# The QCD Analysis Of The World Data On Structure Functions $g_1^{p,d,n}$ For Proton, Deuteron And Neutron

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**Abstract.** The fits of all published data on $g_1$, including the new COMPASS measurements of $g_1^d(x,Q^2)$, have been performed by using two different QCD evolution formalisms in the next-to-leading-order (NLO) approximation. In both methods we obtain two solutions for fitted parameters of the parton distribution functions (PDFs), one with $\Delta G>0$ and the other - with $\Delta G<0$, where $\Delta G$ is the first moment of the polarized gluon distribution in nucleon.

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## 1. INPUT DATA AND FITTING PROCEDURES

The NLO QCD fits on $g_1(x, Q^2)$, performed by the COMPASS Collaboration, have been motivated by two reasons: first - to see an impact of the new COMPASS data [1] on the parametrizations of polarized PDFs [2], second - to cross check the QCD results by using two different methods of the analysis.

All published data (see [1] and references therein) on spin-dependent structure functions $g_1^p(x,Q^2), g_1^n(x,Q^2)$ and $g_1^d(x,Q^2)$ were used as an input. The data have included 230 $g_1(x, Q^2)$ points and covered a DIS region from 1 to about 100 $GeV^2$ in $Q^2$. Usual DIS cut $Q^2 > 1$ $GeV^2$ limits the $x$ range. For the COMPASS data this limit corresponds to $x>0.004$. This is the lowest $x$-value for all the data. The highest values of $x$ are limited by acceptance and statistics at $x \approx 0.75$.

In the quark-parton model the $g_1$ is expressed as convolutions of the non-singlet ($\Delta q_3$ and $\Delta q_8$), singlet ($\Delta\Sigma$) and gluon ($\Delta G$) parton distribution functions and corresponding Wilson coefficient functions. In the 3 quark limit the PDFs are related to the $u$-, $d$- and $s$-quark distributions:

$\Delta\Sigma = \Delta u + \Delta d + \Delta s$, $\Delta q_3 = \Delta u - \Delta d$, $\Delta q_8 = \Delta u + \Delta d - 2\Delta s$.

Fits have been performed by two programs. The Program 1 [3] is based on the numerical solution of the DGLAP integro-differential evolution equations for PDFs. The Program 2 [4], first, finds the analytical solution of differential equations for the PDFs moments and, second, performs the inverse Mellin transformation of moments for the final PDFs reconstruction.

The programs work in the $\overline{MS}$ renormalization and factorization scheme in the NLO approximation and require the input $x$-parametrization of PDFs at the initial $Q_0^2 = 3GeV^2$, which is close to the weighted average of the present COMPASS data.

The PDFs $\Delta\Sigma$, $\Delta q_3$, $\Delta q_8$ and $\Delta G$ are parametrized as:

$$\Delta F(x,Q_0^2) = \eta_k \frac{x^{\alpha_k}(1-x)^{\beta_k}(1+\gamma_k x)}{\int_0^1 x^{\alpha_k}(1-x)^{\beta_k}(1+\gamma_k x)dx}, \qquad \eta_k = \int_0^1 \Delta F_k(x,Q_o^2)dx$$

and 11 parameters $\eta_k$, $\alpha_k$, $\beta_k$ *and* $\gamma_k$ are found by minimization of the sum:

$$\chi^2 = \sum_{i=1}^{230} \frac{\left[g_1^{fit}\left(x_i,Q_i^2\right) - g_1^{\exp}\left(x_i,Q_i^2\right)\right]^2}{\left[\sigma\left(x_i,Q_i^2\right)\right]^2},$$

where $g_1^{fit}$ and $g_1^{\exp}$ is the evolved and experimental value of the structure function, respectively, and $\sigma$ is a statistical error. Fits are performed under conditions that: parameters $\eta_3$ and $\eta_8$ are fixed by the baryon octet constants F and D assuming the $SU(3)_f$ flavor symmetry; linear term $\gamma_k x$ is used for the singlet PDFs only; $\alpha_s(Q^2)$ evolved from the PDG value $\alpha_s\left(M_z^2\right) = 0.1187 \pm 0.005$; positivity limits $|\Delta s(x)| \le s(x)$ and $|\Delta G(x)| \le G(x)$ are imposed at each step of optimizations and the unpolarized PDFs are taken from MRST parametrizations [5].

## 2. RESULTS OF THE FITS

The both programs give consistent shapes of the PDFs and values of its fitted parameters (see the list in [1]) with similar $\chi^2$ for the two solutions, one - with $\Delta G$>0 ( $\chi^2$/ndf = 233/219) and the other - with $\Delta G$<0 ( $\chi^2$/ndf = 247/219).

The parameters $\eta_\Sigma$, or the first moments of the singlet quark distributions, for the two solutions and two programs, are very close to each other and can be averaged: $\eta_\Sigma$ =0.30±0.01(stat.) ±0.03(syst), where the systematic error represents the difference between the fits.

The fits reproduce trends of the world $g_1$ data rather well (see Fig.1) but precisions of the measurements, especially for $g_1^d$ and $g_1^n$, are still poor.

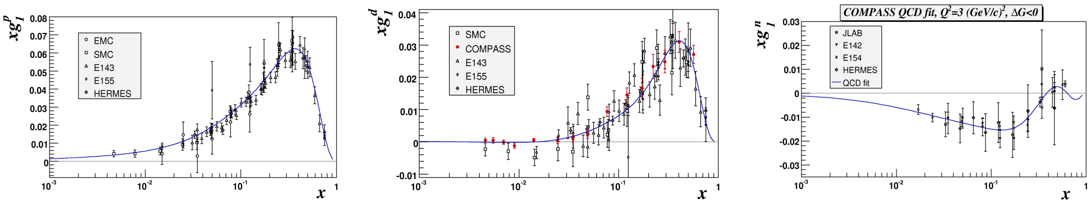


**FIGURE 1.** The world data on $g_1(x)$ at $Q^2$=3 $GeV^2$. The solid line is the fit for $\Delta G$<0, the fit for $\Delta G$>0 is not distinguished in this scale.

The fitted $g_1^N(x) = g_1^d(x,Q^2)/(1-1.5\omega_D)$ at $Q_0^2 = 3GeV^2$, where $\omega_D$ is a probability for the D-wave state of the deuteron, are compared in Fig.2 with the COMPASS data [1] and with the published PDFs parametrizations [2] obtained without the new COMPASS measurements of $g_1^d$.

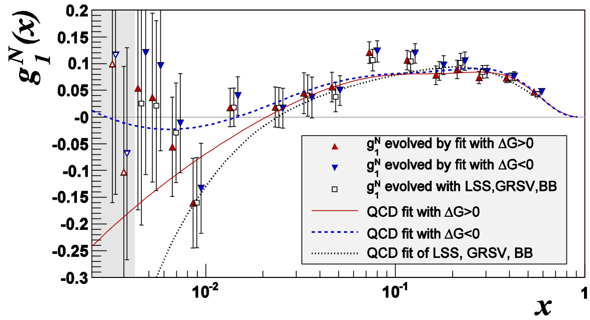


**FIGURE 2.** The COMPASS values of $g_1^N(x)$ evolved to $Q^2$ = 3 $GeV^2$ with $\Delta G$ >0 and $\Delta G$< 0, the corresponding results of the world data fits and the published LSS, GRSV and BB parametrizations, are shown. These parametrizations lead to almost the same values of $g_1^N$ and have been averaged. Two additional COMPASS points at the smallest $x$ obtained with $Q^2$ >0.7 $GeV^2$ (not included in the fits), are also shown. The data points evolved with different fits are shifted in $x$ for clarity. Errors are statistical only.

From this figure one can see that (1) even additional two points at the smallest $x$ do not help to choose between the $\Delta G$ solutions; (2) previous parametrizations do not reproduce the trend of the COMPASS data at $x\to 0$ and (3) the fit with $\Delta G$ >0 shows a dip at $x$~0.25 related to the shape of $\Delta G(x)$.

The shapes of $\Delta G(x)$ obtained by two Programs are shown in Fig.3 for $\Delta G>0$ and $\Delta G<0$. They are almost identical. For the positive case the shape of $\Delta G(x)$ is restricted from two sides: it must be close to zero at low $x$, to avoid pushing $g_1^N(x)$ down to negative values, and limited at higher $x$ by the positivity constraint $|\Delta G(x)|<G(x)$. As a consequence, the whole $\Delta G(x)$ is squeezed in a narrow interval of $x$ around the

maximum at $x \approx 0.25$. For the negative case the shape of $\Delta G(x)$ is rather smooth in the whole range of $x$.

Although the gluon distributions strongly differ in two fits, the fit parameters $\eta_G$, i.e. the first moments of the gluon distributions, are both small and almost equal in the absolute value: $|\eta_G| \approx 0.2$-$0.3$. So, the gluon contribution to the nucleon spin is rather small.

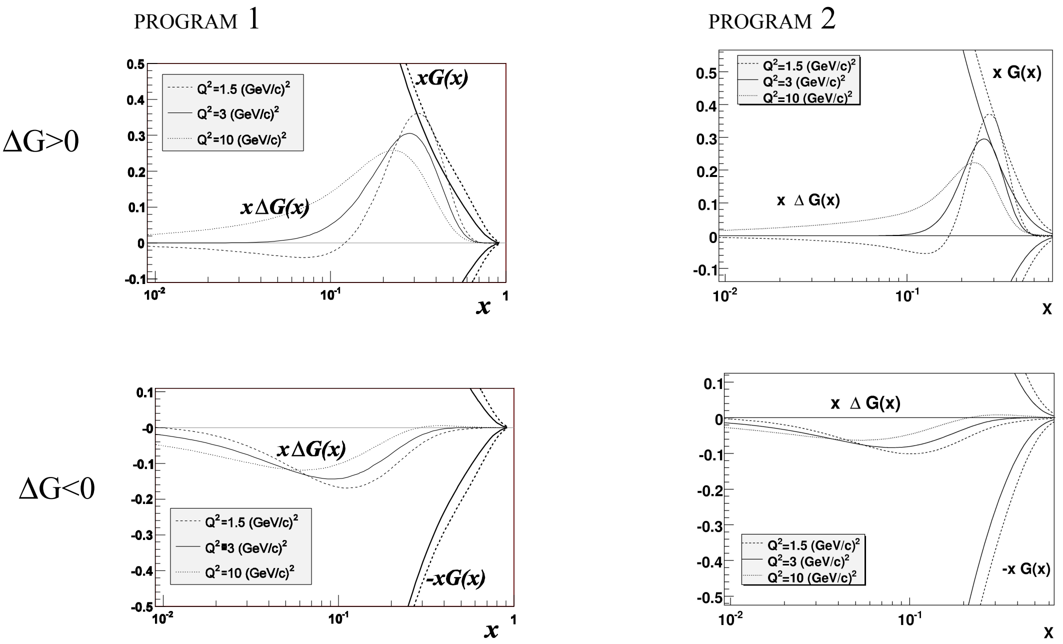


**FIGURE 3.** The gluon distributions $x \cdot \Delta G(x)$ corresponding to the fits with $\Delta G > 0$ (upper plots) and $\Delta G < 0$ (lower plots) obtained with Program 1 (left) and Program 2 (right). The dashed, solid and dotted lines correspond to $Q^2 = 1.5, 3$ and $10\ GeV^2$, respectively. The unpolarized distributions $x \cdot G(x)$, used as constraints, are shown for $Q^2 = 1.5$ and $3\ GeV^2$.

The direct measurements of the gluon polarization could help to choose between the two solutions for $\Delta G$. In Fig.4 the available data are compared to the fitted $\Delta G(x)/G(x)$. The most precise COMPASS – high $p_T$ – $Q^2 < 1\ GeV^2$ – point is closer to the $\Delta G > 0$ solution, although it is only $1.3\sigma$ away from $\Delta G < 0$. Further improvement of the precision is needed.

The polarized strange quark distributions, obtained from the fits, are almost identical for $\Delta G > 0$ and $\Delta G < 0$. The first moment of the polarized strange quark distribution found from the fits at $Q^2 = 3GeV^2$, is $(\Delta s + \Delta\bar{s}) = -0.10 \pm 0.01(stat) \pm 0.01(evol)$, where the evolution error is estimated from the results obtained by the two programs.

In conclusion, the new QCD NLO fits of the world data on $g_1$ have produced consistent results and yielded the two solutions for the PDFs parameters with $\Delta G(x) > 0$

and $\Delta G(x)$ <0, which equally well describe the present $g_1$ data. Direct measurements of the gluon polarization could help to choose between them. The first moments of the polarized singlet quark, gluon and strange quark distributions found from fits at $Q^2$=3$GeV^2$, are equal to: $\eta_\Sigma = 0.30 \pm 0.01(stat) \pm 0.03(syst), |\eta_G| \approx 0.2 - 0.3$, $(\Delta s + \Delta\bar{s}) = -0.10 \pm 0.01(stat) \pm 0.01(syst)$, respectively, where systematic errors are estimated from the difference of the results obtained by two programs.

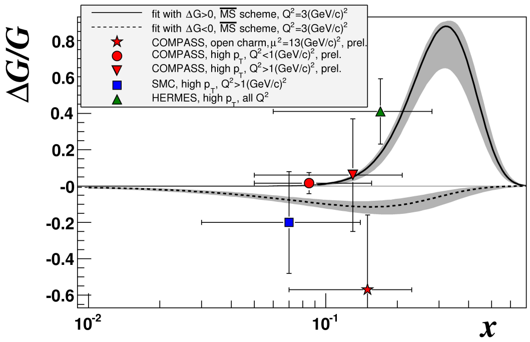


**FIGURE 4**. The gluon polarization $\Delta G(x)/G(x)$ at $Q^2$ = 3 $GeV^2$ for the fits with $\Delta G$>0 (solid line) and $\Delta G$ <0 (dashed line). The bands correspond to the statistical errors.

# 3. REFERENCES

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