

# Diffractive Pion Dissociation Into $3\pi$ and $5\pi$ Final States

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BMBF, EU



## Constituent Quark Model of Mesons – Spin Parity Rules

$$P = (-1)^{l+1} \quad C = (-1)^{l+s}$$

e.g.  $P = (-1)^J \Rightarrow s = 1 \Rightarrow CP = +1$

$J^{PC}$  Multiplets:

$0^{++}, 0^{-+}, 1^{--}, 1^{+-}, 1^{++}, 2^{++}, \dots$

**Forbidden:**

$0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, \dots$

## Spin Exotic States

- $J^{PC}$  forbidden in constituent quark model  $\Rightarrow$  cannot be a  $q\bar{q}$ -state
- No mixing with quark model states

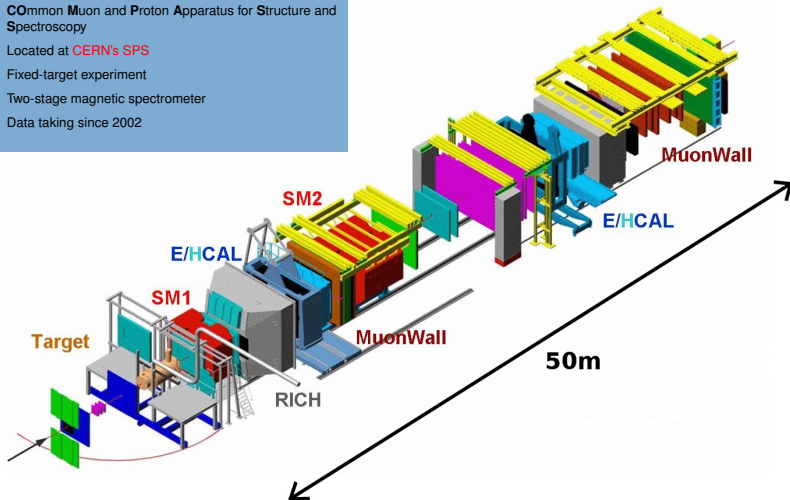
$$\underbrace{J^{PC} = 1^{-+}}_{\text{Smoking Gun}} + \underbrace{\text{Resonant Phase Motion}}_{\text{Bullet}} + \underbrace{\text{Quark Model}}_{\text{Victim}}$$

## Challenges and Opportunities in the light (u,d) Quark Sector

- High density of states; broad, overlapping states
- **Exploit interference** effects  $\rightarrow$  phase motion
- Requires **high statistics, complete PS coverage**

## Overview

- **CO**mmun **M**uon and **P**roton **A**pparatus for **S**tructure and **S**pectroscopy
- Located at **CERN's SPS**
- Fixed-target experiment
- Two-stage magnetic spectrometer
- Data taking since 2002

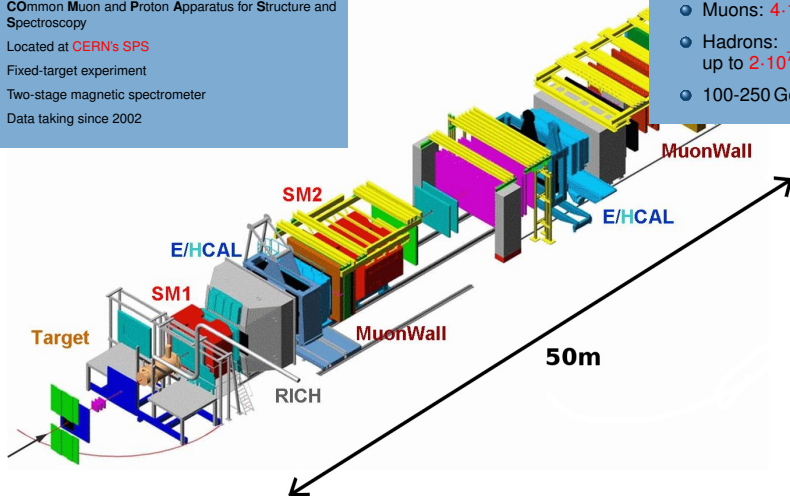


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## Second./Tert. SPS Beams

- Muons:  $4 \cdot 10^7 \text{s}^{-1}$
- Hadrons: up to  $2 \cdot 10^7 \text{s}^{-1}$
- 100-250 GeV

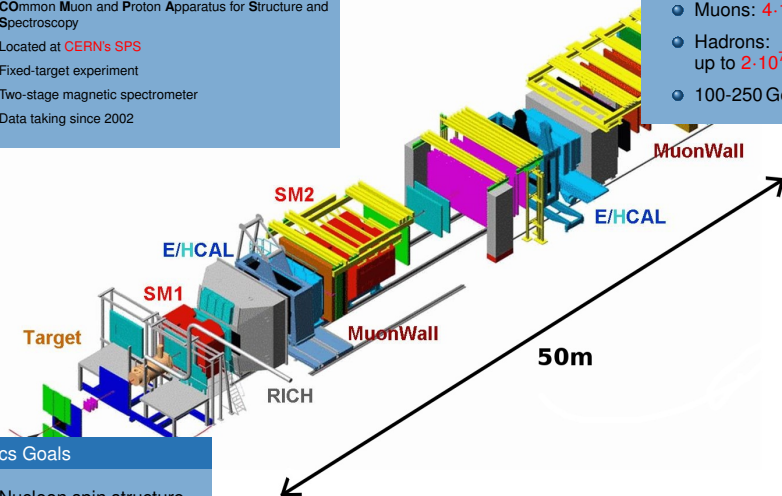


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## Physics Goals

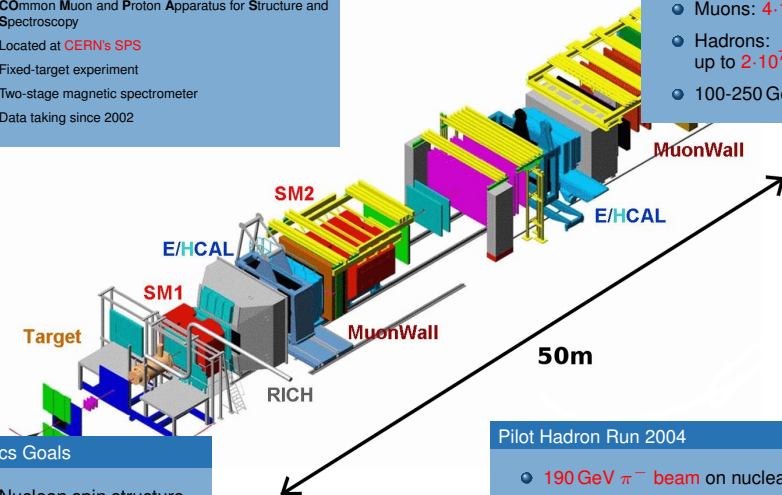
- Nucleon spin structure
- **Hadron spectroscopy**

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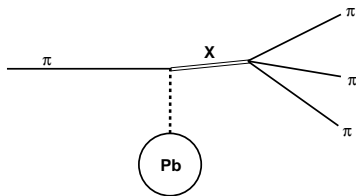
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- **Hadron spectroscopy**

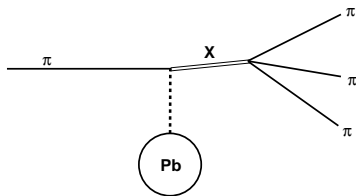
## Pilot Hadron Run 2004

- **190 GeV  $\pi^-$  beam** on nuclear targets
- Tracking: Silicons, SciFis, GEMs, MicroMegs, MWPCs, Drift Chambers

- Diffraction:** target particle remains intact  
**Reggeon t-channel exchange**  
**Assumptions:** Factorization of meson and Pb vertex, no final state interaction
- Dissociation:** beam pion is excited to some resonance  $X^-$ , which subsequently decays  
 $\Rightarrow$  e.g.  $\pi^- \text{Pb} \rightarrow X^- \text{Pb} \rightarrow \pi^- \pi^- \pi^+ \text{Pb}$   
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## Meson Spectroscopy and Search for Exotic States (non- $q\bar{q}$ )

- Evidences for spin-exotic  $\pi_1(1600)$  state with  $J^{PC} = 1^{-+}$  in  $\rho\pi^- \rightarrow \pi^- \pi^+ \pi^-$  ?

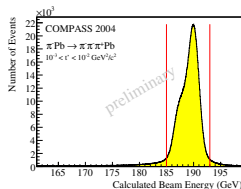
[BNL-E852, Phys. Rev. **D65**, 072001, 2002], [VES, Nucl. Phys. **A663**, 596, 2000]

**controversial situation!**



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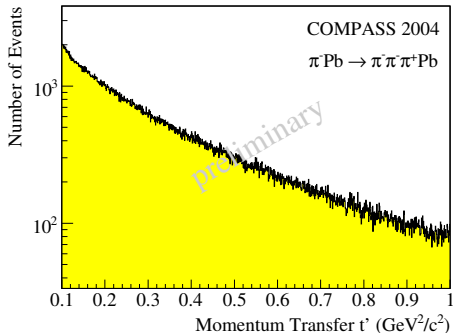
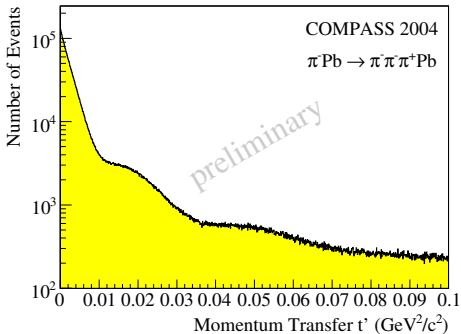
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[BNL-E852, Phys. Rev. **D65**, 072001, 2002], [VES, Nucl. Phys. **A663**, 596, 2000]

**controversial situation!**

- Exclusive  $3\pi$  or  $5\pi$  final state events (pion hypothesis).
- **COMPASS 2004 (few days of data taking)**:
  - ▶  $\sim 4\,000\,000$   $3\pi$  events  
 $\sim 400\,000$  events with  $0.1 < t' < 1.0 \text{ GeV}^2/c^2$
  - ▶  $\sim 380\,000$   $5\pi$  events

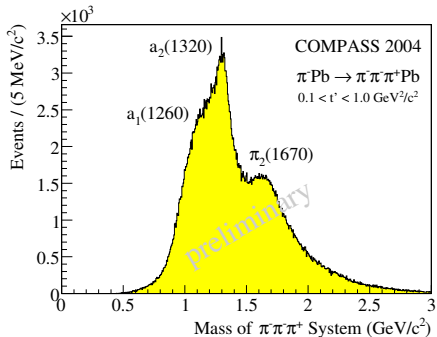
Momentum transfer from target:  $-t = -(\mathbf{p}_{\text{beam}} - \mathbf{p}_{(\pi^- \pi^- \pi^+)})^2$   
 $\Rightarrow t' = |t| - |t|_{\text{min}}$



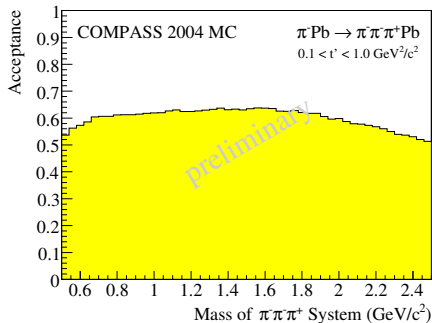
**Diffraction pattern:** Pb nucleus acts like "black disc" in optics

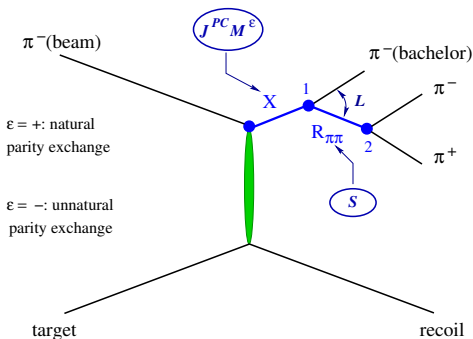
**High- $t'$ :** scattering on single nucleons inside Pb nucleus

High- $t'$  Spectrum



Flat acceptance: 55-60%





- Isobar model: Intermediate 2-particle decays
- Zemach / Helicity formalism, reflectivity basis
- Reggeon exchange
- Partial wave:  $J^{PC} M^\epsilon [isobar] L$
- Nucleon target  $\Rightarrow$  rank 2

• **Mass-independent PWA** (40 MeV/ $c^2$  mass bins): **42 waves**

- ▶ Extended log-likelihood method (Ascoli/Kachaev fitter)
- ▶ Acceptance corrections included
- ▶  $\rho(770)$ ,  $f_2(1270)$ ,  $\rho_3(1690)$  from PDG,  $(\pi\pi)_S$  with separated  $f_0(980)$  from VES
- ▶ Multiple solutions ( $\Delta \ln L \leq 1$ ) added as additional error

• **Mass-dependent  $\chi^2$ -fit**: **6 waves**

- ▶ Only positive reflectivity waves (natural parity exchange dominant)
- ▶  $X$  parameterized by Breit-Wigner (BW) functions
- ▶ Coherent background added for some waves:  $\exp(-\alpha p^2)$

# Partial Wave Set for Mass-Independent Fit (42 Waves)

Description of possible Decay Amplitudes



$J^{PC} M^{\epsilon}$	$L$	Isobar $\pi$	Thresh. [GeV]
$0^{-+}0^{+}$	$S$	$f_0\pi$	1.40
$0^{-+}0^{+}$	$S$	$(\pi\pi)_S\pi$	-
$0^{-+}0^{+}$	$P$	$\rho\pi$	-
$1^{-+}1^{+}$	$P$	$\rho\pi$	-
$1^{++}0^{+}$	$S$	$\rho\pi$	-
$1^{++}0^{+}$	$P$	$f_2\pi$	1.20
$1^{++}0^{+}$	$P$	$(\pi\pi)_S\pi$	0.84
$1^{++}0^{+}$	$D$	$\rho\pi$	1.30
$1^{++}1^{+}$	$S$	$\rho\pi$	-
$1^{++}1^{+}$	$P$	$f_2\pi$	1.40
$1^{++}1^{+}$	$P$	$(\pi\pi)_S\pi$	1.40
$1^{++}1^{+}$	$D$	$\rho\pi$	1.40
$2^{-+}0^{+}$	$S$	$f_2\pi$	1.20
$2^{-+}0^{+}$	$P$	$\rho\pi$	0.80
$2^{-+}0^{+}$	$D$	$f_2\pi$	1.50
$2^{-+}0^{+}$	$D$	$(\pi\pi)_S\pi$	0.80
$2^{-+}0^{+}$	$F$	$\rho\pi$	1.20
$2^{-+}1^{+}$	$S$	$f_2\pi$	1.20
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$2^{-+}1^{+}$	$D$	$f_2\pi$	1.50
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$2^{++}1^{+}$	$P$	$f_2\pi$	1.50
$2^{++}1^{+}$	$D$	$\rho\pi$	-
$3^{++}0^{+}$	$S$	$\rho_3\pi$	1.50
$3^{++}0^{+}$	$P$	$f_2\pi$	1.20
$3^{++}0^{+}$	$D$	$\rho\pi$	1.50
$3^{++}1^{+}$	$S$	$\rho_3\pi$	1.50
$3^{++}1^{+}$	$P$	$f_2\pi$	1.20
$3^{++}1^{+}$	$D$	$\rho\pi$	1.50
$4^{-+}0^{+}$	$F$	$\rho\pi$	1.20
$4^{-+}1^{+}$	$F$	$\rho\pi$	1.20
$4^{++}1^{+}$	$F$	$f_2\pi$	1.60
$4^{++}1^{+}$	$G$	$\rho\pi$	1.64
$1^{-+}0^{-}$	$P$	$\rho\pi$	-
$1^{-+}1^{-}$	$P$	$\rho\pi$	-
$1^{++}1^{-}$	$S$	$\rho\pi$	-
$2^{-+}1^{-}$	$S$	$f_2\pi$	1.20
$2^{++}0^{-}$	$P$	$f_2\pi$	1.30
$2^{++}0^{-}$	$D$	$\rho\pi$	-
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FLAT			

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$0^{-+}0^{+}$	$S$	$f_0\pi$	1.40				1.50
$0^{-+}0^{+}$	$S$						-
$0^{-+}0^{+}$	$F$						1.50
$1^{-+}1^{+}$	$F$						1.20
$1^{++}0^{+}$	$S$						1.50
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$2^{-+}0^{+}$	$S$						-
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$2^{-+}0^{+}$	$D$	$f_2\pi$	1.50	$1^{++}1^{-}$	$S$	$\rho\pi$	-
$2^{-+}0^{+}$	$D$	$(\pi\pi)_S\pi$	0.80	$2^{-+}1^{-}$	$S$	$f_2\pi$	1.20
$2^{-+}0^{+}$	$F$	$\rho\pi$	1.20	$2^{++}0^{-}$	$P$	$f_2\pi$	1.30
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$2^{-+}1^{+}$	$F$	$\rho\pi$	1.20				

## Waveset Features

- 41 Waves + Flat Background
- Isobars:
  - ▶  $\sigma(600), \rho(770), f_0(980), f_2(1270), \rho_3$
- **positive reflectivity** dominates
- 7 negative reflectivity waves included
- More  $M = 1$  waves than previous (e.g. BNL) analyses
- $2^{-+}F$  waves included

FLAT

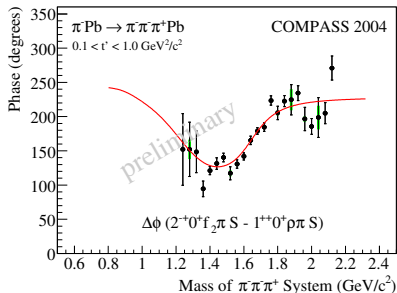
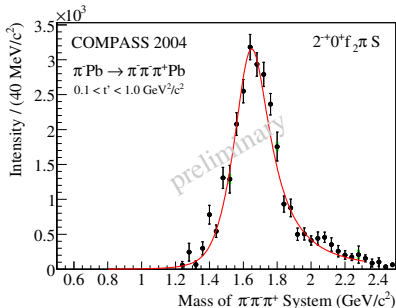
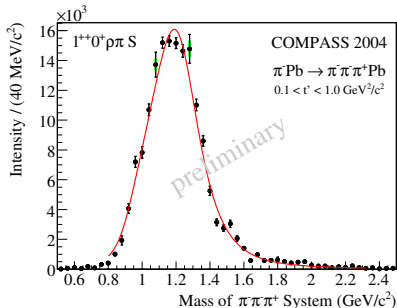
# Partial Wave Set for Mass-Dependent. Fit (6 Waves)

Extraction of Resonance Parameters from Intensities and Interferences



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$2^{++}0^{-}$	$D$	$\rho\pi$	-
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FLAT			



- BW for  $a_1(1260)$  + background:

$$M = (1.255 \pm 0.006^{+0.007}_{-0.017}) \text{ GeV}/c^2$$

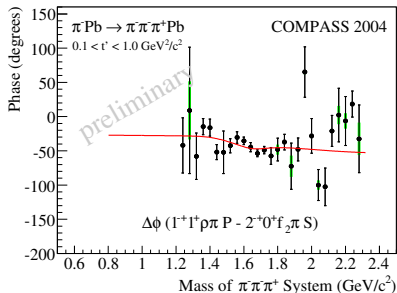
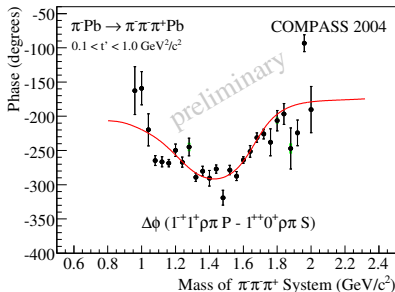
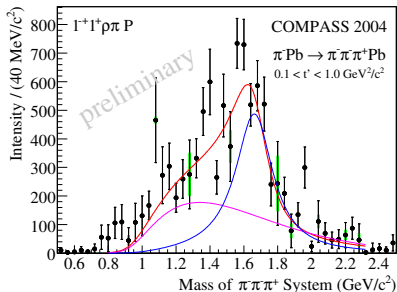
$$\Gamma = (0.367 \pm 0.009^{+0.028}_{-0.025}) \text{ GeV}/c^2$$

- BW for  $\pi_2(1670)$ :

$$M = (1.658 \pm 0.003^{+0.024}_{-0.008}) \text{ GeV}/c^2$$

$$\Gamma = (0.271 \pm 0.009^{+0.022}_{-0.024}) \text{ GeV}/c^2$$

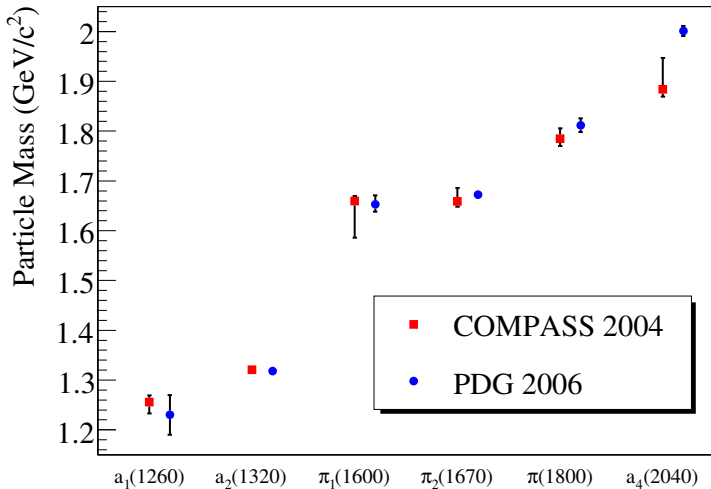




- Significant  $1^{-+}$  amplitude consistent with resonance at  $\sim 1.7 \text{ GeV}/c^2$
- No leakage observed ( $< 5\%$ )
- BW for  $\pi_1(1600)$  + background:  

$$M = (1.660 \pm 0.010^{+0.000}_{-0.064}) \text{ GeV}/c^2$$

$$\Gamma = (0.269 \pm 0.021^{+0.042}_{-0.064}) \text{ GeV}/c^2$$



## Motivation:

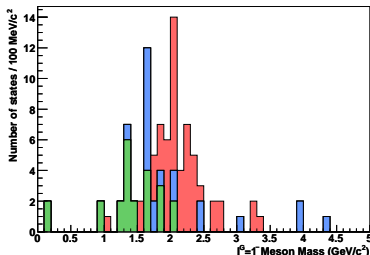
- Access to **mass-range  $> 2 \text{ GeV}/c^2$**
- Interesting accessible quantum numbers:
  - ▶  $1^-(1^{--})$  spin exotic
  - ▶  $1^-(0^{++})$  **not accessible in  $3\pi$ !**
  - ▶  $1^-(3^{--})$  high J spin exotic?
- BNL:  $\pi^- p \rightarrow f_1 \pi \rightarrow \eta \pi^+ \pi^- \pi^-$   
**2 exotic  $1^{--}$  states**  
 (Kuhn et al., Phys. Lett. B, 595(2004)109)
- Interesting decay modes  $b_1 \pi$ ,  $f_1 \pi$ ,  $\rho' \pi$
- *Light meson frontier*:  
 many **disputed states** in this region  
 $(0^{-+})(1^{++})(2^{-+})...$

## $\pi_1(1^{--})$ branching ratios

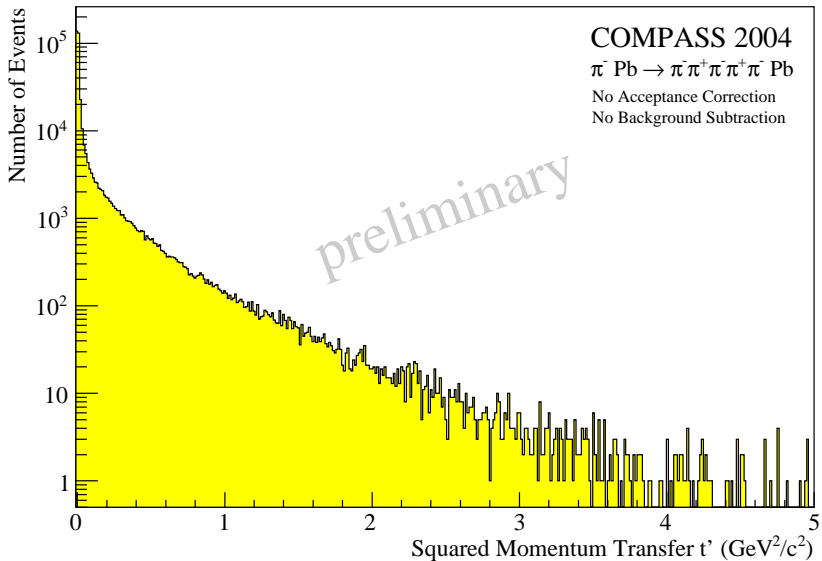
Flux-Tube model predictions:

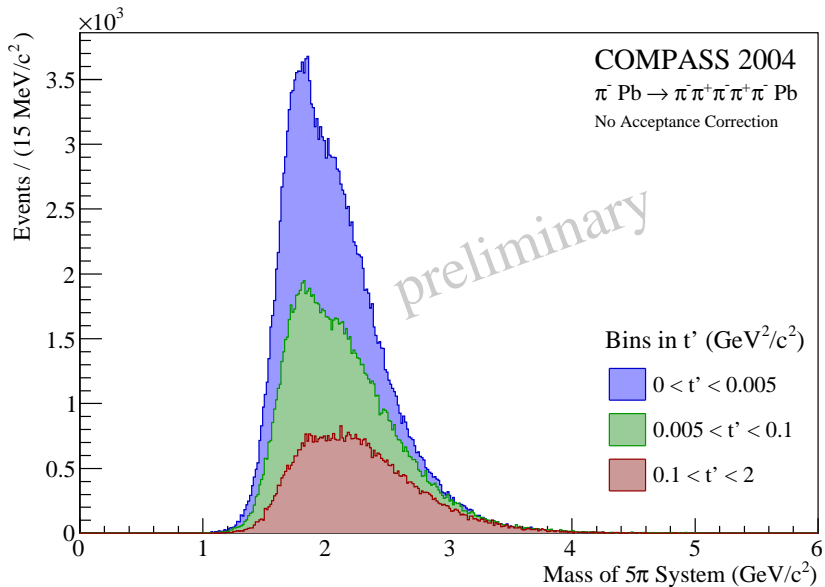
(Page, Swanson, Szczepaniak, Phys. Rev. D59, 034016(1999))

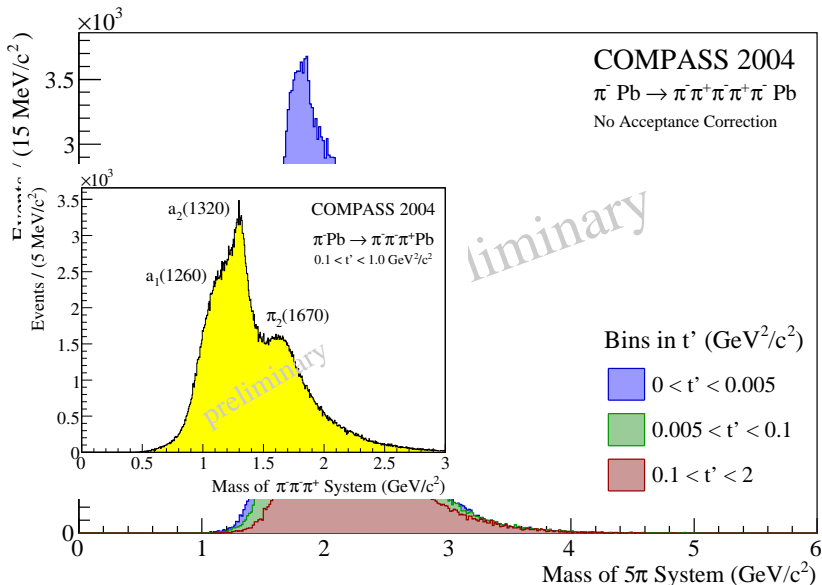
$m_{\pi_1}$	$b_1 \pi$	$f_1 \pi$	$\eta' \pi$	$\rho(1450)\pi$
$1.6 \text{ GeV}/c^2$	24:	5:	2	
$2.0 \text{ GeV}/c^2$	43:	10:	27:	12

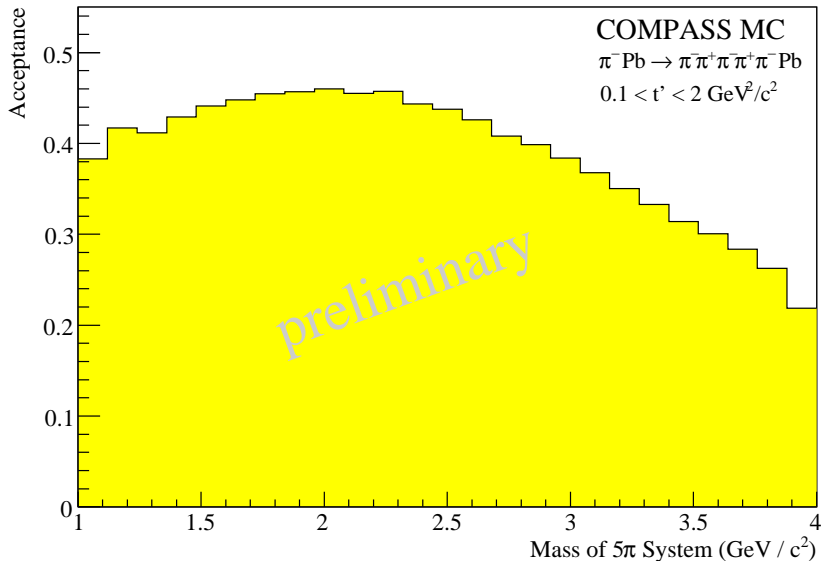


Meson states with  $J^G = 1^-$  listed in the PDG.  
 Green = established, blue = need confirmation,  
 red = "further states". The histogram is stacked.



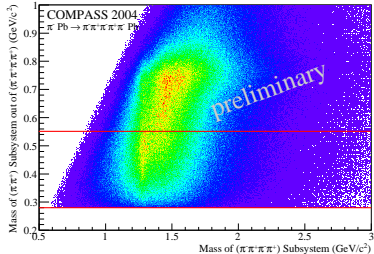
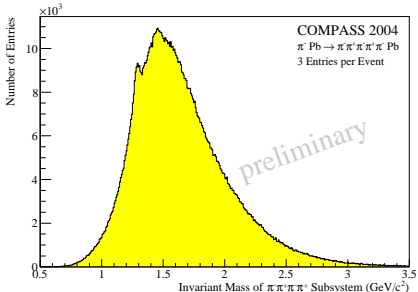




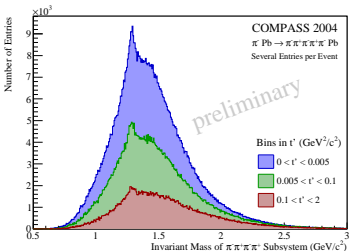


# 4 $\pi$ Subsystem – the $f_1$ and Friends

Isobar Candidates



Cut 4 $\pi$  spectrum:



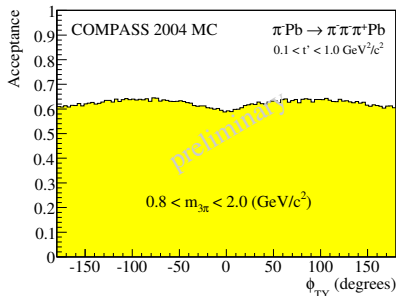
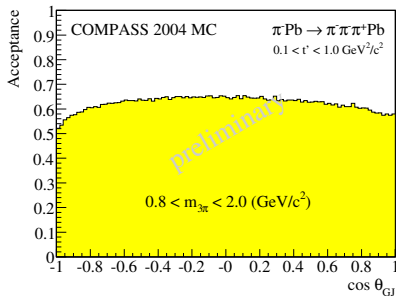
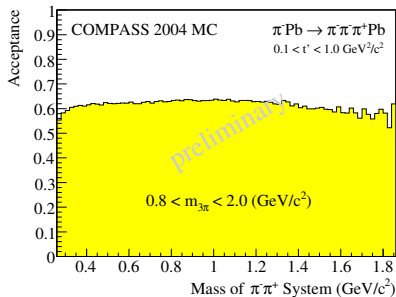
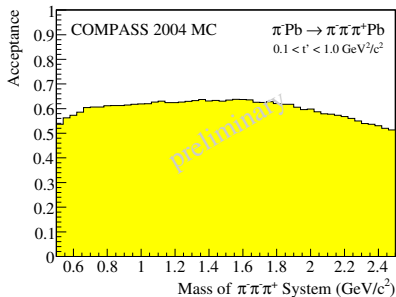
Isobar candidates:

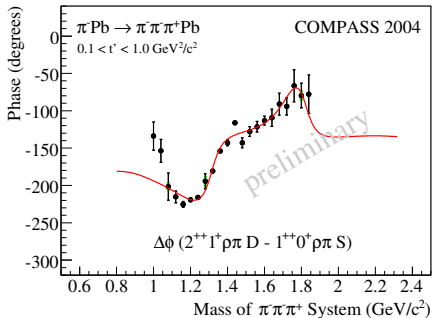
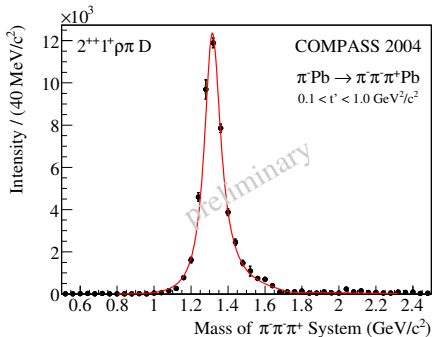
Name	Mass (GeV / c <sup>2</sup> )	$J^{PC}$
$f_0$	1370 / 1700	$0^+(0^{++})$
$\eta'$	1403	$0^+(0^{-+})$
$\rho'$	1450	$1^+(1^{--})$
$b_1$	1235 / 1800	$1^+(1^{+-})$
$f_1$	1285 / 1450	$0^+(1^{++})$
$\eta'_2$	1645	$0^+(2^{--})$
$f_2$	1565	$0^+(2^{++})$
$\rho_3$	1690	$1^+(3^{--})$



- **COMPASS 2004** pilot run using a 190 GeV  $\pi^-$  beam
  - ▶ **Diffraction dissociation** on lead targets exploited for meson production
  - ▶  $3\pi \sim 4\,000\,000$  events recorded within a **few days of data taking**
  - ▶  $5\pi \sim 370\,000$  events
  - ▶ **Excellent acceptance** for diffractive charged pion events ( $\sim 55\text{-}60\%$  for  $3\pi$ )
  
- **Partial wave analysis** on  $\sim 400\,000$   $\pi^- \pi^- \pi^+$  events with  $0.1 < t' < 1.0$  GeV<sup>2</sup>/c<sup>2</sup>
  - ▶ Dominant  $a_1(1260)$ ,  $a_2(1320)$  and  $\pi_2(1670)$  states resolved
  - ▶ Excellent agreement with PDG
  - ▶ Also small, well-known resonances  $\pi(1800)$  and  $a_4(2040)$  can be fitted
  
- **Spin-exotic  $1^{-+}$  state** observed in  $\rho\pi$  decay channel both in intensity and phase motion
  - ▶  $\Rightarrow$  consistent with  $\pi_1(1600)$  resonance
  - ▶ Publication about to be submitted

- Analysis of  $5\pi^\pm$  final state in progress
- Clear  $f_1$  signal in  $4\pi$  subsystem
- New PWA software: <http://ospdev.org/projects/rootpwa>
  - ▶ Users or collaborators welcome!
  
- COMPASS Hadron Run 2008
  - ▶ Change-over to liquid hydrogen target
  - ▶ Spectrometer upgrade (Recoil Detector, PID, ECAL ...)
  - ▶  $\sim 2$  orders of magnitude more high- $t'$  statistics has been collected
  - ▶ Analysis in progress
  
- COMPASS Hadron Run 2009 topics:
  - ▶ Central Production (proton and pion beam)
  - ▶ Diffractive Dissociation
  - ▶ Repeat measurements on Pb with upgraded spectrometer; more statistics
  - ▶ Primakoff



$2^{++}1^+\rho\pi D$ 

- Two Breit-Wigners needed to describe  $2^{++}1^+\rho\pi D$  phase motion:  
 BW1 for  $a_2(1320)$  + BW2 for  $a_2(1700)$
- $M = (1.321 \pm 0.001^{+0.000}_{-0.007}) \text{ GeV}$ ,  $\Gamma = (0.110 \pm 0.002^{+0.002}_{-0.015}) \text{ GeV}$
- $a_2(1700)$  parameters fixed to PDG values:  $M = 1.732 \text{ GeV}$ ,  $\Gamma = 0.194 \text{ GeV}$

## Mass-Independent Cross-Section and Spin Density Matrix

$$\sigma_{\text{indep}}(\tau) = \sum_{\epsilon} \sum_r \left| \sum_i T_{ir}^{\epsilon} \psi_i^{\epsilon}(\tau) / \sqrt{\int |\psi_i^{\epsilon}(\tau')|^2 d\tau'} \right|^2, \quad \rho_{ij}^{\epsilon} = \sum_r T_{ir}^{\epsilon} T_{jr}^{\epsilon*}$$

- $\epsilon$ : reflectivity,  $r$ : rank of density matrix,  $i$ : different partial waves
- $T$ : complex production amplitudes (fit parameters!)
- $\psi$ : complex decay amplitudes
- $\tau$ : phase space coordinates (5 parameters for 3-body decay)

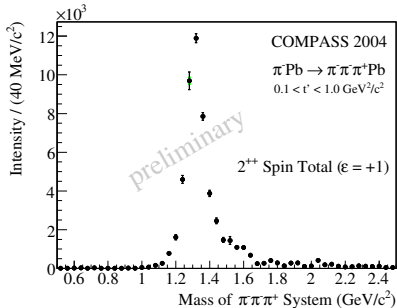
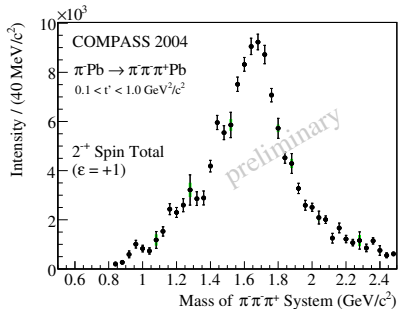
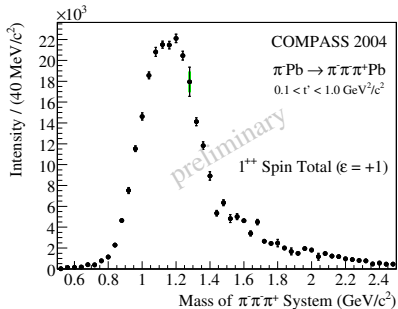
## Likelihood Function

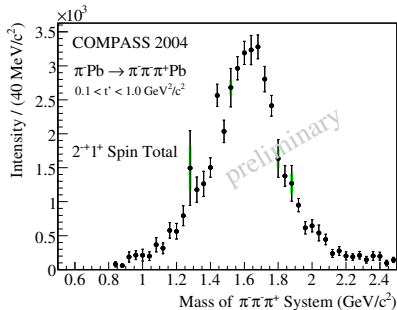
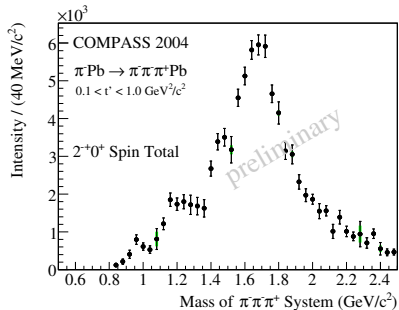
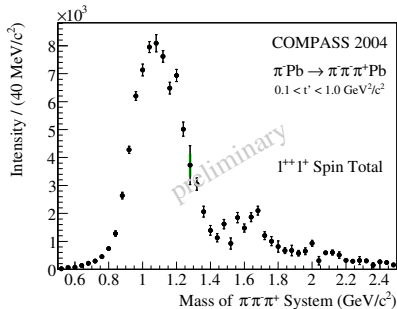
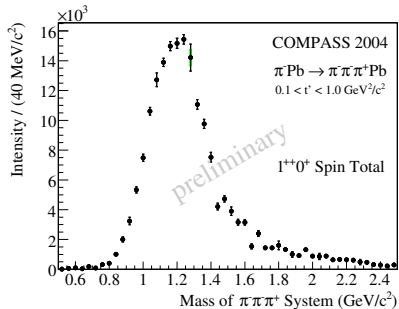
$$\ln L = \sum_n \ln \sigma_{\text{indep}}(\tau_n) - \int \sigma_{\text{indep}}(\tau') \text{Acc}(\tau') d\tau'$$

- $n$ : analyzed events, **Acc**: Acceptance

## Mass-Dependent Fit

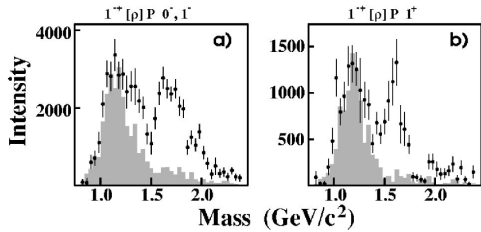
$$\rho_{ij}^{\epsilon} = \sum_r \left( \sum_k C_{ikr}^{\epsilon} \text{BW}_k(m) \sqrt{\int |\psi_i^{\epsilon}(\tau)|^2 d\tau} \right) \left( \sum_l C_{jlr}^{\epsilon} \text{BW}_l(m) \sqrt{\int |\psi_j^{\epsilon}(\tau)|^2 d\tau} \right)^*$$



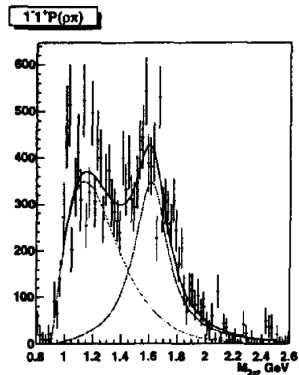


State	(GeV)	COMPASS $\pm$ stat $\pm$ syst	PDG
$a_1(1260)$	$M$	$1.256 \pm 0.006 + 0.007 - 0.017$	$1.230 \pm 0.040$
	$\Gamma$	$0.366 \pm 0.009 + 0.028 - 0.025$	0.250 to 0.600
$a_2(1320)$	$M$	$1.321 \pm 0.001 + 0.000 - 0.007$	$1.3183 \pm 0.0006$
	$\Gamma$	$0.110 \pm 0.002 + 0.002 - 0.015$	$0.107 \pm 0.005$
$\pi_1(1600)$	$M$	$1.660 \pm 0.010 + 0.000 - 0.064$	$1.653^{+0.018}_{-0.015}$
	$\Gamma$	$0.269 \pm 0.021 + 0.042 - 0.064$	$0.225^{+0.045}_{-0.028}$
$\pi_2(1670)$	$M$	$1.659 \pm 0.003 + 0.024 - 0.008$	$1.6724 \pm 0.0032$
	$\Gamma$	$0.271 \pm 0.009 + 0.022 - 0.024$	$0.259 \pm 0.009$
$\pi(1800)$	$M$	$1.785 \pm 0.009 + 0.012 - 0.006$	$1.812 \pm 0.014$
	$\Gamma$	$0.208 \pm 0.022 + 0.021 - 0.037$	$0.207 \pm 0.013$
$a_4(2040)$	$M$	$1.884 \pm 0.013 + 0.050 - 0.002$	$2.001 \pm 0.010$
	$\Gamma$	$0.295 \pm 0.024 + 0.046 - 0.019$	$0.313 \pm 0.031$

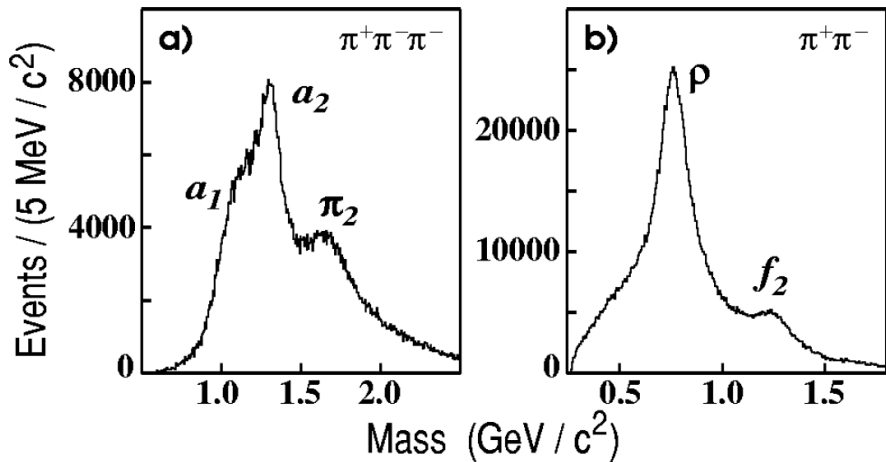




BNL-E852, Phys. Rev. **D65**, 072001, 2002



VES, Nucl. Phys. **A663**, 596, 2000

Phys. Rev. **D65**, 072001, 2002

Single pion decay angle in  $X^-$  rest frame (Gottfried-Jackson frame).

