POLARIZED PDFs and HIGHER TWIST from NLO ANALYSIS of DIS and SIDIS

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CONTENTS

1) From experiment to $g_1(x, Q^2)$ in DIS——kinematics.

2) DIS theoretical expression for $g_1(x, Q^2)$.

3) Extension to SIDIS.

4) Data sample.

5) Results and comparison with DSSV.

6) Controversy about Higher Twist.

7) Spin sum rule.

From experiment to $g_1(x, Q^2)$ in DIS

Measured asymmetries:

$$A_{\parallel} = \frac{d\sigma^{\overset{\rightarrow}{\Leftarrow}} - d\sigma^{\overset{\rightarrow}{\Rightarrow}}}{2\, d\sigma_{unpold}} \qquad A_{\perp} \equiv \frac{d\sigma^{\rightarrow\Uparrow} - d\sigma^{\rightarrow\Downarrow}}{2 d\sigma_{unpold}}$$

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If both $A_{\|}$ and $A_{\perp}$ measured: $\Rightarrow \frac{g_1}{F_1}$

If only $A_{\|}$ measured:

$$\frac{A_{\|}}{D} = (1 + \gamma^2)\left[\frac{g_1}{F_1}\right] + (\eta - \gamma)\, A_2$$

$$\frac{A_{\|}}{D} \approx (1 + \gamma^2)\left[\frac{g_1}{F_1}\right] \qquad \gamma^2 = \frac{4M^2x^2}{Q^2}$$

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Taking F_1 from experiment $\Rightarrow g_1(x, Q^2)_{exp}$

We utilize (in $\overline{MS}$ scheme)

$$
\begin{aligned}
g_1(x,Q^2)_{exp} &= g_1(x,Q^2)_{LT} + g_1(x,Q^2)_{TMC} + g_1(x,Q^2)_{HT} \\
&= g_1(x,Q^2)_{LT} + g_1(x,Q^2)_{TMC} + \frac{h(x)}{Q^2}
\end{aligned}
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$$g_1(x,Q^2)_{LT} = \frac{1}{2}\sum_{flavors} e_q^2 \Big\{ [\Delta q(x,Q^2) + \Delta\bar{q}(x,Q^2)]$$
$$+ \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \{\Delta C_q(x/y)\, [\Delta q(y,Q^2) + \Delta\bar{q}(y,Q^2)]$$
$$+ \Delta C_G(x/y)\, \Delta G(y,Q^2)\} \Big\}$$

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Inclusive DIS determines ONLY the sum of quark and antiquark densities

Important difference between UNPOLARIZED and POLARIZED DIS:

About half of data are at MODERATE Q^2 and W^2 i.e.

$$1 \lesssim Q^2 \lesssim 4 GeV^2 \qquad 4 \lesssim W^2 \lesssim 10 GeV^2$$

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We believe Higher Twist corrections are important.
γ^2 term should not be neglected!

Extension to SIDIS

Aside from a kinematic factor, the SIDIS polarized cross-section, in NLO is

$$\begin{aligned}\Delta\sigma_p^h|_{NLO} &= \sum_i e_i^2\,\Delta q_i\Big[1 + \otimes\frac{\alpha_s}{2\pi}\Delta C_{qq}\otimes\Big]D_{q_i}^h \\ &+ \Big(\sum_i e_i^2\Delta q_i\Big)\otimes\frac{\alpha_s}{2\pi}\Delta C_{qg}\otimes D_G^h \\ &+ \Delta G\otimes\frac{\alpha_s}{2\pi}\Delta C_{gq}\otimes\Big(\sum_i e_i^2 D_{q_i}^h\Big)\end{aligned}$$

This involves a double convolution and thus a double Mellin Transform.

The measured asymmetry is

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Use DSS Fragmentation Functions.....will use others as well

Note that DSS FFs are significantly different from others:

$D_g^{\pi^+} \gg$ Krezer (KRE) or Albino, Kniehl and Kramer (AKK) at large x.

$D_{s+\bar{s}}^{\pi^+} \gg$ AKK for $x \leq 0.7$

$D_{s+\bar{s}}^{K^+} \gg$ KRE, $\ll$ AKK

$D_g^{K^+} \ll$ KRE and AKK

This needs study!

Parametrization

$$\Delta u + \Delta \bar{u} = A_U x^{\alpha_U} (1-x)^{\beta_U} (1 + \epsilon_U \sqrt{x} + \gamma_U x)$$

$$\Delta \bar{u} = A_{\bar{u}} x^{\alpha_U} (1-x)^{\beta} (1 + \gamma_{\bar{u}} x)$$

$$\Delta d + \Delta \bar{d} = A_D x^{\alpha_D} (1-x)^{\beta_D} (1 + \gamma_D x)$$

$$\Delta \bar{d} = A_{\bar{d}} x^{\alpha_D} (1-x)^{\beta}$$

$$\Delta s = \Delta \bar{s} = A_s x^{\alpha_s} (1-x)^{\beta} (1 + \gamma_s x)$$

$$\Delta G = A_G x^{\alpha_G} (1-x)^{\beta} (1 + \gamma_G x)$$

16 free parameters

The Data Sample

Inclusive DIS: 841 experimental points

Semi-inclusve DIS: 202 experimental points

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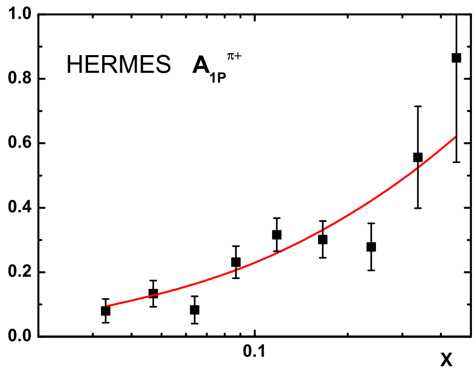
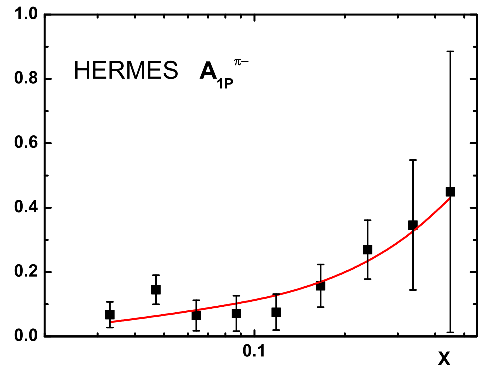
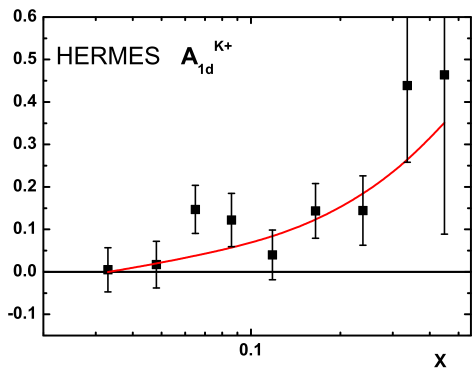
DIS: $\chi^2_{NExpP} = 0.85$ SIDIS: $\chi^2_{NExpP} = 0.90$

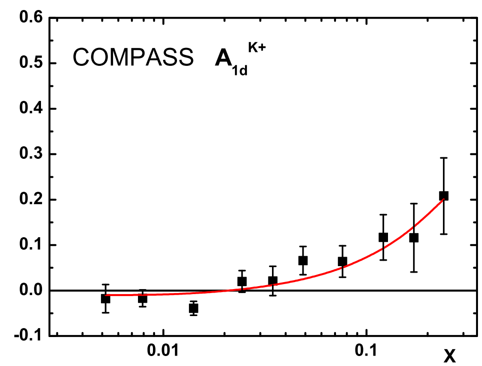
Overall $\chi^2_{DOF} = 0.88$

Fits to SIDIS data









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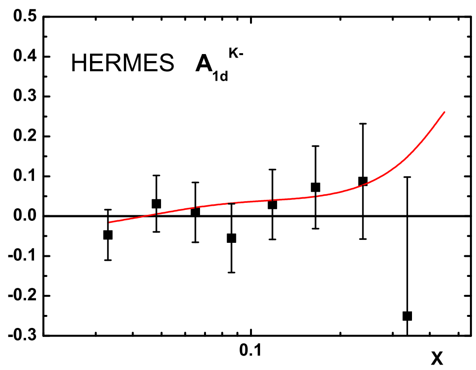
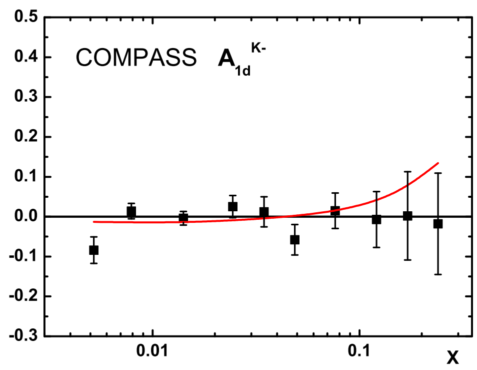
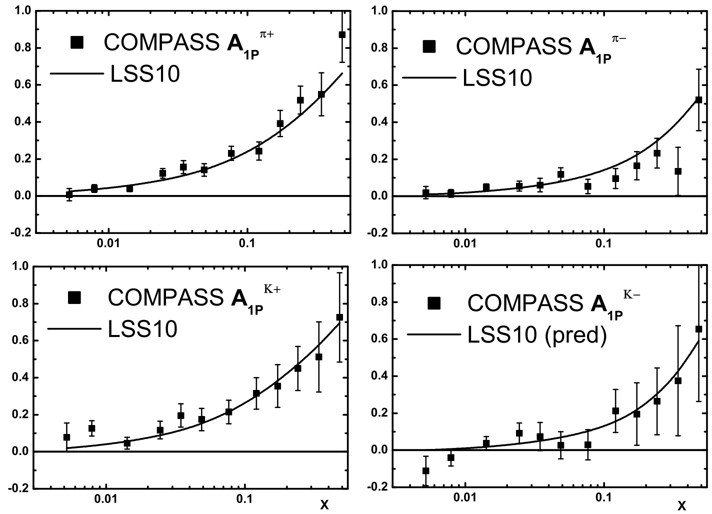
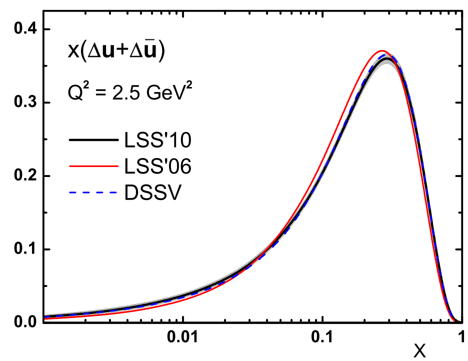


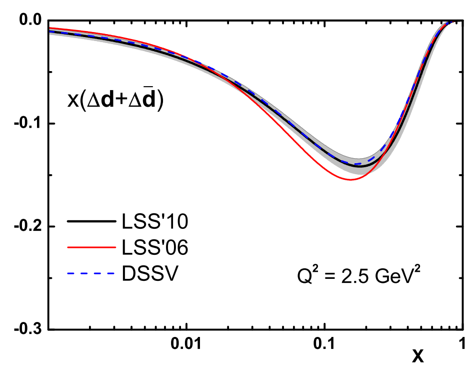


Predictions for COMPASS proton SIDIS data



Results and comparison with DSSV





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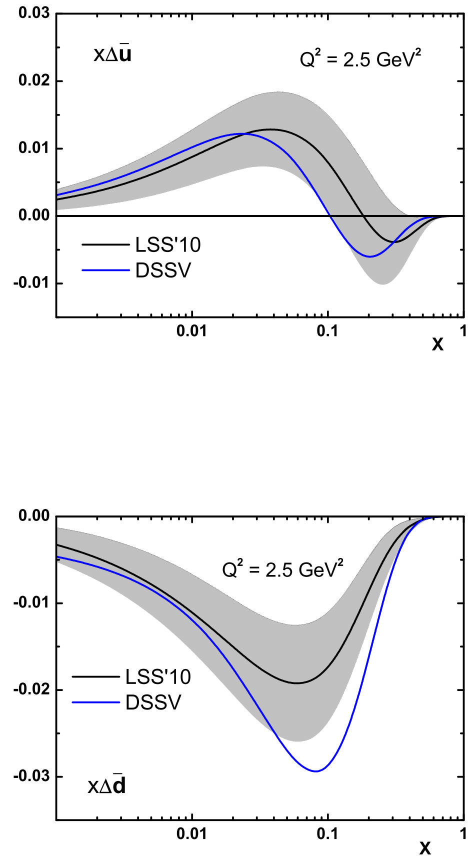

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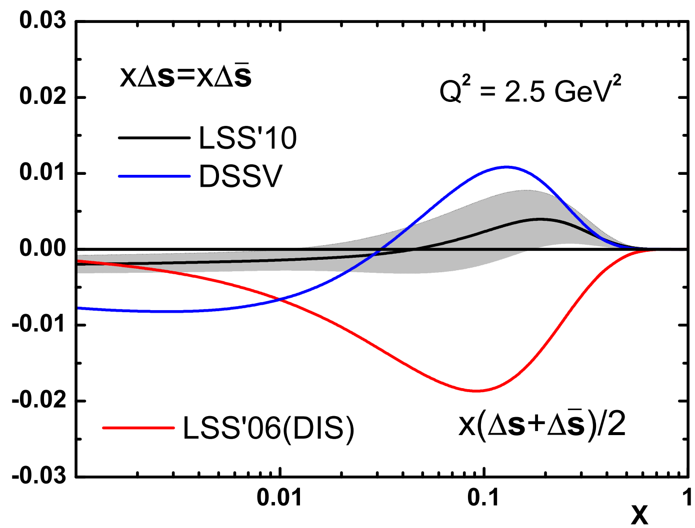
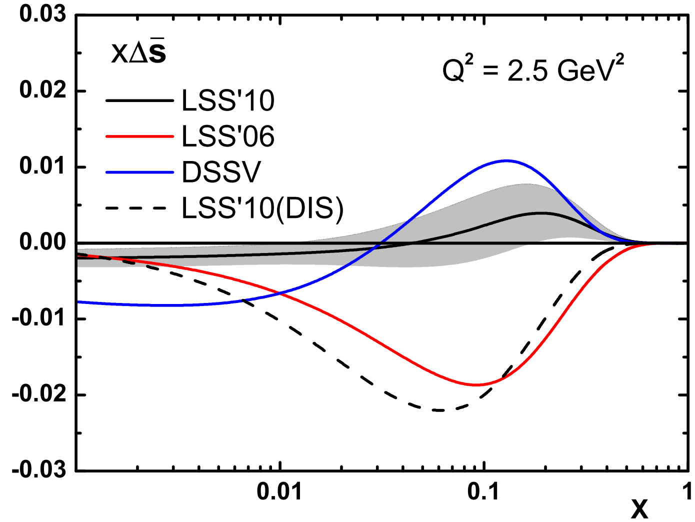


Note: DSSV use $\alpha_{\bar{s}} = \alpha_{\bar{d}}$ and find $= 0.16$

LSS find: $\alpha_{\bar{s}} = 0.05 \pm 0.02 \quad \alpha_{\bar{d}} = 0.55 \pm 0.12$

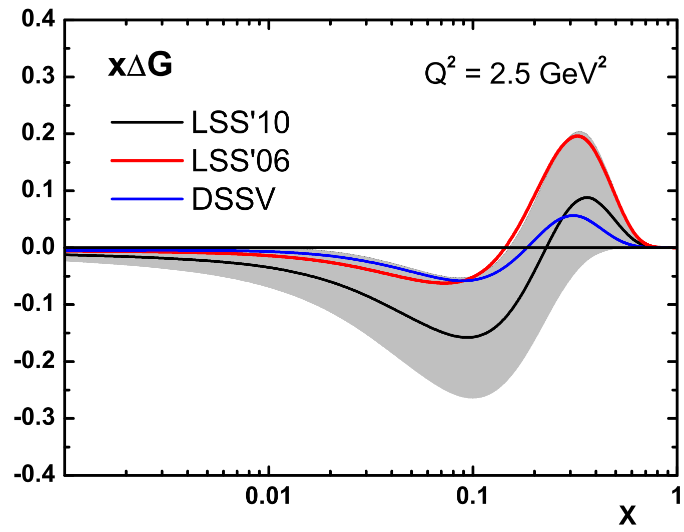
Results and comparison with DSSV

Redo DIS including term $(1+\gamma x)$ to permit sign change.

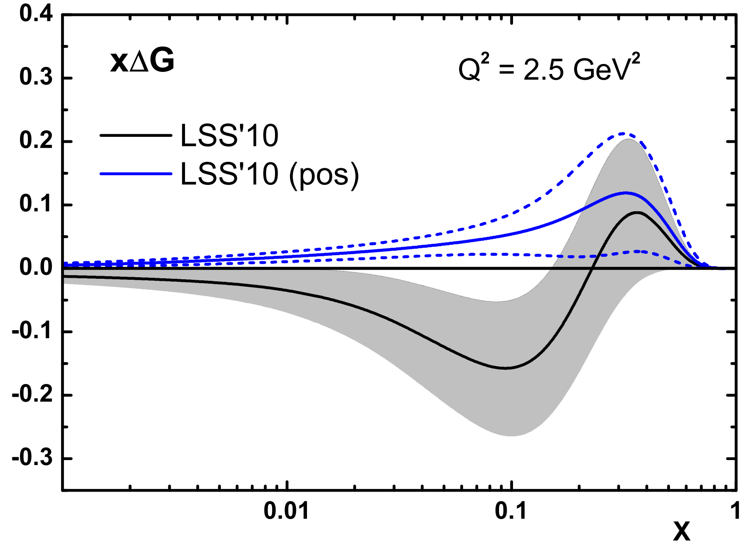


Δs is controversial

Results and comparison with DSSV



We also find an acceptable solution with positive ΔG



NB: has very little effect on $\Delta\bar{u}, \Delta\bar{d}, \Delta\bar{s}$.

Dashed lines: error bands —NB: Warning: error bands do not reflect functional uncertainty!!!

The controversy about Higher Twist

Following Operator Product Expansion (OPE), LSS use

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Higher twist corrections: the exactly known kinematical target mass corrections (TMC) and genuine dynamical higher twist terms (HT).

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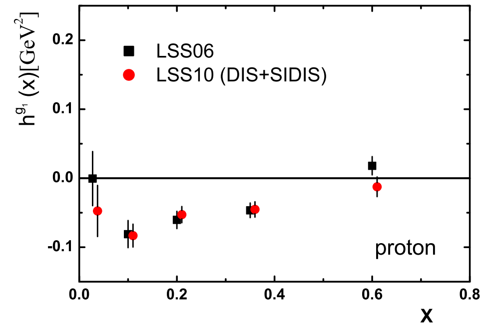
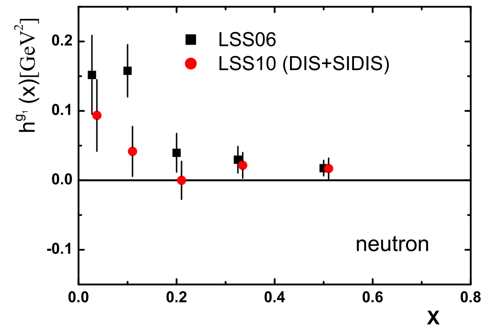
Higher twist corrections: the exactly known kinematical target mass corrections (TMC) and genuine dynamical higher twist terms (HT).

Possible slow scale i.e. Q^2 dependence in $h(x)$, the precise form of which is unknown, neglected compared to $1/Q^2$ variation.

We find significant HT contribution



Very important for CLAS data.



Blümlein and Böttcher (BB) (arXiv:1005.3113 v1)
disagree

They use

$$g_1(x, Q^2)_{exp} = g_1(x, Q^2)_{LT}\Big[1 + \frac{C(x)}{Q^2}\Big]$$

where any Q^2 dependence in $C(x)$ is neglected.

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where any Q^2 dependence in $C(x)$ is neglected.

BB find no evidence for HT i.e.their $C(x)$ for protons and neutrons is compatible with zero.

Thus

$$C(x) = \frac{h(x)}{g_1(x, Q^2)_{LT}}$$

If legitimate to neglect the scale dependence in $h(x)$ then $C(x)$ must vary considerably with Q^2, contradicting the use of $C(x)$ as Q^2-independent.

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Two approaches incompatible and their results incommensurate. One of the two methods (or perhaps both) has to be incorrect.

Since LSS formulation is closer in structure to the OPE we believe it to be the correct way to implement HT corrections.

Another problem: BB utilize above for proton and deuteron data and extract the neutron value of $C(x)$ via

$$C_n(x) = \frac{2}{1 - 1.5\omega_D} C_d - C_p$$

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This is incorrect. The correct relation should be

$$\begin{aligned} C_n(x) &= \frac{1}{g_{1n}(x,Q^2)_{LT}} \Big[\frac{2}{1 - 1.5\omega_D} g_{1d}(x,Q^2)_{LT} C_d(x) \\ &- g_{1p}(x,Q^2)_{LT} C_p(x) \Big] \end{aligned}$$

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Dangerous, since $g_{1n}(x,Q^2)_{LT}$ has a zero!

LSS Letter to BB—no response—so
(arXiv:1007.4781) "Comments on BB paper"

followed by Version 2 of BB, abandoning factorized form for HT

"We prefer the additive case, since the twist-2 scaling violations of $g_1(X, Q^2)$ do not influence $C_{p,d,n}(x)$."

No reference to LSS

Claim no evidence for HT, but central values essentially same as LSS. BB use only statistical errors, but, more important, define error bars by $\Delta\chi^2 = 9.3$.

LSS method agrees with approach to HT of moments.

$$\bar{h}^N \equiv \int_{0.0045}^{0.75} dx\, h^N(x) \qquad N = p, n$$

$$\bar{h}^p = (-0.028 \pm 0.005) GeV^2 \quad \bar{h}^n = (0.018 \pm 0.008) GeV^2$$

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$$\overline{h}^p + \overline{h}^n = (-0.013 \pm 0.009) GeV^2$$

$$|\overline{h}^p + \overline{h}^n| < |\overline{h}^p - \overline{h}^n|$$

Agrees $1/N_C$ expansion.

The spin sum rule: $\overline{MS}$: $Q^2 = 4GeV^2$

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma(Q^2) + \Delta G(Q^2) + \text{OAM}$$

Positive ΔG

$$\Delta G = 0.316 \pm 0.190 \qquad \Delta\Sigma = 0.207 \pm 0.034$$

$$J_z = (0.42 \pm 0.19) + \text{OAM}$$

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Changing sign ΔG

$$\Delta G = -0.339 \pm 0.458 \qquad \Delta\Sigma = 0.254 \pm 0.042$$

$$J_z = (-0.21 \pm 0.46) + \text{OAM}$$

Summary

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- Higher Twist: LSS disagrees with BB, but agrees with moment studies
- ΔG still ambiguous. EIC, large Q^2 and small x could resolve.

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