Final COMPASS results on the spin-dependent structure functions $g_1^d$ and $g_1^p$

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The COMPASS experiment at the CERN SPS took data with a polarised muon beam scattering off a polarised NH$_2$ target in 2007 and 2011 and scattering off a $^6$LiD target in 2002-2004 and 2006. The 2006 measurement increased the statistics from the 2002-2004 data taking by about 50%. For the measurement in 2011 the beam energy was increased from 160 GeV to 200 GeV, thus higher values of $Q^2$ and lower values of $x$ were reached. The new results from the 2011 and 2006 data taking on the longitudinal double spin asymmetry $A_1^{p,d}$ and the spin-dependent structure function $g_1^{p,d}$ are presented and compared to the previous results. Using the combined deuteron data set, the first moment of $g_1^d$ is calculated. From its first moment, the quark contribution to the nucleon spin is calculated. This quantity is also obtained from a NLO QCD fit to the world data including our data. The final deuteron results together with the two measurements on the proton are used to update the results on the Bjorken sum rule connecting the first moment of the non-singlet structure function to the ratio of the weak coupling constants. The recent results on the spin structure function $g_1^p$ and $g_1^d$ constitute the COMPASS legacy on the measurements of $g_1$ through inclusive spin-dependent deep inelastic scattering.

I. INTRODUCTION

The COMPASS experiment is investigating the composition of the spin of the nucleon, which is of special interest since only about 30% of it is carried by the quarks. In order to study the spin structure, the spin-dependent structure function $g_1$ is measured. In addition to the structure function, also its first moment, $\Gamma_1 = \int_0^1 g_1(x) dx$, is of interest. The one of the deuteron spin-dependent structure function is related to the total contribution from quark spins to the nucleon spin. Furthermore, the difference between the first moment of the proton and neutron spin-dependent structure function is connected to the ratio of the weak coupling constants. This relation is called Bjorken sum rule. Using only the COMPASS data, this sum rule is tested.

For the measurement of the structure function $g_1$, data were taken in 2002-2004 using a $^6$LiD target. In addition also data from 2006 are available, which increase the statistics by about 50%. The measurement using a NH$_3$ target was performed in 2007 and 2011. For the latter, the beam energy was increased from 160 GeV to 200 GeV. This gives access to higher values of $Q^2$ and smaller values of $x$.

II. THE COMPASS EXPERIMENT

A detailed description of the COMPASS experiment is given in Ref. [1]. The COMPASS collaboration runs a fixed target experiment located at the M2 beamline of the CERN SPS. The nominal beam energy of the muon beam is 160 GeV. For the data taking in 2011 it was increased to 200 GeV. The muon beam is naturally polarised (polarisation $\sim 80\%$). Since 2006, the target used for the measurement of the spin-dependent structure function consists of three cells. The two outer cells (30 cm long) were polarised in the opposite direction compared to the central cell (60 cm long). The diameter of the cells was 3 cm during the 2006 data taking and 4 cm during the 2011 one. For a better cancellation of the spectrometer acceptance, the polarisation of the cells was changed by rotating the solenoid field on a regular basis.

III. RESULTS ON $A_1$ AND $g_1$

The longitudinal double spin asymmetry $A_1$ is extracted using a simultaneous measurement of deep inelastic scattering for both target polarisations. The collected data is subdivided into periods of stable data taking that contains data for both polarisations of each target cell. For each of these periods, the asymmetry is calculated taking into account the beam and target polarisations together with the dilution and depolarisation factors. The individual results are merged to obtain the final results. In case of the measurement using the LiD target, the results from the 2002-2004 data taking are given in Ref. [2]. The results obtained from the 2006 data taking are compared to the previous results in Figure 1 showing very good agreement between both sets of measurements.

The combined COMPASS results on the double spin asymmetry are obtained by using the weighted mean. The combined data set is the most precise measurement of $A_1^d$ at low $x$. This is shown in Figure 2 where the combined COMPASS deuteron data are shown together with the world data at their measured kinematics showing good agreement. This illustrates the well-known weak
**FIG. 1.** Comparison between the results on the longitudinal double spin asymmetry $A_d^1$ obtained from the 2002-2004 data and the 2006 data. The band at the bottom corresponds to the systematic uncertainty.

**FIG. 2.** Comparison of the combined COMPASS results on $A_d^1$ to the world data (CLAS [3], HERMES [4], SMC [5], E155 [6] and E143 [7]). All data points are at their measured kinematics. The band at the bottom shows the systematic uncertainty of the COMPASS data.

**FIG. 3.** Comparison of the mean values of $Q^2$ as a function of $x$ for the 2007 and 2011 data.

**FIG. 4.** Comparison of the two COMPASS results on $A_p^1$ to the world data (CLAS [3], HERMES [4], SMC [5], E155 [10], E143 [7] and EMC [11]). All data points are at their measured kinematics. The bands at the bottom show the systematic uncertainty of the COMPASS data.

$Q^2$ dependence of the asymmetry. The data cover the range of about $Q^2 = 6$ (GeV/c)$^2$ to $Q^2 = 60$ (GeV/c)$^2$ at high $x$.

The measurement of the double spin asymmetry $A_d^1$ using the NH$_3$ target was performed in 2007 [8] and in 2011 [9]. For the measurement in 2011, the beam energy was increased to 200 GeV. This results in higher values of $Q^2$ and the possibility to reach lower values in $x$. A comparison between the mean values of those two kinematic variables is shown in Figure 3 for the two data sets. The effect of the increase beam energy is visible in each $x$ bin.

The results from the 2007 and 2011 data taking are compared to the world data on $A_p^1$ in Figure 4. All data sets are shown at their measured kinematics. The two data sets of COMPASS agree very well with each other. In addition, the results agree well with the world data illustrating again the weak dependence of the asymmetry on $Q^2$.

The spin-dependent structure function is related to the double spin asymmetry via $g_1 = F_2 A_1/(2x(1 + R))$. Here, the parametrisation of $F_2$ from SMC [5] is used together with the ratio $R = \sigma_L/\sigma_T$ of the absorption cross-sections for longitudinal and transverse virtual photons taken from Ref. [12]. The results on the spin-dependent structure function obtained from the COMPASS data are compared to the world data in Figures 5 and 6. In addition, also the results from our NLO QCD fit are shown. All details on the analysis of the 2006 data are given in Ref. [13] and the ones for the analysis of the 2011 data are given in Ref. [9].
IV. QCD FIT TO THE \( g_1 \) WORLD DATA

The COMPASS results on the spin-dependent structure functions are used in a NLO QCD fit, together with the world data on \( g_1^p, g_1^n \) and \( g_1^d \) with \( Q^2 > 1 \text{(GeV}/c^2)^2 \) and \( W^2 > 10 \text{(GeV}/c^2)^2 \) in a NLO QCD fit. The fit was performed in the \( \overline{\text{MS}} \) renormalisation and factorisation scheme. The \( Q^2 \) evolution was performed using the DGLAP equations for the \( Q^2 \) evolution. As they only describe the dependence on \( Q^2 \) and not the \( x \) dependence, a functional form for the quark single distribution \( q^i \), the two non-singlet quark distributions \( q_3 \) and \( q_8 \) and the gluon spin distribution was introduced. The functional form assumed at an input scale of \( Q_0^2 = 1 \text{(GeV}/c^2)^2 \) is

\[
f = \eta \frac{x^\alpha(1-x)^\beta(1+\gamma x)}{\int_0^1 x^\alpha(1-x)^\beta(1+\gamma x) dx}.
\]

Here, \( f \) represent the various quark distributions and \( \eta \) represents the first moment of a particular distribution. They are fixed in case of the non-singlet distribution by the baryon decay constants, \( \eta_3 = 3F - D \) and \( \eta_8 = F + D \). In case of the non-singlet distributions the parameter \( \gamma \) was fixed to zero. This was also done for the gluon distribution. The parameter \( \beta \) of the gluon distribution was fixed to the value of the unpolarised distribution from MSTW [11] as it is not constrained by the data. In each iteration step of the fit, the positivity constrain \( |\Delta q(x)| < q(x) \) was tested. An extra term was added to the \( \chi^2 \) to penalise violations.

A detailed study of the systematic uncertainty was performed taking into account the dependence on different parametrisations and the dependence on the input scale. The dependence on the input scale was studied in various fit for input scales between \( Q_0^2 = 1 \text{(GeV}/c^2)^2 \) and \( Q_0^2 \approx 70 \text{(GeV}/c^2)^2 \). The systematic studies result in two kinds of functional shapes, which describe the data equally well. One solution with \( \gamma_8 \) fixed to zero, which results in a negative gluon distribution, and one with \( \gamma_8 \) as a free parameter resulting in a positive gluon distribution. This studies show that the contribution from systematics is much larger than the statistical uncertainty of the fit.

The results from the NLO QCD fit are shown in Figure 7 for the singlet distribution and the polarised parton distribution functions. The contribution from the gluons to the nucleon spin is not well constrained by the fit, whereas the contribution from the quarks was found to be:

\[
0.26 < \Delta \Sigma < 0.36.
\]

V. FIRST MOMENT OF \( g_1 \) AND NUCLEON AXIAL CHARGE

The combined COMPASS deuteron data are used to calculate the first moments of the spin-dependent structure function. For the calculation, all data points are evolved to a common \( Q^2 \) of \( 3 \text{(GeV}/c^2)^2 \). The evolution was performed using the results from the NLO QCD fit described before. In addition to the contribution from the data, also a contribution from the unmeasured region needs to be considered. This contribution was cal-
calculated using an extrapolation of the results from the NLO QCD fit to \( x = 0 \) and \( x = 1 \). The contribution from the unmeasured region to the full first moment is only 3\%. The result for the first moment of the nucleon, \( g_1^N = g_1^N/(1 - 1.5\omega d) \) with \( \omega_d = 0.05 \pm 0.01 \) \[15\], at \( Q^2 = 3 \text{ (GeV/c)}^2 \) is:

\[
\Gamma_1^N = 0.046 \pm 0.002 \text{(stat.)} \pm 0.004 \text{(syst.)} \pm 0.005 \text{(evol.)}. \tag{3}
\]

The evolution uncertainty is connected to the uncertainty of the NLO QCD fit. The systematic uncertainty takes into account the uncertainty on the target and beam polarisations, the uncertainty on the dilution and depolarisation factors and the \( F_2 \) parametrisation. The new result is in good agreement with the one obtained from the 2002-2004 data set. The 2006 data reduced again the statistical uncertainty. The 2006 data yield a reduced statistical error.

The first moment of the nucleon spin-dependent structure function is of special interest since it is connected to the singlet charge \( a_0 \). It is identified with \( \Delta \Sigma \) in the MS scheme:

\[
\Gamma_1^N(Q^2) = \frac{1}{36} [a_8 C^{NS}(Q^2) + 4a_0 C^S(Q^2)]. \tag{4}
\]

\( C^{NS} \) and \( C^S \) are the non-singlet and singlet coefficients, which are calculated in in the NLO approximation of QCD \[16\ 17\]. For the calculation of \( a_0 \), the axial charge \( a_8 = 0.585 \pm 0.025 \) is used \[18\]. The result obtained at \( Q^2 = 3 \text{ (GeV/c)}^2 \) is:

\[
a_0 = 0.32 \pm 0.02 \text{(stat.)} \pm 0.04 \text{(syst.)} \pm 0.05 \text{(evol.)}. \tag{5}
\]

This result is in good agreement with the one \( a_0 = 0.33 \pm 0.03 \text{(stat.)} \pm 0.05 \text{(syst.)} \) obtained from the 2002-2004 data, where the systematic uncertainty includes the evolution uncertainty. The 2006 data reduced again the statistical error. The result is also in good agreement with the result from the NLO QCD fit to the world data.

**VI. BJORKEN SUM RULE**

The results on the proton and deuteron spin-dependent structure function from COMPASS are used to calculate the non-singlet structure function:

\[
g_1^{NS} = g_1^p - g_1^n = 2 \left( g_1^p - \frac{g_1^d}{1 - 1.5\omega_D} \right). \tag{6}
\]

The first moment of this structure function is connected to the ratio of the weak coupling constants:

\[
\Gamma_1^{NS} = \frac{1}{6} \left| g_A \right| C^{NS}(Q^2). \tag{7}
\]

This relation is the Bjorken sum rule. The ratio \( g_A/g_V \) is obtained from the neutron \( \beta \) decay, \( g_A/g_V = 1.2723 \pm 0.0023 \) \[19\].

The calculation of the non-singlet first moment is done differently compared to the one of the nucleon. The various data points from the three COMPASS data sets are evolved to the \( Q^2 \) of the 2007 data using the results from the NLO QCD fit. Afterwards, the non-singlet structure function was calculated. A separate NLO QCD fit was performed to the resulting non-singlet structure function, where only the non-singlet distribution \( \Delta g_3 \) was fitted. This required only three parameters to describe the data. A comparison between the result of the non-singlet NLO QCD fit and the data points is shown in Figure 8. The result from the non-singlet fit was used to evolve the data points to a common \( Q^2 \) of 3 (GeV/c)^2. Afterwards the first moment was calculated for the measured region. The contribution from the unmeasured region was calculated using the results from the non-singlet NLO QCD fit. Their contribution is about 6\% to the full first moment. The obtained first moment for \( Q^2 = 3 \text{ (GeV/c)}^2 \) is:

\[
\Gamma_1^{NS} = 0.192 \pm 0.007 \text{(stat.)} \pm 0.015 \text{(syst.)}. \tag{8}
\]
The contributions to the systematic uncertainty are the same as in case of the first moment of the nucleon. Using the non-singlet coefficient $|g_A/g_V|_{\text{NLO}} = 1.29 \pm 0.05_{\text{(stat.)}} \pm 0.10_{\text{(syst.)}} \quad (9)$

The dominant contribution to the systematic uncertainty is the uncertainty on the beam polarisation. The result is in good agreement with the results obtained from the neutron $\beta$ decay. Thus the Bjorken sum rule is verified at the level of 9%.

VII. CONCLUSION

The new 2006 data on the double spin asymmetry $A_1^p$ and the spin-dependent structure function $g_1^p$ increase the statistics of the 2002-2004 results by about 50%. This is important at low $x$, where only an additional measurement from SMC exists. The combined data is used to calculate the first moment of the spin-dependent structure function, which is connected to the singlet charge $a_0$. It is of special interest since it represents the contribution from the quarks to the nucleon spin in the $\overline{\text{MS}}$ scheme. The obtained results is $a_0 = 0.32 \pm 0.02_{\text{(stat.)}} \pm 0.04_{\text{(syst.)}} \pm 0.05_{\text{(evol.)}}$.

The new results of the 2011 data taking increase the kinematic coverage of the COMPASS data on the spin-dependent structure function $g_1^p$ and the double spin asymmetry $A_1^p$ towards larger values of $Q^2$ and lower values of $x$. Using these results together with the ones from the deuteron, a validation of the Bjorken sum rule was performed at the level of 9%. The obtained ratio of the weak coupling constants is $|g_A/g_V|_{\text{NLO}} = 1.29 \pm 0.05_{\text{(stat.)}} \pm 0.10_{\text{(syst.)}}$.

All results on the spin-dependent structure function measured at COMPASS were used together with the world data on $g_1$ in a NLO QCD fit to extract the polarised parton distribution functions. The fit results in a contribution from the quarks to the nucleon spin in the range of: $0.26 < \Delta \Sigma < 0.36$. This result is in good agreement with the results for $a_0$ using only the COMPASS deuteron data with similar precision.

The recent results on the spin structure function $g_1^p$ and $g_1^d$ constitute the COMPASS legacy on the measurements of $g_1$ through inclusive spin-dependent deep inelastic scattering.