

# Hadron spectroscopy at COMPASS: First results on diffractive dissociation



**Frank Nerling**

*Universität Freiburg, Physikalisches Institut*

on behalf of the  
**COMPASS Collaboration**

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Paris, France, 22-28 July 2010*

## Outline:

- **Introduction**
  - The COMPASS experiment
  - PWA method
- **First results on diffractive production** (2008 data)
  - neutral channel:  $3\pi$  final states neutral vs. charged mode  
→ *First PWA fits - main wave*
  - kaonic channels: Kaon diffraction & first glimpse on  $(K\bar{K}\pi)^-$   
→ *Further ongoing analyses*
- **Conclusions & outlook**



bmb+f - Förderschwerpunkt

**COMPASS**

Großgeräte der physikalischen  
Grundlagenforschung



# The COMPASS experiment

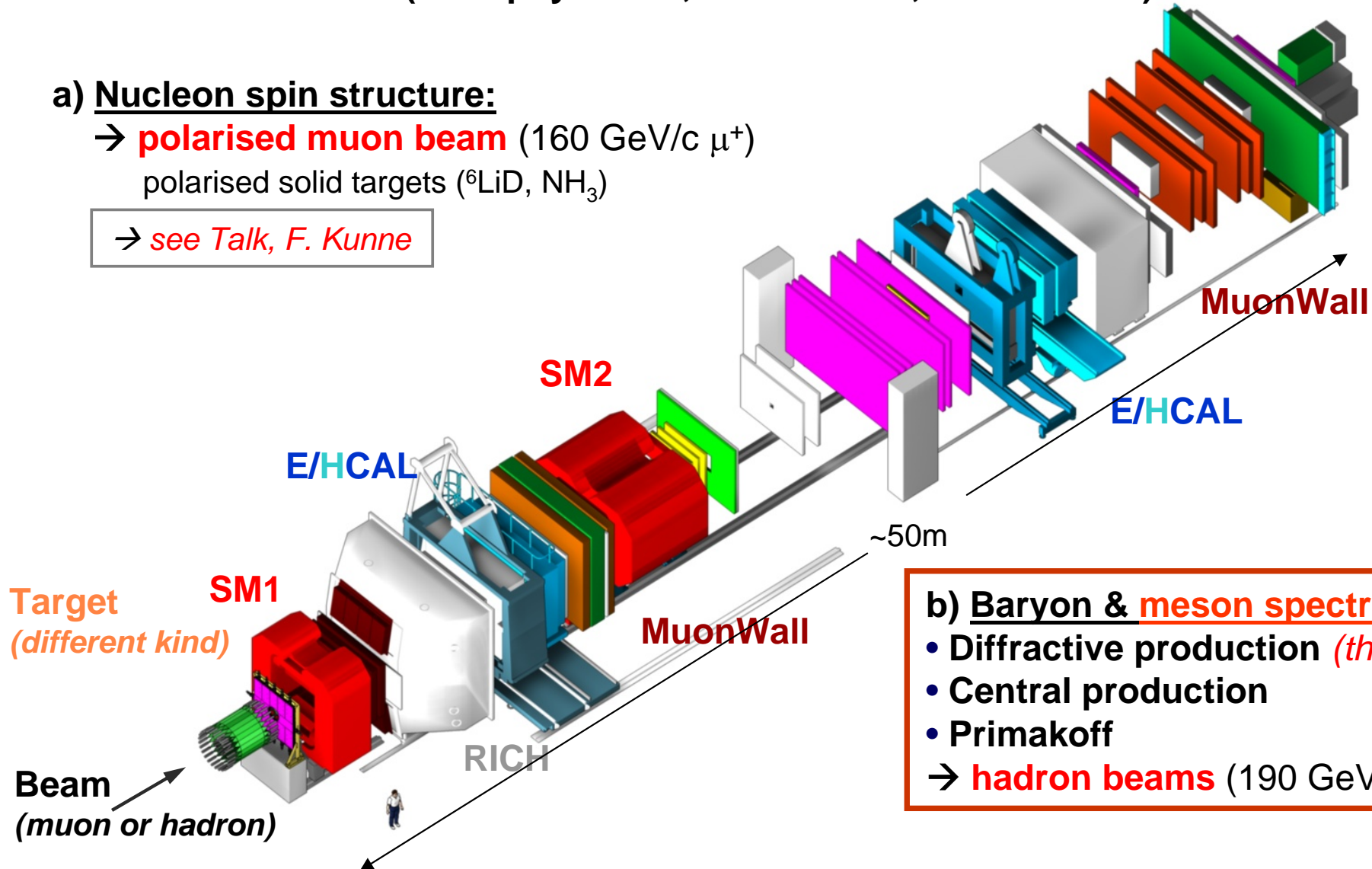


**CO**mmun **Muon** **P**roton **A**pparatus for **S**tructure and **S**pectroscopy  
(~250 physicists, 25 institutes, 10 countries)

## a) Nucleon spin structure:

→ **polarised muon beam** (160 GeV/c  $\mu^+$ )  
polarised solid targets ( ${}^6\text{LiD}$ ,  $\text{NH}_3$ )

→ see Talk, F. Kunne



## b) Baryon & meson spectroscopy:

- Diffractive production (*this talk*)
  - Central production
  - Primakoff
- **hadron beams** (190 GeV/c  $\pi^-$ ,  $K^-$ )

[hep-ex/0703049, NIM A 577, 455 (2007)]  
[NIM A Hadron Set-up 2008/09 under preparation]



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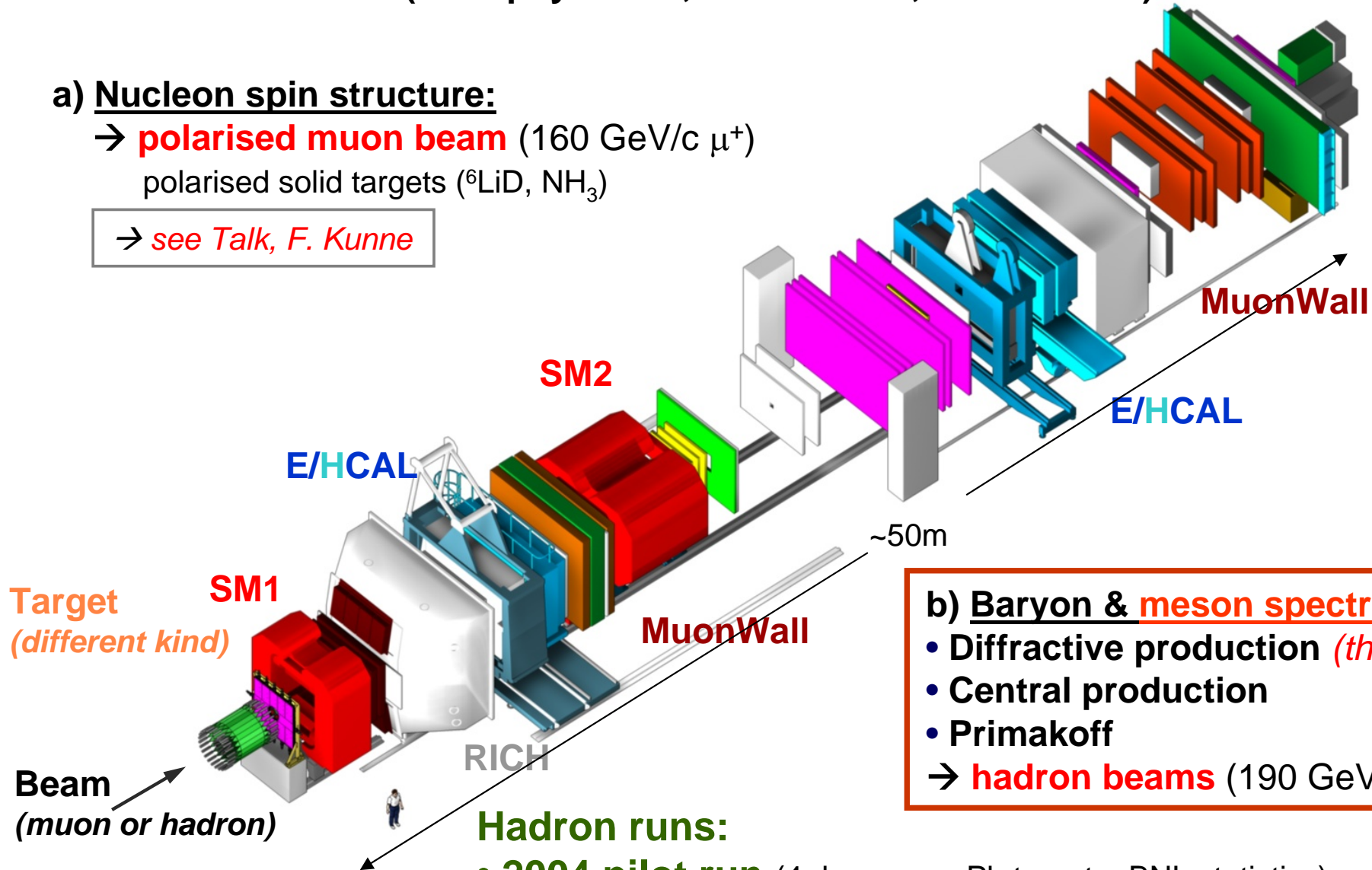


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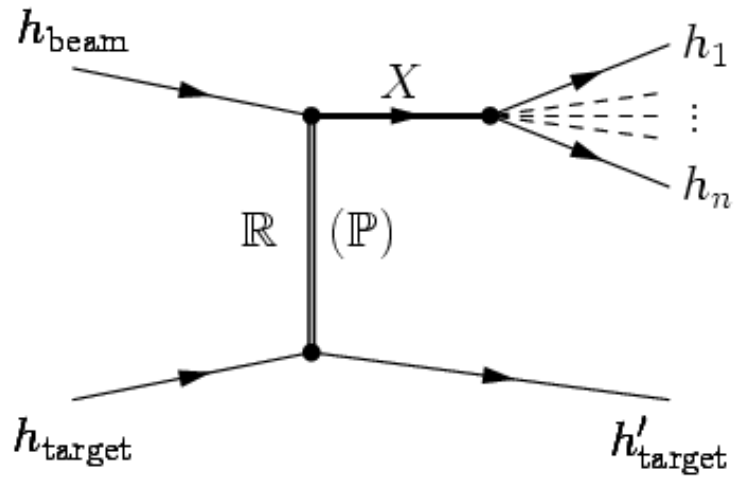
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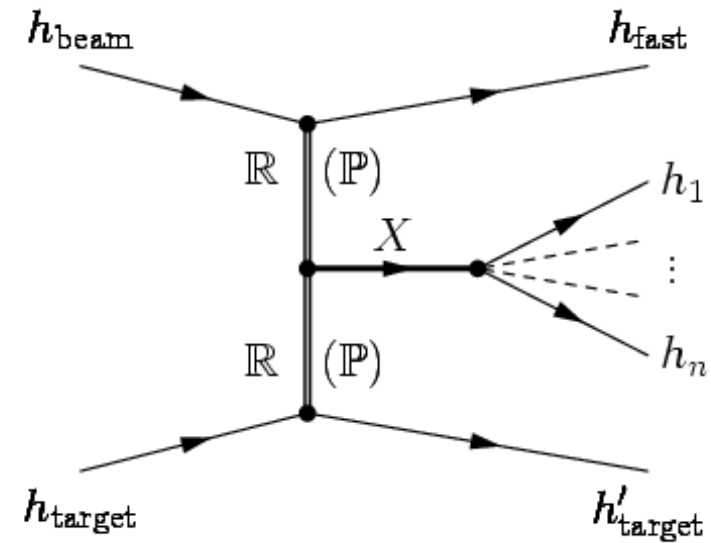
## Hadron runs:

- **2004 pilot run** (4 days  $\pi^-$  on Pb target: ~BNL statistics)
- **2008/09** (10-100x world stats ( $\text{LH}_2$ ),  $\pi^\pm$ ,  $K^\pm$ ,  $p^\pm$  beam, plus nuclear targets)

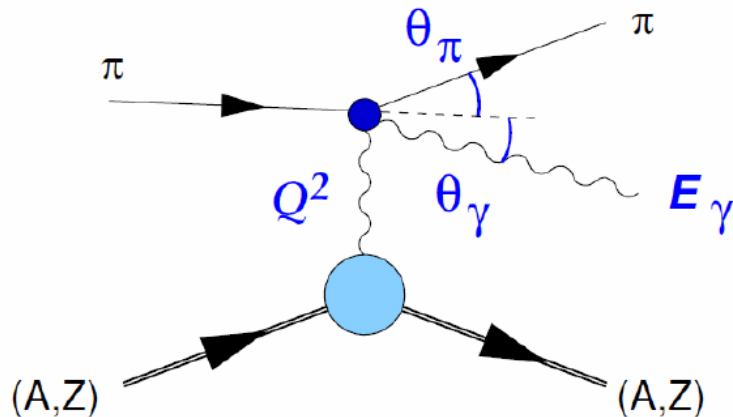
## a) Diffractive production



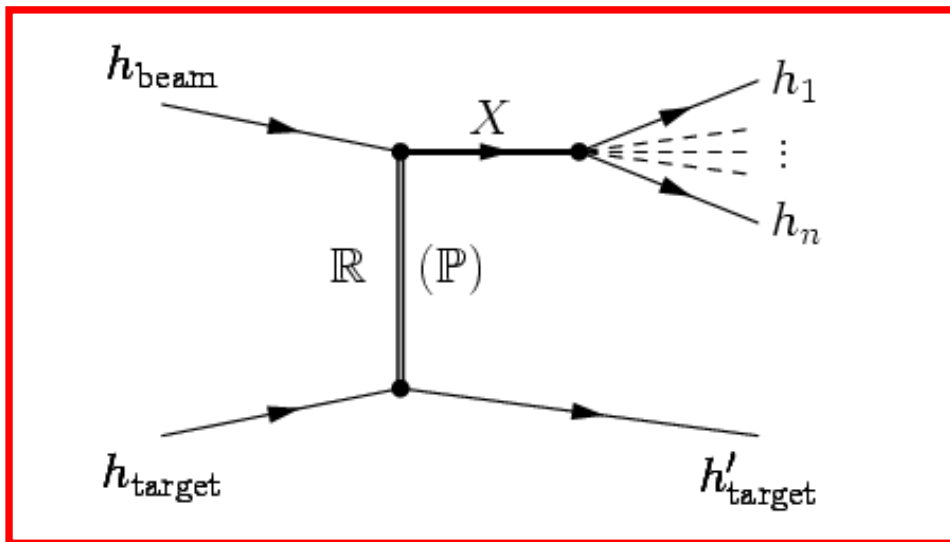
## b) Central production



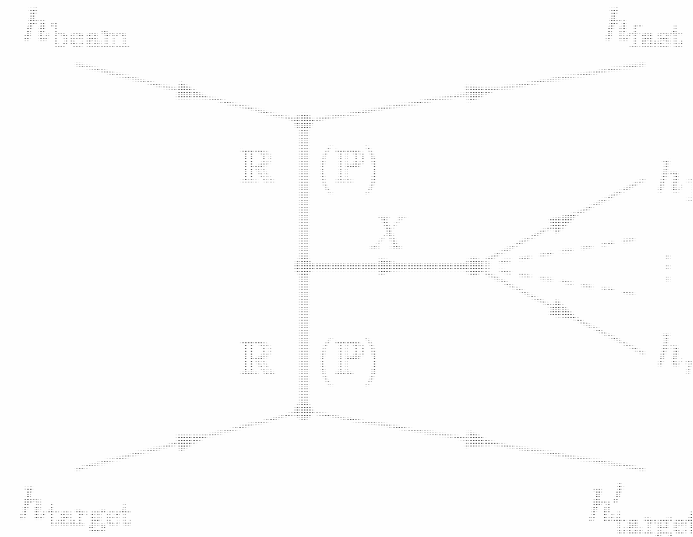
## c) Photo production (Primakoff reactions)



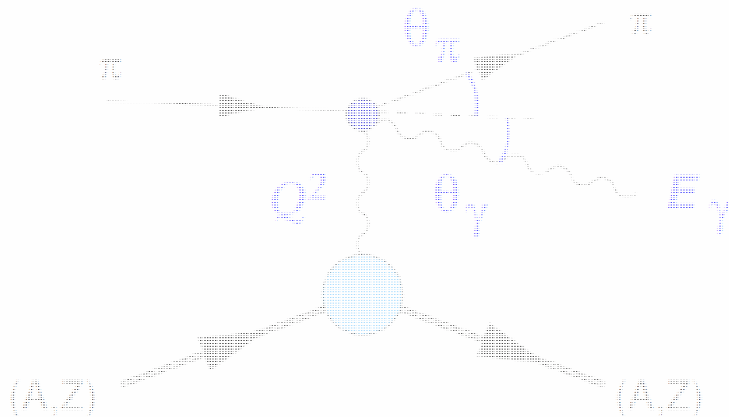
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## b) Central production



## c) Photo production (Primakoff reactions)



Focus on first results from diffractive dissociation



# Mesons and Spin Exotic States



## Constituent quark model

- color neutral  $q\bar{q}$  systems
- Quantum numbers  $I^G J^{PC}$
- $P = (-1)^{L+1}$     $C = (-1)^{L+S}$     $G = (-1)^{L+L+1}$
- $J^{PC}$  multiplets:  $0^{++}$ ,  $0^{-+}$ ,  $1^{--}$ ,  $1^{+-}$ ,  $1^{++}$ ,  $2^{++}$ , ...
- **Forbidden:**  $0^{--}$ ,  $0^{+-}$ ,  $1^{-+}$ ,  $2^{+-}$ ,  $3^{-+}$ , ...

Hybrid candidates (1.3 - 2.2 GeV/c<sup>2</sup>):

lightest hybrid predicted: exotic  $J^{PC} = 1^{-+}$

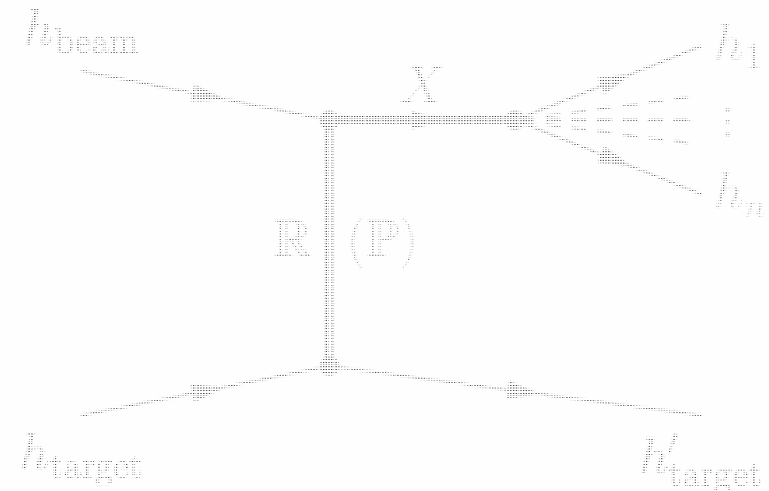
- $\pi_1(1400)$ : VES, E852, Crystal Barrel  $\rightarrow \eta\pi$
- $\pi_1(1600)$ : E852, VES  $\rightarrow \rho\pi, \eta'\pi, f_1\pi, b_1\pi$
- $\pi_1(2000)$ : E852  $\rightarrow f_1(1285)\pi, b_1(1235)\pi$
- ... still controversial  $\rightarrow$  COMPASS

## QCD: meson states beyond

- Glueballs:  $gg, ggg$
- Hybrids:  $q\bar{q}g$
- Tetraquarks:  $(q\bar{q})(q\bar{q})$

## Diffractive scattering

- study of  $J^{PC}$  exotic mesons
- t-channel Reggeon exchange
- forwards kinematics, target stays intact





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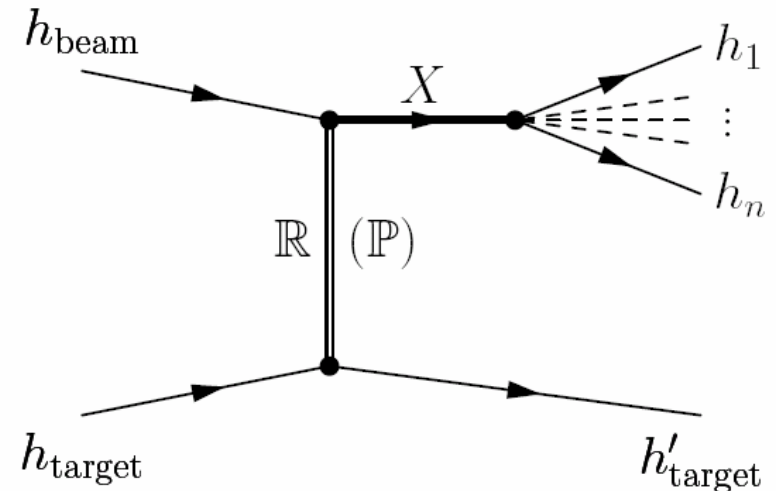
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## Diffractive pion dissociation

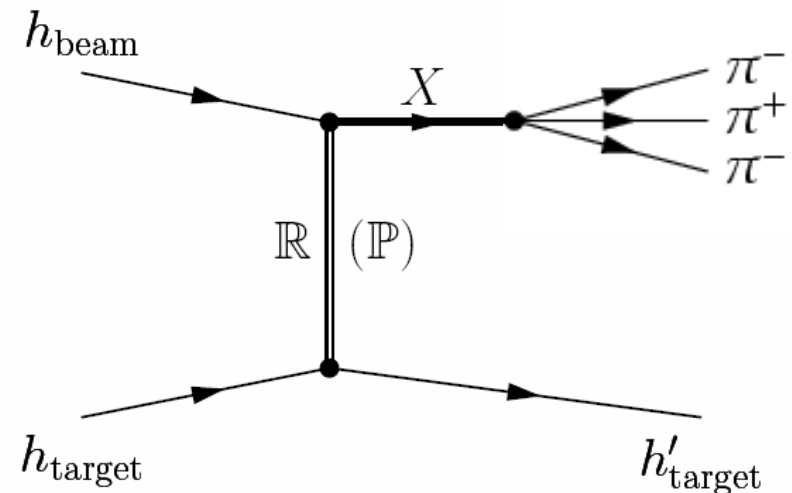
- incoming  $\pi^-$  excited to resonance  $X^-$
- $X^-$  decays into final state, e.g.  $(3\pi)^-$ :  
 $\pi^- \mathbf{p} \rightarrow \pi^- \pi^+ \pi^- \mathbf{p}$  (charged mode)
- **small** momentum transfer

## QCD: meson states beyond:

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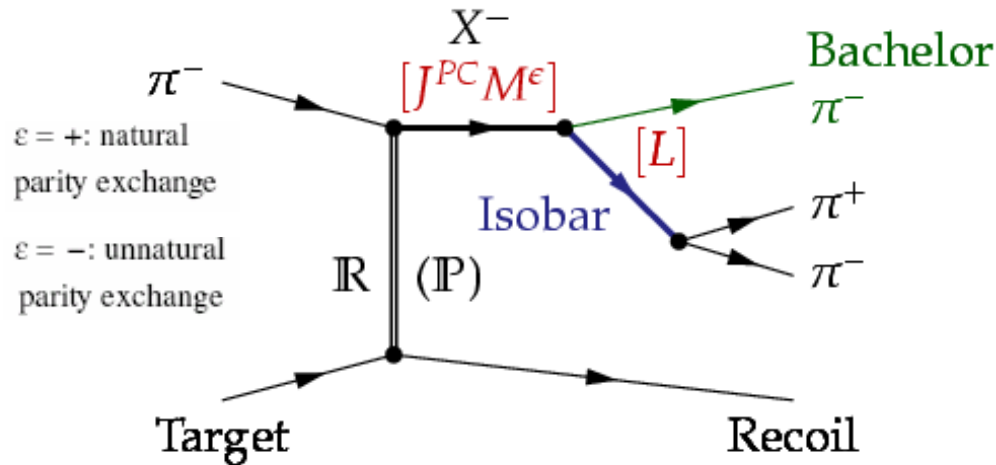
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# PWA using isobar model



**X<sup>-</sup> decay described using isobar model:**

- Intermediate di-pion resonance (isobar)
  - *Spin S* and rel. *orbital angular momentum L* w.r.t *bachelor π*
  - *L+S* couple to *J*
- Partial waves (reflectivity basis):  $J^{PC} M^\epsilon$  [isobar] L

## Partial wave analysis:

- **program:** Illinois/Protvino/Munich (D.Ryabchikov) software (IHEP/VES, TUM/COMPASS)
- **Isobars:**  $(\pi\pi)_S$  [broad  $f_0(600)+f_0(1370)$ ],  $f_0(980)$ ,  $\rho(770)$ ,  $f_2(1270)$ ,  $\rho_3(1690)$
- **Acceptance:** corrections included (2004: ~60%, rather flat)

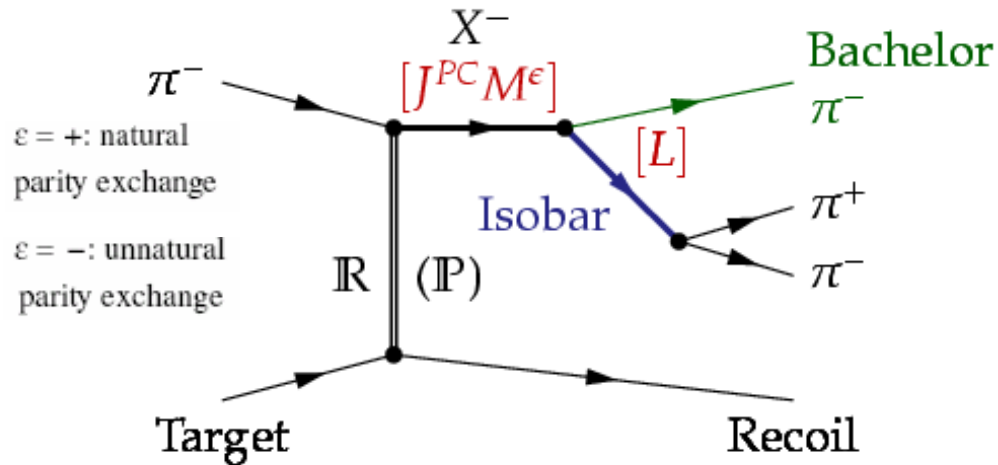
## Step 1) Mass independent PWA: (40MeV/c<sup>2</sup> bins, 41+1 partial waves)

$$\sigma_{indep}(\tau, m, t') = \sum_{\epsilon=\pm 1} \sum_{r=1}^{N_r} \left| \sum_i T_{ir}^\epsilon f_i^\epsilon(t') \psi_i^\epsilon(\tau, m) / \sqrt{\int |\psi_i^\epsilon(\tau', m)|^2 d\tau'} \right|^2$$

- Production amplitudes  $T_{ir}^\epsilon \rightarrow$  extended maximum likelihood fit
- Decay amplitudes  $\psi_i^\epsilon(\tau, m)$  (Zemach tensors, D functions)



# PWA using isobar model



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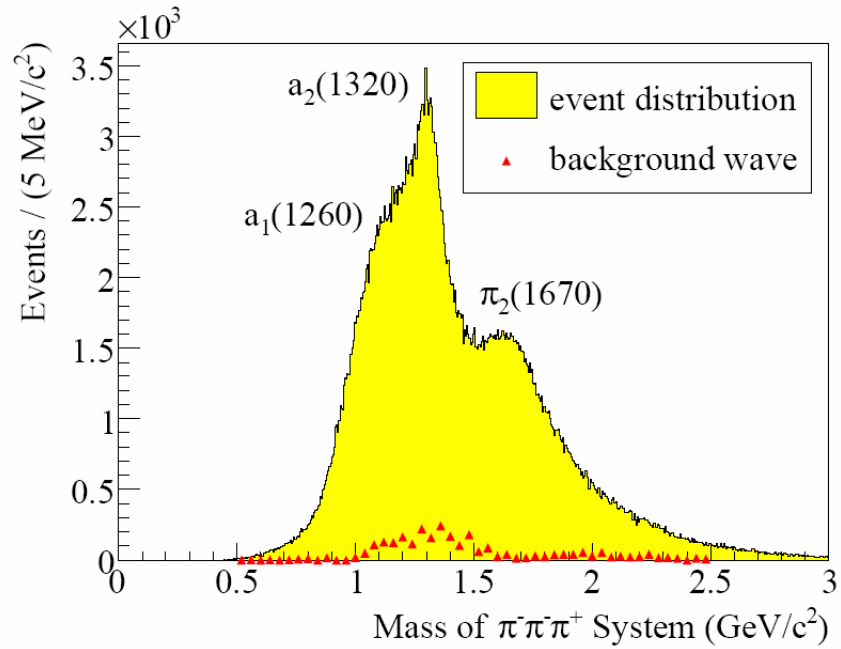
## Step 1) Mass independent PWA: (40MeV/c<sup>2</sup> bins, 41+1 partial waves)

## Step 2) Mass dependent $\chi^2$ fit: (to mass independent result)

- **6 main partial waves chosen**, parameterised by Breit-Wigner
- **Coherent background** for some waves

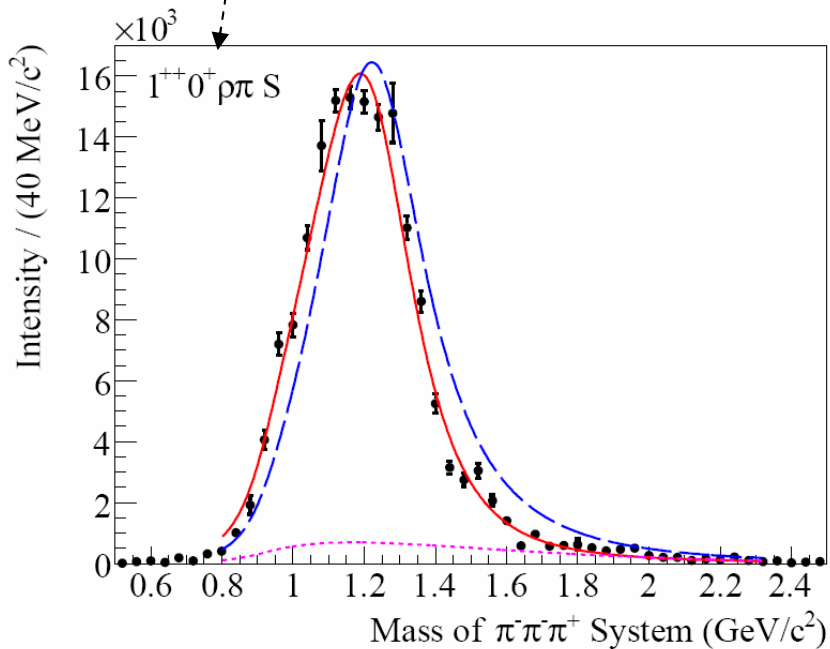
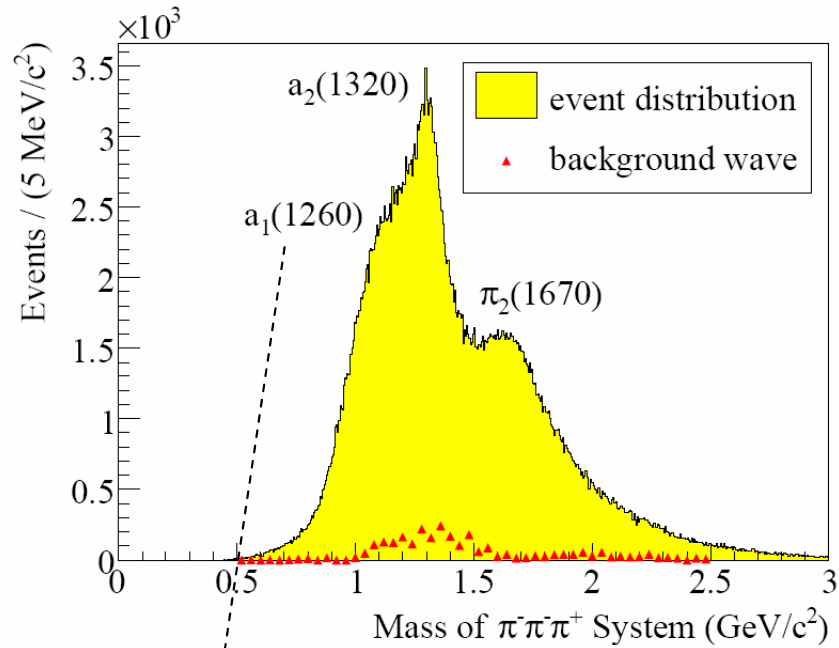


# Diffraction dissociation into $3\pi$ final states (2004 data, Pb target) [PRL 104 (2010) 241803]



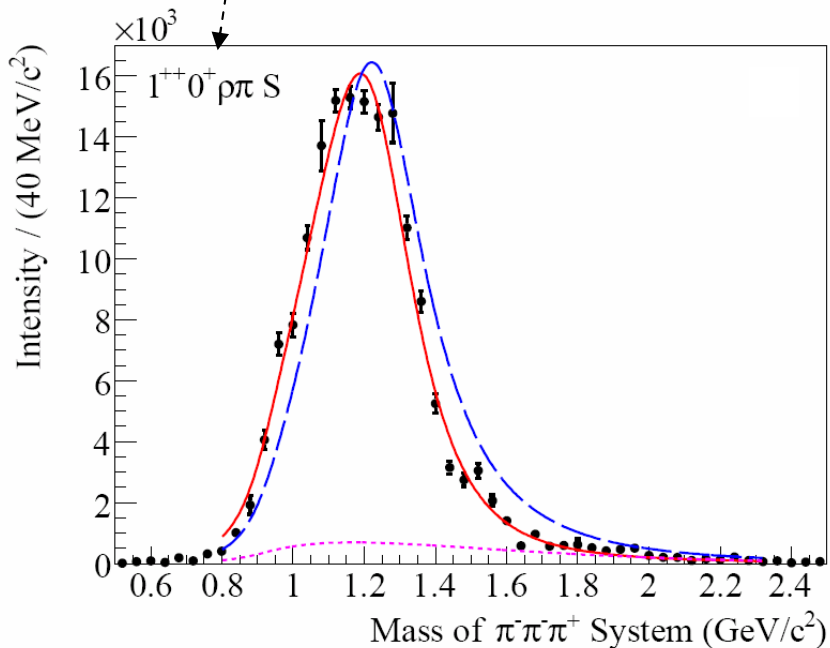
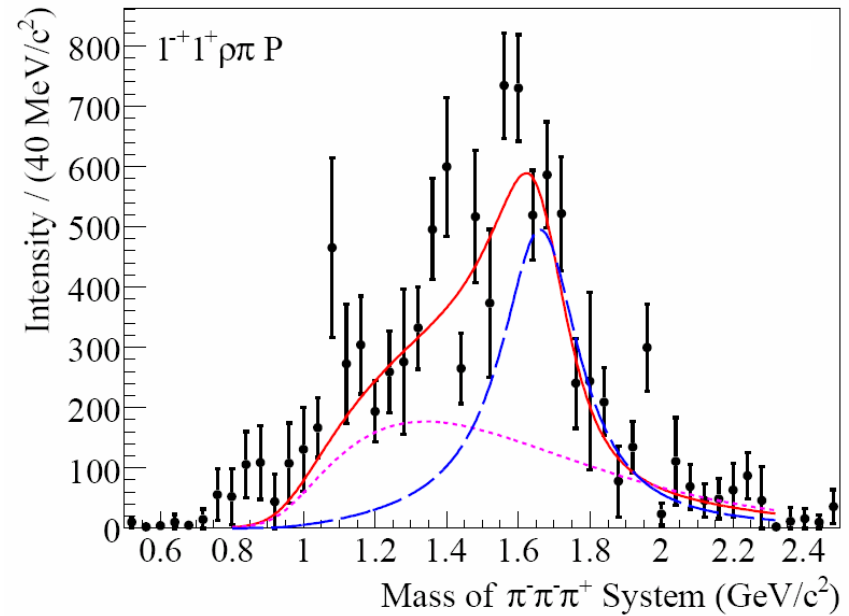
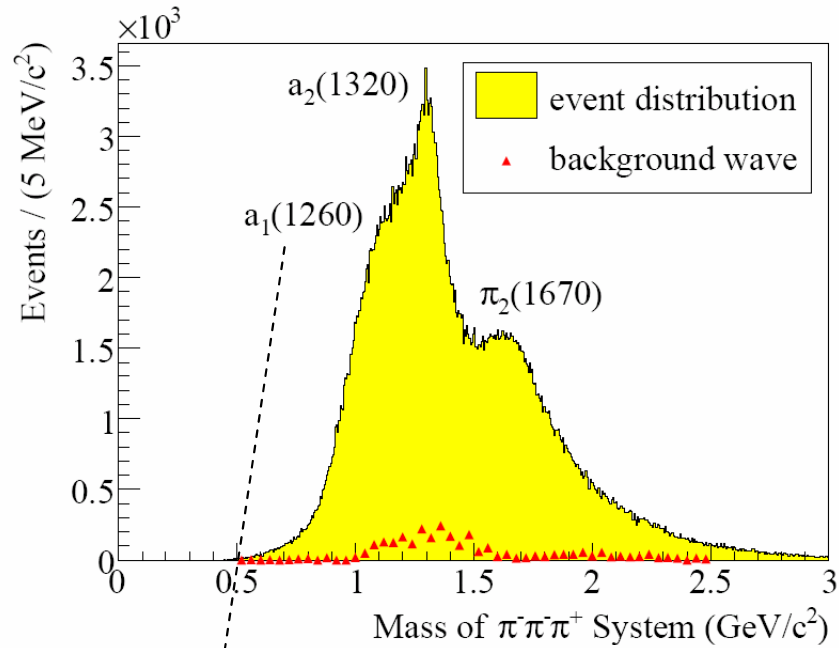


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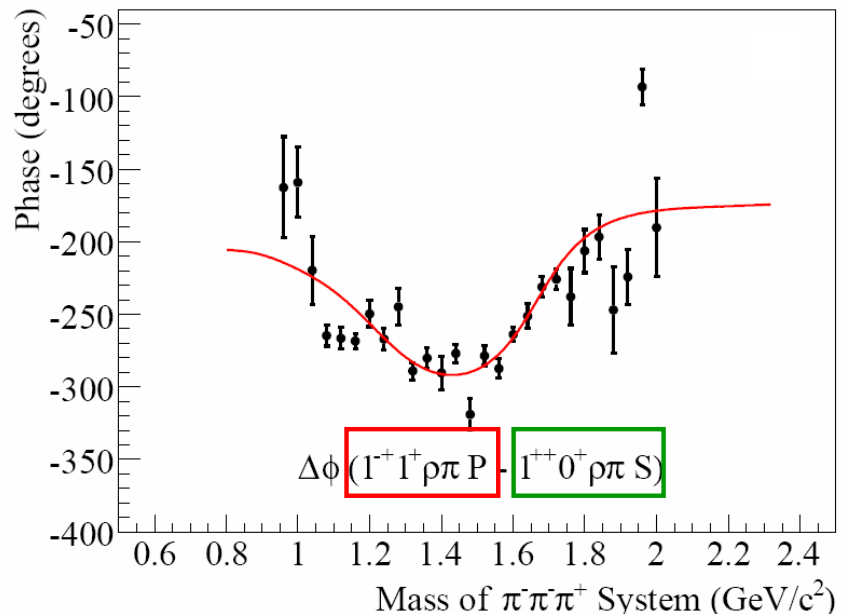
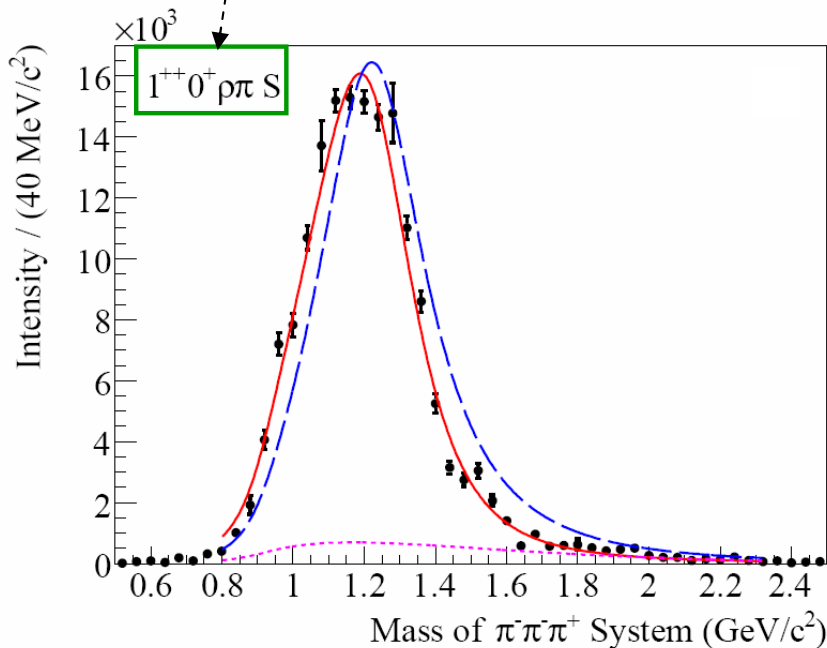
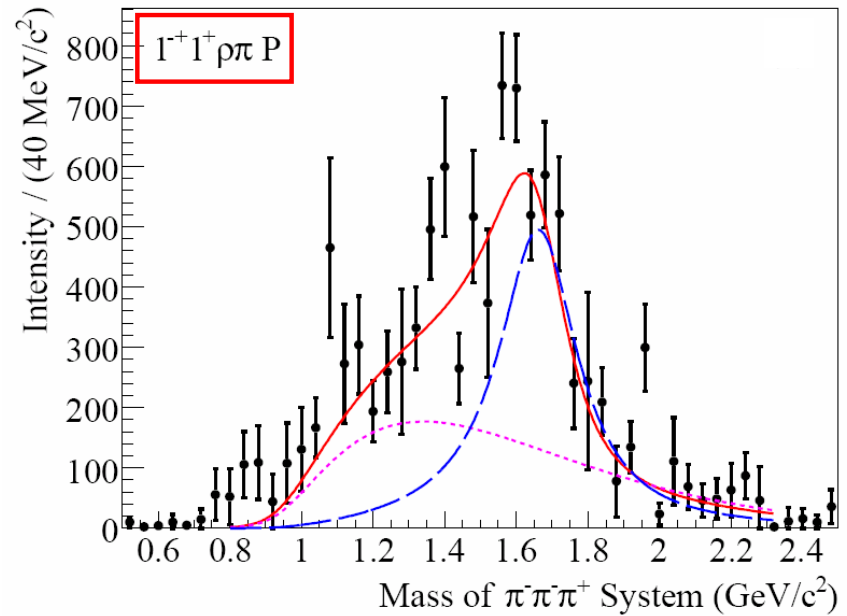
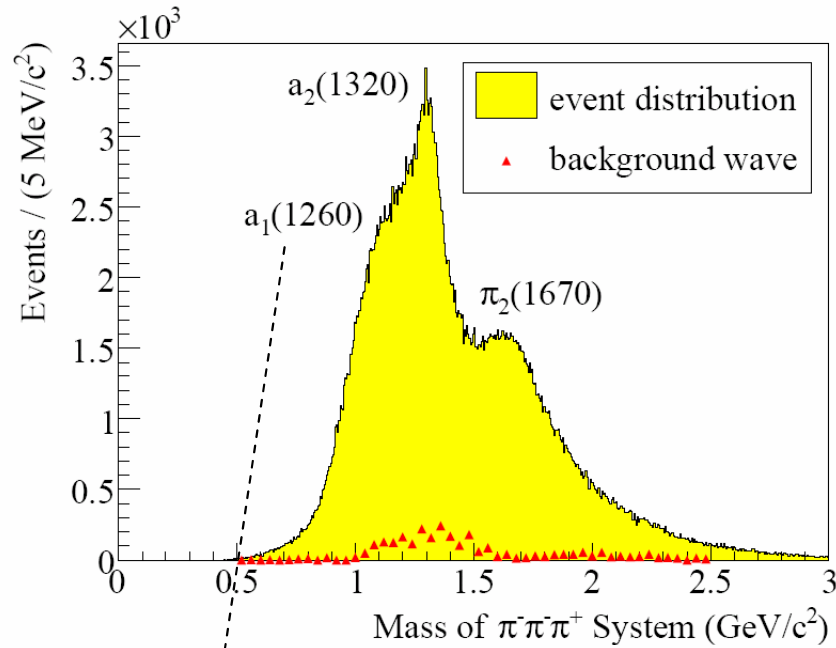


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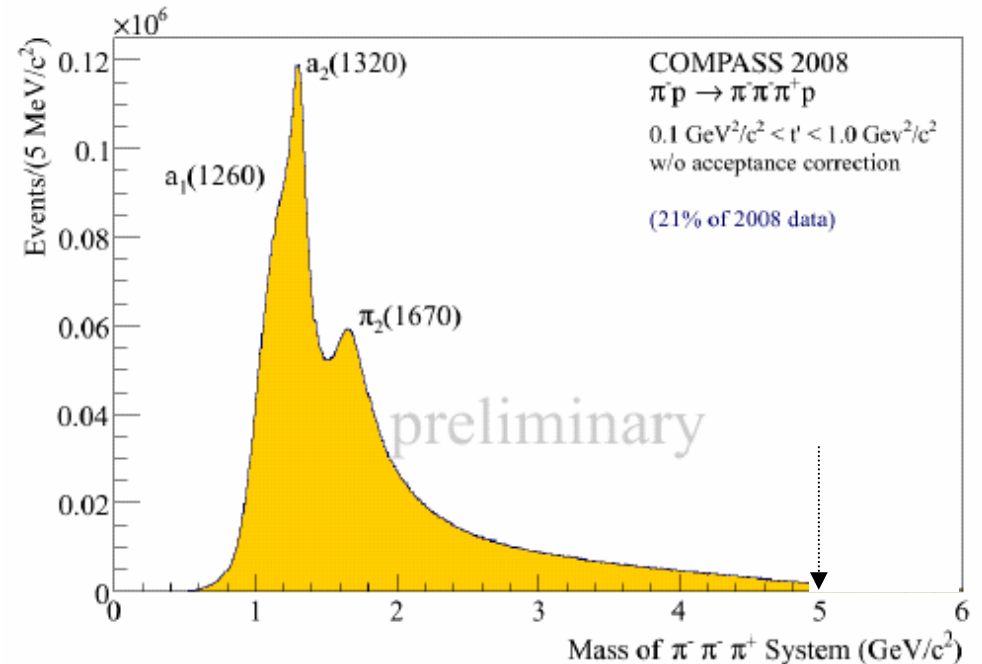
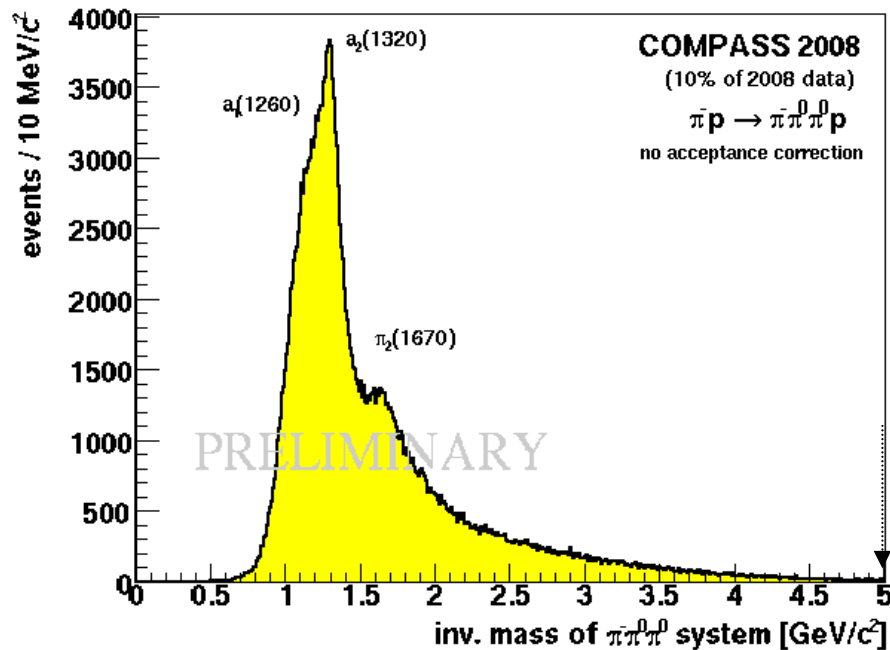


# Diffraction dissociation into $3\pi$ final states (2008 data, $\text{LH}_2$ target)



Mass of outgoing  $3\pi$  system – **neutral**  
**mode:**  $\pi^- p \rightarrow \pi^- \pi^0 \pi^0 p$

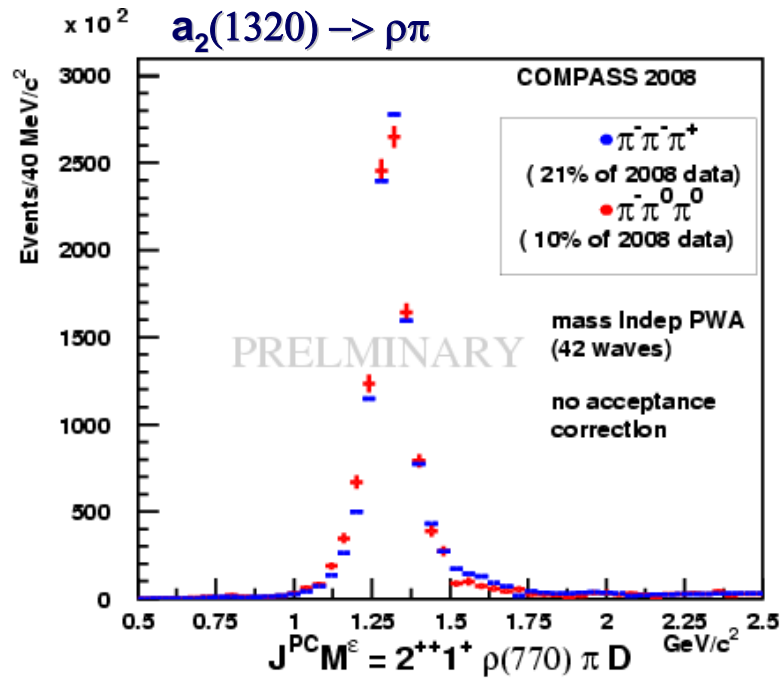
Mass of outgoing  $3\pi$  system – **charged**  
**mode:**  $\pi^- p \rightarrow \pi^- \pi^+ \pi^- p$





# First comparison: Neutral vs. charged mode

## simple isospin symmetry check



$a_2(1320)$  used as a standard candle for normalisation

**Isospin symmetry:** neutral / charged mode

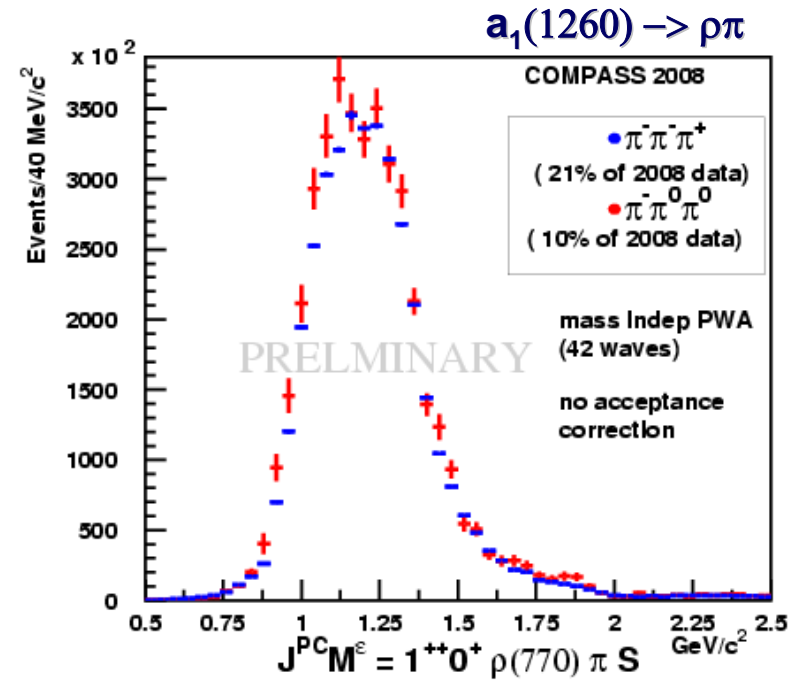
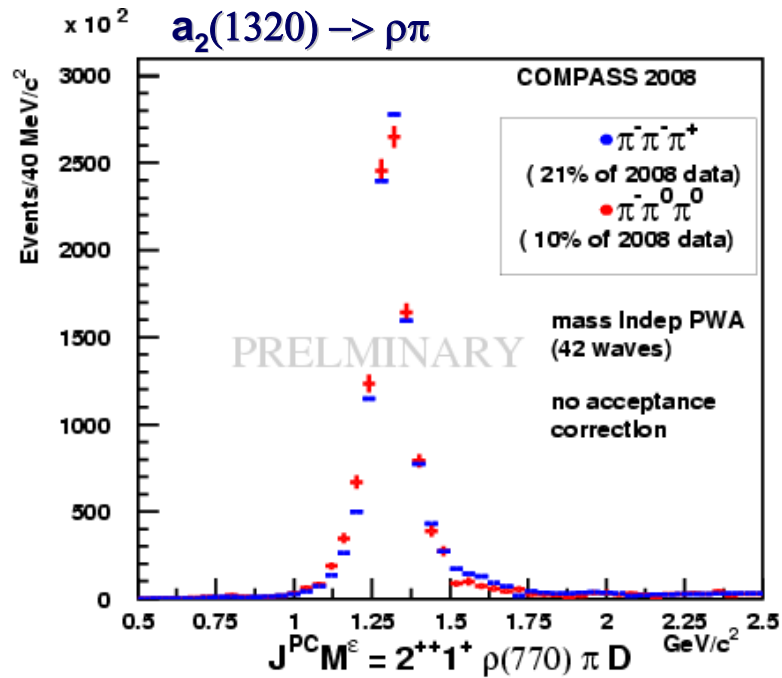
- isobar decaying into  $f_2 \pi$ : 1/2 intensity expected
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# First comparison: Neutral vs. charged mode

## simple isospin symmetry check



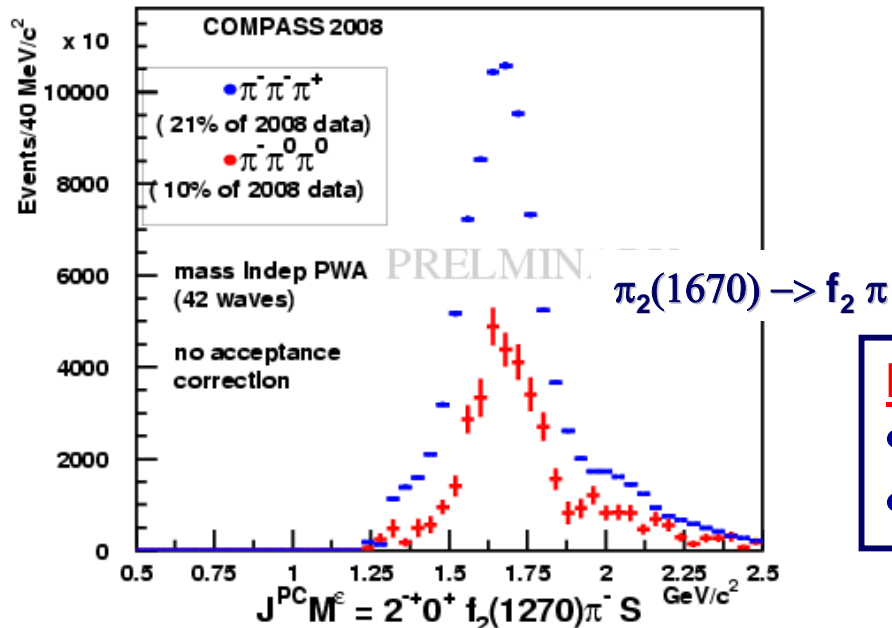
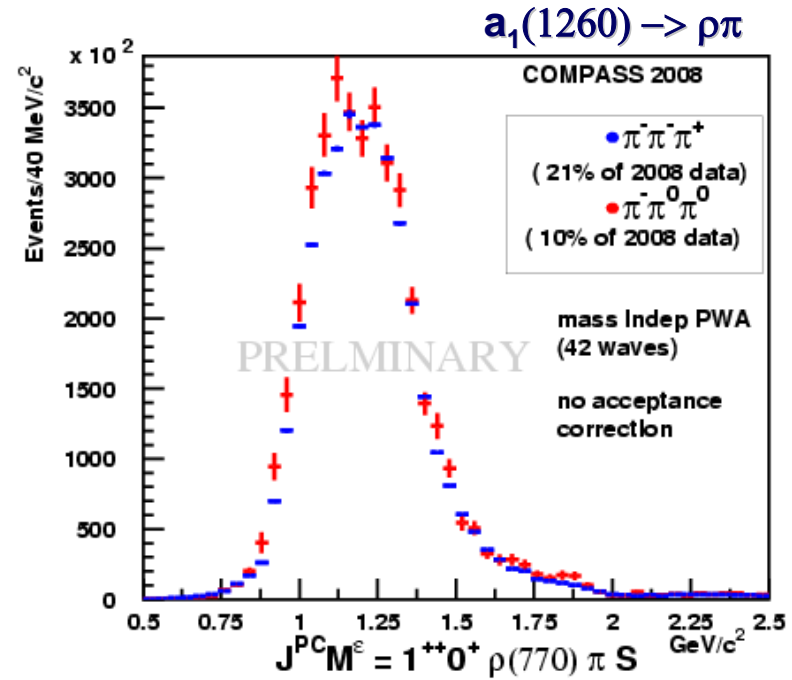
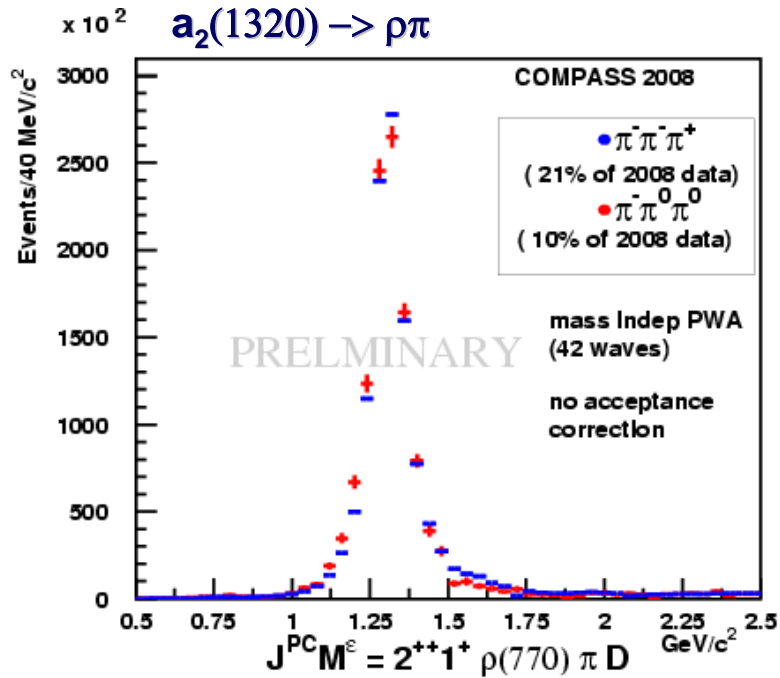
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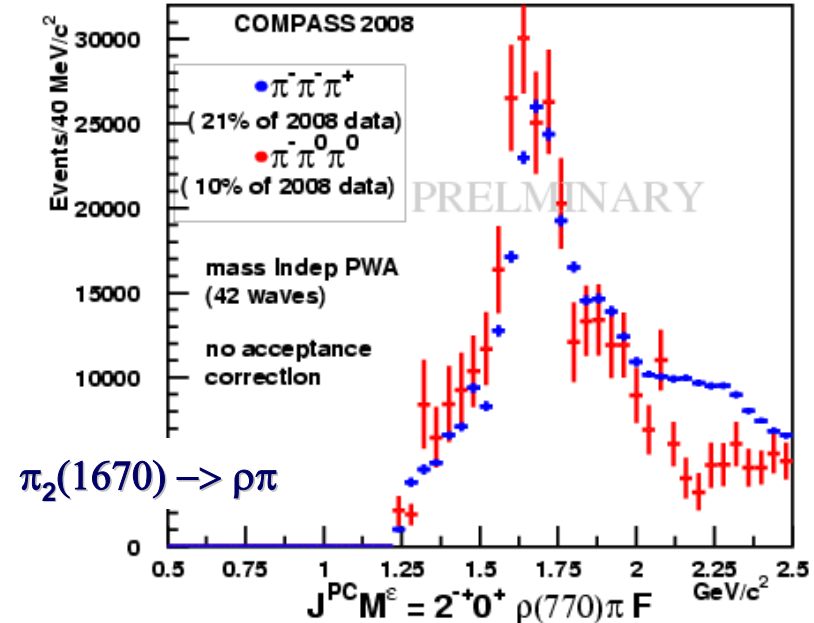
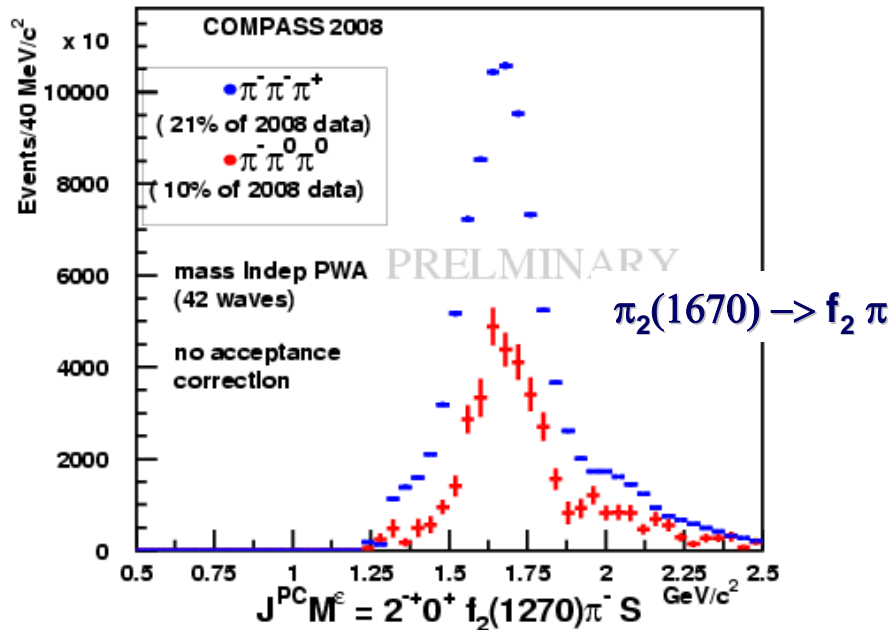
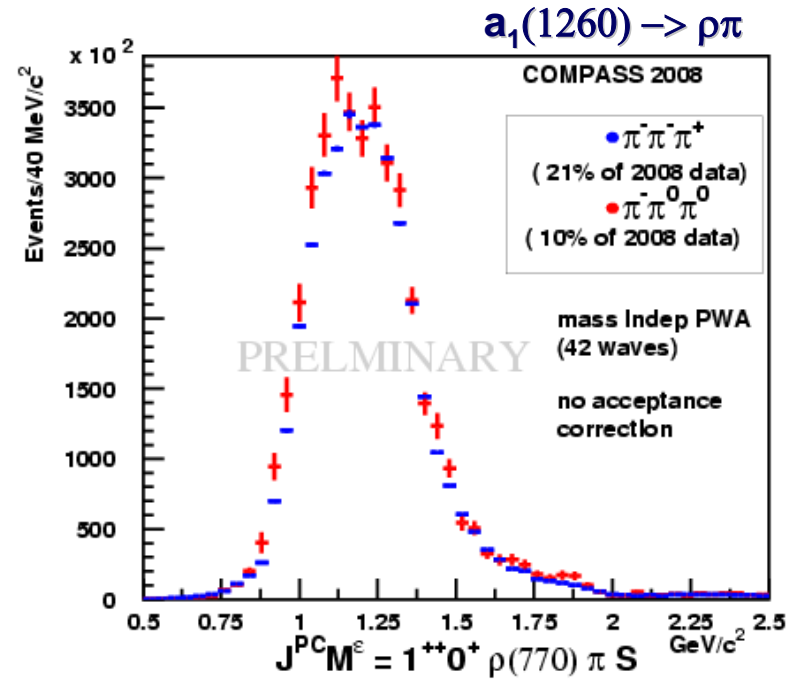
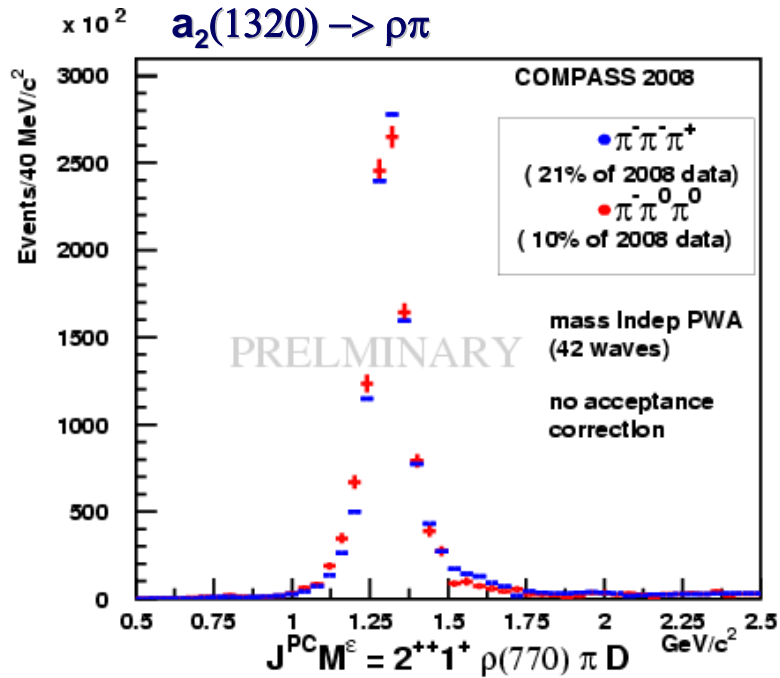
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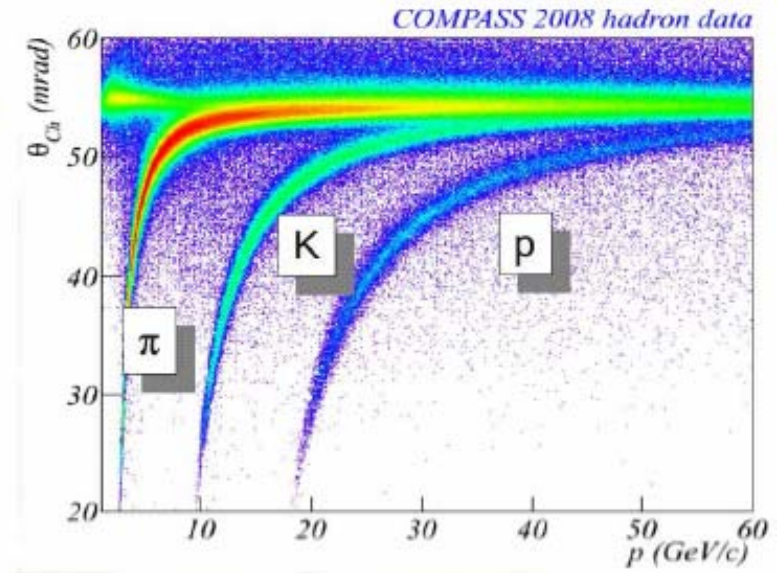
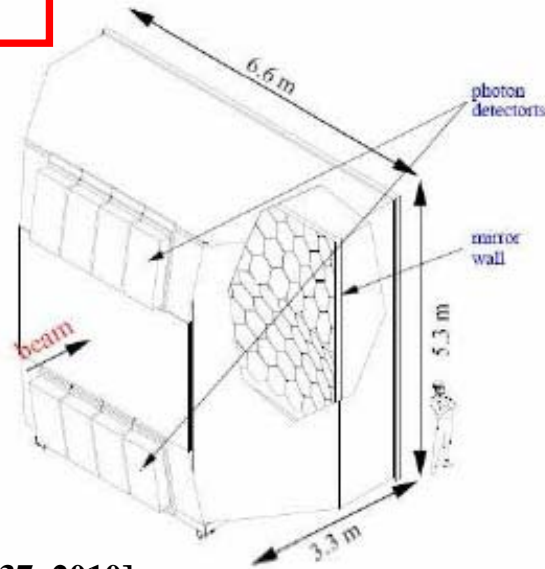
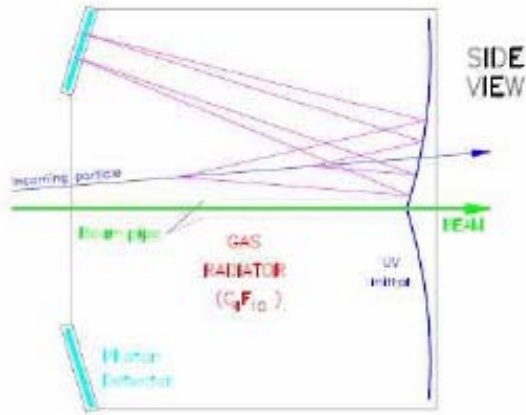




# Studies of final states with (hidden) strangeness -- Kaonic channels

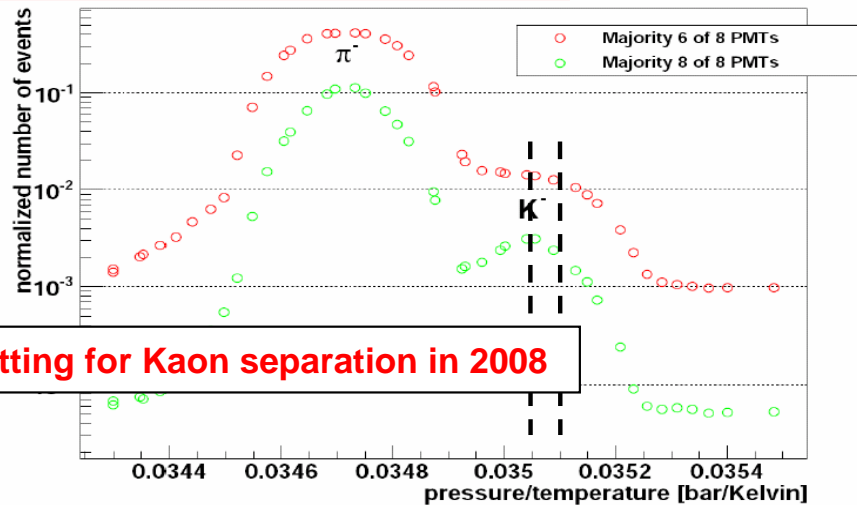


## RICH: Final state particle ID



Major upgrade in 2006:  
[NIM-A587:371-387, 2008], [NIM-A616:21-37, 2010]

## CEDARs: Beam particle ID



Setting for Kaon separation in 2008

- Motivation:**
- Search for  $J^{PC}$  spin exotic mesons decaying via  $K\bar{K}\pi$
  - Study glueball candidates predicted to decay into  $K\bar{K}$
  - New insights on kaonic spectrum in general

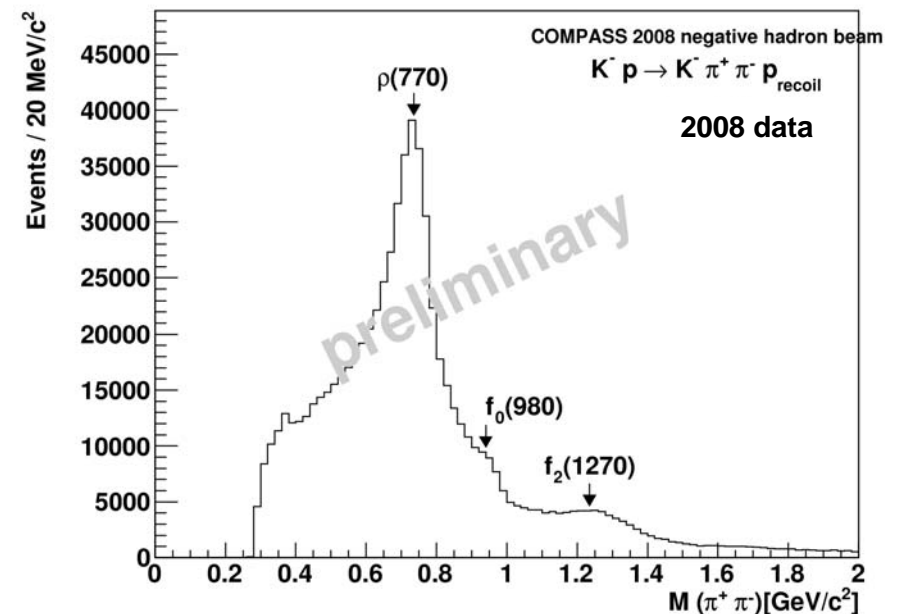
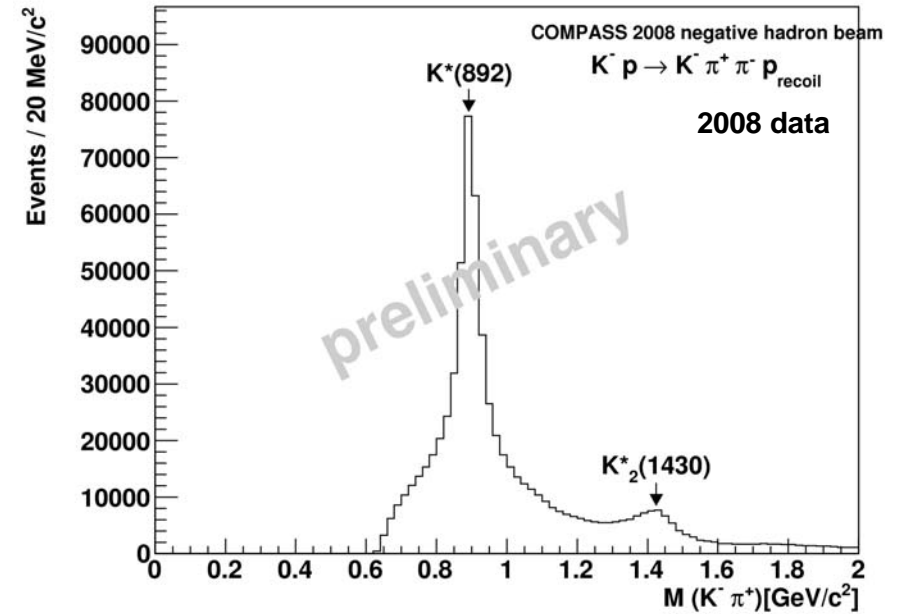
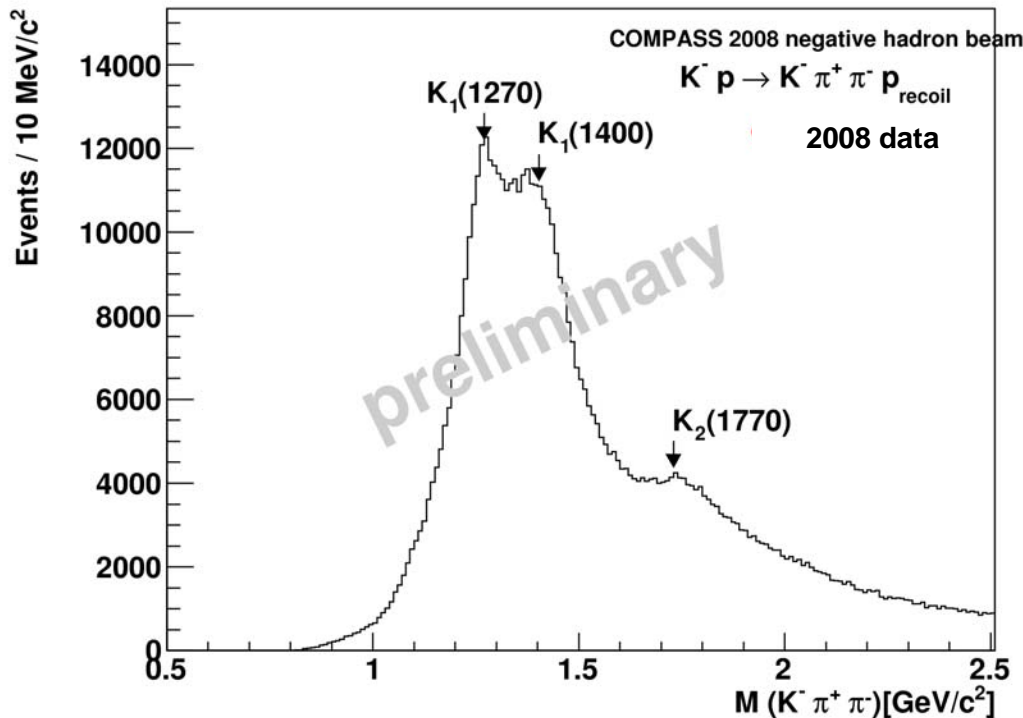


# Kaonic channels I: Kaon diffraction



## Main issues of selection:

- Beam kaon tagged by CEDARs
- Final state kaon identified by RICH
- ~600 k events on tape (2008 data only)  
→ to be compared with ~200 k events WA3





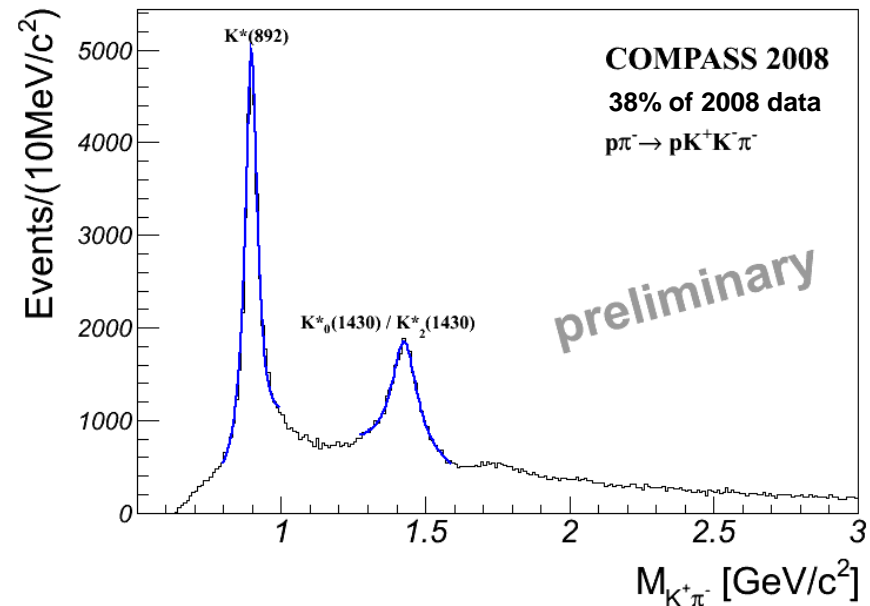
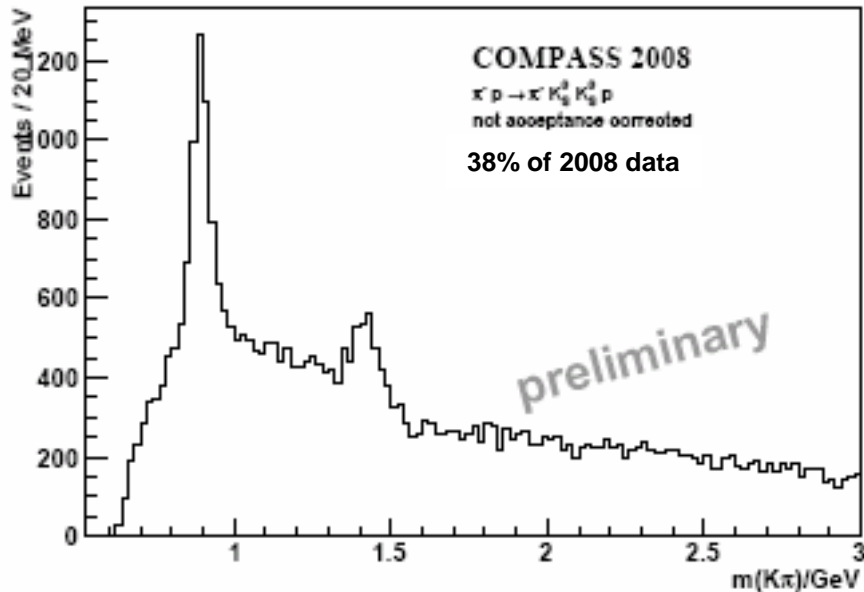
# Kaonic channels II: $(K\bar{K}\pi)^-$

$\pi^- p \rightarrow \pi^- K_s K_s p$  vs.  $\pi^- p \rightarrow \pi^- K^+ K^- p$



## Main issues of selection:

- Beam kaon anti-tagged by CEDARs
- Final state kaon identified by V0 vertex ( $K_s K_s$ ) or by RICH ( $K^+ K^-$ )



- Combinatorics in  $K_s\pi^-$  case
- Resonances:  $K^*(892)$ ,  $K_2^*(1430)$ ,  $K_3^*(1780)$ , also probably  $K_4^*(2045)$



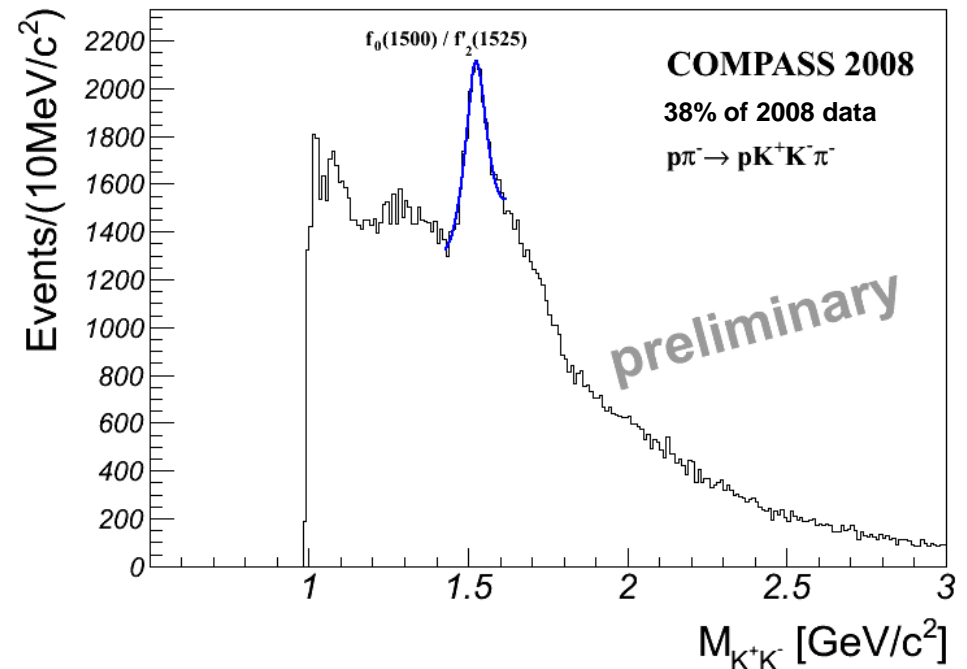
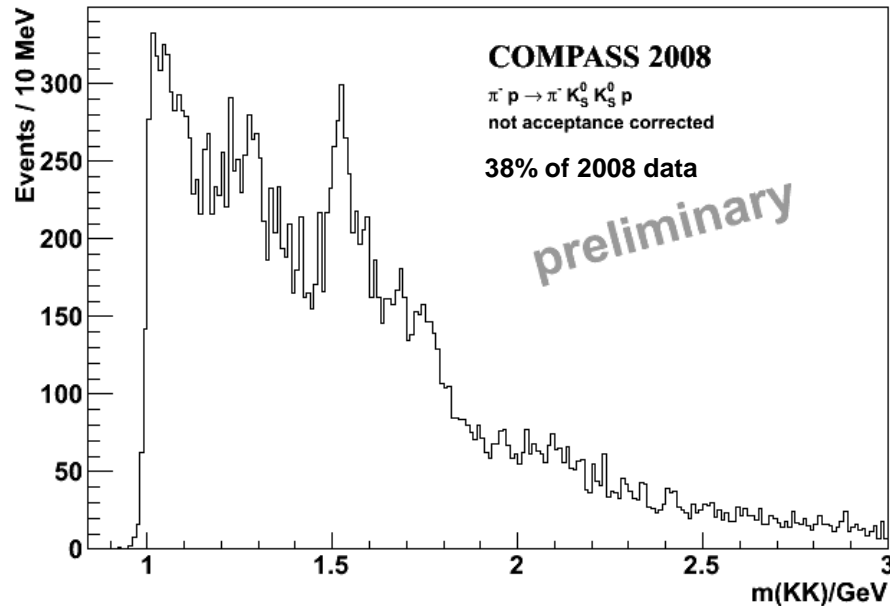
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- Difference near threshold  $\rightarrow$  momentum cut due to RICH
- known resonances seen as expected



# Summary & conclusions



- **COMPASS: high potential for Hadron Spectroscopy**
  - ✓ 2004 data: **Observed exotic  $J^{PC} \rightarrow \pi_1(1600)$**  [PRL 104 (2010) 241803]
  - ✓ 2008/09: **Data taken with hadron beams on proton & nuclear targets**
  - ✓ **Very high statistics** (10-100x world statistics)
- **COMPASS measures Neutral & Charged channels**
  - ✓ **First results on  $3\pi$  final state 2008 data** (diffr. dissociation)
    - *First comparison neutral vs. charged mode* (PWA: isospin symmetry)
    - => independent confirmation of new states within same experiment
  - => **Ongoing:** Ecal calibration (*for full usage of hardware upgrades*)
- **COMPASS measures kaonic final states** (diffr. dissociation)
  - ✓ **Kaon diffraction:** First look into  $K^-\pi^+\pi^-$  **promising** (2008 only: 3x WA32))
  - ✓  $(KK\pi)^-$ : First comparison  $K_S K_S \pi^-$  **vs.**  $K^- K^+ \pi^-$  (PWA underway)

**Further:**

  - $(KK\pi)^0$ : in  $K_S K^{+/-} \pi^{-/+} \pi^-$  final states (PWA started, higher masses (> 2.2 GeV)
    - PWA of  $\underline{f_1(1285)\pi}$  &  $\underline{f_1(1420)\pi}$  (→ never done before)

=> **more results soon**





# Conclusions & outlook

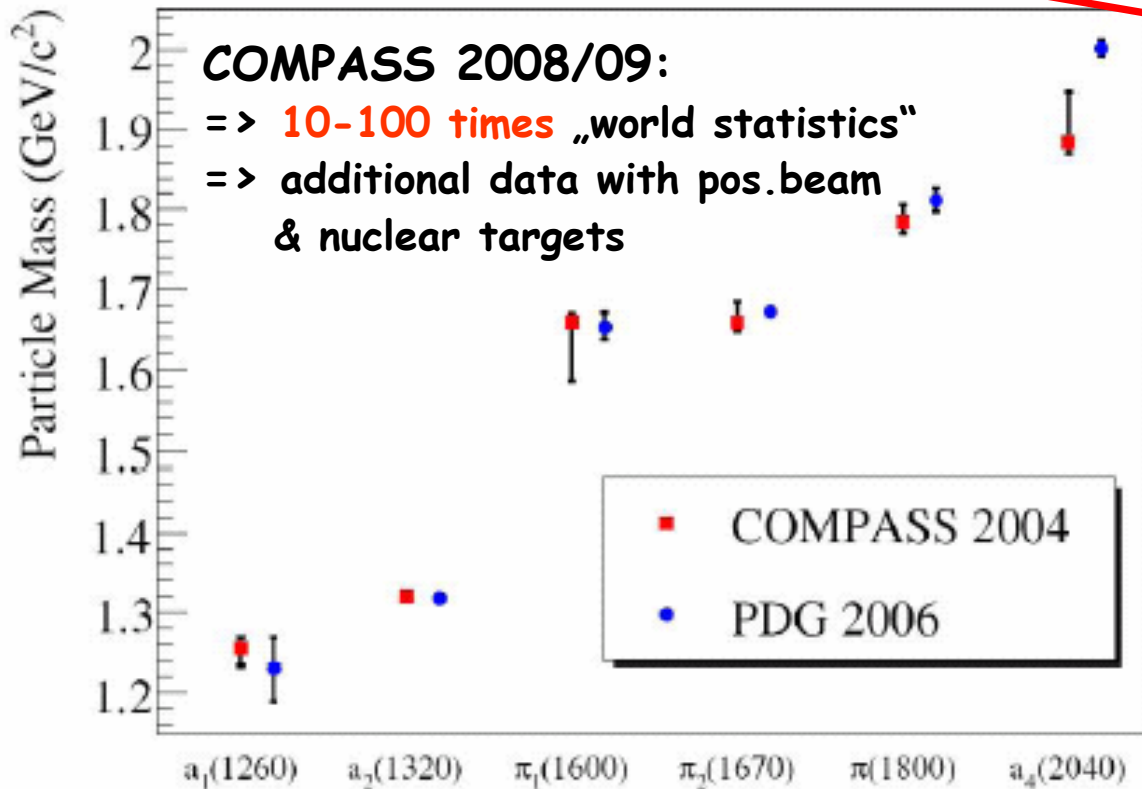
## Further

- **Central production** (*charged & neutral, pionic & kaonic*),
  - **Baryon spectroscopy** (*charged & neutral, pionic & kaonic*),
  - **Primakoff & low  $t'$**  => transition region of production mechanisms
- .... different targets also  
    → detailed study of different production mechanisms



# Conclusions & outlook

**Stay tuned for interesting COMPASS results ...**



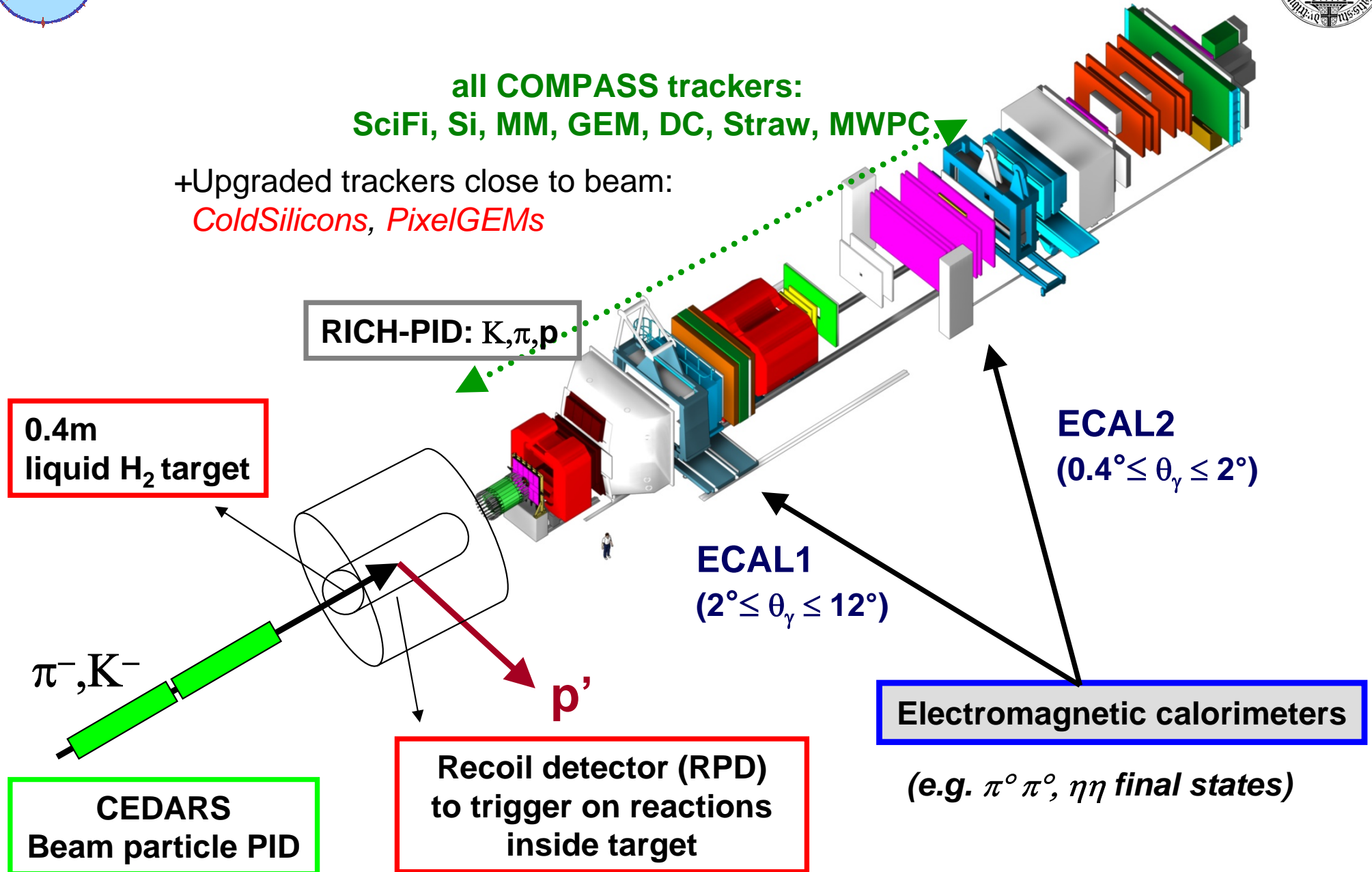
**THANK YOU !!!**



# Backup



# COMPASS spectrometer: Hadron setup 2008/09



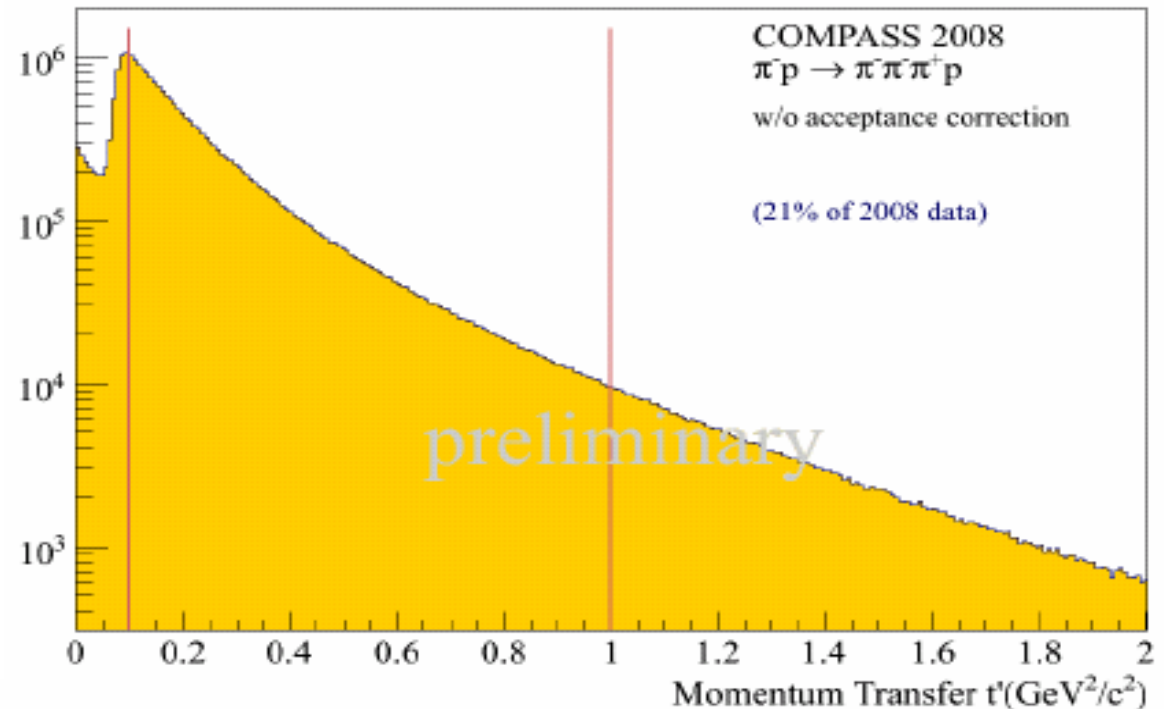
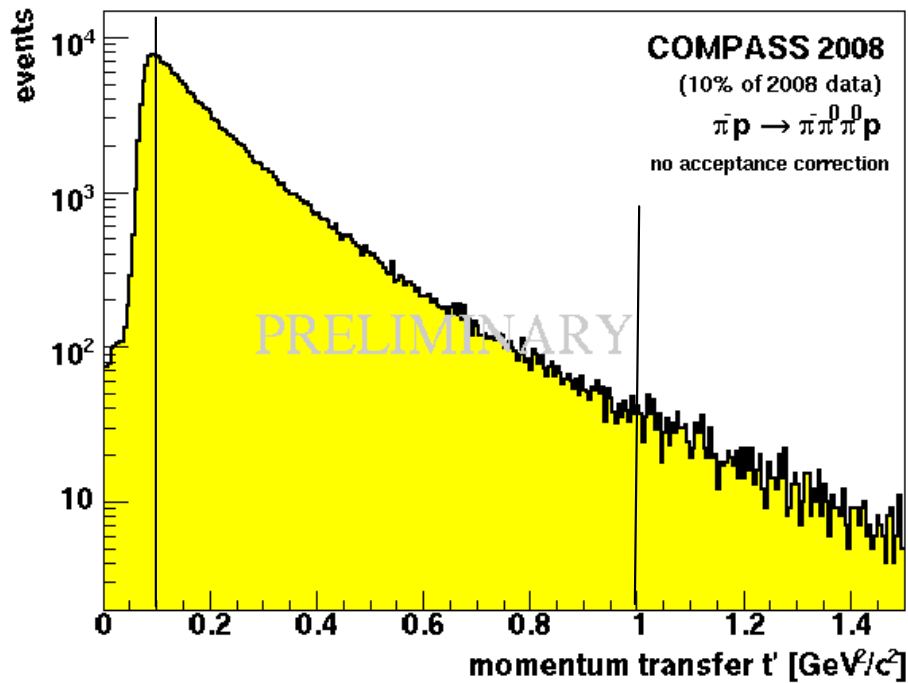


# Diffractive dissociation into $3\pi$ final states (2008 data, $\text{LH}_2$ target)



neutral mode

charged mode



$$t = (p_{\text{beam}} - p_X)^2 < 0$$

$$t' \equiv |t| - |t|_{\text{min}} > 0$$

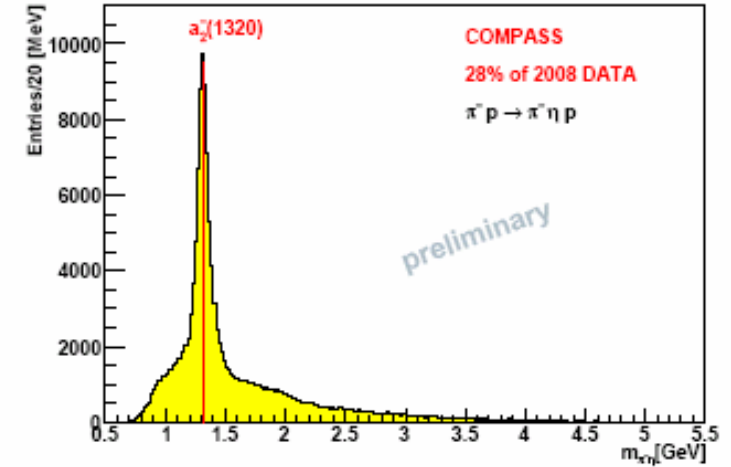
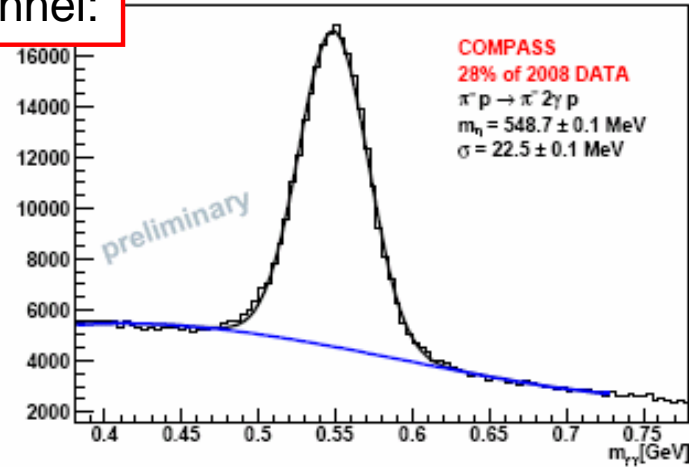


# Further neutral channels (PWA ongoing):

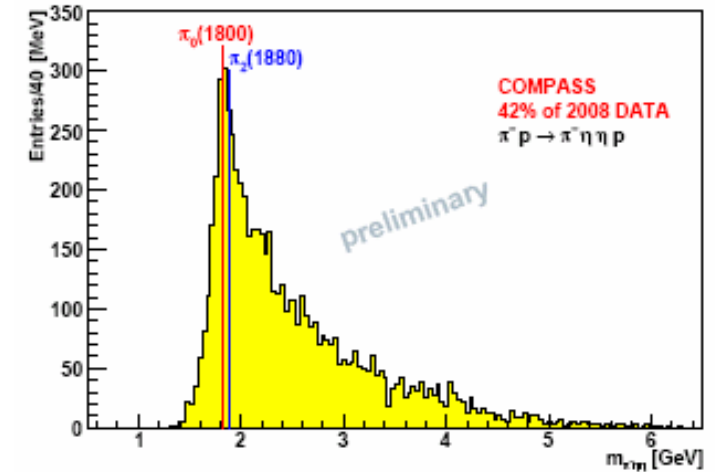
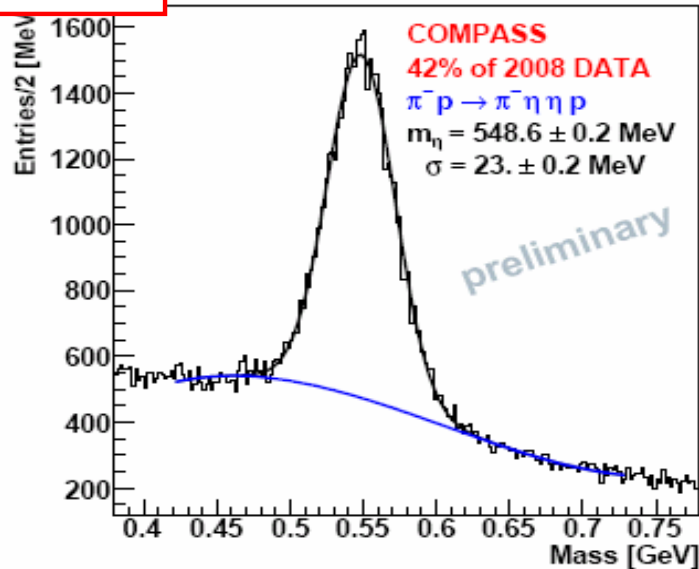


$$\pi^- p \rightarrow \pi^- \eta p \quad \& \quad \pi^- p \rightarrow \pi^- \eta \eta p$$

$\eta$  masses in 2  $\gamma$  channel:



$\eta$  masses in 4  $\gamma$  channel:



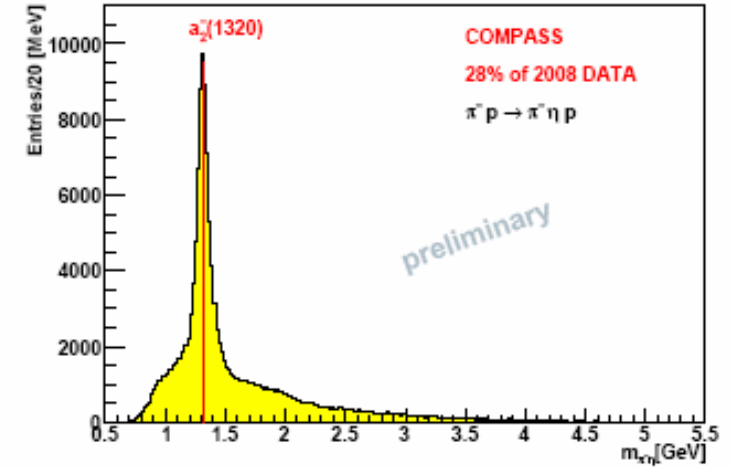
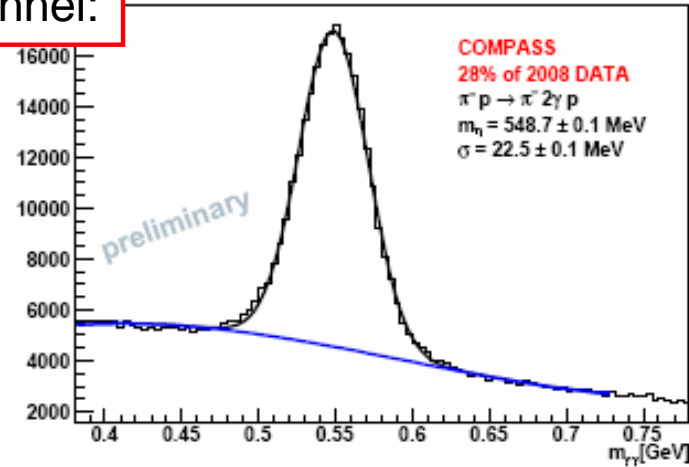


# Further neutral channels (PWA ongoing):

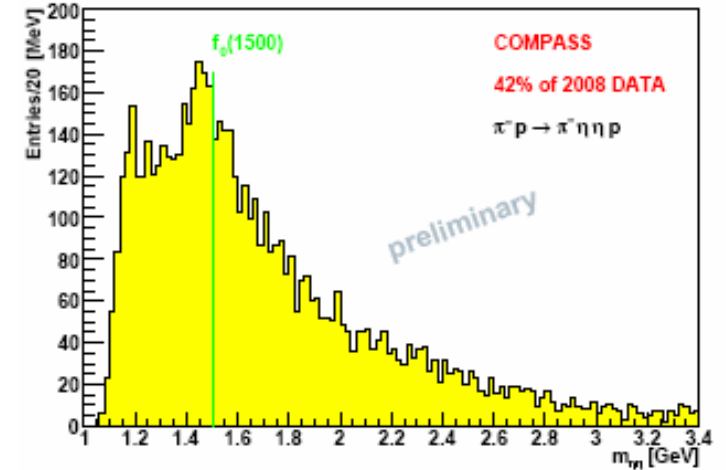
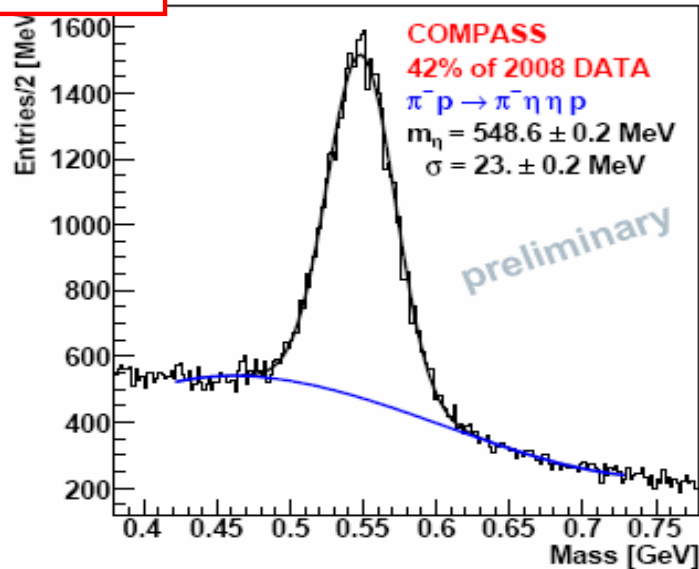
$$\pi^- p \rightarrow \pi^- \eta p \quad \& \quad \pi^- p \rightarrow \pi^- \eta \eta p$$



$\eta$  masses in 2  $\gamma$  channel:



$\eta$  masses in 4  $\gamma$  channel:

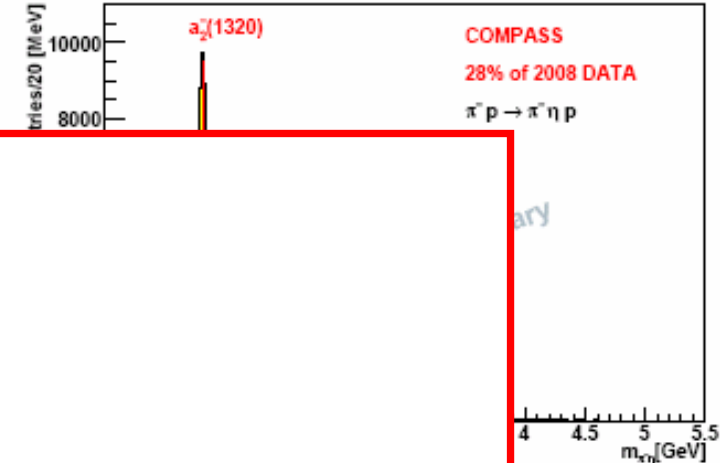




# Further neutral channels (PWA ongoing):



$\eta$  masses in 2  $\gamma$  channel:



## Further ongoing analyses (involving neutrals)

- $\pi^- \eta$ ,  $\pi^- \eta \eta$  &  $\pi^- \pi^- \pi^+ \eta$  ( $\eta \rightarrow \gamma\gamma$  &  $\eta \rightarrow 3\pi$ )  
 $\rightarrow \pi_1(1400) \rightarrow \pi^- \eta$ ,

$\rightarrow$  lightest glueball candidate  $0^{++} \rightarrow \eta \eta$

- $\pi^- \pi^- \pi^+ \pi^0$ ,  $\pi^- \pi^- \pi^+ \eta$  &  $\pi^- \pi^- \pi^+ \pi^0 \pi^0$

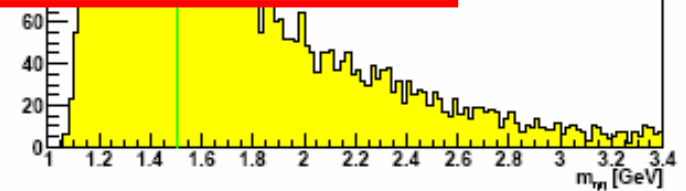
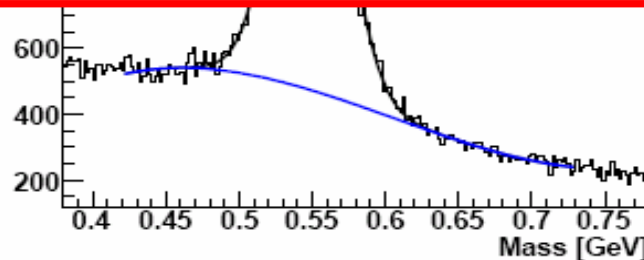
$\Rightarrow$  accessible intermediate isobars:  $f_1$ ,  $b_1$ ,  $\eta$ ,  $\eta'$ ,  $\omega \rightarrow$  search for spin exotic states

$\eta$  ma  $\rightarrow$  COMPASS: significantly **more statistics** w.r.t. previous experiments

## After hardware upgrades introduced in 2008/09:

- Optimisation of ECals reconstruction (*under development*)
- important for all final states involving neutrals

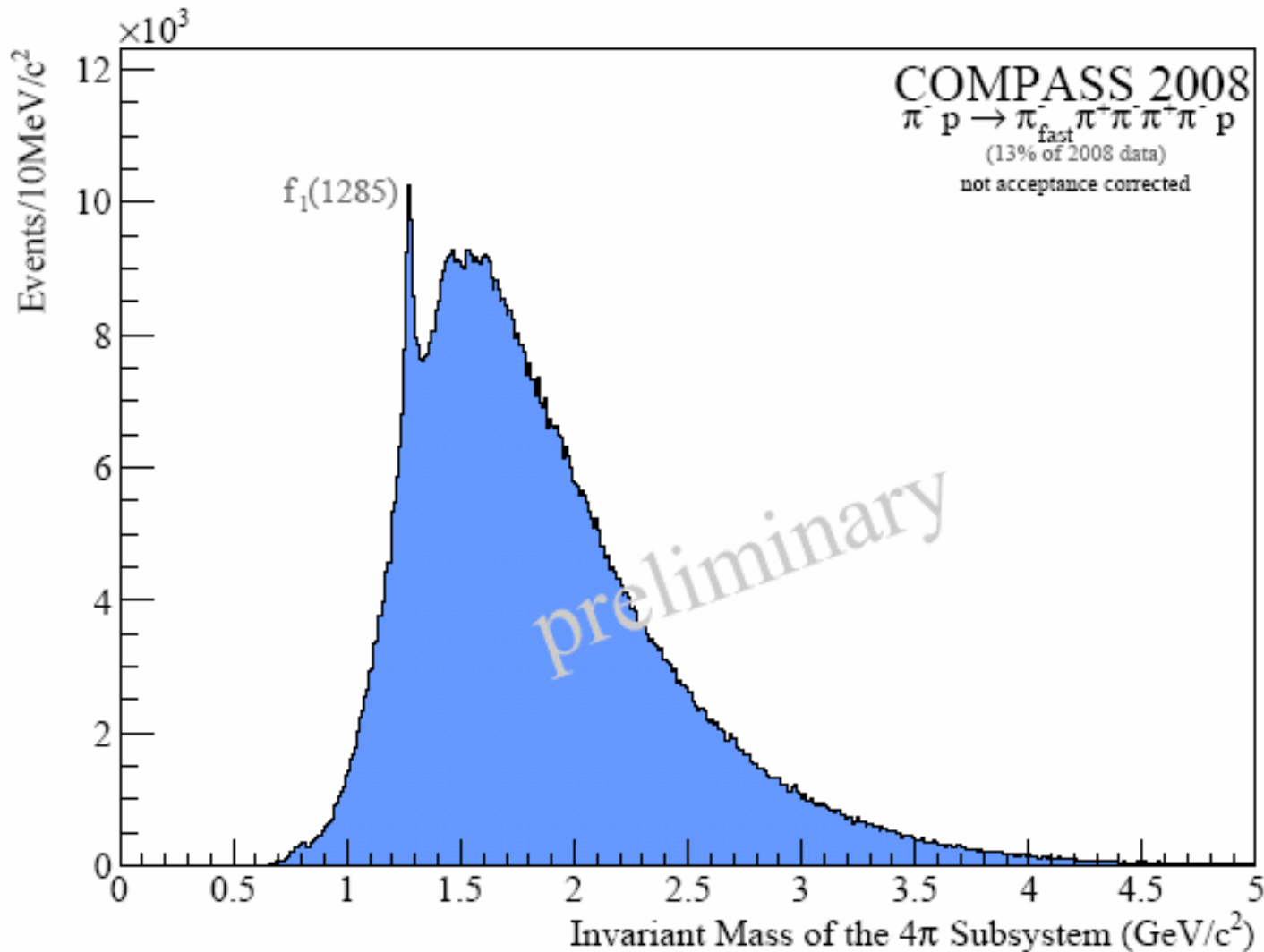
$\Rightarrow$  **Will improve statistics outcome & resolutions**







# Another channel for $\pi_1(1600)$ search: $f_1(1285)\pi$ (also $b_1\pi$ in $\pi^-\pi^+\pi^-\pi^0\pi^0$ final states)

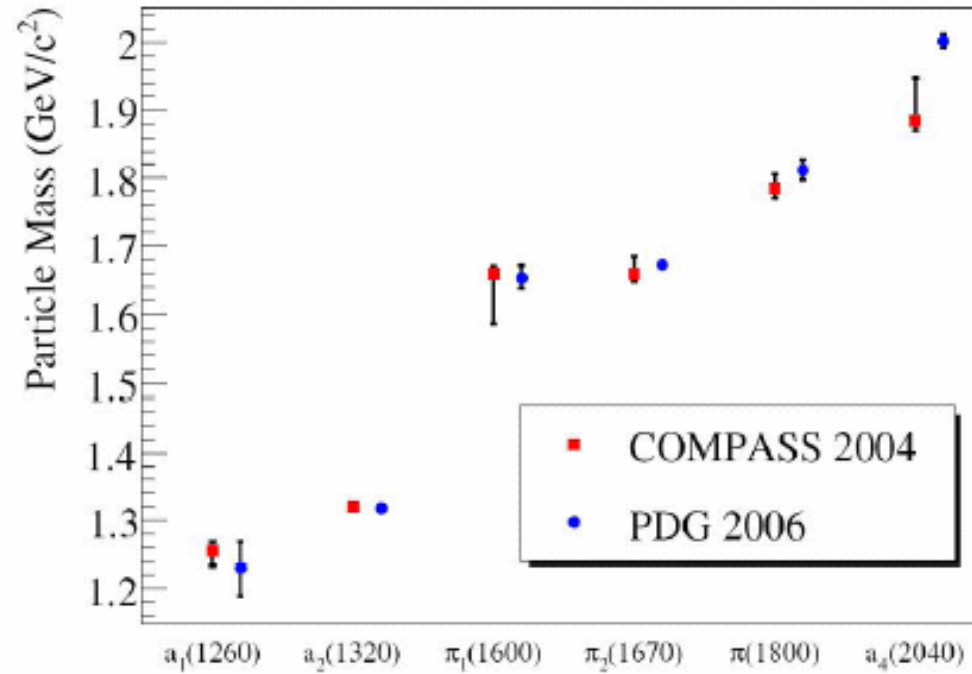


Hybrid nature:

Flux tube model:  $BR(\pi_1 \rightarrow f_1 \pi) / BR(\pi_1 \rightarrow b_1 \pi)$  [Isgur, Kokoski, Paton, PRL54, 869-872, 1985]



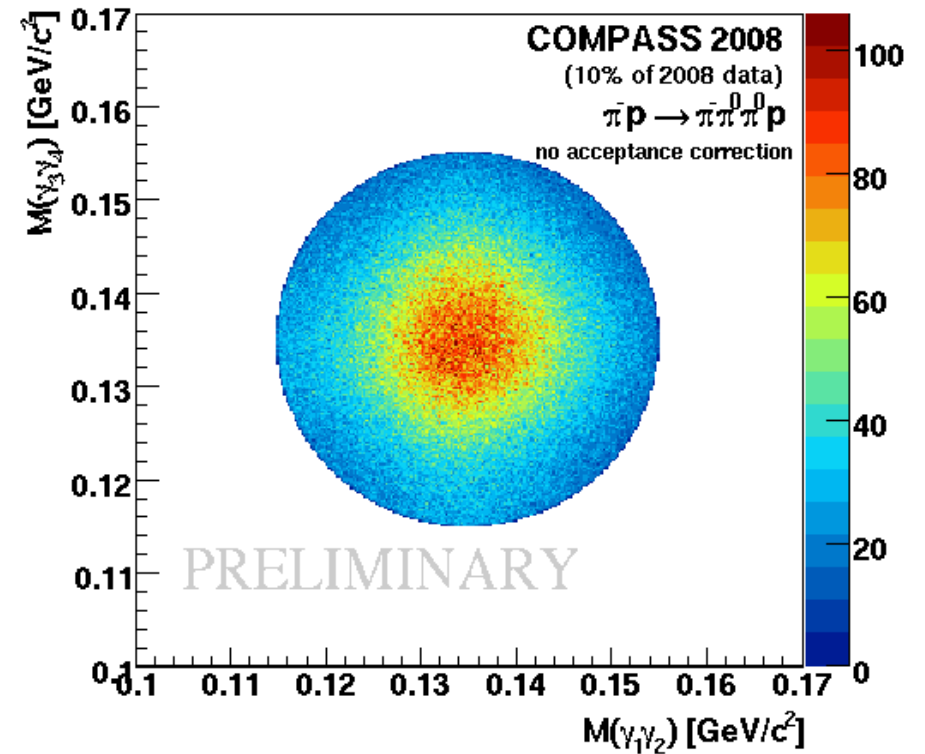
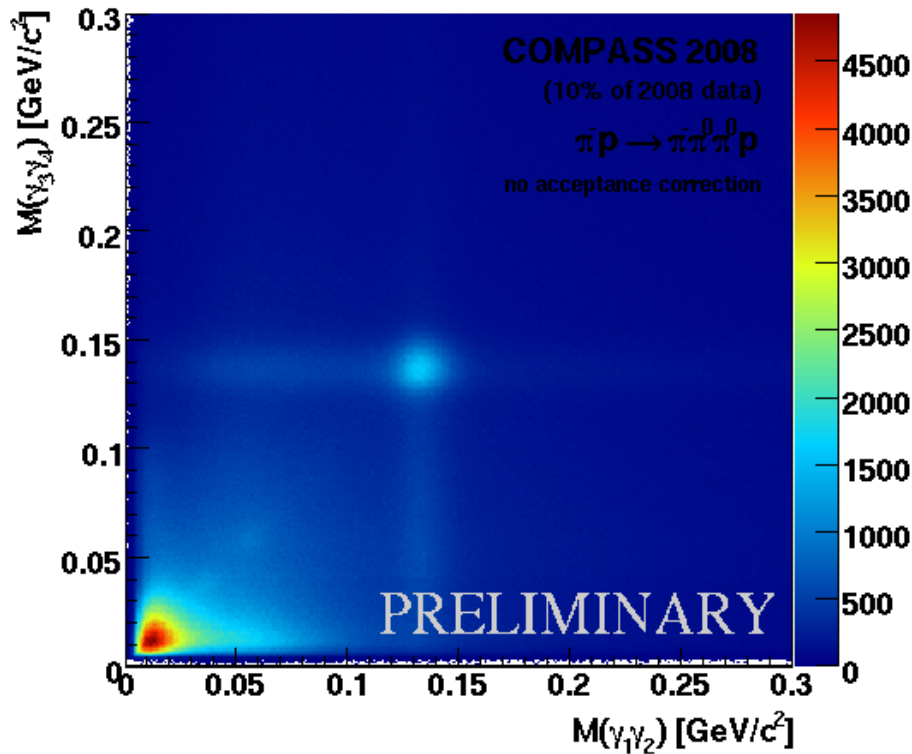
# Fitted resonances (2004 data)



Resonance	Mass (MeV/ $c^2$ )	Width (MeV/ $c^2$ )	Intensity (%)	Channel $J^{PC} M^\epsilon$ [isobar] $L$
$a_1(1260)$	$1255 \pm 6_{-17}^{+7}$	$367 \pm 9_{-25}^{+28}$	$67 \pm 3_{-20}^{+4}$	$1^{++}0^+ \rho\pi S$
$a_2(1320)$	$1321 \pm 1_{-7}^{+0}$	$110 \pm 2_{-15}^{+2}$	$19.2 \pm 0.6_{-2.2}^{+0.3}$	$2^{++}1^+ \rho\pi D$
$\pi_1(1600)$	$1660 \pm 10_{-64}^{+0}$	$269 \pm 21_{-64}^{+42}$	$1.7 \pm 0.2_{-0.1}^{+0.9}$	$1^{-+}1^+ \rho\pi P$
$\pi_2(1670)$	$1658 \pm 3_{-8}^{+24}$	$271 \pm 9_{-24}^{+22}$	$10.0 \pm 0.4_{-0.7}^{+0.7}$	$2^{-+}0^+ f_2\pi S$
$\pi(1800)$	$1785 \pm 9_{-6}^{+12}$	$208 \pm 22_{-37}^{+21}$	$0.8 \pm 0.1_{-0.1}^{+0.3}$	$0^{-+}0^+ f_0\pi S$
$a_4(2040)$	$1885 \pm 13_{-2}^{+50}$	$294 \pm 25_{-19}^{+46}$	$1.0 \pm 0.3_{-0.1}^{+0.1}$	$4^{++}1^+ \rho\pi G$



# All & Preselected gg pairs, circular cut on PDG $\pi^0$ mass

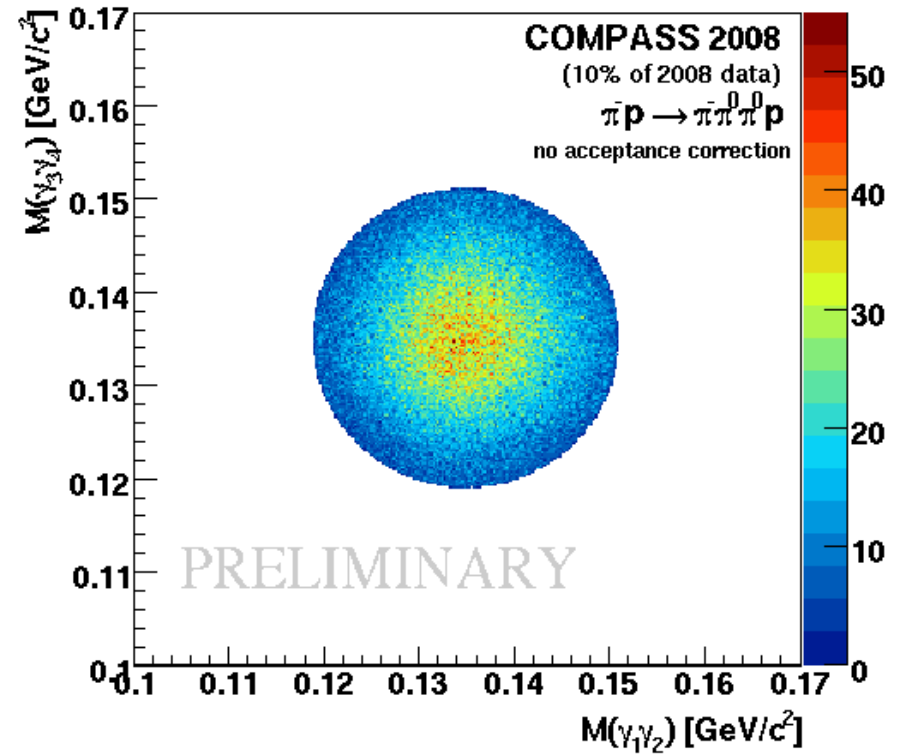
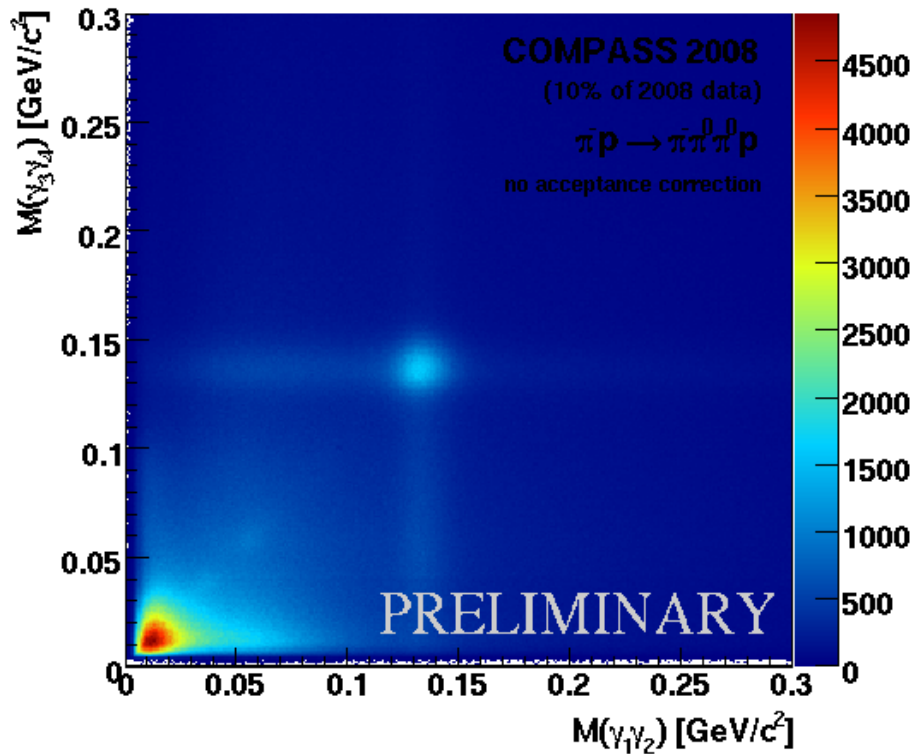


$2\pi^0$  evt := exactly 4 clusters, exactly one  $2\pi^0$  combi within PDG  $\pm$  20 MeV



# All & Preselected gg pairs, circular cut on PDG $\pi^0$ mass

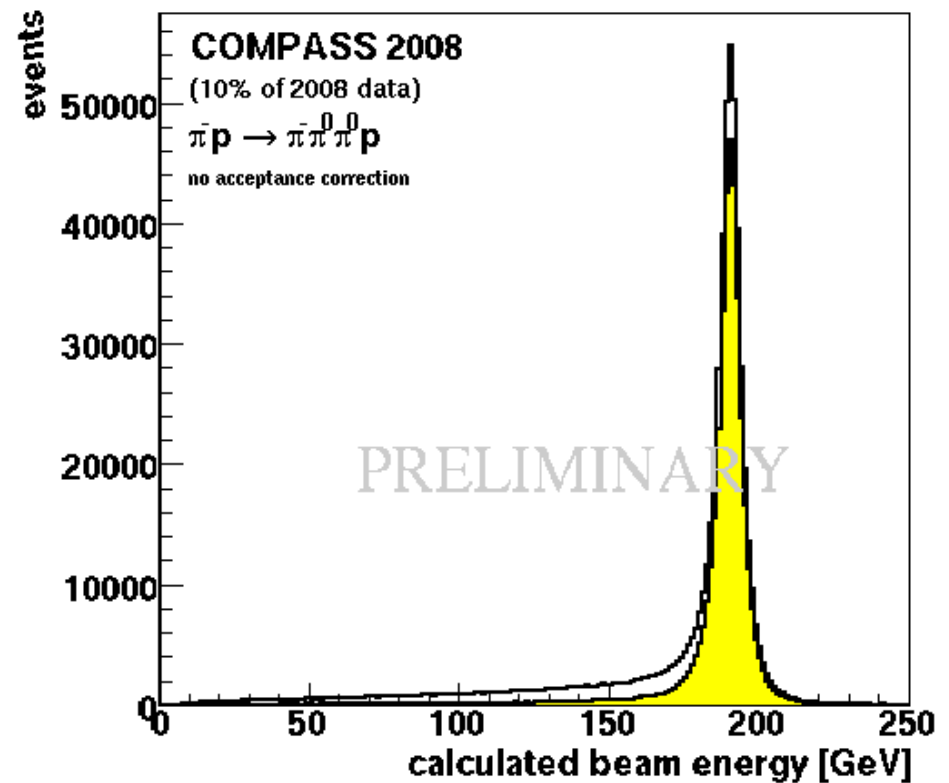
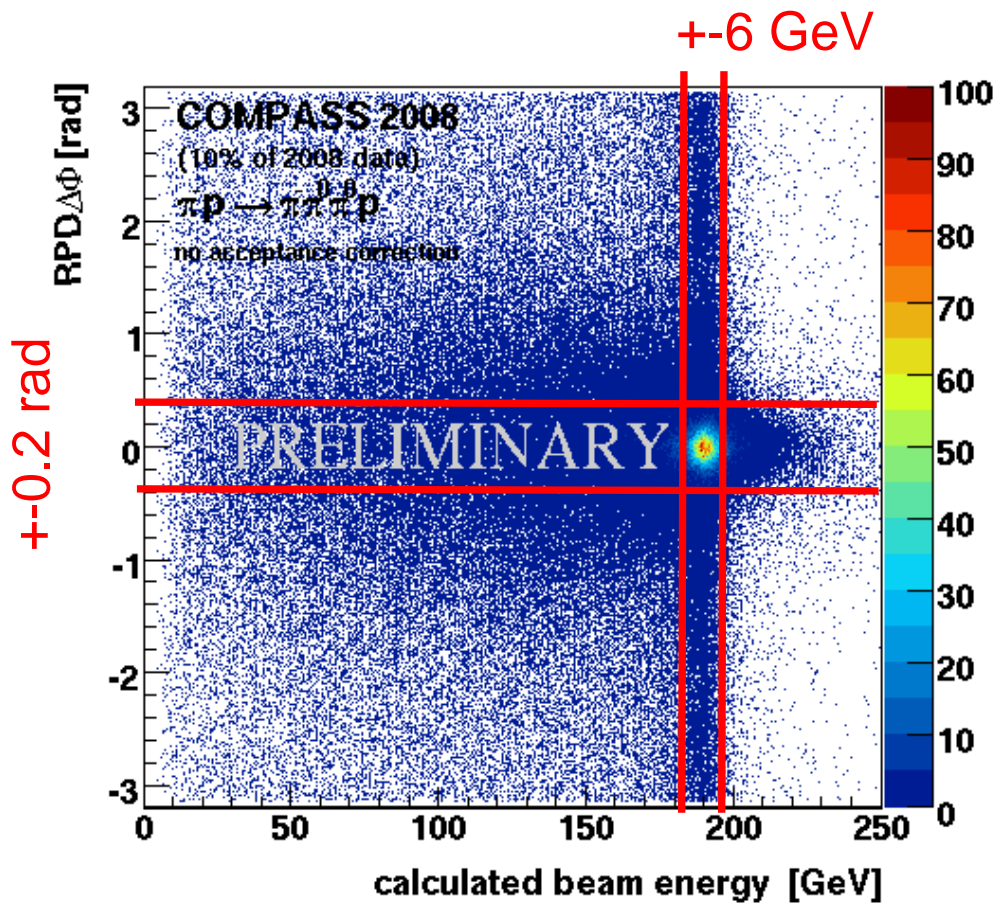
After final cuts on  $\Delta\Phi$  and exclusivity,  
see next slides



$2\pi^0$  evt := exactly 4 clusters, exactly one  $2\pi^0$  combi within PDG  $\pm 20$  MeV

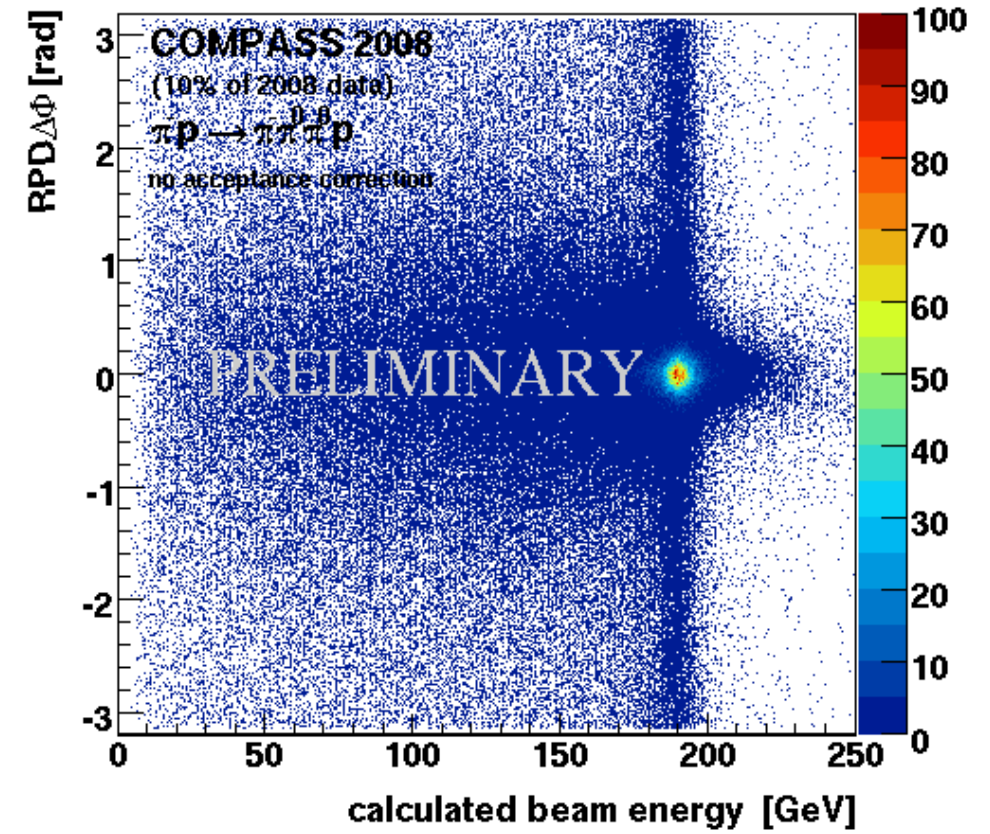
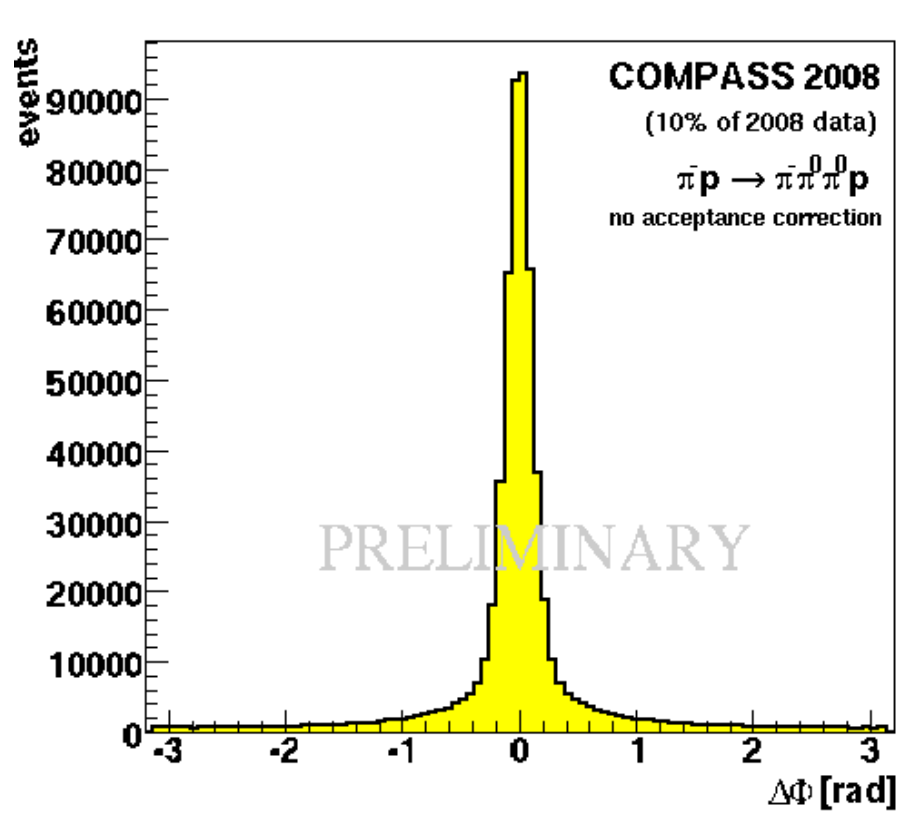


# $\Delta\Phi$ (RPD-Spectro) vs. $E_{\text{beam}}$



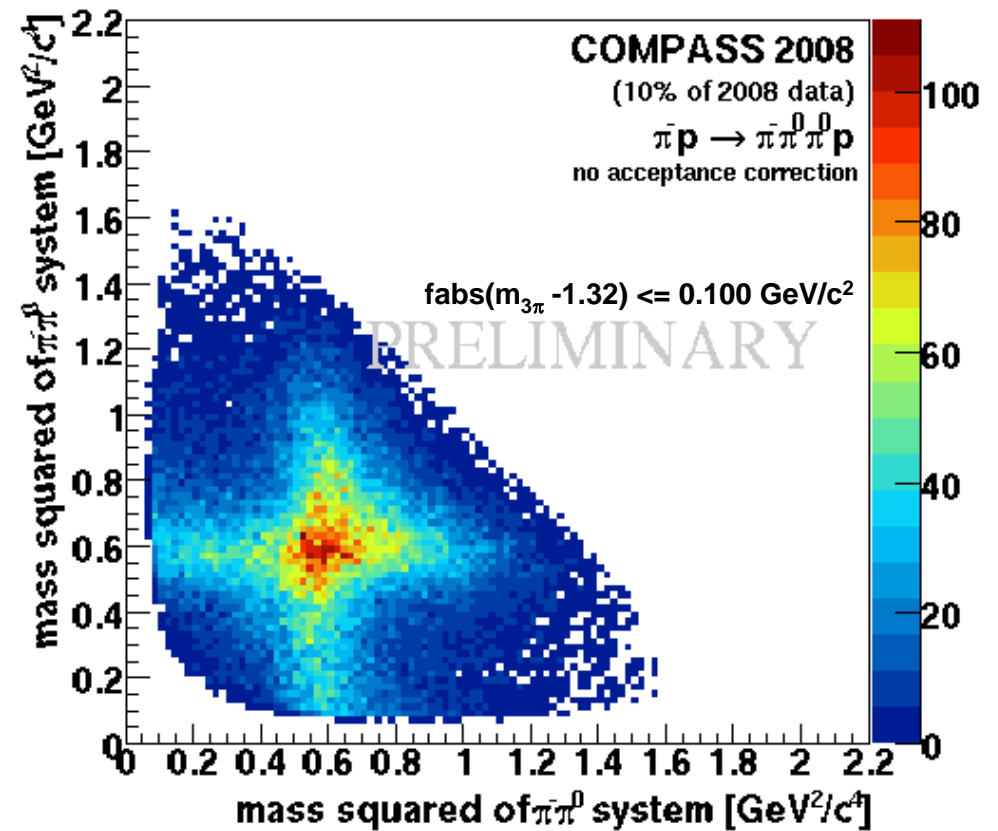
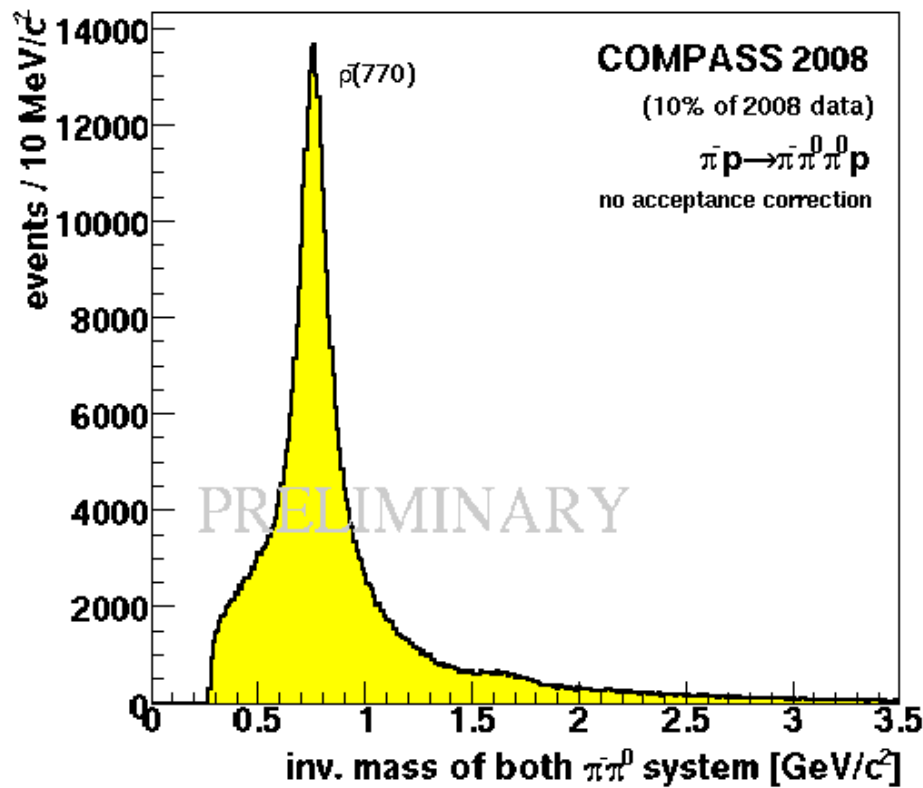


# $\Delta\Phi$ (RPD-Spectro) vs. $E_{\text{beam}}$



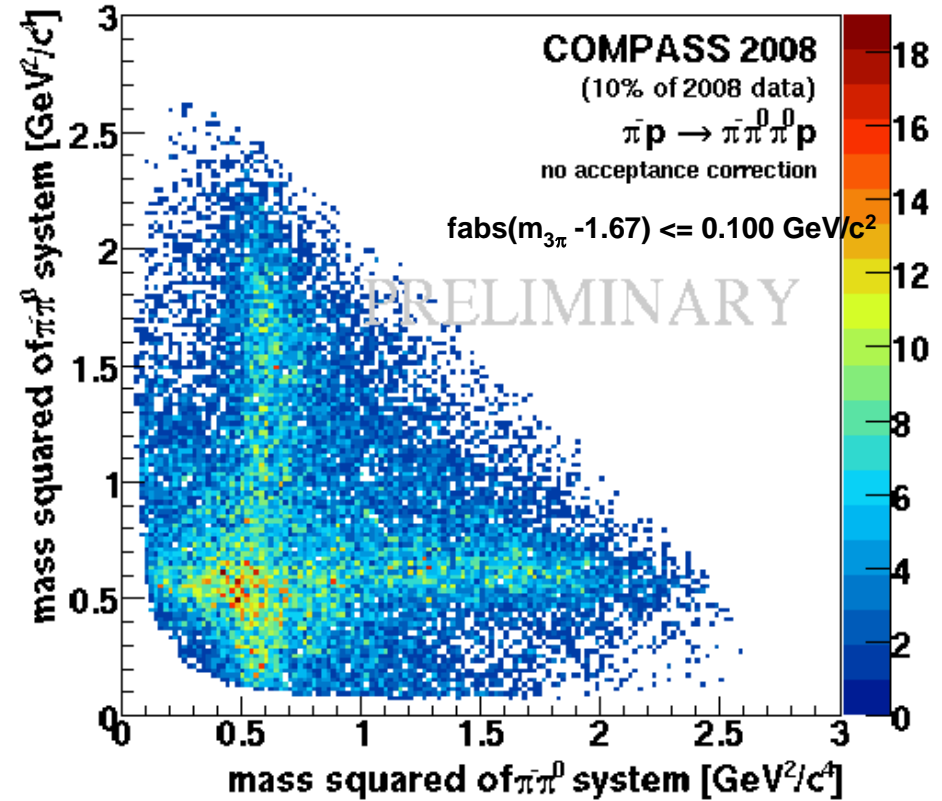
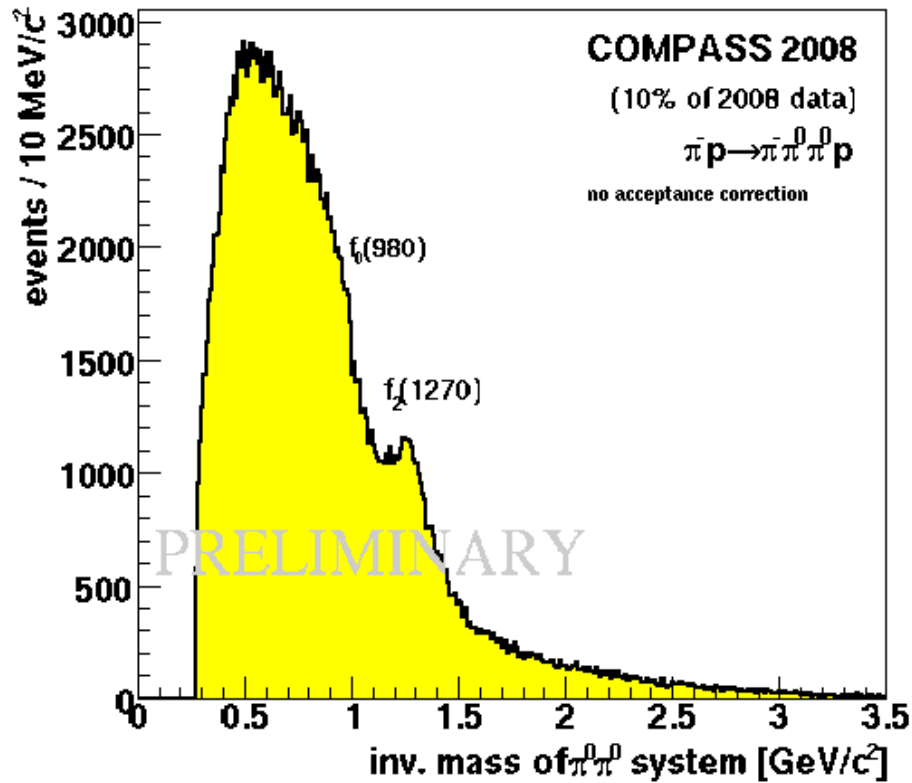


# Mass spectrum of both $\pi^-\pi^0$ systems & Dalitz plot, $a_2$ region





# Dalitz plots: $\pi_2$ region



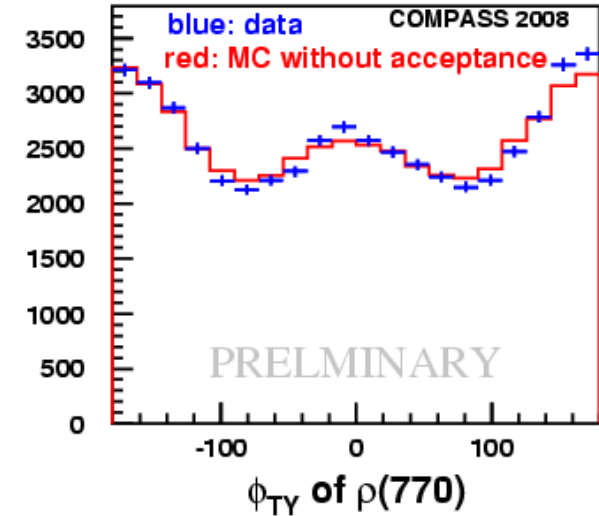
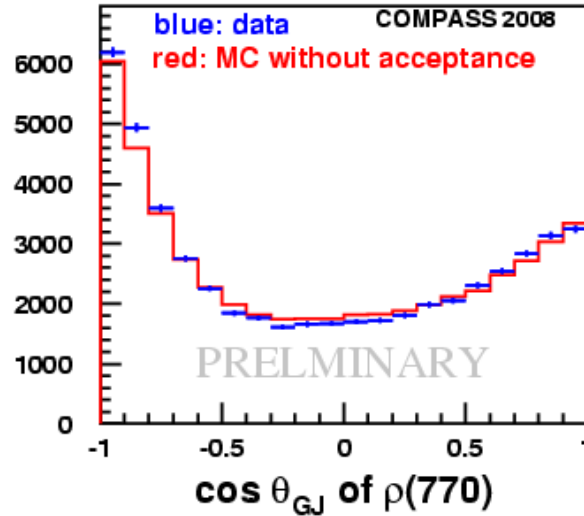




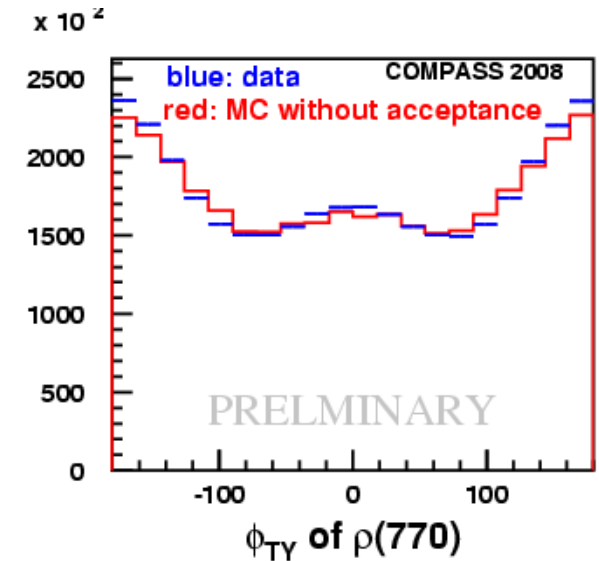
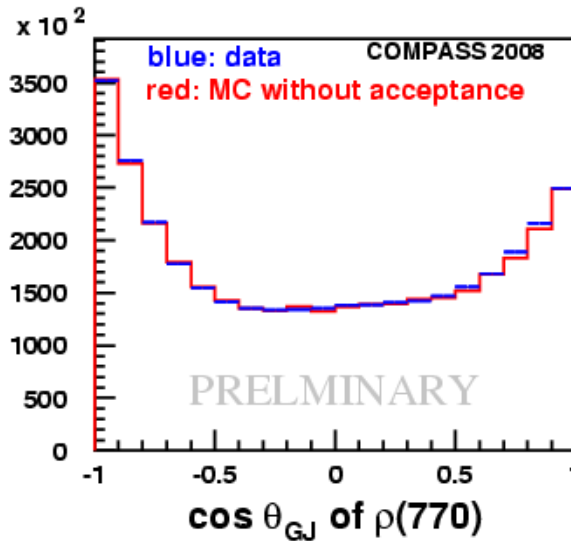
# Decay angles in G.J. frame: Full PhaseSpace Generated Prediction vs. fitted data



a1/a2 mass region - neutral  
(1.22 - 1.38 GeV/c<sup>2</sup>)



a1/a2 mass region - charged  
(1.22 - 1.38 GeV/c<sup>2</sup>)





# Waveset used for the PWA

$J^{PC} M^\epsilon$	$L$	Isobar $\pi$	Threshold (GeV/ $c^2$ )
→ $0^{-+}0^+$	$S$	$f_0(980)\pi$	1.25
$0^{-+}0^+$	$S$	$(\pi\pi)_s\pi$	-
$0^{-+}0^+$	$P$	$\rho\pi$	-
→ $1^{-+}1^+$	$P$	$\rho\pi$	-
→ $1^{++}0^+$	$S$	$\rho\pi$	-
$1^{++}0^+$	$P$	$f_2\pi$	1.20
$1^{++}0^+$	$P$	$(\pi\pi)_s\pi$	0.94
$1^{++}0^+$	$D$	$\rho\pi$	1.30
$1^{++}1^+$	$S$	$\rho\pi$	-
$1^{++}1^+$	$P$	$f_2\pi$	1.40
$1^{++}1^+$	$P$	$(\pi\pi)_s\pi$	1.20
$1^{++}1^+$	$D$	$\rho\pi$	1.40
→ $2^{-+}0^+$	$S$	$f_2\pi$	1.20
$2^{-+}0^+$	$P$	$\rho\pi$	0.80
$2^{-+}0^+$	$D$	$(\pi\pi)_s\pi$	0.80
$2^{-+}0^+$	$D$	$f_2\pi$	1.50
$2^{-+}0^+$	$F$	$\rho\pi$	1.20
$2^{-+}1^+$	$S$	$f_2\pi$	1.20
$2^{-+}1^+$	$P$	$\rho\pi$	0.80
$2^{-+}1^+$	$D$	$(\pi\pi)_s\pi$	1.20
$2^{-+}1^+$	$D$	$f_2\pi$	1.50
$2^{-+}1^+$	$F$	$\rho\pi$	1.20
→ $2^{++}1^+$	$P$	$f_2\pi$	1.20
$2^{++}1^+$	$D$	$\rho\pi$	-
$3^{++}0^+$	$S$	$\rho_3\pi$	1.76
$3^{++}0^+$	$P$	$f_2\pi$	1.20
$3^{++}0^+$	$D$	$\rho\pi$	1.20
$3^{++}1^+$	$S$	$\rho_3\pi$	1.76
$3^{++}1^+$	$P$	$f_2\pi$	1.20
$3^{++}1^+$	$D$	$\rho\pi$	1.50
$4^{-+}0^+$	$F$	$\rho\pi$	1.00
$4^{-+}1^+$	$F$	$\rho\pi$	1.20
→ $4^{++}1^+$	$F$	$f_2\pi$	1.60
→ $4^{++}1^+$	$G$	$\rho\pi$	1.40
$1^{-+}0^-$	$P$	$\rho\pi$	-
$1^{-+}1^-$	$P$	$\rho\pi$	-
$1^{++}1^-$	$S$	$\rho\pi$	-
$2^{-+}1^-$	$S$	$f_2\pi$	1.20
$2^{++}0^-$	$P$	$f_2\pi$	1.30
$2^{++}0^-$	$D$	$\rho\pi$	-
$2^{++}1^-$	$P$	$f_2\pi$	1.30
FLAT			

Table 5: List of the 42 waves used for the mass independent PWA.