Exclusive processes beyond leading twist: $\gamma_T^* \to \rho_T$ impact factor with twist-3 accuracy

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ECT*, Trento 2010,

Introduction: phenomenology of exclusive processes within collinear factorization

- Experimental tests are possible in fixed target experiments
  - $e^{\pm}p$, $\mu^{\pm}p$: HERA (HERMES), JLab, COMPASS...

  as well as in colliders, mainly for medium s
  - $e^{\pm}p$ colliders: HERA (H1, ZEUS)
  - $e^{+}e^{-}$ colliders: LEP, Belle, BaBar, BEPC
- Collinear factorization has been proven only for specific cases:
  e.g.: ρ_T production cannot directly be factorized (appearance of end point singularities)
  $\Rightarrow$ improvement needed for a consistent approach of exclusive processes

QCD in the perturbative Regge limit with k_T-factorization

- At the same time, at large s, the interest for phenomenological tests of hard Pomeron and related resummed approaches has become pretty wide:
  - inclusive tests (total cross-section) and semi-inclusive tests (diffraction, forward jets, ...)
  - exclusive tests (meson production)
- These tests concern all type of collider experiments:
  - $e^{\pm}p$: HERA: (H1, ZEUS)
  - $p\bar{p}$ and pp: TEVATRON (CDF, D0); LHC (CMS, ATLAS, ALICE)
  - e^+e^-: (LEP, ILC)
- These high energy exclusive processes in the perturbative Regge limit may provide new ideas when dealing with collinear factorization

Introduction

Exclusive ρ-production

Polarization effects in $\gamma^* P \to \rho P$ at HERA

- one can experimentally measure all spin density matrix elements

- at $t = t_{min}$ one can experimentally distinguish

$$\left\{ \begin{array}{lll} \gamma_L^* \to \rho_L : & \text{dominates} & \text{(twist 2 dominance)} \\ \gamma_T^* \to \rho_T : & \text{sizable} & \text{(twist 3)} \end{array} \right.$$

- S-channel helicity conservation:

$$\left\{ \begin{array}{ll} \gamma_L^* \to \rho_L & (\equiv T_{00}) \\ \gamma_T^* \to \rho_T , & \end{array} \right.$$

Dominate with respect to all other transitions.
Experimentally, $\gamma_T^* \to \rho_T$ is dominated by $\gamma_{T(-)}^* \to \rho_{T(-)}$ and $\gamma_{T(+)}^* \to \rho_{T(+)}$ $(\equiv T_{11})$

Introduction
Exclusive ρ-production

The processes with vector particle such as rho-meson probe deeper into the fine features of QCD.
It deserves theoretical developpement to describe HERA data in its special kinematical range:

- large $s_{\gamma^* P}$ $\Rightarrow$ small-x effects expected, within k_t-factorization
- large Q^2 $\Rightarrow$ hard scale $\Rightarrow$ perturbative approach and collinear factorization $\Rightarrow$ the ρ can be described through its chiral even Distribution Amplitudes

$$\begin{cases} \rho_L & \text{twist 2} \\ \rho_T & \text{twist 3} \end{cases}$$

The main ingredient is the $\gamma^* \to \rho$ impact factor
SIMPLEST OBJECT: ONLY 1 SOFT PART

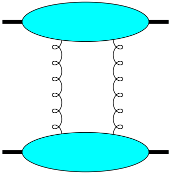
- For ρ_T, special care is needed: a pure 2-body description would violate gauge invariance.
- We show that:
  - Including in a consistent way all twist 3 contributions, i.e. 2-body and 3-body correlators, gives a gauge invariant impact factor
  - Our treatment is free of end-point singularities and does not violates the QCD factorization

Impact factor for exclusive processes
Theoretical motivations

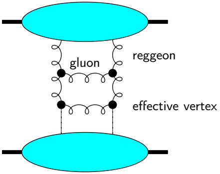
QCD in perturbative Regge limit

- In this limit, the dynamics is dominated by gluons (dominance of spin 1 exchange in t channel)
- BFKL (and extensions: NLL, saturations effects, ...) is expected to dominate with respect to Born order at large relative rapidity.

Born order:



BFKL ladder:



Impact factor for exclusive processes

k_T factorization

impact representation

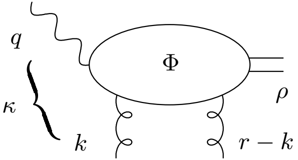
Sudakov decomp.: $k = \alpha\, p_1 + \beta\, p_2 + k_\perp$ $\underline{k}$ = Eucl. $\leftrightarrow$ $k_\perp$ = Mink.

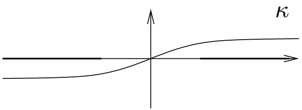
$$\mathcal{M} = is \int \frac{d^2\,\underline{k}}{(2\pi)^2 \underline{k}^2\,(\underline{r}-\underline{k})^2}\, \Phi^{\gamma^*(q_1)\to\rho(p_1^\rho)}(\underline{k},\underline{r}-\underline{k})\; \Phi^{\gamma^*(q_2)\to\rho(p_2^\rho)}(-\underline{k},-\underline{r}+\underline{k})$$

The $\gamma^*_{L,T}(q)g(k_1) \to \rho_{L,T}\, g(k_2)$ impact factor is normalized as

$$\Phi^{\gamma^*\to\rho}(\underline{k}^2) = e^{\gamma^*\mu}\,\frac{1}{2\,s}\int \frac{d\kappa}{2\pi}\,\mathrm{Disc}_\kappa\,\mathcal{S}_\mu^{\gamma^*\,g\to\rho\,g}(\underline{k}^2),$$

with $\kappa = (q+k)^2 = \beta\, s - Q^2 - \underline{k}^2$



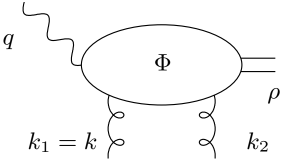


Impact factor for exclusive processes
Gauge invariance within subleading twists

Gauge invariance

- QCD gauge invariance (probes are colorless)
  $\Rightarrow$ impact factor should vanish when $\underline{k} \to 0$ or $\underline{r} - \underline{k} \to 0$

- In the following we will restrict ourselve to the case $t = t_{min}$, i.e. to $\underline{r} = 0$



$$k_1 = \frac{\kappa + Q^2 + \underline{k}^2}{s} p_2 + k_\perp$$

$$k_2 = \frac{\kappa + \underline{k}^2}{s} p_2 + k_\perp ,$$

$$k_1^2 = k_2^2 = -\underline{k}^2$$

This kinematics takes into account skewedness effects along p_2
$t = t_{min}$ $\Rightarrow$ restriction to the transitions

$$\begin{cases} 0 \quad \rightarrow \quad 0 & \text{(twist 2)} \\ (+ \text{ or } -) \quad \rightarrow \quad (+ \text{ or } -) & \text{(twist 3)} \end{cases}$$

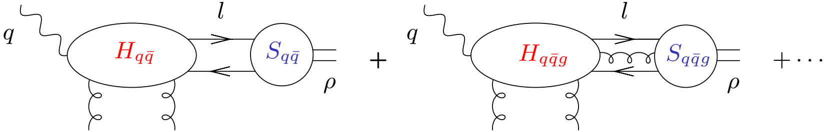
- At twist 3 level (for $\gamma_T^* \to \rho_T$ transition), gauge invariance is a non trivial statement which requires 2 *and* 3 body correlators

Collinear factorization
Light-Cone Collinear approach

Ellis+Furmanski+Petronzio 83; Efremov+Teryaev 84; Anikin+Teryaev 03

- The impact factor can be written as

$$\Phi = \int d^4l \cdots \mathrm{tr}[\underbrace{H(l\cdots)}_{\text{hard part}} \quad \underbrace{S(l\cdots)}_{\text{soft part}}]$$



- At the 2-body level:

$$S_{q\bar{q}}(l) = \int d^4z\, e^{-il\cdot z} \langle \rho(p)|\psi(0)\,\bar{\psi}(z)|0\rangle,$$

- H and S are related by $\int d^4l$ and by the summation over spinor indices

Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (2-body case)

1 - Momentum factorization (1)

- Use Sudakov decomposition in the form $(p = p_1,\, n = 2\,p_2/s \Rightarrow p \cdot n = 1)$

$$l_\mu \quad = \quad y\,p_\mu \quad + \quad l_\mu^\perp \quad + \quad (l \cdot p)\,n_\mu\,, \qquad y = l \cdot n$$

scaling: $\quad 1 \quad 1/Q \quad 1/Q^2$

- decompose $H(k)$ around the p direction:

$$H(l) = H(yp) \quad + \quad \left.\frac{\partial H(l)}{\partial l_\alpha}\right|_{l=yp} (l - y\,p)_\alpha + \dots \quad \text{with} \quad (l - y\,p)_\alpha \approx l_\alpha^\perp$$

- In Fourier space, the twist 3 term $l_\alpha^\perp$ turns into a derivative of the soft term
  $\Rightarrow$ one will deal with $\int d^4z\; e^{-il\cdot z} \langle \rho(p)|\psi(0)\, i\, \overleftrightarrow{\partial}_{\alpha\perp} \bar{\psi}(z)|0\rangle$

Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (2-body case)

1 - Momentum factorization (2)

- write

$$d^4 l \longrightarrow d^4 l\ \delta(y - l\cdot n)\ dy$$

- $\int d^4 l\, \delta(y - l\cdot n)$ is then absorbed in the soft term:

$$\begin{aligned}
(\tilde{S}_{q\bar{q}}, \partial_\perp \tilde{S}_{q\bar{q}}) &\equiv \int d^4 l\, \delta(y - l\cdot n) \int d^4 z\, e^{-il\cdot z} \langle \rho(p)|\psi(0)\,(1,\ i\ \overleftrightarrow{\partial_\perp})\bar{\psi}(z)|0\rangle \\
\left(\delta(y-l\cdot n) = \int \frac{d\lambda}{2\pi} e^{-i\lambda(y-l\cdot n)} \Rightarrow\right) &= \int \frac{d\lambda}{2\pi}\, e^{-i\lambda y} \int d^4 z\, \delta^{(4)}(z - \lambda n)\, \langle \rho(p)|\psi(0)\,(1,\ i\ \overleftrightarrow{\partial_\perp})\bar{\psi}(z)|0\rangle \\
&= \int \frac{d\lambda}{2\pi}\, e^{-i\lambda y} \langle \rho(p)|\psi(0)\,(1,\ i\ \overleftrightarrow{\partial_\perp})\bar{\psi}(\lambda n)|0\rangle
\end{aligned}$$

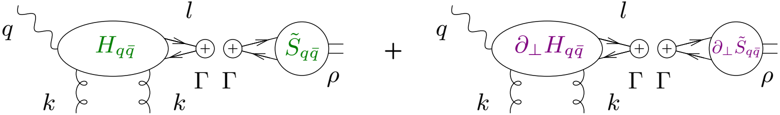
- $\int dy$ performs the longitudinal momentum factorization

Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (2-body case)

2 - Spinorial (and color) factorization

- Use Fierz decomposition of the Dirac (and color) matrices $\psi(0)\,\bar\psi(z)$ and $\psi(0)\, i\, \overleftrightarrow{\partial_\perp}\, \bar\psi(z)$:



- Φ has now the simple factorized form:

$$\Phi = \int dx \, \left\{ \mathrm{tr}\left[H_{q\bar q}(x\,p)\,\Gamma\right]\, S^{\Gamma}_{q\bar q}(x) + \mathrm{tr}\left[\partial_\perp H_{q\bar q}(x\,p)\,\Gamma\right]\, \partial_\perp S^{\Gamma}_{q\bar q}(x) \right\}$$

$\Gamma = \gamma^\mu$ and $\gamma^\mu\,\gamma^5$ matrices

$$S^{\Gamma}_{q\bar q}(x) = \int \frac{d\lambda}{2\pi}\, e^{-i\lambda x} \langle \rho(p)|\bar\psi(\lambda n)\,\Gamma\,\psi(0)|0\rangle$$

$$\partial_\perp S^{\Gamma}_{q\bar q}(x) = \int \frac{d\lambda}{2\pi}\, e^{-i\lambda x} \langle \rho(p)|\bar\psi(\lambda n)\,\Gamma\, i\, \overleftrightarrow{\partial_\perp}\, \psi(0)|0\rangle$$

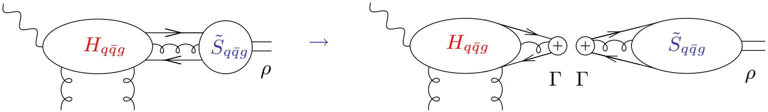
- choose axial gauge condition for gluons, *i.e.* $n\cdot A = 0 \Rightarrow$ no Wilson line

Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (3-body case)

Factorization of 3-body contributions

- 3-body contributions start at genuine twist 3
  $\Rightarrow$ no need for Taylor expansion
- Momentum factorization goes in the same way as for 2-body case
- Spinorial (and color) factorization is similar:



Collinear factorization

Parametrization of vacuum–to–rho-meson matrix elements (DAs): 2-body correlators

2-body non-local correlators

ρ_L twist 2

ρ_T kinematical twist 3 (WW)
genuine twist 3
genuine + kinematical twist 3

- vector correlator

$$\langle \rho(p)|\bar\psi(z)\gamma_\mu\psi(0)|0\rangle \stackrel{\mathcal{F}}{=} m_\rho\, f_\rho\, \left[\varphi_1(y)\,(e^*\cdot n)p_\mu + \varphi_3(y)\, e^{*T}_\mu\right]$$

- axial correlator

$$\langle \rho(p)|\bar\psi(z)\gamma_5\gamma_\mu\psi(0)|0\rangle \stackrel{\mathcal{F}}{=} m_\rho\, f_\rho\, i\,\varphi_A(y)\,\varepsilon_{\mu\lambda\beta\delta}\, e^{*T}_\lambda\, p_\beta\, n_\delta$$

- vector correlator with transverse derivative

$$\langle \rho(p)|\bar\psi(z)\gamma_\mu\, i\,\overleftrightarrow{\partial^\perp_\alpha}\,\psi(0)|0\rangle \stackrel{\mathcal{F}}{=} m_\rho\, f_\rho\, \varphi_1^T(y)\, p_\mu e^{*T}_\alpha$$

- axial correlator with transverse derivative

$$\langle \rho(p)|\bar\psi(z)\gamma_5\gamma_\mu\, i\,\overleftrightarrow{\partial^\perp_\alpha}\,\psi(0)|0\rangle \stackrel{\mathcal{F}}{=} m_\rho\, f_\rho\, i\,\varphi_A^T(y)\, p_\mu\, \varepsilon_{\alpha\lambda\beta\delta}\, e^{*T}_\lambda\, p_\beta\, n_\delta,$$

where y ($\bar y \equiv 1-y$) = momentum fraction along $p \equiv p_1$ of the quark (antiquark) and $\stackrel{\mathcal{F}}{=} \int_0^1 dy\,\exp\,[i\,y\,p\cdot z]$, with $z = \lambda n$

$\Rightarrow$ 5 2-body DAs

Collinear factorization

Parametrization of vacuum–to–rho-meson matrix elements: 3-body correlators

3-body non-local correlators **genuine twist 3**

- vector correlator

$$\langle \rho(p)|\bar{\psi}(z_1)\gamma_\mu g A^T_\alpha(z_2)\psi(0)|0\rangle \stackrel{\mathcal{F}_2}{=} m_\rho\, f_3^V\, B(y_1,y_2)\, p_\mu\, e^{*T}_\alpha,$$

- axial correlator

$$\langle \rho(p)|\bar{\psi}(z_1)\gamma_5\gamma_\mu g A^T_\alpha(z_2)\psi(0)|0\rangle \stackrel{\mathcal{F}_2}{=} m_\rho\, f_3^A\, i\, D(y_1,y_2)\, p_\mu\, \varepsilon_{\alpha\lambda\beta\delta}\, e^{*T}_\lambda\, p_\beta\, n_\delta,$$

where y_1, $\bar{y}_2$, $y_2 - y_1$ = quark, antiquark, gluon momentum fraction

and $\stackrel{\mathcal{F}_2}{=} \int\limits_0^1 dy_1 \int\limits_0^1 dy_2\ \exp\left[i\, y_1\, p\cdot z_1 + i(y_2-y_1)\, p\cdot z_2\right]$, with $z_{1,2} = \lambda n$

$\Rightarrow$ 2 3-body DAs

Collinear factorization
Symmetry properties

From C-conjugation on the previous correlators, one gets:

- 2-body correlators:

$$
\begin{aligned}
\varphi_1(y) &= \varphi_1(1-y) \\
\varphi_3(y) &= \varphi_3(1-y) \\
\varphi_A(y) &= -\varphi_A(1-y) \\
\varphi_1^T(y) &= -\varphi_1^T(1-y) \\
\varphi_A^T(y) &= \varphi_A^T(1-y)
\end{aligned}
$$

- 3-body correlators:

$$
\begin{aligned}
B(y_1, y_2) &= -B(1-y_2, 1-y_1) \\
D(y_1, y_2) &= D(1-y_2, 1-y_1)
\end{aligned}
$$

Collinear factorization
Equations of motion

Equations of motion

twist 2
kinematical twist 3 (WW)
genuine twist 3
genuine + kinematical twist 3

- Dirac equation leads to

$$\langle i(\overrightarrow{\not{D}}(0)\psi(0))_\alpha\,\bar{\psi}_\beta(z)\rangle = 0 \qquad (i\overrightarrow{D}_\mu = i\overrightarrow{\partial}_\mu + A_\mu)$$

- Apply the Fierz decomposition to the above 2 and 3-body correlators

$$-\langle\psi(x)\,\bar{\psi}(z)\rangle = \frac{1}{4}\langle\bar{\psi}(z)\gamma_\mu\psi(x)\rangle\gamma_\mu + \frac{1}{4}\langle\bar{\psi}(z)\gamma_5\gamma_\mu\psi(x)\rangle\gamma_\mu\gamma_5.$$

- $\Rightarrow$ 2 Equations of motion:

$$\bar{y}_1\,\varphi_3(y_1) + \bar{y}_1\,\varphi_A(y_1) + \varphi_1^T(y_1) + \varphi_A^T(y_1)$$
$$+\int dy_2\left[\zeta_3^V\,B(y_1,\,y_2) + \zeta_3^A\,D(y_1,\,y_2)\right] = 0 \qquad \text{and} \quad (\bar{y}_1 \leftrightarrow y_1)$$

- In WW approximation: genuine twist 3 = 0 i.e. $B = D = 0$

$$\begin{cases} \varphi_A^T(y) = \frac{1}{2}[(y-\bar{y})\,\varphi_A^{WW}(y) - \varphi_3^{WW}(y)] \\ \varphi_1^T(y) = \frac{1}{2}[(y-\bar{y})\,\varphi_3^{WW}(y) - \varphi_A^{WW}(y)] \end{cases}$$

Collinear factorization
$n-$independence

A minimal set of DAs

- The non-perturbative correlators cannot be obtained from perturbative QCD (!)
- one should reduce them to a minimal set before using any model
- this can be achieved by using an additional condition: independency of the full amplitude with respect to the light-cone direction n

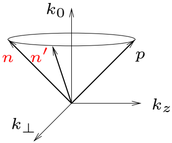
$\Rightarrow$ we prove that 3 independent Distribution Amplitudes are needed:

$\phi_1(y)$ $\longleftarrow$ 2 body twist 2 correlator

$B(y_1,\, y_2)$ $\longleftarrow$ 3 body genuine twist 3 vector correlator

$D(y_1,\, y_2)$ $\longleftarrow$ 3 body genuine twist 3 axial correlator

Collinear factorization
$n-$independence



$n-$independence in practice

- n^μ, with $n^2 = 0,\ n \cdot p = 1$ is not fixed uniquely

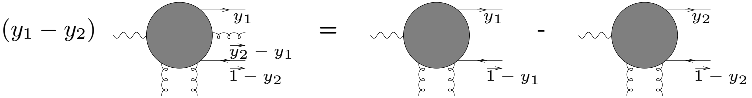
$$n^\mu \to n'^\mu = n^\mu + \frac{\vec{n}^2}{2} p^\mu + n_T^\mu$$

- ρ_T polarization: $e_\mu^{*T} = e_\mu^* - p_\mu\, e^* \cdot n$
- for the full factorized amplitude:

$$\mathcal{A} = H \otimes S \qquad \frac{d\mathcal{A}}{dn^\mu} = 0\,, \qquad \text{where} \quad \frac{d}{dn^\mu} = \frac{\partial}{\partial n^\mu} + e_\mu^* \frac{\partial}{\partial (e^* \cdot n)}$$

- rewrite hard terms in one single form, of 2-body type: use Ward identities
  Example: hard 3-body $\longrightarrow$ hard 2-body

$$\mathrm{tr}\left[H_{3\rho}(y_1, y_2)\, p^\rho\, \not{p}\right]\ B(y_1,\, y_2) = \frac{1}{y_1 - y_2}\left(\mathrm{tr}\left[H_2(y_1)\,\not{p}\right] - \mathrm{tr}\left[H_2(y_2)\,\not{p}\right]\right) B(y_1,\, y_2)\,,$$



- thus, symbolically,

$$\frac{dS}{dn^\mu} = 0$$

Collinear factorization

$n-$independence

Constraints from $n-$independence

twist 2
kinematical twist 3 (WW)
genuine twist 3
genuine + kinematical twist 3

- vector correlators

$$\frac{d}{dy_1}\varphi_1^T(y_1) = -\varphi_1(y_1) + \varphi_3(y_1)$$
$$-\zeta_3^V \int_0^1 \frac{dy_2}{y_2 - y_1}\left(B(y_1, y_2) + B(y_2, y_1)\right)$$

- axial correlators

$$\frac{d}{dy_1}\varphi_A^T(y_1) = \varphi_A(y_1) - \zeta_3^A \int_0^1 \frac{dy_2}{y_2 - y_1}\left(D(y_1, y_2) + D(y_2, y_1)\right)$$

Collinear factorization

A set of independent non-perturbative correlators

Solution

twist 2
kinematical twist 3 (WW)
genuine twist 3
genuine + kinematical twist 3

- the set of 4 equations (2 EOM + 2 n-independence relations) can be solved analytically
- $7 \longrightarrow$ 3 independent DAs

Wandzura-Wilczek

$$\varphi(y) = \varphi^{WW}(y) + \varphi^{gen}(y)\,, \qquad \varphi(y) = \; \varphi_3(y),\; \varphi_A(y),\; \varphi_1^T(y),\; \varphi_A^T(y)$$

where $\varphi^{WW}(y)$ and $\varphi^{gen}(y)$ are contributions in the so called Wandzura-Wilczek approximation and the genuine twist-3 contributions.

WW = vanishing 3-parton distributions $B(y_1, y_2)$ and $D(y_1, y_2)$, i.e. which satisfy the equations

$$\bar{y}_1\, \varphi_3^{WW}(y_1) + \bar{y}_1\, \varphi_A^{WW}(y_1) + \varphi_1^{T\;WW}(y_1) + \varphi_A^{T\;WW}(y_1) = 0$$
$$y_1\, \varphi_3^{WW}(y_1) - y_1\, \varphi_A^{WW}(y_1) - \varphi_1^{T\;WW}(y_1) + \varphi_A^{T\;WW}(y_1) = 0\,.$$
$$\frac{d}{dy_1}\varphi_1^{T\;WW}(y_1) = -\varphi_1(y_1) + \varphi_3^{WW}(y_1)\,, \qquad \frac{d}{dy_1}\varphi_A^{T\;WW}(y_1) = \varphi_A^{WW}(y_1)\,.$$

Solutions:

$$\varphi_A^{WW}(y_1) = \frac{1}{2}\left[\int_0^{y_1}\frac{dv}{\bar{v}}\varphi_1(v) - \int_{y_1}^1 \frac{dv}{v}\varphi_1(v)\right]\,, \qquad \varphi_3^{WW}(y_1) = \frac{1}{2}\left[\int_0^{y_1}\frac{dv}{\bar{v}}\varphi_1(v) + \int_{y_1}^1 \frac{dv}{v}\varphi_1(v)\right]$$

From these expr. the remaining $\varphi_A^{WW\;T}$ and $\varphi_1^{WW\;T}$ are

$$\varphi_A^{T\;WW}(y_1) = \frac{1}{2}\left[-\bar{y}_1\int_0^{y_1}\frac{dv}{\bar{v}}\varphi_1(v) - y_1\int_{y_1}^1 \frac{dv}{v}\varphi_1(v)\right]\,,$$

$$\varphi_1^{T\;WW}(y_1) = \frac{1}{2}\left[-\bar{y}_1\int_0^{y_1}\frac{dv}{\bar{v}}\varphi_1(v) + y_1\int_{y_1}^1 \frac{dv}{v}\varphi_1(v)\right]$$

Genuine twist-3

$$\bar{y}_1\,\varphi_3^{gen}(y_1) + \bar{y}_1\,\varphi_A^{gen}(y_1) + \varphi_1^{T\;gen}(y_1) + \varphi_A^{T\;gen}(y_1)$$
$$= -\int_0^1 dy_2\left[\zeta_3^V\,B(y_1,\,y_2) + \zeta_3^A\,D(y_1,\,y_2)\right]$$

$$y_1\,\varphi_3^{gen}(y_1) - y_1\,\varphi_A^{gen}(y_1) - \varphi_1^{T\;gen}(y_1) + \varphi_A^{T\;gen}(y_1)$$
$$= -\int_0^1 dy_2\left[-\zeta_3^V\,B(y_2,\,y_1) + \zeta_3^A\,D(y_2,\,y_1)\right]\,.$$

$$\frac{d}{dy_1}\varphi_1^{T\;gen}(y_1) = \varphi_3^{gen}(y_1) - \zeta_3^V\int_0^1 \frac{dy_2}{y_2-y_1}\left(B(y_1,y_2)+B(y_2,y_1)\right)\,,$$
$$\frac{d}{dy_1}\varphi_A^{T\;gen}(y_1) = \varphi_A^{gen}(y_1) - \zeta_3^A\int_0^1 \frac{dy_2}{y_2-y_1}\left(D(y_1,y_2)+D(y_2,y_1)\right)\,.$$

Solution for genuine twist-3

$$\varphi_3^{gen}(y) =$$

$$-\frac{1}{2}\int_y^1 \frac{du}{u}\Big[\int_0^u dy_2 \frac{d}{du}(\zeta_3^V B - \zeta_3^A D)(y_2,\, u) - \int_u^1 \frac{dy_2}{y_2 - u}(\zeta_3^V B - \zeta_3^A D)(u,\, y_2)$$

$$- \int_0^u \frac{dy_2}{y_2 - u}(\zeta_3^V B - \zeta_3^A D)(y_2,\, u)\Big]$$

$$-\frac{1}{2}\int_0^{y_1} \frac{du}{\bar{u}}\Big[\int_u^1 dy_2 \frac{d}{du}(\zeta_3^V B + \zeta_3^A D)(u,\, y_2) - \int_u^1 \frac{dy_2}{y_2 - u}(\zeta_3^V B + \zeta_3^A D)(u,\, y_2)$$

$$- \int_0^u \frac{dy_2}{y_2 - u}(\zeta_3^V B + \zeta_3^A D)(y_2,\, u)\Big].$$

Finally, the solution for $\varphi_1^{T\; gen}$

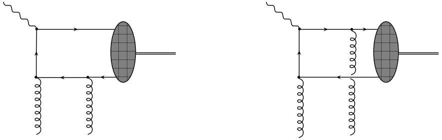
$$\varphi_1^{T\; gen}(y) = \int_0^y du\, \varphi_3^{gen}(u) - \zeta_3^V \int_0^y dy_1 \int_y^1 dy_2 \frac{B(y_1,\, y_2)}{y_2 - y_1}\,.$$

Computation and results

Computation of the hard part

2-body diagrams

- without derivative

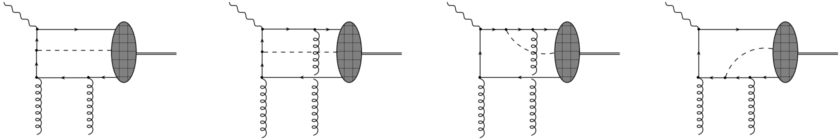


twist 2 $(\gamma^*_L \to \rho_L)$

twist 3 $(\gamma^*_T \to \rho_T)$

- practical trick for computing $\partial_\perp H$: use the Ward identity

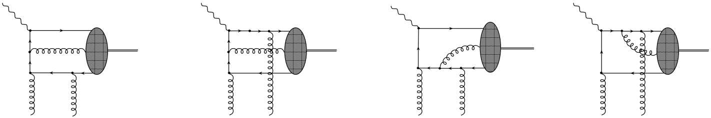
$$\frac{\partial}{p_\mu} \underset{p}{\longrightarrow} = \underset{p}{\longrightarrow} \underset{\gamma^\mu}{\bullet} \underset{p}{\longrightarrow} \qquad \text{where} \quad \underset{p}{\longrightarrow} = \frac{1}{m - \not{p} - i\epsilon}$$



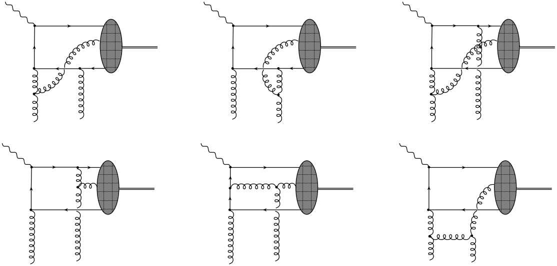
Computation and results
Computation of the hard part

3-body diagrams

- “abelian” type



- “non-abelian” type



Computation and results

Recall: $\gamma_L^* \to \rho_L$ impact factor

$\gamma_L^* \to \rho_L$ impact factor

$$\Phi^{\gamma_L^* \to \rho_L}(\underline{k}^2) = \frac{2\, e\, g^2\, f_\rho}{Q} \frac{\delta^{ab}}{2\, N_c} \int dy\, \varphi_1(y) \frac{\underline{k}^2}{y\, \bar{y}\, Q^2 + \underline{k}^2}$$

pure twist 2 scaling (from ρ-factorization point of view)

Computation and results

Results: $\gamma_T^* \to \rho_T$ impact factor

$\gamma_T^* \to \rho_T$ impact factor:

Spin Non-Flip/Flip separation appears

$$\Phi^{\gamma_T^* \to \rho_T}(\underline{k}^2) = \Phi_{n.f.}^{\gamma_T^* \to \rho_T}(\underline{k}^2)\, T_{n.f.} + \Phi_{f.}^{\gamma_T^* \to \rho_T}(\underline{k}^2)\, T_{f.}$$

where

$$T_{n.f.} = -(e_\gamma \cdot e^*) \quad \text{and} \quad T_{f.} = \frac{(e_\gamma \cdot k)(e^* k)}{\underline{k}^2} + \frac{(e_\gamma \cdot e^*)}{2}$$

non-flip transitions $\left\{ \begin{array}{l} + \to + \\ - \to - \end{array} \right.$ flip transitions $\left\{ \begin{array}{l} + \to - \\ - \to + \end{array} \right.$

Computation and results

Results: $\gamma_T^* \to \rho_T$ impact factor

pure twist 3 scaling (from ρ-factorization point of view)

$$
\begin{aligned}
&\Phi^{\gamma_T^* \to \rho_T}_{n.f.}(\underline{k}^2) \\
&= -\frac{e\, g^2 m_\rho f_\rho}{2\sqrt{2}\, Q^2} \frac{\delta^{ab}}{2\, N_c} \left\{ -2 \int dy_1 \frac{\left(\underline{k}^2 + 2\, Q^2\, y_1\, (1-y_1)\right) \underline{k}^2}{y_1\,(1-y_1)\,(\underline{k}^2 + Q^2\, y_1\,(1-y_1))^2} \left[(2y_1 - 1)\, \varphi_1^T(y_1) + \varphi_A^T(y_1)\right] \right. \\
&+2 \int dy_1\, dy_2 \left[\zeta_3^V\; B(y_1, y_2) - \zeta_3^A\; D(y_1, y_2)\right] \frac{y_1\,(1-y_1)\,\underline{k}^2}{\underline{k}^2 + Q^2\, y_1\,(1-y_1)} \left[\frac{(2 - N_c/C_F) Q^2}{\underline{k}^2\,(y_1 - y_2 + 1) + Q^2\, y_1\,(1-y_2)} \right. \\
&\left. -\frac{N_c}{C_F} \frac{Q^2}{y_2 \underline{k}^2 + Q^2\, y_1\,(y_2 - y_1)} \right] - 2 \int dy_1\, dy_2 \left[\zeta_3^V\, B(y_1, y_2) + \zeta_3^A\, D(y_1, y_2)\right] \left[\frac{2 + N_c/C_F}{1 - y_1} \right. \\
&+ \frac{y_1\, Q^2}{\underline{k}^2 + Q^2 y_1\,(1-y_1)} \left(\frac{(2 - N_c/C_F)\; y_1\, \underline{k}^2}{\underline{k}^2\,(y_1 - y_2 + 1) + Q^2 y_1\,(1-y_2)} - 2 \right) \\
&\left.\left. + \frac{N_c}{C_F} \frac{(y_1 - y_2)\,(1-y_2)}{1 - y_1} \frac{Q^2}{\underline{k}^2\,(1-y_1) + Q^2\,(y_2 - y_1)\,(1-y_2)} \right] \right\}
\end{aligned}
$$

and

$$
\begin{aligned}
\Phi^{\gamma_T^* \to \rho_T}_{f.}(\underline{k}^2) &= -\frac{e\, g^2 m_\rho f_\rho}{2\sqrt{2}\, Q^2} \frac{\delta^{ab}}{2\, N_c} \left\{ 4 \int dy_1 \frac{\underline{k}^2\, Q^2}{(\underline{k}^2 + Q^2\, y_1\,(1-y_1))^2} \left[\varphi_A^T(y_1) - (2y_1 - 1)\, \varphi_1^T(y_1)\right] \right. \\
&- 4 \int dy_1\, dy_2 \frac{y_1\, \underline{k}^2}{\underline{k}^2 + Q^2\, y_1\,(1-y_1)} \left[\zeta_3^A\, D(y_1, y_2)\,(-y_1 + y_2 - 1) + \zeta_3^V\; B(y_1, y_2)\,(y_1 + y_2 - 1)\right] \\
&\left. \times \left[\frac{(2 - N_c/C_F) Q^2}{\underline{k}^2\,(y_1 - y_2 + 1) + Q^2\, y_1\,(1-y_2)} - \frac{N_c}{C_F} \frac{Q^2}{y_2\, \underline{k}^2 + Q^2 y_1\,(y_2 - y_1)} \right] \right\}
\end{aligned}
$$

Computation and results

Results: $\gamma_T^* \to \rho_T$ impact factor

WW limit

- WW limit: keep only twist 2 + kinematical twist 3 terms (i.e $B = D = 0$)
- The only remaining contributions come from the two-body correlators
- non-flip transition

$$\Phi^{\gamma_T^* \to \rho_T}_{n.f.}(\underline{k}^2) = \frac{-e\, m_\rho\, f_\rho}{2\sqrt{2}\, Q^2} \frac{\delta^{ab}}{2\, N_c} \int_0^1 dy \left\{ \frac{(y-\bar{y})\varphi_1^{T\,WW}(y) + 2\, y\, \bar{y}\, \varphi_3^{WW}(y) + \varphi_A^{T\,WW}(y)}{y\, \bar{y}} - \frac{2\,\underline{k}^2 \left(\underline{k}^2 + 2\, Q^2\, y\, \bar{y}\right) \left((y-\bar{y})\, \varphi_1^{T\,WW}(y) + \varphi_A^{T\,WW}(y)\right)}{y\, \bar{y}\, (\underline{k}^2 + Q^2\, y\,(1-y))^2} \right\}$$

which simplifies, using equation of motion:

$$\int dy\, [(y-\bar{y})\varphi_1^{T\,WW}(y) + 2\, y\, \bar{y}\, \varphi_3^{WW}(y) + \varphi_A^{T\,WW}(y)] = 0$$

$$\Phi^{\gamma_T^* \to \rho_T}_{n.f.}(\underline{k}^2) = \frac{e\, m_\rho\, f_\rho}{\sqrt{2}\, Q^2} \frac{\delta^{ab}}{2\, N_c} \int_0^1 dy \frac{2\,\underline{k}^2 \left(\underline{k}^2 + 2\, Q^2\, y\, \bar{y}\right)}{y\, \bar{y}\, (\underline{k}^2 + Q^2\, y\, \bar{y})^2} \left[(2\, y - 1)\, \varphi_1^{T\,WW}(y) + \varphi_A^{T\,WW}(y)\right].$$

- flip transition:

$$\Phi^{\gamma_T^* \to \rho_T}_{f.}(\underline{k}^2) = -\frac{e\, m_\rho\, f_\rho}{\sqrt{2}\, Q^2} \frac{\delta^{ab}}{2\, N_c} \int_0^1 \frac{2\,\underline{k}^2\, Q^2}{(\underline{k}^2 + Q^2\, y\, \bar{y})^2} \left[(1 - 2\, y)\varphi_1^{T\,WW}(y) + \varphi_A^{T\,WW}(y)\right].$$

Computation and results

- The obtained results are gauge invariant

$$\Phi^{\gamma_T^* \to \rho_T} \to 0 \quad \text{when} \quad \underline{k} \to 0$$

- $\gamma_T^* \to \rho_T$ impact factor is gauge-invariant only provided the 2 and 3-body contributions have been taken into account in a consistant way

- Our results are free of end-point singularities, in both WW approximation and full twist-3 order calculation

Computations and results

- Comparison with a fully covariant approach by Ball+Braun et al:
The dictionnary between the two approaches within a full twist 3 treatment is now established:

$$
\begin{aligned}
B(y_1,\,y_2) &= -\frac{V(y_1,\,1-y_2,\,y_2-y_1)}{y_2-y_1}\,,\\
D(y_1,\,y_2) &= -\frac{A(y_1,\,1-y_2,\,y_2-y_1)}{y_2-y_1}\\
\varphi_1(y) &= \phi_{||}(y)\\
\varphi_3(y) &= g^{(v)}(y)\,,\\
\varphi_A(y) &= -\frac{1}{4}\frac{\partial g^{(a)}(y)}{\partial y}
\end{aligned}
$$

- We performed calculations of the same impact factor within the covariant approach by Ball+Braun et al:

  calculations proceed in quite different way : eg. no $\varphi^T_{1,A}$ −DAs but Wilson line effects are important !!

  We got a full agreement between two approaches

Conclusions

- We have performed a full up to twist 3 computation of the $\gamma^* \to \rho$ impact factor, in the $t = t_{min}$ limit.

- Our impact factor respects gauge invariance. This is achieved ONLY after including 2 and 3 body correlators.

- It is free of end-point singularities
(this should be contrasted with standard collinear treatment, at moderate s, where k_T-factorization is NOT applicable: see Mankiewicz-Piller).

- We relied on the Light-Cone Collinear approach
(Ellis + Furmanski + Petronzio; Efremov + Teryaev; Anikin + Teryaev), which is non-covariant, but very efficient for practical computations.

  Agreement with the covariant approach by Ball et al

- This Light-Cone Collinear approach is systematic, and can be extended to any process, including higher twist effects (but does not preclude potential end-point singularities)

Phenomenological prospects:

- We have all ingredients necessary to estimate:
  - $\frac{\sigma_L}{\sigma_T}$
  - elements of the density matrix
  - how important are $\bar{q}\,q\,g$ contributions compared to $\bar{q}\,q$ ones
  - generalizations for $t \neq 0$

THANK YOU FOR ATTENTION