

# Hadron Spectroscopy with COMPASS

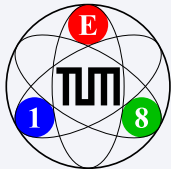
Boris Grube  
for the COMPASS Collaboration

Physik-Department E18  
Technische Universität München,  
Garching, Germany

PANIC11

MIT

26 July 2011, 3C-1



# The COMPASS Physics Program

## Common Muon and Proton Apparatus for Structure and Spectroscopy

### Goal

- Study **non-perturbative regime of QCD**
- Probe structure and dynamics of hadrons

### Very low $Q^2$ : chiral dynamics

- $\pi$  and  $K$  polarizabilities
- Chiral anomaly  
 $F_{3\pi}$

### Intermediate $Q^2$ : spectroscopy

- Mass spectrum of hadrons
- Gluonic excitations

### Large $Q^2$ : nucleon structure

- Helicity, transversity PDFs
- Generalized PDFs  
→ 3B-4 and 4B-1

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# Outline

- 1 The experimental setup
- 2 Search for spin-exotic mesons
  - Introduction
  - Partial-wave analysis of  $(3\pi)^-$  final state from  $\pi^-$  diffraction
  - Partial-wave analysis of  $\pi^- p \rightarrow \eta' \pi^- p$
- 3 Kaon diffraction into  $K^- \pi^+ \pi^-$  final state

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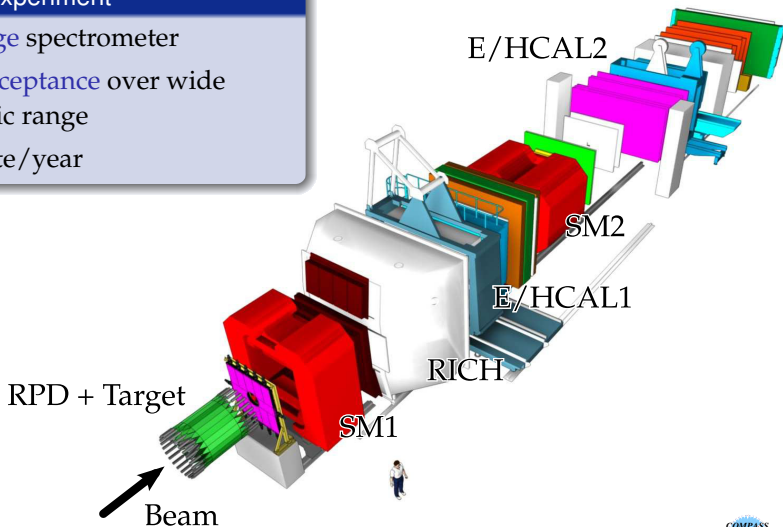
# The COMPASS Experiment at the CERN SPS

Experimental Setup

NIM A 577, 455 (2007)

## Fixed-target experiment

- Two-stage spectrometer
- Large acceptance over wide kinematic range
- $> 1$  PByte/year





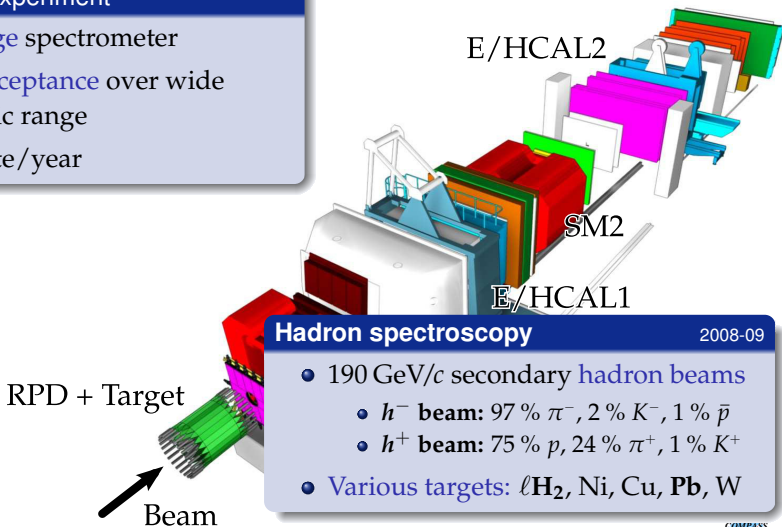
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## Hadron spectroscopy

2008-09

- 190 GeV/c secondary hadron beams
  - $h^-$  beam: 97 %  $\pi^-$ , 2 %  $K^-$ , 1 %  $\bar{p}$
  - $h^+$  beam: 75 %  $p$ , 24 %  $\pi^+$ , 1 %  $K^+$
- Various targets:  $\ell$ H<sub>2</sub>, Ni, Cu, Pb, W

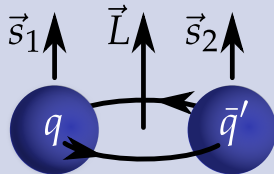
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# Mesons in the Constituent Quark Model

## Spin-parity rules for bound $q\bar{q}'$ system

- Quark spins couple to **total intrinsic spin**  
 $S = 0$  (singlet) or 1 (triplet)
- Relative **orbital angular Momentum**  $\vec{L}$   
and total spin  $\vec{S}$  couple to  
**meson spin**  $\vec{J} = \vec{L} + \vec{S}$
- Parity  $P = (-1)^{L+1}$
- Charge conjugation  $C = (-1)^{L+S}$
- **Forbidden  $J^{PC}$  combinations:**  $0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, \dots$



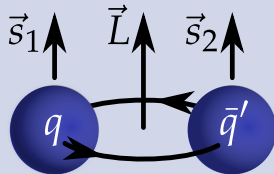
## QCD allows for states beyond the CQM

- Hybrids  $|q\bar{q}g\rangle$ , glueballs  $|gg\rangle$ , multi-quark states  $|q^2\bar{q}^2\rangle, \dots$
- Physical mesons: linear superposition of all allowed basis states
- **“Exotic” mesons** have quantum numbers forbidden for  $|q\bar{q}\rangle$ 
  - Particularly interesting:  $J^{PC}$ -exotic states

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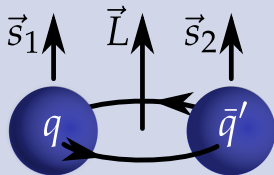
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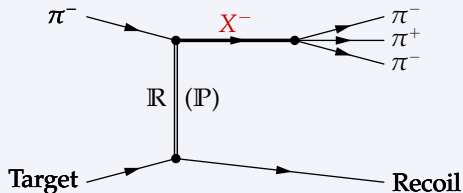
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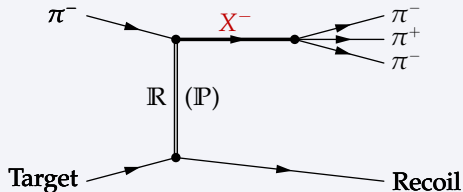
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# Production of Hadrons in Diffractive Dissociation



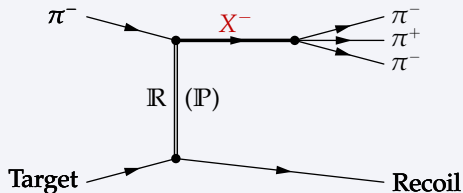
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  - **Beam particle is excited** into some resonance  $X$
  - $X$  decays into  $n$ -body **final state**
- At high energies **Pomeron exchange** dominates
- Rich spectrum of intermediate states  $X$
- **Goal:** use kinematic distribution of the final-state particles
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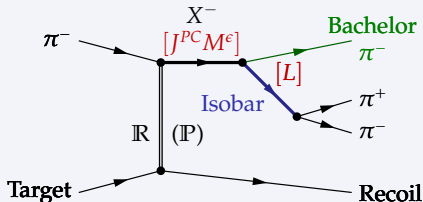
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# Diffractive Dissociation of $\pi^-$ into $\pi^- \pi^+ \pi^-$ Final State



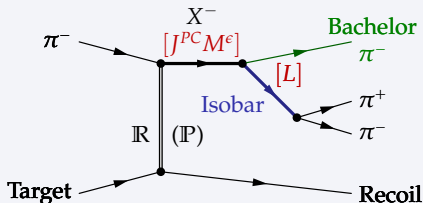
## Isobar model: $X^-$ decay is chain of successive two-body decays

- Isobar has spin  $S$  and relative orbital angular momentum  $L$  w.r.t. bachelor  $\pi^-$ ;  $L$  and  $S$  couple to spin  $J$  of  $X^-$
- “Wave”: unique combination of isobar and quantum numbers
  - Full wave specification (in reflectivity basis):  $J^{PC} M^e [\text{isobar}] L$

Fit model: 
$$\sigma(m_X, \tau) = \sigma_0 \left| \sum_{\text{waves}} T_{\text{wave}}(m_X) A_{\text{wave}}(m_X, \tau) \right|^2$$

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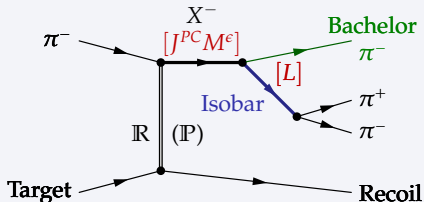
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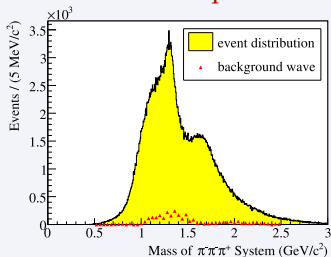
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PRL 104 (2010) 241803

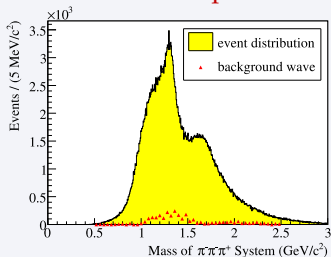
## $\pi^- \pi^+ \pi^-$ mass spectrum



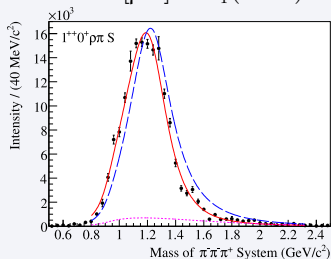
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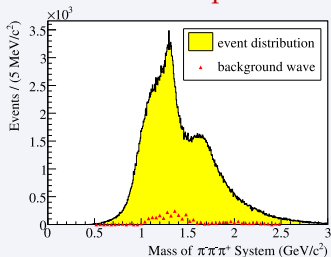
## $1^{++} 0^+ [\rho\pi] S : a_1(1260)$



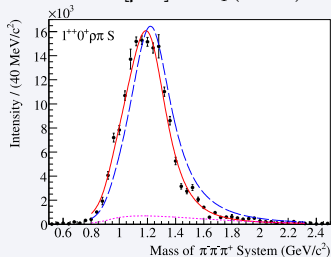
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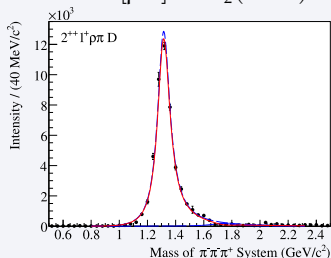
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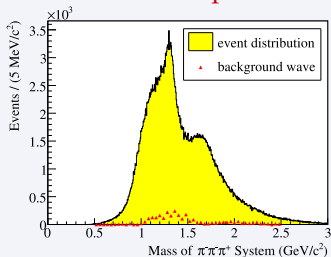
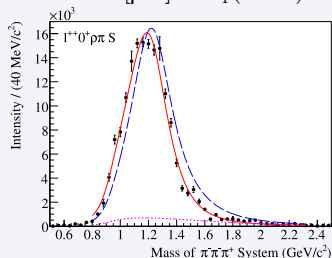
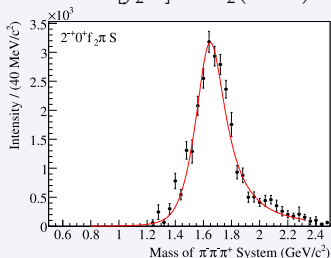
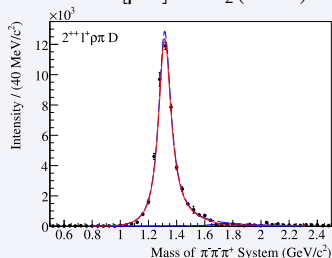


## $2^{++} 1^+ [\rho\pi] D : a_2(1320)$



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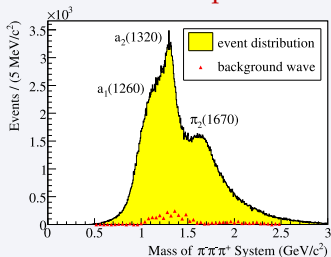
PRL 104 (2010) 241803

 $\pi^- \pi^+ \pi^-$  mass spectrum $1^{++} 0^+ [\rho\pi]S : a_1(1260)$  $2^{-+} 0^+ [f_2\pi]S : \pi_2(1670)$  $2^{++} 1^+ [\rho\pi]D : a_2(1320)$ 

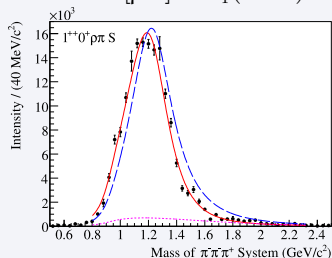
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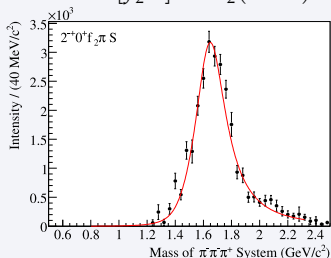
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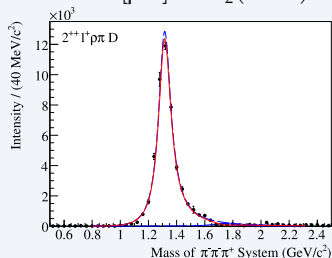
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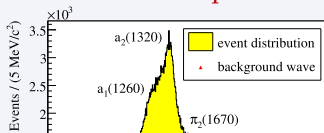
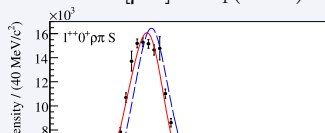
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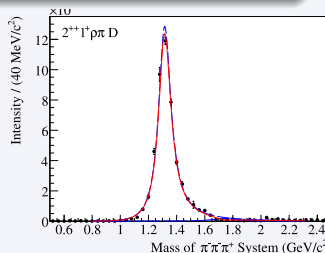
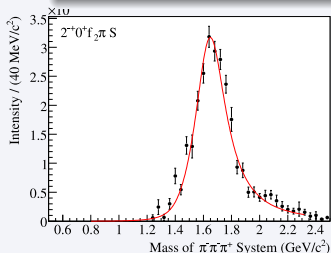


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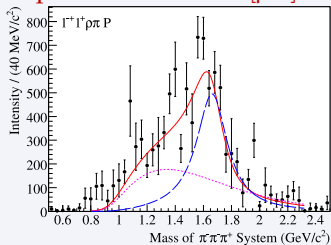
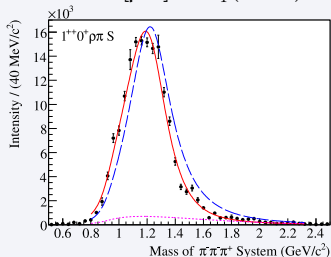
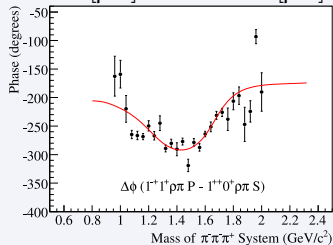
 $\pi^- \pi^+ \pi^-$  mass spectrum $1^{++} 0^+ [\rho\pi] S : a_1(1260)$ 

- Momentum transfer:  $0.1 < t' < 1.0$  ( $\text{GeV}/c^2$ )<sup>2</sup>
- Data described by model consisting of 41 waves + incoherent isotropic background
- Isobars:  $(\pi\pi)_S, f_0(980), \rho(770), f_2(1270),$  and  $\rho_3(1690)$



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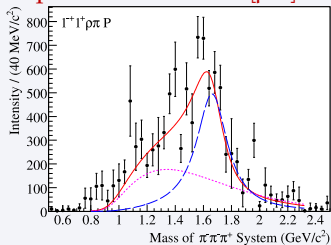
Spin-exotic  $1^{-+} 1^+ [\rho\pi]P$  $1^{++} 0^+ [\rho\pi]S : a_1(1260)$  $1^{-+} 1^+ [\rho\pi]P - 1^{++} 0^+ [\rho\pi]S$ 

- Significant  $1^{-+}$  amplitude consistent with resonance around  $1.7 \text{ GeV}/c^2$
- $\pi_1(1600)$  BW + background  
 $m = 1660 \pm 10^{+0}_{-64} \text{ MeV}/c^2$   
 $\Gamma = 269 \pm 21^{+42}_{-64} \text{ MeV}$

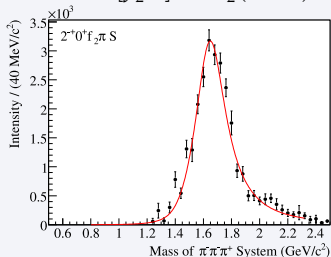
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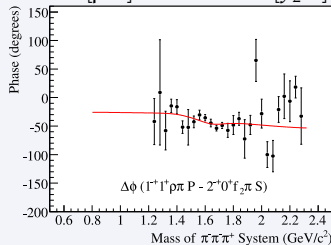
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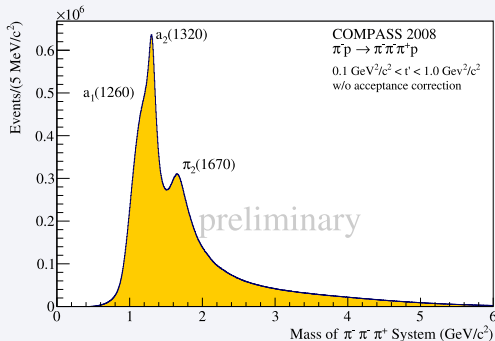
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- Liquid hydrogen target
- **Spectrometer upgrade:** recoil proton detector, beam PID, calorimetry, and tracking
- 190 GeV/c negative hadron beam: 97 %  $\pi^-$ , 2 %  $K^-$ , 1 %  $\bar{p}$
- 200 × 2004 statistics: 96 M events  $\implies$  challenging analysis

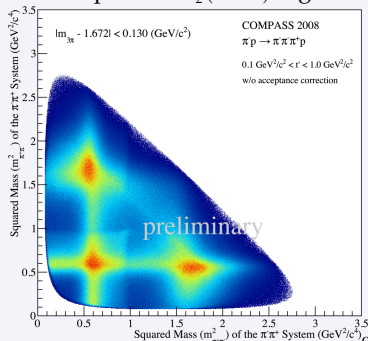
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$\pi^- \pi^+ \pi^-$  mass distribution

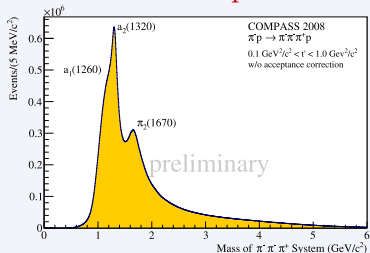


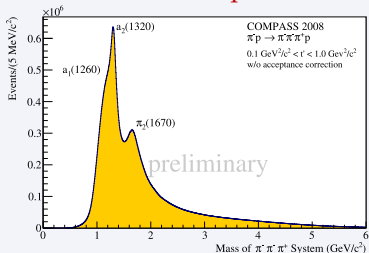
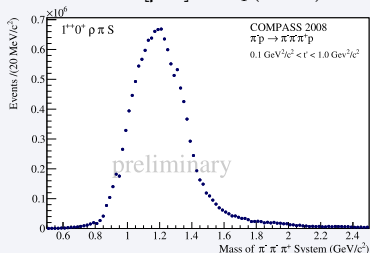
Dalitz plot for  $\pi_2(1670)$  region

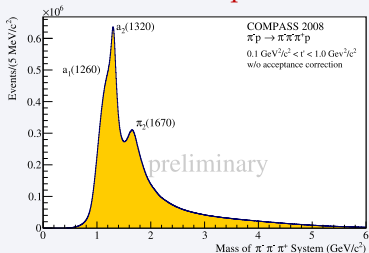
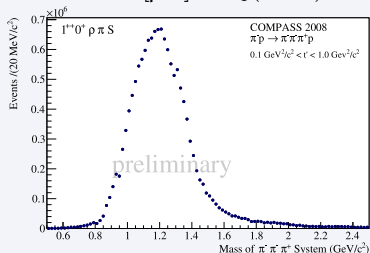
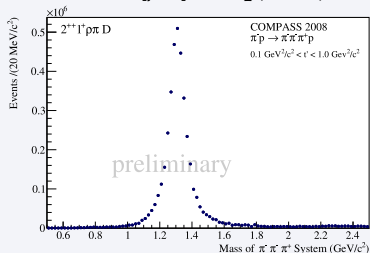


# PWA of $\pi^- p \rightarrow \pi^- \pi^+ \pi^- p$ (2008 Data)

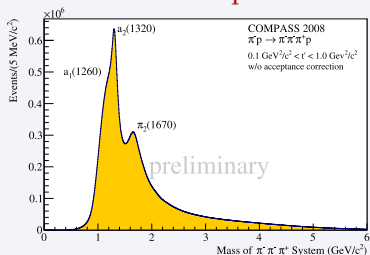
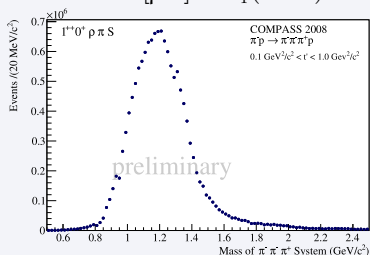
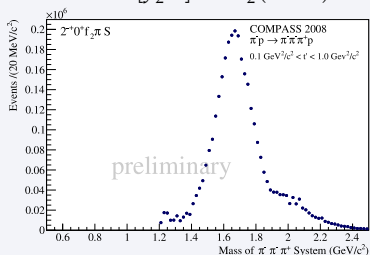
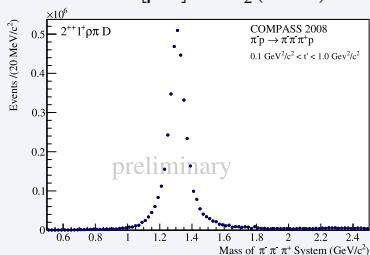
## $\pi^- \pi^+ \pi^-$ mass spectrum



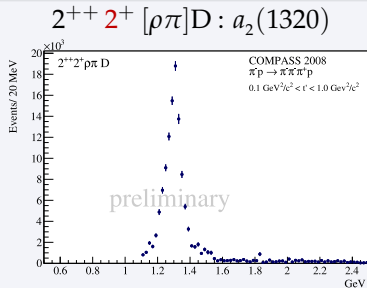
PWA of  $\pi^- p \rightarrow \pi^- \pi^+ \pi^- p$  (2008 Data) $\pi^- \pi^+ \pi^-$  mass spectrum $1^{++} 0^+ [\rho\pi]S : a_1(1260)$ 

PWA of  $\pi^- p \rightarrow \pi^- \pi^+ \pi^- p$  (2008 Data) $\pi^- \pi^+ \pi^-$  mass spectrum $1^{++} 0^+ [\rho\pi]S : a_1(1260)$  $2^{++} 1^+ [\rho\pi]D : a_2(1320)$ 

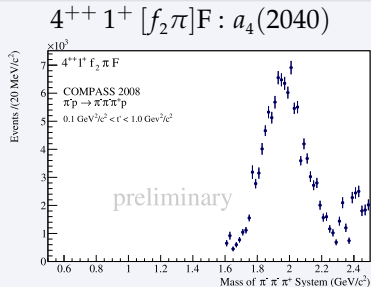
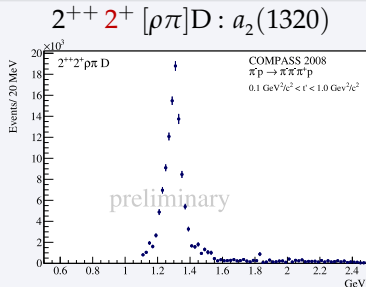


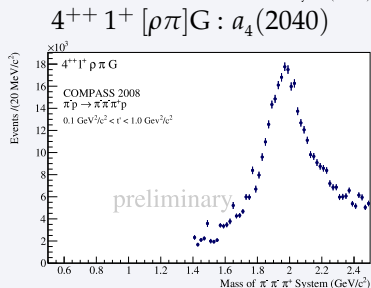
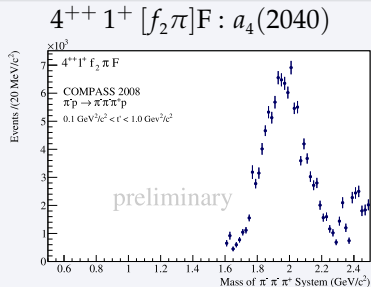
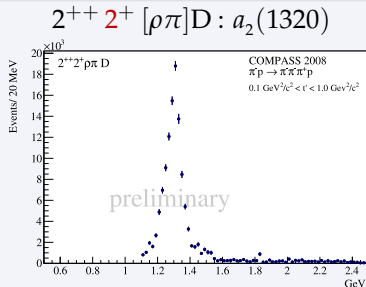
PWA of  $\pi^- p \rightarrow \pi^- \pi^+ \pi^- p$  (2008 Data) $\pi^- \pi^+ \pi^-$  mass spectrum $1^{++} 0^+ [\rho\pi]S : a_1(1260)$  $2^{-+} 0^+ [f_2\pi]S : \pi_2(1670)$  $2^{++} 1^+ [\rho\pi]D : a_2(1320)$ 

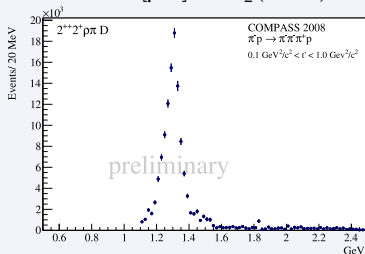
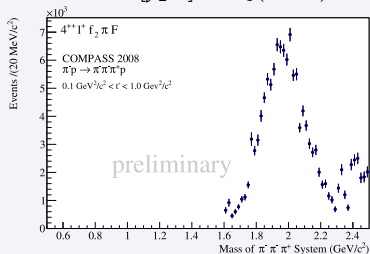
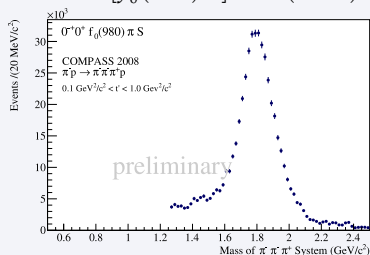
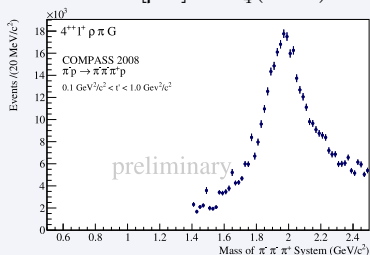
# PWA of $\pi^- p \rightarrow \pi^- \pi^+ \pi^- p$ (2008 Data)

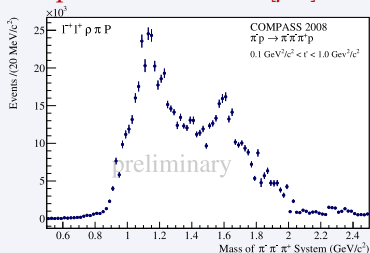


# PWA of $\pi^- p \rightarrow \pi^- \pi^+ \pi^- p$ (2008 Data)



PWA of  $\pi^- p \rightarrow \pi^- \pi^+ \pi^- p$  (2008 Data)

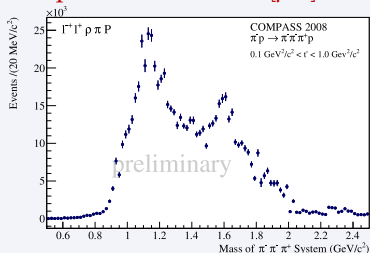
PWA of  $\pi^- p \rightarrow \pi^- \pi^+ \pi^- p$  (2008 Data) $2^{++} 2^+ [\rho\pi]D : a_2(1320)$  $4^{++} 1^+ [f_2\pi]F : a_4(2040)$  $0^{-+} 0^+ [f_0(980)\pi]S : \pi(1800)$  $4^{++} 1^+ [\rho\pi]G : a_4(2040)$ 

PWA of  $\pi^- p \rightarrow \pi^- \pi^+ \pi^- p$  (2008 Data)Spin-exotic  $1^{--} 1^+ [\rho\pi]P$ 

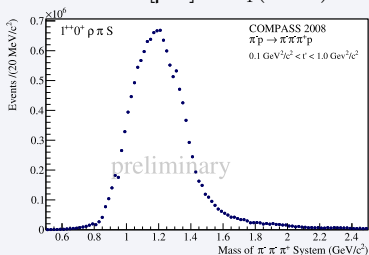
- Structure around  $1.1 \text{ GeV}/c^2$  unstable w.r.t. fit model
- Stable **enhancement around  $1.6 \text{ GeV}/c^2$**
- Phase motion w.r.t. to tail of  $a_1(1260)$
- Phase locked w.r.t.  $\pi_2(1670)$
- **Ongoing analysis**

# PWA of $\pi^- p \rightarrow \pi^- \pi^+ \pi^- p$ (2008 Data)

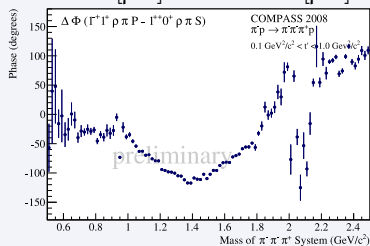
## Spin-exotic $1^{-+} 1^+ [\rho\pi]P$



## $1^{++} 0^+ [\rho\pi]S : a_1(1260)$



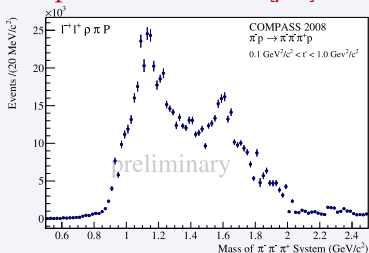
## $1^{-+} 1^+ [\rho\pi]P - 1^{++} 0^+ [\rho\pi]S$



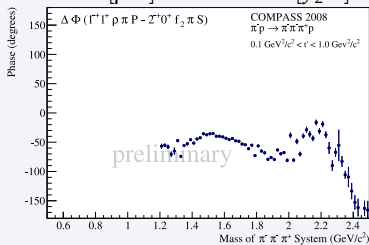
- Structure around  $1.1 \text{ GeV}/c^2$  unstable w.r.t. fit model
- Stable **enhancement around  $1.6 \text{ GeV}/c^2$**
- Phase motion w.r.t. to tail of  $a_1(1260)$
- Phase locked w.r.t.  $\pi_2(1670)$
- **Ongoing analysis**

# PWA of $\pi^- p \rightarrow \pi^- \pi^+ \pi^- p$ (2008 Data)

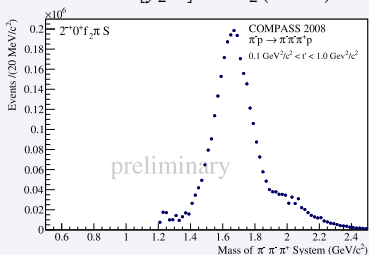
## Spin-exotic $1^{-+} 1^+ [\rho\pi]P$



## $1^{-+} 1^+ [\rho\pi]P - 2^{-+} 0^+ [f_2\pi]S$



## $2^{-+} 0^+ [f_2\pi]S: \pi_2(1670)$



- Structure around  $1.1 \text{ GeV}/c^2$  unstable w.r.t. fit model
- Stable **enhancement around  $1.6 \text{ GeV}/c^2$**
- Phase motion w.r.t. to tail of  $a_1(1260)$
- Phase locked w.r.t.  $\pi_2(1670)$
- **Ongoing analysis**



# Nuclear Effect in Diffractive $\pi^- \pi^+ \pi^-$ Production

## Comparison of PWA results using different target materials

- 2008:  $H_2$  target ( $\approx 23$  M events)
- 2009:  $Pb$  target ( $\approx 1.2$  M events)
- Normalized to  $a_2(1320)$  region  $[1.1, 1.6]$   $GeV/c^2$

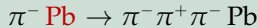
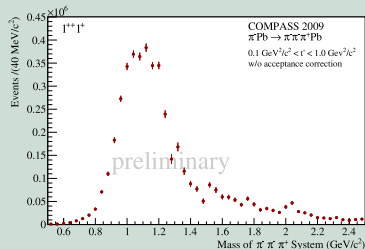
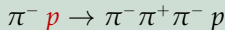
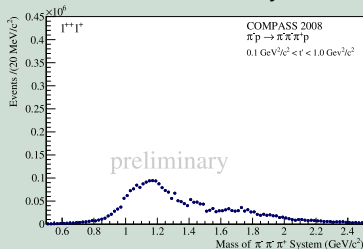
# Nuclear Effect in Diffractive $\pi^- \pi^+ \pi^-$ Production

## Comparison of PWA results using different target materials

- 2008:  $H_2$  target ( $\approx 23$  M events)
- 2009:  $Pb$  target ( $\approx 1.2$  M events)
- Normalized to  $a_2(1320)$  region  $[1.1, 1.6]$   $GeV/c^2$

## Enhancement of $M = 1$ states on $Pb$ target

Total intensity of  $J^{PC} = 1^{++}$  waves with  $M^e = 1^+$



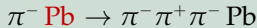
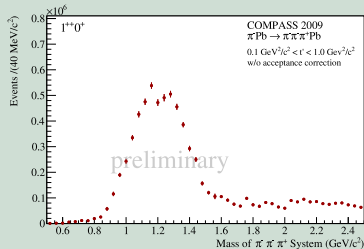
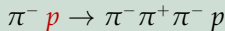
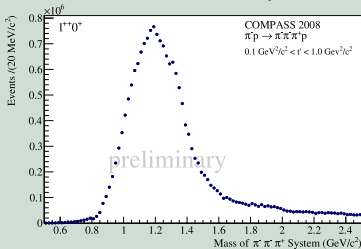
# Nuclear Effect in Diffractive $\pi^- \pi^+ \pi^-$ Production

## Comparison of PWA results using different target materials

- 2008:  $H_2$  target ( $\approx 23$  M events)
- 2009:  $Pb$  target ( $\approx 1.2$  M events)
- Normalized to  $a_2(1320)$  region  $[1.1, 1.6]$   $GeV/c^2$

## Suppression of $M = 0$ states on $Pb$ target

Total intensity of  $J^{PC} = 1^{++}$  waves with  $M^e = 0^+$



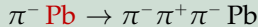
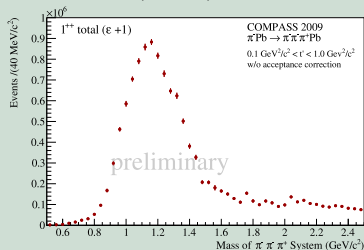
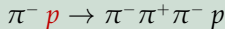
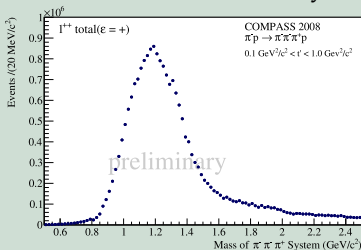
# Nuclear Effect in Diffractive $\pi^- \pi^+ \pi^-$ Production

## Comparison of PWA results using different target materials

- 2008:  $H_2$  target ( $\approx 23$  M events)
- 2009:  $Pb$  target ( $\approx 1.2$  M events)
- Normalized to  $a_2(1320)$  region  $[1.1, 1.6]$   $GeV/c^2$

## Total intensity of sum over all $M$ states comparable

Total intensity of  $J^{PC} = 1^{++}$  waves (all  $M$ )

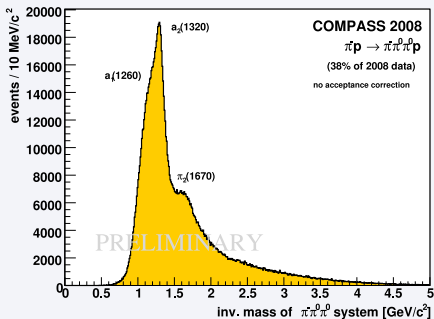


# Comparison to PWA of $\pi^- p \rightarrow \pi^- \pi^0 \pi^0 p$ (2008 Data)

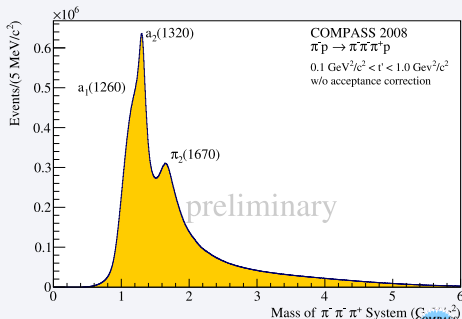
## Isospin partner to $\pi^- p \rightarrow \pi^- \pi^+ \pi^- p$

- Important consistency check
- Comparison with  $\pi^- \pi^+ \pi^-$  data set: normalization to  $a_2(1320)$
- Isospin symmetry:  $I = 1$  isobar  $\implies$  same intensity  
 $I = 0$  isobar  $\implies$  half intensity

### $\pi^- \pi^0 \pi^0$ mass distribution



### $\pi^- \pi^+ \pi^-$ mass distribution

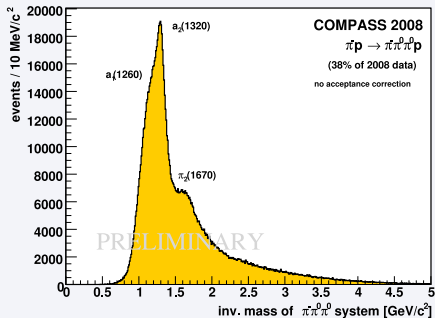


# Comparison to PWA of $\pi^- p \rightarrow \pi^- \pi^0 \pi^0 p$ (2008 Data)

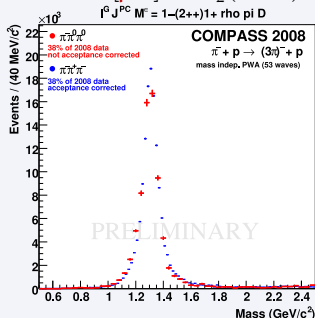
## Isospin partner to $\pi^- p \rightarrow \pi^- \pi^+ \pi^- p$

- Important consistency check
- Comparison with  $\pi^- \pi^+ \pi^-$  data set: normalization to  $a_2(1320)$
- Isospin symmetry:  $I = 1$  isobar  $\implies$  same intensity  
 $I = 0$  isobar  $\implies$  half intensity

## $\pi^- \pi^0 \pi^0$ mass distribution



## $2^{++} 1^+ [\rho\pi] D: a_2(1320)$

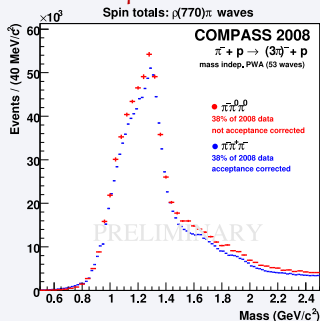


# Comparison to PWA of $\pi^- p \rightarrow \pi^- \pi^0 \pi^0 p$ (2008 Data)

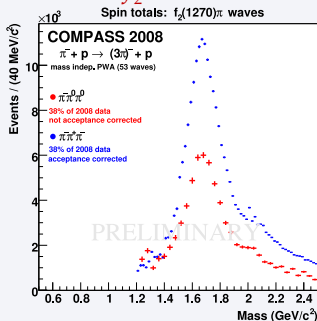
## Isospin partner to $\pi^- p \rightarrow \pi^- \pi^+ \pi^- p$

- Important **consistency check**
- Comparison with  $\pi^- \pi^+ \pi^-$  data set: **normalization to  $a_2(1320)$**
- **Isospin symmetry**:  $I = 1$  isobar  $\implies$  **same** intensity  
 $I = 0$  isobar  $\implies$  **half** intensity

### All $\rho\pi$ waves



### All $f_2\pi$ waves

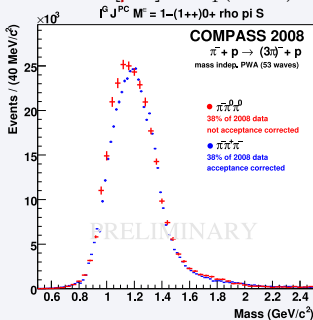


# Comparison to PWA of $\pi^- p \rightarrow \pi^- \pi^0 \pi^0 p$ (2008 Data)

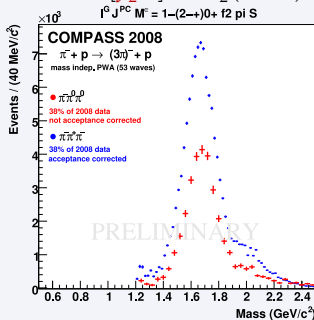
## Isospin partner to $\pi^- p \rightarrow \pi^- \pi^+ \pi^- p$

- Important **consistency check**
- Comparison with  $\pi^- \pi^+ \pi^-$  data set: **normalization to  $a_2(1320)$**
- **Isospin symmetry**:  $I = 1$  isobar  $\implies$  **same** intensity  
 $I = 0$  isobar  $\implies$  **half** intensity

$1^{++} 0^+ [\rho\pi]S : a_1(1260)$



$2^{-+} 0^+ [f_2\pi]S : \pi_2(1670)$





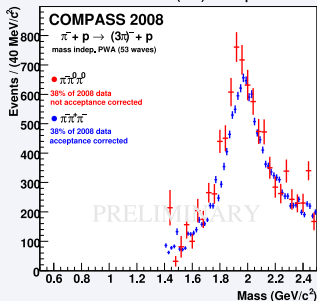
# Comparison to PWA of $\pi^- p \rightarrow \pi^- \pi^0 \pi^0 p$ (2008 Data)

## Isospin partner to $\pi^- p \rightarrow \pi^- \pi^+ \pi^- p$

- Important consistency check
- Comparison with  $\pi^- \pi^+ \pi^-$  data set: normalization to  $a_2(1320)$
- Isospin symmetry:  $I = 1$  isobar  $\implies$  same intensity  
 $I = 0$  isobar  $\implies$  half intensity

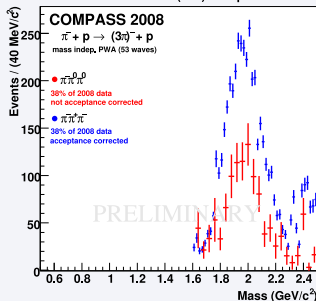
$$4^{++} 1^+ [\rho\pi]G : a_4(2040)$$

$I^G J^{PC} M^{\pi} = 1-(4^{++})1^+ \rho \pi \text{ pi } G$



$$4^{++} 1^+ [f_2\pi]F : a_4(2040)$$

$I^G J^{PC} M^{\pi} = 1-(4^{++})1^+ f_2 \text{ pi } F$

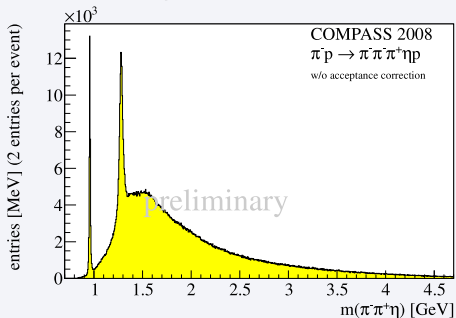


# PWA of $\pi^- p \rightarrow \eta' \pi^- p$ (2008 Data)

## Selection of exclusive 3 charged tracks + 2 photons events

- $\eta'$  reconstructed via  $\pi^+ \pi^- \eta$  decay
- $\eta$  reconstructed from  $\eta \rightarrow \gamma\gamma$
- 35 000 events after selection

$\pi^+ \pi^- \eta$  mass distribution

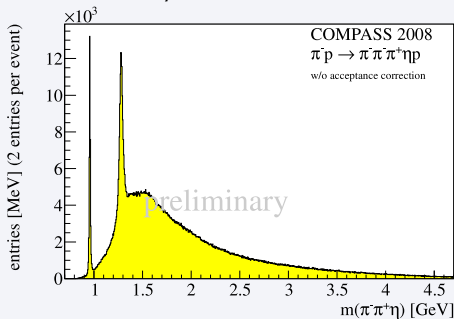


# PWA of $\pi^- p \rightarrow \eta' \pi^- p$ (2008 Data)

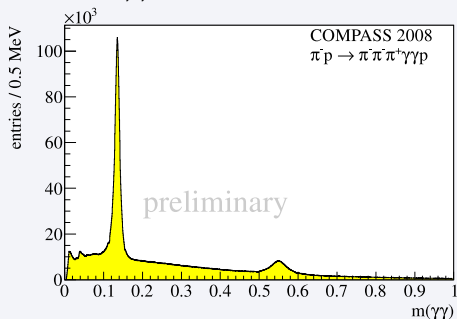
Selection of exclusive 3 charged tracks + 2 photons events

- $\eta'$  reconstructed via  $\pi^+ \pi^- \eta$  decay
- $\eta$  reconstructed from  $\eta \rightarrow \gamma\gamma$
- 35 000 events after selection

$\pi^+ \pi^- \eta$  mass distribution



$\gamma\gamma$  mass distribution

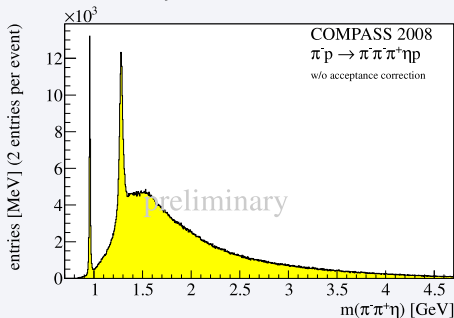


# PWA of $\pi^- p \rightarrow \eta' \pi^- p$ (2008 Data)

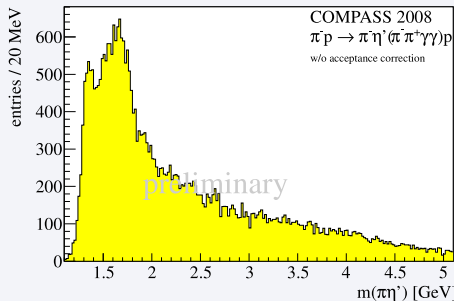
## Selection of exclusive 3 charged tracks + 2 photons events

- $\eta'$  reconstructed via  $\pi^+ \pi^- \eta$  decay
- $\eta$  reconstructed from  $\eta \rightarrow \gamma\gamma$
- 35 000 events after selection

$\pi^+ \pi^- \eta$  mass distribution

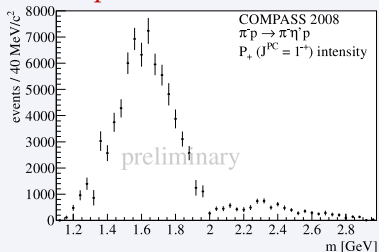


$\eta' \pi^-$  mass distribution



# PWA of $\pi^- p \rightarrow \eta' \pi^- p$ (2008 Data)

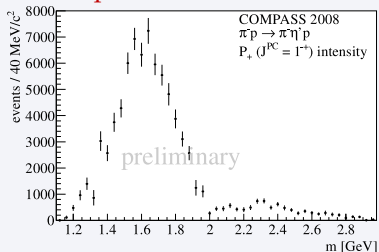
## Spin-exotic $1^{-+} 1^{+}$



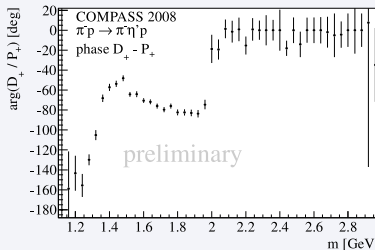
- $1^{-+} 1^{+}$  is dominant wave
- Phase motion w.r.t.  $2^{++} 1^{+}$
- No phase motion in the  $2 \text{ GeV}/c^2$  region
- Ongoing analysis

# PWA of $\pi^- p \rightarrow \eta' \pi^- p$ (2008 Data)

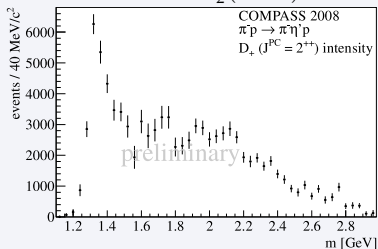
## Spin-exotic $1^{-+} 1^{+}$



## $2^{++} 1^{+} - 1^{-+} 1^{+}$



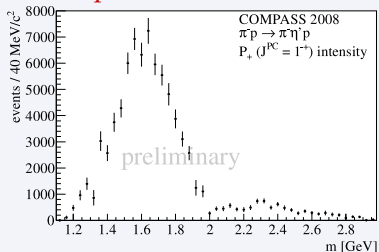
## $2^{++} 1^{+} : a_2(1320)$



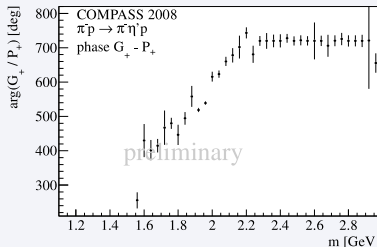
- $1^{-+} 1^{+}$  is dominant wave
- Phase motion w.r.t.  $2^{++} 1^{+}$
- No phase motion in the 2 GeV/ $c^2$  region
- Ongoing analysis

# PWA of $\pi^- p \rightarrow \eta' \pi^- p$ (2008 Data)

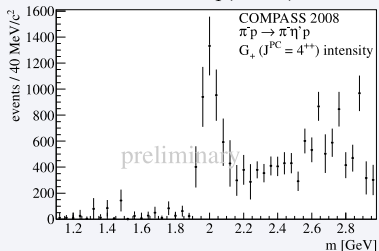
## Spin-exotic $1^{-+} 1^{+}$



## $4^{++} 1^+ - 1^{-+} 1^+$



## $4^{++} 1^+ : a_4(2040)$



- $1^{-+} 1^+$  is dominant wave
- Phase motion w.r.t.  $2^{++} 1^+$
- No phase motion in the  $2 \text{ GeV}/c^2$  region
- Ongoing analysis

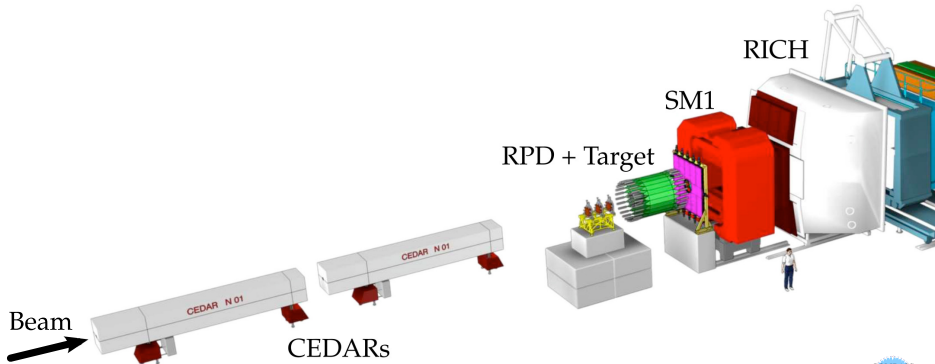
# Outline

- 1 The experimental setup
- 2 Search for spin-exotic mesons
  - Introduction
  - Partial-wave analysis of  $(3\pi)^-$  final state from  $\pi^-$  diffraction
  - Partial-wave analysis of  $\pi^- p \rightarrow \eta' \pi^- p$
- 3 Kaon diffraction into  $K^- \pi^+ \pi^-$  final state



# Kaon Diffraction into $K^- \pi^+ \pi^-$ Final State

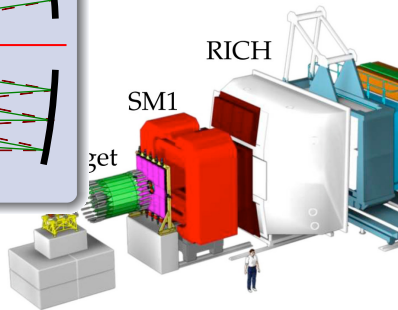
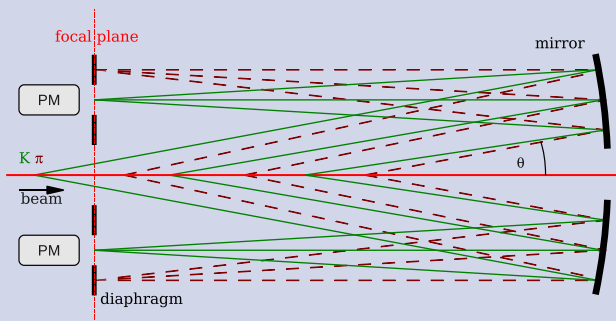
## Experimental Setup



# Kaon Diffraction into $K^- \pi^+ \pi^-$ Final State

## Experimental Setup

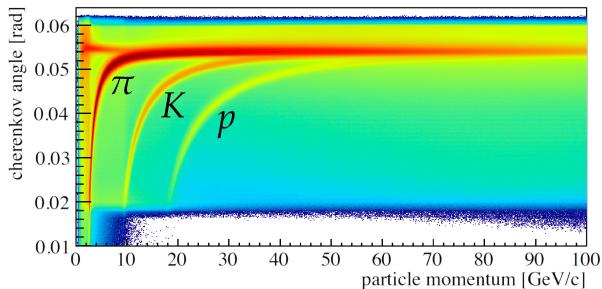
2%  $K^-$  beam content tagged by CEDARs



# Kaon Diffraction into $K^- \pi^+ \pi^-$ Final State

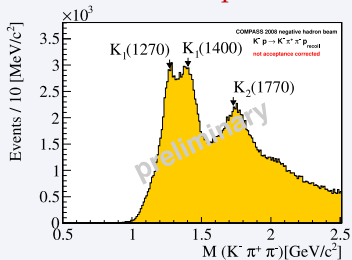
## Experimental Setup

### Final-state $K^-$ identified by RICH



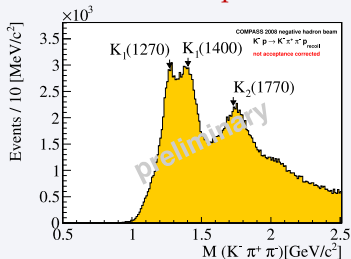
# PWA of $K^- p \rightarrow K^- \pi^+ \pi^- p$ (2008 Data)

## $K^- \pi^+ \pi^-$ mass spectrum

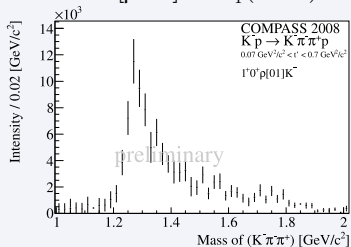


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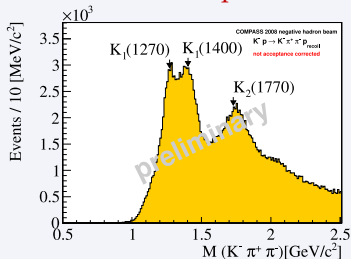


## $1^+ 0^+ [\rho K^-] S : K_1(1270)$

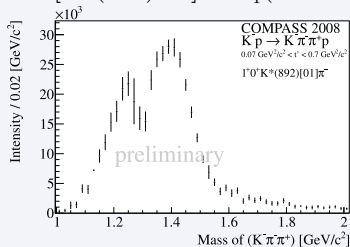


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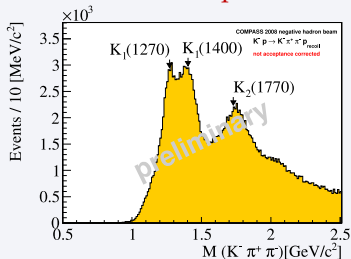


## $1^+ 0^+ [K^*(892)\pi^-]S : K_1(1270, 1400)$

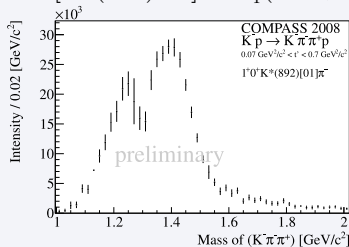


# PWA of $K^- p \rightarrow K^- \pi^+ \pi^- p$ (2008 Data)

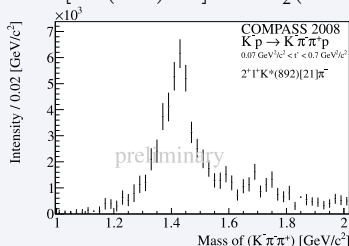
## $K^- \pi^+ \pi^-$ mass spectrum



## $1^+ 0^+ [K^*(892)\pi^-]S : K_1(1270, 1400)$

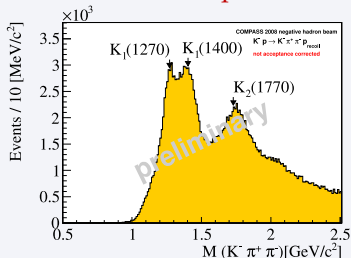


## $2^+ 1^+ [K^*(892)\pi^-]D : K_2^*(1430)$

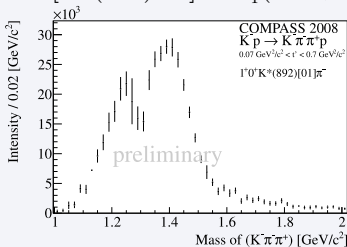


# PWA of $K^- p \rightarrow K^- \pi^+ \pi^- p$ (2008 Data)

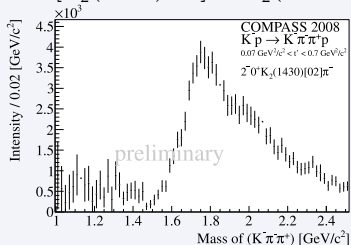
## $K^- \pi^+ \pi^-$ mass spectrum



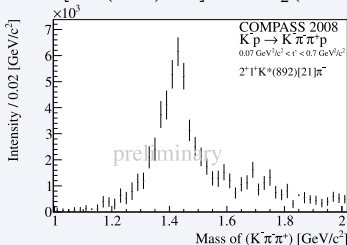
## $1^+ 0^+ [K^*(892)\pi^-]S : K_1(1270, 1400)$



## $2^- 0^+ [K_2(1430)\pi^-]S : K_2(1770, 1820)$



## $2^+ 1^+ [K^*(892)\pi^-]D : K_2^*(1430)$





# Conclusions and Outlook

## Huge amount of data from 2008-09 runs

- Analysis ongoing
  - Main focus on search for  $J^{PC}$ -exotic mesons
    - Pilot run: significant  $J^{PC} = 1^{-+}$  signal consistent with  $\pi_1(1600)$  seen in  $\pi^-\pi^+\pi^-$  data on Pb target
    - Detailed study of  $\pi^-\pi^+\pi^-$  final state
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    - Extend analysis to  $\eta \pi^-$
  - Other channels

## Poster session

- Started analysis
  - #25: Study of the  $\pi^- \text{Pb} \rightarrow \pi^- \pi^+ \pi^- \text{Pb}$  Primakoff Reaction at COMPASS
  - #16: OZI Test and Spin Alignment of Vector Mesons with the COMPASS Experiment
- 2012 precision pion (using Primakoff)





# Outline

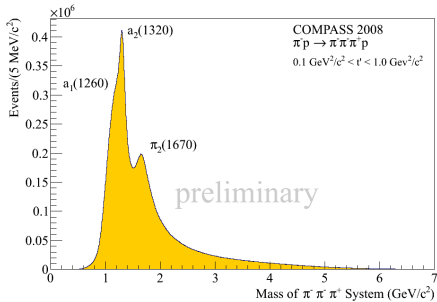
- 4 Backup slides
  - $\pi^- \pi^+ \pi^-$  final state
  - $\eta' \pi^-$  final state
  - $K^- \pi^+ \pi^-$  final state



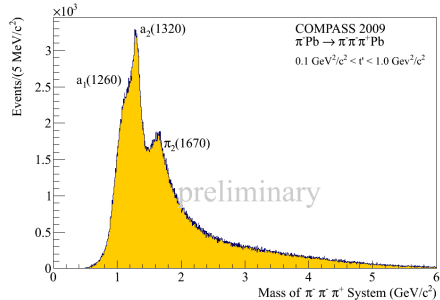
# $\pi^- \pi^+ \pi^-$ Final State

$\pi^- \pi^+ \pi^-$  Invariant Mass Distribution

*p* target



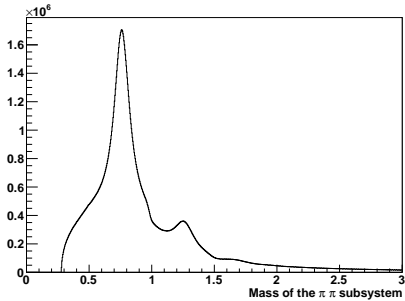
Pb target



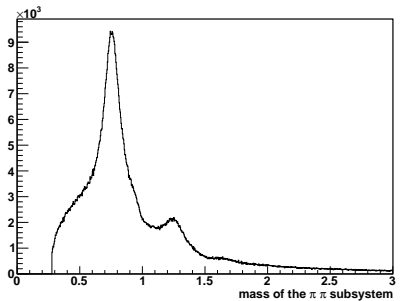
# $\pi^- \pi^+ \pi^-$ Final State

$\pi^+ \pi^-$  Invariant Mass Distribution

*p* target



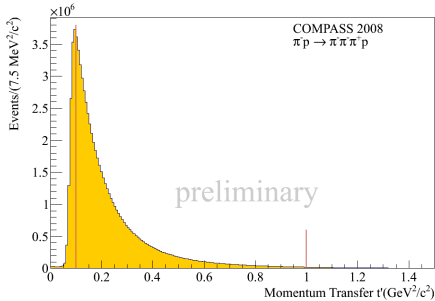
Pb target



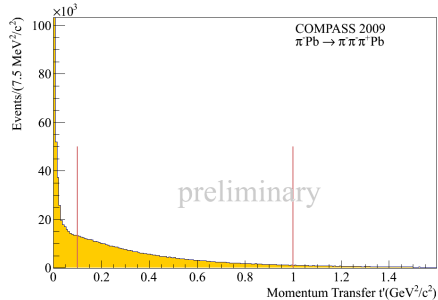
# $\pi^- \pi^+ \pi^-$ Final State

## Momentum Transfer Distribution

$p$  target



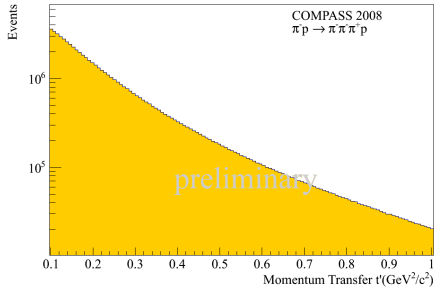
Pb target



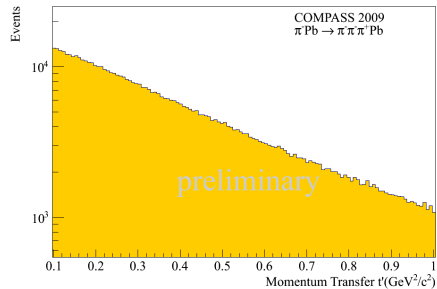
# $\pi^- \pi^+ \pi^-$ Final State

## Momentum Transfer Distribution

$p$  target



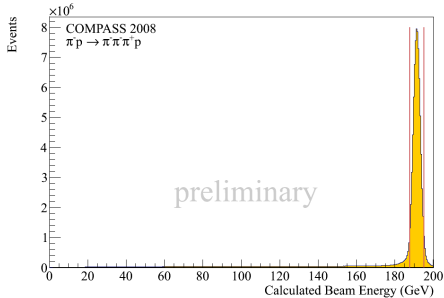
Pb target



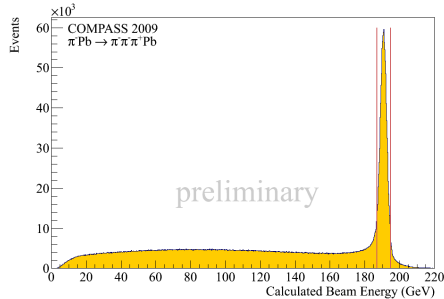
# $\pi^- \pi^+ \pi^-$ Final State

$\pi^- \pi^+ \pi^-$  Exclusivity

*p* target



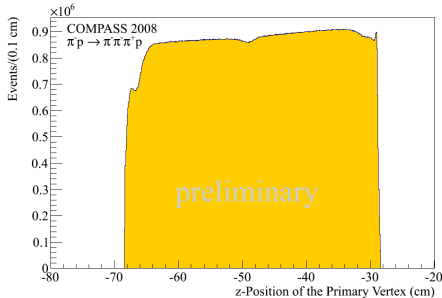
Pb target



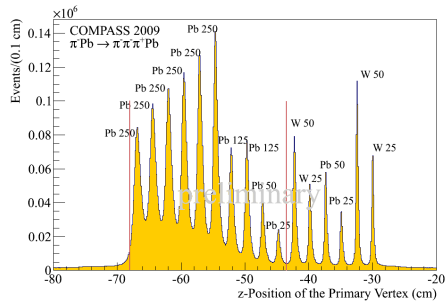
# $\pi^- \pi^+ \pi^-$ Final State

## Primary Vertex Distribution

*p* target



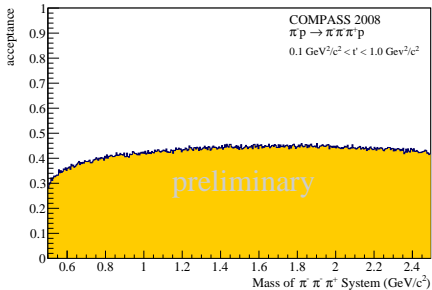
Pb target



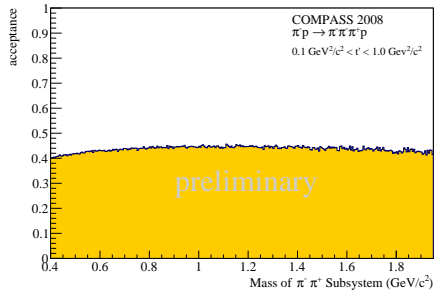
# $\pi^- \pi^+ \pi^-$ Final State

Acceptance ( $p$  Target)

$\pi^- \pi^+ \pi^-$  mass

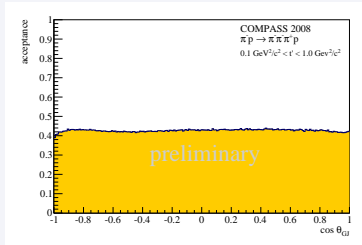
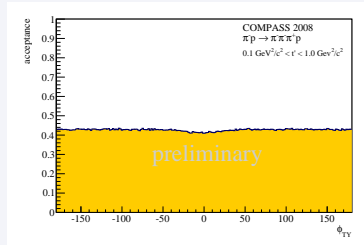
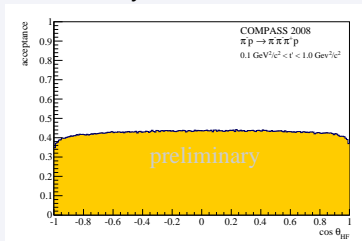
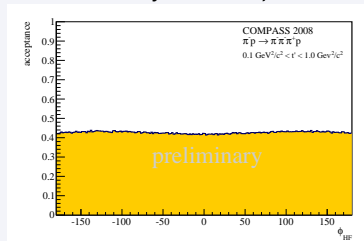


$\pi^+ \pi^-$  mass



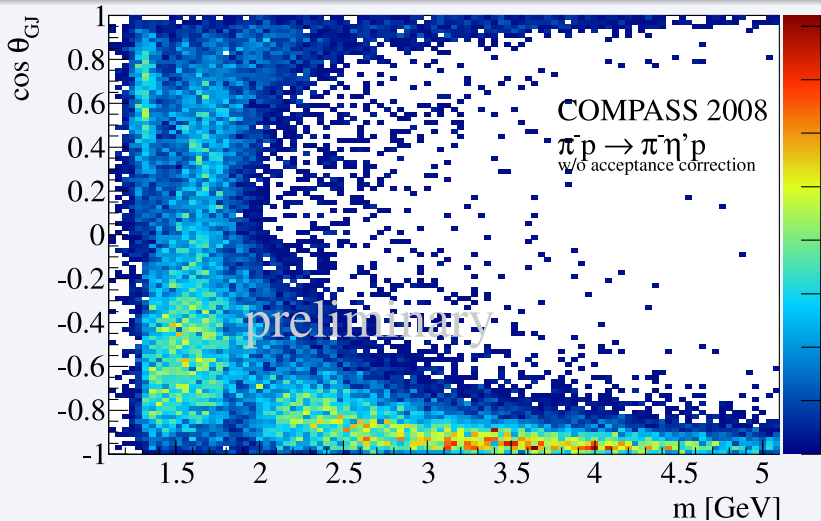


# $\pi^- \pi^+ \pi^-$ Final State

Acceptance ( $p$  Target)Gottfried-Jackson frame:  $\cos \theta_{GJ}$ Gottfried-Jackson frame:  $\phi_{TY}$ Helicity frame:  $\cos \theta_{HF}$ Helicity frame:  $\phi_{HF}$ 

# $\eta' \pi^-$ Final State

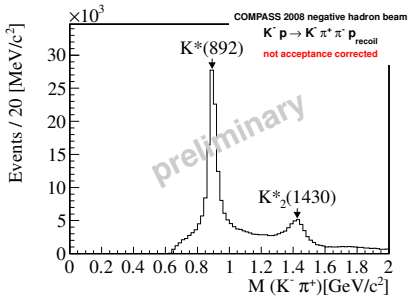
$\cos \theta_{GJ}$  vs.  $\eta' \pi^-$  Invariant Mass



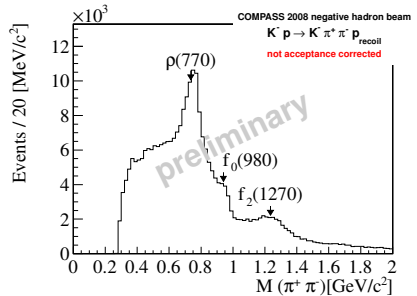
# $K^- \pi^+ \pi^-$ Final State

## Subsystem Invariant Mass

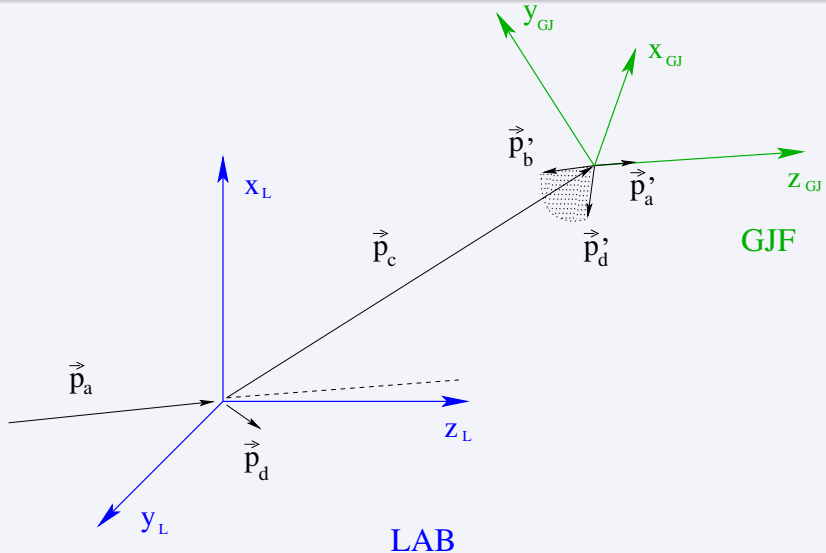
### $K^- \pi^+$ mass



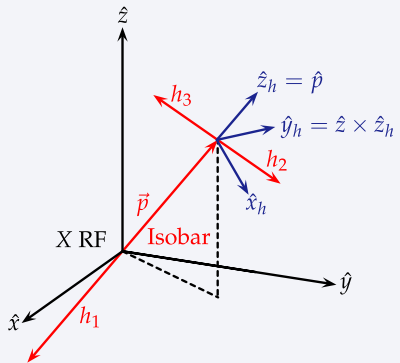
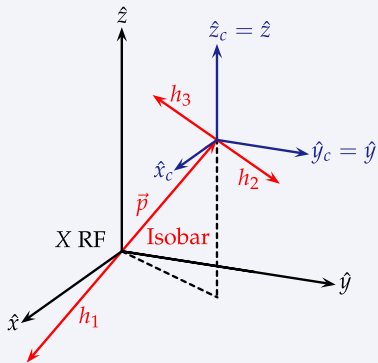
### $\pi^+ \pi^-$ mass



# Gottfried-Jackson Coordinate System



# Canonical vs. Helicity Coordinate System



# Partial-Wave Analysis Formalism

## Cross section parameterization in mass-independent PWA

$$\sigma(\tau; m_X) = \sum_{\epsilon=\pm 1} \sum_{r=1}^{N_r} \left| \sum_i^{\text{waves}} T_{ir}^\epsilon(m_X) A_i^\epsilon(\tau) \right|^2$$

- $\epsilon, i$ : quantum numbers of partial wave ( $J^{PC} M^\epsilon$  [isobar]  $L$ )
- $T_{ir}^\epsilon$ : complex production amplitudes; fit parameters
- $A_i^\epsilon$ : complex decay amplitudes
- $\tau$ : phase space coordinates

## Spin-density matrix

$$\rho_{ij}^\epsilon = \sum_{r=1}^{N_r} T_{ir}^\epsilon T_{jr}^{\epsilon*} \quad \sigma(\tau; m_X) = \sum_{\epsilon=\pm 1} \sum_{i,j}^{\text{waves}} \rho_{ij}^\epsilon(m_X) A_i^\epsilon(\tau) A_j^{\epsilon*}(\tau)$$

- Diagonal elements  $\rho_{ii}$ : intensities
- Off-diagonal elements  $\rho_{ii}, i \neq j$ : interference terms

# Partial-Wave Analysis Formalism

## Two-body decay amplitude in helicity formalism

- Decay  $X(w, J, \lambda) \rightarrow 1(J_1, \lambda_1) [L, S] 2(J_2, \lambda_2)$

$$A_X^{\text{hel}} = \sqrt{2L+1} \sum_{\lambda_1, \lambda_2} (J_1 \lambda_1 J_2 - \lambda_2 | S \delta) (L 0 S \delta | J \delta) D_{\lambda \delta}^{J*}(\theta, \phi, 0) F_L(q) \Delta(w) A_1 A_2$$

- $\delta = \lambda_1 - \lambda_2$
- $D_{\lambda \delta}^{J*}(\theta, \phi, 0)$  — Wigner  $D$ -function describes rotational properties of helicity states
- $\theta, \phi$  — polar angles of decay daughter 1 in  $X$  rest frame (GJ or helicity frame)
- $F_L(q)$  — Blatt-Weisskopf barrier factor
- $\Delta(w)$  — amplitude that describes resonance shape of  $X$
- $A_{1,2}$  — decay amplitudes of (unstable) daughter particles 1 and 2

# Partial-Wave Analysis Formalism

## Two-body decay amplitude in canonical formalism

- Decay  $X(w, J, M) \rightarrow 1(J_1, M_1) [L, S] 2(J_2, M_2)$

$$A_X^{\text{can}} = \sqrt{2J+1} \sum_{M_1, M_2} (J_1 M_1 J_2 M_2 | S M_S) \sum_{M_L} (L M_L S M_S | J M) \\ \sqrt{\frac{4\pi}{2L+1}} Y_{M_L}^L(\theta, \phi) F_L(q) \Delta(w) A_1 A_2$$

- $Y_{M_L}^L(\theta, \phi)$  — Spherical harmonic describes rotational property of  $|L M_L\rangle$  state
- $\theta, \phi$  — polar angles of decay daughter 1 in  $X$  rest frame (reached by simple boost, no rotations)
- $F_L(q)$  — Blatt-Weisskopf barrier factor
- $\Delta(w)$  — amplitude that describes resonance shape of  $X$
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# Partial-Wave Analysis Formalism

## Extended maximum-likelihood method

- Likelihood  $\mathcal{L}$  to observe  $N$  events distributed according to  $\sigma(\tau; m_X)$  and acceptance  $\text{Acc}(\tau; m_X)$

$$\mathcal{L} = \underbrace{\left[ \frac{\bar{N}^N}{N!} e^{-\bar{N}} \right]}_{\text{Poisson likelihood}} \prod_{n=1}^N \underbrace{\left[ \frac{\sigma(\tau_n; m_X)}{\int d\tau \sigma(\tau; m_X) \text{Acc}(\tau; m_X)} \right]}_{\text{Likelihood of event } n}$$

$$\text{with } \bar{N} \propto \int d\tau \sigma(\tau; m_X) \text{Acc}(\tau; m_X)$$

$$\mathcal{L} \propto \left[ \frac{\bar{N}^N}{N!} e^{-\bar{N}} \right] \left[ \frac{1}{\bar{N}^N} \prod_{n=1}^N \sigma(\tau_n; m_X) \right]$$

$$\mathcal{L} \propto e^{-\int d\tau \sigma(\tau; m_X) \text{Acc}(\tau; m_X)} \prod_{n=1}^N \sigma(\tau_n; m_X)$$

# Partial-Wave Analysis Formalism

## Extended maximum-likelihood method (cont.)

- Insert **parameterization** of cross section for  $\sigma(\tau_n; m_X)$

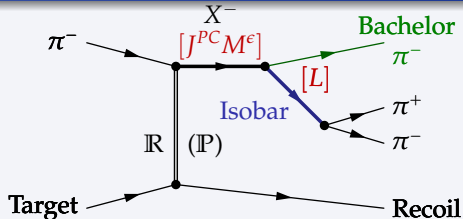
$$\mathcal{L} \propto e^{-\int d\tau \sigma(\tau; m_X) \text{Acc}(\tau; m_X)} \prod_{n=1}^N \sum_{r=1}^{N_r} \left| \sum_{\text{waves}} T_{r,\text{wave}}(m_X) A_{\text{wave}}(\tau_n; m_X) \right|^2$$

- Make expression less unwieldy by **taking logarithm**

$$\ln \mathcal{L} = \sum_{n=1}^N \ln \left[ \sum_{r=1}^{N_r} \left| \sum_{\text{waves}} T_{r,\text{wave}}(m_X) A_{\text{wave}}(\tau_n; m_X) \right|^2 \right] - \underbrace{\int d\tau \sigma(\tau; m_X) \text{Acc}(\tau; m_X)}_{\text{Normalization integral}}$$

- **Normalization integral** estimated using phase space **Monte Carlo**

# Diffractive Dissociation of $\pi^-$ into $\pi^- \pi^+ \pi^-$ Final State



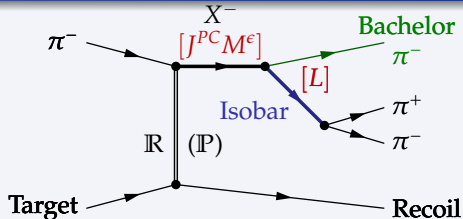
## Partial-wave analysis (PWA)

- Cross section parameterization

$$\sigma(\tau; m_X) \propto \left| \sum_{\text{waves}} T_{\text{wave}}(m_X) A_{\text{wave}}(\tau; m_X) \right|^2$$

- Determination of transition amplitudes  $T_{\text{wave}}(m_X)$ 
  - 1 Bin data in  $m_X$  (neglect  $m_X$  dependence within mass bin)
  - 2 Calculate decay amplitudes  $A_{\text{wave}}(\tau; m_X)$  for every event
  - 3 Extended maximum likelihood fit of  $\tau$  distribution in each mass bin taking into account detector acceptance  $\implies T_{\text{wave}}(m_X)$
- Resonance parameters from  $\chi^2$ -fit of  $T_{\text{wave}}(m_X)$

# Diffractive Dissociation of $\pi^-$ into $\pi^- \pi^+ \pi^-$ Final State



## Partial-wave analysis (PWA)

- Cross section parameterization

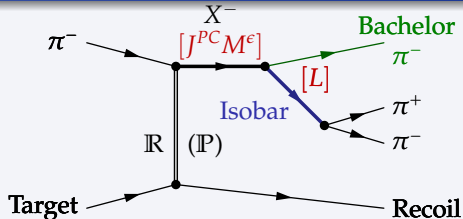
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