AdS/CFT Correspondence: the high energy limit

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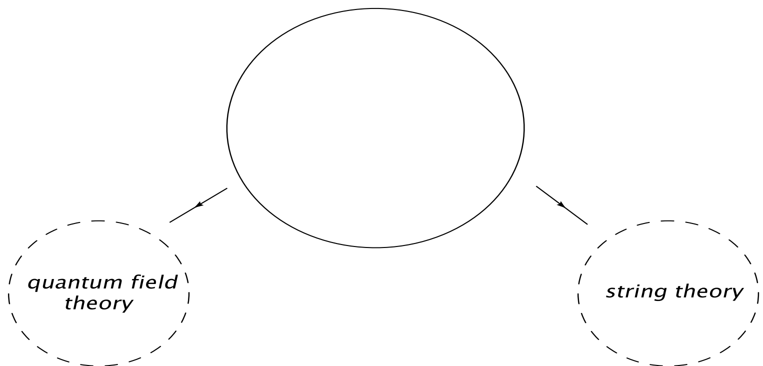
GPD Workshop, Trento, October 13, 2010

- Introduction: realistic expectations
- High energy scattering of planar amplitudes
- The Pomeron in AdS/CFT
- Conclusions

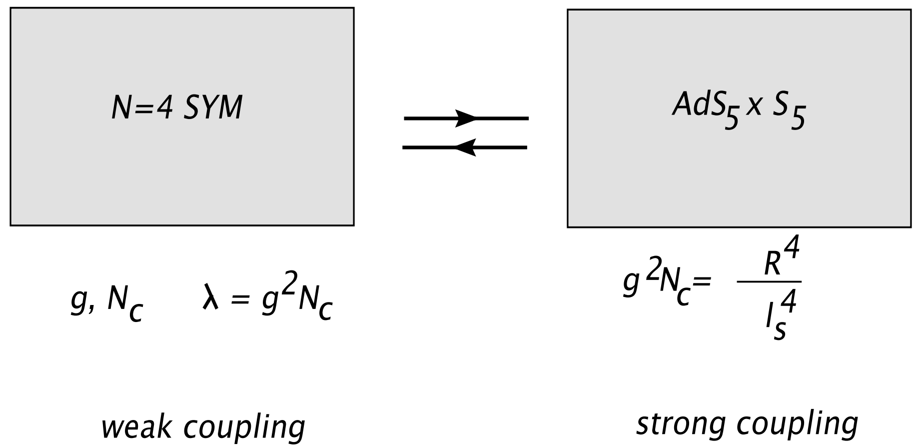
Introduction

Hypothesis of AdS/CFT correspondence:
certain quantum field theories and string theories are two different limits of the same theory:



Hopes connected with this conjecture:
- solve quantum field theory beyond perturbation theory
- solve QCD beyond weak coupling (need to know the dual analogue)
- connnect string theory with the real world

Since we do not know the dual of QCD: begin with $N = 4$ SYM:
The most symmmetric gauge theory (β-function vanishes).
Differs from QCD (particle content, no running of the coupling constant)
Hope: theory is soluble (integrable), plays role of 'harmonic oscillater in Quantum mechanics.



On both sides expansion in $1/N_c$ (expansion in toplogy).
This talk: analyse high energy scattering amplitudes
(after the recent successes in anomalous dimensions of gauge invariant operators).
History: Regge limit stimulated string theory (Veneziano amplitudes),

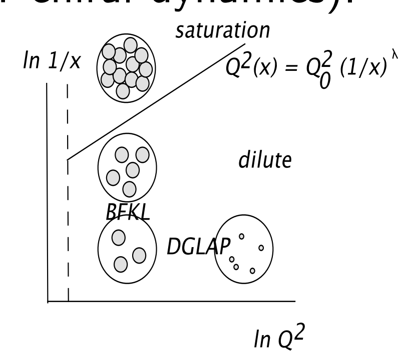
Three lines of investigations:

(a) scattering amplitudes in the planar limit.
Main interest: n point amplitudes in $N = 4$, guide for multiloop/multileg amplitudes in QCD.
Important starting point: BDS formula. Recent attempts to find corrections.
Is $N = 4$SYM soluble: integrability?

(b) Gauge invariant scattering amplitudes: Vacuum exchange (Pomeron-Graviton duality)

(c) Modelling the infrared, beyond $N = 4$ SYM:
(Soft) Pomeron in hadron-hadron scattering is non-pertubative: need methods other the pQCD.
Physical Pomeron is also sensitive to low-energy features of QCD (slope α': chiral dynamics).

Hard Pomeron: in scattering of small-size projectiles (virtual photon)
Soft Pomeron: in hadron-hadron scattering
Transition in deep inelastic scattering (saturation, unitarization)

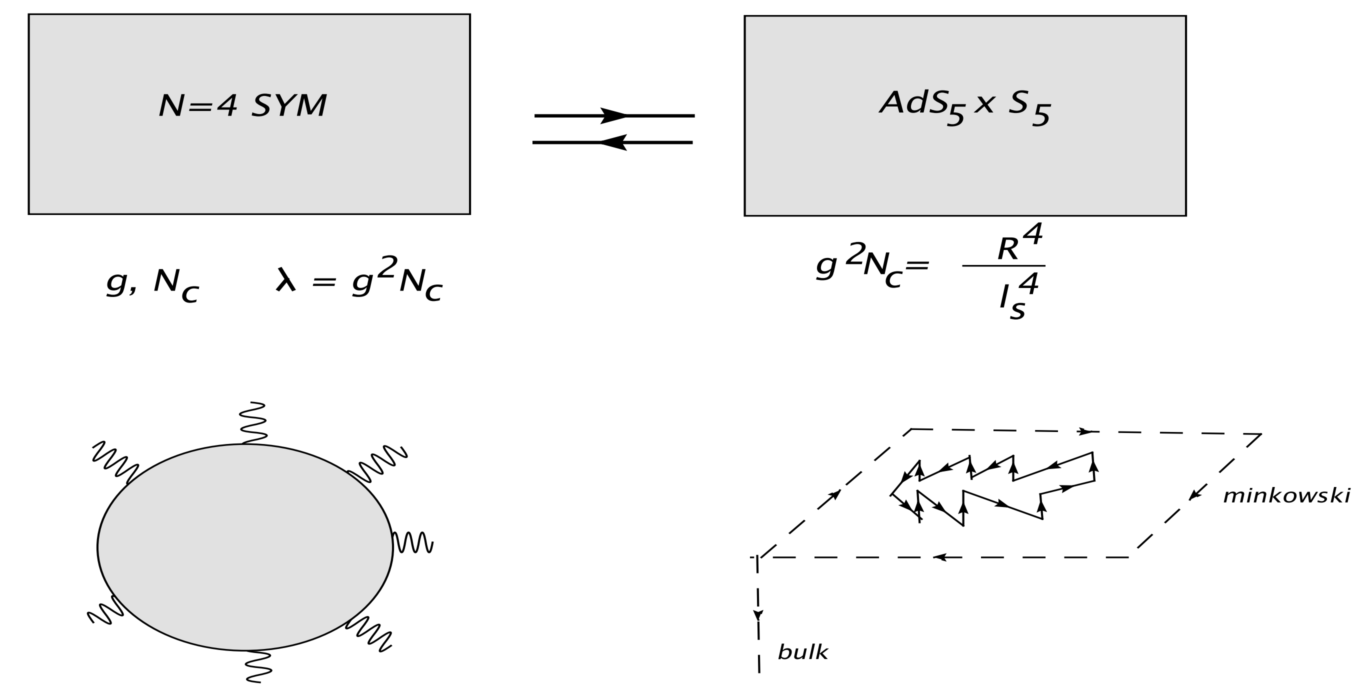


transition from hard to soft

Within AdS/CFT : hard Pomeron $\longrightarrow$ unitarization $\longrightarrow$ more sophisticated geometry on the string

Planar scattering amplitudes at high energies

$N = 4$, MHV amplitudes. Duality:



Compute planar amplitudes

Compute minimal surfaces
Amplitude has exponential form

Gauge theory side: enormous activity, in particular recent progress in two loop calculations.
Most remarkable: Bern-Dixon-Smirnow (BDS) formula for planar n-gluon scattering amplitude:

Remove color factors, factor out tree amplitude, IR singular:

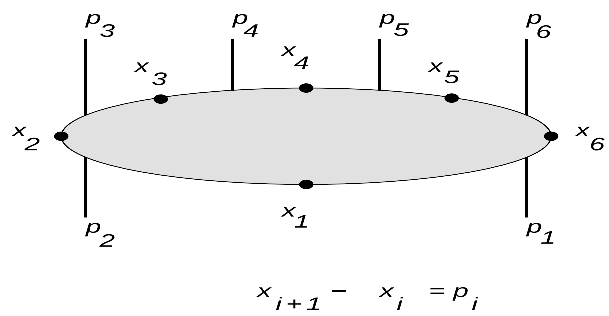
$$tr(T^{a1}...T^{an}) + noncycl.perm, \quad A_n = A_n^{tree} \cdot M_n(\epsilon)$$

$$\ln M_n = \sum_l a^l \Big[\Big(f^{(l)}(\epsilon) I_n(l\epsilon) + F_n(0) \Big) + C^{(l)} + E_n^{(l)}[\epsilon] \Big]$$

$$a = \frac{N_c \alpha}{2\pi} (4\pi e^{-\gamma})^\epsilon, \; d = 4 - 2\epsilon$$

Present understanding: formula correct for $n = 4$ and $n = 5$.
Needs corrections for $n \geq 6$.

Dual conformal symmetry:



Invariance under conformal transformations in dual space x_i.
Present believe: in euklidean region (all invariants are negative)

$$M_n \sim \exp[\ln M_n^{BDS} + R^{(n)}]$$

Remainder function $R^{(n)}$ function $(n \geq 6)$ depends upon unharmonic cross ratios, e.g. $R^{(6)}(u_1, u$

$$u_1 = \frac{x_{13}^2 x_{46}^2}{x_{14}^2 x_{36}^2}, u_2 = \frac{x_{24}^2 x_{15}^2}{x_{25}^2 x_{41}^2}, u_3 = \frac{x_{26}^2 x_{35}^2}{x_{25}^2 x_{36}^2} = \frac{s_2 s}{s_{345} s_{456}}$$

Strong interest: find the remainder function $R^{(n)}$ Holy Grail Function

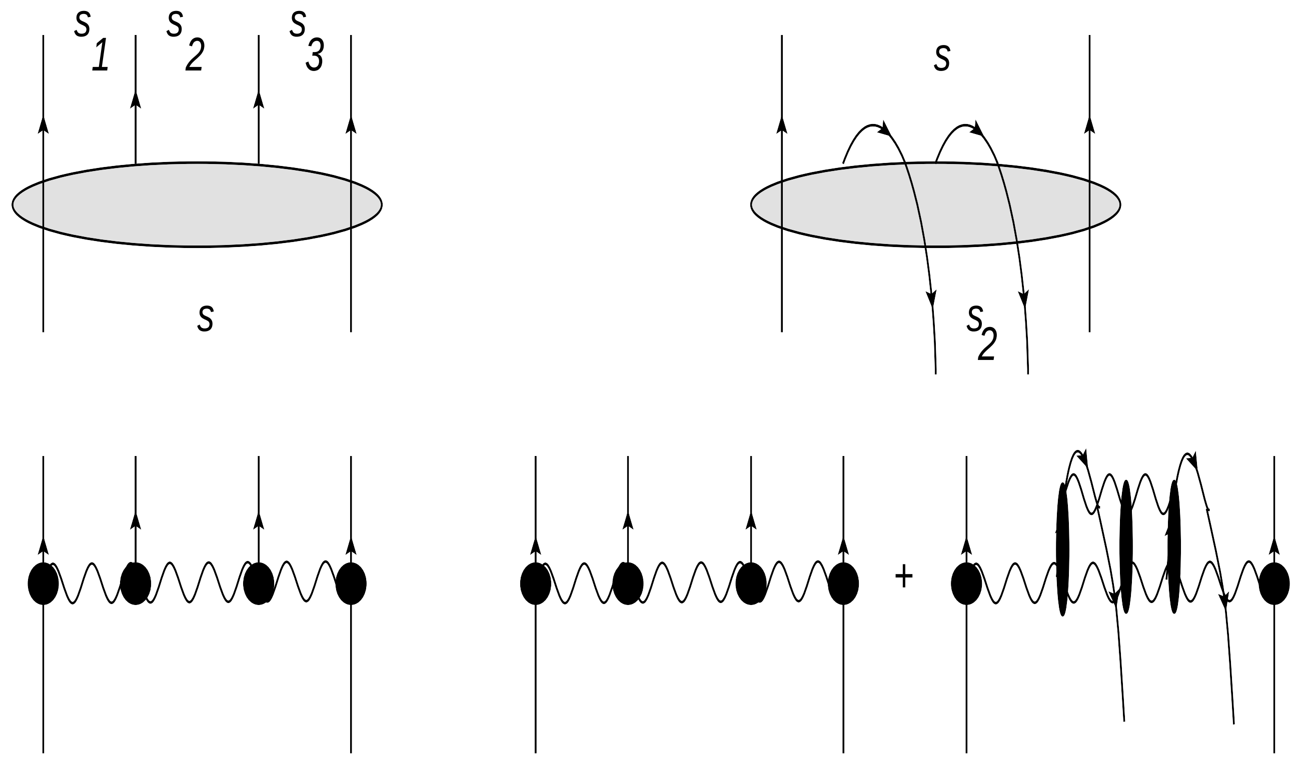
How much do we know about $R^{(n)}$:

- vanishes for $n = 4, 5$ (no anharmonic ratios):
  consistency test: $R^{(6)}$ should vanish in certain 'collinear' limits
- we have exact two loop results (Del Duca et al; Goncharov et al)
- new input from strong coupling (see below)

Some help from the Regge limit (JB, Lipatov, Sabio-Vera):
compare with leading log calculations

- BDS not correct, identify missing piece.
- define "mixed" physical region (some energies positive, others negative there exists a special Regge-cut piece, visible just in this region.
- remainder function $R^{(6)}$ should correct the BDS formula in this region

More on this “Regge cut contribtion” (first in a 1979 paper):

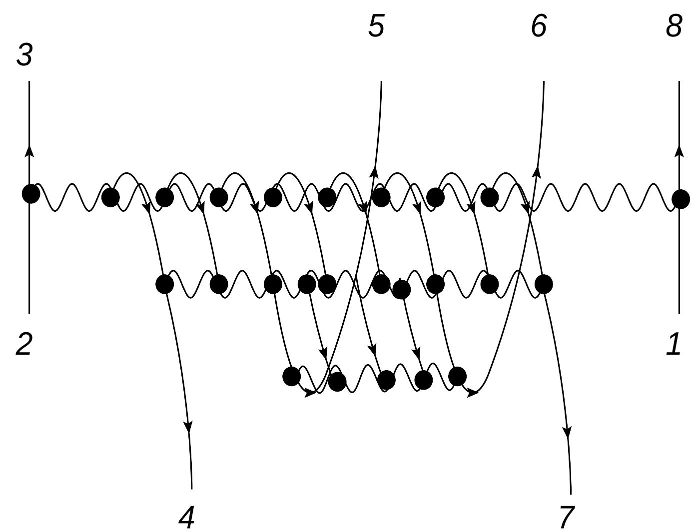


Appears in the 'mixed physical region' (nonplanarity: Mandelstam cut) and in energy discontinuities.

Important property: integrability

$$\Delta T_{2\to 4} \sim s_2^{-E}$$

where E is lowest eigenvalue of the color octet BFKL Hamiltonian $H^{(8)}_{BFKL}$.
Generalization to $n > 6$:



BKP-octet hamiltonian is integrable (open spin chain)(Lipatov)
Direct evidence for integrability at weak coupling, related to $R^{(n)}$.

On the strong coupling side:

AdS/CFT correspondence:
at strong coupling, the scattering amplitude is given by a minimal area A:

$$\text{Amp} \quad \sim \quad \langle W \rangle \sim \exp\left[-\frac{\sqrt{\lambda}}{2\pi} A\right] = \exp\left[-\frac{\sqrt{\lambda}}{2\pi}(A_{\text{div}} + A_{\text{BDS}} - R)\right]$$

Contours (light-like polygon) of the area are determined by kinematics

Euler-Lagrange equations very complicated: solved for $n = 4$. (Alday,Maldacena):
4-point amplitude is known for all values of λ!

For $n \geq 6$: instead of solving Euler-Lagrange equations use auxiliary quantum integrable system: mi
is related to free energy of this system (Alday,Maldacena,Sever,Vieira; Alday,Gaiotto,Maldacena).
Concretely: area is obtained from a family of functions (Y functions) which obey set of nonline
equations.
Task: solve these equations, as function of the polygon.

The Y equations:

$$\log Y_2(\theta) = -m\sqrt{2}\cosh(\theta - i\phi) - 2\int_{-\infty}^{\infty} d\theta' K_1(\theta-\theta')\log(1+Y_2(\theta'))$$
$$- \int_{-\infty}^{\infty} d\theta' K_2(\theta-\theta')\log((1+Y_1(\theta'))(1+Y_3(\theta')))$$

$$\log Y_{2\pm 1}(\theta) = -m\cosh(\theta - i\phi) \pm C - \int_{-\infty}^{\infty} d\theta' K_2(\theta-\theta')\log(1+Y_2(\theta'))$$
$$- \int_{-\infty}^{\infty} d\theta' K_1(\theta-\theta')\log((1+Y_1(\theta'))(1+Y_3(\theta'))) \ .$$

$$u_1 = \frac{x_{13}^2 x_{46}^2}{x_{14}^2 x_{36}^2} = \left(1 + \frac{1}{Y_2(\theta = -i\pi/4)}\right)^{-1}$$
$$u_2 = \frac{x_{24}^2 x_{15}^2}{x_{25}^2 x_{14}^2} = \left(1 + \frac{1}{Y_2(\theta = i\pi/4)}\right)^{-1}$$
$$u_3 = \frac{x_{35}^2 x_{26}^2}{x_{36}^2 x_{25}^2} = \left(1 + \frac{1}{Y_2(\theta = -3i\pi/4)}\right)^{-1}$$

with $m = m(u_1, u_2, u_3)$, $\phi = \phi(u_1, u_2, u_3)$, $C = C(u_1, u_2, u_3)$

What we have done (JB,Kotanski,Schomerus) numerical solution, Regge limit helps

- have computed the remainder function $R^{(6)}$ in the Regge limit (in the physical/euklidean region)
  $R^{(6)} \longrightarrow const$
- performed the analytic continuation into the 'mixed region':
  a new term appears which has Regge behavior
  (and can be attributed to an excitation of the TBA system)

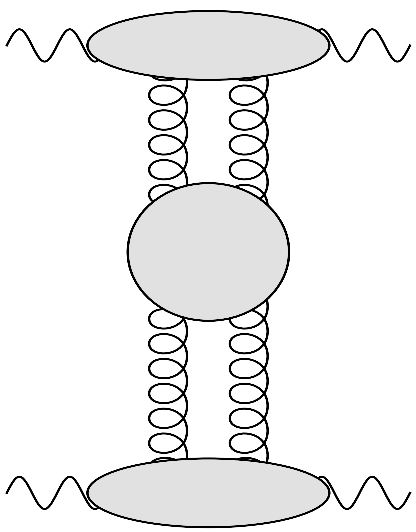
Still to do:

- check Steinmann relations
- is exponentiation correct?

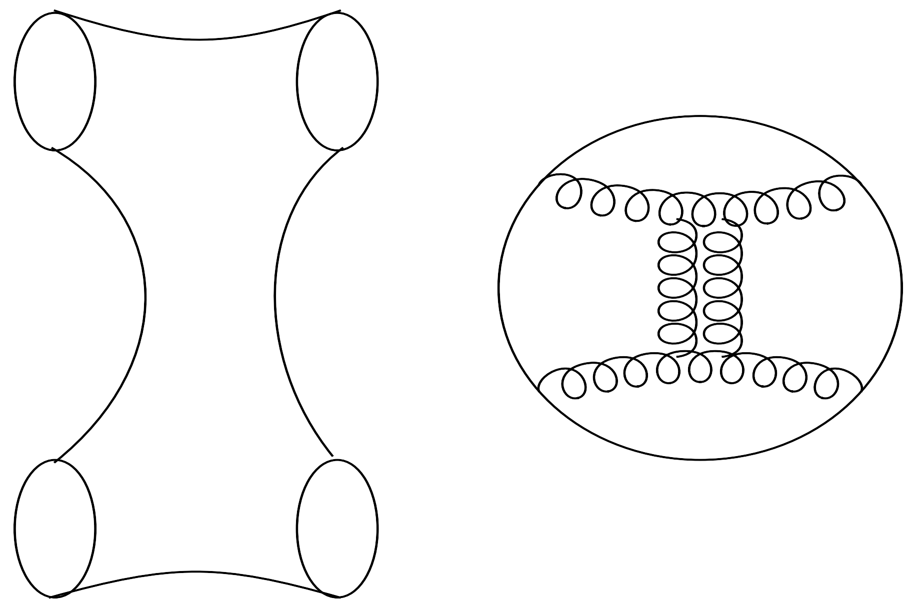
Resume:
Evidence that we can construct n-point amplitudes for weak and strong coupling,
but there is still work to be done.

“Phenomenology” in AdS/CFT: Pomeron and DIS

A. Basic message: BFKL in $N = 4$ SYM is dual to the graviton in AdS_5



weak coupling: BFKL



strong coupling: graviton

B. In more detail:

correlator of R-currents (global $SU(4)$ symmetry): analogue of $\gamma^*\gamma^*$-scattering in QCD:

$$< J_{\mu_1}(x_1) J_{\mu_2}(x_2) J_{\mu_3}(x_3) J_{\mu_4}(x_4) > \sim < R_{\mu_1}(x_1) R_{\mu_2}(x_2) R_{\mu_3}(x_3) R_{\mu_4}(x_4) >$$

First: weak coupling side the BFKL amplitude

$$A(s,t) = is \int \frac{d\omega}{2\pi i} \left(\frac{s}{kk'}\right)^{\omega} \Phi_1(Q_A^2, k, q-k) \otimes G_\omega(k, q-k; k', q-k') \otimes \Phi_2(Q_B^2, k', c$$

Impact factors (JB et al; Balitski,Chirilli), characteristic BFKL function (Lipatov et al) are known i

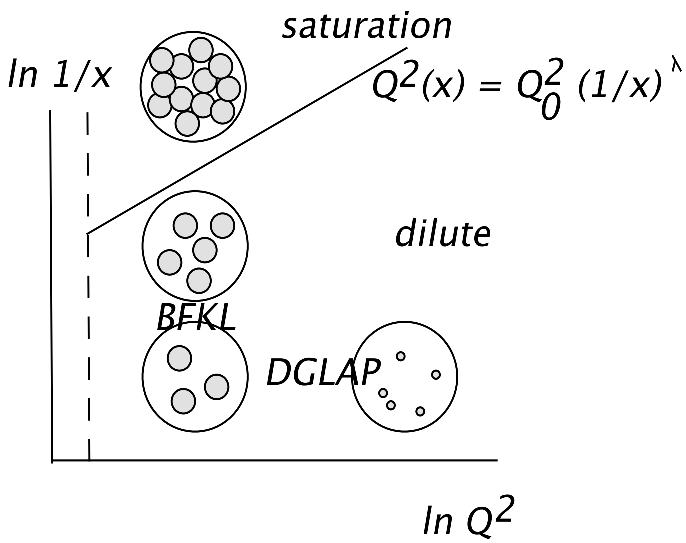
$$G_\omega(k, q-k; k', q-k') \sim \frac{1}{\omega - \chi(n,\nu})$$

Connection between small x-limit and short distance limit (DIS):
leading twist anomalous dimension near $\omega = j - 1 \approx 0$

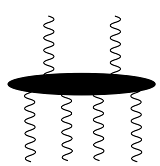
$$A(s, t=0) \sim \frac{is}{Q^2} \int \frac{d\omega}{2\pi i} \left(\frac{s}{Q_1^2}\right)^{\omega} \int \frac{d\nu}{2\pi i} \left(\frac{Q_1^2}{Q_2^2}\right)^{i\nu+\omega/2} \Phi_1(n,\nu) \frac{1}{\omega - \chi(\nu, 0)} \Phi_2(n, \nu$$

Beyond the BFKL: unitarization problem: as old as strong interactions.

Best understood in deep inelastic scattering:



transition from hard to soft



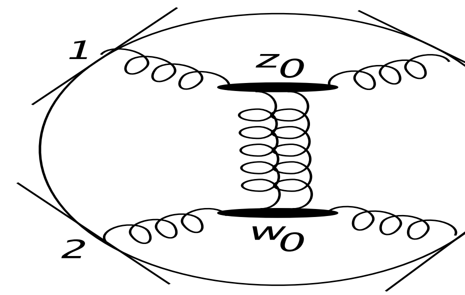
From large x, Q^2 to small x, Q^2, three regions:

dilute (hard), saturation (dense),
strong interaction (soft Pomeron).

Near the saturation region:
vital role of triple Pomeron vertex (BK-kernel)
Appealing physical picture

Next: the strong coupling side:
The leading term (in $1/\lambda$) is given by supergravity (Witten diagram): graviton exchange.
Calculation (JB, Kotansk, Schomerus) gives:

$$I^{\mathrm{GR}} = \frac{1}{4}\int \frac{d^4 z dz_0}{z_0}\int \frac{d^4 w dw_0}{w_0} T_{(13)\mu\nu}(z) G_{\mu\nu;\mu'\nu'}(z,w) T_{(24)\mu'\nu'}(w)\,.$$



Fouriertransform, high energy limit, polarization vectors, helicity structure of the exchanged gravitor

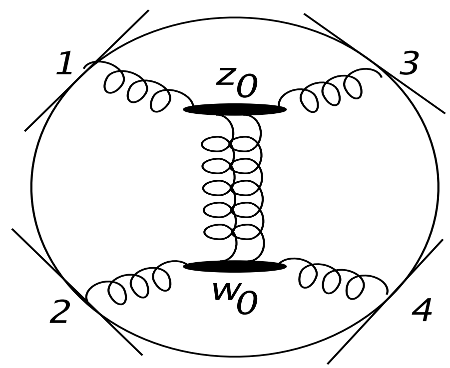
$$\frac{2p_{2;\mu}p_{1;\mu'}}{s}\frac{2p_{2;\nu}p_{1;\nu'}}{s}$$

leads to

$$\mathcal{A}^{\mathrm{GR}}_{\lambda_1\lambda_2\lambda_3\lambda_4}(s,t) = s^2 \int dz_0 dw_0 \Phi_{\lambda_1\lambda_3}(|\vec{p}_1|,|\vec{p}_3|;z_0)\,\Sigma(|\vec{p}_1+\vec{p}_3|,z_0,w_0)\,\Phi_{\lambda_2\lambda_4}(|\vec{p}_2|,|\vec{p}_4|;$$

'Impact factors', integral over fifth coordinate analogous to transverse momentum.

Limit of $Q_A^2 \gg Q_B^2$: dominant region close to the boundary ($z_0 \ll w_0$):

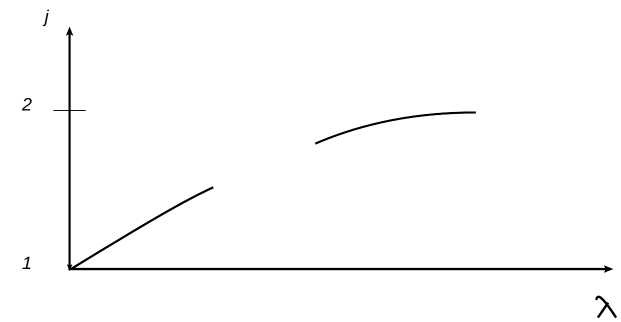


'hard physics' lives close to the boundary, 'soft physics' close to the center.
Consequence: attempts to get closer to QCD will modify the center (hard wall...)

Further details:
find powers of $\ln Q_A^2/Q_B^2$, beginning of OPE expansion?
Dependence upon polarization: similar to QCD.

Cannot see in Witten diagram: reggeization of the graviton. $j = 2 \longrightarrow j = 2 - \frac{2}{\sqrt{\lambda}} + \mathcal{O}(\lambda)$
More general (Lipatov et al, Polchinski et al, Brower et al):
existence of function $j(\nu, \lambda)$

$$1 + \chi(\nu, \lambda) < j(\nu, \lambda) < 2 - \frac{4+\nu^2}{2\sqrt{\lambda}} + \dots$$



Diffusion in $\ln z$ (Brower et al.).

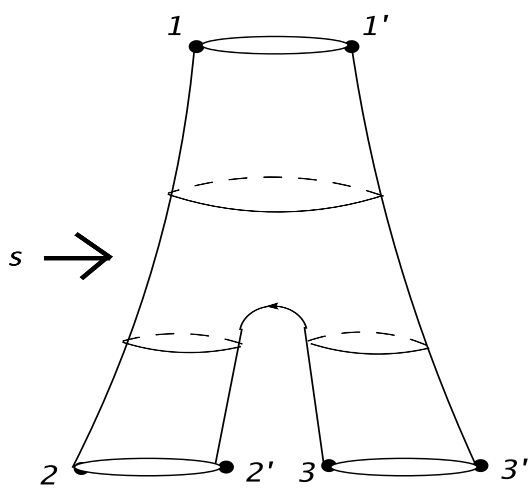
Result for $\gamma^*\gamma^*$/R current-R current scattering:

- intercept: function $j(\nu, \lambda)$ interpolates between weak and strong coupling: $1 < j(\nu, \lambda) < 2$.
  We know the first two corrections for $\lambda \longrightarrow 0$, first correction at $\lambda \longrightarrow \infty$.
  Connection with anomalous dimension.
- impact factor: we know the first term at $\lambda \longrightarrow 0$, the first term at $\lambda \longrightarrow \infty$.
- need string calculation

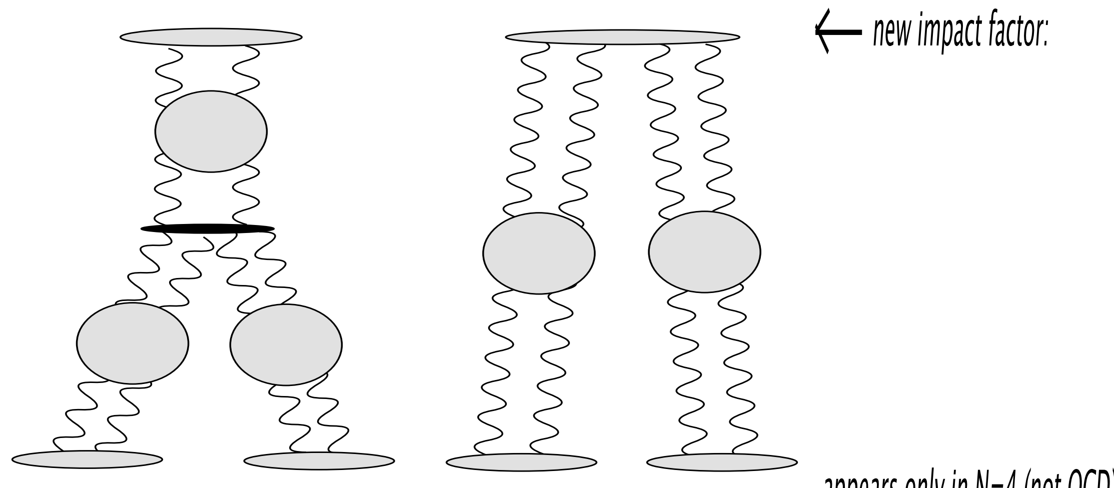
What next: unitarization. Eikonalization?

Problem of unitarization worse than BFKL: single graviton $\sim s^2$, double graviton $\sim s^3$,...
Need to go beyond planar (large-N_c limit): as first step study six-point function.

On the gauge theory side: pair-of pants topology:

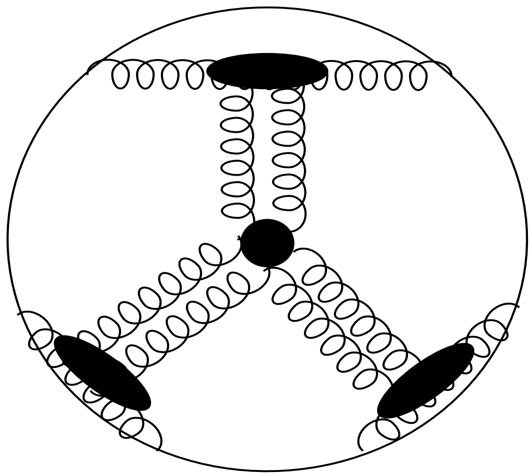


color diagram

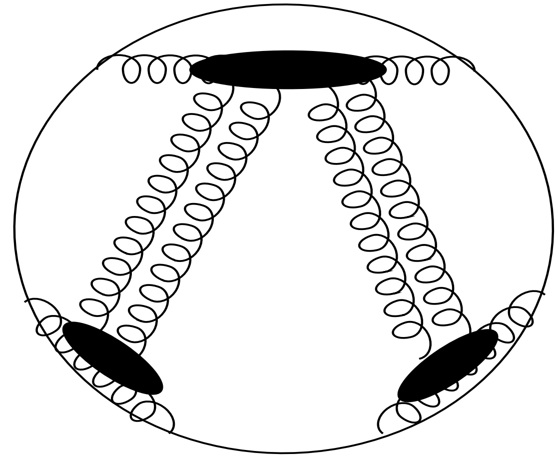


momentum space

On the string theory side:



triple graviton vertex vanishes:
need string theory calculation



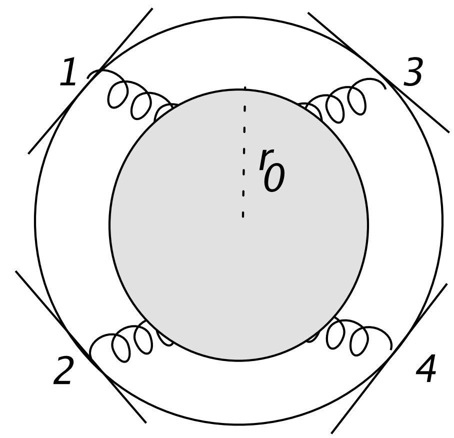
compute new impact factor
(leading order in $1/\lambda$)

To leading order: triple graviton vertex vanishes!

C. A more ambitious approach: a 'soft' Pomeron in a 'confining' theory (Polchinski et al)

Observation: 'soft' Pomeron comes from larger values of fifth coordinate z_0. (smaller r):

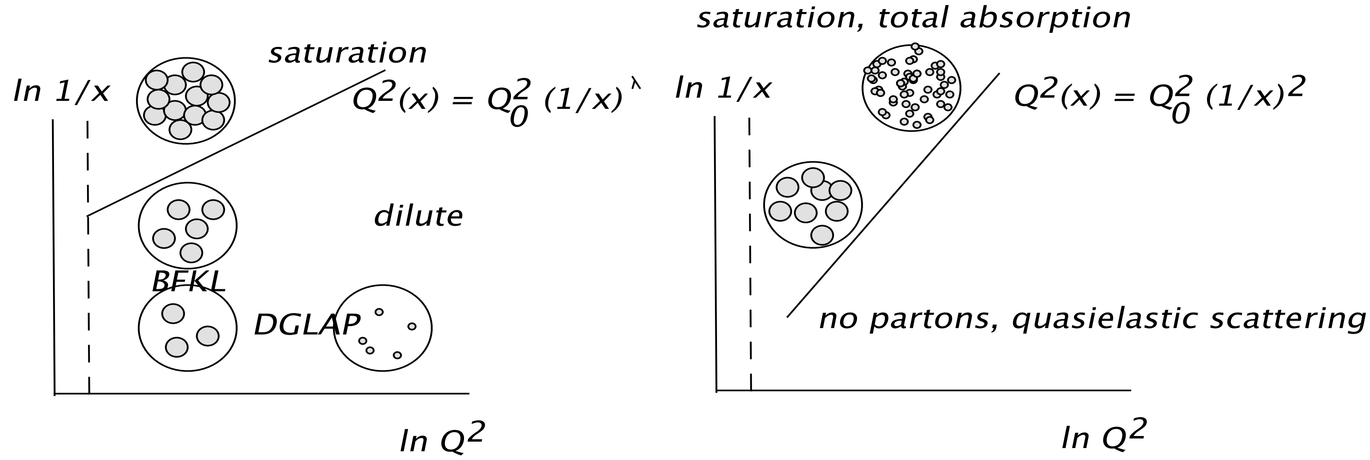
Modify the $AdS_5 \times W$: boundary $\longrightarrow$ scale.
Compute glueball, continue in t.
Obtain slope parameter.



Questions:
how to connect this soft 'Pomeron' with the hard Pomeron (=reggeized graviton)?
Is there 'saturation'?

D. Deep Inelastic scattering (Polchinski et al; Mueller,Hatta,Iancu)

Goal: deep inelastic scattering for all x.
Framework: $N = 4$ DIS on hot plasma, or DIS on dilaton field



real QCD (weak coupling) — N=4 SYM with scale (strong coupling)

Most striking results:
- no partons at finite x
- saturation line $Q_s^2 \sim (T/x)^2$ (multiple graviton exchange).

Conclusions

Exciting investigations:
scattering amplitudes in $N = 4$ SYM from weak to strong coupling:

- weak coupling: BDS formula, exponentiation, remainder function
  strong coupling: Y equations. First attempts to solve.
  important role: integrability
- Pomeron-Graviton duality: control the weak coupling limit (NLO)
  and the strong coupling limit (LO)
  need string calculations
- Steps towards phenomenology

$$\mathsf{Amp}' = \mathsf{Amp}^{BDS}_{2\to 4}\ \ (1 + i\Delta_{2\to 4})$$

$$i\Delta_{2\to 4} \ = \ \frac{a}{2}\sum_{n=-\infty}^{n=\infty}(-1)^n \int \frac{d\nu}{\nu^2+\frac{n^2}{4}} \left(\frac{q_2^* p_4^*}{p_5^* q_1^*}\right)^{i\nu-\frac{n}{2}} (s_2^{\omega(\nu,n)} - 1) \left(\frac{q_2 p_4}{p_5 q_1}\right)^{i\nu+\frac{n}{2}} \ .$$

$$\omega(\nu,n) \ = \ 4a\mathcal{R}\left(2\psi(1) - \psi(1+i\nu+\frac{n}{2}) - \psi(1+i\nu-\frac{n}{2})\right) \ .$$

Leading: $n = 1$, $\nu = 0$: $\omega(0,1) = \frac{\lambda}{\pi^2}(2\ln 2 - 1)$

What about exponentiation?

Strong coupling result $((1-u_3)\sim 1/s_2)$

$$\text{Amp}' \quad \sim \quad \langle W'\rangle \sim \exp\left[-\frac{\sqrt{\lambda}}{2\pi}A'\right] = \exp\left[-\frac{\sqrt{\lambda}}{2\pi}(A'_{\rm div}+A'_{\rm BDS}-R')\right]$$

$$e^{\frac{\sqrt{\lambda}}{2\pi}R'} \;\sim\; \Big((1-u_3)\sqrt{\tilde u_1\tilde u_2}\Big)^{\frac{\sqrt{\lambda}}{2\pi}e_2}\, e^{-i\frac{\pi}{2}\frac{\sqrt{\lambda}}{4\pi}\ln(\tilde u_1\tilde u_2)}\, e^{-\frac{\sqrt{\lambda}}{\sqrt{2}\pi}|\log(\tilde u_1/\tilde u_2)|}$$

$$e_2 \;=\; \left(\sqrt{2}+\frac{1}{2}\log(3+2\sqrt{2})\right)$$

Weak coupling:

$$\text{Amp}' = \text{Amp}^{BDS}_{2\to 4}\;\;(1+i\Delta_{2\to 4})$$

$$i\Delta_{2\to 4} \;=\; \frac{a}{2}\sum_{n=0}^{n=\infty}(-1)^n\int\frac{d\nu}{\nu^2+\frac{n^2}{4}}\left((1-u_3)^{-\omega(\nu,n)}-1\right)|w|^{2i\nu}\cosh nC$$

$$w \;\approx\; \sqrt{\frac{u_1}{u_2}} \qquad \text{and} \qquad \cosh C \;=\; \frac{1-u_1-u_2-u_3}{2\sqrt{u_1u_2u_3}} \qquad \text{and} \qquad \omega(0,1) \;=\; -E_2 \;=\; \frac{\lambda}{\pi^2}(2\ln 2-1)\;.$$