EXOTICS AND GLUEBALLS ON THE LATTICE

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Abstract
I review the results from lattice gauge theory for the properties of the light 1^{-+} exotic state and 0^{++} glueball.

1. INTRODUCTION

High-energy experiments have confirmed that QCD is a simple elegant theory that contains quarks and gluons. At low energy the states observed in experiments are messy hadrons. It is hard to relate the world of quarks and gluons to the ‘real-world’ practicalities of hadrons, because QCD is such a hard theory to solve. A particularly good test of our understanding of the non-perturbative aspects of QCD is to study particles where the gauge field is excited somehow, and hence playing a more important dynamic role than in ‘standard’ hadrons. Examples of such particles are glueballs (particles made out of the gauge fields) and hybrid mesons ($\overline{q}q$ and excited glue).

Quantities, such as masses, depend on the coupling (g) like $M \sim e^{-1/g^2}$ [1], hence perturbation theory can’t be used to compute the masses of hadrons such as the proton. The only technique that offers any prospect of computing masses and matrix elements non-perturbatively, from first principles, is lattice QCD. I review the results from the lattice for the glueballs and mesons with exotic quantum numbers, Other recent reviews [2–4] of lattice results for hybrids and glueballs focus on different aspects of the subject.

2. LATTICE QCD CALCULATIONS

In this section, I briefly describe the formalism for lattice QCD calculations. The lecture notes by Gupta [5] provide specific details about lattice QCD calculations.

Many bound state properties of QCD can be determined from the path integral

$$c(t) \sim \int dU \int d\psi \int d\overline{\psi} \sum_{\underline{x}} O(\underline{0}, 0) O(\underline{x}, t)^{\dagger} e^{-S_F - S_G} \tag{1}$$

where S_F is the fermion action (some lattice version of the continuum Dirac action) and S_G is the pure gauge action. The path integral in Eq. (1) is put on the computer using a clever finite difference formalism [1], due to Wilson, that maintains gauge invariance. The path integral in Eq. (1) is evaluated using algorithms that are generalizations of the Monte Carlo methods used to compute low-dimensional integrals. The algorithms produce samples of gauge fields, that are essentially snapshots of the vacuum. The physical picture for Eq. (1) is that a hadron is created at time 0 using an interpolating operator. The quarks then propagate to time t in the background of the gauge fields, where the hadron is destroyed. The physics from the calculation is extracted using a fit model [1]:

$$c(t) = a_0 e^{-m_0 t} + a_1 e^{-m_1 t} + \cdots \tag{2}$$

where m_0 (m_1) is the ground (first excited) state mass and the dots represent higher excitations. Although in principle excited state masses can be extracted from a multiple exponential fit, in practise this is a numerically non-trivial task, because of the noise in the data from the Monte Carlo calculation. More sophisticated fitting techniques are starting to be used to extract masses from the data. For example the CP-PACS Collaboration have used maximum entropy fitting techniques [6] to look at excited states of the rho and pion in quenched QCD. CP-PACS obtained the masses of the first excited rho and pion to be

1540 (570) MeV and 660 (590) MeV respectively [6]. More experience is needed with these 'advanced fitting' methods before they can be used to make physical predictions.

Any gauge invariant combination of quark fields and gauge links can be used as interpolating operators ($O(\underline{x}, t)$) in Eq. (1). Interpolating operators that are similar to the state will couple strongly to the state. For example, a $\overline{q}\overline{q}qq$ state may not couple strongly to an interpolating operator with only a valence content of $\overline{q}q$.

The fermion integration can be done exactly in Eq. (1) to produce the quark determinant. The determinant describes the dynamics of the sea quarks. In quenched QCD calculations, the quark determinant is set to a constant. Quenched calculations are roughly 1000 times computationally cheaper than the calculations that include the dynamics of the sea quarks.

There are a variety of heuristic ways of understanding quenched QCD. One way is to view quenched QCD as QCD with infinitely heavy sea quarks. The connection between quenched QCD and the large N_c (number of colours) limit of QCD has recently been discussed by Chen [7]. Perhaps, surprisingly quenched QCD gives quite a reasonable description of experiment. For example, the most accurate quenched calculation of the hadron spectrum, to date, has been completed by the CP-PACS Collaboration [8]. CP-PACS [8] found that the masses of 11 light hadrons disagree with experiment by at most 11%. Quenched QCD is not a consistent theory and problems with the formalism have been found in calculations [9].

In an individual lattice calculation there are errors from the finite size of the lattice spacing and the finite lattice volume. State-of-the-art lattice calculations in quenched QCD run at a number of different lattice spacings and physical volumes and extrapolate the results to the continuum and infinite volume [8] limit. The increased computational costs of unquenched calculations means that most calculations are currently done at fixed lattice spacings [10] or an extrapolation to the continuum limit is attempted from coarse lattice spacings [11]. One of the most interesting unquenched calculations is being performed by the MILC Collaboration [12]. MILC's calculations include 2+1 flavours of sea quarks with a lattice spacing of 0.09 fm, box size of 2.6 fm, and the lightest ratio of the pseudo-scalar to vector mass is 0.4.

3. RESULTS FOR GLUEBALLS IN QUENCHED QCD

Interpolating operators for glueballs are constructed for Eq. (1) from closed loops of gauge links with specific J^{PC} quantum numbers. Some highlights of the results are that the lightest glueball is the 0^{++} state with a mass of 1.611(30)(160) GeV [2,13] (where the second error is systematic). The next lightest glueball is 2^{++}. The ratio of the tensor to scalar glueball mass is $M_{2^{++}}/M_{0^{++}}$ = 1.42(6) [13]. The spectrum of glueball states for other J^{PC} quantum numbers with masses under 4 GeV has been comprehensively mapped out by Morningstar and Peardon [14].

In the real world glueballs will decay to two mesons, hence they will have a decay width. Lattice QCD calculations are performed in Euclidean space, for convergence of the path integral in Eq. (1). The Euclidean nature of lattice calculations makes the computation of inherently complex quantities such as decay widths more involved [15].

The GF11 lattice group computed the decay widths for the decay of the 0^{++} glueball to two pseudoscalars [16] to be 108(28) MeV. Although the error is only statistical, it is encouraging that the width was small relative to the mass, so the 0^{++} glueball may exist as a well-defined state. The calculation was done at a coarse lattice spacing. The decay widths for individual meson pairs [17] did not agree with the predictions from the 'flavour democratic' assumption.

The experimental situation [18] for light 0^{++} scalars is very interesting, because there are too many states to put into SU(3) nonets, as other particles with different J^{PC} quantum numbers, such as the pseudoscalars, can be. The $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ hadrons have masses close to the mass of the 0^{++} glueball from quenched QCD. Potentially one way of identifying one of the f_0's with a quenched glueball would be to reduce the errors on the value of the mass of the 0^{++} glueball until the value agrees with one of the experimental masses and the error is at least 3σ away from the other masses. This requires the error on the 0^{++} glueball mass to be below 50 MeV. The detailed error for the mass of the 0^{++} mass from Morningstar and Peardon [14] is 1730(50)(80) MeV. The second error is from the different ways of choosing the lattice spacing in quenched QCD calculations and reflects the fact the quenched QCD is not the real world. As the 80 MeV systematic error can not be reduced in quenched QCD, the quenched glueball spectrum is known as accurately as it will ever be. There are some preliminary indications that this ambiguity in the choice of lattice spacing has been reduced in the unquenched calculations from the MILC Collaboration [12, 19].

In unquenched QCD interpolating operators with 0^{++} can be constructed from quarks and anti-quarks, such as $\overline{q}q$. In full QCD, the pure glue 0^{++} operators will mix with the fermionic 0^{++} operators. If the mixing is very strong, then the final 0^{++} masses will have little to do with the glueball masses from quenched QCD.

Weingarten and Lee [20] studied the effect of mixing between the glueball and $\overline{q}q$ states in quenched QCD. They measured the correlation between the 0^{++} glueball states and $\overline{q}q$ states in Eq. (1). The results were expressed as a mixing matrix

$$\begin{pmatrix} m_g & E(s) \\ E(s) & m_\sigma(s) \end{pmatrix} \tag{3}$$

where m_g is the glueball mass, $m_\sigma(s)$ is the mass of the non-singlet 0^{++} state at the strange quark mass, and $E(s)$ is the mixing energy. Weingarten and Lee measured: m_g = 1648(58) MeV, $m_\sigma(s)$ = 1322(42) MeV, and $E(s)$ = 61(58) MeV in the continuum limit. The qualitative picture that emerges is that the $f_0(1710)$ is ‘mostly’ 0^{++} glueball, and the $f_0(1500)$ is ‘mostly’ $\overline{s}s$. It is not clear whether $f_0(1500)$ being $\overline{s}s$ is consistent with its decay width [21]. The mixing energy $E(s)$ has large lattice spacing errors. For example at a lattice spacing of $a^{-1} \sim$ 1.2 GeV, the Weingarten and Lee [20] result is $E(s) \sim$ 0.36 GeV. This has been checked by another group’s result [22] of $E(s) \sim$ 0.44 GeV.

The analysis of Weingarten and Lee [20] depends on the 0^{++} states being well defined in quenched QCD. Bardeen [9] *et al.* have shown that there is a problem with the non-singlet 0^{++} correlator in quenched QCD. The problem can be understood using quenched chiral perturbation theory. The non-singlet 0^{++} propagator contains an intermediate state of $\eta' - \pi$. The removal of fermion loops in quenched QCD has a big effect on the η' propagator. The result is that a ghost state contributes to the scalar correlator, that makes the expression in Eq. (2) inappropriate to extract masses from the calculation. Eichten *et al.* [9] predict that the ghost state will make the a_0 mass increase as the quark mass is reduced below a certain point. This behaviour was observed by Weingarten and Lee [20] for small box sizes ($L \leq 1.6$ fm) for quark masses below strange. It is not clear how the problem with the non-singlet 0^{++} correlator in the quenched approximation affects the results of Weingarten and Lee [20], however, their most important results come from masses above the strange quark mass where the ghost diagram will make a smaller contribution that may be negligible.

Lattice QCD calculations are sometimes criticized for just producing numbers, but no insight. Increasingly, lattice QCD methods are used to provide intuition about hadronic physics. For example the large N_c limit of QCD has been a place where analytical calculations are possible, however, the calculation of the $1/N_c$ corrections has turned out to be hard.

Teper and Lucini [23] have systematically studied the glueball spectrum for N_c = 2,3,4 and 5. They found that the dependence of the glueball spectrum on N_c is weak. To determine the N_c dependence of the glueball masses, the systematic errors, such as lattice spacing errors, had to be quantified and controlled. This type of lattice study is very useful to the attempts to compute the glueball spectrum using the ADS super-gravity duality (for example see Brower *et al.* [24] and the references within), as the glueball spectrum is obtained in the large N_c limit.

The light scalar mesons seem to be full of surprises. There are lighter 0^{++} states, such as the $f_0(980)$ and the $a_0(980)$, and the enigmatic $f_0(400-1200)$. The $f_0(980)$ and $a_0(980)$ states are considered by some people to be kaon molecules or $\overline{q}\overline{q}qq$ states, although there are dissenting opinions. There has been some recent work by Alford and Jaffe [25] on $\overline{q}\overline{q}qq$ quark states.

4. RESULTS FOR GLUEBALL MASSES IN TWO-FLAVOUR QCD

The Weingarten and Lee [20] analysis predicted that the mixing of the 0^{++} glueball and $\overline{q}q$ states is small. Parts of their calculation have been criticized in Ref. [22], however, the problems with the non-singlet 0^{++} correlator [9] in the quenched QCD will make further progress in mixing in the quenched QCD difficult. There are attempts to take into account the quenched artifact in the a_0 correlator [26].

A lattice QCD calculation that included the dynamics of the sea quarks should just reproduce the physical spectrum of 0^{++} states. Some insight into the composition of individual 0^{++} states, such as whether a physical particle couples to $\overline{q}q$ or pure glue operators, could be studied by looking at the effect of decreasing the sea quark mass. For very heavy sea quark masses the theory is more like quenched QCD, where glueballs are distinct from $\overline{q}q$ operators.

Figure 1 shows a compendium of recent results for the mass of singlet 0^{++} states from two-flavour unquenched QCD versus the square of the lattice spacing.

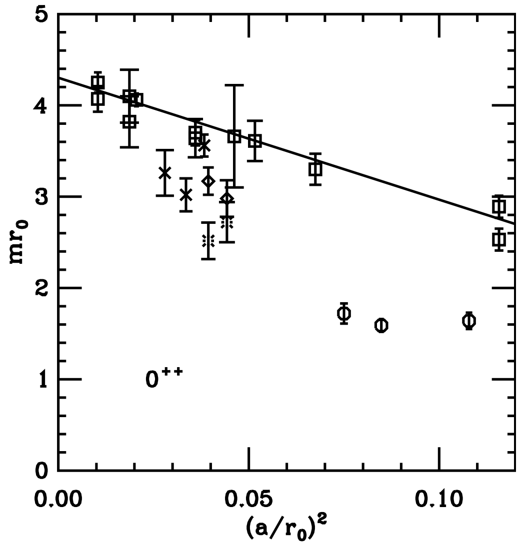

Fig. 1: Singlet 0^{++} mass in units of r_0 as a function of lattice spacing. The crosses are from SESAM [27]. The octagons are from UKQCD's [22] first $n_f = 2$ data set. The diamonds are the results from Hart and Teper [28]. The bursts are from a combined analysis of glueball and $\overline{q}q$ interpolating operators. The squares are the results from quenched calculations (see Ref. [22] for references).

Hart and Teper [28] found that the ratio of the 0^{++} glueball mass in $n_f = 2$ QCD to the quenched QCD result was: $M^{0^{++}}_{n_f=2}/M^{0^{++}}_{quenched}$ = 0.84 ± 0.03 at a fixed lattice spacing of 0.1 fm. The n_f = 2 results [28] for the mass of the 2^{++} were consistent with the quenched value. As the lattice spacing dependence of the mass of the singlet 0^{++} state in two-flavour QCD and quenched QCD could be different, a definitive result will only come after a continuum extrapolation of the unquenched masses. In quenched QCD [13], the difference between the continuum extrapolated mass of the 0^{++} glueball and the mass at 0.1 fm is of the order of 200 MeV. This is the same magnitude of the mass splittings between the masses of the experimentally observed particles $f_0(1500)$ and $f_0(1710)$. Although the current results for singlet 0^{++} states are starting to be interesting, the lattice spacing used in unquenched calculations must be reduced before direct contact can be made to phenomenology.

Traditionally, glueball calculations have been done with Wilson loop type operators. However, singlet quark operators of the form $\overline{q}q$ also have the quantum numbers of 0^{++}. The UKQCD Collaboration were the first to attempt a joint analysis of 0^{++} states that included glueball and $\overline{q}q$ operators [22]. Preliminary results are now available for a finer lattice spacing [29]. In Fig. 1 we plot the masses from the calculation by Hart and Teper (diamonds) with the masses obtained in this analysis (bursts). The value of $1/r_0$ is $\sim$ 373 MeV from the string tension [2]. The inclusion of the $\overline{q}q$ operators with the Wilson loop operators has produced a further suppression of the mass of the singlet 0^{++} state at the lattice spacing used.

The mass of the 0^{++} singlet meson on the lightest UKQCD data set are degenerate with the mass of two pions [28]. As the mass of the sea quarks is reduced, two-pion states may affect the physics of singlet 0^{++} states. Two-pion interpolating operators may also need to be included in the basis of interpolating operators.

5. RESULTS FOR LIGHT 1^{-+} EXOTIC MESONS

The quark model predicts the charge conjugation (C = $(-1)^{L+S}$) and parity (P= $(-1)^{L+1}$) of a meson with spin S and orbital angular momentum L. States with quantum numbers not predicted by the quark model, such as J^{PC}_{exotic} = 1^{-+}, 0^{+-}, 2^{+-}, 0^{--} are known as exotics [30]. Exotic states are allowed by QCD. Morningstar and Peardon [14] claim that there are no glueballs with exotic quantum numbers with masses less than 4 GeV in quenched lattice QCD.

There are a number of different possibilities for the structure of an exotic state. An exotic state could be a hybrid meson, that is a quark and anti-quark with excited glue, or bound state of two quarks and two anti-quarks ($\overline{q}\overline{q}qq$).

One possible interpolating operator [31], that can be used in Eq. (1), for a hybrid 1^{-+} particle is

$$O_{1^{-+}}(\underline{x}, t) = \overline{q}(\underline{x}, t)\gamma_j F_{ij}(\underline{x}, t) q(\underline{x}, t) \tag{4}$$

where F is the QCD field strength tensor. If F is removed from Eq. (4), the operator creates the ρ particle. In this formalism a gauge invariant interpolating operator, for any possible exotic hybrid particle or four-particle state can be constructed. The dynamics then determines whether the resulting state has a narrow decay width, hence it can be detected experimentally. In the large N_c (number of colours) limit [30, 32] both exotic hybrid mesons and non-exotic mesons have widths that are small compared to their masses.

There have not been many new calculations of the mass of the light 1^{-+} hybrid recently. All the results from the various lattice QCD calculations, by UKQCD [33, 34], MILC [31, 35] and SESAM [36] are essentially consistent with the mass of the 1^{-+} state around $1.9(2)$ GeV [2]. The interpolating operators used to create the exotic meson states in the MILC calculations [31] are different from those used in the UKQCD [33] and SESAM simulations [36], hence giving confidence that the systematic errors are under control. The results for the hybrid masses reported by Lacock and Schilling [36], include some effects from dynamical sea quarks. The recent results for the 1^{-+} mass from calculations that used an asymmetric [37] lattice in time (for a better signal to noise ratio) are consistent with the older results.

The MILC Collaboration have started the first serious study of the exotic meson spectrum in unquenched QCD [38]. The MILC Collaboration use a formalism called improved staggered fermions for the quarks. This formulation can study much lighter quarks than competitive fermion actions. The main disadvantage of this formalism is that flavour symmetry is broken. The preliminary result from MILC for the mass of the lightest 1^{-+} state is consistent with (or perhaps slightly lower than) earlier estimates from MILC and UKQCD. MILC found problems extracting the 1^{-+} state from the lightest unquenched calculations [38]. Their preliminary speculation is that this is due to mixing with $\overline{q}\overline{q}qq$ states. Further work is required to test this.

There are a number of experimental candidates for light 1^{-+} states [18]. The E852 Collaboration have reported [39] a signal for 1^{-+} state around 1.6 GeV. There is also an experimental signal for a 1^{-+} state at 1.4 GeV [18].

There has been some recent work [40] on the quark mass dependence of the 1^{-+} states. The lattice calculations are usually done at large quark masses and the results extrapolated to the physical quark masses. The conclusion of Ref. [40] was that the inclusion of the decay of the hybrid in the quark mass dependence of the exotic mass could reduce the final answer by 100 MeV. The predictions in Ref. [40] will be tested as the quark masses used in lattice calculations are reduced. The 1^{-+} state at 1.4 GeV seems low relative to the lattice results.

It is possible that the states seen experimentally are really $\overline{q}\overline{q}qq$ states, in which case the operators used in the lattice simulations [Eq. (4)] might not couple strongly to them. Alford and Jaffe [25] studied $\overline{q}\overline{q}qq$ operators with J^{PC} = 0^{++} in a recent lattice calculation. The motivation was to gain insight into states such as the $f_0(980)$ that some people believe is not a $\overline{q}q$ meson, but a $\overline{q}\overline{q}qq$ state. A similar lattice calculation could in principle be done for the J^{PC} = 1^{-+} exotic.

To definitely identify a particle requires both the calculation of the mass as well as the decay widths. There has been very little work on strong decays on the lattice. The most obvious hadronic process to study using lattice gauge theory is the $\rho \to \pi\pi$ decay, however, there have only been a few attempts to calculate the $g_{\rho\pi\pi}$ coupling [41, 42]. Michael discusses the problems with the formalism for hadronic decays on the lattice [15].

In the static quark limit the exotic states on the lattice are described by adiabatic potentials. The ground state of the static potential (A_{1g}) is the familiar Coulomb plus linear potential. The excited potential (E_u) is a very flat potential, that can be used with Schrödinger's equation to predict the spectrum of heavy-heavy hybrids [2]. UKQCD [43] have investigated the de-excitation of the E_u potential to the A_{1g} potential by the emission of a light quark loop. In the real world, the decays would correspond to $1^{-+} \to \chi_b\eta$ and $1^{-+} \to \chi_b S$ with S a scalar and η a pseudo-scalar. The decay width of $1^{-+} \to \chi_b\eta$ and $1^{-+} \to \chi_b S$ transitions were less than 1 MeV and around 80 MeV respectively. The various approximations in the static limit mean that these widths have no direct relevance to experiment.

The MILC Collaboration [31] have investigated the mixing between the operator in Eq. (4) and the operator ($\pi \otimes a_1$) Eq. (5).

$$\overline{q}^a\gamma_5 q^a \overline{q}^b \gamma_5\gamma_i q^b \tag{5}$$

that has the quantum numbers 1^{-+}. This type of correlator is part of the calculation required to compute the decay width of the 1^{-+} state to ρ, and a_1. The more complicated part is to use Eq. (5) in Eq. (1) which requires some clever numerical work.

6. CONCLUSIONS

The glueball spectrum from quenched QCD has been stable for a number of years and has provided useful hints to experiments that are trying to find experimental evidence for glueballs. The mixing between glueball and $\overline{q}q$ states has been studied by Lee and Weingarten [20] and the UKQCD Collaboration [22]. Further checks on the seminal calculations of Lee and Weingarten [20] will be hampered by formalism problems in quenched QCD [9]. It is better to study the mixing using unquenched QCD calculations [22].

Progress in glueball and hybrid meson spectroscopy will be dependent on how close the masses of the sea quarks are to their physical values. The physical mass of the singlet 0^{++} in unquenched QCD is obscured by lattice artefacts. To reduce the systematic errors on the mass of the singlet 0^{++} requires lattice calculations at finer lattice spacings. This is computationally expensive, but possible.

The computation of decay widths from a Euclidean lattice calculation is a tough problem. The UKQCD Collaboration have recently computed the coupling for the rho to decay to two pions [44]. This formalism may be able to compute couplings for decays relevant to scalar and exotic meson decays.

There is a sizeable community of lattice people in the UKQCD Collaboration who are interested in glueball and exotic meson physics. At the end of 2003 the UKQCD Collaboration will get a QCDOC [45] (QCD On a Chip) computer that has essentially the computational power of 10 000 PCs. So the COMPASS Collaboration may expect improved lattice calculations of pertinent hadronic masses from the UKQCD Collaboration in the next few years.

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