Introduction to the PWA Isobar Technique

The Illinois-Protvino-Munich Program

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Physics and Methods in Meson Spectroscopy

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Motivation and Introduction

Physics Case and Task of the PWA

- Hadron spectroscopy: find (all) resonances and determine their properties (like mass M, width Γ, Spin J, Parity P, partial decay widths)

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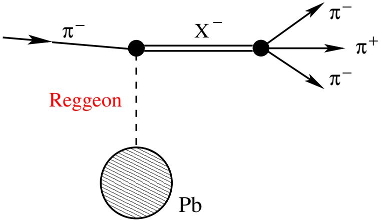
Different Production Mechanisms:

- $\bar{p}p$ annihilation
- Diffractive dissociation
- Central production
- Photo production
- ...



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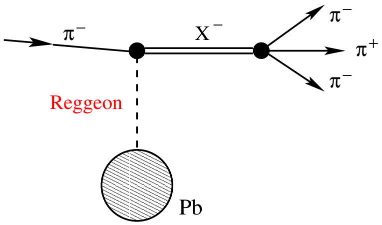
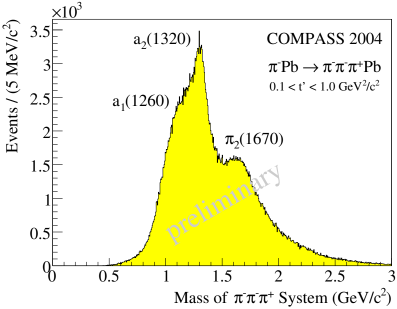


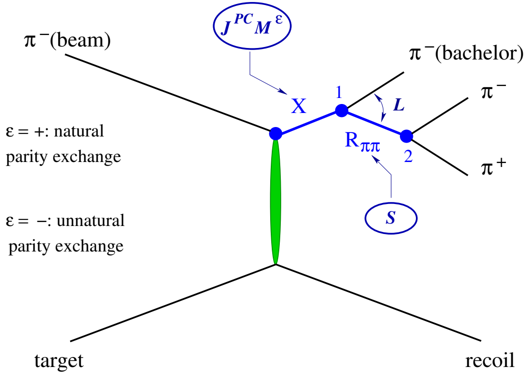


Partial Wave Analysis:

- Find and disentangle overlapping resonances X
- Determine J^P from angular distributions of input events
- Take into account exp. acceptance

→ Establish a model to fit the data

The Isobar Model and Decay Amplitudes



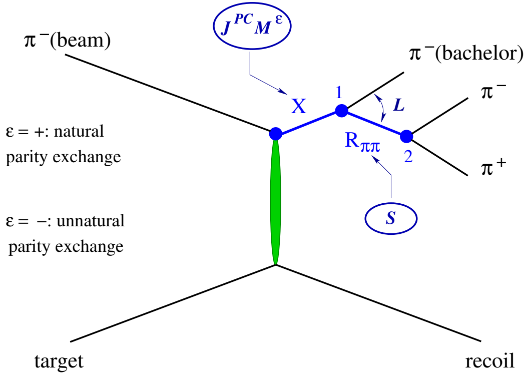
Isobar Model:

- How to describe the decay? → only two-body decays assumed
- L-S coupling
- (Known) intermediate resonances: isobars

→ Partial wave: $J^{PC}M^{\epsilon}[isobar]L$

(π^- beam: I^G fixed to 1^-, $C = +1$)

The Isobar Model and Decay Amplitudes



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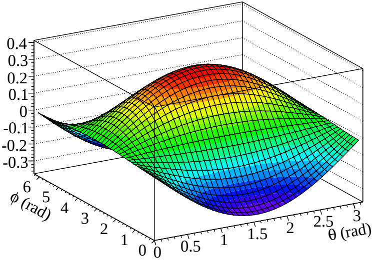
Decay Amplitudes $\psi(\tau, m)$ τ: phase space, $m = m_X$

- Choose a spin formalism: e. g. helicity, Zemach, ... (includes ref. system)
- reflectivity basis:

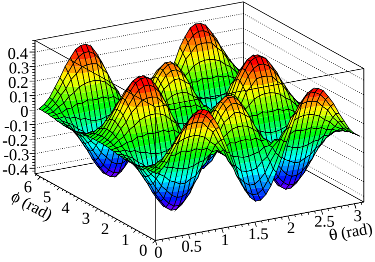
$$\psi^{\epsilon}_{JM}(\tau, m) = c(M)\left[\psi_{JM}(\tau, m) - \epsilon P(-1)^{J-M}\psi_{J(-M)}(\tau, m)\right]$$

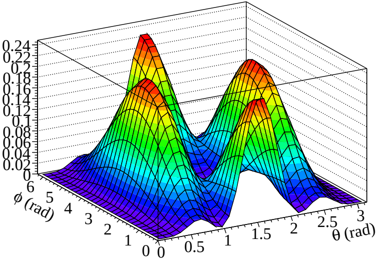
- Multiply sub-amplitudes for each tree node/isobar → contain angular distr. (D-funct./sph. harmonics) and isobar description

$\Re(Y_1^1)$



$\Re(Y_4^2)$





- Total intensity of Y_1^1 and Y_4^2
- Real life more complicated!

(Our) PWA Technique

General Ansatz

Model: The total PWA Cross-Section

$$\sigma(\tau, m) = \sum_{\epsilon=\pm1} \sum_{r=1}^{N_r} \left| \sum_i \sum_k C_{ikr}^{\epsilon} \, \mathrm{BW}_k(m, M_{0\,k}, \Gamma_{0\,k}) \; \psi_i^{\epsilon}(\tau, m) \right|^2$$

- ϵ: reflectivity, N_r: rank of spin density matrix
- i: different partial waves
- k: Breit-Wigner functions (or background): $\mathrm{BW}_k(m, M_{0\,k}, \Gamma_{0\,k})$
- C: complex production amplitudes
- ψ: complex decay amplitudes

- Waves with different ϵ do not interfere (parity conservation)
- π^- scattering on nucleons: $N_r = 2$ (spin-flip/non-flip processes)

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- Practical approach to determine production amplitudes and BW parameters $\rightarrow$ task split into two parts: mass-indep. PWA and mass-dep. fit

Mass-Independent PWA

Mass-Independent Cross-Section and Spin Density Matrix ρ

$$\sigma_{\text{indep}}(\tau, m) = \sum_{\epsilon=\pm1} \sum_{r=1}^{N_r} \left| \sum_i T_{ir}^{\epsilon} \psi_i^{\epsilon}(\tau, m) \Big/ \sqrt{\int |\psi_i^{\epsilon}(\tau', m)|^2 \mathrm{d}\tau'} \right|^2 \quad , \quad \rho_{ij}^{\epsilon} = \sum_{r=1}^{N_r} T_{ir}^{\epsilon} T_{jr}^{\epsilon *}$$

- Within (chosen) mass bins the mass-dependence of X is dropped
- Fit parameters are the complex amplitudes $T_{ir}^{\epsilon} \rightarrow N_r$ production vectors
- For each mass bin a separate fit is performed

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Extended Maximum Likelihood Method

$$\ln L = \sum_{n=1}^{N_{\text{events}}} \ln \sigma_{\text{indep}}(\tau_n, m_n) - \int \int \sigma_{\text{indep}}(\tau, m)\, \text{Acc}(\tau, m)\, \mathrm{d}\tau\, \mathrm{d}m$$

- N_{events}: event sample (real data), Acc: Acceptance (from MC)

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Fitting Package

- "Fumili/Ascoli/Kachaev" fitter: first and second derivatives used
- Multiple solutions ($\Delta \ln L \leq 1$) tested $\rightarrow$ additional error

Physics Observables from Mass-Indep. PWA

- Output of mass-indep. PWA is spin density matrix in each mass bin

Structure of Spin Density Matrix $\rho^{\epsilon}_{ij} = \sum_{r=1}^{N_r} T^{\epsilon}_{ir} T^{\epsilon *}_{jr}$ (hermitian)

- Block-diagonal in ϵ
- Chung-Trueman parameterization: [S. U. Chung and T. L. Trueman, Phys. Rev. **D11, 633]**
  - First $r - 1$ elements in each T^{ϵ}_{ir} put to zero
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Physics Observables

- Intensity of a partial wave: ρ_{ii}^{ϵ}
- Interference between partial waves described by off-diagonal elements ρ_{ij}^{ϵ}

$$\rho_{ij}^{\epsilon} = r_{ij}^{\epsilon}\, e^{i\phi_{ij}^{\epsilon}} \quad \text{and} \quad \mathrm{Coh}_{ij}^{\epsilon} = \frac{r_{ij}^{\epsilon}}{\sqrt{\rho_{ii}^{\epsilon}\rho_{jj}^{\epsilon}}}$$

Mass-Dependent Fit

- Input: spin density matrix in each mass bin (no events anymore!)
- Task: fit (subset of) spin density matrix as function of mass
  $\rightarrow$ find a model to describe PW intensities and their interferences

Decomposition of the mass-dep. Spin Density Matrix

$$\rho^{\epsilon}_{ij}(m) = \sum_{r=1}^{N_r} \left(\sum_k C^{\epsilon}_{ikr} \mathrm{BW}_k(m) \sqrt{\int |\psi^{\epsilon}_i(\tau)|^2 \mathrm{d}\tau} \right) \left(\sum_l C^{\epsilon}_{jlr} \mathrm{BW}_l(m) \sqrt{\int |\psi^{\epsilon}_j(\tau)|^2 \mathrm{d}\tau} \right)^*$$

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Further Remarks

- "BW" functions can also stand for background (like Deck effect)
- MINUIT used for fitting: χ^2 fit

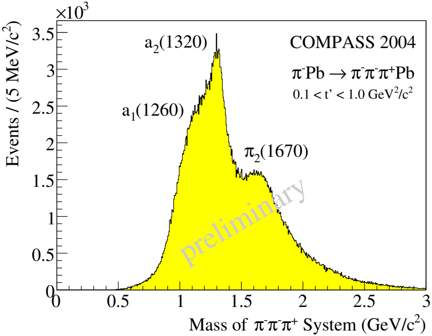
Example: Diffractive Meson Production at COMPAS

(Selected Results from 2004)

Analysis Overview

Event Selection:

- 190 GeV/c π^- beam on Pb
- Diffractive trigger
- Primary vertex in target with 3 outgoing particles $(- - +)$
- Exclusivity: target stays intact
- $\sim$ 400 000 events with $0.1 < t' < 1.0$ GeV$^2/c^2$



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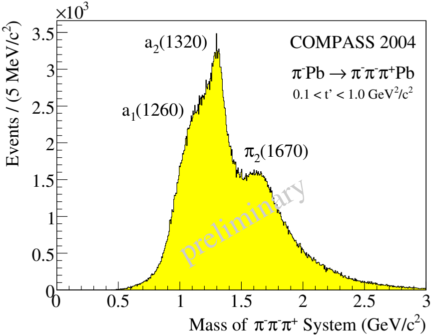

PWA Details

- Mass-independent fit:
  - 40 MeV/c^2 mass bins, 42 waves (lower mass thresholds)
  - Isobars: $\rho(770)$, $f_2(1270)$, $\rho_3(1690)$, $(\pi\pi)_s$ (separated $f_0(980)$), $f_0(980)$
  - Nucleon target $\Rightarrow$ rank 2, Pomeron exchange $\Rightarrow$ more $\epsilon = +1$ waves
  - Large t' range $\Rightarrow$ t'-dependence of PWs modelled
- Mass-dependent fit: 7 waves (all with $\epsilon = +1$)

Partial Wave Set for Mass-Indep. Fit (42 Waves)

$J^{PC}M^{\epsilon}$	L	Isobar π	Cut [GeV]
$0^{-+}0^{+}$	S	$f_0\pi$	1.40
$0^{-+}0^{+}$	S	$(\pi\pi)_s\pi$	-
$0^{-+}0^{+}$	P	$\rho\pi$	-
$1^{-+}1^{+}$	P	$\rho\pi$	-
$1^{++}0^{+}$	S	$\rho\pi$	-
$1^{++}0^{+}$	P	$f_2\pi$	1.20
$1^{++}0^{+}$	P	$(\pi\pi)_s\pi$	0.84
$1^{++}0^{+}$	D	$\rho\pi$	1.30
$1^{++}1^{+}$	S	$\rho\pi$	-
$1^{++}1^{+}$	P	$f_2\pi$	1.40
$1^{++}1^{+}$	P	$(\pi\pi)_s\pi$	1.40
$1^{++}1^{+}$	D	$\rho\pi$	1.40
$2^{-+}0^{+}$	S	$f_2\pi$	1.20
$2^{-+}0^{+}$	P	$\rho\pi$	0.80
$2^{-+}0^{+}$	D	$f_2\pi$	1.50
$2^{-+}0^{+}$	D	$(\pi\pi)_s\pi$	0.80
$2^{-+}0^{+}$	F	$\rho\pi$	1.20
$2^{-+}1^{+}$	S	$f_2\pi$	1.20
$2^{-+}1^{+}$	P	$\rho\pi$	0.80
$2^{-+}1^{+}$	D	$f_2\pi$	1.50
$2^{-+}1^{+}$	D	$(\pi\pi)_s\pi$	1.20
$2^{-+}1^{+}$	F	$\rho\pi$	1.20

$J^{PC}M^{\epsilon}$	L	Isobar π	Cut [GeV]
$2^{++}1^{+}$	P	$f_2\pi$	1.50
$2^{++}1^{+}$	D	$\rho\pi$	-
$3^{++}0^{+}$	S	$\rho_3\pi$	1.50
$3^{++}0^{+}$	P	$f_2\pi$	1.20
$3^{++}0^{+}$	D	$\rho\pi$	1.50
$3^{++}1^{+}$	S	$\rho_3\pi$	1.50
$3^{++}1^{+}$	P	$f_2\pi$	1.20
$3^{++}1^{+}$	D	$\rho\pi$	1.50
$4^{-+}0^{+}$	F	$\rho\pi$	1.20
$4^{-+}1^{+}$	F	$\rho\pi$	1.20
$4^{++}1^{+}$	F	$f_2\pi$	1.60
$4^{++}1^{+}$	G	$\rho\pi$	1.64
$1^{-+}0^{-}$	P	$\rho\pi$	-
$1^{-+}1^{-}$	P	$\rho\pi$	-
$1^{++}1^{-}$	S	$\rho\pi$	-
$2^{-+}1^{-}$	S	$f_2\pi$	1.20
$2^{++}0^{-}$	P	$f_2\pi$	1.30
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FLAT			

Partial Waves used in Mass-Dep. Fit (7 Waves)

$J^{PC} M^{\epsilon}$	L	Isobar π	Cut [GeV]
$0^{-+}0^{+}$	S	$f_0\pi$	1.40
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$0^{-+}0^{+}$	P	$\rho\pi$	-
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$1^{++}0^{+}$	P	$f_2\pi$	1.20
$1^{++}0^{+}$	P	$(\pi\pi)_s\pi$	0.84
$1^{++}0^{+}$	D	$\rho\pi$	1.30
$1^{++}1^{+}$	S	$\rho\pi$	-
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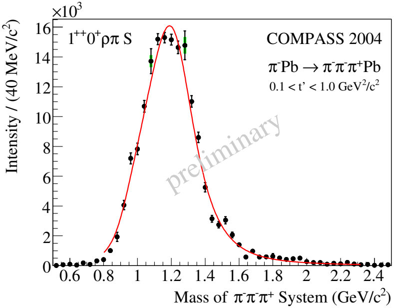
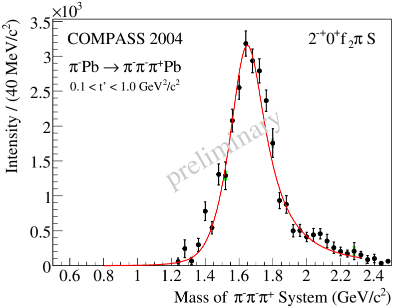
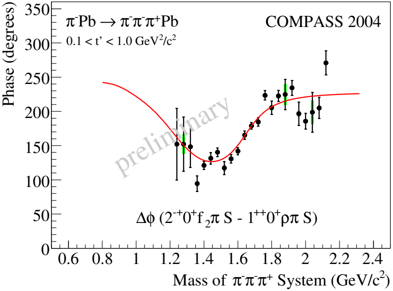
$1^{++}0^{+}\rho\pi S$ and $2^{-+}0^{+}f_2\pi S$







- BW for $a_1(1260)$ + background:

  $M = (1.256 \pm 0.006\ ^{+0.007}_{-0.017})$ GeV

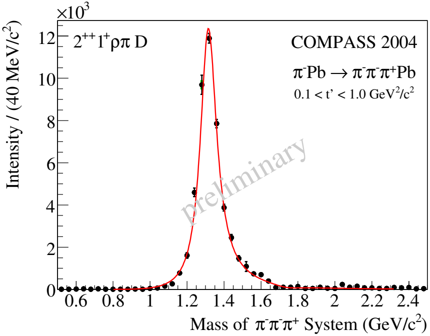
  $\Gamma = (0.366 \pm 0.009\ ^{+0.028}_{-0.025})$ GeV

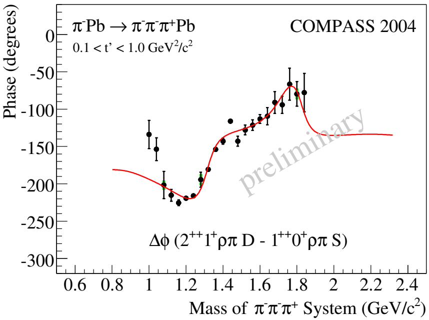
- BW for $\pi_2(1670)$:

  $M = (1.659 \pm 0.003\ ^{+0.024}_{-0.008})$ GeV

  $\Gamma = (0.271 \pm 0.009\ ^{+0.022}_{-0.024})$ GeV

$2^{++}1^{+}\rho\pi D$





- Two Breit-Wigners needed to describe $2^{++}1^{+}\rho\pi D$ phase motion: BW1 for $a_2(1320)$ + BW2 for $a_2(1700)$
- $M = (1.321 \pm 0.001\ ^{+0.000}_{-0.007})$ GeV, $\Gamma = (0.110 \pm 0.002\ ^{+0.002}_{-0.015})$ GeV
- $a_2(1700)$ parameters fixed to PDG values: $M = 1.732$ GeV, $\Gamma = 0.194$ GeV

Summary and Outlook

- PWA technique (Illinois-Protvino-Munich Program)
  - Basic assumption: isobar model
  - Two-step procedure: mass-independent PWA and mass-dependent fit
  - Physics observables derived from spin density matrix
- Examples from COMPASS 2004 data
- Crosscheck with BNL-E852 Program planned
- Different fitting packages could be tried
  → A. Caldwell, "Bayesian Fitting - BAT, a new tool", Friday, 11:20
- Include relativistic effects in PWA (decay amplitudes)
  → J. Friedrich, "Relativistic Effects in PW Formulation", Friday, 12:00

THANK YOU!