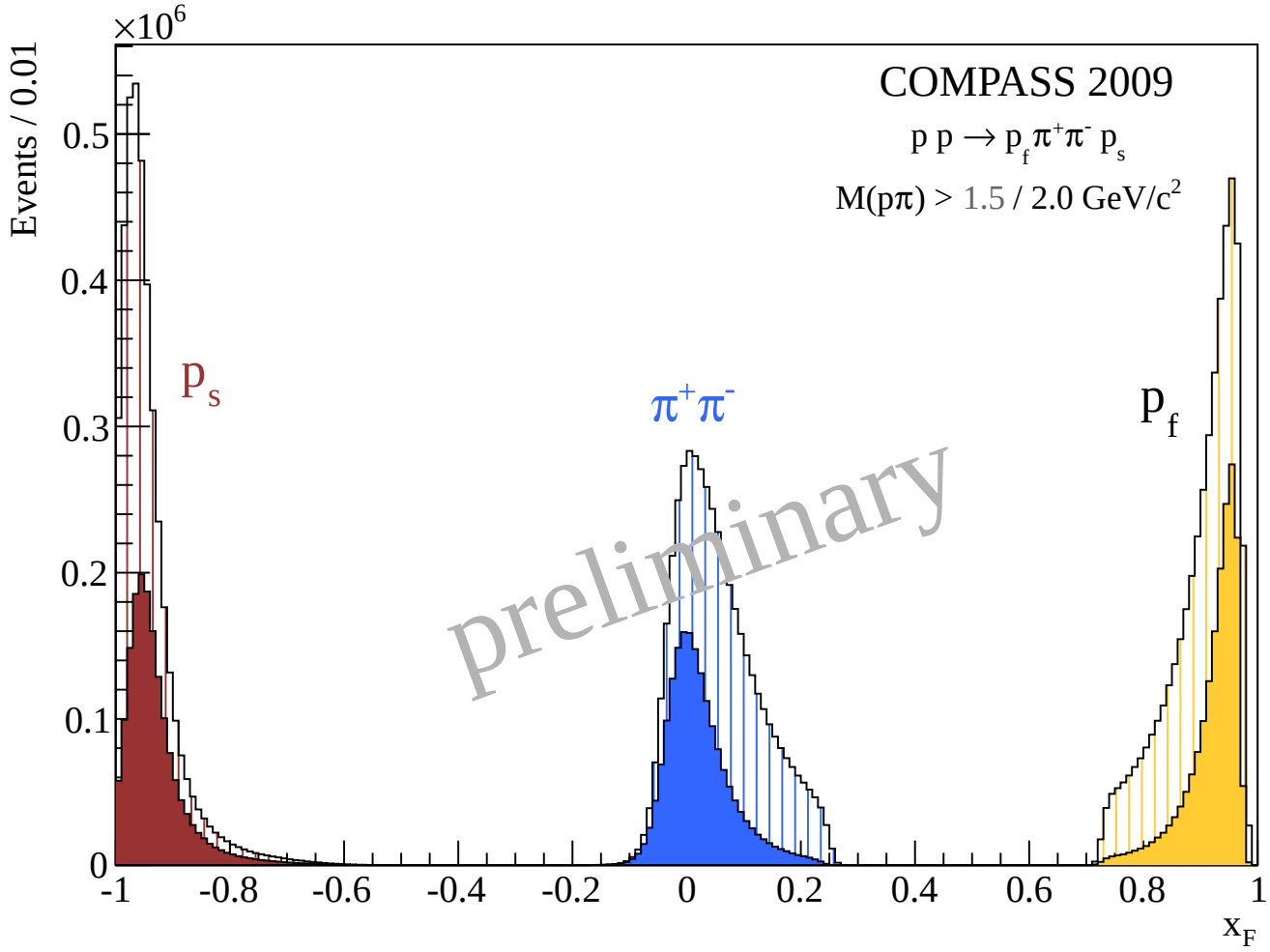
- Diffractive production of baryon resonances
- Kinematic overlap between production mechanisms
  $\to$ mass dependence





$pp \to p_{\text{fast}}\pi^+\pi^- + p_{\text{slow}}$ (2008)

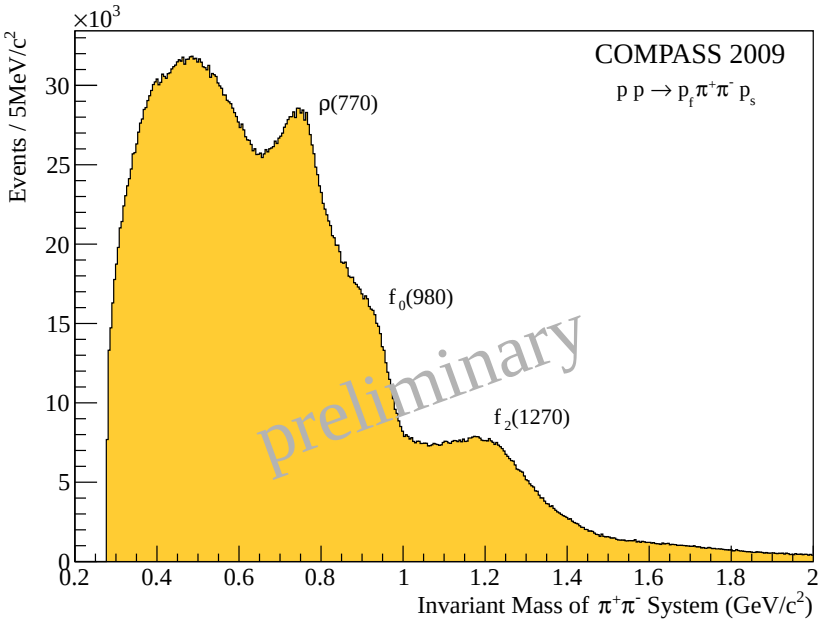
Central Production – $\pi^+\pi^-$ System

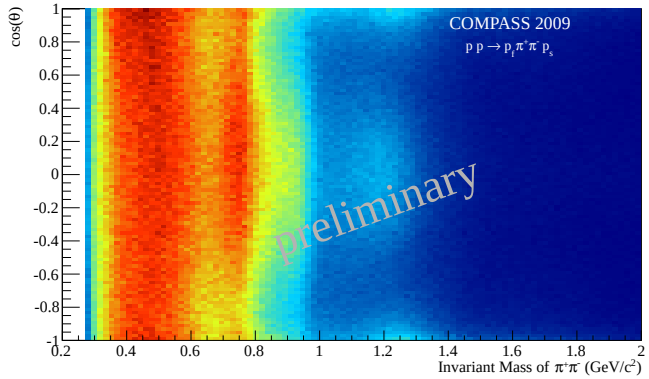


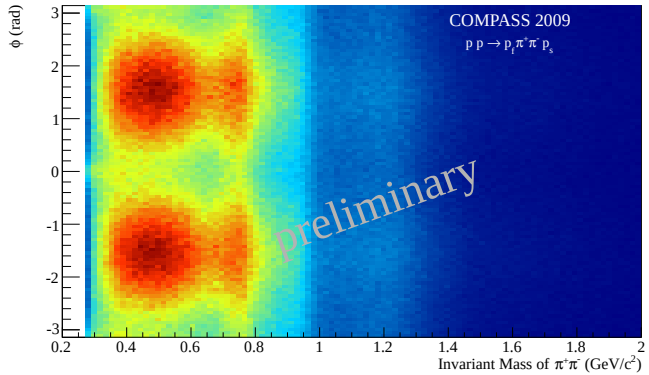
$$x_{\text{Feynman}} = 2p_l/\sqrt{s}$$

$pp \to p_{\text{fast}}\pi^+\pi^- + p_{\text{slow}}$ (2008)

Central Production – $\pi^+\pi^-$ System

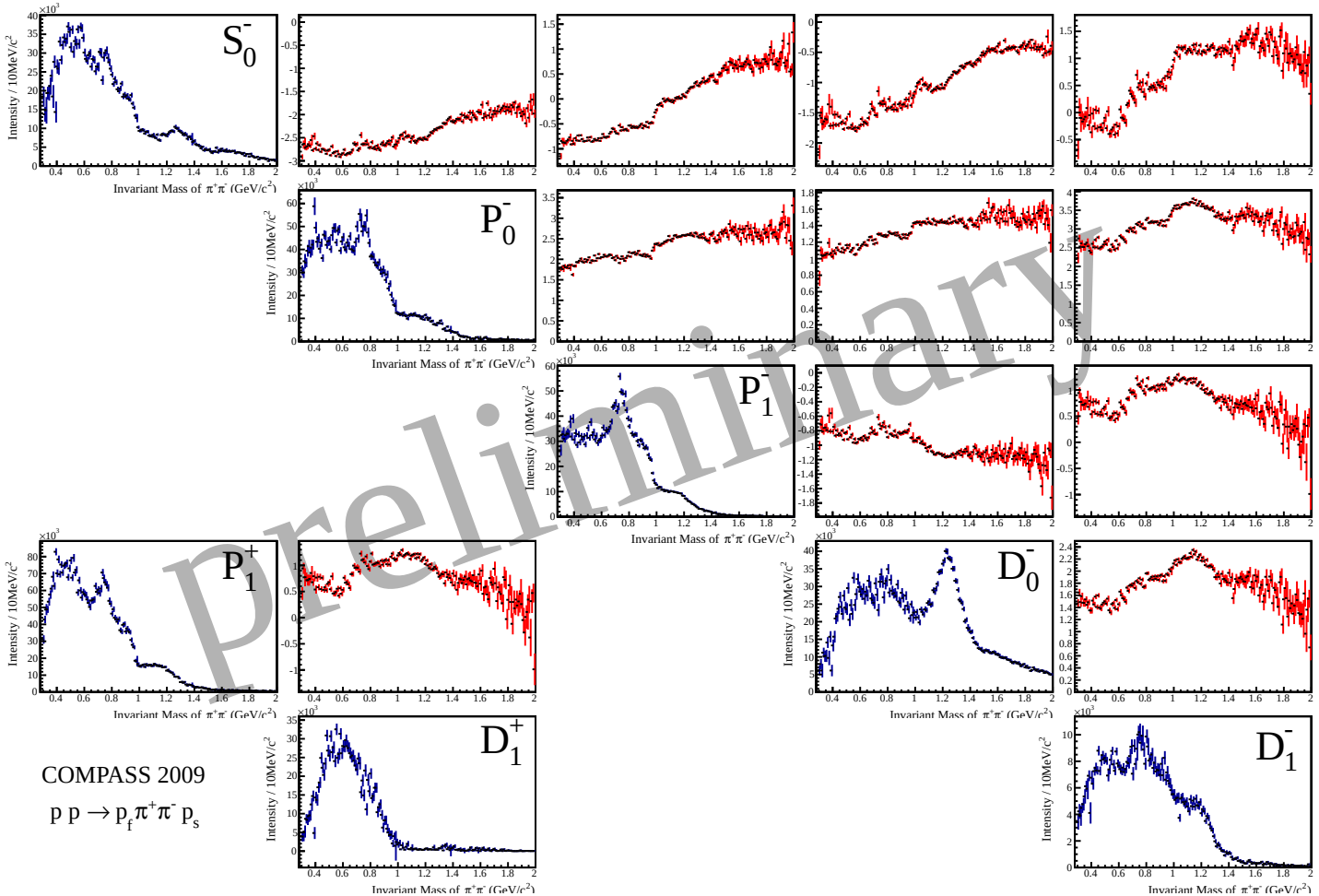




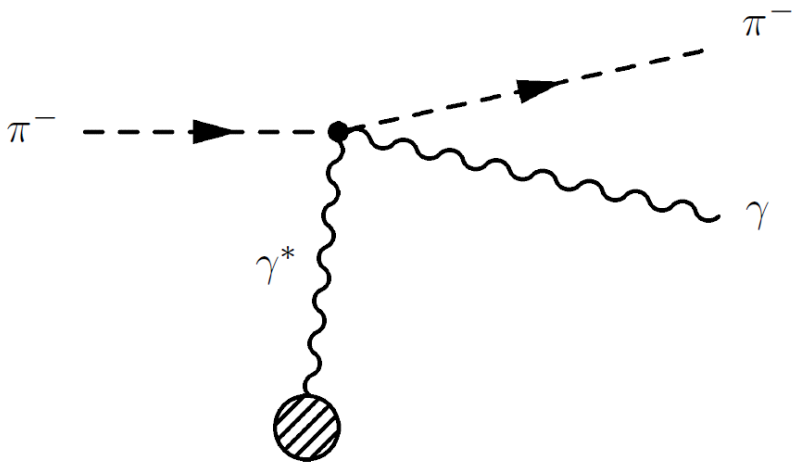


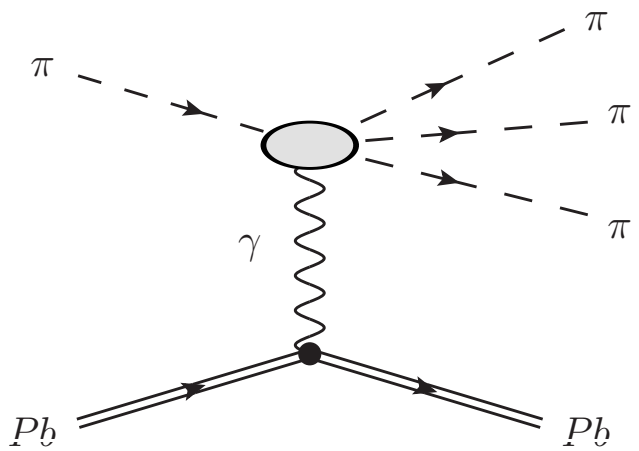
$pp \to p_{\text{fast}}\pi^+\pi^- + p_{\text{slow}}$ (2008)

Amplitude Analysis of $\pi^+\pi^-$ System – Physical Solution after Disambiguation



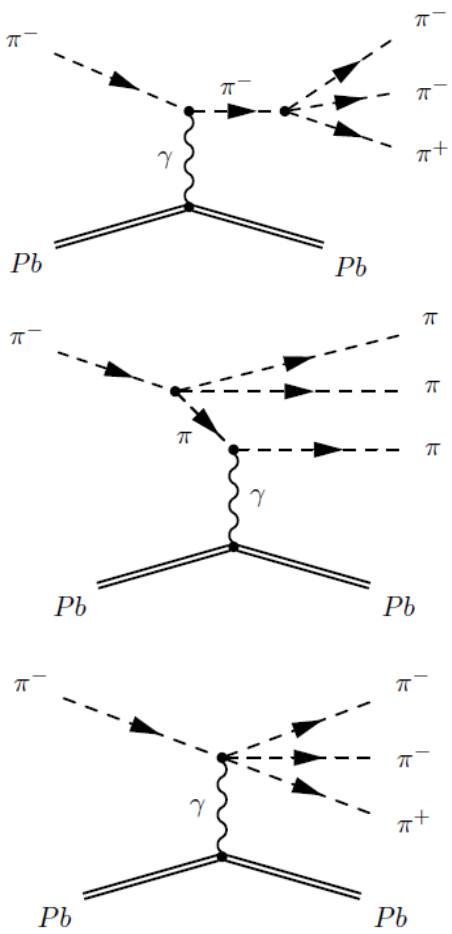
Tests of Chiral Dynamics



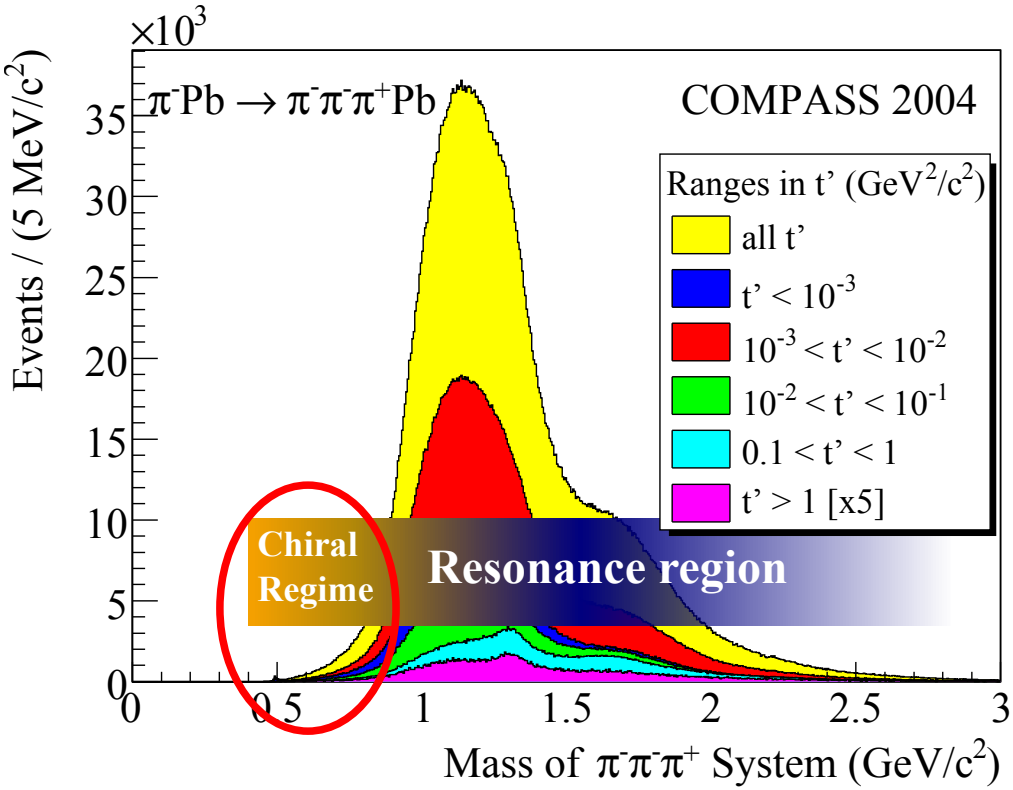


Primakoff 3π Spectral Function from χPT

PRL 108, 192001 (2012)



- Heavy nucleus acts as a quasi-real photon source
- Chiral regime (low masses, $t' < 0.001(\mathrm{GeV}/c)^2$)
  $\rightarrow$ fraction of final state events photon produced



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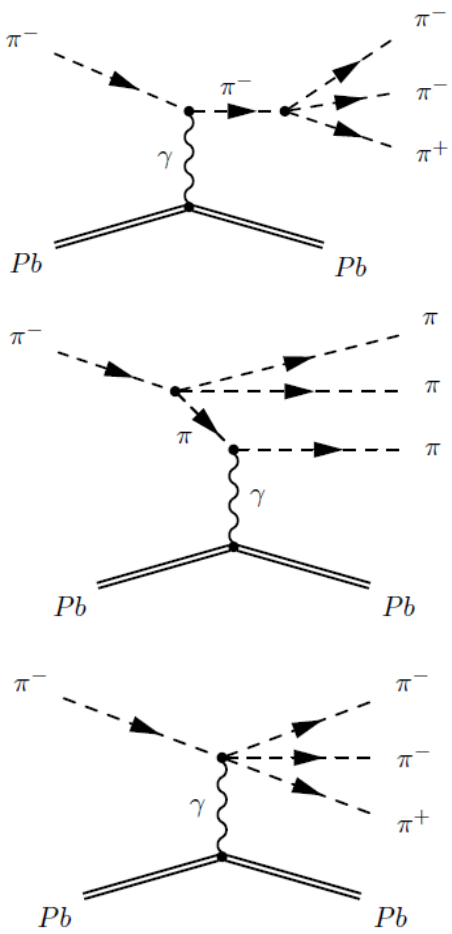

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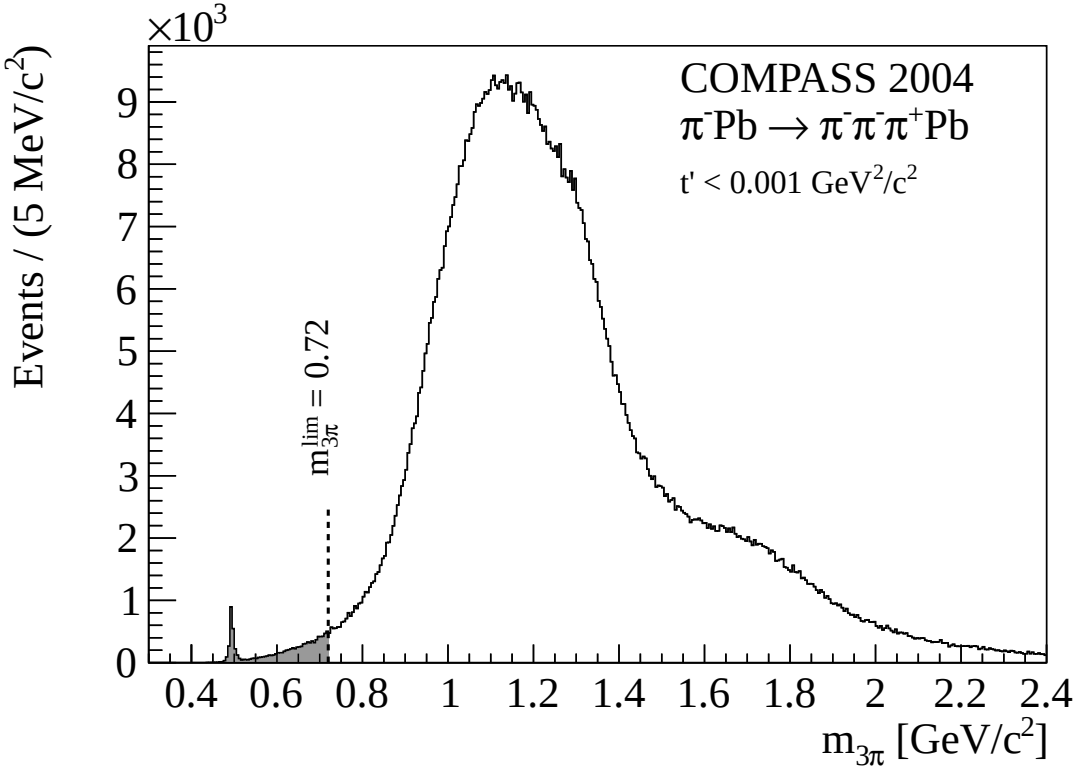

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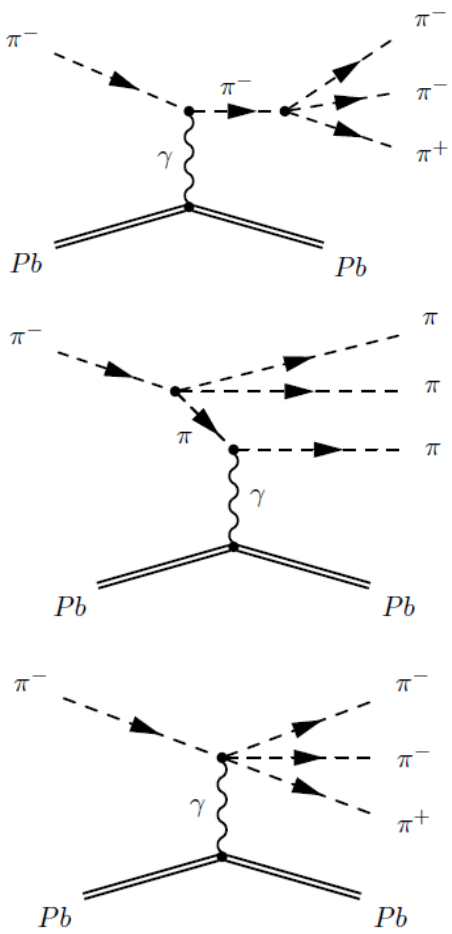
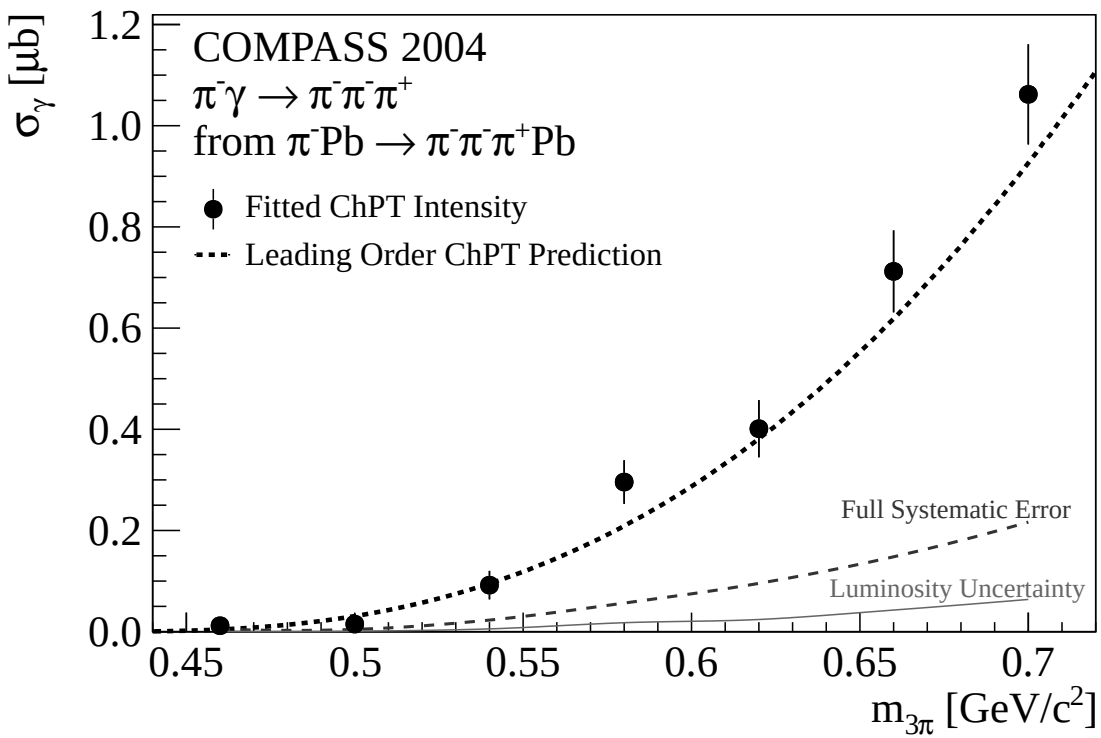
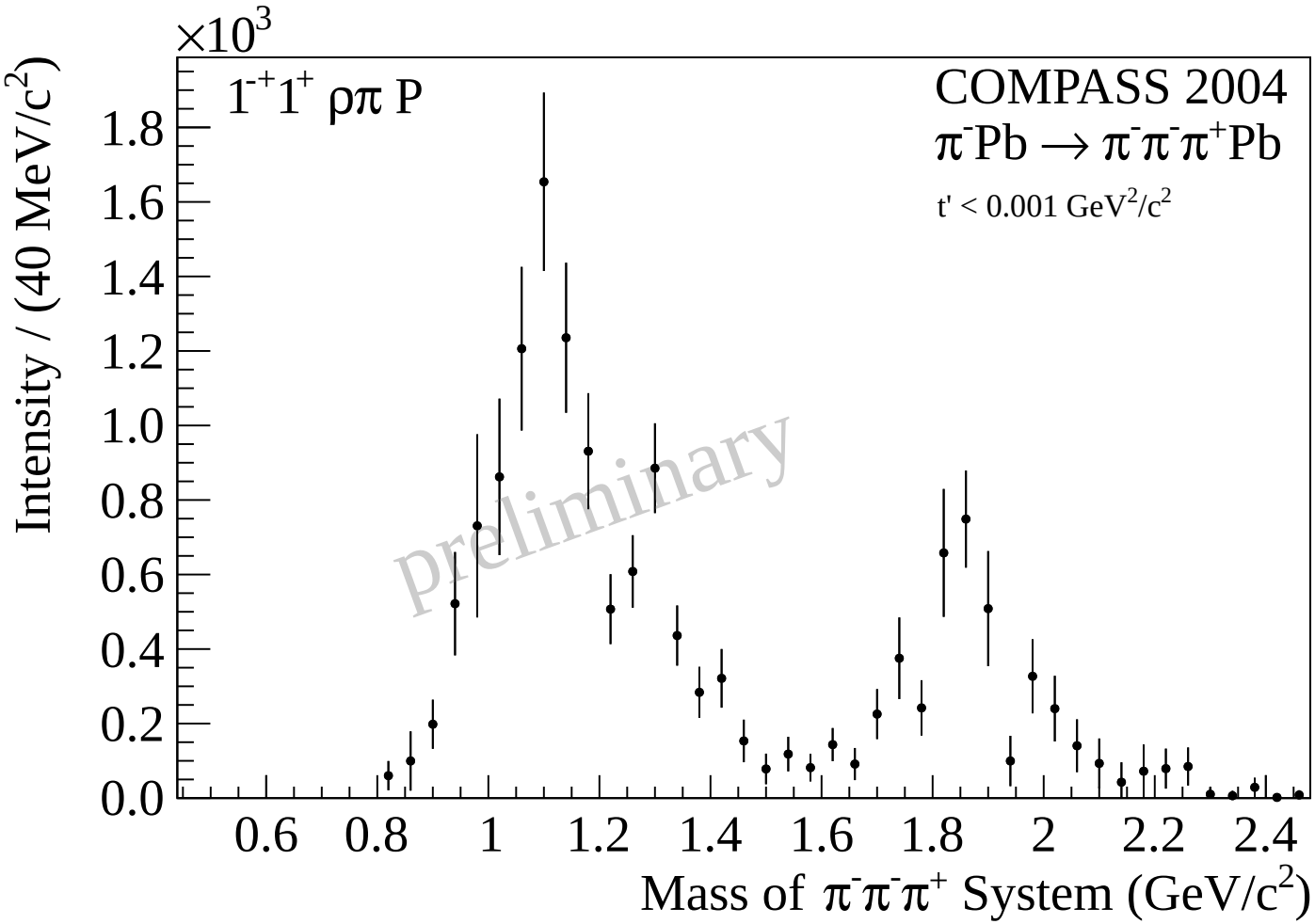


- Heavy nucleus acts as a quasi-real photon source
- Chiral regime (low masses, $t' < 0.001(\mathrm{GeV}/c)^2$) $\rightarrow$ fraction of final state events photon produced
- Analysis ansatz: χPT amplitude included in PWA
-



$\pi^- Pb \to \pi^-\pi^+\pi^- Pb$ (2004)

$1^{-+}\rho\pi$ in Primakoff Production



Conclusion and Outlook

Conclusion

- COMPASS 2008/2009: large data sets in
  - diffractive $\pi^-/K^-/p$ dissociation (up to 2 orders of magnitude improvement)
  - Primakoff
- Chiral dynamics: 3π–amplitude
- $\pi^-\pi^+\pi^-$, $\pi^-\pi^0\pi^0$, $\eta\pi^-$, $\eta'\pi^-$, $K^-\pi^+\pi^-$, 5π, $\pi^-\pi^+_{\text{central/isobar}}$
  - Where is the $I = 0$ partner $\eta_1(1600)$ of the $\pi_1(1600)$?
    Hybrid Supermultiplet?

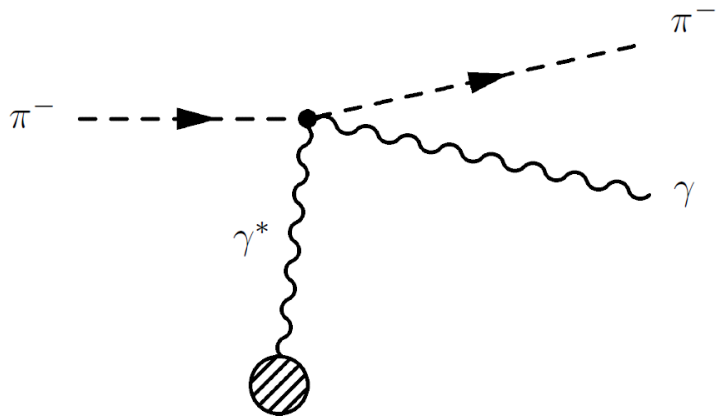
Conclusion and Outlook

Outlook

- Dedicated Primakoff run 2012

Pion Polarizability
in Primakoff–Compton Scattering

- Electric polarizability
  $\alpha_\pi =^{\chi_{\mathrm{pt}}} (2.93 \pm 0.5) \times 10^{-4}\,\mathrm{fm}^3$
- Magnetic polarizability
  $\beta_\pi =^{\chi_{\mathrm{pt}}} (-2.77 \pm 0.5) \times 10^{-4}\,\mathrm{fm}^3$



Pion Polarizability
in Primakoff–Compton Scattering

- $\gamma^{(*)}\,\gamma^{(*)} \longrightarrow \pi^+\,\pi^-$ [via $e^+e^- \to e^+e^-\,\pi^+\pi^-$]

  PLUTO, MARKII,...: $\alpha_\pi - \beta_\pi = 4 - 20$

  rather insensitive to polarisability, dominated by loops

- Radiative π^+ production on the proton:

  $\gamma\,\pi^* \longrightarrow \pi\,\gamma$ [via $\gamma\,p \to n\,\pi^+\,\gamma$]

  Mainz (2005) measurement: $\alpha_\pi - \beta_\pi = 11.6 \pm 1.5 \pm 3.0 \pm 0.5$

  "± 0.5": model error *only within the used ansatz*,

  full systematics not under control

- Primakoff Compton reaction:

  $\gamma^{(*)}\,\pi \longrightarrow \pi\,\gamma$ [via $\pi\,Z \to Z\,\pi\,\gamma$]

  tiny extrapolation $\gamma^* \to \gamma$ $\mathcal{O}(10^{-3}\,m_\pi^2)$

  fully under theoretical control

Pion Polarizability

in Primakoff–Compton Scattering

Days	π beam, days	μ beam, days	$\alpha_\pi - \beta_\pi$ σ_{tot}	$\alpha_\pi + \beta_\pi$ σ_{tot}	$\alpha_2 - \beta_2$ σ_{tot}
120	90	30	± 0.27 ± 0.26 ± 0.66	fixed ± 0.016 ± 0.025	fixed fixed ± 1.94
			ChPT prediction		
			5.70	0.16	4

Conclusion and Outlook

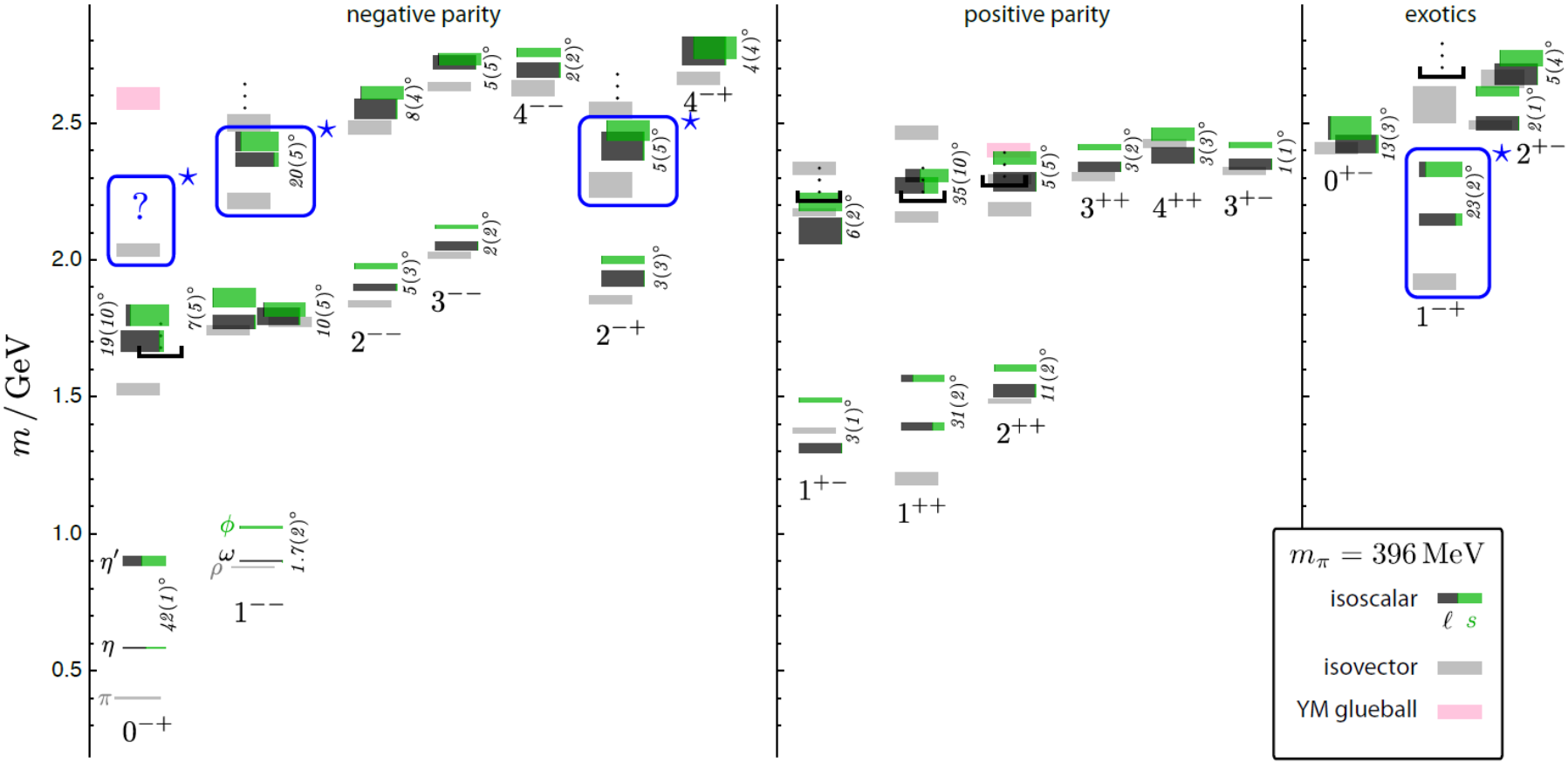
Outlook

- Dedicated Primakoff run 2012
- Beyond standard analysis:
  - Non-Resonating Production (Deck)
  - Study 2π and 4π systems, Isobar-fits
  - Rescattering
- Further measurements: OZI-violation, Multi-particle final states...
- Consolidate Data $\rightarrow$ Global Meson Analysis Working Group

Backup

Light Meson Spectrum on the Lattice

Dudek et al. [arXiv:1106.5515v1] (@ $m_\pi = 400$MeV)



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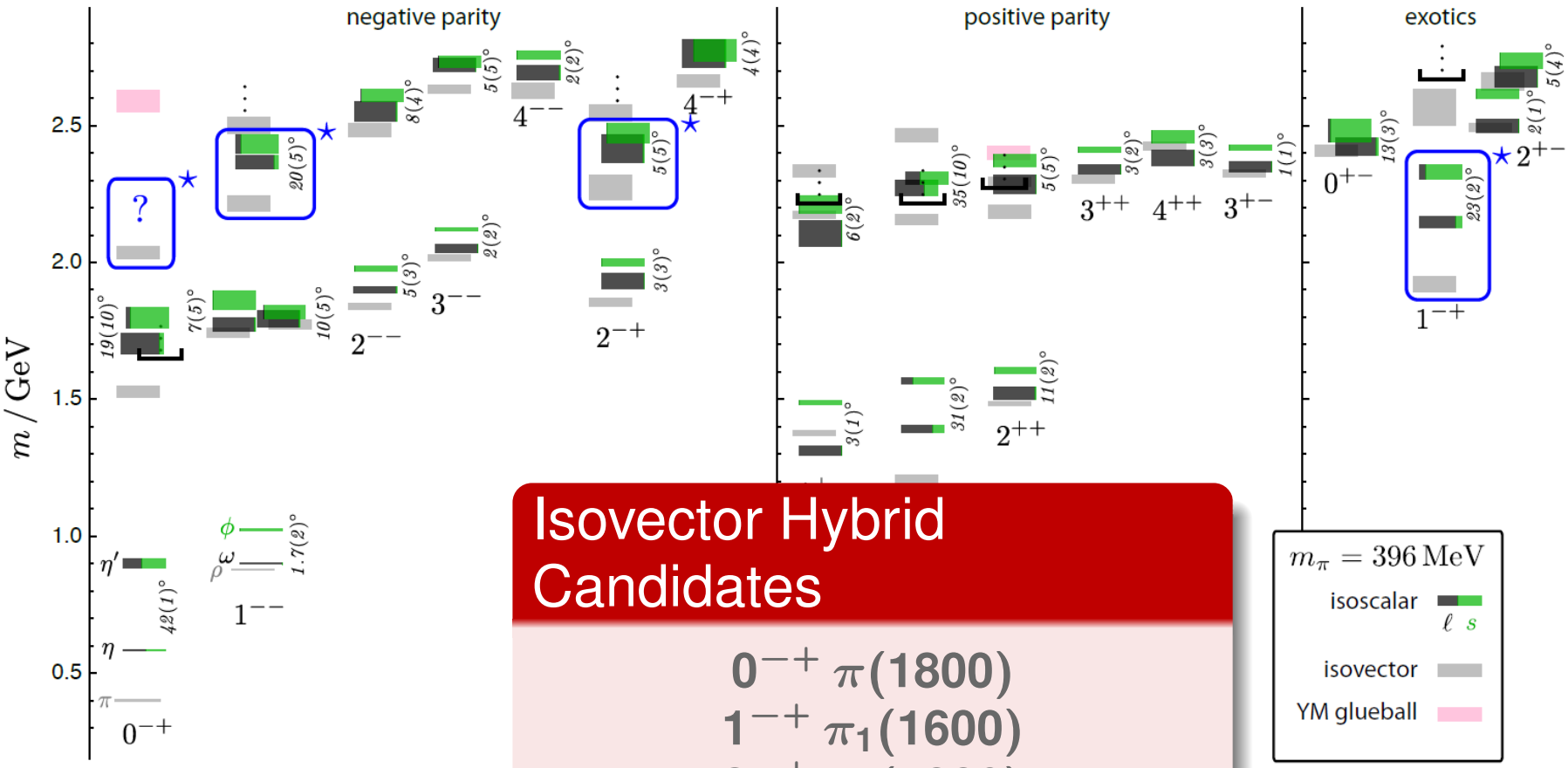


Isovector Hybrid Candidates

0^{-+} $\pi(1800)$

1^{-+} $\pi_1(1600)$

2^{-+} $\pi_2(1880)$

5π Resonance Parameters

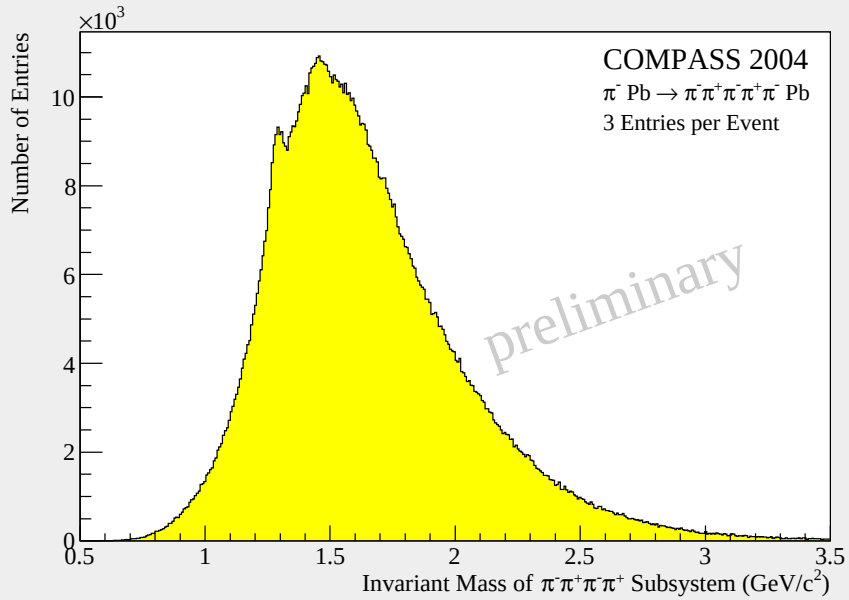
Comparison to PDG

Parameter				Fit	PDG
Resonance		J^{PC}		(MeV/c^2)	
	$\pi(1300)$	0^{-+}	M	1400^{*}	1300 ± 100
			Γ	$500^{\dagger}$	200…600
	$\pi(1800)$	0^{-+}	M	$1781 \pm 5^{+1(+8)}_{-6(-6)}$	1816 ± 14
			Γ	$168 \pm 9^{+5(+62)}_{-14(-15)}$	208 ± 12
○	$a_1(1900)$	1^{++}	M	$1853 \pm 7^{+36(+36)}_{-6(-49)}$	1930^{+30}_{-70}
			Γ	$443 \pm 14^{+12(+98)}_{-45(-65)}$	155 ± 45
○	$a_1(2200)$	1^{++}	M	$2202 \pm 8^{+15(+53)}_{-8(-11)}$	$2096 \pm 17 \pm 121$
			Γ	$402 \pm 17^{+41(+125)}_{-52(-51)}$	$451 \pm 41 \pm 81$
	$\pi_2(1670)$	2^{-+}	M	$1719.0^{\dagger}$	1672.4 ± 3.2
			Γ	$251.4^{\dagger}$	259 ± 9
	$\pi_2(1880)$	2^{-+}	M	$1854 \pm 6^{+6(+6)}_{-4(-9)}$	1895 ± 16
			Γ	$259 \pm 13^{+7(+7)}_{-17(-31)}$	235 ± 34
○	$\pi_2(2100)$	2^{-+}	M	$2133 \pm 12^{+7(+43)}_{-18(-18)}$	2090 ± 29

Isobars that have been used

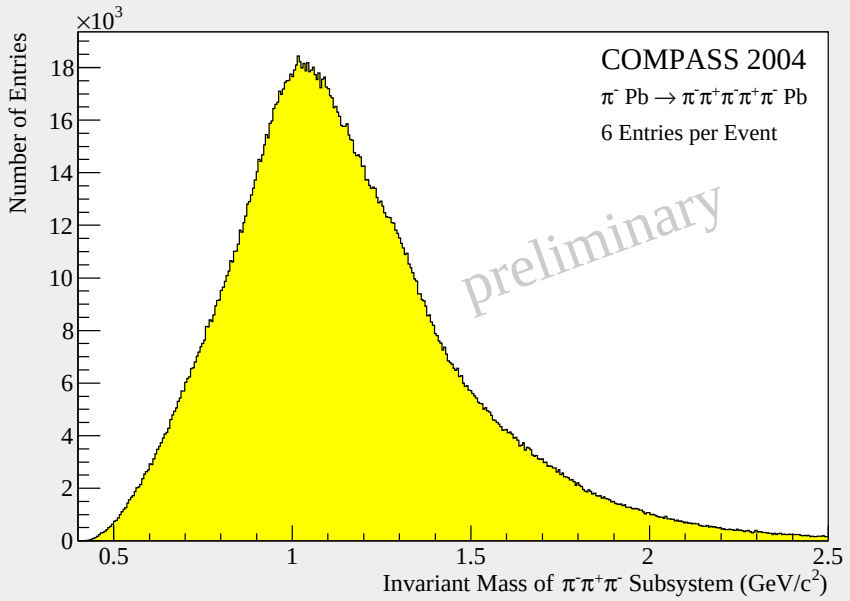
4π Isobars ($G = +$)			3π Isobars ($G = -$)
Name	Mass / GeV	$I^G J^{PC}$	
f_0	1370 / 1500 / 1700	$0^+(0^{++})$	
η	1405	$0^+(0^{-+})$	
ρ'	1450 / 1700	$1^+(1^{--})$	
b_1	1235 / 1800	$1^+(1^{+-})$	
f_1	1285 / 1420	$0^+(1^{++})$	
f_2	1270 / 1565	$0^+(2^{++})$	
η_2'	1645	$0^+(2^{-+})$	
ρ_3	1690	$1^+(3^{--})$	
η_1	1600	$0^+(1^{-+})$	
b_0	1800	$1^+(0^{+-})$	
b_2	1800	$2^+(2^{+-})$	

4π subsystem



Isobars that have been used

3π subsystem



4π Isobars ($G = +$)			3π Isobars ($G = -$)		
			Name	Mass / GeV	$I^G J^{PC}$
			a_1	1270	$1^-(1^{++})$
			a_2	1320	$1^-(2^{++})$
			π'	1300	$1^-(0^{-+})$
			π_2	1670	$1^-(2^{-+})$
b_2	1800	$2^+(2^{++})$	π_1	1600	$1^-(1^{-+})$

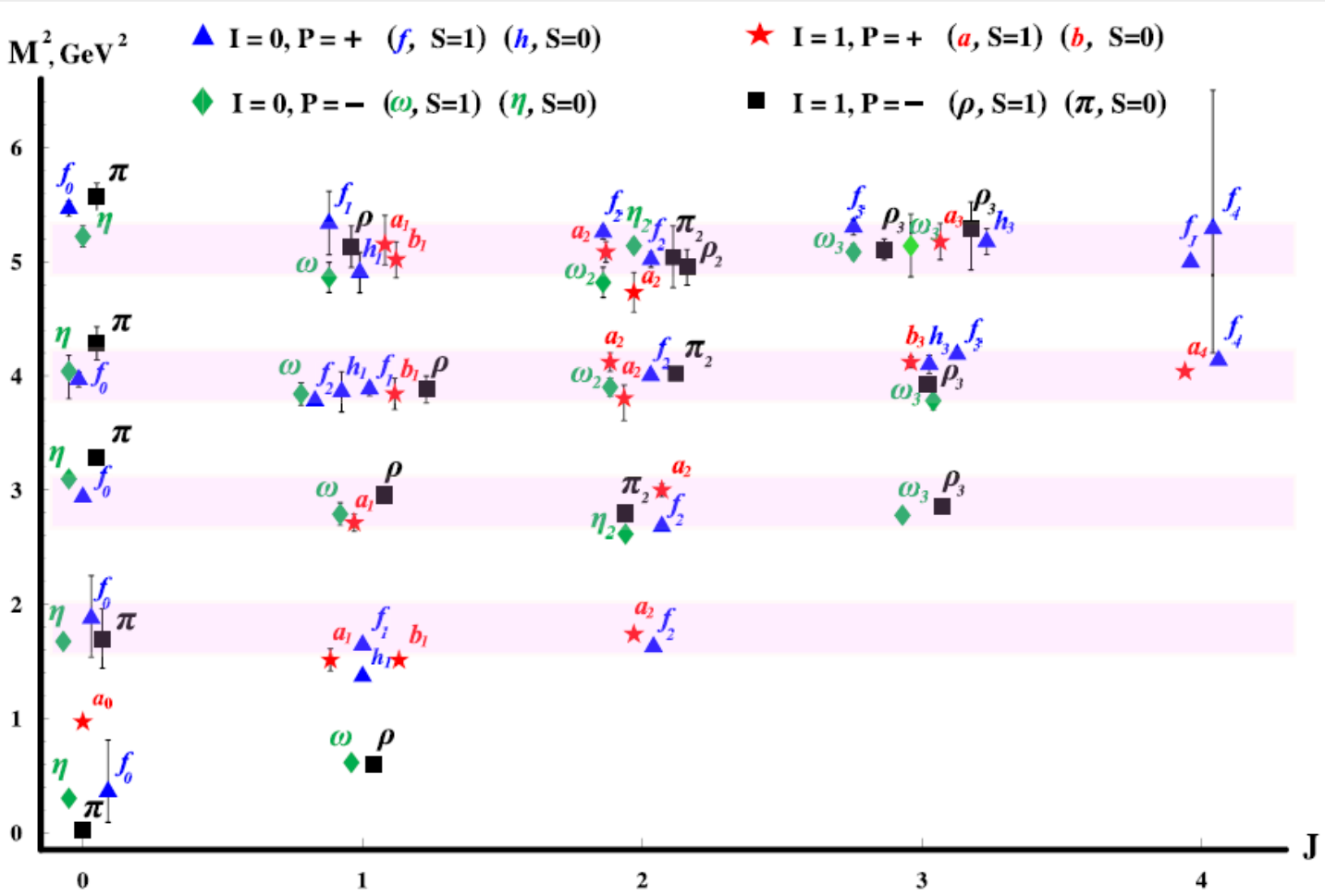
Isobars that have been used

4π Isobars ($G = +$)			3π Isobars ($G = -$)		
Name	Mass / GeV	$I^G J^{PC}$	Name	Mass / GeV	$I^G J^{PC}$
f_0	1370 / 1500 / 1700	$0^+(0^{++})$			
η	1405	$0^+(0^{-+})$	a_1	1270	$1^-(1^{++})$
ρ'	1450 / 1700	$1^+(1^{--})$	a_2	1320	$1^-(2^{++})$
b_1	1235 / 1800	$1^+(1^{+-})$	π'	1300	$1^-(0^{-+})$
f_1	1285 / 1420	$0^+(1^{++})$	π_2	1670	$1^-(2^{-+})$
f_2	1270 / 1565	$0^+(2^{++})$			
η_2'	1645	$0^+(2^{-+})$			
ρ_3	1690	$1^+(3^{--})$			
η_1	1600	$0^+(1^{-+})$			
b_0	1800	$1^+(0^{+-})$	π_1	1600	$1^-(1^{-+})$
b_2	1800	$2^+(2^{+-})$			

2π subsystem: σ, $\rho(770)$, $f_2(1270)$

Excited Mesons in Parity Doublets?

[PRD **77** (2008) 034002]



plot from M. Shifman and A. Vainshtein [PRD **77** (2008) 034002]

See also: R. F. Wagenbrunn and L. Ya. Glozman [PRD **75** (2007) 036007] and references therein

Excited Mesons in Parity Doublets?

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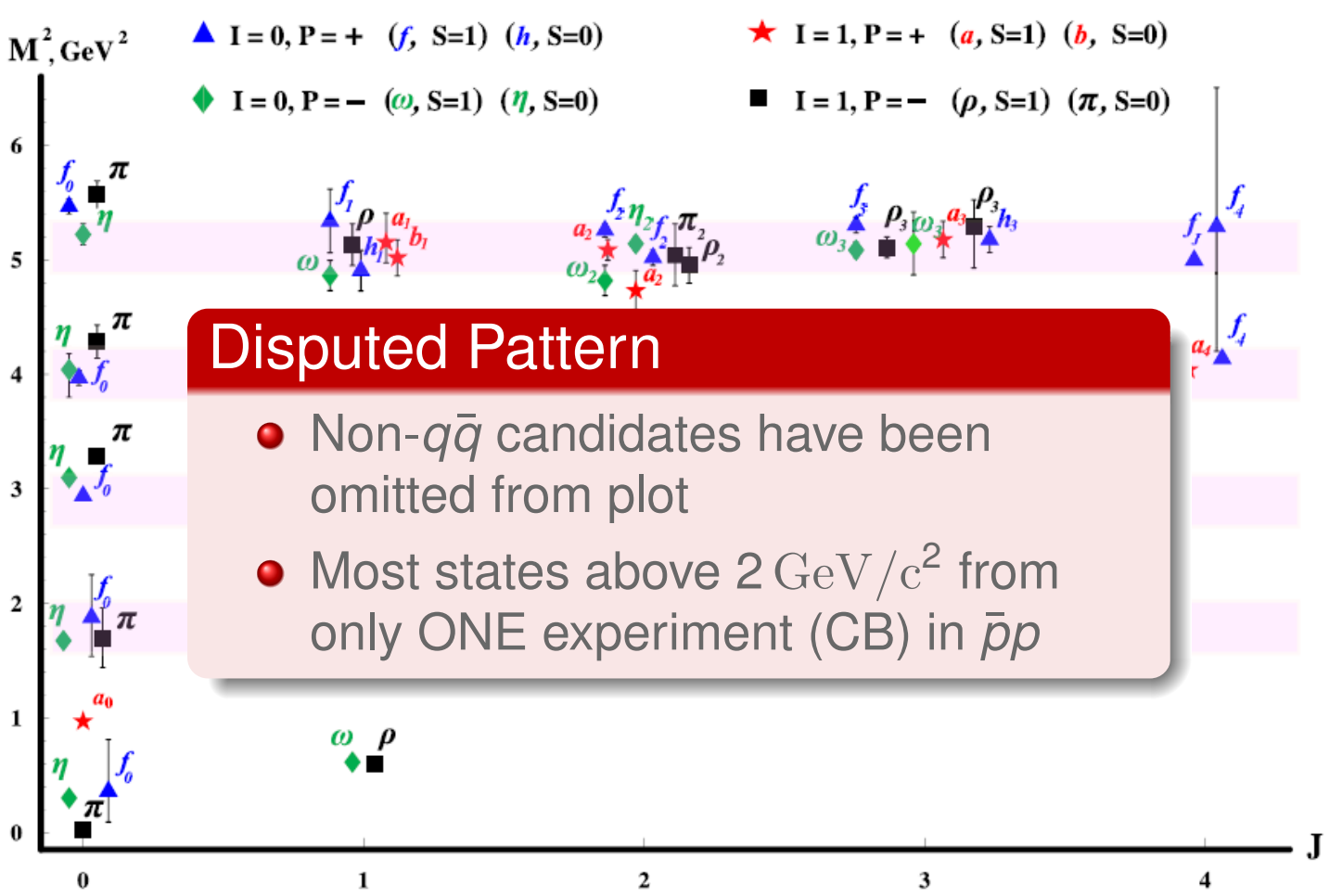

Disputed Pattern

- Non-$q\bar{q}$ candidates have been omitted from plot
- Most states above 2 $\mathrm{GeV}/\mathrm{c}^2$ from only ONE experiment (CB) in $\bar{p}p$

plot from M. Shifman and A. Vainshtein [PRD **77** (2008) 034002]

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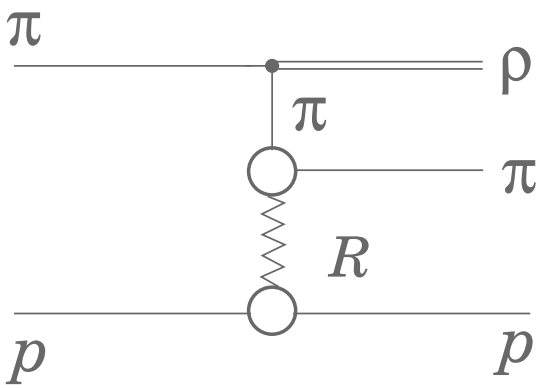
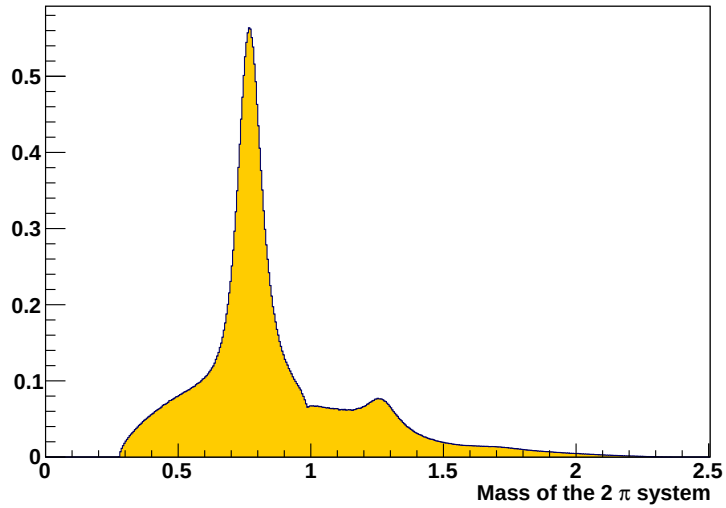
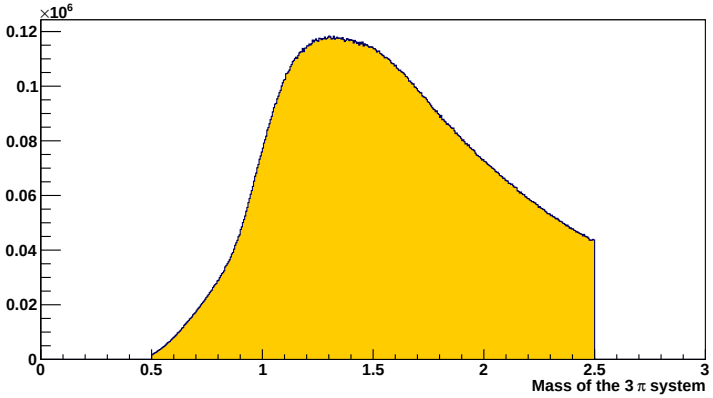
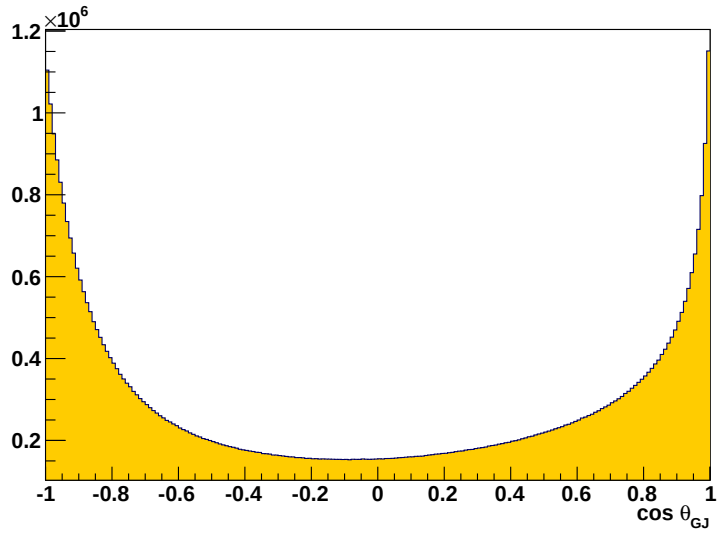
Nonresonant Scattering — Deck Effect

Monte Carlo Simulation



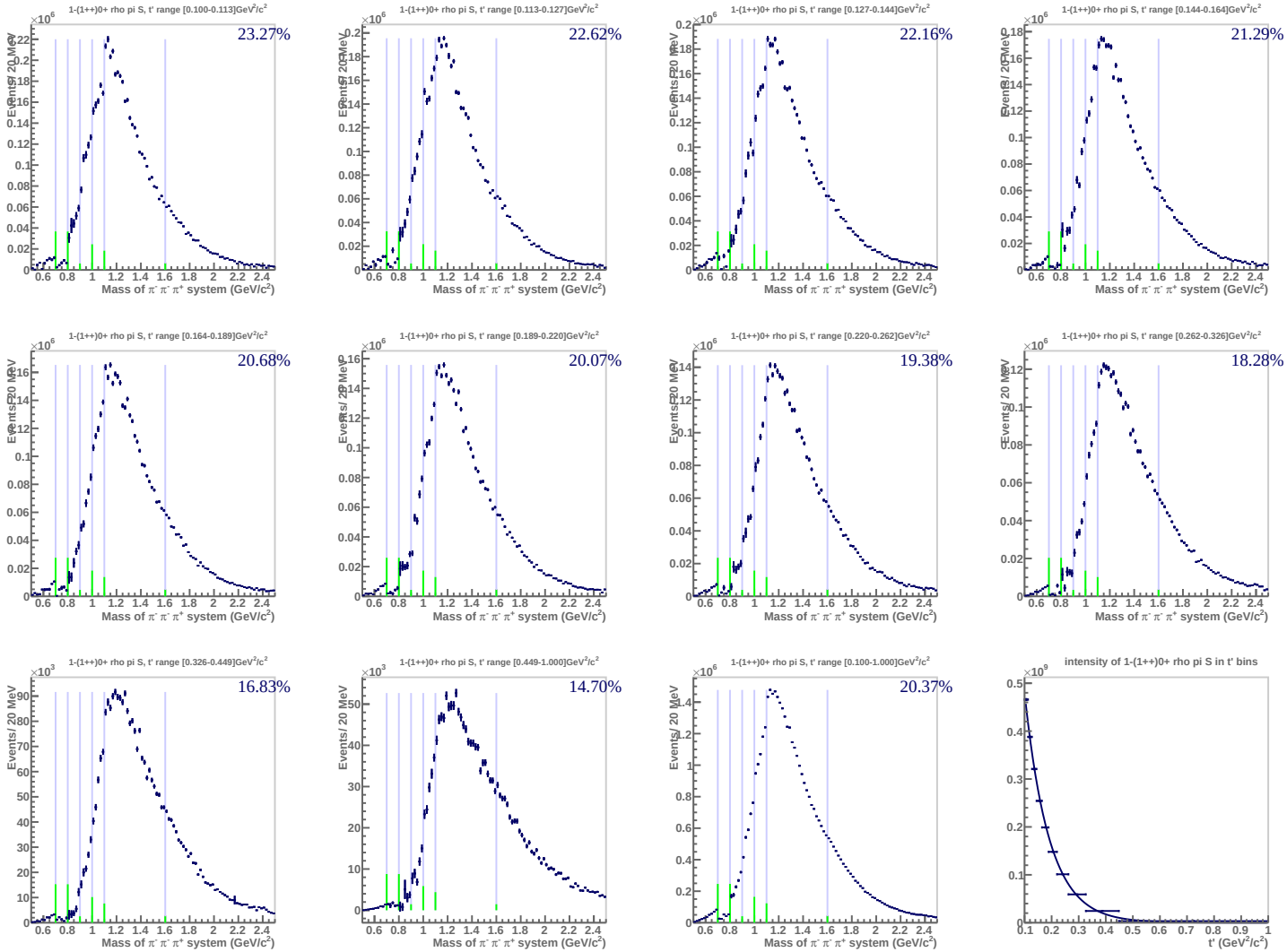






Deck Effect $J^{PC} = 1^{++}$ Component

Partial-Wave Decomposition in $m_{3\pi}$ and t' Bins



The Quark Model of (light) Mesons

Combining $q\bar{q}$ – there are some forbidden states!

Mesons:

- Color neutral objects,
- made from a fermion-antifermion ($q\bar{q}$) pair
- characterized by $I^G(J^{PC})$(mass)

Potential model:

$$V = H_{\text{conf}} + H_{\text{SS}} + H_{\text{LS}} + H_{\text{Annih}}$$

Godfrey, Isgur, Phys. Rev. D32(1985)189

$$G = (-1)^{I+\ell+s} \qquad P = (-1)^{\ell+1} \qquad C = (-1)^{\ell+s}$$

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Godfrey, Isgur, Phys. Rev. D32(1985)189

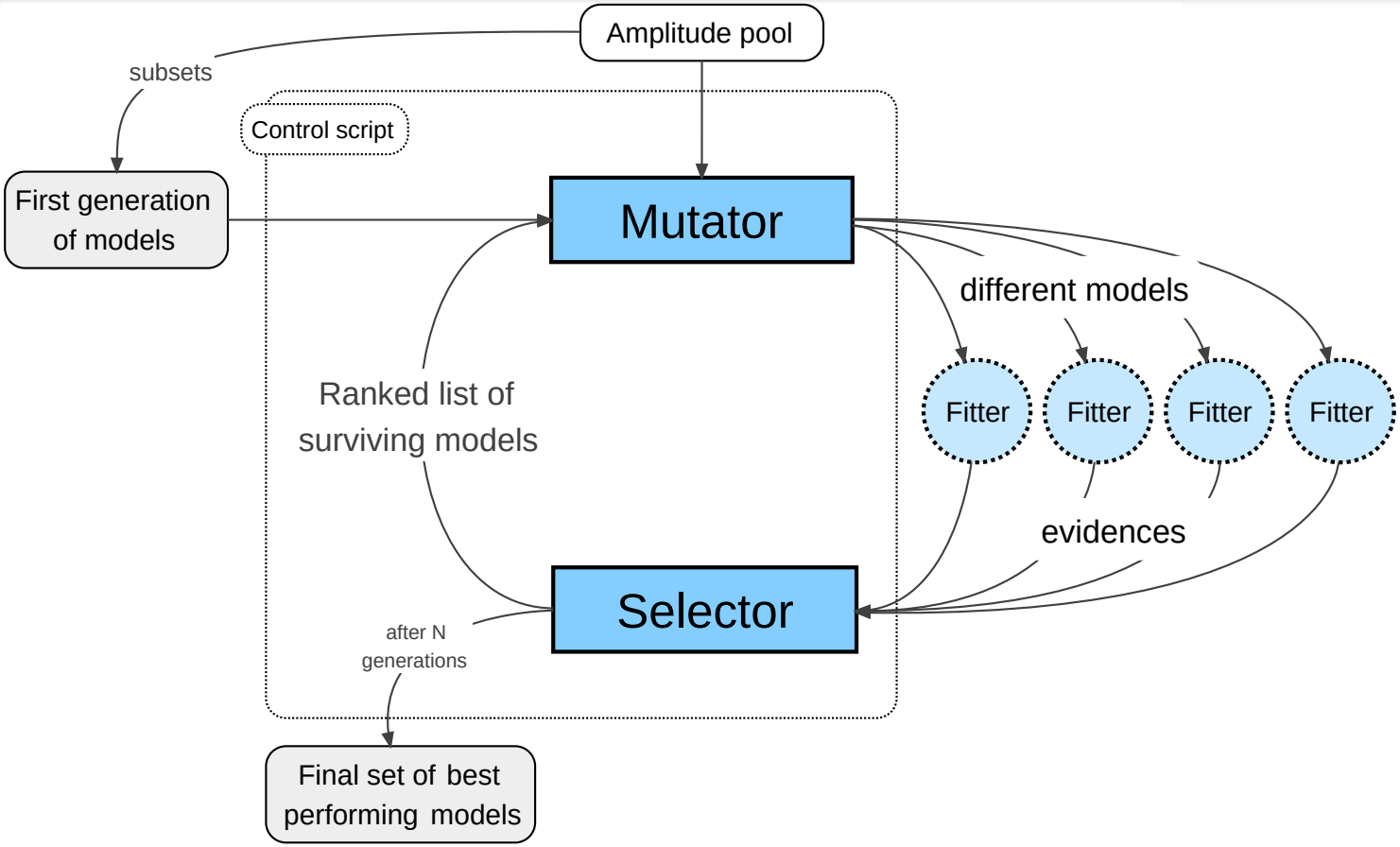
$$G = (-1)^{I+\ell+s} \qquad P = (-1)^{\ell+1} \qquad C = (-1)^{\ell+s}$$

J^{PC} Multiplets

- $\ell = 0 \quad \Rightarrow \quad$ pseudoscalar 0^{-+}, vector 1^{--} states
- $\ell = 1 \quad \Rightarrow \quad$ scalar 0^{++}, axial vector 1^{+-}, 1^{++} and tensor 2^{++} states
- Same $J^{PC} \Rightarrow$ mixing!
- Forbidden: $0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, \ldots \rightarrow$ *spin exotic* states

Evolutionary Waveset Exploration

Genetic Algorithm — 284 Waves in Pool



Evidence = Goodness of fit

- Bayesian Statistics → regularized Log-Likelihood
- Takes into account model complexity