Description of polarized $\pi^- + N$ Drell-Yan processes

by Aram Kotzinian

February 8, 2010

Abstract

The general description of polarized Drell-Yan process is presented in the article by Arnold, Metz and Schlegel [1]. Since at COMPASS we can have only target nucleon polarized, here we consider only the reaction with unpolarized beam and polarized target. We present the simplified expressions for general expression of cross-section in one photon approximation, the leading order parton model expressions for structure functions. We define also the asymmetries and present the expressions for statistical error calculations in the case of flat acceptances.

1 Description of Drell–Yan processes

Here we will describe the notations, choice of reference frames and the general expression for Drell–Yan cross-section closely following the article by Arnold, Metz and Schlegel [1]. Since at COMPASS we will not have a polarized beam here we consider only the reaction with unpolarized beam (H_a)and polarized target (H_b)

$$H_a(P_a) + H_b(P_b, S) \to \gamma^*(q) + X \to l^-(l) + l^+(l') + X \tag{1}$$

where $P_{1(2)}$ is a momentum of beam (target) hadron, $q = l + l'$, l and l' are the momenta of virtual photon, lepton and anti-lepton and S is four-vector of the target polarization.

1.1 Kinematic variables in different reference frames

Several frames are commonly used in description of Drell–Yan processes:

- Target rest frame (TF) where the the unit vectors of coordinate system are chosen as follows: $\hat{z}$ is chosen along beam momentum, $\hat{x}$ – along virtual photon transverse to beam direction component and $\hat{y} = \hat{z} \times \hat{x}$. Then

$$P^\mu_{1,TF} = (E,\, 0,\, 0,\, P^3_{1,TF})\,, \tag{2}$$
$$P^\mu_{b,TF} = (M_b,\, 0,\, 0,\, 0)\,, \tag{3}$$
$$q^\mu_{TF} = (q_{0,TF},\, q_T,\, 0,\, q_{L,TF})\,, \tag{4}$$
$$S^\mu_{TF} = \left(0,\, |\vec{S}_T|\, \cos\phi_S,\, |\vec{S}_T|\, \sin\phi_S,\, S_L\right), \tag{5}$$

- Initial hadrons center of mass frame (CM) obtained by Lorentz boost in the direction opposite to beam hadron momentum.

$$P^\mu_{a,CM} = (P^0_{a,CM},\, 0,\, 0,\, P^3_{a,CM})\,, \approx \frac{\sqrt{s}}{2}(1,\, 0,\, 0,\, 1)\,, \tag{6}$$
$$P^\mu_{b,CM} = (P^0_{b,CM},\, 0,\, 0,\, P^3_{b,CM}) \approx \frac{\sqrt{s}}{2}(1,\, 0,\, 0,\, -1)\,, \tag{7}$$

$$q^\mu_{CM} = (q_{0,CM},\, q_T,\, 0,\, q_{L,CM})\,, \tag{8}$$

$$S^\mu_{CM} = \Big(-S_L\,\frac{|\vec{P}_{b,CM}|}{M_b},\, |\vec{S}_T|\,\cos\phi_S,\, |\vec{S}_T|\,\sin\phi_S,\, S_L\,\frac{P^0_{b,CM}}{M_b}\Big)\,, \tag{9}$$

where $s = (P_a + P_b)^2$ is the total energy squared in the incoming hadrons CM frame[1)].

– Collins–Soper frame (CS) can be reached from CM or TF by two subsequent Lorentz boost. In a first step one boosts along the z-axis such that the virtual photon no longer has a longitudinal momentum component. In a second step one boosts along the x-axis such that also the transverse momentum of the virtual photon disappears. Neglecting the leptons mass the lepton and anti-lepton momenta in this frame are given as

$$l^\mu_{CS} = \frac{q}{2}\Big(1,\, \sin\theta\,\cos\phi,\, \sin\theta\,\sin\phi,\, \cos\theta\Big)\,, \tag{10}$$

$$l'^\mu_{CS} = \frac{q}{2}\Big(1,\, -\sin\theta\,\cos\phi,\, -\sin\theta\,\sin\phi,\, -\cos\theta\Big)\,. \tag{11}$$

In following as angular variables we will use the azimuthal angle of the target transverse polarization in TF, ϕ_S, and polar and azimuthal angles of lepton momentum in CS frame, θ and ϕ.

1.2 General expression for cross-section

The general form for Drell–Yan cross-section expressed in above mentioned angular variables is derived in [1]. We are interested by the case when only target nucleon is polarized. In this case the general expression can be presented as

$$\begin{aligned}
\frac{d\sigma}{d^4q\,d\Omega} &= \frac{\alpha^2_{em}}{F\,q^2}\Big\{\Big((1+\cos^2\theta)\,F^1_U + (1-\cos^2\theta)\,F^2_U \\
&+ \sin 2\theta\,F^{\cos\phi}_U\cos\phi + \sin^2\theta\,F^{\cos 2\phi}_U\cos 2\phi\Big) \\
&+ S_L\Big(\sin 2\theta\,F^{\sin\phi}_L\sin\phi + \sin^2\theta\,F^{\sin 2\phi}_L\sin 2\phi\Big) \\
&+ |\vec{S}_T|\Big[\Big(F^{\sin\phi_S}_T + \cos^2\theta\,\tilde{F}^{\sin\phi_S}_T\Big)\,\sin\phi_S \\
&+ \sin 2\theta\Big(F^{\sin(\phi+\phi_S)}_T\sin(\phi+\phi_S) + F^{\sin(\phi-\phi_S)}_T\sin(\phi-\phi_S)\Big) \\
&+ \sin^2\theta\Big(F^{\sin(2\phi+\phi_S)}_T\sin(2\phi+\phi_S) + F^{\sin(2\phi-\phi_S)}_T\sin(2\phi-\phi_S)\Big)\Big]\Big\}\,,
\end{aligned} \tag{12}$$

where $F = 4\sqrt{(P_a\cdot P_b)^2 - M_a^2 M_b^2}$ represents the flux of incoming hadrons and the solid angle Ω specifies the orientation of the lepton. In general, the structure functions depend on the three independent variables, for example, the Lorentz invariant ones $P_a\cdot q$, $P_b\cdot q$, and q^2, i.e., $F^1_U = F^1_U(P_a\cdot q,\, P_b\cdot q,\, q^2)$ and so on and do not depend on lepton polar angle θ and azimuthal angles ϕ and ϕ_S. In Eq. (12) the subscript of structure function corresponds to polarization state of target nucleon (U for target polarization independent contribution, L and T for contribution from target longitudinal and transverse polarizations) and superscript – to azimuthal modulation. We have used the following relations

[1)] Note that with our notation in Eq. (5) the sign of zeroth and third component of spin four-vector in Eq. (9) is opposite to that used in [1]. This brings to change of sign in Eq. (25) comparing with the sign in [1].

with the structure functions introduced in [1]:

$$
\begin{aligned}
F_T^{\sin\phi_S} &= F_{UT}^1 + F_{UT}^2\,, & \tilde{F}_T^{\sin\phi_S} &= F_{UT}^1 - F_{UT}^2\,,\\
F_T^{\sin(\phi+\phi_S)} &= \tfrac{1}{2}(F_{UT}^{\sin\phi} + F_{UT}^{\cos\phi})\,, & F_T^{\sin(\phi-\phi_S)} &= \frac{1}{2}(F_{UT}^{\sin\phi} - F_{UT}^{\cos\phi})\,,\\
F_T^{\sin(2\phi+\phi_S)} &= \tfrac{1}{2}(F_{UT}^{\sin 2\phi} + F_{UT}^{\cos 2\phi})\,, & F_T^{\sin(2\phi-\phi_S)} &= \frac{1}{2}(F_{UT}^{\sin 2\phi} - F_{UT}^{\cos 2\phi})\,.
\end{aligned}
\tag{13}
$$

1.3 Definition of azimuthal asymmetry observables

Factorizing in Eq. (12) the part of cross-section which survives after integration over azimuthal angles ϕ and ϕ_S

$$
\hat{\sigma}_U = (F_U^1 + F_U^2)(1 + A_U^1 \cos^2\theta)\,,
\tag{14}
$$

we can rewrite the general expression in the form

$$
\begin{aligned}
\frac{d\sigma}{d^4q\, d\Omega} &= \frac{\alpha_{em}^2}{F\,q^2}\hat{\sigma}_U \Big\{ \Big(1 + D_{[\sin 2\theta]}\, A_U^{\cos\phi}\cos\phi + D_{[\sin^2\theta]}\, A_U^{\cos 2\phi}\cos 2\phi\Big)\\
&+ S_L \Big(D_{[\sin 2\theta]}\, A_L^{\sin\phi}\sin\phi + D_{[\sin^2\theta]}\, A_L^{\sin 2\phi}\sin 2\phi\Big)\\
&+ |\vec{S}_T| \Big[\Big(D_{[1]}\, A_T^{\sin\phi_S} + D_{[\cos^2\theta]}\, \tilde{A}_T^{\sin\phi_S}\Big)\sin\phi_S\\
&+ D_{[\sin 2\theta]}\Big(A_T^{\sin(\phi+\phi_S)}\sin(\phi+\phi_S) + A_T^{\sin(\phi-\phi_S)}\sin(\phi-\phi_S)\Big)\\
&+ D_{[\sin^2\theta]}\Big(A_T^{\sin(2\phi+\phi_S)}\sin(2\phi+\phi_S) + A_T^{\sin(2\phi-\phi_S)}\sin(2\phi-\phi_S)\Big)\Big]\Big\}\,,
\end{aligned}
\tag{15}
$$

where

$$
D_{[f(\theta)]} = \frac{f(\theta)}{1 + A_U^1\cos^2\theta}.
\tag{16}
$$

In analogy with polarized SIDIS case we will call $D_{f(\theta)}$ depolarization factors. The asymmetries $A_{U,L,T}^{f(\phi,\phi_S)}$ are defined as

$$
\begin{aligned}
A_U^1 &= \frac{F_U^1 - F_U^2}{F_U^1 + F_U^2}\,, & A_U^{\cos\phi} &= \frac{F_U^{\cos\phi}}{F_U^1 + F_U^2}\,, & A_U^{\cos 2\phi} &= \frac{F_U^{\cos 2\phi}}{F_U^1 + F_U^2}\,,\\
A_L^{\sin\phi} &= \frac{F_L^{\sin\phi}}{F_U^1 + F_U^2}\,, & A_L^{\sin 2\phi} &= \frac{F_L^{\sin 2\phi}}{F_U^1 + F_U^2}\,,\\
A_T^{\sin\phi_S} &= \frac{F_T^1 + F_T^2}{F_U^1 + F_U^2}\,, & \tilde{A}_T^{\sin\phi_S} &= \frac{F_T^1 - F_T^2}{F_U^1 + F_U^2}\,,\\
A_T^{\sin(\phi+\phi_S)} &= \frac{F_T^{\sin\phi} + F_T^{\cos\phi}}{2(F_U^1 + F_U^2)}\,, & A_T^{\sin(\phi-\phi_S)} &= \frac{F_T^{\sin\phi} - F_T^{\cos\phi}}{2(F_U^1 + F_U^2)}\,,\\
A_T^{\sin(2\phi+\phi_S)} &= \frac{F_T^{\sin 2\phi} + F_T^{\cos 2\phi}}{2(F_U^1 + F_U^2)}\,, & A_T^{\sin(2\phi-\phi_S)} &= \frac{F_T^{\sin 2\phi} - F_T^{\cos 2\phi}}{2(F_U^1 + F_U^2)}\,.
\end{aligned}
\tag{17}
$$

For unpolarized azimuthal asymmetries often differen notations are used: $\lambda = A_U^1$, $\mu = A_U^{\cos\phi}$ and $\nu = 2\,A_U^{\cos 2\phi}$. In LO QCD parton model the asymmetries are directly related to convolutions of LO TMD PDFs, see Sec. 1.4.

We conclude this section by noting, that when one integrates over the lepton azimuthal angle ϕ, only one azimuthal asymmetry survives in Eq. (15), namely, the transverse spin dependent $\sin\phi_S$ asymmetry.

1.4 Leading order QCD parton model

The structure functions entering in Drell-Yan precess cross-section can be expressed at high energies ($s^2, q^2 \gg M_a^2, M_b^2$) and small transverse momentum of virtual photon ($q_T \ll q$) as a convolution over TMD DFs [1]. We adopt the the following notation for the convolution of TMDs in the transverse momentum space:

$$\begin{aligned} \mathcal{C}\left[w(\vec{k}_{aT}, \vec{k}_{bT}) f_1 \bar{f}_2\right] \equiv \frac{1}{N_c} \sum_q e_q^2 \int d^2\vec{k}_{aT}\, d^2\vec{k}_{bT}\, \delta^{(2)}(\vec{q}_T - \vec{k}_{aT} - \vec{k}_{bT})\, w(\vec{k}_{aT}, \vec{k}_{bT}) \times \\ \left[f_1^q(x_a, \vec{k}_{aT}^2)\, f_2^{\bar{q}}(x_b, \vec{k}_{bT}^2) + f_1^{\bar{q}}(x_a, \vec{k}_{aT}^2)\, f_2^q(x_b, \vec{k}_{bT}^2)\right], \end{aligned} \tag{18}$$

where $N_c = 3$ is the number of colors, the argument of PDFs

$$x_a = \frac{q^2}{2P_a \cdot q}, \quad x_b = \frac{q^2}{2P_b \cdot q} \tag{19}$$

can be interpreted as the fractions of light-cone momentum of partons from initial hadron H_a and H_b. Then using the unit vector $\vec{h} \equiv \vec{q}_T / q_T$ one eventually finds the following LO structure functions in the CS-frame:

$$F_U^1 = \mathcal{C}\left[f_a\, \bar{f}_a\right], \tag{20}$$

$$F_U^2 = 0, \tag{21}$$

$$F_U^{\cos\phi} = 0, \tag{22}$$

$$F_U^{\cos 2\phi} = \mathcal{C}\left[\frac{2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}}{M_a M_b}\, h_1^\perp\, \bar{h}_1^\perp\right], \tag{23}$$

$$F_L^{\sin\phi} = 0 \tag{24}$$

$$F_L^{\sin 2\phi} = \mathcal{C}\left[\frac{2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}}{M_a M_b}\, h_1^\perp\, \bar{h}_{1L}^\perp\right], \tag{25}$$

$$F_T^1 = \mathcal{C}\left[\frac{\vec{h}\cdot\vec{k}_{bT}}{M_b}\, f_1\, \bar{f}_{1T}^\perp\right], \tag{26}$$

$$F_T^2 = 0, \tag{27}$$

$$F_T^{\sin(\phi-\phi_S)} = 0, \tag{28}$$

$$F_T^{\sin(\phi+\phi_S)} = 0, \tag{29}$$

$$F_T^{\sin(2\phi+\phi_S)} = -\mathcal{C}\left[\frac{2(\vec{h}\cdot\vec{k}_{bT})[2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}] - \vec{k}_{bT}^2(\vec{h}\cdot\vec{k}_{aT})}{2M_a M_b^2}\, h_1^\perp\, \bar{h}_{1T}^\perp\right], \tag{30}$$

$$F_T^{\sin(2\phi-\phi_S)} = -\mathcal{C}\left[\frac{\vec{h}\cdot\vec{k}_{aT}}{M_a}\, h_1^\perp\, \bar{h}_1\right]. \tag{31}$$

As one can see from Eqs. (20–31), 6 out of 12 structure functions describing single polarized Drell-Yan process vanish at twist-two. In this approximation

$$\hat{\sigma}_U^{LO} = F_U^1 (1 + \cos^2\theta), \tag{32}$$

and Eq. (15) simplifies to

$$\frac{d\sigma^{LO}}{d^4q\, d\Omega} = \frac{\alpha_{em}^2}{F\, q^2} \hat{\sigma}_U^{LO} \Big\{ \Big(1 + D_{[\sin^2\theta]}^{LO}\, A_U^{\cos 2\phi} \cos 2\phi\Big)$$

$$
\begin{aligned}
&+ \quad S_L D^{LO}_{[\sin^2\theta]} A_L^{\sin 2\phi} \sin 2\phi \\
&+ \quad |\vec{S}_T| \Big[A_T^{\sin\phi_S} \sin\phi_S + D^{LO}_{[\sin^2\theta]} \Big(A_T^{\sin(2\phi+\phi_S)} \sin(2\phi+\phi_S) \qquad (33) \\
&+ \qquad A_T^{\sin(2\phi-\phi_S)} \sin(2\phi-\phi_S) \Big) \Big] \Big\} ,
\end{aligned}
$$

and the depolarization factors at LO depends only on lepton polar angle θ:

$$
D^{LO}_{[f(\theta)]} = \frac{f(\theta)}{1+\cos^2\theta} . \qquad (34)
$$

For completeness we present here also the expressions for nonzero asymmetries at LO:

$$
\begin{aligned}
A_U^{\cos 2\phi}(LO) &= \frac{F_U^{\cos 2\phi}}{F_U^1} , \\
A_L^{\sin 2\phi}(LO) &= \frac{F_L^{\sin 2\phi}}{F_U^1} , \qquad (35) \\
A_T^{\sin\phi_S}(LO) &= \tilde{A}_T^{\sin\phi_S}(LO) = \frac{F_T^1}{F_U^1} , \\
A_T^{\sin(2\phi+\phi_S)}(LO) &= \frac{F_T^{\sin(2\phi+\phi_S)}}{2\,F_U^1} , \qquad (36) \\
A_T^{\sin(2\phi-\phi_S)}(LO) &= \frac{F_T^{\sin(2\phi-\phi_S)}}{2\,F_U^1} .
\end{aligned}
$$

In principle with longitudinally and transversely polarized targets COMPASS will be able to extract all 12 structure functions. The simple interpretation of these structure functions within LO QCD parton model is possible only for 6 of them. We see from Eqs. (23), (25), (26), (30) and (31) that the measurement of asymmetry

– $A_U^{\cos 2\phi}$ gives access to Boer-Mulders functions of incoming hadrons,
– $A_L^{\sin 2\phi}$ – to Boer-Mulders functions of beam hadron and $h_{1L}^{\perp}$ function of the target nucleon,
– $A_T^{\sin\phi_S}$ – to Sivers function of the target nucleon,
– $A_T^{\sin(2\phi+\phi_S)}$ – to Boer-Mulders functions of beam hadron and $h_{1T}^{\perp}$ (pretzelosity) function of the target nucleon,
– $A_T^{\sin(2\phi-\phi_S)}$ – to Boer-Mulders functions of beam hadron and h_1 (transversity) function of the target nucleon.

Within QCD TMD PDFs approach the remaining asymmetries can be interpreted as higher order in q_T/q kinematic corrections and as contribution of non-leading twist PDFs.

1.5 Statistical error estimation of asymmetries

Here we will present the simple expressions for LO asymmetry extraction and statistical error estimations assuming for simplicity flat acceptance of COMPASS apparatus in azimuthal angles ϕ_S, ϕ and $cos\,\theta$. We will take into account so called dilution factor, f, which describes the relative number of polarizable nucleons in the target material. From Eqs. (32) – (34) after integration over the virtual photon transverse momentum q_T and lepton polar angle θ the azimuthal angles distribution in a given bin (say (x_a, x_b) bin) is given as[2)]

$$
\begin{aligned}
\frac{d\,N(x_a,x_b,\phi,\phi_S)}{d\,\phi\,d\,\phi_S} &= N(x_a,x_b)\Big\{\Big(1+\frac{1}{2}\,A_U^{\cos 2\phi}\,\cos 2\phi\Big) \\
&+ f\,S_L\frac{1}{2}\,A_L^{\sin 2\phi}\,\sin 2\phi \\
&+ f\,|\vec{S}_T|\Big[A_T^{\sin\phi_S}\,\sin\phi_S+\frac{1}{2}\Big(A_T^{\sin(2\phi+\phi_S)}\,\sin(2\phi+\phi_S) \\
&+ A_T^{\sin(2\phi-\phi_S)}\,\sin(2\phi-\phi_S)\Big)\Big]\Big\}\,,
\end{aligned}
\tag{37}
$$

where the total number of event in the given bin (x_a, x_b)

$$
N(x_a,x_b)\propto\int dq_T^2\,d\phi_S\,d\cos\theta\,d\phi\frac{d\sigma}{dx_a\,dx_b\,dq_T^2\,d\phi_S\,d\cos\theta\,d\phi}. \tag{38}
$$

Note that in Eq. (37) the relative contribution of virtual photon depolarization factors $D^{LO}_{\sin^2\theta}$ after integration of Eq. (33) over θ are equal to $1/2$ for flat in θ acceptance.

To single out different asymmetries one can use the Fourier projection on corresponding modulation:

$$
\begin{aligned}
A_U^{\cos 2\phi}(x_a,x_b) &= 4\,\frac{\int\,d\phi_S\,d\phi\,\frac{d\,N(x_a,x_b,\phi,\phi_S)}{d\,\phi\,d\,\phi_S}\,\cos 2\phi}{N(x_a,x_b)}\,, \\
A_L^{\sin 2\phi}(x_a,x_b) &= \frac{4}{f\,S_L}\,\frac{\int\,d\phi_S\,d\phi\,\frac{d\,N(x_a,x_b,\phi,\phi_S)}{d\,\phi\,d\,\phi_S}\,\sin 2\phi}{N(x_a,x_b)}\,, \\
A_T^{\sin\phi_S}(x_a,x_b) &= \frac{2}{f\,|\vec{S}_T|}\,\frac{\int\,d\phi_S\,d\phi\,\frac{d\,N(x_a,x_b,\phi,\phi_S)}{d\,\phi\,d\,\phi_S}\,\sin\phi_S}{N(x_a,x_b)}\,, \\
A_T^{\sin(2\phi+\phi_S)}(x_a,x_b) &= \frac{4}{f\,|\vec{S}_T|}\,\frac{\int\,d\phi_S\,d\phi\,\frac{d\,N(x_a,x_b,\phi,\phi_S)}{d\,\phi\,d\,\phi_S}\,\sin(2\phi+\phi_S)}{N(x_a,x_b)}\,, \\
A_T^{\sin(2\phi-\phi_S)}(x_a,x_b) &= \frac{4}{f\,|\vec{S}_T|}\,\frac{\int\,d\phi_S\,d\phi\,\frac{d\,N(x_a,x_b,\phi,\phi_S)}{d\,\phi\,d\,\phi_S}\,\sin(2\phi-\phi_S)}{N(x_a,x_b)}\,.
\end{aligned}
\tag{39}
$$

Finally, the statistical accuracy for asymmetries are given as

$$
\begin{aligned}
\delta\,A_U^{\cos 2\phi}(x_a,x_b) &= 2\,\frac{\sqrt{2}}{\sqrt{N(x_a,x_b)}}\,, \\
\delta\,A_L^{\sin 2\phi}(x_a,x_b) &= \frac{2}{f\,S_L}\,\frac{\sqrt{2}}{\sqrt{N(x_a,x_b)}}\,, \\
\delta\,A_T^{\sin\phi_S}(x_a,x_b) &= \frac{1}{f\,|\vec{S}_T|}\,\frac{\sqrt{2}}{\sqrt{N(x_a,x_b)}}\,, \\
\delta\,A_T^{\sin(2\phi+\phi_S)}(x_a,x_b) &= \frac{2}{f\,|\vec{S}_T|}\,\frac{\sqrt{2}}{\sqrt{N(x_a,x_b)}}\,, \\
\delta\,A_T^{\sin(2\phi-\phi_S)}(x_a,x_b) &= \frac{2}{f\,|\vec{S}_T|}\,\frac{\sqrt{2}}{\sqrt{N(x_a,x_b)}}\,.
\end{aligned}
\tag{40}
$$

[2)] We have used the relation $d^4q=(s/4)\,dx_a\,dx_b\,dq_T^2\,d\phi_S$.

References

[1] S. Arnold, A. Metz and M. Schlegel, Phys. Rev. D **79**, 034005 (2009).